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PROJECT PRESENTATIONS

RESEARCH PAPERS

Positioning of the Teacher in the Improvement of Mathematics Classroom Practice

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This paper is a hybrid between a discussion essay and a theoretical paper. Our concern is the positioning of teachers in contexts where the reshaping of educational institutions along with the commercialisation and commodification of research carried out at universities, increasingly interferes with the intellectual freedom of both teachers and researchers. Funding of research in these contexts privileges 'findings' with direct implications for developing teachers' classroom work. We discuss examples drawn from a range of studies, including classroom observation, curriculum design, and professional development settings in terms of the subjectivities attributed to the unauthorised position in the relations established in these practices.

Introduction

While "teachers are regarded as key persons of educational change" (Kieran, Krainer & Shaughnessy, 2013, p. 363), they are still commonly construed by researchers and educational stakeholders as "passive recipients" of research outcomes, while at the same time providing the empirical data for the research (ibid., p. 365). With references to research papers from the 1990s, White, Jaworski, Agudelo-Valderrama, & Gooya (2013) noticed that:

> The term "professional development" of teachers has often, in the past, implied a deficit view of teachers, emphasizing elements of knowledge which teachers lack, or ways in which teachers need to be developed (p. 396).

Consequently, researchers in mathematics education are positioned as authors and teachers as unauthorised audience. Amongst many others, Kieran et al. (2013) address this by suggesting that teachers should be involved as "key stakeholders who co-produce professional and scientific knowledge" (p. 387).

Our concern is the positioning of teachers in contexts where the "remaking of the school as a business" (Pinar, 2004, p. 5), increasingly interferes with the intellectual freedom of both teachers and researchers and implies a "de-skilling" of teachers (Apple, 1990). In England, for example, anti-theoretical vocationalism (e.g. teacher *education* became teacher *training*) substantially narrowed down the knowledge base on which teachers' judgements about curriculum, pedagogy and assessment can be exercised. In addition, sociology, philosophy and history of education have been taken out of initial teacher education programmes in many countries, so that teachers have few resources with which to produce critical responses to policy changes (Lerman, 2014). While research in mathematics education always has been conceptualised as having a face towards practice, the reshaping of educational institutions along with the commercialisation and commodification of research carried out at universities (Radder, 2010), might lead to researchers feeling more obliged to participate in practices with direct implications for developing and evaluating teachers' classroom work. In this contribution, we explore the positioning of teachers in these practices.

Levels of Subjectivity Attributed to Teachers

In most activities, in which researchers are involved in practices with direct implications for developing teachers' classroom work (such as in the role of curriculum designers, didacticians, and teacher evaluators) teachers and researchers are in what Dowling (2009) describes as a "pedagogic relation", as opposed to an "exchange relation". Pedagogic relations can be recognised by the establishment of an author, an audience and a privileged "content", that is, a hegemonising practice/discourse aiming at closure, the evaluation principles of which are controlled by the author. In "exchange relations" the principles of evaluation of performances are located within the audience. Dowling (2009, p. 244) distinguishes three levels of subjectivity attributed to "unauthorised positions" in both relations. For our purpose the levels attributed in pedagogic relations are of relevance. These are “apprenticeship” (high level), “dependency” (low level) and “objectification” (no subjectivity), which may amount to different hierarchical positions in relation to access to the principles of the hegemonising practice/discourse. We draw on these notions in our exploration.

Observing, Evaluating, and Quantifying Teachers

There is a long tradition of carrying out studies that seek to identify how pedagogy and curriculum systematically relate to students’ learning, aiming to evaluate the quality of pedagogic practice. As early as in 1891-92 Joseph Mayer Rice conducted lesson observations in classrooms in primary and grammar schools in 36 US cities, where he also talked to teachers, parents and staff in education authorities and visited teacher education institutions, collected student productions and tested year-3 pupils in arithmetic. He classified schools into levels of excellence, ranging from a mechanical ‘antiquated’ approach of drill-and-practice to a ‘scientific’ approach (Rice, 1893). The report is explicitly evaluative, with clear preferences for a ‘progressive’ (the ‘scientific’) curriculum and pedagogy. In mathematics education research the activity of evaluation lingers on. As Morgan (2013) observes, “The moralising of the student implicated in the regulative discourse of the classroom is paralleled by the moralising of teachers in the discourse of mathematics education.” (p. 61).

Measuring the Quality of Mathematics Teachers’ Practice

Scales based on ratings of a range of aspects of teacher performance are increasingly used in the USA for formative teacher assessment, evaluation of curriculum policy and professional development (Hill et al., 2012). One such scale is the Mathematical Quality of Instruction (MQI) score that aims at “independent estimates of the mathematical quality and the pedagogical quality of instruction” (Learning Mathematics for Teaching Project, 2011, p. 27). The separation does not seem to be feasible, as it turns out that the "conceptualization is deeply disciplinary, but coordinates mathematical and pedagogical perspectives" (p. 31).

The scales (each with several codes marked as present/ absent and appropriate/ inappropriate) comprise *Richness and development of the mathematics, Responding to students, Connecting classroom practice to mathematics, Language, Equity, and Presence of unmitigated mathematical errors*. The framework is an ad-hoc construction of categories (as opposed to being informed by an analytical framework) and the criteria for the code marking are not discursively available; hence they require 'expert rating'. The expert is either a researcher of the Learning Mathematics or Teaching Project team or some apprentice, who might have been introduced into this practice through observing the lesson observers and learning how they marked the codes. The observed teachers are in a non-negotiable unauthorised position in a hegemonic discourse of good teaching, if this evaluation is not embedded in some form of professional development.

Even though measures of instructional quality originate in the idea that students' scores on mathematics tests are an inappropriate measure of the quality of teaching and hence classroom teaching needs to be looked at, correlations with some measures of student outcomes are still often incorporated in studies that use such measures or are used as an argument for their validity. In discussing the MQI measure, the authors write that "it is unclear whether and how well the various elements of this instrument correlate with student outcomes. Ideally, a next validation study would also compare student outcomes with both teacher scores on the MQI instrument and on more pedagogically focused instruments." (Learning Mathematics for Teaching Project, 2011, p. 44). What the observers engage in, is an evaluation of teaching practice with largely implicit criteria for the 'good' teacher. The performance criteria could possibly be acquired by the teachers through observing a 'master teacher' who gets full score on the MQI scale.

Measuring the Efficacy of Mathematics Teaching

Quantitative measures of teaching are most prominently used in the USA. One example is 'value-added' modelling (VAM):

> Value-added modeling has been widely accepted as a more objective approach to estimating the value of teachers than other methods since it expresses a teacher's unique contribution to student learning in precise, quantitative terms (Wei, Hembry, Murphy, & McBride, 2012, p. 25).

In this statistical apparatus, teaching *as such* is essentialised as a quantifiable teacher attribute (their 'value') that exists to a variable degree in teachers, who are producing gain scores in students, conceptualised in terms of profiles with differing quantifiable characteristics (such as gender, ethnicity, language) that amount to differences in speed when their teachers steer them in a race through a curriculum that leads to their achievement gains on standardised tests. The students' characteristics can be accounted for as 'noise' in the model in order to get to the essence of the teacher 'value'. This is their 'unique' contribution.

When looking at table 3 in Wei et al. (2012, p. 14 ff), one sees that different more or less complex versions of such models amount to very different rankings of the same mathematics teachers. Hill, Kapitula and Umland (2011, p. 826), amongst others, offer some internal critique, but appear to sympathise with the form of the accountability procedure:

> Although we do recommend the use of value-added scores in combination with discriminating observation systems, evidence presented here suggests that value-added scores alone are not sufficient to identify teachers for reward, remediation, or removal.

These procedures establish accountability relations, which include both standards and standardised procedures for monitoring the standards that are defined and developed without involvement of the party made accountable (and become punished or rewarded). Researchers are obviously involved in developing the procedures for monitoring the standards and funded by those who purchase their research.

Teachers as Operators of Proprietary Curriculum Schemes

The increased amount of studies of *instructional effectiveness* of different teaching approaches by means of randomised controlled trials (RCT) mark a comeback of experimentalism. The UK government has recently recruited proponents of RCT for promoting their use in evaluating education and public policy, with the intention to identify interventions with large effects for low cost. Examples include the Department for Business, Innovation and Skills's (BIS) new project about the relative effectiveness of different approaches to 'delivering' adults' English and mathematics learning.

Experimental curriculum development studies occasionally include classroom observations in order to check the fidelity of the teachers' 'dispending' of the intervention (the 'treatment'), or to complement measurement of gain scores with scores from classroom observations (e.g. Clements et al., 2011; Ross & Bruce, 2007). Teaching is only relevant in relation to the statistical regularity the black box of classroom practise produces as its achievement outcomes. The teacher is then clearly objectified.

Developing and evaluating curriculum schemes may attribute some level of subjectivity to the involved teachers in the form of dependency. We looked at an example from the USA (again), which included a large scale study that used a cluster randomized trial with data from 1305 pre-school children for the evaluation of a curriculum based on mathematics learning trajectories (Clements et al., 2011); the children in the 'treatment group' "outperformed those in the control group on the total mathematics test score, with an effect size of 0.72" (ibid., p. 153). As the control group consisted of schools introducing other pre-school curricula, the explanation provided for the effect size emphasised that the teachers enrolled for the experimental treatment were not only trained to use the material appropriately but also introduced to the principles of its construction. The authors argue that the requirement of fidelity of implementation does not imply that teachers "should implement curricula in routinized ways" (p. 157). However, the teachers were not apprenticed into the theoretical and empirical base for the learning trajectories assumed in the scheme and had no influence on the design of the materials with respect to the principles for their construction.

In this project, development of teaching quality (in recruiting William James as authority) is constituted as based on "the science of learning and instruction" which "continues to lay down increasingly specific and useful guidelines" (p. 158). Curriculum is then a technology derived from this 'science' involving teachers who will never have full access to it, except a singular short professional development and access to a website with exemplary teaching activities along with some explanations. This approach is opposed to "focusing primarily on teachers' autonomously inventing individual curricula" or "idiosyncratic 'creativity' that does not build on extant science, and learning and instruction is less likely to serve either the profession or the classroom's students" (p. 158).

The importance of early-years intervention is argued from a "human capital perspective" (p. 128). In the RCT-part of the study of this curriculum, the teachers are objects to the same exact science (of teaching), measuring 'fidelity of implementation' and other variables, including 'teacher personal attributes'. The project is based on the assumption of relatively unskilled teachers, "especially those in early childhood, [who] have limited time and knowledge of mathematics and mathematics education research (Sarama, 2002; Sarama & DiBiase, 2004) required to plan, research, and write truly research-based curricula (as defined in Clements, 2007)." (p. 158). The production of the curriculum material is perpetuating the de-skilling of teachers. The teachers remain dependent on using the proprietary material.

Teachers and Researchers as 'Collaborators'

Kieran et al. (2013) claim that teachers are the key stakeholders in educational research in three "important dimensions": "reflective, inquiry-based activity with respect to teaching action; a significant action-research component accompanied by the creation of research artefacts by the teachers (sometimes assisted by university researchers); and the dynamic duality of research and professional development" (p. 361). The following examples have been selected to illustrate some of these dimensions.

Hatch and Shiu (1998) discuss several types of what they call *practitioner research*, which is an example of 'empirical action research' in mathematics education. This is conceptualised as a division of labour between researchers and teachers, either in the form of a virtual collaboration across time and space, or as in one setting (e.g. professional development). They explain, amongst other things, how transcripts from a teacher's audio recording of their own classroom might serve as a source for both "personal knowledge" and "general knowledge" when analysed and discussed from the perspective of both a practitioner and a researcher. They note that "vivid accounts of the particular […] speak more directly to a practitioner" (ibid., p. 311) and are offered "to the research community for replication and response" (ibid., p. 314). This statement points to a potential distributive mechanism; teachers produce exemplars of pedagogic activities and student work (e.g., published in professional journals), while researchers produce generalised accounts. The status differentiation between researchers at universities and 'practitioner researchers' is not challenged by this move, even though this might have been intended.

A learning study, which has become a common activity in professional development in Sweden, is "a form of action research made by teachers with the specific goal of finding out what matters for student learning of a particular capability" (Runesson & Kullberg, 2010, p. 308). Teams of teachers (often in collaboration with a researcher) design, conduct and discuss lessons in iterative cycles, with pre- and post-tests of student's achievement, based on principles from 'variation theory'. They note that experiences from many learning studies indicate that "by deeply investigating the particular", teachers can also gain knowledge of "general character". Teachers' potential subjectivity here is apprenticeship into variation theory.

Kieran et al. (2013) illustrate 'the dynamic duality of research and professional development' by a range of examples, including 'Learning Communities in Mathematics' (LCM). In the following we refer to Jaworski (2006) in our discussion of the LCM project, which involved collaborations between researchers in mathematics education ('didacticians') and mathematics teachers at project schools (that volunteered to participate), underpinned by 'community of inquiry' as a general guideline for the work. 'Community' might suggest a sense of solidarity arising from a shared aim or signify informal relationships between the participants. The communities of inquiry are formally established. They include mutual obligations and are constructed as mutually beneficial for both categories of people involved, the teachers and the researchers, through establishing a 'co-learner partnership' between them. "[P]articipants grow into and contribute to continual reconstitution of the community through critical reflection" (Jaworski, 2006, p. 202). Activities within the project took place both at the schools and at the university, including workshops with researchers introducing inquiry tasks and teachers reporting from classroom activities. In the work with mathematics tasks (mostly designed by the didacticians), the 'inquiry stance' points to an openness of the tasks, which affords cooperation, but does not imply it. The agenda was set by the researchers, positioning the teachers as apprentices into a pedagogic action with "inquiry as a fundamental theoretical principle" (ibid., p. 187). The establishment of a community of inquiry implies an acceptance of a social order where an inquiry stance "is a form of social positioning taken in a community of teachers in which inquiry has become one of the social norms in practice" (ibid., p. 201).

In another example provided by Kieran et al. (2013), "teacher researchers collaborated with university researchers in reflecting on their own teaching and in conducting cycles of action research that focussed on improving the mathematical discourse of their classrooms" (p. 369). In the project, a group of teachers were guided through carrying out analyses of their own classroom discourse, with a particular focus of their own choice. The group shared the goal of learning, reflecting upon and changing mathematical discourse in classrooms. The project had as a main goal "to give teachers the opportunity to find their own research voice" (p. 370). Quantitative and qualitative discourse analytical methods were introduced by the researchers, using data from the teachers' classrooms. The project amounted to a publication including teachers' analyses and contributions by the researchers leading the project (Herbel-Eisenmann & Cirillo, 2009). Teachers also presented at meetings and conferences. In this project, the teachers were apprenticed into a particular mode of research, and eventually became subjects of the activity.

Discussion

The different positions in relation to access to the principles of a particular hegemonising discourse of 'good teaching' in pedagogic relations between teachers and researchers relate to different constellations in professional development settings. One question arising from the outline of these examples pertains to modes of involvement in the discourse constituted in the projects where teachers are positioned as apprentices or dependent. All aim at improving teaching practice and assume commitment to both aims and means by the teachers. The means appear to be handed over to the teachers by the researchers and might not be open to critique; pursuing a shared educational project is assumed. A reflection on how discourses of 'good teaching' relate to projects of education in the political and economic contexts of the settings is deferred, as the focus is on improving classroom practice *as such*, based on a projection of what researchers aim to achieve on teachers, who are constructed as the key "stakeholders […] who *can affect to the greatest extent the achievement of one of the main purposes of the research enterprise, that is, the improvement of students' learning of mathematics*" (Kieran et al., p. 365; italics in original). Coherence in the group and commitment to the means might not be established by this deferment. As Gellert, Becerra Hernández and Chapman (2013 p. 346) note, action research is dominated by professional development programmes for teachers aiming to further their personal knowledge, and a search for "a deeper ecological validity" of their practice, while the "transformative potential of action research in terms of emancipatory educational, cultural and political processes" has not been much pursued.

In the curriculum trial the participation of teachers was not voluntary (schools were selected randomly). The aims of this curriculum to increase pre-school children's achievement gains in numbers, shape and measurement and the increasing use of computers were not negotiable. With respect to access to the means, the teachers were positioned as dependent on the 'scientific' underpinnings of this particular approach, although they were expected to use their professional knowledge as pre-school teachers to adapt their interaction with the children, according to what they were taught in the introduction to the learning trajectories and the mathematical concepts involved. They were not provided with any means or resources with which to critically reflect on the economic context in which the activity took place. It was assumed that they had not acquired the competencies for developing their own curriculum approaches in their teacher education. Their status as teachers is one of a skilled operator of proprietary curriculum schemes. In this context, one might envisage some teachers continuing to work with and discuss this particular curriculum scheme connected via a website to a virtual community of its users, and others picking up another curriculum package, which they get for free as an exchange for participating in another randomised controlled trial.

While classroom observations might all envelope an evaluative component, the objectification of the teacher is evident in disembedded observation studies concerned with gap-gazing in teachers' knowledge and skills, whose classrooms might exhibit impoverished, undeveloped mathematics (in contrast to *Richness and development of the mathematics*), who do not interpret student productions and do not use student errors (in contrast to *Responding to students* appropriately), and do not correct their mathematical errors (*Presence of unmitigated mathematical errors*) etc. (categories in the MQI). When such observational categories are quantified and linked to student scores on standardised achievement tests, the ground is laid for the objectification of teachers and students in the bureaucratic accountability machinery based on quantified standards and standardised measurement procedures for monitoring the standards. Mathematics is implicated in this social technology, as 'mathematicoscience', which represents the 'bureaucratic public voice' (Dowling, 2009, p. 142). Researchers producing observation scales and other quantitative measures of teaching quality are allies in this process of increasing bureaucratisation, while alternative modes of classroom research are at risk of being "written off for their alleged 'ideological posturing'" in the hegemonic discourse of neo-classical experimentalism (Howe, 2004, p. 57).

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Mathematical Success in Culturally Diverse Mathematics Classrooms

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This paper discusses the initial thematic analysis of practices used in remote classrooms where there are high percentages of Australia's First People. The case studies from which the data are drawn are from a large project that explores successful teaching of numeracy/mathematics in remote and very remote contexts of Australia. There is considerable diversity in the case studies due to the variety within the contexts and the needs of those contexts. However, there are also similarities emerging across cases. This paper discusses the emerging similarities that are evident in many of the sites. The project uses grounded theory to explore the similarities. There appear to be three levels of practice that need to be taken into account when building a successful numeracy program for Australia's First People learners.

Sucess in Remote Indigenous Communities

Overall, First Australians represent about 3% of the national population. This measure is somewhat crude as it is based on the self-identification of First People. Contrary to popular belief where there is often an exoticism that First People live in remote outback lands, First Australians live predominantly in well-populated areas with approximately 60% living in major cities or major regional areas, and just over 20% living in remote and very remote areas (Australian Bureau of Statistics, 2013). This figure however, must be explained as there are quite nuanced statistics for various regions. For example in the Northern Territory where the overall population is just over 200,000, First Australians are 30% of the population and that there is a much younger demographic of First Australians than there are older First Australians (Australian Bureau of Statistics, 2014). The Northern Territory has the highest proportion of First Australians of all states, while Victoria has only 1% of their population as identified First Australians. The age structure of First Australians is also different from the non-Indigenous Australian population. The median age for First Australians is 22 years while for the non-Indigenous population it is 38 years. Such differences have impact on education provision. In many remote and very remote communities, the proportion of the population is close to 100% First Australians with the non-Indigenous people being in community in service roles and only in transitional roles. The project discussed herein draws on communities in remote and very remote settings. Aboriginal communities in the Australian context are suffering from a wide mix of social, economic, infrastructural, health and criminal issues. The issues, identified in numerous reports to government through to the general media, are widespread. The issues are complex and are contextually bounded by the communities themselves. Many remote communities serve a number of different family groups, and this also creates particular challenges in some communities. Many remote communities are built to cater for the familial/tribal differences within a given community; and often with the non-Indigenous people being separate from the main community.

The roles of non-Indigenous people in remote communities generally involve service roles such as teachers, police, health workers, social security providers and the like. Teachers are often early career teachers, young and remain in community for the length of their contract of employment – usually 2-3 years – and use the position to levy for a 'better' position in an urban setting. As neophyte teachers, usually in their first teaching position (Goos, Dole, & Geiger, 2011), the teachers are not only confronting the first year of teaching but also in a remote, isolated context working with families whose language and culture are very different from their own (Howard, Cooke, Lowe, & Perry, 2011). Increasingly, systems and employers are building programs to help support teachers in this critical period of their teaching career and in their cultural induction into remote community life. But as the majority of the teachers are very early career teachers the possibilities for mentoring are limited in situ. Furthermore, the tyranny of distance means that the provision of professional development and support is limited. This creates quite unique circumstances for the induction and on-going development of early career teachers in remote settings. The high turnover of teachers and leaders in remote communities also means that there is often a loss of knowledge as staff continually moves through communities (Helmer, Harper, Lea, Wolgemuth, & Chalkiti, 2013).

What is well known and documented is the lack of success (as measured on traditional assessment and reporting measures) for remote students whose culture is different from that of the Western education system. In the Australian context culture and geographical location compound the (limiting) possibilities for success in mathematics. There is a substantial literature that demonstrates the issues that are central to this paper are common in other countries such as Canada (Ezeife, 2002). Some of the key issue within current education policy and thinking are issues of attendance (Purdie & Buckley, 2010), language and assessment (Mushin, Gardner, & Munro, 2013), and the upskilling of teachers who commence work in remote areas (Auld, Charles, Dyer, Levy, & Marshall, 2013).

Within the Australian context, there are a range of approaches that have sought to address the poor educational outcomes for Australia's First People. Some of these have included approaches that build bridges between the two cultures (Robinson & Nichol, 1998) where others have sought to use artefacts (such as witchetty grubs) (Treacy, Frid, & Jacob, 2014) to provide a meaningful context for learning mathematical concepts. More recently, there has been a strong push for more explicit teaching approaches such as that undertaken by the Cape York Academy (Cape York Aboriginal Australian Academy, 2014) where students have been exposed to a curriculum and pedagogy that seeks to remove areas where there is potential for misunderstanding in the instructional process and students are grouped by "ability". There are also approaches that have sought to transpose successful approaches in other contexts into the Australian remote context (Sullivan, Jorgensen, Boaler, & Lerman, 2013) to identify salient features of such programs that offer potential success for Indigenous learners.

What is clear, however, that despite considerable funding being allocated to interventionist programs to bring about success in mathematics learning for the most at-risk learners in Australia, there has been very little success in terms of measurable learning gains. In contrast, this project has taken as foundational, the possibility of those working in remote contexts to bring about success. To achieve this, a large national study is in process that is seeking to identify elements that may be contributing to the success of Australia's First People who are most likely to perform poorly on standard measures of success in mathematics.

Defining "Remote" and "Very Remote"

Schools selected for this study are either 'remote' or 'very remote" as defined by the Australian Curriculum and Assessment Reporting Authority. The concept of remote is very Australian-centric and refers to geographical isolation. For example, remote education is defined by The Queensland Department of Education Training and Employment (2014) being over "3 hours from a regional/provincial or larger town" (np). With this isolation, many other influences are felt such as access to resources (hospitals, shops); the psychological effect of remoteness; the communication possibilities (often no mobile or internet connections), mode of transport to the site (often on unsealed roads); being cut off in the wet season due to floods (can be for 3 months).

Defining "Success in Numeracy"

While it is recognised from the outset that there are considerable limitations with a National Testing program, the first round of selections of schools has been through the National Assessment Plan for Literacy and Numeracy (NAPLAN). This is the only "objective" measure in Australia where schools can be compared. All schools sit the same tests so there is some comparability across the country. The tests are "designed to show a snapshot of typical achievement and do not describe the full range of what students are taught" (Australia Curriculum Assessment and Reporting Authority, 2014, np). Initially schools were selected on the basis of their performance on numeracy where they performed better or significantly better than similar schools. The MySchool site provides information publicly on school performance so data were used to identify schools who were performing better than similar schools usually across at least 2-3 years so that the data for one year was not aberrant data, that the school was listed as remote or very remote, and had at least 80% Indigenous population.

The project has also incorporated a process of personal recommendation on the basis of inclusion. Many schools, for various reasons, do not produce data that shows performance on NAPLAN testing. As such, regional directors and state authorities have recommended the inclusion of some schools. In all cases, schools are asked to provide evidence of their success. School-based data is more reliable and robust than relying solely on NAPLAN scores. There have been two cases where the school was unable to provide data when the ethnographic case studies were undertaken and these have not been included in the study.

Success in Remote Numeracy: The Project

The study is funded by the Australian Research Council through its Discover grants scheme. The project is funded for three years and seeks to document at least thirty-two case studies of remote and very remote schools that have been identified as being successful in the teaching of numeracy. Across the Australian research community there have been numerous funded studies that have sought to bring about change and success. In contrast, this project has been founded on a number of premises – first that there are numerous teachers/schools who are succeeding, who know the communities and know what is working. This project seeks to document their practices and understanding of what works in that community. Many interventionist studies have met with minimal success for a range of reasons. This project is about documenting and developing richer understandings of what works in these contexts while recognising that each context is highly nuanced.

Field trips are undertaken to each site. Depending on the size of the school and the focus of the work at the school, the visits can be up to one week for larger schools. The data collection consists of interviews with key staff, lesson observations, and collection of school artefacts. A story is developed for each site post the visit and is then negotiated with the relevant personnel at the school. Once the story is approved by the school it is uploaded to a website for public sharing.

Now that the project has a substantial database of data, it is now moving into the formal and on-going analysis stage for the overall project. This has included the design of the nodes for the coding of data via NVivo10 - (QSR International, 2014). This is a qualitative data analysis program using grounded theory (Strauss & Corbin, 1997) that enables a theory to be developed from the themes in the data. Prior to the coding, a system of categories (for the nodes) was developed. This is based on the emerging themes that the research team has noted in the data. These nodes provide the initial starting point for the coding of the data but the program allows for more nodes to be added. The coding process allows for initial 'hunches' to be confirmed or invalidated, as well as for new nodes to be developed as the data are coded. Collectively this process enables the research team to develop themes across the thirty-two or so cases that will be developed once the project has been completed.

At this halfway point, there are now themes emerging from across the sites, where it is clear that there are some commonalities across some sites. It is not the case that there are common themes in across all schools, and this is to be expected given the diversity of schools, contexts and systems within which the schools operate. It is not the intent of the project to derive a common set of themes from all the schools.

Findings

What is emerging from the data is a series of themes that can be found in some schools, while other schools may have common themes with and across other sites. It is emerging that the practices of effective mathematics teaching are multileveled and it is important that this layering of practices is taken into considerations.

Multiple Levels of Practice

What is emerging from the case studies is the recognition that quality practice in mathematics education for remote and very remote communities is multi-levelled. The schools in the project may have stories that are focused on particular level/s but there is reference to other levels in their stories.

Envisioned Practices—School-level Practices

This level refers to those practices that build a strong culture at the school. These practices focus on the building of cultures around learning and cultures of support. This level of practice highlights the importance of quality leadership at the school to build an environment where learning is the focus and where practices are informed by 'big ideas' in education—such as a coherent philosophy which is used to inform the work across the school. There are also practices at this level that create a strong sense of the core business of the school and how this business can be enacted and realised.

Enabling Practices

This level of practice is operationalised as a mediator between the vision of the school and the grass-roots classroom practice. While leaders may have a vision of what they see as the key business of the school, there are practices that need to be enabled for the vision to filter through to the level of classroom practice.

Enacted Practices—Classroom Practices

The final level of practice is at the level of classroom. At this level, the classroom teachers and support staff work to build quality practices to enable students access to, and success in, mathematics. These practices are those around pedagogy, curriculum, and assessment. They are informed by, and facilitated by, the other levels of practices across the school.

The levels are interactive rather than hierarchical since the practices at any level are informed by and shaped by the practices at other levels.

To further expand the model, Figure 1 highlights the various attributes that can be seen within the different levels. For example, the school may have challenges with attendance so there are practices envisioned at the macro level. The school may decide that one of the issues around attendance is that students have not felt welcomed at the school or that if there have been behaviour issues during the day that a student may not feel keen to return to school. At one school, the administration team stand at the school gate and welcome every student who comes to school and wishes them (in a very genuine manner) a great, successful and happy day at the school. At the end of the day, every child is farewelled and congratulated for being at school, for successes and encouraged to return the following day. This has created significant changes in attendance. The envisioned practice is that school should be a place where students are welcomed and wanted and this must be made genuinely apparent to students (for whom school is quite a different culture from that of their homes). At the level of the classroom, teachers adopt similar practices to ensure students feel welcome in their classrooms. To ensure synergy between the practices of the administration team and the vision of the school and that which is enacted at the level of the classroom, a culture has been built at the school through considerable professional development and the allocation of resources to build a very transparent culture at the school (the enabling practices).

Consider the classrooms practices at a school where there are groups for learning. At this school, the teachers have been working towards having targeted learning for the students. Recognising that attendance is a significant factor in achievement, students who attend regular are generally achieving significantly better than those whose attendance is poor, sporadic and/or irregular. To help support the learning of teachers, there have been many approaches being undertaken at schools and across systems to help build teachers' knowledge of assessment and recording as well as their knowledge of how to scaffold from identified starting points. Collectively these skill sets are achieved through various processes adopted by schools. In some sites, the schools recognise that professional learning of teachers (often early career) is needed so funds are re-allocated within schools to have devolved models of leadership where there are 'coaches' to work with the teachers in their curriculum learning and planning. A number of schools have models of leadership in place in order to build communities of learners (and learning) – for teachers and Aboriginal support workers. In this scenario, teachers and leaders at various schools have adopted a range of practices to enable such interventions to occur (and succeed). It is by undertaking range of case studies, that the themes are beginning to emerge. They are not in all schools but there are synergies across sites that are becoming apparent.

For example, in many schools, there is a strong vision for recognition of the culture and languages of the communities in which the schools are located. Having high expectations of learners—students, teachers and Aboriginal Education Workers (AEW)—is foundational to the goal of the schools. Various strategies are implemented on how to achieve this goal. In some schools, there is considerable investment of resources in the AEWs, and in the teachers to work alongside AEWs. In one site, for example, the school recognised the difference between the two languages spoken by the students—their home language and the need to be fluent in Standard Australian English. The vision of the school was to establish a strong early years program in home language and then scaffold the students into SAE. The early years sector of the school relied heavily on the AEWs to develop resources in home language that focused on mathematics concepts—such as spatial language, prepositions, etc. so that students could be scaffolded into learning SAE while also valuing the home language of the students. Teachers developed many classroom-based practices to enable the learning in two languages in mathematics, aided by the AEWs. Across other sites different scenarios were developed that also employed the skills of the AEWs at classroom level practice, but also at the level of the enabling practices in terms of the upskilling of the AEWs in order that they were empowered to work alongside the classroom teachers. Collectively these stories highlight various features of the practices employed by the schools to enable students to have greater access to mathematics and mathematics learning.

Furthermore, at most sites, there was a strong recognition that teaching needed to be targeted at the learners. This targeted teaching was premised on a number of issues, depending on the context. Most commonly the issues were related to attendance (or lack of); the language of the students, the culture of the students, and the resources that were available in community. For many communities, there are very different experiences for the students upon which they can draw in terms of understanding mathematics. The taken-for-granted experiences of urban students are worlds apart for remote students. For example, the language of money is one that can be problematic. A task in which coins are differentiated by gold and silver colourings (dollars and cents) may be obvious for urban students, but in remote contexts where the home language may be a Creole, then a term such as "silver" is used to refer to "coins" as opposed to notes. Posing a question where students may be required to identify silver coins to make up a dollar value may be nonsensical for some Aboriginal students. Knowing, recognising and celebrating the culture and language of remote students were also built into programs, particularly when targeting learning. Knowing why students perform mathematically as identified on

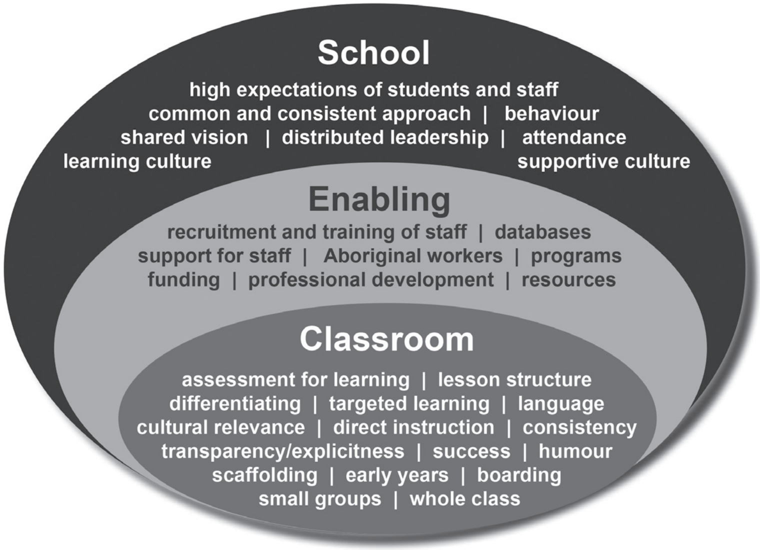

Figure 1: Levels of practices

various assessments helped teachers target learning. But communities varied in how they might take such targeted teaching. In some cases, strategies included grouping students by attendance so that teaching could be targeted for the diversity within/across achievement/age levels. In some sites, the teaching was targeted by drawing on cultural knowledges and how these impacted on achievement—in these cases, the knowledge of AEWs was critical.

Collectively, what we can see emerging across the study are the intersections and differences across sites. A model is needed in which points of intersections across sites can be mapped. This will be an on-going refinement in the project. More sites will be needed before this can be achieved. But what is emerging is that there are significant—as in both importance and frequency—of common themes. NVivo is a rigorous tool that will help in the development of this model. At this point, it is beginning to show the frequency of some constructs, and hence their relative importance in terms of thematic analysis. Other tools for analysis of these themes will be explored in 2015 as more case studies are completed and the data set is increased. It is anticipated that both confirmatory and contradictory cases are likely to emerge in the 2015 round of site visits.

Conclusion

At this stage, it is only possible to draw on emerging themes, as per a grounded theory approach. What is becoming clear from the study is that there are differences and synergies across sites. More exploration of these themes is necessary to better understand what is working in successful sites. There have been a number of contradictory cases—such as one school that relied heavily on direct instruction and worksheets while other cases involved small group work that was less orientated to the recording of results. In the initial stages of the research, this contradiction was stark, but as more cases are undertaken, what is becoming more obvious is that even with such a stark difference in pedagogy, the school using worksheets was also relying heavily on targeted learning. In the case of the worksheets, the macro practices at the school were centred on ensuring that the learning experiences were commensurate with the student's level of understanding. This is a theme that is emerging in nearly every school. What is different is the classroom level practice (such as worksheets, or group work, or language-based activities) where teachers carefully plan teaching activities to meet the learner's needs. The differences in the classroom (enacted) practice are how the envisioned practice (targeted learning) is being realised.

It is not the intent for the overall project to develop a common model for all remote schools. Across all case studies, the context in which the practices are developed are acknowledged from the outset. The conditions of that context help to shape the possibilities for various practices and this is critical for what can be developed. A "one-size-fits-all" model is not a desired outcome, as has been argued in other national contexts (Meaney, Trinick, & Fairhall.Uenuku, 2010). However, there is likely to be a set of principles that can inform practice in remote schools, some of which are likely to be more relevant to one context than another.

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Scaffolding Early Indigenous Learners into the Language of Mathematics

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This paper derives from a large project that explores successful practices in the teaching of mathematics in remote and very remote Indigenous communities in Australia. The focus of this paper is from one case study where the community speaks a shared language – Kriol - across a large region while also trying to preserve its own languages. The school has adopted a number of strategic practices to help young learners gain access to mathematics through both the language and concepts of mathematics. As students progress through the school, scaffolds are removed and the induction into Standard Australian English is facilitated.

Language and Mathematics in Remote Australia

In the Australian context, unlike many other nations, the country is largely monolingual with the language of instruction being Standard Australian English (SAE). However, there are many other languages spoken by migrants as well as the First People. In the late 18th Century it was estimated that there were between 350 and 750 distinct Aboriginal languages and dialects across Australia. Many of the Indigenous languages have died so that by the start of the 21st century it is estimated that there are fewer than 150 languages remaining. Those that are surviving tend to be in remote areas most of which are common in the Northern Territory. These languages include Pitjantjartjara, Warlpiri, Tiwi and Yolngu groups of languages. In these remote communities, the Indigenous languages are used in the homes and across the communities so that the language of instruction is a seen as a "foreign language" since it is only spoken in school and with government agencies. The home language pervades all other interactions outside these venues.

There are languages, such as Kriol spoken across the Kimberley region in Western Australia, which have evolved as language groups have interacted with others and with English speakers. Across the Kimberley, Kriol is the language spoken by most Aboriginal people and with SAE being the language of instruction at schools and in contexts outside communities. It is a recognized language with its own form and rules. Inflection has been identified as being a key part of many of the Kriol languages (Meakins, 2011)

A further language spoken by First People is that known as Aboriginal English (AE) which has many English words but also very nuanced by Aboriginal terms (Eades, 2013). AE may sound very similar to SAE but is a recognized language with its own special features (Malcolm, 2013). As languages develop, there is a cross influences so that AE evolves but increasingly there are AE words that are being taken up in SAE. An example of this is the term "deadly" which is used an adjective. It is best described to convey meaning of something of great value. It conveys greater, and a more emotive, meaning than English equivalents so is being adopted into SAE. In Australian research where there has been a focus on Indigenous learners and language has recognized the importance of teachers paying attention to scaffolding learners between their home language and the language of instruction (Oliver, Rochecouste, Vanderford, & Grote, 2011) as well as resources developed to support learners and learning (Disbray & Loakes, 2013). Other countries also recognize that importance of recognizing the importance of bi-lingual approaches to working with Indigenous students whose languages are different from that of the instructional discourse (Wiltse, 2011).

The early years are important for both language development and in shaping the dispositions of learners towards mathematics. It is also important for the development of fundamental mathematical concepts particularly for students whose home language is not that of the school language (Lee, Lee, & Amaro-Jiménez, 2011). There is a movement towards having instruction in the early years in the home language (Chitera, 2011) and with a progressive scaffolding to the language (usually English) as the students progress through school.

While there are quite strong disparities between diverse groups, it is also acknowledged that even for English language speakers, coming to learn mathematics in English for students who live in poverty whose home language is English, there are quite profound challenges (Brown, Cady, & Lubinski, 2011). In their work, Brown et al, found that poverty is inextricably bound to academic language development and discourse diversities. Developing practices that support the mathematical and language development becomes critical for success for students living in poverty and for those whose language is different from the dominant discourse of classroom instruction.

One of the main challenges for shifting towards a process where mathematics teachers also focus on the language demands of mathematics instruction is with the preparedness of teachers to undertake this task (Tan, 2011). In classrooms where there are language disparities between the home language and that of the language of instruction (such as English) teachers need to be aware of the nuances in the two languages in order to specifically target and address language demands for English Language Learners (ELLs). Teachers need to be aware of the language demands of mathematics if they are able to successfully transition speakers whose home language is different from school mathematics instruction into successful learning of mathematics (Chitera, 2012).

The Remote Numeracy Project

The larger project from which this paper is generated seeks to develop 32 case studies from across Australia that demonstrated success in numeracy/mathematics. The cases are in remote and very remote areas across Queensland, Northern Territory, Western Australia, South Australia, and New South Wales. The schools are selected on the basis of their success in numeracy/mathematics, that they have high percentages of Aboriginal/Torres Strait Islander people (Indigenous Australians), and are in remote or very remote areas of the country. Success in numeracy is defined through either performance on national testing since this is the only way of comparing schools across Australia. Personal recommendations from district personnel are also considered since it is recognized that there are limits to the national testing scheme. Schools are able to provide their own evidence of success which extends beyond the limited testing protocols of the national testing scheme. The field work consists of interviews with staff and lesson observations as well as collection of school documents. The cases are largely ethnographic in form and are not evaluative. It is expected that the stories will be quite different for a wide range of reasons given the very diverse contexts within which the schools are located. Each school is written up as an individual case study and the stories are shared through a project website. The data presented in this paper is drawn from one school where there was a very strong emphasis on language.

Bardulu: A Case Study

Bardulu School (a pseudonym to protect the identity of the school in accordance with University Ethics) is a small school in the Kimberley region of Western Australia. The students speak a local language—Kriol—which is a creole that has developed across the Kimberley region. Kriol is spoken at home and across the communities while the school emphasizes the use of Standard Australian English (SAE) in the classroom. The school has adopted a both-ways approach to teaching where there is a strong focus on language, and in particular recognizing and valuing the home language of the students, and then building transitions to assist students from their home language into SAE. The approach is underpinned by a strong valuing of Kriol alongside recognition that coming to learn SAE is empowering for students and a key function of formal schooling. Such an approach is often referred to as a "two-ways" or "both ways" approach in many Australian schools working with remote Aboriginal (and Torres Strait Islander) communities (Frawley & Fasoli, 2012). The approach seeks to value both cultures (and languages) while ensuring that students become empowered through being proficient in both cultures. Bardulu has adopted an approach where there is a strong immersion in the home language in the early years and resources have been made (usually by the local people working at the school – AIEOs) that make explicit links between Kriol and Standard Australian English (SAE). As the students progress up the school, they are scaffolded into the use of SAE so that by the time they have completed their primary years of schooling, they should be competent in both SAE and Kriol. The school also teaches the home language as a dedicated language program. While Kriol is a commonly used language across the Kimberley, this has arisen through the death of many of the Indigenous languages of the region. There is a movement across the area to try to keep the home languages alive so many schools are implementing programs where elders who still speak the home language of the communities are teaching language to the students. Bardulu also has an active language program operating at the school.

Exploring the Data

The transcribed data has been entered into NVivo and then coded using an evolving set of nodes. Once the data are coded, various analyses can be commenced, of which one is query in which relationships between and among concepts can be undertaken. NVivo also allowed for queries to be undertaken around particular constructs. For example, in one case, the school had a very strong emphasis on transitioning between the home language (Kriol) and the language of instruction (Standard Australian English – SAE). In the early years, there was an immersion process where students would be exposed to many texts in both Kriol and SAE. As they progressed through school, the scaffolds were progressively removed so that students could gain a proficiency in SAE. In querying how teachers and staff spoke of the use of Kriol (a Kimberley version of a creole), NVivo creates maps to enable pathways for understanding the use and relationships around constructs. Figure 1 shows how NVivo creates these relationships of constructs.

What is clear from this query is that there are many ways in which Kriol is used in the context of this one site. This can be seen from the visual representation of the NVivo query. Here it is possible to start to see how Kriol is used in the school, how it is scaffolded by the teachers and the Aboriginal Indigenous Education Officers (AIEO), and the issues around code switching. There are many aspects of Kriol being articulated by the participants. In scanning through the two sides of the inquiry, there is frequent use of terms and discussions around the linking of home language (Kriol) and school language (SAE).

The valuing of Kriol at Bardulu School is important in a both-ways program. Students' home language is valued and seen as a legitimate language. It helps build strong foundations of many mathematical concepts through the use of a language with which the students are familiar. In the early years, teachers work very closely with the Aboriginal staff. Collectively they are able to do considerably work in code switching and language development for the younger students. This is important given that rich spatial and prepositional language of school mathematics that is quite different in Kriol and home languages.

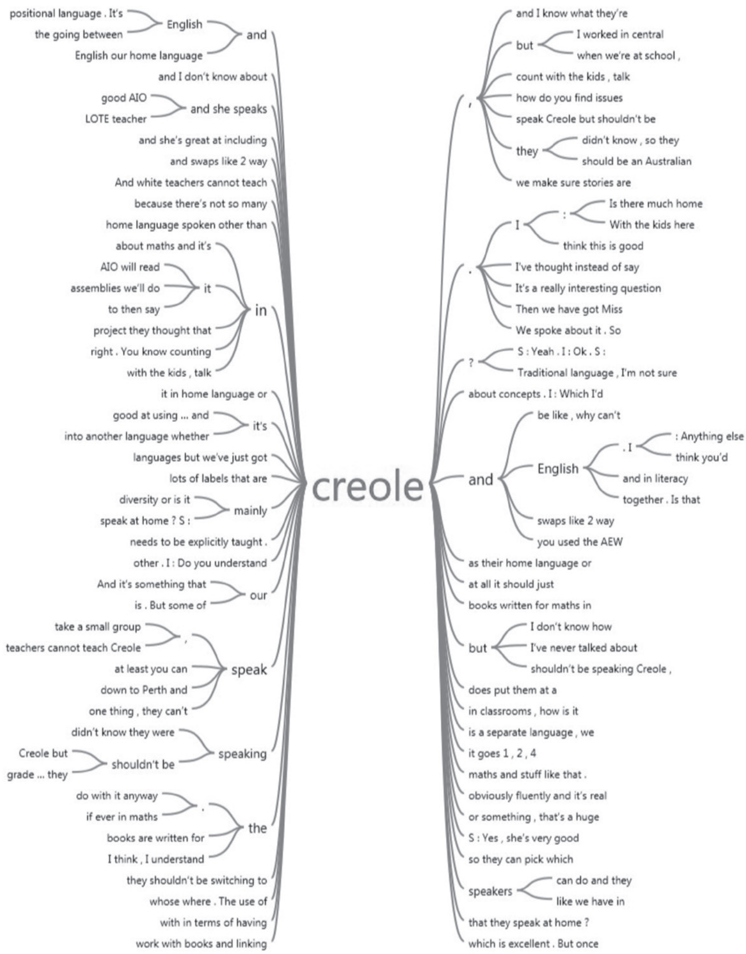

Figure One: NVivo word frequency query on "Creole" - results preview

The AIEOs play a key role in the language shifts since they are members of the community and fluent in Kriol. Their role in scaffolding both-ways language programs is integral to the success in mathematics.

The AIEOs are employed to undertake a range of strategies to help build bridges between the home language (Kriol) and the language practices within the classroom. These include modelling language use, translation, code switching, building resources in both languages, and providing advice for teachers. The AIEOs are a key and integral role in the both ways programs.

Language Practices

The approach taking in the classrooms was one where the teachers explicitly acknowledged that the students spoke a different language from that used for instruction. There was a very strong emphasis on the explicit teaching of SAE but also recognising and valuing home language. As many teachers remarked, it was important for the students to know that their home language was a valued language but different from that used in school:

> Valuing home language is very important, because that's their knowledge that they bring to the classroom. We explicitly teach SAE. We teach kids that Kriol is a different language.

There was a strong use of Kriol in the early years of school so that the students are scaffolded into the use of SAE while also valuing the home language of the students.

> We use lots of Kriol in junior years, but not much after Grade 3.

The way in which the both-ways approach was adopted was that teachers would have wrist bands—one red which represented Kriol and one black which represented English. Teachers would often direct attention to one band—for example, the black band, which then directed students' attention to speaking in that language. In this way, the lesson pacing was not disrupted but students were aware that they needed to use a particular code—either SAE or Kriol.

Teachers were also very positive in terms of the capacity of the students to learn mathematics. Recognising that there were challenges in terms of language and culture, teachers believed that these were not excuses for not providing a quality learning experience with high expectations of the students. The teacher's comment below was representative of the views of the teachers where they did not offer excuses for an impoverished curriculum, or for reducing the demands on language learning in mathematics.

> Nothing can't be overcome. I don't dumb down my speaking; I prefer them to come to my level.

Similarly, another teacher offered the comment where she felt that it was an injustice if teachers did not provide good role models in the use of correct and deep language in mathematics.

> I teach the proper words in maths e.g., numerator and denominator. I'm doing an injustice if I say well only one of you will know it, so I'm not going to teach it.

The comments offered by the teachers were also observed in their teaching practices. Most of the lessons observed at the school demonstrated that the teachers put into practice what they articulated in the interviews. Classrooms were language-rich with numerous displays on walls and with large banks of resources, often made by the staff, that met the needs of the students and built bridges between Kriol and SAE/Mathematics.

Creating Language-based Resources for Mathematics

Throughout the interviews there was a strong recognition of the inter-relationship between Kriol and SAE. The teachers were all able to articulate various areas of language use that impacted on the possibilities for learning and/or creating struggles in learning mathematics. Much like Pitjantjatjara, there are challenges in the use of prepositions and comparatives. A teacher noted this in her comments about the limits created through Kriol in terms of many mathematical concepts:

> Difficulties with Kriol in maths – struggle with prepositions, and increase / decrease. This disadvantages them in terms of testing.

One of the strategies employed in the junior years of schooling was the strong linking between Kriol and SAE. Many of the AIEOs had a key role in creating rich resources for scaffolding learning mathematical language. One of the key resources was student-focussed books. The AIEO would take the students outside to model the language – for example, one book focused on spatial prepositions – above, under, beside etc – and the students positioned themselves on play equipment to represent the words. Photographs were taken and the AIEO then wrote text in both Kriol and SAE to demonstrate the words. The key word was written in a different colour so could be identified easily. The children also enjoyed the books as they had photos of themselves in it. An example, taken from one of the books on positional language is shown below. The accompanying photo had a group of children inside the cubby house:

> Olu pre primary insiedwai la kubbi house.
>
> The Pre-Primaries are inside the cubby house

On another page a group of students were standing underneath the bridge in the climbing frame.

> Orlu pre primary standingup bud ununeet.
>
> The Pre primaries are standing under the bridge.

Similarly, the AIEOs also made resources that had relevance to the students' home lives. For example, they had created a song "Five pies in the Bardulu shop" which was a version of "Mother Duck" where, in the pie song, a pie is eaten and the students are learning to count backwards from five. Other resources were made where mathematical songs were translated into Kriol and the students sang the Kriol and English versions.

There had been considerable support for the AIEOs to develop their skills in order to construct the books. The AIEOs use cameras, word processing, laminating and binding. The books are often of the "big book" genre so that they are read aloud as a group activity.

> AIEOs from the valley used to get together and type the books (Kriol / English). AIEO takes the kids out, takes the photos of the kids.

The schools across the region were all part of the government school sector. The regional office supported the professional learning of the AIEOs. The AIEOs were taken to a central location and professional development workshops were made available to build their skills and knowledge in order to construct stories to support the students.

In constructing the resources for the students, teachers also recognised the importance of the students being part of the resources. This helped to engage the learners. As the resources have been made over a number of years, young students enjoy seeing family and siblings in the books in their classroom. The books are both in the large "big book" format as well as smaller books that the students have as a reader.

The Importance of Local People: Aboriginal Indigenous Education Officers

The AIEOs are an integral part of the school and there is one AIEO in each classroom (except for one classroom). The role of the AIEO is to support the teacher with teaching but at this school their role is seen to be central to the operation of the school. This is summed up in the following comment from one of the administration team members. The school has proactively sought to ensure that their AIEOs are well trained and supported so that they are able to work alongside their teachers and that they also have key roles in the school.

> AIEOs all have to have Cert 3 now. Three of our AIEOs are going for Cert 4. Also, we have two qualified language teachers, qualified to take class on their own. We also have one trainee language teacher. AIEOs run NAIDOC day—last year we went to the billabong and had a bbq and invited community. AIEOs have to give rewards on assembly. We expect them to model behaviour to kids. In a lot of Aboriginal schools, the kids get shame (shy) and won't stand up in front of people. We try to instil in them that there's no shame, it's about being brave and giving it a go. If our Aboriginal staff are doing it, there's no reason the kids can't do it.

As the AIEOs are members of the three communities that are served by the school, they are also very fluent in the home language (Kriol) and also live in the communities. Their cultural knowledge and community knowledge are an integral part of the operations of the school programs.

One of the key roles within the school of the AIEOs that is a strength of the school is their role in language. The AIEOs have strong language skills that the students hear in their homes and communities so the AIEOs take on a role of translation and modelling code-switching to the students. Most of the teachers in remote schools do not remain in the communities for long periods of time, so very rarely speak the home languages of the students. The AIEOs have an integral role in building language skills in mathematics.

> I am trying to get AIEOs to model code switching. Teacher speaks in SAE; AIEO models in home language. When I was a kid, my parents taught home language. I'm now learning home languages of this area, and the kids are impressed. I tease the kids, "It's not my language but I'm going to beat you in it!" My daughter is learning one of the home languages; I'm learning the other; we teach each other at home.

The teachers recognized that the AIEOs had very specific, and valuable, roles in the mathematics classroom. One of the teachers described the role that her AIEO played in the lower primary mathematics lessons. In this classroom, the AIEO was very important in the code switching between Kriol and SAE. She modelled the use of Kriol in mathematics lessons. Often the practice was one where the teacher would speak in SAE and then the AIEO would repeat the teacher's speech in Kriol. This enabled the students to hear the instructions or concepts in SAE but could also access meaning through Kriol. This was a very common practice in the lower primary where the students were encountering many mathematical concepts that were not commonplace in the family and/or community lives of the students.

> Delta's [pseudonym in accordance with University Ethics] role in numeracy – she takes a small group, speaks Kriol, counts with the kids. She also does code switching mainly in lower grades. By the upper years, the kids should speak English.

The AIEOs also knew what was happening in the community, had significant cultural resources to assist with planning, and were a valuable aide to the teachers. All the AIEOs had undertaken training and were well qualified to work alongside the teachers.

Conclusion

The value of language-based practices, and the valuable support offered by the AIEOs helps build a very strong community at the school in which language support is critical. At Bardulu, the AIEOs have a central role in the school in terms of supporting teachers in their planning of mathematics teaching and resources, as well as providing contextual information to explain behaviours and customs of the students. The school places a high status on the home languages of the students creating jumping boards from which the teachers build effective learning experiences in mathematics. The many concepts and terms of mathematics can create potential for misunderstandings but the practices adopted by the school are aimed at reducing this potential. As teachers are frequently "tourist teachers"—where they come into the school and stay a minimal period—here is a need to build sustainable practices within the school. To this end, the school has invested heavily in the training of the AIEOs and in so doing has built a culture whereby the AIEOs are a valued part of the teaching community within the school.

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Longevity of Math Teachers in Urban Schools

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This paper investigates reasons behind urban math teachers' job satisfaction and longevity, and examines if there are differences by teachers' race. The study site is a Title I urban public high school with many math teachers of color with long tenures at the school. Teachers of color are more likely than white teachers to remain in "hard to staff" urban schools. Therefore, retention of math teachers of color in urban schools may serve as a strategy to resolve math teacher shortages. Findings suggest the importance of administrative support for math teacher retention especially in urban schools. In addition, participants find deep meaning teaching in an urban school they say they would not experience teaching in a suburban school. Trends by teachers' race and suggestions for urban math teacher retention are discussed.

Objectives

Urban schools with diverse student bodies often struggle to find certified math teachers who remain in the classroom (Hanushek, Kain, O'Brien, & Rivkin, 2005; Jacob, 2007; Murnane & Steele, 2007). However, teachers of color are more likely to work in diverse urban schools (Borman & Dowling, 2008; Hanushek, Kain, & Rivkin, 2004; Ingersoll & May, 2011), making their retention a potential means of reducing math teacher shortages in such schools. Studies also indicate that teachers of color produce better academic results for students of color as measured by standardized tests, attendance, and advanced level course enrollment than white teachers (Achinstein et al., 2010; Hanushek et al., 2005).

This study investigates reasons for longevity of math teachers in one traditional urban Title I public school with high retention rates of math teachers, Beachside High School (pseudonym). Beachside High serves 98.5% students of color where 100% of students are considered socioeconomically disadvantaged (California Department of Education, 2013). Specifically, I focus on these research questions.

1. What reasons do participants give for job satisfaction (and how might this differ by race)?
2. What conditions, factors, and/or experiences influence participants' longevity in the classroom (and how might this differ by race)?

Theoretical Framework

Students in urban setting schools, with high populations of students of color, have less than a 50% chance of being taught by a math (or science) teacher with teaching certification specific to their field of instruction (Oakes, 1990, in Darling-Hammond & Berry, 1999). Such schools typically serve poor students of color in under-resourced urban districts (Darling-Hammond & Sykes, 2003). Not only do urban schools struggle to find certified teachers, their teachers also exit at greater rates (Ingersoll, 2001) primarily due to job dissatisfaction, not retirement (Ingersoll & Perda, 2009). Half of teachers in high poverty schools leave within the first five years of teaching, and as soon as the first three years in some urban districts (Haberman, 2005). On average, teachers who remain in urban schools are as effective or better than those who exit, and high teacher turnover is detrimental to student achievement (Hanushek et al., 2005).

White teachers demonstrate "white flight," where the percentage of students of color in a school is independently and significantly related to (white) teacher turnover, even when holding school poverty constant (Ingersoll & May, 2011). For teachers of color, on the other hand, student demographics are not correlated with reasons for their transition to a different school nor to their exit from the field (Ingersoll & May, 2011). Unfortunately, teachers of color exhibit greater attrition rates because they are two to three times more likely to choose to teach in an under-resourced urban school with challenging work conditions (e.g. lack of support for new teachers, availability of classroom resources) (Ingersoll & May, 2011).

Retention of teachers of color may improve the shortage of math teachers in urban schools. The present study investigates what keeps math teachers in one urban Title I public high school, examining differences by teachers' race.

Methods

All math teachers with more than five years' experience at Beachside High School, or BHS, were invited to sit for a 60-minute interview, and all those invited participated. All interviews were audio recorded with participants' consent. Interviews were transcribed through a transcription service. To ensure accuracy of transcription, I re-listened to audio files several times and made corrections to transcripts.

Using a grounded theory approach (Charmaz, 2006) I developed *initial codes* through line-by-line coding of transcript data (Thornberg & Charmaz, 2014). I used Atlas.ti to code and re-code transcripts. I synthesized data through *focused coding*. I created a matrix display (Miles & Huberman, 1994) with frequency and percentage of codes by participant, with participants of similar ethnic backgrounds in columns next to one another. I then returned to the coded transcripts I created in Atlas.ti to analyze and write narrative profiles for each teacher, pulling out illustrative vignettes and quotations. This perspective proved helpful to my analysis in developing *theoretical codes* and establishing overall themes.

To attend to concerns of validity I wrote self-reflective memos to be mindful of the potential bias my perspective brings to the work (Creswell, 2013) as a former urban math teacher of color (Asian American) of 11 years. To consider reliability concerns I re-listened to audio files multiples times to correct interview transcripts, conducted several rounds of coding and re-coding, and engaged with an interpretive community to discuss codes and themes.

Data Sources

Interviews were conducted with all eight math teachers who met the study criterion of more than five years' experience at Beachside High School. Because half of teachers in high poverty schools exit within the first five years, or even as low as three years in some urban districts (Haberman, 2005), I sought a site with many math teachers above their fifth year. Eight of the total nine math teachers met the study criterion of teaching more than five years at BHS. The eight study participants have been teaching at Beachside High for 10 to 24 years.

Six of the eight participants are teachers of color (four are Asian American and two are African American), much more diverse than the national or California averages, where teachers of color comprise only 16.5% and 33.2% of the teaching force respectively (California Department of Education, 2013; Ingersoll & May 2010).

Beachside High School is a large traditional Title I public school in the Brookdale Unified School District, not a charter or specialized high school. Beachside High serves approximately 1,700 students, of whom 98.5% are students of color, 100% are socioeconomically disadvantaged, and 35.1% are English Language Learners (California Department of Education, 2013). The student body is 34.4% African American, 40.1% Asian American, 1.6% Filipino, 20.5% Latin@, 0.4% mixed race, 0.3% Native American, 0.6% Native Hawaiian or Pacific Islander, and 1.5% white (California Department of Education, 2013).

Beachside High School (pseudonym) is situated in Brookdale (pseudonym), an urban California city with 65.5% residents of color. Brookdale is one of the most diverse cities in the nation, with no majority ethnic group (U.S. Census Bureau, 2010).

Results

In this section I first describe what threatens participants' satisfaction and longevity specific to urban schools. Next I explain three factors that improve satisfaction and longevity, two that may also apply to suburban teachers and one factor specific to urban teachers. Lastly, I examine trends by teachers' race.

Lack of Administrative Support

The purpose of this study is to determine reasons urban math teachers stay in the classroom. To fully understand what keeps urban math teachers in the classroom, I first discuss barriers to longevity before discussing factors that improve retention. In interviews, six of the eight participants brought up the lack of administrative support as challenges to their work. Participants' expectations for support centered solely around their desire for assistance with classroom management or safety concerns. Participants did not mention a desire for support from administration for any other matters, such as curricular design or opportunities for leadership development. Rather, participants expected administrators to "back them up" if needed for help with classroom management. For example, Mrs. Cao, 35, a Vietnamese American teacher of 13 years at BHS, who grew up in Brookdale, mentioned her struggles with classroom management. At times she needs to call security, but the school administrative systems did not providing the support she desires.

While BHS math teachers recounted stories that illustrate their management or safety concerns at BHS, due to what they characterized as poor administration, they did not express fear of the community itself. BHS math teachers expressed anger, frustration, and disappointment with administrators' "laissez faire" disciplinary approach that they felt "enabled" students. Ms. Allen (pseudonym), an African American math teacher with 10 years experience at BHS, mentioned safety concerns such as the following. "Kids are getting robbed, beat up and then instead of the kid getting consequences, the kid's sitting next to the kid who just stole their iPhone and beat them up in class with no consequence for the kid." She attributes the presence of such safety concerns to poor administration.

> I'd say the biggest, hugest, worst problem at Beachside High is not at all the kids. The kids are always fine. I don't even care if half of them are coming out of jail. They're fine. They're all fine. They're kids, you know. They just want to come here and enjoy their high school experience and talk to a girl and you know I mean silly stuff, right? Our biggest problem is they are children and they need consistency and our administrators are completely and totally inconsistent. There are no rules. There is no structure. There are no guidelines. There are no punishments. There are no rewards. There are no consequences. There's nothing, and it's just letting our kids know that nobody cares.

Like Ms. Allen, participants expressed frustration and disappointment with their administration's approach to management, adding that they believe it communicates low expectations to students. While this threatens their longevity, the eight participants have been at BHS for 10 to 24 years. Their longevity is improved by three factors. The first two factors may also relate to suburban math teachers, and the third factor is specific to urban math teachers.

Structural Features

Common to teachers of various school settings (e.g. suburban teachers), BHS math teachers enjoy structural features of the profession, such as their autonomy, work hours, reliable employment, and summer vacations. For instance, Mr. Liu mentioned the unpredictability of his previous careers and his desire to find steady employment.

All eight participants felt they had autonomy in their classes to develop lessons, projects, and exams. Teachers described having "total autonomy" and "lots of freedom." Previous research indicates that autonomy influences teacher retention and attrition (Borman & Dowling, 2008; Boyd et al., 2009; Hanushek et al., 2004; Ingersoll, 2001). Autonomy is critical to job satisfaction not only for teachers (Ingersoll, 2001; Ingersoll & May, 2011) but overall for any profession (Spector, 1986). Autonomy in instruction also offered opportunities for BHS math teachers to be creative to find ways to "make the math accessible to all students." In addition to autonomy, teachers discussed their enjoyment of a "fresh start," "every day is a new day," and that their jobs are "never boring." Participants also mentioned this aspect as fitting their personalities, where they "get bored easily."

Student Interactions

In addition to structural features of the profession, participants, both of color and white, enjoy interactions with students. First, participants enjoy students' "aha" moments." For instance when asked what she likes best about teaching, Mrs. Talbott, 34, an African American teacher of 12 years at BHS who grew up in Brookdale, responds enthusiastically, " When students say, 'Oh I get it!' You know, I love that. I love that, I love that. It makes me feel like I've done my job and that I've opened up a new world for someone." When asked what he likes best about teaching Mr. Ngô, 35, a Vietnamese American math teacher of 16 years at BHS, responded, "It's interacting with the students. Students ask me high level, critical thinking questions."

Second, participants enjoy learning from students. Mrs. Guo, for example, 61 years old and a Chinese American math teacher of 15 years, told me that she is "learning every day." When asked what keeps her motivated she said, "I'm learning from my kids, knowing humanity and knowing each kid can bring something." Mrs. Cao, 35, a Vietnamese American math teacher who grew up in Brookdale and attended Title I Brookdale public schools, described how she feels "inspired" and "empowered" by her students.

Third, not surprisingly, participants enjoy gratitude from students. Mr. Gardner, 59, a white teacher of 24 years at Beachside High responded, "The biggest impact is when students come back and are like, 'I am so glad you did this or I'm so glad you did that. You have no idea what you've done (for me).'" Even small gestures are meaningful to participants. Mrs. Talbott mentioned that getting a candy cane from a student at Christmas time "means the world" to her. This is consistent with social emotional rewards of teaching generally of various settings, subjects, and grade levels (Lortie, 1975).

Meaningful Work in an Urban School

Participants explained that they felt more useful, or more needed, in an urban under-resourced school like BHS. These findings are specific to teachers in an urban school. When asked if participants chose to teach at BHS intentionally or if by coincidence they affirmed their commitment to teaching in an urban school. For example, white Jewish math teacher Mr. Levi, 59, who has been at Beachside High for 15 years, explained why he teaches at BHS: "Because that is where the most need is. It's that simple. That's why I'm here. Being a math teacher I could go teach in almost any district in the country, but this is where I feel I have the most influence."

Brookdale native Mrs. Talbott told me that she would only teach in Brookdale schools. When I probed and asked if she would ever consider teaching in a suburban school, she responded quickly, "Oh heck no. No, nothing against that. That's fine, but I feel that I can have more impact in here, you know, in the neighbourhoods, in the schools that I'm from because that's where the struggle is." Mrs. Talbott's personal experience growing up in Brookdale, attending Brookdale public schools, and currently living in Brookdale, makes her all too aware of the city's challenges. For Mrs. Talbott, Beachside High is "where the struggle is." Her sentiment indicates a social justice focus to her work as well as a commitment to her community. Like Mrs. Talbott, participants expressed personal meaning they gain from teaching in a Title I urban school rather than a suburban low poverty school.

Trends by Teachers' Race

African American teacher Ms. Allen described her experiences in a math fellowship program as the only black woman. She explained that she felt she was perceived as "the angry black person" by others in the fellowship. "When you're the only black person then you start thinking you're crazy unless there's another black person there because you can't process all these feelings, and then when you step out and dissect it you realize, 'That's super foul.'" Ms. Allen explicitly mentioned that she feels more comfortable and more respected teaching amongst her diverse colleagues at BHS, very unlike the environment of her math fellowship program. She explained how these experiences make her more understanding of others' struggles. "I think being an outside person makes you sensitive to other outsiders. It makes you more respectful, whether it's the money they have or don't have, or the countries they're coming from, or the language they have or don't have." She also noted that her ethnic identity influences her work with students.

> I think that my identity as a black person - if I see kids cursing on the bus I tell them to stop. A lot of people are afraid of black kids, so they don't want to correct them. They are children and need to be corrected. But because they're afraid that they might all pull out their guns, and I don't care even if they've got a gun, "Put the gun away and stop cursing. You're not going to shoot me, I know your mom. I'll call your mother." [laughter] I'm not afraid of black kids. I can look at them and know, "You don't do that at home, so don't even think about trying it here."

Similarly, Brookdale native Mrs. Talbott explained wanting to be a role model to African American students in her community:

> Growing up I think I only had one black math teacher, and I want show other black kids that, "We can do math too you know, like hello!" and I'm from the neighborhoods they're from. I want to be an inspiration for them like, "She's just like me," you know, and I still talk like them. I still live in the same neighborhoods as them.

Interestingly, Mr. Ngô, who like Mrs. Talbott grew up in Brookdale and attended Brookdale public schools, did not mention wanting to contribute to his community as an influential factor in his job satisfaction or longevity. Mrs. Cao, also a Brookdale native, mentioned being "inspired by her students" as influential to her job satisfaction, but did not express a strong commitment to the Brookdale community as Mrs. Talbott did. Mrs. Guo expressed a "special interest in helping immigrant students." Chinese American teacher Mr. Liu, who grew up in a city near Brookdale, also did not express a desire to serve as a role model or give back to the community. Further investigation may help uncover reasons behind Mr. Ngô, Mr. Liu, and Mrs. Cao's commitment to BHS, as well as white Jewish teacher Mr. Levi's relationship to the community.

White teacher Mr. Gardner described a transformative, or seminal (Warren, 2010), experience being immersed in a community of color through his student-teaching placement in a Brookdale middle school. He explained his initial discomfort leaving his "lily white" hometown to student teach in a school with 90% African American students. He described the racism of his hometown, explaining that there were laws regulating to whom homes could be sold. In his student-teaching placement, two African American teachers took him under their wings. He reflected fondly on his student-teaching experience:

> It was the best thing that could have happened to me. It scared the poop out of me when I was told I was going to student-teach there, but it opened my eyes to this whole world that I had no clue about growing up. It changed who I am and a lot about how I view the world. That's how I came in to Beachside High.

These findings highlight trends by teachers' race of how participants gain meaning teaching in an urban school like BHS.

Discussion

Findings suggest the universality of certain factors of teachers' professional satisfaction, such as structural features of the profession and social emotional rewards gained through student interactions (Lortie, 1975). Participants' appreciation of seemingly small gestures by students (e.g. graduates returning to say hello, saying thank you) is consistent with Hargreaves's (2000) study of elementary and secondary teachers' perception of interactions with students.

However, findings also highlight the importance of other factors specific to a diverse, urban, high poverty high school. First, for math teachers at BHS, administrative support is critically important to their longevity. Second, study participants find deep meaning working in an urban school that they say they would not experience at a suburban school.

The Importance of Administrative Support

Nationwide, administrative support is a significant factor of teacher attrition, especially in urban, high poverty public schools (Borman & Dowling, 2008; Ingersoll, 2001; Ingersoll & Perda, 2009). An interview study of 12 teachers who left urban schools by Smith and Smith (2006) suggests that teachers who exit urban schools leave out of fear of the community and unsafe incidents that may make their way into the school setting. However, participants in the present study didn't express fear of the community.

BHS teachers may not be afraid for several reasons. First, participants have been teaching at BHS for ten to 24 years, and have experienced safe school climates with different administrators. Second, participants mentioned close relationships with students who engaged in unsafe actions. Third, participants from the present study either grew up in Brookdale, currently live in Brookdale, grew up in a community of similar demographics, or have had transformative experiences where they view the city of Brookdale positively.

The Importance of Finding Meaning Teaching in an Urban School

Study participants found considerable meaning teaching at BHS. Participants explicitly stated that they would not achieve these social emotional rewards teaching in a suburban low poverty school. These findings are consistent with research on the personal meaning dedicated teachers gain from their work in urban schools (Jupp, 2013; Nieto, 2003; Stanford, 2001).

How participants found meaning teaching in an urban school varied by teachers' race. A body of research suggests that teachers of color enter the profession for "humanistic" reasons and choose to teach in urban schools with students of color (Ainstein et al., 2001) in order to "raise the race" and "give back" to the community (Villegas & Irvine, 2010). Mrs. Allen, Mrs. Talbott and, to some degree, Mrs. Guo and Mrs. Cao expressed reasons that align with these findings. However, Mr. Liu and Mr. Ngô did not express a desire to give back to the community. This contrasts Su's (1997) case study of teacher candidates, where commitment to urban schools for participants of color was influenced by their awareness of educational inequalities. Su's work is relevant because although results are not parsed out by ethnicity, (participants of color are compared to white teachers) the majority of the participants of color in the study were Asian American. Quite possibly, because Mr. Ngô grew up in Brookdale and even attended BHS, his lack of exposure to schools with more resources makes him unaware of inequities. Likewise, Mr. Liu grew up in a city very close to Brookdale. Research on Asian American teachers, much less Asian American math teachers, is extremely sparse (Ng, Lee, & Pak, 2007), and further investigation may yield interesting findings about Mr. Liu and Mr. Ngô's commitment to Brookdale youth.

Significance

This study sheds light on what keeps math teachers in urban schools with predominantly students of color. Participants' longevity, for teachers of color and white teachers, is threatened by administrative issues, but improved by structural features of the profession, student interactions, and meaningful work in an urban school. Math teacher longevity in urban schools may be improved with stronger administrative support and by cultivating positive relationships between teachers and administrators and between students and teachers. Schools that can support such relationship building may be able to better retain math teachers in urban schools.

Future research may address areas of the study's limitations. Participant observation and student interviews may add data to triangulate with interview data. Adding surveys, ranking activities (Stanford, 2001), or focus groups may also enrich data on math teacher satisfaction and longevity. Additional interviews with the same participants may illuminate more reasons behind teacher longevity (Seidman, 2006). Inclusion of data collection with teachers who recently left BHS may help understand the "tipping point" of how and when math teachers decide to exit an urban school.

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Mathematics Education as a Disciplinary Institution

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Sociological theories of school see the social function of school not only in the qualification of students, but also in their selection and the legitimisation of contemporary social mechanisms. I argue that a sociological approach towards mathematics education based on Michel Foucault's theory of disciplinary institutions and dispositifs helps to link these general functions of school to the theory and praxis of mathematics education, providing a language to express connections between school mathematics and society and to critically address issues of selection and legitimisation in the mathematics classroom. In this paper, I will present a reception of Foucault's theory and its interpretation and use for critical mathematics education. This presentation will be supported by two exemplary studies on mathematics and legitimacy.

Introduction

During the last three centuries, mathematics education has become a worldwide enterprise which consumes an impressive amount of monetary and human resources. During all this time, mathematics educators, mathematics education researchers and educational politicians have presented ever-new discourses for the legitimisation of compulsory mathematics education. These discourses (e.g. NCTM, 2000) typically present mathematics education in an idealistic form as a school subject which allows equal access, fair assessment, and is relevant to students' lives in regard to both the content (such as algebra) and meta-abilities (such as reasoning or problem solving).

In contrast to these expressions of common desire, there is little research on what contemporary mathematics education really does in our societies, which role it plays, what it allows, what it prevents, which functions it serves. However, since the 1970s mathematics education has been increasingly perceived as a socially and politically sensitive endeavour (Lerman, 2000). Thereby mathematics education research follows a trend that originated in sociology. It were sociologists such as Jean E. Floud (1956) in Britain, Helmut Schelsky (1959) in Germany and Pierre Bourdieu (1961) in France, who made school an object of sociological studies, questioning traditional pedagogical discourses on education and discussing school-inherent mechanisms for the reproduction of social class structures. While these approaches were mainly based on a Marxist sociology, later studies followed more structuralist paradigms. In the German case, Fend (1974) argued that school has primarily three social functions, namely *qualification* preparing students for well-describable situations in their life, s*election and allocation* selecting students along certain criteria and allocating them to educational or professional opportunities, and *legitimisation and integration* justifying the current mechanisms in society and installing the students into them.

Mathematics education research has traditionally focussed on the function of qualification, incorporating psychological approaches to search more "effective" methods of qualifying students, while studies on the role of mathematics education in selection and legitimisation have become more frequent only recently (Lerman, 2000). Some scholars question the social function and legitimacy of the current state of selection and allocation through mathematics education by discussing the role of mathematics as a "gatekeeper" excluding certain ethnic or socio-economic groups from educational success and privileged life opportunities (Stinson, 2004; Gellert & Jablonka, 2007) or as a device for the administration of the bio-capital of a state through its education system (Popkewitz, 2002). Other scholars analyse how contemporary social mechanisms depend on mathematics education, using sociological concepts such as socialisation and ideology (e.g. Skovsmose, 2005; Ullmann, 2008).

In this paper I argue that mathematics education can be understood as a disciplinary institution within the sociological framework of the French philosopher Michel Foucault. Examining mathematics education within this framework allows both to sociologically substantiate socio-critical mathematics education research that originally does not follow a well-cut sociological paradigm, and to provide a more differentiated access to the legitimisation function of school, connecting it to knowledge and learning. The sociology of Foucault proves to be a good choice as it does not only connect issues of power, knowledge and subjectivity, but helps to question values and commonly held beliefs. While several studies in education and especially in mathematics education (e.g. Baker & Heyning, 2004; Walshaw, 2004; Walshaw, 2010) have incorporated theories from Foucault, no study has yet attempted to provide a general sociological approach to the role mathematics education plays in legitimising contemporary social mechanisms. Therefore, the intention of this paper is to present the Foucaultian framework and to show how it can be used to understand connections between mathematics, education and society.

Two Studies on Mathematics and Legitimisation

While contemporary anthologies give a good introduction to the field of socio-critical mathematics education (Alrø, Ravn, & Valero, 2010; Boaler, 2000; Ernest, Greer, & Sriraman, 2009; Freitas & Nolan, 2008; Lerman, 1994; Restivo, van Bendegem, & Fischer, 1993; Valero & Zevenbergen, 2004), I will only discuss two exemplary studies here. The first example is the work of Philipp Ullmann (2008) who authored a "critical study of the legitimacy and praxis of modern mathematics". The second example is part of my own research on the connections between logic, mathematics education and society (cf. Kollosche, 2013; Kollosche, 2014).

Ullmann's work is mainly inspired by sociology. After discussing his understanding of modernity, Ullmann presents his use of the Marxist concept of ideology as a body of knowledge that lacks a material basis and is used to rule people. The main claim, to the substantiation of which he devotes his study, is that mathematics is accompanied by the following ideology:

> *Mathematics, and that is mathematical knowledge, is secured, true, rational, objective and universally valid. As an agent of this unique knowledge mathematics is a value-free and therefore liberal science and legitimised to raise a universal claim to truth, validity and responsibility.* With this claim, mathematics becomes the legitimising foundation of modernity, ultimately promising to be the product of modernity which solves its crisis.
>
> (Ullmann, 2008, p. 11, my translation, Italics in the original)

Ullmann then shows in well-analysed examples how the efficiency of the application of mathematics in science and administration relies on that ideology. Furthermore, he illuminates through a study of contemporary schoolbooks how this ideology is fostered in mathematics school education. He concludes that, “mathematics is ideology of modernity” (p. 12, my translation).

In Ullmann’s understanding, mathematics and its ideology are central mechanisms in the functioning of contemporary society. Therefore, developing that ideology in class serves the function of legitimisation and integration. Unfortunately, Ullmann neither looks at processes in the mathematics classroom in more detail, nor does he analyse “pure” mathematics. However, one of my studies may partly fill this gap and eventually help to answer the question how such an ideology is linked to mathematics and how it is socialised in the mathematics classroom.

My study mainly aimed on analysing connections between society, education and “pure” mathematics. Logic was held to be one characteristic of mathematics and therefore analysed (Kollosche, 2013). The analysis was conducted following a genealogic approach (Foucault, 1971/1984; Lightbody, 2010), which identifies social dimensions of phenomena that have become taken for granted by analysing their genesis. The object of study was a set of four principles of logic which Scholasticism identifies in the opus of Aristotle and which form the basis for Ancient and most Modern logics. They also can be recognised in the mathematics classroom, most obviously in discourses on arguing and proofing, but also in the structure of school mathematics itself.

The genealogic analysis of these principles of logic shows that the social function of logic can be understood in terms of dialectics of benefits and sacrifices in three dimensions. In a religious dimension the promise of an eternal, universal and ever-reliable truth causes comfort for some, while the determinism of the new-born idea of “truth” frightens others. In an epistemological dimension logic provides a system to order thoughts in terms of right, wrong and origin, but it also demands the thinkable to be reduced to that very form, putting many aspects of life out of intellectual range. In a political dimension logic allows for a physically non-violent search for consensus in democratic debates, whereas it constitutes a rhetorical tool to enforce one’s wishes and to subjugate the thoughts of others.

Eventually it shows that logic is inseparably bound to several essential fields of society in that it provides possibilities to think and act within these fields.

So, logic serves society in various dimensions. In order to fulfil its functions, logical thinking needs to be legitimised and every new generation has to be integrated in the discourses that build on logic. While logical reasoning may take place everywhere in- and outside school, I argue that logic has a special role in mathematics education. Mathematics as an academic discipline can be more logical than any other academic discipline as it does not have to represent any empirical objects but can change and does change its objects of study towards forms that fit the logical paradigm. As an example for this phenomenon, consider the definitions for the concept of continuity, which have changed dramatically in the history of calculus due to logical inconsistencies. This strict logical structure of academic mathematics enters school mathematics not only through proofs and argumentations in class, but also through the constitution of mathematical knowledge itself, e.g. through classifications. Eventually, in mathematics classrooms students are obliged to interact with logically shaped structures from early on and for a long period of lifetime. Therefore, it can be argued that mathematics education has a special role in introducing students to a logical style of thought, preparing the students to accept and support logic-related mechanisms in society.

The examples presented show that research on the legitimisation and integration function of mathematics education clearly has explanatory potential, but it also becomes clear that it is difficult to describe the connections between power, mathematics or logic and the individual in naive terms. The following sociological framework helps to make underlying assumptions about the social explicit and to express the ideas presented above in well-informed concepts.

Sociological Framework

Michel Foucault has contributed in many areas of 20th century humanities. Among these is the attempt to explain the connections of power, knowledge and subjectivity in different fields of society. Foucault (1980a) wanted to overcome some constraints inherited in Marxist sociology (p. 100ff.), such as the rigid antagonism between bourgeoisie and working class or the role of the individual as a passive victim of the camouflaging knowledge of ideology. Foucault (1982) understands power not as a good that people possess but as the command over *techniques for the conduct of the self or others.* With this differentiation, Foucault wants to give an answer to the question how power is executed, and he wants to acknowledge that people can have power over themselves, influencing the ways in which they behave, think and act. For example, while commanding is a widespread technique for the conduct of others in the military, it requires many techniques for the conduct of the self to be a soldier – the ability to follow commands regardless of one's own feelings and thoughts.

Foucault (1979) is most interested in what he calls *disciplinary techniques*, that is, techniques that serve for the conduct of others through their conducts of the self. For example, employers ask employees to start working on time, but they do not provide the means to achieve punctuality. Consequently, employees have to develop techniques for the conduct of the self that ensure punctuality, for example buying and relying on an alarm clock, sticking to a neat time table in the morning or going to bed early. As there is no pre-defined way of how to cope with disciplinary demands, the individual *ascesis*, that is the individual development of appropriate techniques for the conduct of the self, constitutes a considerable aspect of the individual's identity. The ways we dress or the styles in which we talk are altogether our individual techniques to comply with techniques for the conduct of our dress code and language use. Foucault highlights that our conduct of the self, even if developed in single domains such as the workplace, eventually becomes an integral part of our personality, so that we might turn out to be punctual in private life too and that we might even demand this punctuality from others. It is in this internalisation of originally external demands that Foucault sees the efficiency of disciplinary techniques and the reason for their spread throughout society in the Modern age.

Foucault (1984) uses a wide interpretation of the concept of knowledge, including beliefs, values, morals and presumptions (p. 334f.). He then regards knowledge as inseparably linked to techniques of conduct (1979) and coins the concept of *power-knowledge* relations. On the one hand, knowledge may produce, improve and justify certain techniques of conduct. For example, Foucault (1965) shows how research on insanity allowed early Modern societies to isolate and "treat" people who before were considered ordinary members of society. On the other hand, knowledge itself needs a basis of legitimisation, that is, techniques of conduct which validate it as truth. For example, research on insanity relied on certain academic techniques and methods that were held to produce truth. It is therefore impossible to separate knowledge from power. Indeed, knowledge requires power in order to become accepted, just as power needs knowledge in order to be executed:

> Perhaps, too, we should abandon a whole tradition that allows us to imagine that knowledge can exist only where the power relations are suspended and that knowledge can develop only outside its injunctions, its demands and its interests. Perhaps we should abandon the belief that power makes mad and that, by the same token, the renunciation of power is one of the conditions of knowledge. We should admit rather that power produces knowledge; that power and knowledge directly imply one another; that there is no power relation without the correlative constitution of a field of knowledge, nor any knowledge that does not presuppose and constitute at the same time power relations. (FOUCAULT, 1975/1979, P. 27)

Foucault (1979) analyses the prison as a *disciplinary institution* and later talks about *dispositifs* (sometimes translated as "devices") of power. A dispositif is a "system of relations that can be established between" the techniques for the conduct of others, the techniques for the conduct of the self, the forms of ascesis, the knowledge supported and being supported by these techniques including commonly held values and convictions as well as academic support, and the institutions relying on these techniques and on this knowledge in a certain field of social practice (Foucault, 1980b, p. 194). One of his best-known fields of study is his work on the dispositif of delinquency. Foucault (1979) presents a genealogy of the prison, connecting the modern idea of a disciplined and productive conduct of the self with the emergence of techniques for the conduct of prisoners, with the modern discourse which understands delinquency as a pathology that requires treatment, with the development of academic disciplines which produce knowledge on delinquency and with the spread of institutions such as the police and prisons. Most interesting is his answer to the accusation that "the prison, in its reality and visible effects" was a "great failure" (p. 264), as it does not diminish the crime rate and isolates even occasional perpetrators and innocents within a milieu of severe delinquency. The traditional critique of the prison states "either that the prison was insufficiently corrective, and that the penitentiary technique was still at the rudimentary stage; or that in attempting to be corrective it lost its power as punishment" (p. 268). However, Foucault searches for the positive effects of the failure of the prison:

> For the observation that the prison fails to eliminate crime, one should perhaps substitute the hypothesis that the prison has succeeded extremely well in producing delinquency, a specific type, a politically or economically less dangerous [...] form of illegality; in producing delinquents, in an apparently marginal, but in fact centrally supervised milieu; in producing the delinquent as a pathologized subject. The success of the prison, in the struggles around the law and illegalities, has been to specify a "delinquency". (FOUCAULT, 1975/1979, P. 277)

In the case of both insanity and delinquency, advocates of reason and social discipline have used their power to introduce new concepts of thread, of seemingly incurable pathologies, which can only be avoided by a reasonable and obedient conduct of the self. This is how power is used to cultivate new knowledge, while this new knowledge itself legitimises, demands and develops new techniques for the conduct of the sane and disciplined self as well as of the insane and delinquent others. At various points of his work, Foucault shows that the emergence of the human sciences in modernity, especially of psychology and pedagogy, is on the one hand justified by an upcoming social need for the custody and treatment of the insane, the ill-behaving and the uneducated, while on the other hand these human sciences justify the need and develop techniques for such custody and treatment.

Besides the prison, also monasteries, barracks, asylums and schools may be understood as disciplinary institutions. In case of the school, we observe techniques for the control, teaching and motivation of students, students' techniques for behaviour and learning and academic disciplines which contribute to the theory and legitimisation of teaching. However, I argue that mathematics education is not only embedded in a dispositif of school, but also in dispositifs that are more closely linked to the subject of mathematics.

Mathematics Education as a Disciplinary Institution

The social functions of school as described by Fend (1974) correspond to the development of specific techniques for the conduct of the self. Solving quadratic equations, perceiving two-dimensional projections of three-dimensional shapes, accepting marks for achievements, reducing real life problems to mathematics or conceiving mathematics as a legitimate tool to handle any situation in society are altogether techniques for the conduct of the self which do not come naturally but have been acquired somewhere and somewhen. As these issues are addressed regularly over many years in mathematics education, mathematics education is the dominant institution in which this ascesis takes place and can be understood as an institution in which teachers apply disciplinary techniques, that is techniques for the conduct of the students in order to initiate their "mathematical" ascesis.

These "mathematical" techniques for the conduct of the student's self then have a social dimension in that they allow society to install techniques for the conduct of others which then build on these "mathematical" techniques for the conduct of the self. For example, the selection of applicants by educational institutions or companies on the basis of the applicants' marks in the subject of mathematics requires the public acceptance of the legitimacy of this mechanism of selection. Or, if mathematics education makes students perceive mathematics as a technique that can help to decide any problem, society can use this prestige of mathematics to legitimise social decisions. Ullmann (2008) has shown that mathematics is widely used in society to legitimise social decisions and that these decisions build on the concept of mathematics that has been acquired in school. As I showed elsewhere for the case of logic in school mathematics (Kollosche, 2013), more detailed studies can help illuminate the connections between the organisation of society and the mathematical.

So, the effectiveness, which Foucault attributes to disciplinary techniques, also shows in the mathematics classroom. During years of learning, students learn to behave and to perceive in specific, politically biased ways. In their ascetical processes students produce mathematical identities that may vary between the extremes of unquestioned complicity or mathophobic avoidance. Eventually, instead of understanding that avoidance as the "great failure" of mathematics education (which is certainly is from a pedagogical perspective), "one should perhaps substitute the hypothesis" that mathematics education has succeeded extremely well in producing complicity to mathematics (in the case of those who want to follow) or in provoking autonomous exclusion from mathematical discourses (in the case of those who do not want to follow). Mathematics education then would be a disciplinary institution playing a central role in introducing mathematics as a technique for social power.

In the end it becomes clear that preparing students to accept and participate in the mathematical organisation of our society is one of the main functions of mathematics education. Nevertheless, the mainstream of mathematics education research is still concerned with the development of more effective ways of teaching mathematical content knowledge and techniques – qualifications which are (from a certain grade onwards) hardly needed in future life (for the German case cf. Heymann, 1996, p. 153). Conceptualising mathematics education as an institution whose primary function is to qualify students, produces a power-knowledge that makes society perceive and legitimise mathematics education on the ground of supposedly relevant contents while it effectively fulfils social functions which are much more fundamental for the organisation of contemporary Western society.

Consequences

The Foucaultian paradigm proves to be successful in providing concepts to describe how mathematics education is involved in the school functions of qualification, selection and allocation, and legitimisation and integration. Socio-critical research on mathematics education may use this paradigm to pose research questions on different social layers:

– Which techniques for the conduct of the self are connected to mathematics education, which disciplinary techniques initiate their development, and how can they be used for the government of our society?

– What are the possible ways of ascesis (e.g. self-exclusion) with which a student can react to the disciplinary techniques he is subjected to in the mathematics classroom?

– In how far do school mathematics contents represent a certain power-knowledge? Which techniques of conduct does this knowledge presuppose and which techniques of conduct does it allow?

– Lastly, in how far does mathematics education research represent a certain power-knowledge, developing, improving and legitimising certain techniques for the conduct of students in class?

While some of these questions have already been partly answered by socio-critical studies on mathematics education, the Foucaultian paradigm bears the possibility to interrelate these often isolated findings into an integrated view on mathematics education as a socially sensitive dispositif of power. Such an understand of mathematics education may not only help those students "left behind" and teachers struggling to teach mathematics, it may eventually help to redefine what mathematics education actually is and what we want it to be.

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Toward a Socio-spatial Framework for Urban Mathematics Education Scholarship

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The purpose of this paper is to re-engage the task of conceptualizing urban mathematics education by proposing a theoretical framework for scholarship, policy, and practice in urban mathematics education. The authors engage scholarship in mathematics education, urban education, critical geography, and urban sociology to consider a socio-spatial framework for urban mathematics education, which includes a visual schematic that locates mathematics teaching and learning—vis-à-vis a now-classic math-instructional triad—within a system of socio-spatial considerations relevant to urban contexts in the United States. The authors also consider the potential for such a framework for considering a more global perspective of urban within mathematics education scholarship.

> The challenge is to build theories and models that realistically reflect how geography and opportunity in mathematics education interact. If this challenge is addressed, the field will be one step closer to making scholarship in urban mathematics education visible (Tate, 2008, p. 7).

During the past two decades, urban mathematics education has emerged as a vibrant new area of scholarship in the United States—evinced most recently by the arrival and proceedings of the *Journal of Urban Mathematics Education (JUME)*. The roots of this subdomain of mathematics education extend back at least to efforts during the 1980s (see Tate, 1996), concurrent with the development and publication of standards by the National Council of Teachers of Mathematics (NCTM) for mathematics curriculum and evaluation (1989) and for the practice of mathematics teaching (1991). These developments also coincided with commensurable shifts in research; mathematics education scholarship around the world was entering its much-discussed social turn (e.g., Meyer & Secada, 1989; also see Lerman, 2000; Martin & Larnell, 2013; Stinson & Bullock, 2012). For researchers, teachers, policymakers, and education-interested foundations in the United States (e.g., Ford, National Science Foundation), a crucial new question emerged: How would the then-new vision for school mathematics reform extend to and take shape in urban districts and classrooms (Tate, 2008)? This question remains central in the latest shift to the Common Core State Standards for School Mathematics.

Our aim in this presentation is to broaden the discourse in urban mathematics education in ways indicated by the above epigraph excerpted from Tate's (2008) commentary in the inaugural issue of *JUME*. Urban mathematics education scholarship has advanced to the point at which we may now begin to evaluate the production of knowledge in this subdomain—and, particularly, the building of "theories and models that realistically reflect how geography and opportunity in mathematics education interact" (p. 7). What has the study of urban mathematics education entailed? What can it become? The purpose of the present paper is to take "one step closer" toward addressing these questions and toward new directions for urban mathematics education scholarship and practice.

Overview of the Socio-spatial Framework for Urban Mathematics Education Scholarship

In the spirit of addressing Tate's challenge (also see Anderson, 2014), our objective is to posit a new theoretical framing for scholarship in urban mathematics education—the first of its kind (Figure 1). In this section, we detail the theoretical concepts undergirding the framework. We situate this framing squarely (but not entirely) in mathematics education scholarship—using as our central unit of analysis the well-regarded math-instructional triad of teacher(s), learner(s), and mathematics (Cohen, Raudenbush, & Ball, 2003; NCTM, 1991; Stein, Smith, Henningsen, & Silver, 2009).

Extending beyond mathematics education, we look toward the interdisciplinary areas of urban sociology, critical geography, and urban education scholarship to consider the various forces that influence

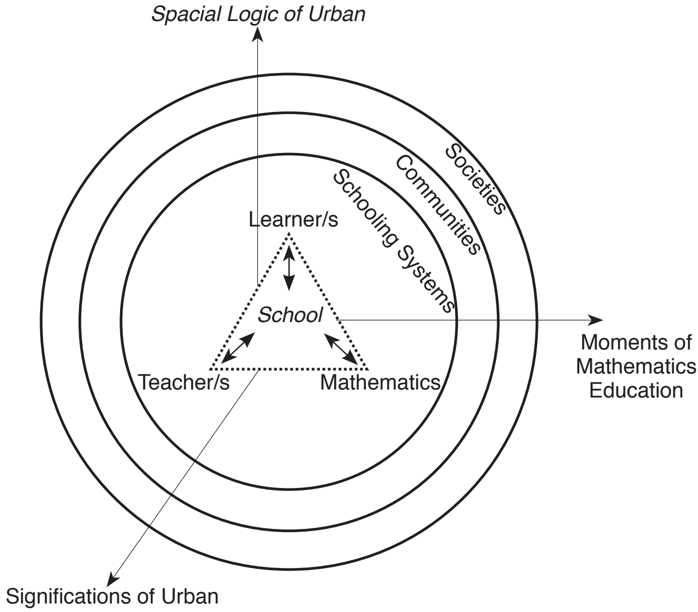

Figure 1: Socio-spatial framework for urban mathematics education scholarship

mathematics teaching and learning in urban spaces as well as the social significations that shape interactions in urban settings—and to emphasize the ways in which urban spaces and their meanings are reciprocally constituted. We recognize, however, that the task of defining urban has been an overwhelming challenge across disciplines, and our attempt here is to incorporate what is known inasmuch as we can given what is available to us contemporarily (Milner & Lomotey, 2013).

To inform the framework with respect to the social meanings that shape urban mathematics education, we draw on Leonardo and Hunter's (2007) typology of significations that circumscribe urban education (also see Martin & Larnell, 2013). We represent that typology as an axis of the framework that intersects with spatial considerations of urban, drawn from scholarship in human and critical geography (e.g., Soja, 1980; Thrift, 2003) and urban sociology (e.g., Johnson, 2012). These two axes, when taken together, are intended to signal a "socio-spatial dialectic" regarding urban education (also see Soja, 2012). By socio-spatial dialectic, we mean that the social significations and spatial considerations necessarily interact to determine meaning for urban such that, as Tate (2008) suggested: "to realistically reflect how [spatial] geography and [social] opportunity in mathematics education interact." We then add a third axis to situate the socio-spatial elements in relation to the evolution of mathematics education. The third axis incorporates the various theoretical orientations—e.g., cognitivism/behaviorism, constructivism, sociocultural perspectives—that have emerged amid "moments" of mathematics education during the past century (Stinson & Bullock, 2012).

Math-instructional Triad as the Central Element of the Framework

At the center of our framework are the interactions and participation by and among learners, teachers, learners and teachers together, and other interactions permuted according to learners, teachers, and mathematics curriculum (see Figure 2). As Cohen and colleagues (2003) suggest, "Teaching is what teachers do, say, and think *with* learners, concerning content, in particular organizations and other environments, in time" (p. 124, emphasis added). This depiction of a triadic relationship is traceable beyond mathematics education scholarship to the works of John Dewey, Jerome Bruner, Theodore Sizer, and others (Cohen & Ball, 2000). In terms of the diagrammatic representation of the framework, the triad represents a kind of coordinate point with respect to the social, spatial, and math-education "theory-moment" axes that we describe in the following sections. Furthermore, we embed this triad within these multiple levels and axes to acknowledge that the math-instructional triad along is a limited representation of the ways in which mathematics education unfolds amid sociohistorical and contemporary contexts (see Weissglass, 2002).

Spatial Axis of the Framework

To substantiate the spatial aspect of this framing, we draw primarily on and discuss Thrift's (2003) four conceptions of space in relation to the other aspects of the framework: (a) empirical-constructing space, or the ways in which space is rendered measurable or objective

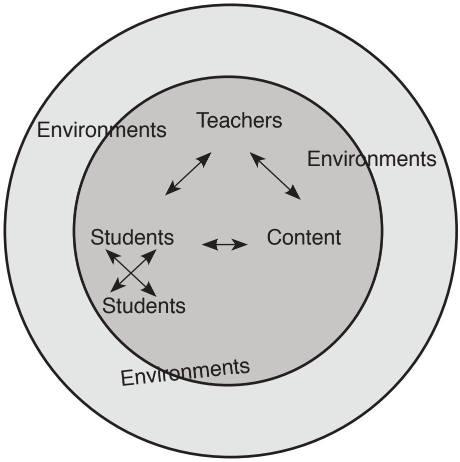

Figure 2: Math-instructional triad,
with Cohen & Ball's (2000) focus on interaction

(b) interactive-connective space, or the pathways and networks that constitute space (c) image space, the visual artifacts that we readily associate with certain kinds of spaces, and (d) place space, or our everyday notion of spaces in which human beings reside--even if notions of "human" and "being" are actively being reconsidered (p. 102). Each of these types refers to ways in which space is conceptualized in relation to human geography, and not necessarily with respect to either a strictly geographical sense of urban spaces or the meanings that are derived from them. This allows us to avoid constraints of a spatial logic that is determined solely by, for instance, characterizations based on population density or physical geography (see Milner, 2012). The strength of articulating four distinctive conceptions of urban allows one to look across their various permutations in ways that provide a nuanced perspective on space.

Social-signification Axis of the Framework

It is clear that urban is not simply geospatial; it also carries social and political meanings. Therefore, considerations of the urban in mathematics education must engage these social and political dimensions directly because "'place matters' in the study of urban mathematics education" (Anderson, 2014, p. 10). The social-signification axis of our framework includes Leonardo and Hunter's (2007) three significations of urban: urban-as-sophistication (or cosmopolitan space), urban-as-pathological (or urban as "dirty, criminal, and dangerous;" p. 789), and urban-as-authenticity (or the politics of authenticity). It has become apparent that "urban" does not always refer to the geographical urban space; rather, studies have used the label urban as a proxy descriptor for poor, Black, and Brown populations who inhabit these spaces and disproportionately fall victim to the segregation and concentrated poverty that often characterize these spaces (Darling-Hammond, 2013). Such employment of "urban" ignores the heterogeneity of urban space, its politics, its people, and their experiences (Fischer, 2013).

Theory-moment Axis of the Framework

With a third axis in the framing, we attempt to construct (at least initially) what could be called a mathematical-socio-spatial dialectic. That is, we situate the math-instructional triad within the dimensional space of not only the socio-spatial dialectic but also with respect to the ongoing "moments" of mathematics education theory and practice (Stinson & Bullock, 2012; also see Martin & Larnell, 2013). Put differently, the axes represent the intersectionality of geography (or spatiality), social opportunity, and the development of mathematics education, which is what Tate (2008) originally outlined. The moments of mathematics education—the "process-product," "constructivist-interpretivist," "social turn," and most recently perhaps, "sociopolitical turn"—are overlapping categorical periods of research, practice, and policy (also see Gutierrez, 2013). These periods have often been indexed by a crisis metaphor within mathematics education scholarship (Washington, Torres, Gholson & Martin, 2012); this notion of crisis also connects to particular significations of urban life and contexts.

Ecological Rings of the Framework

In addition to the axes that situate the math-instructional triad amid social, spatial, and sociohistorical considerations, we also locate the triad—and its associated network of practices—amid nested and reciprocally formative organizational fields in which mathematics teaching and learning occur (Arum, 2000; Martin, 2000; Weissglass, 2002). For the purposes of our initial framing, we consider the activity of mathematics teaching and learning within schooling systems (e.g., classrooms, schools, districts), communities, and at broader societal levels. Our attention to these levels also incorporates issues related to state regulation (e.g., Common Core State Standards), professional associations (e.g., National Council of Teachers of Mathematics), market competition (e.g., choice and charter movements), and other institutional forces that shape and circumscribe school-level practices (Arum, 2000). We recognize, however, that this aspect of the framework should be further developed to address the nuances of particular contexts to which the framework is applied—particularly, the various global contexts beyond the United States (from which this current articulation emerges).

Objectives and Global Considerations

The primary objective of the paper is to engage the task of conceptualizing urban mathematics education scholarship by offering a theoretical framing that pushes beyond traditional notions of "urban" in relation to education and mathematics education particularly. Our hope is to continue building and refining this framing toward application in research involving the "network of mathematics education practices" (Valero, 2012, p. 374) in urban spaces. We recognize that access to quality mathematics opportunities is a global issue given increased disparities in wealth and resource allocation (English et al., 2008; Thomas, 2001), and we posit that it is necessary to locate the network of mathematics education practices in a socio-spatial context in order to address these disparities.

However, we do not intend to present this framework as universal to mathematics education as a global enterprise. Toward that end, we acknowledge two key issues as we offer this conceptual framework for consideration among the MES community. First, we acknowledge that our present notion of urban relies heavily on U.S. scholarship. As such, we recognize that elements of the framework—particularly the three significations of urban—may not map onto other socio-political contexts, thus limiting the framework's global applicability. Secondly, we acknowledge the related importance of avoiding an "uncritical globalization of issues" (Atweh & Clarkson, 2001, p. 86), and we hope to expand the framing to encompass a more global sense of urban. Based on these acknowledgements, the purpose of this paper is not only to present a conceptual framework for urban mathematics education, but also to engage an international audience about its global potential.

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"Math is not for us, not an indigenous thing, you know": Empowering Taiwanese Indigenous Learners of Mathematics through the Values Approach

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The South-eastern region of Taiwan, Taitung, is home to the highest concentration of indigenous communities. Their status on indicators such as education level, SES, and life span are worse than other ethnic groups. They are also found to underperform in school mathematics. This paper reports on a study that had been designed to understand more about the indigenous students' want to learn, embodied by what they value in mathematics learning experiences. Eight Grade 4 indigenous students from a carving class were interviewed. It was found that the indigenous students do not value mathematics learning, although they value innate intelligence at the same time that they value effort and out-of-school support. Of concern is the resultant belief that doing well in mathematics is out of reach of the indigenous people.

The Indigenous People of Taiwan

The indigenous people of Taiwan refers to those who have been living in Taiwan before the ethnic Han Chinese migrated there from South-eastern China in the 17th century. They currently make up about 1.5% of the total population of Taiwan (Tsou, 2010). According to anthropologist research, they are one of the Austronesian peoples sharing the same linguistic and cultural ties. Within Taiwan, they also exhibit cultural diversity. Belonging to different tribes, these indigenous Taiwanese speak different languages and observe different cultural rituals and living styles. Their dwellings are also spread across diverse terrains, ranging from high altitude mountains to coastal areas and islands.

Like many other indigenous groups around the world, the history of the indigenous Taiwanese is marked by different periods of colonization, involving the Portuguese, Spanish, Dutch, Japanese and the Chinese. Over these various colonization periods, the indigenous peoples have not only gradually lost their languages, traditions, and cultural rituals (e.g., hunting, weaving, embroidering, carving), but also their identities and willpower (see, for example, Hsieh, 2013).

The Taiwanese government's concern for the welfare of the indigenous population led to the establishment of the Council of Aboriginal Affairs in 1996, which then became the Council of Indigenous Peoples in 2002. The number of officially recognized indigenous tribes has also increased from 9 (in the Japanese colonization period) to 16 in 2014. Over the years, much governmental investment has been made into indigenous people's affairs, in such forms as affirmative action, stipends, scholarships as well as counselling and remediation programs in schools. Yet, the outlooks of the indigenous peoples' employment rate, SES, and life spans are less than rosy. Lee, Chao, Lee and Chang (2011) as well as Tsou (2010) provide relevant context for this.

The same may also be said of the performance of the indigenous students in schools, whether this might be school attendance, dropout rates, education level, motivation, or achievement. For example, the dropout rates of indigenous students are higher than other ethnic groups at all levels of schooling. The education level for the indigenous students in general is much lower than that of the other ethnic groups. Their test results also fall way behind their non-indigenous peers (Taiwan Ministry of Education, 2014). Indeed, many indigenous students from early on in school dislike mathematics and experience difficulties learning it.

It does seem that the government's various attempts at improving the educational attainment of indigenous Taiwanese have not paid off. Critics may say that whatever the Taiwanese government does now is too little, if not futile. On the other hand, the approaches might have been a case of treating the symptoms but not addressing the roots, as reflected in a Chinese saying, "heal the head when experiencing headache, and heal the foot when the foot hurts" (頭痛醫頭腳痛醫腳).

This paper reports on a research study that explores the root causes of under-achievement in mathematics of indigenous Taiwanese students. To do so, we will next argue for the need to consider the volitional approach, rather than the usual cognitive and/or affective approaches. The empowering nature of students' want to learn (Kytle, 2004) will be discussed, and we will argue that values underpin this want to learn. With this values framework, the study was designed and executed. The data analysis will be presented, and we will explore the participants' values that contributed to the ways the indigenous Taiwanese students regarded mathematics and mathematics learning, and to the ways they perceived mathematical performance.

The Want to Learn

Utterances such as the one cited in the title of this paper reflect culturally-based volitional perspectives not just about one's own potential to succeed in (mathematics) education, but also one's own agency and empowerment in life. These perspectives are volitional because they stimulate actions, such as being engaged in mathematics lessons, employing home tutors, or planning for more personal daily mathematics practice. This (co-) construction of and participation in one's own life reflects a sense of willpower to persevere with what one finds to be important, and which one values. If a learner believes that she and/or members of her community has no access to mathematical knowledge, or that the mathematical knowledge she has acquired is not going to enable her to control aspects of her life, then it is reasonable to argue that she is not likely going to keep on trying to make a difference in the livelihood of herself and of her community. Without this will, the cognitive tools that she might have acquired would not be of much use. Similarly, her emotional wellbeing might be affected as well.

This sense of willpower is an expression of a learner's want to learn, and paves the way for cognitive and affective tools to play their respective roles in facilitating (mathematics) learning and teaching. Following from Seah and Andersson (2015), we assert that the things which learners value in their pedagogical activities embody this want to learn. That is to say, the drivers for our want to learn are the values which we subscribe to. These values reflect what we emphasise and regard as important, and they shape the way our own wants to learn looks like, in terms of motivation, engagement, and perseverance. For example, one may value *obligation*, such that her want to learn is shaped by the importance she accords to doing well at school in order to bring honour to her family. On the other hand, another student who might value (relational) *understanding* (see Skemp, 1976) is likely going to develop her want to learn in a rather less instrumental manner.

Values in Mathematics Learning

Indeed, Seah and Andersson (2015) define values as:

> The convictions which an individual has internalised as being the things of importance and worth. What an individual values defines for her/him a window through which s/he views the world around her/him. Valuing provides the individual with the will and determination to maintain any course of action chosen in the learning and teaching of mathematics. They regulate the ways in which a learner's/teacher's cognitive skills and emotional dispositions are aligned to learning/teaching in any given educational context. (p. 169)

In this context, we have adopted the perspective of empowering the mathematics learning experiences of the indigenous students through fostering in them the valuing of enabling convictions related to the nature of mathematics, the pedagogy of mathematics, and education in general. This categorization of values in mathematics education has been based on Bishop's (1996) conception, in which he had labelled these as mathematical values, mathematics educational values, and general educational values respectively.

In particular, Bishop's (1988) seminal book, "Mathematical enculturation: a cultural perspective on mathematics education" had proposed three pairs of complementary mathematical values which he believed underpin the discipline of "western" mathematics, which would have included the mathematics that is taught in schools across Taiwan. These mathematical values are *rationalism* and *objectivism*, *control* and *progress*, and *mystery* and *openness*.

Mathematics educational values reflect what are being regarded as important in the process of mathematics learning, and are thus much more exposed to cultural influences than mathematical values. In the Taiwanese context, mathematics educational values include *achievement*, *practice*, and *relevance* (Seah et al., 2014). Of particular importance here is the recognition that the *relevance* that is valued in one culture may not be similar to what is considered relevant in another culture, such as the indigenous Taiwanese culture.

The third category of values that may be found in mathematics classrooms, general educational values, would include the range of convictions that are embraced in the local mathematics curriculum and in the wider society. Such values might include *creativity, problem-solving* and *achievement*.

Bishop's (1988) notion of complementarity, as well as Hofstede's (1997) conceptualization of value continua, both suggest that convictions are not valued in an either-or manner. Thus, for example, even though *process* and *product* may be opposite ideas which we may imagine to occupy two ends of a value continuum, it is rare for us to come across any student or teacher who values either of these only. It is more probable that we see the importance of both *process* and *product* in our mathematics work, although our valuing of one of these may be more dominant than that of the other.

Given the relative infancy of values research in the context of mathematics education, and the subsequent dearth of systematic research into what are valued by indigenous mathematics learners in Taiwan, we seek here to open up an enacted research agenda for the improvement of mathematics education in the Taiwanese indigenous communities, by establishing baseline understandings of what are being valued (and not valued) by indigenous Taiwanese mathematics learners.

Research Design

Given that we do not yet have much knowledge of what indigenous students (in Taiwan or anywhere else) value in their mathematics learning experience, this exploratory study has chosen to adopt the interview as the research method. After all, a strength of the interview method is its adaptability (Bell, 2005). The affordances to probe further, to attend to non-verbal reactions, and to respond to any culturally-based misunderstandings are seen to be important features for the purpose of our study. Yet, we are mindful of the power asymmetry that is likely to exist in the interview context (Kvale, 2006), possibly arising from the difference in social status (teacher-student, adult-child) and in ethnicities. As such, the first author spent considerable amount of time in the research site, both to allow for the student participants to get familiarized with her, and also to socialize herself to understand the indigenous culture more.

This study was conducted in a primary school in Taitung, in the South-eastern region of Taiwan. Taitung is home to the highest concentration of indigenous Taiwanese communities. This school had been selected for the value it places on the preservation and celebration of indigenous culture, as evidenced through such programs as the inclusion in its school curriculum of indigenous language classes, traditional arts and craft teaching, as well as hunting rituals. Concerts would also be organized to celebrate ethnic cultural events such as the annual harvesting season. The school was physically located in a tribal village in Taitung.

Data sources were the eight Grade 4 indigenous students (6 boys and 2 girls) attending the weekly after-school carving class in the school. The carving master teaching this class was a respected elder in the tribe who has dedicated his time to the preservation of the indigenous culture. Indeed, he was also often found using the indigenous language in his interaction with his students. According to him, he wanted the children to learn the indigenous language as well as the traditional skills. The first author had spent a few months in the class, observing the lessons and interacting with the students.

Most of the boys in the school had chosen to join the baseball team, while most of the girls chose to take part in the traditional cross stitch activity. Indeed, culturally-based gender roles with regards to the learning of traditional skills were still strong. Boys were expected to observe their fathers or other male elders in the tribe to learn carving skills, whereas girls would expect themselves to learn how to weave or do cross-stitch. Nevertheless, the gender equity education that was introduced in the school might have gone some distance in explaining why girls were found in the carving class at all!

The interview was in the form of a semi-structured, focus-group session. The interview protocol was made up of the following main questions:

1. What do you consider to be important in mathematics learning?

2. What are important factors contributing to a poor performance in mathematics at school?
3. What do you consider to be important if one wishes to improve one's mathematics learning?
4. What do you consider to be important to be good at mathematics in school?

The interview session was audio-recorded, and the audio file transcribed thereafter to facilitate analysis by the researchers. The content analysis of the interview transcript was guided by a three-stage coding process that was conceptualised by Strauss (1987), made up of open coding, axial coding and selective coding.

Results

All the 8 student participants did not see the value of learning mathematics. They questioned the need to study the discipline, given that calculators and computers were so inexpensive and accessible these days. In addition, they also found mathematics to be boring. In fact, if given a choice, they would rather clean the toilet than do any mathematical activity! The indigenous students also shared that they could neither see nor imagine any relationship between – and the benefit of – mathematics learning on the one hand, and a sense of joy or any prospect of improving the quality of life on the other. Subsequently, mathematics teachers and mathematicians were not their role models. In fact, in the indigenous Taiwanese culture, singers were role models. The indigenous peoples were generally good at singing and dancing, and becoming a singer has since become an avenue to fame and riches, these being symbols of success in the indigenous culture. As such, for the indigenous student participants, they were learning mathematics because it was part of the school curriculum which they could not avoid. Otherwise, they could not see much connection, if at all, between learning mathematics and preparing themselves for a better life in the future.

The interview also invited the student participants to think about the important factors that would contribute to poor mathematical performance. This was a topic that brought out a lot of liveliness in the students, with the initial response being a fair bit of laughter and pointing to one another as being a living example of someone who did poorly at school mathematics. In fact, they seemed to be extra eager to contribute their opinions! They all agreed that a poor performance in mathematics implies lack of intelligence. In other words, mathematical competency equates intelligence. But what did intelligence or smartness mean to the indigenous students? When they were asked if they thought they were smart, they either looked down or simply avoided eye contact with the interviewer. Yet, when they were asked if anyone of them was good at something other than mathematics, such as dancing or singing, the students were quick to act out different poses and to voice mimic their singer idols. They didn't agree that one needed to be smart in order to be good at something. Smartness seems to have been engrained in their minds as being associated with schoolwork, of which mathematics was the most prominent. In this regard, none of the students considered that they would ever be intelligent. Not that any of them wanted to be intelligent, since it would be related to excelling in mathematics, which was in turn associated with being nerdy and not being cool.

But, what did it mean to them to be poor at school mathematics? All the focus group student participants attributed the reasons to "don't study" and "don't care". Student perception of lack of intelligence was to be a contributing reason. In the words of a male student, "because if you're not smart and not good at math, why care and why study? It won't change anything. Why waste time? Fool!" This response was met with signs of agreement (e.g. approving nods) by his peers in the focus group.

On the other hand, the student participants were able to articulate what could be done to help somebody who had performed poorly in mathematics. They emphasised the importance of providing the person with extra help at home. These out-of-school assistances would include the provision of the financial means to employ a private tutor and to participate in after-school programs. When the students were asked why they nominated *out-of-school assistance,* they referred to their observation that the "good" mathematics students in school had access to such support mechanism.

The student participants were also asked what would be valued when they considered the improvement of their own mathematical ability. All of them felt that mathematical ability can't be changed, and thus it can't be improved. One of the boys' parents had in fact told him that "math is not for us, not an indigenous thing, you know"! When the student shared this in the focus group, his peers went into a discussion on this with him, ending up with all of them agreeing that this was a real possibility. The students were then asked further if mathematical ability might become better if one studies harder, concentrates in class more, is more careful, and practices harder. They did not seem to think so, judging by the low groans we could hear, followed by silence while a few of them shook their heads.

What did the student participants consider to be important factors of being good at mathematics in school? Only the girls in the focus group responded that *studying hard* and *listening* carefully were important factors. However, all of them considered one's innate ability to be smart to be important, and that this was not gender-biased in any way. Attendance of after school programs was also valued by all the student participants.

Other values were also mentioned, and these referred to teachers and schools. In particular, being good at mathematics was also related to a valuing of school *effectiveness*, of teacher *competency,* and of teacher-student *relationship*.

One of the students had nominated as one of the important factors the assistance given by family members. It worth noting that the other students did not mention this valuing because they had considered it to be unrealistic in the context of the indigenous Taiwanese communities. Just like in many parts of rural mainland China, one or both parents was likely to have moved into the cities where there were greater employment opportunities, leaving behind the grandparents and young children in the tribal villages. Thus, in the indigenous Taiwanese communities, the major caregivers in families were often single parents or grandparents (Lee, Chao, Lee & Chang, 2011).

Discussion

It was striking to notice that even though the student participants did not feel that they were as intelligent as other peers, they were at the same time wanting assistance to support their mathematics learning. These students desired to learn mathematics and to do well in it. They appeared to place importance on – and value – *nature* (intelligence as a given), but one which reflects a belief in what Dweck (2007) called the growth mindset rather than a fixed mindset. That is, at the same time that the students valued innate *intelligence* as a condition for mathematical performance in school, their growth mindset has also led to a valuing of convictions such as *change*, *development* and *effort*, achieved through such means as out-of-school support structures such as home tutoring. While what one is endowed with might be important, what one tries to develop and accomplish with what one is endowed with also plays a role in defining how good one gets to be in mathematics learning. This reflects our understanding that what one values can be located along value continua, as we discussed above. Here, the students' valuing was positioned somewhere along the *nature / nurture* value continuum.

The students' valuing of *nature* was also reflected in their belief that mathematics is "not an indigenous thing, you know". We found ourselves asking what might (their) mathematics teachers do to alter this student belief. Assuming that a mathematics teacher does not subscribe to a similar value, current literature suggests that it is in the professional interest of the teacher to establish some form of values alignment in class (Seah & Andersson, 2015). Yet, the valuing of *nature* is probably shaped over time and rooted in the wider society, in this case the indigenous Taiwanese culture. It is not clear if teachers with the time and resources they have at hand would be able to align the values in their classes in ways which minimize further the valuing of *nature*, and more importantly from our perspective, how they might successfully do that. At the same time, it appears that values alignment processes within the classroom might benefit from parallel efforts at the institutional and societal levels. In other words, we challenge any notion that values alignment processes at the classroom level alone might be sufficient when we seek to shape students' valuing, and we can imagine the contribution at the institutional and the societal levels as well.

It is also of interest to note that the values that were identified by the indigenous students in this study were all mathematics educational ones (Bishop, 1996), these including *nature, nurture, effectiveness, competency, relationship, listening,* and *out-of-school support*. There was no (explicit) reference to mathematical and general educational values, even though we can imagine some student referring to the importance of *rationalism* (a mathematical value) or of *creativity* (a general educational value in many educational systems) in being good at the subject. Is this dominance of mathematics educational values due to the difficulty for (primary-aged) participants to identify other categories of values, or is it possibly specific to the indigenous cultures only?

Conclusions

Taiwanese students in general may be leading the world in their mathematics performance; with a mean score of 560, Taiwan is positioned joint third in PISA 2012 (OECD, 2013). However, it can be tempting for us to overlook that within these highly performing education systems, there are communities of students who might be underperforming, and whose qualities of lives in future might be at stake. The exploratory study on which this paper is based was conceptualized to provide baseline information on what were considered important by indigenous Taiwanese students with regards to mathematics learning, given that their underperformance in this subject has been a concern for educators and in the wider community. The researching experience has also provided us with invaluable ideas on how the next phase of this study, involving a greater number of schools and indigenous students, might be operationalized. Nevertheless, the qualitative nature of the study has meant that regardless of participant numbers, findings are understood in the context of particular schools and particular (groups of) indigenous students. Thus, the findings are not generalizable.

The analysis suggests that amongst the eight Grade 4 indigenous students at least, mathematics as a school subject was not valued for its own sake. There was the general valuing of *nature* (involving innate intelligence) in the context of doing well in mathematics. At the same time, there was also a valuing of *out-of-school support*, such as home tutoring and family support, although the latter was impacted somewhat by the realities of single-parent and grandparent families in the indigenous Taiwanese communities. The girls amongst the student participants also valued personal *effort*, further supporting the prevalence of what Dweck (2007) called the growth mindset amongst the indigenous Taiwanese students.

This raised for us a question of the extent to which gender (or gender roles) may regulate the indigenous students' valuing. As discussed briefly in the previous section, we are also interested to explore the extent to which the dominance of mathematics educational values (over mathematical and general educational values) might be culturally-related. We suggest here that future research studies might benefit from the parallel collection and interpretation of data from Han Chinese students in Taiwanese schools as well, to facilitate comparison.

As noted above, only a few of the student values were related to teachers. Indeed, Chen and Lee (2013) had reported that students in the Eastern region of Taiwan (an area with a high concentration of indigenous communities) placed lesser demands on teaching sequence and teaching support, when compared with their peers from other parts of Taiwan. We are thus interested in how teachers practicing in indigenous communities deal with differences in what they and their indigenous students value in mathematics and mathematics learning, and how their attempts at aligning the value differences might affect student valuing along continua such as *nature/nurture*. Of concern here is the observed belief amongst the indigenous Taiwanese students that doing well in mathematics is out of reach of the indigenous people, and as a result, they might have given up hope to study and to do well in mathematics. Future research into the role of teachers' values alignment practices in shaping what indigenous Taiwanese students value in mathematics and mathematics learning is expected to go a long way in helping us empower the younger generations of this group of people.

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Preparing Noyce Scholars in the Rocky Mountain West to Teach Mathematics and Science in Rural Schools

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Using Steele's work on stereotype threat among undergraduate students, we are conducting a study to reduce barriers and mitigate factors that may prevent students of color from completing dual majors in STEM and elementary education. This paper focuses on strategies for recruiting and retaining students of color into the Wyoming Interns to Teacher Scholars (WITS) Noyce program through formal and informal mentoring and summer internships. Drawing from the extant literature on mentoring and mitigating stereotype threat, we strategize about how to build programs that promote student resilience, academic success, and communities of trust.

Introduction

The **Wyoming Interns to Teacher Scholars (WITS**) program at the University of Wyoming (UW) is positioned to grow elementary teachers' mathematics and science content knowledge and strengthen their capacity to sustain STEM teaching and learning in rural schools through place-based education (Gruenewald, 2003) and culturally relevant pedagogy (Ladson-Billings, 2009; Leonard, 2008). In order to prepare and support the training of **30 total participants**, the WITS program consists of four components: (1) early recruitment activities with parents and students at regional high schools; (2) training and summer research internships during the sophomore year and scholarship support in the junior, senior, and fifth year of study; (3) mathematics tutoring and mentoring activities that include seminars and retreats with STEM professionals, elders, and community leaders throughout the program; and (4) continued support through professional learning communities (PLCs) and online focus groups (e.g., **blogs** and **Blackboard**) during student teaching.

Wyoming is the ninth largest state in the U.S. geographically, but it has the lowest population of any state with an estimated **582,658 residents**, or 5.8 people per square mile (U.S. Census Bureau, 2014). As such, many of the districts in the state (with the exception of Albany, Laramie, Natrona, and Sheridan) are rural. There are **48 school districts** in Wyoming with a teacher workforce of **7,115** and student enrollment of **89,009 children**. However, enrollment is expected to rise as the number of children under the age of five increased from 31,578 (6.3%) in 2003 to 38,455 (6.6%) in 2013 (U.S. Census, 2014). Thus, new schools and new teachers will be needed in this decade and decades to come.

The **primary goal** of the WITS program is to **recruit and retain diverse teachers** with a background in STEM in order to **increase the number of highly qualified** teachers of mathematics and science who are committed to serving high-need students in rural towns in Wyoming and beyond. The teacher workforce in Wyoming is largely white and female, mirroring the demographics in the United States. Little and Bartlett (2010) describe this workforce as bimodal in terms of age as young (generation Y) and white-haired (Generation X), white women teach primarily at the elementary level. African Americans and Latin@s are underrepresented in the teacher workforce along with immigrants and indigenous teachers of color (Little & Bartlett, 2010). As a result, STEM fields fail to benefit from their potential contributions and alternative perspectives as minority scholars and educators.

Job-related factors that contribute to attrition among all teachers include high-stakes testing, retrenchment of tenure, and increased pressure to improve student outcomes (Moore, 2012). Factors that contributed to fewer minority teachers in the profession have been attributed to inadequate preparation for college (Irvine, 1988), standardized testing for teachers (Leonard & Martin, 2013), and increased access and opportunity to pursue other professional careers (Madkins, 2011). The WITS program is designed as a model in the Rocky Mountain West to study and mitigate factors that deter and inhibit undergraduate students of color from entering STEM and STEM teaching. Placing content specialists who are teachers of color in elementary schools holds great promise for improving STEM education, increasing STEM interest in primary grades, and equipping underrepresented students to persist in STEM content as they progress to middle and high school. Studying barriers such as stereotype threat and mitigating factors by establishing mentoring programs to improve the recruitment and retention of underrepresented minority teachers in Wyoming will add to the literature base.

Rationale

When it comes to mitigating factors that deter students from pursuing STEM and STEM education, the notion of mathematics as a gatekeeper emerges. In order to do well in science, technology, and engineering, students must be literate and knowledgeable of mathematics—calculus in particular. However, school mathematics has traditionally been used as a means to stratify students (Martin, Gholson, & Leonard, 2010) and inadvertently steer underrepresented students away from STEM and STEM-related careers, including STEM teaching. According to Martin et al. (2010), research typically does not focus on "issues of identity, language, power, racialization, and socialization" in mathematics (p. 15). However, we must be cognizant of these issues and address them in programs that aim to recruit and retain students of color in STEM who have been largely underserved in K-12 schools. Efforts to mitigate factors that threaten the recruitment and retention of undergraduate students of color into STEM and STEM-related fields must include scaffolds to enhance their understanding of advanced mathematics. Furthermore, prospective STEM teachers must also be aware of their own mathematics identity as well as engage the children they intend to teach in ways that value their history, community, and place in order to develop mathematics identity in their students.

In the WITS program, we will develop **one-on-one and group mentorship programs** with the aim of creating an encouraging and rewarding environment for STEM undergraduate students. These programs will work toward mitigating systematic disadvantages and boosting self-efficacy, particularly in STEM education. Since stereotypes are systematic and institutional, individuals are often not fully aware of their power. An important first step will be the **formation of support groups** where students share their experiences and bring awareness to the harmful impacts of deeply entrenched biases. A crucial aspect of alleviating the impact of discrimination and prejudice is acknowledging it in institutional spaces and recognizing negative feelings that may result from long-term exposure to such situations. The support groups will create a safe and supportive environment for students to explore these feelings and mitigate their effect. Acknowledging stereotype threats and discussing their impact will help the students to actively combat these stereotypes and to reshape their self-image. The one-on-one mentorship will increase visibility of and access to successful minority scholars in STEM fields (e.g., National Association of Black Engineers, Society of Hispanic Professionals and Engineers, etc.). Such counternarratives will help students become more resilient and able to overcome obstacles. Participants will learn to see intelligence and efficacy as malleable, which require attention and nurturing in order to develop, grow, and expand.

The mentorship program will be an experiential learning process that will build cohesiveness in each cohort. It will cultivate peer-to-peer mentorship and build an additional layer of support for the students from STEM faculty and retired teachers to help mitigate the effects of stereotype threat and improve their academic performance. The program strives to develop a culture of mutual understanding, respect, and mentorship among the students that will extend beyond their current education and will build the foundation for improved STEM programs in Wyoming and elsewhere. The tutoring program will also provide the mathematics support needed for success in advanced mathematics and science content courses.

Our contribution to research and practice is empowering a cadre of elementary teachers with content, cultural, and place-based expertise. We will recruit, support, and prepare a diverse group of STEM majors who are more likely to have interest in teaching in high-need school districts, given characteristics of affinity with these districts (Holloway, 2002; Moore, 2012). We will specifically, target underrepresented minorities from indigenous communities in Riverton and Lander, Wyoming, which are near the Wind River Reservation, and African Americans and Latin@s from nearby Denver, Colorado. In order to prepare this cadre of professionals, we make equity "the central issue" in STEM education (Confrey, 2010) to eliminate deficit-based theories and hegemonic practices that have been pervasive in rural and poor communities of color. To address equity in this broad sense, we will provide WITS scholars with seminars to help them understand the systemic issues and shortcomings of the educational system at large "to provide fair and equal opportunities for large segments of the population" (Confrey, 2010, p. 26). Although academic success is predicated upon teachers having a high level of content and pedagogical content knowledge, learning in mathematics classrooms must be dialogic and include all students as partners in their own learning (Confrey, 2010). Thus, we will employ strategies in the WITS program that will arm elementary teacher candidates with strategies that inhibit their own learning such as racial micro-aggression and stereotype threat.

Literature Review

Underrepresented minority groups are disenfranchised by inadequate support systems and face explicit and implicit biases that negatively affect their academic performance, particularly in high-status courses like mathematics and science (Martin, Gholson & Leonard, 2010; McGee 2013). Stereotype threat results from negative perceptions regarding an individual's social group and often acts as limiting barriers for students in their academic endeavors (Block et al., 2011; Steele, 1997). This effect is especially magnified in engineering and science, which require years of rigorous training. In the case of underrepresented minority students, however, stereotype threat is often compounded by other factors, such as lack of adequate preparation for rigorous pursuit of mathematics and science, lack of supportive social networks that encourage students in their studies, and a systematic lack of financial resources. These various factors create a vicious cycle that denies generations of minority students the opportunity to pursue careers in mathematics and science and to realize potential social and economic upward mobility.

Claude Steele (2010) explained how contextual cues, such as being in an environment where there are few students or faculty of color, or where the curriculum marginalizes the experiences of students of color, are enough to trigger a stereotype threat that undermines performance. Steele (2010) contends that African American student underperformance is a national phenomenon that cuts across the curriculum but is especially prevalent in mathematics. Other groups experiencing this phenomenon include Latin@s, Native Americans, and women in advanced college mathematics classes and other courses. Steele suggests that small, feasible interventions can reduce these threats and dramatically narrow racial and gender achievement gaps. He offers practices that educators can employ to help neutralize these threats. Examples include facilitating faculty-to-student or student-to-student mentoring and cross-racial interactions.

Mentoring programs have become a vehicle to support the recruitment and retention minority students in higher education (Strayhorn & Terrell, 2007) and can be used to mitigate stereotype threat. However, all mentoring programs are not equal or focused on issues specific to African American, Latin@, and Native American students. Mentoring programs may be informal or formal as well as university or nationally focused. These informal or formal programs may include faculty-student mentoring, faculty-faculty mentoring, professional-student mentoring, and peer mentoring (Sandford, Armour, & Stanton, 2010; Strayhorn & Terrell, 2007). However, research findings reveal mentoring programs that lack clear goals and objectives and focus merely upon building informal relationships are less effective (Sandford et al., 2010). Strayhorn and Terrell (2007) found Black students rated mentoring experiences positively when they were connected to research-focused opportunities. Moreover, activities that involve the participants in some kind of physical activity that is interactive can provide benefits for both mentors and mentees (Sandford et al., 2010). In the WITS program, we intend to link faculty to an outdoor field-based internship to develop faculty-student mentoring. We also intend to pair students in peer-peer mentoring relationships to help them to build a sense of belonging (Ek, Cerecer, Alanis, & Rodriguez, 2010). Peer mentors can develop the following attributes: competence, confidence, connection, character, caring, (Lerner, Alberts, Jelicic, & Smith, 2006) and contribution (Karcher et al., 2010). Finally, professional mentoring is critical to providing a source of apprenticeship to help teacher candidates understand the role of teaching and job responsibilities. Thus, WITS scholars will also participate in professional-student mentoring (see Figure 1). Through these three types of mentoring experiences and academic and financial support, we will recruit and retain 8-10 WITS participants as dual STEM and elementary majors per year.

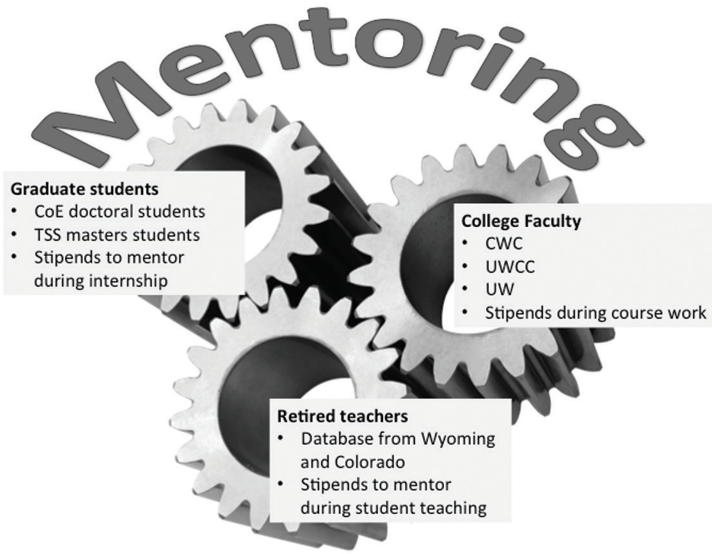

Figure 1. Mentoring model

Theoretical Framework

The theoretical framework that guides the WITS program is self-efficacy theory and the constructs are teacher efficacy and place-based education. Bandura (1977) developed what is now known as self-efficacy theory, which connects the predictive value of an event's success to the confidence that one has to perform it. Efficacy beliefs have tremendous influence over the course of action individuals may take, the amount of effort they will expend, how long they will persevere when challenged, how much stress they will experience, and the level of success they will reach (Bandura, 1997). Bandura identified two factors that affect teacher efficacy: personal efficacy and outcome expectancy. When applied to teaching, personal self-efficacy is defined as perceived judgment about one's ability to teach (Newton, Leonard, Evans, & Eastburn, 2012). Outcome expectancy deals with teacher perceptions of students' ability to learn what is taught. Additionally, Bandura (1997) asserted that there are four factors that influence efficacy beliefs: master experiences, vicarious experiences, verbal persuasion, and affective states. We will address each of these sources of efficacy beliefs through internships/coursework (mastery), seminars/conferences (vicarious), mentoring (persuasion), and mathematics tutoring/support groups (affective). Results of previous research have revealed teacher efficacy in mathematics and science is malleable among pre-service elementary teachers (Leonard, Boakes, & Moore, 2009; Newton et al., 2012).

In addition to teacher efficacy, another construct that undergirds the WITS program is **place-based education**. Placed-based education (PBE) is experiencing resurgence in U.S. schools as a supplement to traditional instruction (Aikenhead, Calabrese Barton, & Chinn, 2006; Akom, 2011; hooks, 2009). PBE can be traced to Dewey (1916) who contended that the experience students bring from out-of-school settings (i.e., place) should be incorporated into meaningful classroom activities (Long, 2009). According to Gruenewald (2003), "place matters to educators, students, and citizens in tangible ways that include providing teachers and students with 'firsthand experience' to link local contexts to learning environments in order to understand sociopolitical processes and shape what happens in the local community"(p. 620). Thus, place and culture are intertwined such that "to live is to live locally, and to know is first of all to know the places one is in" (Casey, 1996, p. 18). Through place, students learn about how the world functions and how their lives fit into the spaces they occupy, which can reinforce motivational ideas such as belonging, reclaiming space, and real-world relevance (Gruenewald, 2003; hooks, 2009). Wyoming's national parks (i.e., Grand Teton and Yellowstone) as well as its wildlife, geysers, and dinosaur fossils provide rich place-based contexts that can be incorporated into informal and formal learning environments.

While no single theory of place exists, place-based practices can be connected to experiential learning, outdoor education, indigenous education, environmental and ecological education, multicultural education, and other approaches that value specific places, locales, and regions where people live and work (Gruenewald, 2003). Underlying these approaches is the notion that pedagogical experiences should be connected to students' lives in meaningful ways (Gay, 2000). Science camps provide a place-based context for children and youth to develop "cognitive abilities to engage in STEM content and problem solving activities" (DeJarnette, 2012, p. 80). However, like other critical pedagogies, many teachers are at a loss regarding how to implement place-based pedagogy in everyday classrooms, especially in mathematics classrooms. We will provide WITS participants with rich experiences via authentic summer research internships to build a foundation to teach place-based mathematics and science education in high-need elementary schools.

The Wyoming Interns to Teacher Scholars' Study

Mathematics tutoring undergirds the WITS program. Success in mathematics coursework is critical not only for mathematics majors, but for engineering and science majors as well. Mathematics identity is formed by a belief in one's ability to do mathematics as well as the motivation and persistence to seek mathematics knowledge (Martin, 2000). There are times when underrepresented students are just not sure that they have done their homework correctly or may have a question about a problem and are too embarrassed to ask questions in the classroom because of stereotype threat. The WITS program will provide mathematics tutoring free of charge for WITS participants. Graduate assistants will be paid stipends to serve as mathematics tutors, offering **face-to-face** tutoring during designated hours, as well as through **telephone hotlines** and **online assistance.**

The following research questions are addressed in the WITS program to study recruitment and retention issues in STEM education:

- What are the barriers to success and mitigating factors for underrepresented minority STEM majors in the WITS program?
- What types of academic support are most effective in alleviating stereotype threat?
- How did the summer research internship influence WITS participants' teacher efficacy and retention in STEM?
- How did the tutoring/mentoring activities provided throughout the WITS program impact Noyce Scholars' teacher efficacy and retention?

Methods

Mixed-methods will be employed to collect and analyze data in this study. Quanti-tative methods will be used to measure change in WITS participants' perceptions of stereotype threat as well as their perceived teacher efficacy in science and mathematics teaching. We will use the Social Identities and Attitudes Scale (SIAS) to measure the degree to which participants are susceptibility to stereotype threat (Picho & Brown, 2011). Additionally, we will use the Science Teaching Efficacy Belief Instrument (STEBI, Enochs & Riggs, 1990) and the Mathematics Teaching Efficacy Belief Instrument (MTEBI, Enochs, Riggs, & Huinker, 2000) as pre-post measures to determine changes in teacher efficacy belief overtime. Qualitative methods will also be used to gather data from field notes, teacher journals, and focus group interviews to determine the impact of the summer internship and mentoring/tutoring programs on retention in the WITS program.

Significance of the Study

The WITS program develops a model that can be replicated in the Rocky Mountain West and elsewhere to address the recruitment and retention of STEM professionals of color, particularly in the education workforce. While the program is in the initial stage of recruiting, responses to our model of building communities of trust and empowerment, managing stereotype threat, and promoting mentoring relationships set us apart from other programs. The findings are mixed concerning the impact that peer-mentoring programs have on mentees as well as the mentors themselves. We engage in social justice by making equity central and by becoming an agency of transformation as we endeavor to change the educational landscape for K-6 students of color by preparing diverse teachers of color in mathematics and science who will learn to embed culture, place, and social justice in mathematics and science content to improve outcomes in high-need classrooms.

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Whose Mathematics? Our Mathematics! A Comment on Mariana Leal Ferreira's Book, *Mapping Time, Space, and the Body*

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In her paper in these proceeding and her new book, "Mapping Time, Space, and the Body", Mariana Leal Ferreira enchants the reader with the creative role that mathematical thinking plays in the lives of Indigenous peoples in Brazil. The Indigenous applications of mathematics make a good starting point for school mathematics. However, the false idea that school mathematics is "white" mathematics remains largely unchallenged. In this commentary, I provide some sources for the multicultural roots of school mathematics.

Mariana Ferreira's paper described some unique, original, and diverse mathematical ideas of Indigenous peoples of Brazil. Her book, "*Mapping Time, Space and the Body: Indigenous Knowledge and Mathematical Thinking in Brazil,*" develops these ideas in greater depth. It was thrilling to feel the pride expressed by Indigenous teachers as they pushed back against racist statements that "Indians could not do mathematics." But it made me sad to read that school mathematics was called the "whites' mathematics." How did we come to believe such a false version of the history of mathematics?

In the colonial period, vast stores of wealth were stolen from the peoples of the Americas, Africa, and Asia. Among the most precious possessions stolen was the people's history. In fact, the whole history of culture and civilization was rewritten to justify colonialism, slavery and modern imperialism (Bernal, 1987, pp. 1-2). The rewritten history claimed that white men in Europe were the founders of mathematics, science, art, culture, law, and philosophy. This version of history served the imperialists who claimed that whites were superior and people of color inferior.

It may sound crude but this false version of history is still taught today. In the case of Africa, the motivation for rewriting history was explained by the great African American scholar, W. E. B. DuBois, back in the 1940s. The Europe-centered "rationalization" that DuBois exposed turns the true history of mathematics on its head.

> The rise and support of capitalism called for rationalization based upon degrading the Negroid peoples. It is especially significant that the science of Egyptology arose and flourished at the very time that the cotton kingdom reached its greatest power on the foundation of American Negro slavery (DuBois, 1974, p. 99).

I believe the study of ethnomathematics shows that all peoples have created mathematics. Would it not be reasonable, then, to look for the origins of mathematical thinking in Africa, the birthplace of the modern human species?

Recently, a few archaeologists have looked in Africa for early evidence of what is called "modern behavior." Modern behavior includes abstract thinking and symbolic representation, essential features of mathematical thinking. The standard history taught today places the origin of modern behavior in Europe (Southern France and Spain) 30,000 to 40,000 BP (before the present era). But archaeologists, digging in Africa, have found much earlier evidence of modern behavior. The artifacts they found included design work and a saw-toothed harpoon that date back to the Middle Stone Age (MSA), 200,000 BP to 40,000 BP. These artifacts root the origin of modern behavior in Africa, before *Homo sapiens* spread to Europe (Lumpkin, 2008).

Even the numerals used in world commerce are not "white" or European. Modern numerals were developed in India with additions by Islamic mathematicians in Asia and Africa. In the US, K-12 school mathematics rests on a foundation created by African and Asian mathematicians, not "whites". Some examples are listed at the end of these comments. Also listed are some theorems developed by Asian and African mathematicians long before they were known in Europe. These theorems have been renamed and are now known by the names of European men. That practice adds to the false idea that mathematics is a "white" subject, rather than the heritage of all humanity.

Amerindian Contribution

What contribution did the peoples of America make to the development of mathematics? I believe that the Indigenous peoples of the Americas made a huge contribution. For example, the Olmec and Maya civilizations of Central America developed a positional numerical system including a zero position holder. That may have been the earliest zero symbol ever used in a positional-value system of numerals. In South America, the Aymara and Inca civilizations also developed the science of astronomy, requiring extensive calculations.

Their knowledge of mathematics and science made it possible for these civilizations to create the great stores of wealth stolen by the European invaders. This stolen wealth provided major funding for the European Industrial Revolution. In turn, the Industrial Revolution led to a great expansion of European science, mathematics, and other branches of knowledge. As Karl Marx put it in *Capital,* on the sources (accumulation) of European capital:

> The discovery of gold and silver in America, the extirpation, enslavement, and entombment in mines of the Indigenous population of that continent, the beginnings of the conquest and plunder of India, and the conversion of Africa into a preserve for the commercial hunting of black-skinned people are all things which characterize the dawn of the era of capitalist production. These idyllic proceedings are the chief moments of primitive accumulation (Marx, 1967, p. 751).

And further, on the birth of European capitalism:

> …capital comes dripping from head to foot, from every pore, with blood and dirt (Marx, 1967, p. 760).

The potential impact on mathematics education of a corrected, truthful version of the origins of school mathematics is enormous. My personal experience in high school and community college teaching has reinforced this conviction. Combined with knowledge of ethnomathematics, it can lead to a truly multicultural mathematics curriculum. Relating instruction to the many cultures that produce mathematics can improve the learning of mathematics and increase educational equity. In this quest, Ferreira's new book, *Mapping Time Space, and the Body,* makes a valuable contribution.

Some African and Asian Roots of Mathematics

Base 10 numerals	Egypt, 3200 BCE
Cipherization of numerals	Egypt (Boyer, 1944)
Positional value numerals	Mesopotamia, China, India
Right-triangle theorem	Babylonia, 1900 BCE
Proofs of right-triangle theorem	China, India, before Pythagoras
Chinese triangle	300 years before Pascal's birth
Formulas for areas and volumes	Babylonia, Egypt c. 1900 BCE
Debate, litigation, deduction	Ancient Egypt, 1850 BCE
Decimal fractions	Syria, 952 CE
Algebra	Mesopotamia, Egypt, India
Cubic equations, trigonometry and non-Euclidean geometry	Central Asia, c. 1000 CE to 1400 CE
Combinatorics	Islamic–Hebrew, North Africa, Spain (Katz, 1993, p. 277)
Infinite series, early calculus	Medieval India (Joseph, 1991, pp. 289-94)
Zero concept	India (Joseph,1991, pp. 241–43), Ancient Egypt (Spalinger, 1985)
Infinity concept	Ancient Egypt[1]

Some Re-Named Theorems

Babylonians knew that the angle inscribed in a semicircle is a right angle. It is now called "Thales' Theorem".

The Chinese triangle is called "Pascal's triangle".

The right triangle theorem was known in Babylonia 1100 years before Pythagoras, and "proved" in China and India 300 to 600 years before Pythagoras.

The Chinese solution of systems of linear equations is called "Kramer's rule".

The second-degree indeterminate equation of Brahmagupta, c. 625, is called "Pell's equation" after John Pell (1610-85).

Nasir-al-Din's quadrilateral is called the "Saccheri quadrilateral" (Boyer, 1968, p. 111).

The root extraction method developed in 3rd century China is called "Horner's method" (Li & Du, 1987, p. 111).

Decimal fractions are credited to Stevin although al-Uqlidisi used them c. 950 (Berggren, 1980, pp. 37-39).

Madhava's series from Kerala, India (c.1400) are known today as the "Gregory series" for arctangent, the "Leibniz series" for pi, and the "Newton power series" for sine and cosine (Joseph, 1991, pp. 289–294).

Notes

1. Ritner, R. K. (Personal email 6/10/13. Quoted with permission). "You are correct that the word "heh" originally meant "million." However, its usage in actual counting fell into disuse before the New Kingdom (Gardiner, Egyptian Grammar, 3rd ed., p. 191) and was replaced by multiples of "hefen" (10,000). As a result, the word "heh" became a term attached to objects signifying "a great number without end" (Woerterbuch der Aegyptischen Sprache, vol. III, p. 158: "unendlich grosse Zahl/ unendlich langen Zeit." This is often inexactly translated as "many" or "millions and millions."

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Learning Mathematics through Birch Bark Biting: Affirming Indigenous Identity

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Focused on supporting mathematics learning in Mi'kmaw communities in Atlantic Canada, this research program examines the implementation of culturally-based inquiry units in Mi'kmaw schools. Drawing from the traditional community practice of birch bark biting, this article demonstrates how students (and their teacher) were invited to learn together, exploring as aspect of Mi'kmaw culture while also connecting this knowledge to relevant mathematics outcomes. The project uses an indigenist methodology to explore the impact of such pedagogical approaches on mathematics teaching and learning in Mi'kmaw schools. Stories of students learning mathematics while birch bark biting are shared.

Context

As Canada's Aboriginal communities re-establish their self-government and self-determination, there is a need to develop sustainable economies and manage natural resources within the context of a growing population and insufficient infrastructure. Aboriginal leaders want the younger generations to acquire the knowledge and skills to address these challenges. Such capacity-building requires adequate education; currently, however, too few Aboriginal students are choosing to pursue studies in essential skill areas, such as mathematics and science. With growing attention on the academic achievement of Aboriginal children, local education authorities and federal and provincial governments are exploring strategies to enhance the learning experiences of Aboriginal youth.

It has been argued that disengagement from mathematics emerges as a result of the conflict between Aboriginal culture and the cultural values embedded in school-based mathematics programs (Cajete, 1994; Hankes & Fast, 2002). The marginalization of Mi'kmaw youth from mathematics has been a long-standing concern in Mi'kmaw communities and this concern extends to many Aboriginal communities across Canada. The Minister's national working group on education (Indian and Northern Affairs Canada, 2002) stated that a key area to be addressed in Aboriginal education in Canada is the development of culturally relevant curricula and resources in areas of mathematics and science. Lunney Borden (2010) has shown that a disconnect between school-based mathematics and Mi'kmaw ways of reasoning mathematically can impact mathematics learning for Mi'kmaw students. The author has argued that the lack of attention to value differences, the use of inappropriate pedagogical strategies, and a failure to attend to ways of knowing embedded in Indigenous languages can make mathematics learning problematic for Mi'kmaw students. As a result, many children choose to opt out of mathematics because the cost of participation is too high, demanding that they deny their own worldview in order to participate in the dominant view of mathematics. Doolittle (2006) has elaborated on this cost of participation for Aboriginal students having cautioned "as something is gained, something might be lost too. We have some idea of the benefit, but do we know anything at all about the cost?" (p. 19).

Mi'kmaw Kina'matnewey (MK), based in Nova Scotia, Canada, stands out as an example of a Regional Aboriginal Education Authority that is experiencing success. These schools boast a graduation rate that in the past 5 years has been between 87.0% and 89.3% (Mi'kmaw Kina'matnewey, 2014) in stark contrast to the often-reported Canadian national graduation rate for Aboriginal children of about 48% (Assembly of First Nations, 2010). MK schools are striving to meet curriculum expectations while maintaining a strong sense of Mi'kmaw cultural identity. One example of such cultural integration is in the area of mathematics. Since 2007, thousands of Mi'kmaw children have been participating in Show Me Your Math (SMYM), an annual program that invites them to investigate the mathematics in their own community context. The SMYM program showcases the mathematics inherent in Mi'kmaw culture through celebrating student projects from all grade levels.

SMYM was developed by myself and Dr. David Wagner along with teachers in MK schools in an attempt to address the marginalization of Mi'kmaw youth from mathematics by helping them to see that there is a considerable amount of mathematical (and scientific) knowledge in their own community heritage. One of the benefits of the program we have seen is the development of a sense of "Wholeness [that] resists fragmentation" and creates "quality mathematics experiences [that] require cultural synthesis bringing together cultures and values from mathematics and the community, personal holism including the child's experiential, conceptual and spiritual development, and intergenerational interaction." (Lunney Borden & Wagner, 2011, p. 379)

In recent years, SMYM has become increasingly more popular with MK students and their teachers, however curriculum pressures have resulted in a call for closer alignment with provincial curriculum expectations so that teachers can integrate SMYM projects into their classroom practices while still addressing specific curriculum outcomes. In an effort to address this expressed need, I had began working with teachers and elders to build units of work from the ideas in SMYM student projects. Inspired by Doolittle (2006) who has suggested it would be helpful to "consider the question of how we might be able to pull mathematics into indigenous culture rather than how mathematics might be pushed onto indigenous culture or how indigenous culture might be pulled onto mathematics" (p. 22) I have aimed to begin with community practice as a starting point.

As a result of recent funding obtained for the expansion of SMYM, we were able to develop community-based inquiry units that begin within community practices. These projects build upon Doolittle's (2006) idea of pulling in mathematics by beginning in aspects of community culture where the already present, inherent ways of reasoning within the culture can help students to make sense of the "school-based" concepts of mathematics in the curriculum. One goal of this work is to have teachers and students learning alongside one another as they explore practices that are relevant to the community. As such, these projects have been called Mawkinumasultinej! Let's Learn Together! as a way of emphasising this focus on learning together. Many projects have been initiated in schools such as eels and eel fishing, quill work designs, paddle-making, and bead work. In this paper I will share stories from the Birch Bark Biting units done with students in grades 5, 6,and 7 and describe how these units have impacted learning for both the teacher and the students.

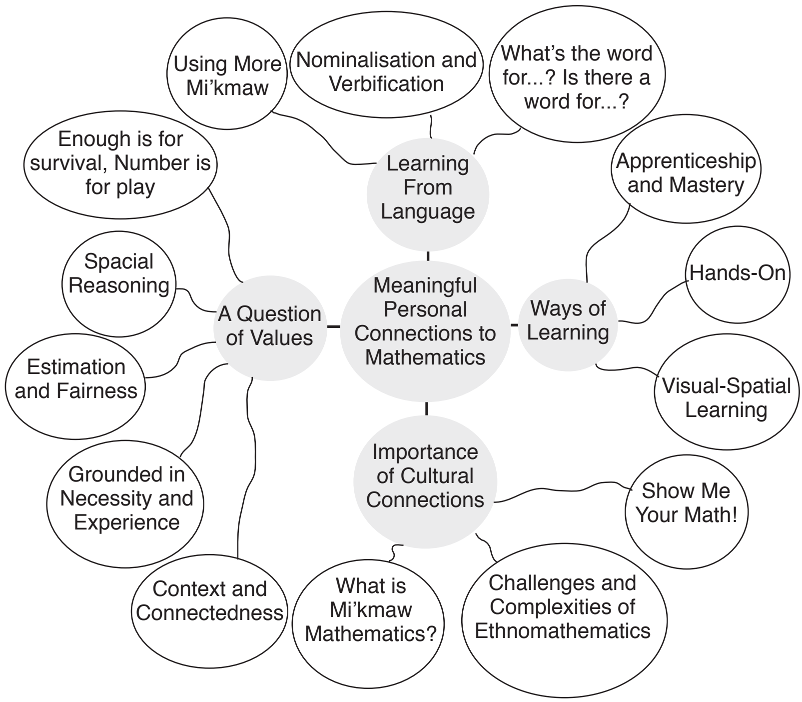

Figure 1: A framework

Building from a Framework

I developed a framework for transforming mathematics education during my doctoral research (Lunney Borden, 2010). The framework can be seen in the model below (see Figure 1) that emerged from conversations with teachers and elders in two Mi'kmaw community schools as we discussed the issues and complexities that arise in mathematics teaching and learning in Mi'kmaw schools. Four key areas of attention for transformation emerged as themes: 1) the need to learn from Mi'kmaw language, 2) the importance of attending to value differences between Mi'kmaw concepts of mathematics and school-based mathematics, 3) the importance of attending to ways of learning and knowing, and 4) the significance of making ethnomathematical connections for students. Each of these will be explained in more detail below.

Learning from Language

Although interconnected, each of the themes can be linked to the idea of learning from language, which emerged as an overarching theme in this work. Examining the Indigenous language of a given community context would provide a starting place for transforming mathematics teaching and learning. Given that the ways of thinking are embedded in Indigenous language, it can be helpful for teachers to understand how the language is structured and used within the community.

It may be helpful to ask questions such as "What is the word for…?" or "Is there a word for…?" to better understand how mathematical concepts are described in the language (See Lunney Borden, 2012). Gathering words that can be used to describe mathematical concepts, provides insight into concepts that may prove to be potential strengths for building a mathematics program. Similarly, awareness of mathematical concepts that have no translation in the Indigenous language exposes the taken-for-granted assumptions that are often present in existing curricula.

Understand the underlying grammar structures of an Indigenous language can also support teaching and learning. The prevalence of nominalisation in mathematics stands in direct contrast to the verb-based ways of thinking inherent in the Mi'kmaw language (see Lunney Borden, 2011). This is an important issue for teachers to consider. Looking to "verbification" as an alternative may help to create a more engaging and rich curriculum for Indigenous learners.

A Question of Values

It is also important for educators to think about how mathematical ideas are used and valued in the community context. It is important to understand how numerical and spatial reasoning emerge in the context of the community culture. This study has shown that spatial reasoning was highly valued as it pertained to matters of survival. Numerical reasoning was seen as useful in play. If we consider mathematics to be about examining quantity, space, and relationships (Barton, 2008) then it becomes important to build learning experiences that value these concepts in a way that is consistent with, rather than in opposition to, the way these concepts are valued within the culture. There is a need to build mathematics learning experiences for Mi'kmaw students from a basis of spatial reasoning.

Ways of Knowing

Language and values also influence the preferred ways of learning in any community context. It was evident in this community context that a mathematics program should provide children with opportunities to be involved in learning focused on apprenticeship with time for mastery, and hands-on engagement with concrete representations of mathematical ideas. Furthermore, building from a valuing of spatial reasoning, a mathematics program should place visual spatial learning approaches on equal footing with the already dominant linear-sequential approaches, providing more ways to learn so that more students can learn.

Cultural Connections

In addition to community language, values, and ways of learning being included in a mathematics program, it is also essential to make meaningful and non-trivializing connections to the community cultural practices. This involves examining how the school-based mathematics can be pulled in through identifying types of reasoning inherent in the community that can help students to make sense of the school-based mathematics. It also means creating learning experiences that help students see that mathematical reasoning is a part of their everyday lives, and has been for generations. The success of Show Me Your Math suggests that inviting students to be mathematicians who investigate mathematics in their own community contexts could also be an important component of a culturally based mathematics program.

Methodology

It is appropriate to first situate myself and explain my role in this work. Although I am not Mi'kmaq, I lived and worked within a Mi'kmaw community for 10 years and during that time I learned the language and culture and become engaed in the overall life of the community. In 2005, when I left the job I loved to begin my doctoral studies I did so with a sense of sadness for leaving but a commitment to continue to work tirelessly alongside community members to improve the educational experiences of Mi'kmaw children. Yet within the academy, I was constantly being questioned as to why I felt it was my place to do such work and I even began to doubt myself. At that point I returned to the community that had become my home to seek advice from respected community members.

I shared my concerns about my role in this work in a conversation with two Mi'kmaw colleagues. Their responses were reassuring. One colleague spoke about the time I had spent in the community, the way I had learned the language and the culture, and assured me that he knew that I had come to work with the community. The other jokingly asked me if I wanted to quit now. His teasing was a way of reminding me that not only did I have the privilege to do this work, I had an obligation to do it. They had shared with me the language, the culture, the ways of knowing and being; they gave to me and now I was in a position to give back in a way that honoured the community. The work I do now honours that commitment.

Understanding the complexities of doing research in Aboriginal communities I worked to mitigate the potential harms, by utilizing an Indigenist methodology. The term Indigenist research was first introduced by Rigney (1999) and is based on three interrelated principles: resistance, political integrity, and privileging Indigenous voices. Adopted by Indigenous and non-Indigenous scholars, it is seen as a paradigm for decolonizing research (Denzin, 2005). This approach "research[es] back to power" (Smith, 2005, p. 90) and holds a "purposeful agenda for transforming the institution of research, the deep underlying structures and taken-for-granted ways of organizing, conducting, and disseminating research and knowledge" (p.88).

Kovach (2009) has argued that "Indigenous methodologies, by their nature, evoke collective responsibility" (p. 178). Furthermore, Kovach has stated:

> Specific responsibilities will depend upon the particular relationship. They may include guidance, direction, and evaluation. They may include conversation, support, and collegiality. Responsibility implies knowledge and action. It seeks

> to genuinely serve others, and is inseparable from respect and reciprocity. (p. 178)

This research is guided by mawikinutimatimk (coming together to learn together) a Mi'kmaw term used to describe the process of people coming together to discuss an issue or solve a problem (Lunney Borden & Wagner, 2013). It implies that everyone comes to the table with gifts and talents to share—everyone has something that they can learn. It conjures an image of a community of learners working in circle where all members are equally important and necessary. Each participant who joins in the circle has something unique to contribute. With mawikinutimatimk there is an embedded understanding that the importance of relationships and the interconnectedness of participants must be honoured. It was through relationship that the birch bark biting unit idea emerged.

Stories of Birch Bark Biting

The idea for the birch bark biting inquiry project emerged somewhat unexpectedly. I was meeting with two elders to discuss some of the projects students had done at the annual math fair to think about ways these might be expanded upon to create classroom teaching ideas. As she spoke with one elder, Josephine, they discussed games that might have been traditionally played by children. The intent of this discussion was to build from games to possible counting and quantity concepts, however this suddenly changed when Josephine declared "You know, when I was young, my mother would peel thin strips of bark off the logs and ask us if we could fold it and bite shapes into it." Aware of the art of birch bark biting in other cultural communities in Canada and intrigued by the mathematical reasoning that would be needed to bite shapes into folded bark, I excitedly asked Josephine more about this. Josephine shared that she recalled some people doing birch bark biting as a pass time when she was young but was unsure if anyone still was able to do it.

She showed me how to fold paper to model how one might fold bark for biting, always folding through the centre. We folded the bark in half and then folded it in half again by folding the original seam onto itself. I asked her if there was a word to describe that action of folding and she replied, "Yes, Tetpaqikatu!" I asked her what that meant and she laughed and said, "Fold it the right way." After sharing ideas and looking up some birch bark biting pictures online together, she told me I should try to learn more about it. So I did.

In following up on the conversation I came across an article that demonstrated that birch bark biting was indeed a historical part of the Mi'kmaw community:

> That she was "the last one that can do it" was the same phrase echoed in 1993 by Margaret Johnson, an Eskasoni Micmac elder from Cape Breton. Continuing research has revealed that two other Micmac women – including Johnson's sister on another reserve – can also do it. (Oberholtzer and Smith, 1995, p.307)

I had known both Margaret, who was affectionately known as Dr. Granny, and her sister, Caroline Gould, who had resided in the community where I had taught and was a well-respected elder who often visited the school. Unfortunately both women had already passed away but both had been highly respected elders in Mi'kmaw communities known for the commitment to language and culture. This inspired me to share what I was learning with teachers and students and by learning together, birch bark biting was revived in Mi'kmaw communities.

Birch bark biting involves folding thin pieces of bark and biting shapes into the bark to create designs. The act of folding the bark presents an opportunity for students to think about fractions, angles, and symmetry. Creating the designs draws in geometric reasoning and visualization of geometric shapes. In our work with students, we have seen a deepened understanding of fractions as part of a whole, of geometric properties of 2D shapes, of symmetry, and of transformational geometry. In fact almost all of the grade 5, 6 and 7 curriculum outcomes for geometry can be discussed when exploring birch bark biting, however we did not begin with the outcomes, we began with the birch bark bitings.

I introduced the unit by sharing the story of the conversation between I had with Josephine. I showed students pictures and videos of birch bark bitings I had found online and shared the story of discovering the article describing the two Mi'kmaw women who had been birch bark biters - women the students themselves knew. I also demonstrated to students what I had learned from Josephine about folding and even brought a few of my own first attempts at creating birch bark bitings. In sharing my story of learning with the students, I invited them and their teacher to learn along with me. We would learn together.

As a group we explored creating our own birch bark bitings and some students found an instant connection to this work while others were initially a bit reluctant to want to bite the bark. Yet soon all students became very engaged in the task, spending time with the bark, working on bitings outside of school time, persistent in their desire to explore new patterns and figure out different designs. As their teacher shared "even the most reluctant ones would keep trying." Initially, as the students were biting the bark, there did not seem to be an intentionality on what they were making, they were simply exploring various bites and examining what resulted when they unfolded the bark. Questions naturally emerged when students spent time working with the bark. For example one student wanted to be able to create an eight-point star. By working on through a series of trials, exploring various angles to make the biting had discovered what to do so that, when unfolded, the eight-point star would emerge. Holly explained that she observed him develop a sense of ownership over it once he figured it out and was then able to create these patterns with intention.

Other questions that emerged took us beyond the creation of the design to wonder about how and when to collect bark, and what would happen to the tree if we were to remove the bark at inappropriate times. Through research we learned that there are two times in the year when Mi'kmaw people would have historically taken bark from a birch tree, late summer and again in early spring when the sap is running. Given that it was not in the appropriate season for gathering bark, we worked with bark that had been previously collected. The bark we used actually emerged in rather serendipitous ways. A community member brought us a large container of bark that he had collected from felled trees that were being logged on the nearby mountain. He shared that he had collected it for Dianne and had it at home for some time. The Dianne he referenced was the late Dianne Toney, who had been a quill box maker and the inspiration for the SMYM program (see Wagner and Lunney Borden, in press). Sadly, Dianne had passed away before the SMYM program began, yet this moment reminded me of her influence on this work. Another batch of bark came from the classroom teacher who, after a birch tree fell on her home during a storm, had kept pieces of the tree believing one day she would create something from them.

Culturally-based Learning

The birch bark biting provided an opportunity to talk about mathematics as an active and dynamic process rather than a fixed and frozen set of facts. I have argued that "verbifying" mathematics, focusing more on action and process, can support students who are thinking more in verbs. With the birch bark biting one could ask about lines of symmetry, the core of the pattern, the fractional pieces and so on, but what proved to be more effective was to shift to discussing how these designs were created. Questions such as "Where was it folded?" or "What did she bite?" or "How many pieces are we creating?" brought students to the same understanding in a way that was more focused on action and process. Students were able to then connect folding it onto itself to the lines of symmetry and the initial shapes that were bitten to the core of the pattern. There was also recognition that this core was flipped and turned to create the design.

One additional benefit of doing this project was to provide students to learn about elders in their own community who had been birch bark biters. It allowed these students to appreciate the knowledge that had been needed to do such complicated designs and to see that their elders were mathematicians too, as they too had used geometric reasoning to create their bitings. Many teachers in the school, upon seeing the designs, recalled having seen Caroline Gould making these bitings at her basket shop when they were children.

Through participation I this activity I saw students, and their teacher, engaged in meaningful learning that emerged in an authentic context. The students showed pride in having learned a nearly lost community practice. They discussed their designs and were able to talk about how they made them referencing mathematical ideas as they did so. They debated whether the core was one-eighth or two-eighths of the pattern with some students arguing that the original biting is made up not only of the top bites but the bottom bites as well. Birch bark biting provided an opportunity to learn in a way that valued wholeness and enabled students to have meaningful connections to the mathematics they were learning.

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A Meta-Research Question about the Lack of Research in Mathematics Education Concerning Students with Physical Disability

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This paper focuses on a "meta-research question" stemming from the first author's PhD research on difference and inclusion/exclusion in mathematics education, namely what is the nature of the relative silence of research in social justice in mathematics education concerning people with physical disabilities. Faced with the paucity of such research, Renato decided to include this questioning in his PhD research where he argues that disability/abnormality is an invention of the normality, inspired by Edward Said who says that the Orient is an invention of the Occident. Renato Marcone approached a few mathematics educators to discuss this issue with them. Bill Atweh, a researcher with publications on social justice spanning several years, agreed to enter into a dialogue with Renato as joint reflections on this question. This paper reflects this dialogue.

Presentation

Renato's inspiration for focusing his PhD research on physical disability in mathematics education came from his experiences in teaching mathematics to blind and deaf students since 2005 and undertaking a masters degree on the topic (Marcone, 2010). Renato commenced his PhD research in 2011, and in 2012 he read Edward Said, Frantz Fanon, Homi K. Bhabha, and Paulo Freire after reading researchers in his own research field (*Mathematics Education, Difference and Inclusion*) such as Lulu Healy, Miriam Penteado, and others. From these authors, Renato developed an understanding of disability as the invention of *normality* (Marcone, 2015) inspired by Said (1979) who, in his book *Orientalism*, argued that the Orient is an invention of the Occident. Renato differentiates between impairment and disability. While one can take an impairment to be a physical condition, disability is the result of having to operate in a socially constructed world and is a reference to how the impairment is seen by others in the world – as abnormality or disability. More often than not, such constructions and viewpoints are from people who do not suffer the impairment – and hence are considered "normal". While the terms abnormality and disability may not be standard in academic and public discourse in some contexts, their deliberate use here is to highlight their pejorative connotations. Incidentally, this understanding matched the views of the participants in the research who almost unanimously agreed that impairment should not necessarily become a disability (Marcone, 2015).

Renato was reading works on social justice and mathematics education, eager to find literature that might inform his work and provide its theoretical framework. He could not find much help in the literature, and then started to ask himself "why?". Instead of trying to raise some hypothesis or speculations, Renato decided to ask directly some authors with backgrounds in research on social justice and mathematics education. This paper reports on the conversation between Bill Atweh and Renato Marcone.

The trigger questions that Renato brought to the dialogue that was started were: Is there a relationship between what you call social justice and the teaching of mathematics for people with disability? Do you agree that there is a lack of research on this issue? Why do you think this group is not often mentioned as a minority in mathematics education publications? The asynchronous dialogue occurred by email. Renato's contribution to the dialogue is written in the first person, bringing Bill's ideas to the conversation as appropriate.

A Conversation about Inclusion, Exclusion, Mathematics Education, and Social Justice

This question came to my mind because I was interested in understanding what makes a minority eligible to be a focus of research, particularly in an area that claims to be seeking social justice for all students. I will start with Bill's first reaction to my invitation to participate in the dialogue. He wrote:

> First I want to thank you for this opportunity for exchange about this very important and worthy question you are investigating. More importantly, the question that you are raising is a question about our own practice as researchers. It is what I might call a meta-research question that we, as a mathematics education community, should engage more often. You are asking, if I understand your email right, *why is there relative silence in the literature about this and possibly other groups of people that are left behind in mathematics education*. Second, let me say that I consider this exchange as a dialogue between us and not as a definite answer to your question.

Bill said my question made him reflect on what are the social justice questions that we raise in mathematics education. The question of gender came to his mind as perhaps one of the first questions that the community raised a few decades ago, although in the past it was not constructed as a social justice issue, but an equity issue. In his previous writings, Bill suggested that social justice is a more general and powerful term to talk about equity/diversity issues. More recently, he had suggested that ethics is even a more comprehensive term that one can use to understand exclusion, and more importantly, points to the obligation to work towards inclusion.

Indeed, I often ask myself "what does social justice mean?" since, it appears to me that we do have to define what justice is and then pursue social justice. However, as is widely believed, justice is not easy to define, as its use depends on the context in which it is used. Consulting a dictionary of philosophy (Abbagnano, 2007), I found that a classic definition of justice could be understood as an adaptation to the current socially accepted standards or, in other words, simply following the law, as Aristotle would say. Also, Hobbes argued that justice is to keep the social pacts under the coercive power of the State, meaning that without the State there are no social pacts and thus there is no justice or injustice. Still, justice, in this sense, is only following standard pre-established norms, and it is not difficult to find many other examples that justice is only about legal status. A second possibility, accordingly to the same dictionary, is that justice is the efficiency of a norm, meaning whether or not this norm makes possible the aspired relationships among a community. The second understanding of social justice necessarily involves politics in its construction of social justice as an arena for the struggle and the pursuing of interests and rights.

Bill replied that the suggestion that knowing what something is before working towards it, "is an interesting one," and he agrees with me saying any construct is controversial and does not lend itself to being settled once for all. However, Bill said, "We still have to act!" meaning we cannot wait for the definition to be settled before acting justly one with another. This is a point made by Ole Skovsmose (2004), that we have no option but to act responsibly towards one another, a concept Bill has taken as an argument for ethics, from Levinas, that is not based on rules and regulations but on an encounter with the other that is not based on, and precedes, knowledge.

Bill also said that we (i.e., the mathematics education community) have considered questions about poverty (mainly in developing countries), ethnicity, cultural diversity, gender, and indigeneity. These have become mainstream in the sense that there is some theoretical and empirical work done on them and they constitute identifiable strands in publications and conferences. A problem about this, according to him, is that, in any attempt to itemize social justice issues/concerns, one is always open to being questioned "but what about this or that"? The point here is that physical disabilities has not become mainstream in the above sense. The question is *"why?"* Having said that, there is a research strand in general education about inclusive education that includes research on the blind, deaf, and similar physical disabilities (of course, the language – in terms of what to call these groups –is always contested and changing). However, there are specific issues concerning mathematics education that are not addressed in this strand in general education literature. For example, we know little about the development of didactical material and/or strategies to teach mathematics (including advanced mathematics) for blind and deaf students to enable them to deal with mathematical situations in everyday life and pursue higher mathematics.

Bill recalled "a very embarrassing recent incident" (in his words) where he was giving some examples about the concept of inclusivity in a talk about "Productive Pedagogy", in the Philippines. Bill mentioned the usual list of social justice concerns – for example, gender, culture, ethnicity, and indigenous education. At the conclusion of the lecture, the inclusive educator of the University approached him, and gently commented that we always forget blind and deaf students in our discussion about mathematics education. Bill stated he was very thankful for the reminder and started thinking about the question I raised, even a few weeks before I wrote to him, and says: "Talking about strange coincidences! He [meaning the inclusive educator] was absolutely right. I think I am a better mathematics educator as a result – even though I still don't know exactly what their needs are and how to meet their needs."

It was heartening to read this. It shows that this question arises naturally in other contexts in the world. However, looking into Bill's statement, perhaps there is an assumption that is possible that we (the mathematics education community) already know exactly what the needs of the so-called *normal* students are. Why do we usually say that we do not know the needs of people with disabilities while we do not know *anybody's* needs, actually? As a matter of fact, there is some research showing that both groups can have the same difficulties in learning mathematical content (see, for instance, Fernandes & Healy, 2007).

I put this question to Bill and he saw it as "another very good point". He said it would be wrong to hope that research would point out a complete list of students' needs once and for all. However, Bill argued that since the excluded are, by definition, often voiceless, silent, forgotten, and invisible in mainstream practice, using Levinas, it points to our own weaknesses as a research community to meet these needs. Perhaps, Bill continued, some of the research in mathematics education may have the ultimate aim of "generalization" of findings and generation of a list of needs that can mechanically be satisfied. "This is not my idea of what research does or should aim to do", he stated.

Let us go back to the question of the "why". Bill added that one could identify many possible reasons. Research into certain areas in terms of social justice has started by people affected by the disadvantages themselves, he observed. Bill pointed out that gender research has become mainstream as the result of a group of women mathematics educators; Edward Said is Palestinian, and Fanon was Afro-French (making reference to some key authors I read for my conceptualisation of my research). He continued, saying many educators come from lower social economic status background and that indigenous rights have been initiated by affected people themselves. Quoting him directly:

> I venture to say, with some hesitation, that blind and deaf people, as well as people with a range of physical and mental disabilities only recently have started to mobilize as identifiable communities with their rights as demonstrated by the Independent Living Movement.

(See http://bancroft.berkeley.edu/collections/drilm/). We should say that particular measures in institutions have been made to cater for the participation of a small number of people with disability, such as ramp access, color of fonts on websites and so on. But these have not been matched by an increase of awareness and interests in mainstream areas of research in mathematics education. We agreed that it seems mathematics educators in general are not the initiators of social justice, but reflectors of issues raised, both in society and in critical social writings. In our social identities, we are mainly concerned with mathematics teaching. Some of us have strong commitment to social justice, however in many instances this commitment is narrowly defined as a commitment to the teaching and the learning of a particular social group, or particular issues of social justice.

I do agree that claims of inclusion are usually raised from the excluded groups; as Spivak (2010) says in her book *Can the Subaltern Speak?*, we cannot give voice to the voiceless, to the oppressed. It is colonizer behaviour to believe we are the owners of the stage and we can give voice to the *other.* A social justice agenda should be based on allowing each group to speak for themselves.

Still, I can say deaf people have been pretty much organized as a social community for a while, perhaps more than any other disability group, even some claiming they belong to a different culture, as they have their own language, for example. Nevertheless, they are not included as an identifiable group in social justice research and educational thought. There is an interesting documentary about this concerning the history of deaf education in United Kingdom (http://www.bslzone.co.uk/bsl-zone/history-of-deaf-education/history-deaf-education-1/). In this documentary, one deaf person asserted that the worst crime against the deaf community was the Milan

Conference in 1880, when a group of hearing people decided that the deaf community should be educated by the oral system only, and the sign language was to be prohibited in schools. It is an example of normal people defining abnormal people. It shows the deaf community struggling for their rights at least since the 19th century in the United Kingdom.

I put this to Bill and he indicated that he was aware of that to a certain extent. Bill recalled that he had a deaf personal friend in the past that inspired him to learn the Australian sign language in order to communicate with him. Bill was pleasantly surprised to learn about the complexity of their community. However, for some reason, Bill thinks it would be safe to say that such organization has not been extended to demands on the educational system itself in the same way that gender issues have been raised by women educators.

Another possible reason raised by Bill for this lack of research on the issue we are discussing here is what he calls "searching for the key in a lit area", a story (joke) he heard as a child about this drunken man who was searching for his house key under a streetlight at 3 am in the morning. A policeman asked him whether he lost his key at that spot. The man said, "No, I actually lost the key around the corner". Surprised, the policemen said, "Then why are you looking for his key here?" The drunken man said, "Well, there is more light here"! At times, it seems to Bill that in mathematics education we often chose research questions where previous research has been done and where theoretical tools have been developed. We feel safer searching in areas that have already been researched. Perhaps very few of us venture into new terrains of research. Bill thinks my research is perhaps going into a "dark" corner to search for new "truth".

This made me feel good about my research. I knew this joke in Portuguese since I was a child; the popular wisdom has always fascinated me. It is true – I do feel in a dark corner occasionally. Bill asked me about the theories I mentioned as my inspirations at the start of our discussion, questioning if these theories make claims that some of the common "remedies" we use to help blind people, for example, (such as adapting mathematical didactical books in braille) are based on our own concept of *normality* and would we be colonizing the world of the blind according to our understanding of *normality*. Here, Bill provoked me with some questions of his own.

> I look forward to reading more about your argument in detail and your data. It sounds a very innovative way to think about the area.

However, the questions in my mind are

1. What are the aspirations of the blind people you dealt with in this regard and how they see the issue of such measures?
2. Are these theories you are using related more to the "recognition" rather than the "distributive" models of social justice as Iris Young and Fraser talk about? What other ways are there to deal with their disadvantage that allows for their "participation" in their society?

Surely I cannot tell all their aspirations, but I can state at least one concerning education based on my own masters degree research (Marcone, 2010): they want access. It is widely known today (looking into official statistics in Brazil and in India, for example) that people with disability do not get access to higher formal education; the numbers of people with disability in the Brazilian universities are derisive. And those who get access want adaptation of widely available knowledge. What I am arguing is that adaptation is not enough, as it is only an imitation of what *normal* people have. As educators, we could think of different activities for different needs, and this is possible only in collaboration with the different groups we are talking about. It is not reasonable to say that this is a teacher's responsibility alone, because it is not – teachers cannot save the world! But it is reasonable to do research on these issues in partnership with researchers who happen to be deaf, blind, or with any other impairment, as their experience of the world is the most important thing for teaching mathematics relevant to their aspirations.

Again, I brought this idea to Bill and he said that he understood my point more now. He guesses he was raising the question above more out of ignorance of my argument than disagreeing with it. According to him, he had the (wrong) impression, from the little that he knew, that I may be arguing that all modifying of our practices, both in education and social life, is colonization of the world of the blind. That's why he raised the question as to aspirations of blind people themselves. Now he hears me saying "it is important but not enough". In other words, if that is the only thing we do then it may be seen as colonization. We totally agreed with each other. Bill gave an example. A few years ago he had a friend from UK who was a drama teacher who had dyslexia. He underwent a master's degree from London University for which he had to write a thesis. He successfully argued that it would be discrimination against people in that condition to have to write a traditional thesis. He was granted his degree based on a "thesis" that was a video rather than a printed document. In our standardized culture in education and normal life, we assume sometimes that everybody should be able to do the same thing in the same way. This is neither social justice nor ethics. This is the same reasoning that gave us standards, outcomes, and standardized testing!

I need to read Iris Young and Fraser first in order to comment on their work and I will do it later. And, as I said, these theories are not dealing with people with disability; they are talking about the colonizer/colonized relationship and I am using this as inspiration to theorize on the relationship between *normal* and *abnormal* people in general, specifically people with, and people without, recognized physical impairments.

About the second aspect of the question, is the need to concentrate efforts on the exclusion apparatus instead of on the disadvantage itself? The problem is not the blindness or the deafness, rather it is the structure that creates a barrier, preventing these students to get access to the contents, to the information. They do have impairment – I am not denying it romantically – but this impairment does not need to become a disability or disadvantage. If one is blind, he or she will be disabled only in a world where is not possible to read with his or her fingers or to listen to text and so on. The idea is to concentrate on the environment, on the exclusion structures such as the school itself, instead of concentrating only on the physical impairment.

To this, Bill replied he likes this point. He said he would pass me papers that he had sent someone else about Nancy Fraser. One of the papers he had written, one is by Fraser herself, and the other by somebody else reviewing her work. Bill finds her model to understand social justice to be useful in analysing social justice action. The way he sees it is that measures of modifying current practices such as distribution and the other measures I am calling for could be based on recognition. Bill finished by saying "We will see where this dialogue would take us. I am really enjoying it and thanks for the opportunity".

Concluding Remarks

Our conversation has not finished yet, as this paper is not finished either. This is the first public version and we do intent to expand it in a short future. I m reading the papers Bill Atweh has indicated and I am discussing these issues with my PhD supervisor, Ole Skovsmose. I have interviewed Swapna Mukhopadhyay too, in order to get more data for the development of this meta-research.

I will finish with an idea from Gilles Deleuze about minority and majority, taken from an interview conducted by Claire Parnet between 1988 and 1989 (https://www.youtube.com/watch?v=_Wer1VGBZi8). Deleuze says that the majority cannot exist, as it is based on a pattern that is not possible to achieve. To be normal, to be part of the majority, one should possess all the majority characteristics – for example, in all the Occident a normal person, belonging to the majority, should be a heterosexual man, white, adult, and a citizen. However, Deleuze says that the majority is never anyone; it is just an empty pattern, even though many people recognize themselves in this empty pattern. "The majority is anyone, and the minority is everybody".

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Raquel Interviews Isolda who Interviews Raquel: A Conversation about Dialogue

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In this paper we characterize a mutual interview between two teachers, Raquel and Isolda, about the concept of dialogue in mathematics education. These teachers have developed a collective practice in a supervised teaching practice course on mathematics based on teacher-students dialogue. The interview was important to know what each teacher thought about the concept. This knowledge was used to design the course. Here we present a definition of mutual interview based both on the concept of dialogue (by Alrø and Skovsmose, and Bohm), and the concept of interview (by Kvale and Brinkmann), as well as the dialogue experience between Raquel and Isolda. This definition also includes dialogic acts and the actions of seeing, thinking, and constructing common knowledge together.

A Research on Dialogue in Pre-Service Mathematics Teacher Education

Dialogue is something that is done with the other. In the educational context, assuming a dialogical stance means that the teacher and the students share the talk, that is, the speech is not monopolized by one party. It is a political stance. Considering equity in dialogue, everyone has the right to express his/her own perspectives. Being engaged in dialogue with the other means listening to him/her, asking the other, being interested in what the other says. Can talking about dialogue with the other also be like that?

This paper describes a conversation in an interview format between Raquel and Isolda, two teachers and researchers of mathematics education, interested in teacher-students dialogue to promote learning. Raquel and Isolda are the authors of this text. It is important to clarify that the interview is part of the research Raquel has been developing on the dialogue between prospective mathematics teachers and their students at schools in the context of a supervised teaching practice. In this text, when we talk about the interview we will be referring to ourselves as Raquel and Isolda. In some moments, however, the first person singular will be used to make Raquel's reflections and clarifications about the research explicit.

Understanding the process of learning to be engaged in dialogue by the prospective mathematics teachers is the central objective of the research. It is difficult for them to establish an open interaction with students in order to ask questions, understand their responses, and use them in the construction of concepts (Almeida & Fernandes, 2010; Moyer & Milewicz, 2002). Being aware of this difficulty and believing that dialogue may promote learning, I think that some actions could be performed in order to make these prospective teachers feel more comfortable about an open interaction in mathematics classes. Therefore, in a supervised teaching practice course in the University where I teach, I have planned some activities with the supervisor teacher (Isolda) about dialogue in mathematics education. Before the prospective teachers started their class planning, they were engaged in dialogue activities. These activities involved the concept of dialogue in moments of reflection, planning, and implementation. This was the environment where the production of research data took place.

In that context, I acted as a supervisor teacher, as I have always been. My acting, however, was different from what I usually performed. At that moment, I assumed the position of the researcher and I was Isolda's collaborator. How the dialogue activities influenced the prospective teachers' pedagogic decisions was an important aspect of the research. Those activities belonged to a collective practice executed by Isolda and myself. In this context of research and practice, I investigated my own collective practice.

Jarvis (1999) calls the professional who develops research on his own practice a practitioner researcher. According to this author, the practitioner researcher knows what works in his/her practice, he/she feels comfortable in relation to the knowledge, skills and attitudes from his/her practice, and knows what problems should be investigated. In the context of my research, I assumed the position of practitioner researcher, and Isolda that of the practitioner. The two teachers are professionals who work in the same area, the supervised teaching practice courses, and therefore are practitioners. Throughout my practice, I often reflect on my actions to assess what needs to be maintained or modified. "Practice is both a site and an opportunity for learning, and reflective practice is a necessary approach to learning how to become an expert practitioner" (Jarvis, 1999, p. 70). Because I reflect on my own practice, I consider myself a reflective practitioner. As a result of this reflection, there were issues of concern related to the investigation that I have been developing. Therefore, besides performing actions of my own practice, I reflected, in a systematic way, about them and their effects on the prospective teachers' decisions.

A collective practice has several elements and among them a collaborative planning of actions and some ideas shared by the actors of this practice. Such common ideas may emerge in various ways, but in this paper we will highlight a special instrument to see what the involved people think about a special subject. It is a mutual interview between Raquel, the practitioner researcher, and Isolda, the practitioner.

Raquel and Isolda in a Mutual Interview

Once the planning and the implementation of dialogue activities were made by Raquel and Isolda (i.e., they would work together), it was not conceivable or ethical to impose one's ideas about dialogue over the other. Rather, there would have to be a proximal discourse for both teachers in order to talk to the prospective teachers about dialogue. Thus, before planning the dialogue activities in detail, the need to know what each teacher understood by dialogue in mathematics education arose. Therefore, Raquel and Isolda arranged a mutual interview about dialogue.

Let us now introduce both teachers properly. Raquel is the researcher who has been developing the present investigation and the collaborator of the supervised teaching practice course. Isolda is the supervisor teacher of this course and who has collaborated with the research. Raquel and Isolda are researchers of mathematics education, they have been working at the same higher education institution, are supervisor teachers, have developed many projects together in mathematics education, and, also importantly, they are friends.

The interview had two aims: to know what each teacher understood about dialogue in mathematics education, and delineate common aspects in these perspectives. It was a mutual and open interview implemented by email. It was mutual, because the two teachers posed and answered questions. The intention was not only to know what the teachers thought about a particular subject, but also to make the teachers' perspective explicit. Raquel and Isolda were, therefore, interviewers and interviewees. The interview was open because there was not a protocol of questions to be followed. The electronic environment facilitated the recording of questions posed, answers provided, specific dates, and waiting time for answers.

The interview had three rounds of questions and answers. In the first round, the teachers elaborated and sent their questions to each other. Each teacher answered questions taking her time, according to her availability, amid daily tasks. The questions were formulated, initially, according to the curiosity of each teacher about what the other thought. The answers informed the other about a subject, in this case the *dialogue*, showing a personal perspective.

In the second and third rounds, besides this curiosity, the questions were formulated based on the answers provided by each teacher in the previous rounds. There was an interest in knowing more about a particular idea and going deeper into a perspective in order to know its origins and foundations. There was time to think about the answers and the new questions. The teachers might read the questions received and think about the answers or write some previous answer, which were on standby to be reviewed and modified at any time. Meanwhile, the teachers thought of new aspects of those answers and supplemented them. These reflective comings and goings characterized the process of providing answers. All questions and answers were stored in a file on their computers, which made it easier to go back and forth improving what had been written earlier.

Dialogic Acts in the Mutual Interview

Throughout the interview it was possible to notice some important actions that assured its development. Those actions are related to the concept of dialogue by Alrø and Skovsmose (2004), in the context of critical mathematics education. The authors have characterized empirically the dialogue between teacher and students, and among students, in terms of the acts that constitute the Inquiry Cooperation

Model: *getting in contact, locating, identifying, advocating, thinking aloud, reformulating, challenging,* and *evaluating*. That concept has been widely used in studies involving communication in mathematics education to promote critical learning, which involves both socio-political aspects related to mathematics, such as those related to the process of discovering mathematics ideas by the students. A recent example is the research by Alrø and Johnsen-Høines (2010) which analyses how prospective teachers put dialogue into action when they question the traditional ways of teaching and plan new possibilities of learning mathematics critically, showing competence to propose innovative strategies.

Some of those acts were also present in Raquel's and Isolda's mutual interview, as is the case for *getting in contact, locating, advocating, thinking aloud, reformulating*, and *challenging*. As we mentioned earlier in this text, in the first round of the interview, the questions showed the teachers' interest in knowing what the other thought about aspects of the dialogue; thus, Raquel and Isolda *got* the necessary *contact* to the development of the interview. Some of the following questions sought to improve their knowledge on a specific idea and know the details about it, since it represented some degree of novelty for the teacher who had asked the question. Deepening and better understanding a perspective are related to the dialogic act of *locating*. This process has permeated the whole development of the interview.

In the mutual interview, the act of *locating* can be illustrated by some moments, as when Isolda asked Raquel for an explanation about the idea that "engaging in a dialogue with the other involves listening carefully to what he says and posing questions." In responding, Raquel explained the concept of active listening, which means "asking questions and giving non-verbal support while finding out what the other is getting at" (Alrø & Skovsmose, 2004, p. 62), and defended its importance in the dialogue to promote learning. Another moment when the dialogic act of *locating* was in action was when Isolda, describing the teacher's talking in the dialogue, referred to some elements of Jean Piaget's learning theory. Interested in knowing more about those elements and their relationship with the dialogue, Raquel posed questions to deepen Isolda's ideas in the following rounds.

The dialogic act of *challenging* appeared in an implicit and explicit way in the conversation among the teachers. In an attempt to *locate* Raquel's perspective, Isolda proposed: "I would want to *challenge* you to make some assumptions on what are disciplinary and bureaucratic questions." Isolda also asked, "Which questions promote learning? Which questions are appropriate for dialogue?" The verb *to challenge* appeared explicitly in Isolda's question. In moments before the interview, the types of questions that are posed in a mathematics class had been a point of reflection for Raquel. Isolda's question was a special opportunity to rethink those kinds of questions. Even when the verb *to challenge* did not appear explicitly in the question, the teachers felt *challenged* to be clear and expose what they thought. Sometimes it was hard to explain something about which they had never written. An example of that was when Isolda asked Raquel to exemplify the dialogue related to the teaching of some mathematical concept. Before asking, Isolda did not know that looking for this example represented a *challenge* for Raquel, precisely because it was one of her uncertainties: when is a conversation considered a dialogue? Raquel used the context of solving equations to illustrate what she believed dialogue was, and, in the next round, she enlisted Isolda's help by asking the question: "what would you call the conversation I described in the context of solving equations to discuss the student's doubt?"

The actions of defending a perspective, agreeing or disagreeing with an idea, and arguing for or against it, are related to the dialogic act of *advocating*. Raquel and Isolda began the interview with individual ideas, certainties and uncertainties. As the rounds went forward, the teachers identified common ideas in their answers, as Isolda stated about the prospective teachers' difficulty of engaging in dialogue with the students in their teaching practice: "I agree that it is difficult for prospective teachers to express and practice the dialogue in class. Which factor(s) do you think cause that difficulty?" Sometimes Raquel also agreed with Isolda. For example, when Raquel judged positively some ideas presented by Isolda by stating: "This idea of intentionality of dialogue is very good" and "The expression 'enhance the dialogue' is very good." Raquel still summed up some ideas of both teachers: "We agree that talking and listening are important actions in the mathematical dialogue."

The dialogic act of *thinking aloud* refers to the verbalization of reasoning to make a perspective public, thus allowing it to be investigated. In the mutual interview, Raquel and Isolda made their perspectives explicit, defended them in writing, and deepened their ideas through explanations and examples. While in a face-to-face conversation the act of *thinking aloud* refers to spoken verbalization, in the mutual interview the quality "aloud" was related to a written explanation of perspectives.

In some questions, Raquel and Isolda paraphrased themselves in seeking to *locate* the perspectives and propose *challenges*. The teachers tried to reproduce what the other had said with their own words. The dialogic act of *reformulating* refers to this attempt to say again what was said by someone in order to know that the perspectives of each side are understood.

In explaining what each teacher meant by dialogue, Raquel and Isolda put many dialogic acts described by Alrø and Skovsmose (2004) into action. In the following section, we highlight common aspects of the teachers' perspectives.

The Creation of a Common Knowledge

In the first round of the interview, Raquel and Isolda made their initial perspectives on the concept of dialogue explicit. With the process of going deeper into the ideas, comparing thoughts, and agreeing and challenging each other, it was possible to see some common aspects between the perspectives presented. However, before presenting them, it is necessary to bring some ideas about dialogue.

Raquel agrees with the definition of dialogue as proposed by Alrø and Skovsmose (2004). The teacher states that dialogue is a kind of conversation with some characteristics (theoretical and empirical), which aims at critical learning. The theoretical characteristics include making an inquiry, running risks, and maintaining equality. In empirical terms, dialogue is characterized by eight dialogic acts that compose the Inquiry Cooperation Model previously discussed. Raquel pointed to the importance of listening carefully to what the other says and posing questions. In a further comment on that idea, Raquel explained the concept of active listening and established relations with the learning process.

> In trying to understand what the other says to me, it seems that I go to meet him and do some intervention so that together we can go somewhere else, new for both. New to the other, because it seems that he has never been there, new to me, because I am going somewhere known to me, but with a different company

> [...]. The teacher engaged in dialogue assumes a stance about what is learning mathematics and how the knowledge is constructed in the classroom. The concepts are not transmitted to the students, as given, but constructed with the students, considering their previous knowledge. The teacher will create an environment in the classroom to promote possibilities of doubting, questioning, and arguing. This teacher believes that the students have to express themselves, put their ideas in public.

When Isolda was asked to write about dialogue and mathematics education, she states that dialogue is:

> an intentional exercise of talking and listening that occurs, producing actions of thought, reflection, and internal constructions. It can be planned with some definite purpose, such as to learn and teach. It is a didactic and pedagogical action, almost a teacher's conduct, which seeks to involve students in actions of thought, to develop or expand the understanding of a concept. The teacher's talking is questioning, and should promote the listening that produces internal operations, cognitive imbalances, assimilations, and accommodations.

From this response, Raquel asked Isolda to explain in more details her ideas about learning based on the theory of Jean Piaget. Therefore, in the following rounds of interview, Isolda refers to the stages of development and to expressions such as "imbalances" and "balances" in her examples of dialogue and other explanations. She says that, while the learning process is in action, one is operating consistently with previous concepts for the construction of a new concept. She still states that, "the mechanisms and constructions are internal and individual, but nothing happens without putting the structures of thought into action".

Considering that Raquel and Isolda are different subjects, their speeches are also different. Each teacher has a career and some maturity in relation to her beliefs about mathematics education. While Isolda assumes Piaget's theory in her speech, Raquel is starting her career as a researcher and making her theoretical choices. Therefore, there would not be a single speech to the prospective teachers, but common aspects that emerged in the process of clarifying perspectives, which will be explained below.

For Raquel and Isolda, dialogue involves assuming a pedagogical stance related to the way that mathematics is learned by students. The teachers believe that knowledge should be discovered by students and not delivered to them. The students must act on the proposed activities and express their ideas and ways of thinking. They should be involved in actions of thought, relating previous knowledge to the situations that they are studying, in order to construct new knowledge. The teacher is the one who guides the process of discovery. Listening and talking are common actions to the teacher and the students. Asking and being attentive to what the other says are part of the dialogue and, therefore, people involved must listen actively. For Raquel and Isolda, the dialogue is intentional. It aims at learning.

Those common ideas emerged in the development of the mutual interview. What one teacher said influenced the other's response in some way. The process of creating a common knowledge is particular to a conversation as the dialogue (Bohm, 1996). For this author, the dialogue has a different sense from the one communication has. One of the senses mentioned by the author for communication is "to make something common, i.e., to convey information or knowledge from one person to another in as accurate a way as possible" (Bohm, 1996, p. 2). On the other hand, in the dialogue,

> When one person says something, the other person does not in general respond with exactly the same meaning as that seen by the first person. Rather, the meanings are only *similar* and not identical. Thus, when the second person replies, the first person sees a *difference* between what he meant to say and what the other person understood (Bohm, 1996, p. 2, emphasis in original).

Therefore, Bohm believes that there is a difference between what is said by a person and what is understood by the other who listens. It is that difference that will make the dialogue go on. The first person who talks,

> may then be able to see something new, which is relevant both to his own views and to those of the other person. And so it can go back and forth, with the continual emergency of a new content that is common to both participants (Bohm, 1996, p. 2).

Unlike every person trying to make his/her ideas common to the other person, in the dialogue, "the two people are making something *in common*, i.e., creating something new together" (Bohm, 1996, p. 2, emphasis in original).

In the mutual interview, each teacher made her initial perspective on dialogue explicit, but there was no intention to make this a true perspective, one that would predominate throughout the interview. "In a dialogue [...] nobody is trying to win" (Bohm, 1996, p. 7). Even trying to be as accurate as possible, it was necessary to return to the answers, ask again, and ask for more explanations and examples in order to understand the particular perspective. In this back and forth movements, common ideas to the two teachers began to emerge, i.e., "something that takes shape in their mutual discussions and actions" (Bohm, 1996, p. 3).

Raquel and Isolda in a Mutual *Inter-view*

The conventional interview is a meeting between two people where one of them, the interviewer, aims to obtain from the interviewee the necessary information on an issue (Lakatos & Marconi, 2010). In Raquel and Isolda's conversation, on the other hand, the roles of interviewer and interviewee were not assigned to one or the other teacher. Both functions were assumed by the two of them who, more than obtaining information about what each one thought about dialogue, mutually exchanged and reflected on their perspectives. Kvale and Brinkmann (2009, p. 2) consider the interview as an interaction between two people, and literally, as "an inter view, an inter-change of views between two persons conversing about a theme of mutual interest." The authors consider that in an interview the participants look at some subject together. The positions of interviewer and interviewee are assumed by different people, who see the object of an interview together. In the case of mutual interview, Raquel and Isolda looked and thought together about the ideas that were being clarified in the conversation. In order to illustrate that action of seeing and thinking together, we emphasize the moment when Raquel presented an idea, Isolda thought about it and then questioned Raquel who, in turn, thought again about the same idea or another aspect related to it. Therefore, there were common objects of reflection, which revolved around the concept of dialogue, which was of mutual interest of the teachers.

Raquel and Isolda thought together about what they were talking about. It was possible to see that action occurred in two dimensions: interpersonal and intrapersonal. The interpersonal level refers to the act of thinking itself during the exchange of questions and answers. The intrapersonal dimension refers to each participant's action of thinking. As an example, let us consider Raquel's report. Initially, she did not imagine the ways that her thoughts would go, because it depended on the clarification of Isolda's perspectives and questions. Besides wanting to know what Isolda thought—the motivation to think together on the interpersonal level—Raquel wanted Isolda to know about her perspective. In order to make it clear to Isolda, Raquel needed to know what she herself thought about it, and in a clear way—the motivation to think together on the intrapersonal level.

In trying to answer one of Isolda's questions, Raquel was involved in an internal process of thinking critically by herself. While she talked to Isolda, she talked to herself, trying to clarify her certainties and uncertainties. Regarding what was uncertain, Raquel opted for the openness, because she knew that in doing so she would have a chance to talk about the issue to Isolda. Raquel changed moments of uncertainty into challenges, trusting that Isolda would perform an active listening. We highlight, therefore, the importance of thinking together on the mutual interview's interpersonal level, as it enables the creation of challenges on the intrapersonal level as listening to yourself, asking yourself, and respecting yourself.

The actions of seeing and thinking together, including the intrapersonal level, resulted in another important action of the mutual interview. Kvale and Brinkmann (2009, p. 2) consider the interview as "a construction site for knowledge", which is "constructed *inter* the *views* of the interviewer and the interviewee." *Inter* the *views* of Raquel and Isolda (i.e., in the interaction of the two teachers) a common knowledge was constructed. Such knowledge did not exist at the beginning of the conversation. There were two perspectives that began to be related as they were clarified and deepened. Some ideas were very close to both teachers, but others were unique to each one. Thus, some of Raquel's conceptions made sense to Isolda and vice versa. Each teacher has incorporated the other teacher's ideas in her speech. Therefore, the interaction made it possible to point to common aspects about dialogue, about learning, and about mathematics education.

The mutual interview between Raquel and Isolda about dialogue was characterized by the actions of seeing, thinking, and constructing common knowledge together. Other more specific actions were also present at the talk of the teachers. This is the case of dialogic acts of *getting in contact, locating, advocating, thinking aloud, reformulating*, and *challenging*. Both the specific and general characteristics of the mutual interview refer to the concept of dialogue by Alrø and Skovsmose (2004). Specifically, the action of creating common knowledge together is associated with the definition of dialogue by Bohm (1996). Considering that characterization, we believe that the mutual interview was, in fact, a *dialogue*. A special reason is that Raquel and Isolda were mutually interested in their perspectives, and did not try to impose their opinion on each other. That interest was expressed in terms of *active listening*. Raquel and Isolda carefully welcomed the questions and answers, and kept in touch by posing new questions.

Clearly the mutual interview between Raquel and Isolda cannot be characterized exactly in the same way as the dialogue by Alrø and Skovsmose (2004), since, for these authors, that concept aims at critical learning of mathematics. The teachers had not intended to learn mathematics, but to learn about dialogue with each other and construct together a new common knowledge. However, the way that Raquel and Isolda mutually posed and answered questions is similar to that concept of dialogue.

Based on theoretical concepts cited in this text and on my own experience with Isolda, we propose a definition of *mutual interview* as: a dialogue between people interested in a subject of mutual interest, who listen actively to each other and assume the roles of both the interviewer and the interviewee, and is characterized by general actions of seeing, thinking, and constructing common knowledge together, as well as by specific dialogic actions of getting in contact, locating and deepening, advocating, thinking aloud, reformulating, and challenging.

Final Remarks

Being engaged in dialogue with the other, especially in the context of mathematics education, is, above all, an act of mutual respect and equity. The one who talks deserves to find someone who listens and shows interest in what is said. That was the case of the mutual interview between Raquel and Isolda. The teachers were able to find out with more details about their perspectives: the other's perspective and their own. Considering the intrapersonal level of thinking together, the process of clarifying ideas was permeated by a sincere and deep self-reflection. This respect for oneself is essential to respect and help the other, the task of every teacher. Once Raquel and Isolda would work in partnership in a supervised teaching practice course, it would not be ethical to impose one's perspective about dialogue over the other. The mutual interview reinforced the respect and trust that the teachers had already felt for each other.

In this paper, based on the experience with Isolda, the concept of interview by Kvale and Brinkmann, and the concept of dialogue by Bohm and by Alrø and Skovsmose, we presented a definition of mutual interview. In contrast to the conventional idea of an interview as a professional meeting between interviewer and interviewee, in which the first obtains information from the second, the mutual interview is a dialogue that aims to know the perspectives of those involved and construct a common knowledge together. This construction process occurs in a dialogic interaction full of actions taking place with the other and which maintains the contact between the participants, particularly because of the active listening.

The mutual interview between Raquel and Isolda was already predestined to be a dialogue from its beginning. Although Raquel has realized this fact just during the interview, Isolda, in the first round, finished her answers by saying: "That is, to begin the conversation, the dialogue!"

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Product and Agent: Two Faces of the Mathematics Teacher

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In this paper we will explore how the "mathematics teacher" becomes a subject and, at the same time, is subjected as part of diverse dispositive of power. We argue that the mathematics teacher becomes both a product and a social agent, which has been set, within current societies, from the ideas of globalization, social progress, and competitive logic. For our approximation, we use the concepts societies of control, dispositive, and discourses from a Foucault–Deleuze toolbox. Our purpose is to cast light on the social and cultural constitution of the ways of thinking about the mathematics teacher. Hence, our critical examination offers understandings about how mathematics teachers are part of the larger cultural politics of schooling and education.

Introduction

Mathematics teachers are important. Or so it is said repetitively in different spheres of society, as well as in mathematics education research. Moreover, mathematical knowledge is considered as an important knowledge for society and its development, since this knowledge is providing tools and skills that help the subject to confront diverse tasks and problems of everyday life and of his/her context. Therefore, mathematics teachers become the ambassadors of mathematics and the ones that bring such knowledge to the new generations. To think mathematically is seen as a powerful means to understand and control one's social and physical reality. The mastery of mathematical knowledge and the possession of mathematical competencies are primordial characteristics of the rational, objective, and universal subject embodied in notions of the cosmopolitan modern citizen (Valero & García, 2014). Mathematics teachers fulfil the Promethean task of bringing light to children for the benefit and progress of humanity.

By studying the diverse discourses that circulate about the mathematics teacher, we can see a centrality in teaching the "right" mathematics and in mathematical knowledge. Diverse enunciations are formulated within trends and interest of society, configuring particular forms of understanding teaching, the teacher, and mathematical knowledge, which leads to shaping a way of reasoning about the mathematics teacher. The mathematics teacher is being constantly (re)formed under such ideas as progress, development, and justice. *Cultural theses* (Popkewitz, 2009) about the mathematics teacher are configured and together with the network of practice of mathematics education (Valero, 2010), are shaping certain possible conditions that allow the generation of the desired subject.

The "mathematics teacher" that we are talking about here is not as such any concrete individual of flesh and bone. It is a discursive construction where power gets actualized in articulating ways of thinking about a desired being. But saying that the "mathematics teacher is not a concrete person" does not mean that we are talking about theoretical constructions that have nothing to do with real people. On the contrary, as we have argued before (Valero, 2009), the forms of thinking about the practices of mathematics education that emerge from, and in, such discourses configure and set frameworks that lead individuals towards the subject they should become or "must be". Understanding the mathematics teacher—as subject—, cultural theses and discourses is an important task for research that takes a critical stance towards the established truths of society. Without that critical stance it is difficult to imagine new possibilities for practice.

In this paper, we engage in a critical study of the cultural thesis about the mathematics teacher that navigates in research and in public discourses. While in recent research there have been studies advancing a political reading of mathematics education practices (Gates & Jorgensen, 2009), there are few studies that turn the gaze towards the mathematics teacher. In this paper we bring into operation some tools from the theoretical toolbox of Foucault (1971, 1972, 1980) and Deleuze (1986, 1992a, 1992b) to explore how the mathematics teacher becomes a subject and, at the same time, is subjected as part of dispositives of power. It is our contention that there is a dominant educational logic whereby the teacher is constructed as the professional that responsibly provides answers to the social demands of our times. Such logic effectively inserts the mathematics teacher in the calculations of power in a double sense. The teacher becomes a desired product of society, a highly valued merchandise whose value is the possession and possibility of transmitting mathematical knowledge. Simultaneously he/she becomes an agent who should be able to conduct children towards the desired knowledge of mathematics, a highly qualified sales person of the mathematical merchandise.

Analytical Strategy

We seek to think the mathematics teacher as a historical-cultural construction, situated in a particular spatio-temporal configuration, wherein the mathematics teacher is a subject immersed in discursive practices. By studying the discourses that circulate in the research and the discourses that are formulated for international agencies, Valero (2014) argues that the research in mathematics education creates the languages for naming study objects and ways of thinking about these objects.

In a Foucaultian approach, discourses are not understood in terms of "a particular instance of language use—a piece of text, an utterance or linguistic performance—but describing rules, divisions, and systems of a particular body of knowledge" (Arribas-Ayllon & Walkerdine, 2008, p. 99). Rather, discourses are generated in spatio-temporal configurations. Furthermore, discourses are not statements by well-respected authors. Rather, it is the repetition of enunciations in certain conditions of possibility that allows the generation of truths and the constitution of forms of reasoning. Hence, a Foucault-inspired discourse analysis seeks the regularities and systematicity that lead to discursive formations, where these statements form a rhizomatic field leading the desired subjects of mathematics education. The role of the analyst is to reveal the convergence of a complex network of discursive practices.

In the case of this investigation, our strategy also deploys some concepts from the theoretical toolboxes of Foucault (1971, 1972, 1980) and Deleuze (1986, 1992a, 1992b), our focus being mainly on societies of control, dispositive and discourses. These concepts allow us to weave together the statements, their conditions of possibility, and their effects.

Specifically, our empirical material is the recent research produced about mathematics teachers published in a volume of "Journal of Mathematics Teacher Education" (JMTE, 2014), and some recent official documents of international agencies published by OECD (2012, 2014) and UNESCO (2009). In this material, enunciations that express the characteristics of the "desired" mathematics teacher were selected and examined to identify the discursive object that we call "the mathematics teacher", and from there some statements about that desired subject were identified as the truths on that ideal subject. But those statements were also seen in relation to other enunciations that talk about the conditions and demands of such desired subject in particular spatio-temporal configurations. The notions of societies of control, dispositive, and discourses allowed us to build a rhizomatic web of enunciations leading us to formulate the statements about the mathematics teacher, allowing us to evidence the forms of reasoning that currently constitute notions of the "mathematics teacher".

Societies of Control, Dispositives and Discourses in the Making of the Desired Mathematics Teacher

Societies of control (Deleuze, 1992b), dispositive (Deleuze, 1992a; Foucault, 1972), and discourses (Deleuze, 1986; Foucault, 1971, 1972, 1980) are entangled to promote an understanding about the fabrication of subjectivity and how certain conditions of possibility are configured to shape the subject: the "mathematics teacher". The research and the international agencies are promoting enunciations and statements—discourses—around an ideal image of the mathematics teacher. The discursive formations respond to particular requirements and demands of spatio-temporal conditions. This ideal image of the mathematics teacher is transformed into the "must be" of the teacher, a desired subject, producing regimes of truth, power relations, which normalize forms of thinking about concrete teachers.

Moreover, discourses have provided and contributed to establish a certain regime of truth and regime of power, in which are circulating ideas that are shaping how we are understanding the diverse issues about the mathematics teacher. In these regimes, some ideas are repeated over and over, some ideas are unquestioned and these ideas do not appear to generate resistances. For example, ideas such as permanent training and lifelong learning currently are considered as natural and innocent, but these ideas are promoting a continuous redefinition of the teacher from constant evaluations. For example, in the research we can find a statement such as:

> Many PPTs (prospective primary teachers) wanted to continue taking another mathematics course because they wanted to improve their mathematics knowledge and skills not only for themselves but also for the sake of their future students. (JMTE, 2014, p. 356)

This kind of statement shows how in many mathematics teachers there emerge a need and an interest in upgrading his/her teaching knowledge and tools—either through specialization or continuing studies—to respond to the challenges. But, why did the need of permanent training and lifelong learning emerge and what is being sought with it? We can find a partial answer in what Deleuze (1992b) argues:

> … in the societies of control one is never finished with anything—the corporation, the educational system, the armed services being metastable states coexisting in one and the same modulation, like a universal system of deformation (Deleuze, 1992b, p. 4)

Therefore, we could understand that the idea of permanent training and lifelong learning is responding to the rationality provided by societies of control. Moreover, societies of control have facilitated the conditions to replace the factory by the corporation, the examination by continuous control, and where the school tends to be replaced by perpetual training (Deleuze, 1992b). Consequently, within societies of control, the process of making all society production, of defining it, of doing it or of creating it is never finished. Therefore, this process is involved in a constant state of becoming. This has led to set truths and discourses that respond to different interest, ideas, and rationalities.

There emerge dispositives and discourses around the mathematics teacher which allow us to think of him/her in terms of development (JMTE, 2014, pp. 1, 86, 180, 303, 380, 405; OECD, 2012, 2014), skills (JMTE, 2014, pp. 15, 87, 110, 284, 337), knowledge (JMTE, 2014, pp. 5, 41, 101, 373, 420), and performance (JMTE, 2014, pp. 66, 150, 202, 240, 299, 409) and where the mathematics teacher is constantly measured and evaluated (JMTE, 2014, pp. 106, 465; OECD, 2012). In the discourses that circulate, it is possible to see increased attention on the mathematics teacher's knowledge—mathematical knowledge, pedagogical knowledge, among others. For example, it is not difficult to find statements in the diverse discourse that circulate, such as:

> [teachers need to] develop professional knowledge in support of their practice (JMTE, 2014, p. 455)

> [T]he teachers' knowledge of functional thinking was below the level expected for teaching middle-school algebra. This provides further evidence of teachers' inadequate understanding of mathematics for teaching (JMTE, 2014, p. 418)

The dispositive is formed as response to an urgent need—"urgency"—(Foucault, 1980) of society and social change. The dispositive defines the space of allowed and prohibited movements. Therefore, dispositives are articulating the network to think, to enunciate, and subjectify the mathematics teacher. Dispositives are promoting diverse statements and enunciations about the mathematics teacher; by creating an image of the desired mathematics teacher and his/her "must be", the dispositive is leading and configuring conditions that favor certain practices and knowledge. In other words, the mathematics teacher is subjected to dispositives and discourses and she/he becomes the product of the predominant system of reason.

Dispositives are shaping discourses. Discourses are composed of enunciations and statements (Foucault, 1972), wherein the regularity in the use of certain statements is prompting to certain discourses that are accepted as true and naturalised, therefore these discourses are not questioned and are accepted without resistance. These are showing ways of knowing and are expressing: a desired vision about the diverse subjects involved in education; the role of mathematics education in building a better world; and some truths established in the diverse practices on teaching and learning of school mathematics (Valero & García, 2014), which are configured within regimens of power and truth.

Foucault (1971, 1980) proposed that there are not inert discourses and an all-powerful subject that is manipulating or is setting it, rather, the subjects are part of a discursive field with a specific position and role. The discursive practices are based in social concerns and social interest and these react to social changes, urgencies, and contingencies.

It is stated that it is urgent that the mathematics teacher has a variety of skills and knowledge, to respond to main urgencies of society, which it will lead to diverse discourses and games of power, in pursuit of the development of a modern society more just and equal. UNESCO (2009) proposed that mathematics teaching should be organized for the development of a conception of mathematics as a tool to understand and change the world, and also as a field of knowledge with objects, rules, and foundations. In this scenario, there emerge tendencies, demands and requirements that the mathematics teacher must face. Changes in the demand for skills and ability have profound implications for the competencies which teachers themselves need to develop, since they will need to acquire tools to effectively teach the 21st century skills that the students need (OECD, 2012):

> The kind of education needed today requires teachers to be high-level knowledge workers who constantly advance their own professional knowledge as well as that of their profession. Teachers need to be agents of innovation [...] [Thus, they] can help to improve learning outcomes and prepare students for the rapidly changing demands of the 21st-century labor market (OECD, 2012, p. 36)

Teacher's Permanent Training and Lifelong Learning as Dispositive in Societies of Control

For example, OECD (2012) pointed out that school leaders reported a lack of qualified mathematics and science teachers. In many countries there is a high demand for qualified mathematics teachers, namely, teachers well-trained to the highest standards of professional knowledge, skills, efficiency, competence, and integrity, who can lead and implement diverse initiatives to improve teaching.

We can see that within diverse places a discussion about educational coverage and high quality education has been started and established. Here emerge at least two issues. Firstly, the increasing demands for professionally qualified teachers and at the same time the lack of mathematics teachers. There is a strong logic of competition between teachers. Teachers need to compete not only in qualifications with others to get a job, but also the teacher must compete with him/herself. By this, teachers are able to show they have the skills and knowledge that are demanded. Secondly, there is a centration on the knowledge and skills of the mathematics teacher for mathematics teaching. Accordingly, this teaching must satisfy established standards of quality, in other words, fulfill the expectations that led to set the need for permanent training and lifelong learning.

The idea of permanent training and lifelong learning is operating as a dispositive, in term of Foucault (1980) and Deleuze (1992a), in other words, these are operating as "an ensemble (set) of strategies of relations of force which condition certain types of knowledge and is conditioned by them" (Bussolini, 2010, p. 92) by setting forms of control, discourses, and forces.

Currently, we can see that the highest mechanism of control in the education system is the use of standardized tests, where the mathematics teacher has a great importance, mathematical knowledge is appreciated by the society, and the teacher is responsible for its teaching. Modern society needed mathematics to establish its foundation on reason and logic. These standardized tests are setting a numerical language of control that marks access to information, and where people have become samples, data, or markets (Deleuze, 1992b), by configuring everything around a marketing logic:

> Marketing has become the center or the "soul" of the corporation […] the operation of markets is now the instrument of social control and forms the impudent breed of our masters. Control is short-term and […] continuous and without limit, while discipline was of long duration (Deleuze, 1992b, p. 6)

In discourses that circulate, we can find how the education system is demanding "good mathematics teachers". The good mathematics teacher is defined from the rationality and desires prevailing in a society, which is setting the parameters and ideas, with which is measured and evaluated the teaching work, and that lead to a desired image to which all teachers should aspire. Currently, for example, statements are favoring the establishment of discourses about the mathematics teacher such as: teachers should have the knowledge (mathematical, pedagogical, didactic, among others) to perform successfully their work, developing fully the student potential; teachers "must be" able to adjust to the requirements and standards established at national and international levels; and teachers should be upgraded constantly and should be in permanent training and lifelong learning.

The "must be" of the teacher is setting diverse dispositives that are excluding all feared and undesired features for a globalized society. Therefore, discourses are promoting a form to conceive, to understand, and to think about the mathematics teacher. The majorities of these discourses are formulated and are sustained in ideas of modern society, for example, globalization, equity, and access to knowledge. For example, permanent training and lifelong learning is leading to particular discourses, where the ongoing evaluation is used as a way to control and is giving the tools to understand if a teacher is "good" or meets the demands.

Moreover, the current education system has been favoring a standardization of mathematics teachers, in others words, the fabrication of a standard subject, which has emerged as a response to social demands and urgencies, for instance, educational equity, where everyone must have access to a equitable education. This standard subject must be able to respond to diverse contexts and realities from a standardized education. Here we can see a paradox because the mathematics teacher will be educated with skills and knowledge that is responding to a ideal context, namely a context very different to the real context. Furthermore, the international standard about the competencies and skills that a student should develop in the school are promoting diverse demands and many of these are focused on the mathematics teacher, in his/her knowledge, skills, and practices.

The emergence of demands, from diverse places, has favored the establishment of a direct relationship between the development of the mathematics teacher and requirements established by the system. Because society needs that the mathematics teacher be able to respond to requirements and to desires. Moreover, professional development and teachers educations are configured in particular epistemologies and practices, responding to the "urgencies" and the dominant rationality of determined spatio-temporal conditions. Currently we can see how such ideas as globalization, equity, and access to knowledge have been promoting different ways of doing, of thinking and of acting, constituting, in the words of Foucault (1980), diverse dispositive and technologies that favor certain truths and subjectivities. But with these ideas (globalization, equity, and access to knowledge), what kinds of mathematics teachers are requested by society through the diverse demands?

Inside a marketing model that is around the mathematics teacher, mathematical knowledge takes great importance, since the mathematics is a precious resource for the society because its development is strongly related to the development of the society and social welfare; moreover, it favors establishing a rationality and the fabrication of the rational, logical, and modern subject that society wants.

Conclusion

Mathematics education has been considered of great importance to ensure social development, but this has led to using mathematics with diverse intentions. For example, to select, to qualify, to segment, to decide, and so on. The mathematics teacher becomes a product of social demands, depending on interests, ideas, rationalities that are subjecting and are regulating his/her teaching practices and work. At the same time, the mathematics teacher becomes an agent within diverse dispositives, favoring diverse interests and rationalities. In both cases, knowledge and knowing are related with power, by promoting ways of being, doing, which lead to discursive practices and to fabricate subjectivities.

Currently, we can appreciate that a large part of discourses are rooted in a logic of competition that responds to globalization, more specifically towards a globalized economy, a globalized knowledge, and a globalized education. Dispositives and discourses formulated are leading and are subjecting the mathematics teacher into permanent training and lifelong learning.

When the mathematics teacher becomes a product, it creates the conditions wherein the mathematics teacher is confronted against other mathematics teachers; the mathematics teacher is confronted against him/her self, and the mathematics teacher is confronted against the ideal mathematics teacher. This final product—the mathematics teacher—must always be in the avant-garde and satisfy all needs. But, how could the mathematics teacher be always in the avant-garde? The one sure way to achieve this is through certain dispositives, and, in this way, to ensure the convergence toward the ideas, ideals, rationalities, and subjectivities that are being promoted by society. Therefore, permanent training and lifelong learning are central for society since the mathematics teacher's development can be controlled.

Moreover, the mathematics teacher becomes an agent. This leads us to think of the teacher as a means for progress and for the success of society. By fabricating subjectivities and reaffirming rationalities and by setting, from discursive practices, the "real" and the accepted, the teacher becomes part of power's dispositives of societies of control.

From discourses, we can think that the mathematics teacher is socially constituted and fabricated to follow norms and standards. But historically, norms and standards have been imposed on the teacher, even if these don't have a relation with his/her history, context, and trajectory. For example, current society is seeking a mathematics teacher's standardization in response to the idea of globalization and social progress, with the promise of a better and brighter future, which has been accepted without resistance, but why don't we resist? A possible answer could be because it is part of our subjectivities as modern citizens.

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Developing a Positive Mathematics Identity for Students of Color: Epistemology and Critical Antiracist Mathematics

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Utilizing critical race theory and antiracist education as the main theoretical frameworks, this paper critically analyzes mathematics education (ME) and problematizes the ways mathematics teaching and learning is approached in many K-12 schools. If mathematics fields are going to diversify, an epistemological shift in how we think about, teach, and learn mathematics is imperative. This conceptual paper: 1) uncovers epistemological racism in mathematics; 2) explores the theoretical frameworks underlying antiracist education (AE) and critical mathematics (CM); and 3) discusses critical antiracist mathematics (CAM) and the promise it holds for facilitating the development of a mathematics identity for students of color.

Critical race theorists contend that one of the major modes for disseminating "master narratives" of the dominant group is through the system of schooling (Zamudio, Russell, Rios, & Bridgeman, 2011). More often than not, these master narratives are presented as objective truths, but in fact are grounded in Western epistemologies, and serve as mechanisms to silence alternative truths or narratives. Zamudio et al state, "Educational institutions present themselves as objective disseminators of knowledge. CRT educators question and interrogate the viability of objectivity in a context of power relations. In doing so, CRT educators work towards broadening truths to include the history and experiences of people of color" (p. 5). As CRT educators and scholars, it is our intention to not only interrogate and critique the racism embedded in the ways mathematics is traditionally approached, understood, and taught, but to provide emancipatory alternatives for educators to approach teaching and learning differently in schools. We contend that critical consciousness, a strong math identity, and opportunities to be seen and understood as knowledge producers is essential for students of color to navigate mathematics, and push mathematics to transform for greater justice.

Epistemological Racism: Decolonizing Mathematics

In 1997, James Scheurich and Michelle Young questioned the racial bias embedded in epistemologies commonly used in educational research. Pleading with scholars to critically analyze epistemological racism, they state, "we educational researchers are unintentionally involved, at the epistemological heart of our research enterprises, in a racism, epistemological racism, that we generally do not see or understand." (Scheurich and Young, 1997, p. 12). This critique, of course, was not new. Scholars of color have argued for decades that dominant research paradigms are based in white male contexts and have marginalized the ways of knowing of diverse people for centuries. This epistemological racism is not limited to just education research(ers). In mathematics, which is overwhelmingly grounded in Western positivist epistemologies, the ways of thinking about, teaching, and learning math are also based in an epistemological racism that scholars generally do not see or understand (Clarkson, 2000).

In the same way that many scholars have articulated a resistance to Euro-Western research methodologies through a process of decolonizing, we believe that we must also decolonize mathematics from Euro-Western epistemologies if children of color are to develop a mathematical identity (Chilisa, 2012; Smith, 2012; Wilson, 2009). According to Chilisa (2012), "decolonization is a process of centering the concerns and worldviews of the colonized Other so that they understand themselves through their own assumptions and perspectives" (p. 13). Decolonizing, therefore, is a process which involves liberating the mind from the oppressive structures and conditions of Western colonization and also the "restoration and development of cultural practices, thinking patterns, beliefs, and values that were suppressed but are still relevant and necessary to the survival and birth of new ideas, thinking, techniques, and lifestyle that contribute to the advancement and empowerment of the historically oppressed and former colonized non-Western societies" (Chilisa, 2012, p. 14). Decolonizing mathematics, therefore, opens up spaces for the historically marginalized (colonized Other) to use their knowledge systems and thinking patterns to solve problems and birth new ideas.

Bishop (1990) explicitly acknowledges Western mathematics as a tool "in the process of cultural invasion in colonised countries," specifically mediated by education. Perpetuating the idea that mathematics is a culture-free endeavor producing culture-free knowledge is a general Euro-Western standpoint, and a method to facilitate cultural imperialism. In his 1988 article, *Mathematics Education in a Cultural Context,* Alan Bishop explains the fallacy of this argument (p. 201):

> ..."a negative times a negative gives a positive" wherever you are, and triangles the world over have angles which add up to 180 degrees. This view though, confuses the "universality of truth" of mathematical ideas with the cultural basis of that knowledge. The ideas are decontextualized and abstracted in such a way that "obviously" they can apply everywhere. In that sense they are clearly universal.

The dissection of this argument reveals the tension involved in the conceptualization of "universality," questioning the origin and necessity of 180 degrees in a triangle and of the idea of negative numbers (Bishop, 1988b, p. 201). Ultimately this calls into question the cultural knowledge base, or master narrative, that is referenced when discussing these mathematical concepts. Despite the introduction of ethnomathematics by D'Ambrosio in 1985, worldmath by Anderson in 1990, and the historico-epistemological analyses of mathematics by Radford in 1997, the idea that mathematics is a culture-free endeavor producing culture-free knowledge persists to be the dominant standpoint. This dominant view, then, is the master narrative that is perpetuated in school and used to silence the narratives and views that mathematics is in fact a "cultured" enterprise.

Similarly, mathematics has also been considered a value-free endeavor producing value-free knowledge due to its seemingly irrefutable nature. Bishop (1991) suggests that values actually play an integral role in the mathematical curriculum and in the mathematical classroom. Valuation happens on multiple levels. At the societal level, mathematics is highly valued because of its control and power in the commercial and industrial domains and can lead to higher status jobs (Bishop, 1991). At the institutional level, such as within the institution of education, mathematics is highly valued and has become a gatekeeper (Stinson, 2004) in which success is necessary in order to complete benchmarks, such as gaining a diploma, in order to access further opportunities. At the pedagogical level, the value of mathematics varies depending on the teacher and is manifested in the mathematical knowledge environment (Bishop, 1991) that they create. As a result, values at the individual student level will be quite diverse, demonstrated in students' attitudes and values statements (Bishop, 1991). These levels do not operate independently of each other, with classroom values being derived from institutional and societal level values.

As such, these institutional and societal values must be derived from somewhere, and Bishop (1991) suggests that it must be "in some way from qualities, beliefs and values of mathematics *itself*" (p. 199). He offers six principle clusters of beliefs and values that permeate Western mathematics, which is the mathematics dominant in our public school curriculum. The ideological values of rationalism and objectism lead to a high valuation of logic, abstraction and the symbolic representation of phenomena. The sentimental values of control and progress lead to a high valuation of the security derived from predictability, generalizability, and controlled change and alternatives. The sociological values of openness and mystery lead to a high valuation of the verification of the truth of propositions representative as universal facts and the mysterious quality of decontextualized and abstract knowledge (Bishop, 1991). He is "not claiming that these values are *only* associated with Western mathematics, nor that these values are superior to any other, although earlier educators certainly *have*, particularly in the Western European colonial era" (Bishop, 1991, p. 205).

When specifically comparing the African and "Western" conceptions of the mathematical concept of objectism, Horton (1967, as cited in Bishop, 1988a) observed the "African preference for explanation using personal idiom and the 'Western' preference for using the impersonal. ...Mathematics favours an objective, rather than a subjective, view of reality" (p. 65). Thus, the mathematical value of objectism is not inherently a Western notion; what is however, based in Western paradigms is the prevailing mathematical conception of objectism. As a result, these Western epistemological frameworks and values associated with mathematics provide the master narrative in schools, by which all other narratives and ways of being are silenced. Delgado Bernal (2002) argues that the insidious nature of Western Eurocentric epistemological perspectives allows it to subtly shape the belief systems and practices of educators and school curriculum while simultaneously adversely affecting the educational experiences of students of color.

Antiracist Education

Unlike multiculturalism writ large, antiracist perspectives believe that racism is institutional not attitudinal (Duarte & Smith, 2000). Meaning, racism is an internalized ideology of white supremacy as manifested in the unequal distribution of resources and power to non-white groups (Grinter, 2000). This system of inequality that privileges whiteness is supported by structures and policies in society resulting in the notion that racism is normal. The ideology that racism is normal and permeates social systems and ideologies is a central tenet of critical race theory (Ladson-Billings, 1998; Zamudio et al, 2011). Uncovering the tacit and overt ways that racial thinking operates to systematically privilege whiteness while subjugating people of color is a central concern of critical race theorists and antiracist activists (Ladson-Billings, 2000; Delgado Bernal, 2002). Therefore, critical race theorists and antiracist educators do not seek prejudice reduction on an individual level, but rather to dismantle systems of oppression through collective action (Duarte & Smith, 2000). Antiracists seek to build solidarity across lines of race, class and gender, for collective political action to redistribute power and economic resources.

Antiracists see this goal being achieved through three strategies: 1) problematizing hegemony rooted in white supremacy; 2) taking collective political action through multiracial coalitions; and 3) antiracist education programs that are anti-assimilationist (Duarte & Smith, 2000). Antiracism, therefore, seeks to develop "learning processes that question the social structure and its basic assumptions, to produce activists against social injustice" (Grinter, 2000, p. 144). Therefore part of antiracist education is to "involve students in the analysis of the whole school ethos and power structure, and in work to identify and remove racism from their educational institution" (Grinter, 2000, p.

144). Critical (multicultural) perspectives grounded in the work of Paulo Freire, such as critical mathematics can be employed to achieve this end.

Critical Mathematics

Critical mathematics owes much of its origin to the work of the Critical Mathematics Educators Group (CMEG), who were highly influenced by the InterAmerican Committee on Mathematics Education (IACME), the International Congress on Mathematical Education (ICME), and the African Mathematical Union (AMU) (Powell, 2012). Initially, these influential organizations placed a focus on the structure and content of curricula, and the contexts surrounding their implementation. There was a growing concern around the social and political issues of mathematics education, specifically around the impacts of colonial and neocolonial educational structures and textbooks (Powell, 2012). The work of Marilyn Frankenstein (1983) further developed the concept of critical mathematics, as she explicitly tied it to Freire's (1970) notion of "critical" and the necessity of critique in emancipatory education.

According to Powell (2012), the purpose of critique is to "challenge and to change taken-for-granted ideas and conditions of life so as to improve social life" (p. 25). Thus, one aim of critical mathematics education is to "engage students, socially marginalized in their societies, in cognitively demanding mathematics in ways that help them succeed in learning that which dominant ideology positions them to believe they are incapable" (Powell, 2012, p.27). Borrowing from Freire's (1970) notion of reading and writing the world, Eric Gutstein (2012) applies this idea specifically to mathematics where students can use "*mathematics as a weapon in the struggle*" (p. 23). Reading the word and the world is about redefining notions of literacy to include not just the reading of text (word), but "the unstated dominant ideologies hidden between the sentences as well" (Kinchloe, 2008, p.16). When students learn to read the world and the word, they question knowledge and the knowledge production process and recognize that they have the power to be change agents in the world around them (Price & Mencke, 2013). Gutstein (2012) explains that through mathematics education, students need to be prepared to investigate and critique injustice, as well as take action against oppressive structures and actions.

As a result, critical mathematics education, according to Paul Ernest (2010), is responsible to "offer values-based criticisms of society, mathematics and the social practices of ME [Mathematics Education], most notably the teaching and learning of mathematics" (p. 3). Given this responsibility, Ernest (2010) describes four main domains critical mathematics education should consider including: 1) clarification of presupposed values, 2) a critique of epistemological issues of mathematics, 3) a critique of society and the role of mathematics in society, and 4) a critique of the teaching and learning of mathematics (p. 3). Critical mathematics research, unfortunately, has placed almost exclusive focus on the latter two of the domains at the expense of deeper examinations of power, privilege, and values embedded within mathematics itself. In other words, more research in the area of uncovering and problematizing the dominant epistemology of mathematics explicitly connected with the domains of teaching and learning mathematics is needed.

It is particularly relevant, then, to explicitly address the epistemological origins, embedded values, and underlying assumptions behind the mathematics taught in schools, which is derived from what Bishop (1990) calls "Western Mathematics." However, it seems insufficient for only researchers and teachers to become aware of the Western European history, culture and values embedded in mathematics and school mathematics. If students are to engage in critiquing and transforming the world using mathematics, they must also become fully aware of the Eurocentric epistemological origins, values and assumptions of the "weapon" that they are armed to use. That is to say that students must be aware that they are "using the master's tool" to attempt to "dismantle the master's house" (Lorde, 1984). Developing critical consciousness to use "mathematics as a weapon" can allow students to become more aware of their potential alignment/discontinuity between the values embedded in school mathematics, Western European values, and their own cultural values Secondly, it also creates space for students to critically reflect on the limitations of this cultured weapon to address issues in their specific context.

In critical mathematics, a mathematics identity is formed by one's belief about one's "(a) ability to do mathematics, (b) the significance of mathematical knowledge, (c) the opportunities and barriers to enter mathematics fields, and (d) the motivation and persistence needed to obtain mathematics knowledge" (Martin, 2000, p. 19). This definition of mathematical identity implies the inclusion the content and practices of mathematics, as well as the importance of a person's epistemological stance. Though the critical mathematics definition of mathematical identity (Martin, 2000) seems to imply a focus on both mathematical content and practices, research on masternarratives, counternarratives, and identity in mathematics thus far has emphasized topics like agency, voice, achievement, persistence, school-based mathematical practices, and participation (Martin, 2007). Understanding how these concepts impact student engagement with mathematics and the development of their mathematical identity is integral; however, all of these concepts are symptomatic of the epistemological racism of Western, and thus, school mathematics. For example, stereotypes in the mathematics classroom are often cited as a barrier to mathematics achievement (Martin, 2007; Nasir, 2013; Steele, 1997). It is our contention that stereotypes and stereotype threat are symptoms of racist ideologies and Eurocentric epistemologies that assume mathematics is neutral; the mis-perception from both teachers and students that students of color cannot achieve at this universal, neutral mathematics reinforces liberal ideologies that deficiency is an individual problem, not a systemic one rooted in white supremacy (Ladson-Billings, 1998). When research only locates the barriers outside of mathematics without critiquing the underlying values and assumptions embedded in the field, it runs the risk of reinforcing the Eurocentric taken for granted assumption that mathematics is neutral and objective. Thus, we must also look *within* mathematics to identify barriers to engagement, achievement and mathematics identity development.

Critical Antiracist Mathematics & Mathematics Identity

Critical Antiracist Mathematics focuses on the development of a mathematical identity, as does critical mathematics, but acknowledges the centrality of race in that development. Haymes (2002) supports a focus on race in the deconstruction of systematic oppression in that the reality of people of color "...in an anti-black racist society is that they live class through race and therefore as 'black people'" (p.155). This is in alignment with our Critical Race Theory (CRT) standpoint which critiques the institutional racism embedded in the epistemologies and practices that dominate schooling, and in our critique, mathematics.

Critical Antiracist Mathematics (CAM) supports the critical mathematics definition of mathematical identity (Martin, 2009), but opens spaces for critiques of the structural and ideological racism that permeates mathematics. Grounded in Critical Race Theory, CAM offers opportunities for students' counternarratives and epistemologies to be validated, as they are also understood and seen as knowledge producers. Delores Delgado Bernal (2002) discusses the importance of critical raced-gendered epistemologies and the use of counternarrative and testimonios to counter the hegemony in Western Eurocentric epistemologies that dominate school curricula and practices.

> Raced-gendered epistemologies also push us to consider pedagogies of the home, which offer culturally specific ways of teaching and learning and embrace ways of knowing that extend beyond the public realm of formal schooling. Because power and politics are the center of all teaching and learning, the application of household knowledge to situations outside of the home becomes a creative process that challenges the transmission of "official knowledge" and dominant ideologies" (pp. 109–110).

Critical raced and gendered epistemologies offer opportunities to see and hear (counter) narratives and experiences that are not visible from Eurocentric epistemological stances. The power of using non-majoritarian narratives in classrooms is not just in their ability to voice different experiences that are often not heard; it is in learning to listen deeply, making those stories matter and challenging the normalcy of epistemological racism in education.

Explicitly recognizing that mathematics is not "value-free" or "culture-free," CAM is counter-hegemonic and does not approach mathematical teaching and learning from the idea of one master-narrative or universal truth. Instead, it recognizes multiple truths and narratives and explicitly names the racism that often serves as barriers for marginalized people to participate in the knowledge production process. Consistent with the Freirean conception of conscientização, developing critical consciousness alongside a mathematics identity allows students to learn and use mathematics to examine power relations and hegemony in social institutions for greater justice, while also developing agency (Freire, 1970). Leonard et al (2010) explain the importance of developing a positive mathematics identity "because mathematics education as a social construction is a gendered and racialized experience" (p.262). Affording students of color the space to critique systems of oppression and develop critical consciousness provides avenues for students to navigate the complex power relations inherent in schooling to think beyond what is, and imagine emancipatory and democratized futures.

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Statistical Education: A Context for Critical Pedagogy

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As our society continues to become more diverse in race, culture, ethnicity, class, and sexual orientation the educational systems we have in place must also revise their purpose and their context. An important conviction of the social justice pedagogy is that teachers as well as students are a part of the solution to injustice. Teacher education programs need to equip teachers with the empathy, consideration, and the skills necessary to appreciate all the children that comprise their classroom, their school, and their community. We merged the research domains of statistics and social justice to design a teacher education course – the key goals was to develop a community enable teachers to investigate social issues with a critical eye, so that the inequities become apparent and ideas for change may be discussed.

A Rationale for Our Inquiry

The UNESCO, in its *World Declaration on Education for All* (1990) calls for "universal access" to education for all children. This declaration emphasizes that the purpose of education should be to help mankind develop "both essential learning tools, such as literacy, oral expression, numeracy and problem solving" in order to enable all citizens to "live and work in dignity, to participate fully in development, to improve the quality of their lives, to make informed decisions, and to continue learning." There are several stakeholders that are responsible for realizing this mission in the society both at the macro and the micro levels. At a macro-level, we have national leaders who are committed to advancing these goals, policy makers who develop supporting agendas and craft proposals, and bureaucrats who act on this agenda and enact the plans. At a more micro level, we have a robust group of champions—teachers—who are in a strong position to realize the educational goals and thereby make a long-term impact in the immediate community.

Although at times unspoken, educators maintain a sphere of influence that allows them to affectively change and promote instruction. Acknowledging the authority a teacher maintains in a classroom, to create systemic change teachers must first experience personal disequilibrium in their own ideology so that they engage in all aspects of a democratic agenda. According to Skovsmose and Valero (2002), democracy:

> Has the purpose of transforming the living conditions of those involved, allows people to engage in a deliberative communication process for problem-solving, and promotes coflection, that is, the thinking process by means of which people, together, bend back on each other's thought and actions in a conscious way. (p. 397)

When a democratic agenda is in place, the full realization of how power is used and maintained in the larger societal picture can be recognized. Therefore when disequilibrium is supported through instruction, students, as well as educators, are stimulated to make accommodation in their belief structures through the use of differentiation, class extension, or reconceptualization (Wertsch & Polman, 2001).

Once a more inclusive view of the world in which we live is recognized, teachers become agents of change within their classroom, opening up communication so that their students experience the full benefit of a democratic education. As teacher educators, we utilized statistics as a tool to transform oppressive ideology and promote self-awareness and self-sustaining behaviors. In merging the research domains of statistics and social justice to design a course for teachers in statistics, it was our priority to establish connections to the chosen research domains and to classroom practices. Our broader research goal is to engage in a self-study (Arizona Group, 1996) to better understand how well we were able to realize this goal. In conceiving, teaching, and reflecting on this course, we were inspired by and heavily drew on what we view as overlapping research domains—statistics education and critical mathematics education. In this paper, we reflect on our personal, practical and professional knowledge experiences grounded in the context of a teacher education course aimed to promote connections to our theoretical ideologies and our practical enactments.

Theoretical Underpinings

Critical Statistics for Social Justice

Building off of the idea first introduced by Frankenstein (1990) of critical mathematics, critical statistics acknowledges the political nature of knowledge and how data can be construed and misrepresented when used in a public arena. Within the realm of statistics, because data and data analysis is not widely understood or questioned, those making the declarations based off of statistical analysis often maintain or claim the power for political stakeholders. Understanding how statistical knowledge is valued and who is making that decision helps individuals to better understand their social realities and how power struggles are enacted and sustained. Frankenstein (1995) when explaining the foundations for critical mathematics states that by asking students to think "critically by examining its underlying interests and methods of collection, description and inference, and by considering historical, philosophical, and other theoretical insights along with statistical knowledge" (p.192) build the capacity to enter into a meaningful conversation about productive change.

Critical pedagogy, the umbrella under which these ideas fall is the "attempt to be discerning and attentive to those places and practices in which social agency has been denied and produced" (Giroux, 2011, p.3). Critical pedagogy brings attention to oppression and allows individuals to become an agent of change that will promote liberatory social change over time (Frankenstein, 1987). We engage in an inquiry into our teaching practices to better understand how we can teach statistics that helps learners attain the content proficiency advocated by proponents of statistics education, but beyond that, equip them with the empathy, consideration, and the skills necessary to appreciate all the children that comprise their classroom, their school, and their community.

Skovsmose (1985) states that when implementing critical education pedagogy two criteria, subjective and objective, need to be used when selecting problems for the classroom. First the subjective requires that the problem appears relevant to the students and within their conceptual understanding. The second, the objective, requires using an existing social issue to build deeper understanding. Therefore, the amalgamation of statistics and social justice provides a platform to have meaningful conversations about issues that impact our world and prepare individuals for active citizenship.

Statistics: Numbers with Context

In today's world, data is omnipresent and practitioners of statistics receive, consume, and interpret data in different ways. An individual awestruck by data seldom questions regarding statistics and places a blind trust in what it may convey; on the other end of this diagonal is a cynical consumer who dismisses any statistical findings and conclusions. In between the two lie the naïve consumer and the critical consumer. A naïve consumer, because of a limited and superficial understanding of data and related ideas is unable to question or critique statistical information whereas a critical consumer is able to ask questions and discern information about data, its collection and analysis methods, and conclusions that are warranted by data (GAISE, 2007; Best, 2001).

Garfield and Ben-Zvi (2008) note that in the realm of statistics, "context provides meaning for the numbers and data cannot be meaningfully analyzed without paying careful consideration to their context" (p. 8). Statistics education takes formal mathematics and provides a context that allows an individual to form questions and conclusions based on a quantified analysis. Statistics education cannot be seen as disconnected rules and ideas but, as a "powerful and relevant tool for understanding complicated, real-world phenomena" (Gutstein, 2006, pg. 30). According to the National Council of Teachers of Mathematics, statistics education must incorporate problem solving, reasoning, and communication, a method that allows mathematics to occur within the context of children's developing understanding of their world, both cognitively and socially (NCTM, 2000). In line with these recommendations, we chose activities and data sets that will enable participants realize the "proficiency goals" but also provide a heightened context for exploring the socio-political dimensions of mathematics and statistics education. We hoped that our teachers will realize that the purpose of teaching statistics is "not only to [enable students] be competitive workers in the economy but also [become] engaged participants in a democracy, able to be critically reflective about the role statistics has played and can play in our society (Lesser, 2007, p. 3).

Contextual Setting

The course in discussion, "Probability and Statistics for high school teachers" was (re) designed to create a community of participants that can work together to negotiate a broader and deeper understanding of statistical ideas, the role of statistics in today's society, and perceptions of the role of social justice in a mathematics classroom. The geographical setting for the course is a large mid-western university in the United States. This course is one of several content courses that we offer for in-service teachers enrolled in a Master of Arts in Teaching (MAT) program in mathematics. It is a 3-credit hour course (approximately 38 contact hours) and is usually taught during the summer term to accommodate teachers' schedules. Over three-weeks of summer, course participants met for approximately three hours a day, five days a week. While both authors were involved in the planning of the course, the first author taught the course to a group of eleven teacher participants. The identified course goals were to help participants a) understand the statistics (and the potential it holds) to explore social justice issues and b) understand and engage in statistical thinking and thereby deepen their content and pedagogical knowledge of statistics.

We used two key resources for this course—Rethinking Mathematics (Gutstein & Peterson, 2013) and Workshop Statistics: Discovery with Data (Rossman & Chance, 2012). We also used data sets from several web-based sources including the world bank, population connection, UNICEF, WHO, and the United States Census Bureau. Activity-based learning was emphasized and course participants engaged in content, pedagogical, and curricular explorations to accomplish the course goals. The focus of the content explorations was to better understand what it means to understand and reason about statistical concepts and thereby emerge and grow as critical and caring consumers of data. Pedagogical explorations were focused on identifying methods for developing students' understanding and reasoning about these concepts, and curricular investigations required teacher participants to identify and develop tasks that will foster a deeper and richer understanding of statistics and social justice issues.

Technological aids such as a TI-83 calculator, Fathom software, and Excel were used during content exploration sessions. Participants completed problem sets that focused on statistical concepts including sampling methods, study design, data displays, descriptive statistics, random variables, correlation, regression and probability. Real data derived from multiple sources (e.g., World Factbook, www.gapminder.org, and world population statistics) were used to investigate many of these concepts. Concurrently, participants read and reflected on chapters from Rethinking Mathematics (e.g., Home Buying While Brown or Black, Sweatshop Accounting, Math, Maps, and Misrepresentation) to generate and foster critical reflection on the use of statistics to explore socially relevant issues.

Practical Enactments

The connections between statistics and social justice are complex and nuanced. Each component holds personal implications that may influence individual progress and therefore when constructing this course we had to think about the statistical content as well as the contextual issues that influence individuals. The ethical guidelines of the American Statistics Association highlight that practitioners of statistics need to realize the "social value of their work and the consequences of how well or poorly it is performed [which] includes respect for the life, liberty, dignity, and property of other people" (www.amstat.org). In our context, this highlights the need to pose questions that will lead both teachers and their students to investigate social issues with a critical eye, so that inequities become apparent and ideas for change discussed.

Addressing Barriers for Implementation

Striving to develop and implement a social justice agenda is not an easy task. Often time there is no context, nor content for a social justice agenda within a conventional mathematics classroom. Limited modes for curricular resources, professional development, or models exist to demonstrate that a social justice agenda will blend with and enhance other content disciplines. One obstacle that must be addressed concerns a widespread belief that students are not mature enough or ready to engage in a discussion centered on sensitive social issues. Another barrier concerns the notion that when using a social justice foundation, meaningful statistics or mathematics, is absent from the curriculum. Many teachers believed that it is necessary "for students to learn the necessary math to deal with and get past the unjust gates in front them—*and* the math necessary to tear down the gates entirely" (Gutstein & Peterson, 2013, xii). However, during a regular school year, they are often inundated with the day-to-day routines and practical ideas of teaching, leaving little time to take on new challenges.

In the context of our research and development for this class, we, the collective whole, had to come together to address these challenges. We understood and embraced the concept that if we failed to produce meaningful dialogue, or if dialogue is absent, change cannot occur. To better cater to the needs of our community, we introduced teachers to "Rethinking Mathematics" to negotiate broader meanings for mathematics and statistics and their role in today's society. Teachers read several chapters from the book and shared their opinions through discussions, debates, seminars, and presentations. As teachers became familiar with new resources on teaching mathematics using the lens of social justice, they too were inspired to develop activities that are relevant to their immediate context. Participation in these activities not only gave members of our community an opportunity to experience personal disequilibrium in their own ideologies but also resulted in a collection of curricular resources that could be used in mathematics classrooms. We provide two activities from our course artifacts (instructor-developed and participant-developed) that pushed us forward in addressing the challenges to our social justice agenda.

Instructor-developed artifact: World population statistics. We used world population data to discuss the potential for a critical analysis and interpretation of statistics. First, we watched a you-tube video "The world is a village" that gave a sneak preview of the world population statistics, condensed to fit a village with 100 citizens (https://www.youtube.com/watch?v=i4639vev1Rw). Next, we presented an activity "Food for thought" from the Population Connection web source (www.Populationconnection.org) and simulated the world population of different continents and became aware of socially relevant statistics on wealth, health, sanitation, and pollution. To uncover and understand our personally-held beliefs about the nations of and the issues facing the world, we watched a TED talk titled "Debunking myths about the third world" and played the Gapminder sorting game (www.gapminder.org/GapminderMedia/GapPDFs/GapminderSort/GapminderSort.pdf,). Next we used the Gapminder free online data visualization software (www.gapminder.org) to investigate and interpret data compiled from various international sources (e.g., UNICEF, World Bank, WHO). In particular, we focused on understanding the dynamics of the graphs and the available options for choosing scales (linear or logarithmic), indicators (e.g., life expectancy, infant mortality) and categorizations (e.g., income, religion) to generate graphical displays. We further explored the Gapminder world graph to better understand the types of statistical analysis exemplified in the dynamic illustrations.

Our preliminary analyses resulted in an investigation of several open-ended questions that could be explored using the world statistics such as: a) How do social and political changes affect literacy rates in West Africa? b) What government changes affect the health and wealth of Iran and China? and c) How have major natural disasters affected the countries of Venezuela and Bangladesh in terms of their political and economic stability as well as their overall development? We encouraged participants to propose their own questions (and possible sub-questions) that they could address by analyzing and interpreting the data using the software. A detailed report of participant-developed artifacts is available elsewhere (Poling & Naresh, 2014). We provide some examples of teacher-generated Gapminder questions here. a) How did the Vietnam War affect life expectancy in the countries involved in the war? b) Look at the Life Expectancy Graphs over time for Vietnam, Cambodia, China, Taiwan and the United States from 1940 to 2012. Describe any trends that you notice, and c) How does the expenditure per student at the primary level affect math achievement in the 8th grade? Compare the following countries: United States and Japan.

Engaging in this activity was an eye-opening experience – as we engaged in an analysis of data about the world nations including the so-called third world nations, we realized that many of our personally-held views turned out to be myths that were thwarted by a critical and careful analyses of data. We are now more attuned to our personal beliefs and their potential impact on our roles as educators in the society.

Teacher-developed artifact: World population statistics. All teacher participants completed a course project that required them to develop a statistical lesson that could be taught to students in grades 7-12. Teachers were encouraged to connect statistical concepts to understand relations of power, resources inequities and disparate opportunities between different social groups. The following sub tasks were assigned.

1. Reflect on the course readings and propose a definition of social justice that makes the most sense.
2. Identify potential data sources that relate to your social justice theme. Identify sources of data that are most useful in exploring key statistical concepts.
3. Identify at least three key statistical concepts that are central to the lesson.
4. Develop a class activity/task. Include procedures for implementing the activity. Discuss one or more approaches to solving the problem.
5. Describe why the chosen activity is appropriate for the chosen grade level.
6. What connections to other mathematical topics / subjects could be made?
7. Establish specific connections to the GAISE guidelines and the CCSS-M standards.

At the end of the first week of classes, teachers completed a course project outline that included responses to the first three tasks. After receiving feedback from their instructor, they incorporated suggested changes (if any) and developed the intended activity. During the last day of classes, they enacted their activity with their peers. Examples of teacher-developed activities include *Pay inequity in the workplaces, Racial disparities and graduation rates, Racial and wealth divide, Disproportionate distribution of a city budget, and A comparison of two school districts budget allocations*. Here is an example of a teacher-developed activity. The teacher chose to remain anonymous but graciously gave us permission to use this artefact.

Title: Racial Disparities and Graduation Rates.

1. Definition of Social Justice: Social Justice is the view that all individuals deserve equal rights, opportunities, and economic growth. Social Justice is a process which seeks fair treatment

and opportunities, challenges prejudice, empowers people to take a stand, and builds more understanding in our worlds.

2. Data Sources
    a. http://www.edweek.org/media/ew/dc/2006/oh_SGB06.pdf – HS Graduation Rate
    b. http://www.radicalmath.org/docs/EducationPays05.pdf – Benefits for Higher Education
    c. http://collegecompletion.chronicle.com/state/#state=oh&sector=public_four – College Completion Rates
    d. http://www.fastweb.com/financial-aid/articles/3930-shocking-student-debt-statistics – Student Loan Debt
    e. http://www.bls.gov/emp/ep_chart_001.htm – Earnings and Employment Rates by College Graduate
    f. http://www.all4ed.org/files/Ohio.pdf – Ohio Dropouts affecting economy
    g. http://www.dispatch.com/content/stories/local/2012/11/28/ohios-grad-rates-show-racial-disparity.html
    h. http://www2.ed.gov/about/pubs/publications-reports.html – Department of Education
3. Key statistical concepts:
    a. Represent data with plots on the real number line (dot plots, histograms, and box plots).
    b. Use statistics appropriate to the shape of the data distribution to compare center and spread of two or more different data sets.
    c. Recognize possible associations and trends in the data.
    d. Distinguish between correlation and causation.
4. – 7) Due to page limitations, we are unable to provide the entire activity in this paper. We will include a comprehensive overview of the lesson in our presentation. Here, we list a set of questions that were central to this discussion.
    a. Describe how you can use data to decide if there a racial gap between the overall black and white graduation rates.
    b. Describe how you can use data to decide if there a racial gap between black and white graduation rates.
    c. How does Ohio compare with the other states? Did Ohio fail at all levels (black, white, overall) with the graduation rate?
    d. What could we do to improve the gap?

Through such activities, we were able to emphasize the significance of engaging in an iterative process of data collection, exploration, analyses, and interpretation (GAISE, 2007) to promote statistical literacy and empower individuals "to use statistics to "talk back" to or change the world" (Lesser, 2007, p.2).

Keeping the Conversation Alive

As our society continues to become more diverse in race, culture, ethnicity, class, and sexual orientation the educational systems we have in place must also revise their purpose and their context. As articulated in Program Ethnomathematics (D'Ambrosio, 2007), a curriculum for critical pedagogy must encompasses three strands—*literacy, matheracy,* and *technoracy*. Literacy is the ability to absorb and process information and could include numeracy, matheracy goes beyond numeracy and enhances an individual's ability to absorb information, develop a hypothesis and draw relevant conclusions, and technocracy equips an individual with the skills necessary to understand the potential that it holds to benefit or destroy mankind. Although we did not set out our course aligned to the key tenets of Program ethnomathematics, in hindsight we realize that, as a community of learners, we have grasped and appropriated many of these principles during this course. We truly believe that our teachers have a much deeper understanding of the role of statistics in today's society and have grown as critical and caring consumers of data. As members of a community that aimed to foster critical statistics pedagogy, they are not only able to think and reason about statistics but also are able to use statistical tools to better explore and understand issues that are significant to their immediate community, the larger society, and the broader world.

It is extremely important to view learners as "democratic citizens" to increase their "awareness of social justice issues" (Lesser, 2007, p. 3). In such a democratic setting, terms and definitions, including that of social justice are subject to multiple connotations. As staunch believers of a critical mathematics pedagogy, we, at times, were inclined to question teacher-proposed definitions and interpretations of social justice and their attempts at translating these constructs into their classroom teaching. However, we have come to realize that it is simply unjust to impose our interpretations suppressing those of the others. Instead, we use the voice and the views of the others' to create instances of cognitive dissonance and use such instances to negotiate and (re) conceive newer meanings for teaching mathematics. It is imperative that we equip teachers with the empathy, consideration, and the skills necessary to appreciate all the children in their classroom, their school, and their community. We need to achieve this by making a genuine attempt in facilitating an inclusive environment to engage others who do not already share our ideologies and visions for realizing the educational goals. Teaching statistics for social justice focuses on the use of statistics as a means to see many different interpretations of the same phenomenon, not only the dominant view. Using Gutstein's (2006) work as an impetus for this trajectory, statistics education "for social justice flows from the broader notion of liberatory education and has two sets of pedagogical goals: one focused on social justice and the other on" statistics (p. 23).

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Who Keeps the Gate? Pre-service Teachers' Perceptions on Teaching and Learning Mathematics

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Mathematics has long been known as a gatekeeper: challenging to learn, tricky to successfully access and navigate, and perhaps not even meant for everyone to learn. Pre-service teachers admit to having experienced such gatekeeper characteristics in their own learning of school mathematics. In a study, pre-service teachers were asked questions about their perceptions of teaching and learning mathematics, as well as their perspectives on students' perceptions of learning mathematics. In this paper, responses to these questions are viewed through two different epistemological lenses: one underscores the role of mathematics as a language and the other ponders the role of the teacher in student learning. Both lenses return the discussion to how mathematics is NOT being demystified in school classrooms, but instead is being maintained as gatekeeper.

Introduction

When asked whether mathematics is easier or harder to teach than other subjects, one middle years pre-service teacher responded that it is "not as hard as grammar but harder than social studies." This response, along with other responses that point to the importance of "telling" or "explaining" mathematics so that students 'get it' had us, as researchers, pondering important epistemological questions regarding knowing (in) mathematics. We believe two specific epistemological questions—*what* is being known and *how* it is being known—have a strong influence on student access to, and success in, school mathematics. By examining middle years pre-service teachers' perceptions of teaching and learning mathematics through the lenses created by these two epistemological questions, we believe that we begin to understand the perpetuation of the gatekeeper status of mathematics. In other words, we examine the epistemological questions: *what* mathematics is being known (underscoring the role of mathematics as language) and *how* is mathematics being known (pondering the role of the teacher in student learning).

The purpose of this paper is to explore the relationship between pre-service teachers' own experiences learning school mathematics and their perceptions on teaching and learning mathematics in middle school (Grades 6-8). The data is drawn from surveys administered to middle years pre-service teachers enrolled in years one through four of a four-year undergraduate teacher education program at a university in a western province of Canada. By reading the survey response data through two different lenses, the paper suggests that the role of teachers in the preservation and maintenance of the gatekeeper status of mathematics cannot go unnoticed, and that there is considerable work to be done in teacher education to dismantle the gate for learners of school mathematics.

What the Literature Says

Some believe that "the birth of mathematics as the privileged discipline or gatekeeper" (Stinson, 2004, p. 9) dates back over 2300 years to the words and writings of Plato. Even though it is assumed that much has changed in society and education since Plato, Volmink (1994) describes mathematics as "an impenetrable mystery to many," clarifying that "in spite of a century of mathematics instruction, most people still feel alienated from the subject" (p. 51). Further, Noyes (2004) confirms that mathematics "has been constructed as the primary gatekeeper ... to future educational and employment opportunities" (p. 278). More subtly, Brown (1997) suggests that even when teachers "encourage students to pursue their own mathematical concerns" as might occur when utilizing non-traditional strategies for teaching mathematics, teachers retain the option of denying students' work as being proper representations of mathematics (p. 1).

Volmink (1994) recommends that there be "a shift from seeing mathematics as involving the 'interpretation of symbolic information' to an emphasis on situating it in the realm of everyday experiences of people" (p. 51). Similarly, Barwell (2008) suggests that students should still "learn to use formal aspects of mathematical discourse" but from a socially discursive position (p. 126). Students' mathematical discourse and thinking becomes more complex over time through cyclical experiences where "new vocabulary is repeatedly encountered and explored in rich, meaningful contexts" (Barwell, 2008, p. 126). Such a shift in the discourses of seeing mathematics differently would necessarily call for a corresponding shift in discourses on ways of being and acting with mathematics. According to Walshaw (2013), "discourses sketch out, for teachers, ways of being in the classroom ... by systematically constituting specific versions of the social and natural worlds for them, all the while obscuring other possibilities from their vision" (p. 76). In this regard, a gate remains closed. Simply put, maintaining the gatekeeper image of mathematics works to secure specific versions of the world of teaching and learning mathematics, while obscuring other possibilities. However, "a door [or gate] can be open or shut, it depends what you want to do with it (Barton, 2008, p. 61).

Context for Research Study

In the study informing this paper, one author/researcher (Nolan), along with a South African researcher and colleague (Junqueira) sought to study what perceptions individual stakeholders, involved in mathematics teaching and learning, hold for the use of Mathematics Specialist Teachers (MST) in Grades 6-8 (Nolan & Junqueira, 2014). In other words, to study from multiple perspectives the phenomenon of MST and how various educational stakeholders perceive this phenomenon. In the larger study, participants from the five stakeholder groups (school administrators, Grades 6-8 classroom teachers, Grades 6-8 students and their parents, as well as pre-service teachers enrolled in the two universities' teacher education programs) were surveyed. This paper and presentation focuses on one particular stakeholder group— the pre-service teachers being educated to teach mathematics at the middle years level (Grades 6-8) in Canada, and specifically in the western province of Saskatchewan.

In the context of Saskatchewan, the education system consists of Grades K-8 (elementary) schools and Grades 9-12 (secondary) schools. Generally, K-8 schools have generalist teachers (a teacher who is responsible for teaching all subjects at a specific grade level) while 9-12 schools have specialist teachers for each subject area. Recently, discourse around new local curriculum, which has a goal of teaching for deep understanding using inquiry-based approaches, and the question of who should teach elementary school mathematics circulates among many of the stakeholders named above. In recent research on the role of teacher education in shaping secondary mathematics teacher identity and agency (Nolan, 2014), novice mathematics teachers reported encountering exceptionally limited skills in, and poor attitudes toward, mathematics in students at the Grade 9 level. One of the key recommendations emerging from that study was that schools should have mathematics teaching specialists at the middle years level. Hence, a study was designed to understand what various stakeholders perceived as the potential role of MST in Grades 6-8, and as identified this paper focuses on one such group of stakeholders: middle years pre-service teachers.

Presentation and Discussion of Data

Approximately 85 middle years pre-service teachers were invited to complete an online survey, consisting of two parts. Part A of the survey focused on current attitudes and practices, including questions about pre-service teachers' experiences of learning mathematics and their goals/priorities for teaching it. Part B specifically addressed the concept of the Mathematics Specialist Teacher (MST). Here, questions focused on pre-service teachers' current understanding of, and comfort with, curriculum; whether they would choose to specialize in the teaching of mathematics if that option were available to them, and their ideas on the benefits and shortcomings of the MST. Due to the length and scope of this paper, the questions used for discussion are drawn from Part A; specifically, on pre-service teachers' responses to the three questions:

1. Overall, would you describe your own experience of learning mathematics in school as positive, negative, or somewhat mixed? Please explain.
2. Based on your experience thus far, do you think mathematics will be easier or harder to teach than other subjects? Please explain.
3. Do you think Grade 6-8 students find mathematics easy to learn or hard to learn? Why?

Admittedly, these questions limit responses through their desire to have respondents select from binary or tertiary options. This design was intentional, positioning pre-service teachers to think carefully about if/why they were drawn to one side or another of the dichotomy. The questions were also worded in this manner to be consistent across the different surveys, especially the survey administered to Grade 6-8 students who, we felt, might be more inclined to respond (and explain the response) when presented with an either-or prompt.

As stated earlier, responses to the three survey questions listed above were viewed through two different researcher lenses: one examines the epistemological question of *what* mathematics is being known, underscoring the role of mathematics as language, and the other epistemological question is that of *how* is mathematics being known, pondering the role of the teacher in student learning. The approach taken in this paper is to present reflections on the survey response data when viewed through these two lenses.

What Mathematics is Being Known: The Role of Mathematics as a Language

The idea that mathematics is related to language or that mathematics functions as a language for the communication of mathematical notions is easy to find within the literature. Oldfield (1996) goes so far as to state that "mathematics can be thought of as a language, just like French or Spanish" (p. 22). Brown (1997) writes that "language is instrumental in developing mathematical understanding" (p. 3). Barwell (2008) confirms that "mathematics is commonly seen as a discipline with no place for linguistic ambiguity … [and] perhaps more than in any other discipline, terms are seen as being precisely defined (p. 118). Thus, perhaps it is logical or understandable in teaching and learning school mathematics to focus on teaching linguistic aspects of mathematics such as vocabulary, definitions, the proper use mathematical symbols and the memorization of such mathematical facts.

In looking at the pre-service teachers' data, the word 'explain' occurs among answers to all three of the online survey questions presented above. It appears that the pedagogy of 'explaining' is a hangover from pre-service teachers' own experience learning mathematics. On the research surveys, pre-service teachers reflect on being students who experienced poor explanations ("I was never taught in a way that I could understand" and "some teachers were not able to explain") and now, as becoming teachers, they reflect on being responsible for producing *better* explanations ("it isn't about explaining again, it is figuring out different ways to explain ideas"; "I do find it difficult to explain things in different ways if students 'don't get it'"; "I think it is harder for me to teach because I want to tell them exactly what to do to get the right answer"). In the minds of these pre-service teachers, explaining occupies a prominent place in teaching and learning mathematics.

In considering the meaning of the word explain, in relation to a linguistic view of mathematics, the data suggests that for some pre-service teachers "mak[ing] (an idea or situation) clear to someone by describing it in more detail or revealing relevant facts" is related, or perhaps most important, to the learning and teaching of mathematics or at least the linguistic aspects of mathematics (http://www.oxforddictionaries.com/definition/english/explain). In unpacking the meaning of 'explain' by continuing with definitions from the Oxford online dictionary, for mathematics to be clear it should be "easy to perceive, understand, or interpret." For mathematics to be easy, it should be "achieved without great effort; presenting few difficulties." If the way to make mathematics clear and easy is to describe mathematics, then one is expected to "give a detailed account in words" of the details (facts). The act of revealing relevant facts implies "making interesting or significant information known, especially of a personal nature" (where a relevant fact is "a thing that is known to be true" and "appropriate to the matter at hand"). In summary, for some students and teachers (including those surveyed), it appears they perceive that learning and teaching mathematics depends upon explanations, easily achievable truths that can simply be revealed if the teacher is knowledgeable.

This formal view of mathematics, reliant upon explanation, is limiting (Barwell, 2008; Renert & Davis, 2010). When only linguistic aspects of mathematics become the focus and when explanation is the preferred method for sharing these aspects, limited mathematical activity becomes stable or traditional in nature. The way mathematics can be 'traditionally taught' might be better understood as the way mathematics is 'normally' taught or learned. For teaching and learning to be 'normal,' it appears that practices should fall within a more narrow range as represented by any normal curve distribution. Brown (1997) warns that "language functions in orienting action within the normative constraints of a given situation" (p.3). However, in this normally linguistic view of mathematics 'novel' dimensions of mathematical practice including "conceptions of mathematics as emergent, embodied, tacit, enacted and participatory" are constrained and can remain hidden and omitted (Renert & Davis, 2010, p. 196). For at least a decade, some mathematics education researchers have suggested that less focus on 'formal mathematics' is necessary in preparing students for the ever-increasing complexities of society (Alrø and Skovsmose, 2004; Barwell, 2008; Renert & Davis, 2010). For these researchers, the language of mathematics is considered more broadly and more encompassing, inclusive of informal mathematical language, dialogue, participation and cyclical tacit experiences.

In expanding the idea of mathematics as (more than) language, the gate should be able to budge; more students and teachers should be able to participate in discussions about mathematics when mathematics moves beyond memorization of established vocabulary, definitions, symbols, facts and procedures. However, the survey responses from pre-service teachers in this study indicate that for some teachers and students, the narrowed and normally linguistic version of mathematics as language is at least most familiar, if not all that is known. The pre-service teacher survey data reflects the same understanding as that in mathematics education research, namely that "the stable, transcendent conception of mathematics dominate mathematics teaching and learning at the moment" (Renert & Davis, 2010, p. 196).

How is Mathematics Being Known: The Role of the Teacher in Student Learning

When the linguistic version of mathematics is the focus in school mathematics classrooms, mathematics becomes, by and large, less accessible to both teachers and students. Teachers and students begin to perceive the subject of mathematics, along with the teaching and learning of it, as comparable to teaching and learning grammar. In fact, research indicates that students' view of the good mathematics teacher is one who explains concepts and executes procedures clearly (Murray, 2011; Walls, 2010). A troubling aspect of this grammar metaphor and language lens is that mathematics becomes less about being a human endeavour of problem posing, solving, and reasoning, and more about making sense of (language) rules and procedures, with total dependence on the teacher to provide careful and correct explanations of these rules and procedures. In studying sources of authority in mathematics classrooms, Wagner & Herbel-Eisenmann (2014) found that "[a]uthority was unquestioned and placed in the teacher and in accepted mathematical procedures instead of being a result of justified statements" (p. 202).

The dependence on the teacher for explanation (and corresponding blame for not having explained things properly) was evidenced in the data when, for example, one pre-service teacher stated: "I found some teachers were not able to explain... and so I had to try and teach myself how to do it". Another pre-service teacher alluded to extensive reliance on the teacher in suggesting that "the teacher can make or break a student's learning experience." With responses such as these, it is no wonder that teachers feel personally responsible for student learning. It is as if the linguistic version of mathematics, being the only version of mathematics known, places teachers in a position of authority and responsibility for student learning. It follows then that good teaching, which 'results in' student learning, is all about getting that language across with clear and correct explanations. In other words, the historical persistence and perpetuation of direct teaching (in the form of clear explanations) as the dominant pedagogy presupposes that a 'good explanation' leads to substantive learning.

Educational Significance of Study and Future Directions

Through these two epistemological lenses of *what* is to be known and *how* it is to be known, we have endeavoured to interpret middle years pre-service teachers' responses to questions on the teaching and learning of mathematics, in particular what they view as 'hard' or 'easy' about the processes. Both lenses return the discussion to how mathematics is NOT being demystified in school classrooms, but instead is being maintained as gatekeeper by teachers' and students' perceptions of what mathematics should be taught and how it should be taught. Access continues to be limited and, even with access, only a limited vision is available for what mathematics is and how it can be known.

In returning to the introduction of this paper, where a middle years pre-service teacher was quoted as saying that mathematics is "not as hard as grammar but harder than social studies" to teach, we reflect on next steps in this research and the implications for teacher education. Firstly, we note that what is needed in working with pre-service teachers is an understanding and recognition that teachers' "own, often troubled relationships with mathematics impact on the ways they interact with learners" (Black, Mendick, & Solomon, 2009, p. 3). More than this, we believe pre-service teachers' understandings of pedagogy, coupled with their understandings of mathematics as a clearly-defined language of rules and procedures, position teachers in a highly regulated environment. Walshaw (2013) proposes that "[t]he teacher's understanding of pedagogy and of the pedagogical relation are both part of a regulatory apparatus, imposing certain meanings that induce that teacher into a particular pedagogical intelligibility" (p. 90). Research drawing on Bourdieu's social field theory (Nolan, 2012) can perhaps help understand the dispositions of new teachers. Using the concept of habitus-field fit, Nolan (2012) draws attention to the highly regulated space where pre-service teachers attempt to negotiate transitions from being students to teachers of mathematics. The analysis points to the realization that mathematics teacher education programs (and, by extension, professional development programs for in-service teachers) are in need of a reconceptualized approach that seeks to disrupt and deconstruct the traditionally performed roles of mathematics teacher and student, while also recognizing the limits of enacting reconceptualized roles within the regulative structures of schools and curriculum.

Asking the question of "[h]ow does a teacher turn herself into a teacher," Walshaw (2013) responds:

> Teachers (as well as others) are not masters of their own thoughts, speech or actions. Their identities are historically and situationally produced by discourses that are often contradictory. The ways in which they teach in the classroom and the ways in which they give meaning to their interactions with students

> are influenced by the discourses made available to them and to the political strength and interest, for the teacher, of those discourses. (p. 80)

With only particular discourses made available to these 'becoming' teachers, in terms of *what* mathematics is being known and *how* it is being known, it is no wonder that pre-service teachers' gaze is limited to envision mere maintenance of the gate to mathematics. The 'explain clearly and correctly' discourse draws teachers to strategies of encouraging their students to believe that they can 'get the right answer' with the proper teacher explanation and student listening disposition. At the same time, teachers and students are expected to accept the reality that mathematics is challenging to learn, tricky to successfully access and navigate, and perhaps not even meant for everyone.

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Long Distance Education: Democratizing Higher Education Access in Brazil

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The authors led a group of diverse students, using long distance technologies to develop mathematical models in relation to their experience with the nationwide protests related to a sudden and steep climb in transportation costs during June 2013 in Brazil. Mathematical modeling became a teaching methodology that focused on the development of a critical and reflexive efficacy that engaged students in a contextualized teaching-learning process that allowed them to become involved in the construction of solutions of social significance. It is important to outline here the theories related to critical mathematical modeling, distance interaction, and transactional distance by using long-distance technologies. This approach coupled with long distance education allows for the democratization of higher education in Brazil.

Introduction

In recent years, Brazil has experienced an accelerated economic growth with accompanying social changes. The country is now the 7th largest economy in the world. It sponsored the 2014 World Cup and it is sponsoring the Olympics in Rio de Janeiro in 2016, and is suffering a tremendous amount of modernization in relation to infrastructure, including that of health and education. Nation-wide a process of upgrading teacher competencies and the training of new teachers on a massive scale is making a difference in school and community quality of life. The most expedient, economical, indeed reasonable method to do this is by integrating the use of long distance and multimedia technologies.

To increase access to a wider audience, we make use of Moodle as the platform and freeware; this has enabled the *Brazilian Open University* system to democratize and increase access to higher education. The study of new educational and methodological proposals became relevant as it promoted social changes resulting from contemporary scientific and technological development. The need to update and upgrade professional development for teachers raises new institutional solutions, methods and resources in order to meet the demand for specialized teacher education programs such as in mathematics as imposed by social and technological developments.

Long Distance Education in Brazil offers teacher education programs for prospective teachers in regions that traditionally have had limited access to higher education and professional development opportunities. This was developed because face to face teacher education programs cannot meet the tremendous need or allow people the time required by traditional instruction to earn degrees. It is is performed by using a variety of technologies as well as special organizational and administrative arrangements between universities and municipalities. Several actions by the Brazilian Ministry of Education and Culture (MEC) were developed for teaching and learning in a long distance modality and forms part of a National plan representing the intention of the government to invest in distance education and a new digital era of informational and communicational technologies that support teaching practices and professional development.

The Long Distance Education program in Brazil aims at meeting the demands of the 21st century. This program is mediated by emerging technologies and methodologies in order to cover all educational levels and social classes. According to the Brazilian Law No. 5622 promulgated in December, 19th 2005, Long Distance Education is characterized as an educational modality in which didactical and pedagogical mediation is facilitated in the teaching and learning processes and occurs through the use of a variety of informational and communicational technologies. In this process, students and teachers develop educational activities in diverse and distinct locales and times.

This context allows federal universities in Brazil to offer *Seminars in Mathematical Modeling* in long distance mathematics undergraduate courses, which are offered entirely in an environment mediated by technology and the internet. The development of the activities in these courses is conducted through the use of the Moodle platform that possesses interactional tools among teachers, tutors, and students. In this regard, the *Centro de Educação Aberta e a Distância* (*CEAD*) at the *Universidade Federal de Ouro Preto* (*UFOP*) have come to integrate instruction, technology, digital media, content and pedagogical methods in order to reach a diversity of students.

In just this university alone, there are over 4500 students enrolled in four undergraduate majors such as Mathematics, Geography, Pedagogy, and Public Administration, CEAD students represent 41% of UFOP students live in three states (Bahia, Minas Gerais, and São Paulo) who access courseware and instruction via 31 *polos* (long distance education centers). UFOP provides one of the largest distance education programs in Brazil.

The Role of Long Distance Education

In Brazil, push back in regards to long distance education is evident, especially in regards to its implementation in higher education. UAB was developed with the mission of providing access to higher education to a population of prospective learners who have not had access to higher education. Article 80 of Law No. 9394/1996, which is the Brazilian guidelines and basis for education, states that the government must encourage the development and diffusion of distance education programs at all educational levels.

Over the past few decades, and in many diverse locations in Brazil, distance education has grown quickly. Beginning initially with the use of mail-order courses, it transitioned to include radio and television. Once associated with mail and printed materials, it facilitated the dissemination and democratization of access and has now moved to the internet and MOOCS. It has become a key element in the democratization for many countries and now allows access to education and professional development opportunities once only given in face to face and to elite members of society. In Brazil it has allowed a portion of the population that traditionally has had difficulty in accessing public education to advance. In a distance education students and teachers are in different locations during all or most of the time in which they either learn or teach (Moore & Kearsley, 2005).

Although this type of education might in some ways hinder traditional teacher-student relationships, strangely enough it also allows students who had never had access to professors or teachers to gain contact. Distance technologies answer the need of a population who deserve initial or continuing education opportunities. Distance education allows for educators and learners to break barriers related to time and space, and allows for interactivity and information dissemination. Many distance education environments are open systems that are composed of flexible mechanisms for participation and decentralization, with control rules discussed by the community and decisions taken by interdisciplinary groups (Moraes, 1997).

This approach allows interactions between teachers who prepare instructional materials and strategies, with tutors, who in the case of Brazil provide hands-on face to face assistance at polos. In Brazil, tutors are tasked to assist students in their activities and tasks, guiding them in their doubts, helping them learn to use search tools, libraries, and offer help in writing and basic math skills. These interactions are triggered by lessons on "platforms" that are virtual learning environments (VLE) and enables the use of technology and the teaching and learning of specific content. These features have enabled the development of large variety of educational methodologies that utilize web interaction channels and aim to provide needed support in the achievement of VLE curricular activities.

Theory of Distance Interaction

Interactional distance learning tools seek to eliminate the gap in respect to the understanding and communication established between teachers, tutors and learners because by either time or geographic distance (Moore & Kearsley, 2005). Distance education is considered an important feature or element of this form of education and is often supplanted by differentiating procedures, instructional and pedagogical tools that facilitate interactions. In this sense, distance education may need to be redefined as a different pedagogical concept for teaching and learning.

According to this theory, there is a real need for distance education students to exercise interactions in VLE that facilitate understanding and the comprehension of new content and activities. This interaction allows students the opportunity to make or answer questions and in most subjects also allows for the expression of opinions. This theory considers that the time and space distances found between teachers, students and tutors needs to be overcome with the inclusion of differentiating instruction and the use of technologies in the process of teaching and learning (Moore & Kearsley, 2005). According to this context, new learning tools are adapted for use in classrooms such as mathematical modeling in mathematics courses.

Theory of Transactional Distance

Before the development of the concept of the Theory of Transactional Distance, definitions for distance education were related to the physical separation of teachers and students. Transactional distance differs from the physical or temporal distance as it refers to the psychological and communicative space that separates teachers from teaching students of transactions triggered in an educational system in distance modality (Moore, 1993), which occurs in planned and structured virtual learning environments.

In the educational process, the Theory of Transactional Distance requires the presence of students, teachers, tutors, and a channel of communication in order to resolve situations of teaching and learning that involve different transactional distances and require different techniques or even specialized forms of instruction (Martindale, 2002). It describes relationships and interactions that exist between teachers, students and tutors and were often established when these individuals were separated by time and space.

However, for this interaction to satisfactorily take place there is a need to discuss the extension of the length of a particular transactional education program, which depends on a set of three distinct qualitative variables: dialogue, program structure and the range of autonomous possibilities for students. It is also emphasized that these many variables are not always technological, as they relate to the interaction between teaching and learning itself (Moore, 1993).

This theory seeks to utilize information, technology, and the inherent communication found in structuring coursework, which prioritizes interactive educational processes centered on the learners themselves. In this educational process, transactional distance is a pedagogical phenomenon and not just a geographical issue. In general, it describes the interrelationship between three categories named dialogue, structure and learner autonomy as well as how these interactions influence the intensity and the quality of the transactional distance.

These theories utilize information technology and communication in structuring coursework, which prioritizes interactive and collaborative educational process centered on learners. This approach investigates the influence that distance education has in the teaching and learning process in curriculum development, and in the organization and management of educational programs (Moore & Kearsley, 2005).

Critical and Reflexive Mathematical Modeling

In the last three decades, critical and reflexive mathematical modeling as a method for teaching and learning mathematics has been a central theme in mathematics education in Brazil. In teacher education programs this is a way to rebuild or restore part of fragmented knowledge students acquired during their previous mathematics learning experiences. Critical and Reflexive Mathematical Modeling has become one of the most important lines of research for processes of teaching and learning of mathematics in Brazil.

This work points out some reasons for teaching and learning of mathematics aimed at solving real world problems that makes use of critical mathematical modeling as a methodology that values and enables connections between mathematics and reality. However, this aspect is not commonly reflected in teaching practices of mathematics teachers. Much of the literature related to mathematical modeling and their critical perspectives contributes to the formation of both critical and reflexive teachers and presents us with opportunities for the meaningful learning of mathematical concepts by students in virtual environments (Rosa & Orey, 2012). As a methodology in a VLE in undergraduate courses for prospective mathematics teachers, it allows for the exploration of issues related to the context and interest of students and thus provides meaning for mathematical content under study.

By using this critical and reflexive mathematical modeling, perspective teachers encourage the examination of a variety of ways in which students develop and use certain mathematical procedures so that they learn to identify and propose solutions to problems faced in everyday life. This process is crucial to the development of an informed, active, and critical citizenship. One of the necessary pedagogical practices for transforming the nature of mathematical teaching is the deployment and implementation of this perspective in long distance mathematics undergraduate courses (Rosa & Orey, 2012). This approach helps prospective teachers to examine, interpret and understand phenomena that affect their daily lives.

Interpreting and understanding these phenomena are due to the power provided by critical and reflexive mathematical modeling, which occurs through the critical analysis of the applications of mathematical concepts during the development of mathematical models in a VLE. Thus, the process of developing mathematical models is not a neutral activity because the solution for modeling a problem situation includes the understanding of how ideas and mathematical concepts are designed in the preparation, analysis, and resolution of these models. Thus, it is important that mathematical results obtained in this process are linked to the reality of the students themselves (Barbosa, 2006).

During the process of constructing a model, it is necessary to describe, analyze, and interpret phenomena present in reality in order to generate critical and reflective discussions about different processes for the resolution of the models, which are prepared by students. Thus, it is important to enable true reflections of reality, which become a transformative action that allows students to practice explanations, sharing their understandings, develop abilities to organize, manage, and find solutions to problems that present themselves (Rosa & Orey, 2007).

This both critical and reflexive discussion triggers a cycle of construction of mathematical knowledge from reality through the process of mathematical modeling. In this process, students develop skills that help them to process information and define essential strategies to perform actions that aim to the transformation of reality (Rosa & Orey, 2007). This kind of discussion provokes in students the ability to comprehend and debate about the implications of mathematical results, which flow from the resolution of problem and situations.

In this regard, critical and reflexive mathematical modeling is considered as an artistic, indeed poetic process because in the process of elaboration of a model, modelers need to possess mathematical knowledge as well as develop a certain a sense of intuition or creativity that enables this interpretation. In this direction, students need to work in a motivating VLE so that they are able to develop and exercise creativity, reflection, and criticality during the process of generation, analyzes, and production of knowledge.

According to this context, mathematical modeling is considered as a learning environment in which students are invited to inquire, investigate, and work with real problems as well as use mathematics as a language for understanding, simplifying, and solving these situations in an interdisciplinary fashion (Barbosa, 2006). In the context of critical and reflexive mathematical modeling process, students communicate by using hermeneutics (written, verbal, and non-verbal communication) that can be developed by using the VLE.

The Brazilian National Curriculum for Mathematics developed in 1998 states that students need to develop their own autonomous ability to solve problems, make decisions, work collaboratively, and communicate effectively. This approach is based on abilities, which help students to face challenges posed by society by turning them into flexible, adaptive, reflexive, critical, and creative citizens. This aspect emphasizes the role of mathematics in society by highlighting the necessity to analyze the role of critical thinking in relation to the nature of mathematical models as well as the function of modeling that solves everyday challenges.

So, when Brazil suddenly erupted in protest, it seemed the perfect opportunity to look at the question of transportation that people were concerned with. Having a number of diverse polos with a diversity of costs, populations and social contexts seemed a rich opportunity, not to be missed. This approach allowed the determination of the main goals for schools that relate to the development of creativity and criticality to help students apply different tools to solve problems faced in their daily lives as well as competencies, abilities, and skills to help them to live in society. This context also allowed for the development of a critical and reflexive mathematical modeling, which enabled students to develop mathematical models related to proposed transportation themes.

The Development of Critical and Reflexive Mathematical Modeling in VLE

Mathematics is often referred to as a language, but it seems that it has become a language that is taught without giving leaners the opportunity to communicate mathematically. It is not until learners reach advanced mathematics that the few that survive this process, are afforded the opportunity to engage in communicating and creating new ideas using the beauty and power found in the language of mathematics. It is no wonder then, that most people detest mathematics. To them mathematics is stuck in endless and boring drills in the use of mechanical and mathematical grammar without being able to write or communicate in this synthetic but powerful language.

In Brazil, a strong culture of inquiry has developed in the mathematics education community by using critical and reflexive mathematical modeling in which students are encouraged to reflect upon, engage in, debate, and dialogue to resolve problems they find in their own contexts. For example, data gleaned from a course offered to mathematics majors in mathematical modeling that made use of an historic event, the nationwide 2013 demonstrations, to develop competency in mathematical modeling and how 110 students in 10 *polos* in two states (Minas Gerais and São Paulo) studied this raise in bus fares in their communities in Brazil and shared their findings with fellow students, faculty, and tutors.

In June 2013, early in the Seminar on Mathematical Modeling, the country erupted in mass demonstrations against the growing problem of corruption and over spending in relation to preparation for the 2014 World Cup tournament. Just in the small college town of Ouro Preto, 10,000 people marched from the university campus to the main square of the city. What sparked this national mass movement was a sudden spike in transportation fares in urban transportation systems. What may seem to those who do not use mass transit as something minor (20 cent rise) created a difficult problem for many who live in the large metropolises of São Paulo, Rio, Salvador, Brasilia, Fortaleza, and Belo Horizonte. Some long daily commutes became R$30 (about U$14) roundtrips five or six times a week and for many became untenable.

Normally a week or so is devoted to bringing consensus with students and generating a number of themes, and to make use of this particular historic circumstance the instructor consulted with the tutors and students and together they agreed that transportation would be the theme. Eight polos were participating in the seminar. The instructor asked the tutors at each polo to organize the students into smaller working groups of 4 or 5 students. Over a period of 5 weeks students were led through the steps, and groups were required to post evidence of their work on line.

Synchronous virtual classes were held. Critical mathematical modeling lessons were transmitted through videoconferences. Lessons were organized and activities and projects were posted in the Moodle platform. Discussion forums were also developed in order to prepare students for the modeling process. By the end of 16 weeks course 4 synchronous meetings were developed in which the elaboration of the mathematical models of each group of students was discussed. The course calendar that contained the description of the course, the terms of the proposed activities, and the dates and times of synchronous was published in the VLE. Approximately, every two weeks there were activities and questions to be worked on by the students and sent to the tutors and the professor through specific links in the Moodle platform.

Although geographically distant from the students during the development of the course, the professor and tutors used Moodle and youtube tools to be connected with them. Pedagogical and didactic strategies were used to promote professor and tutors interactions with the students in order to contribute to the process of teaching and learning the tools of mathematical modeling. The resources used were the discussion forums and videoconferences, which made possible the development of dialogues between all participants in the VLE. In addition to promoting interaction, the professor prepared teaching materials as well as posted information about the structure and policies of the activities available in the VLE.

It is important to highlight the design of the use of digital communication technologies in the development of this course such as videoconferences and VLE guided the selection of procedures and techniques:

Videoconferences

Videoconferences enabled the integration of students, tutors and the professor for socialization and clarification of questionings; which allowed for a collaborative environment for sharing experiences on the proposed themes and promoted students attendance in the polos to develop their modeling projects. The use of videoconference proved to be very effective because it has sufficient teaching resources for conducting synchronous classes. In this perspective, knowledge is translated in a dialogical way so these technological tools can be used as instruments to help students to critically think about problems they face daily.

The Virtual Learning Environment (VLE)

VLE allowed for continuous updates of the course content; the development of discussion forums concerning teaching practices in the critical mathematical modeling process and the elaboration of questions about the pedagogical and technical aspects of this process. VLE also allowed for the integration of students, tutors, and the professor through the tools to deliver messages; the provision of summaries of contents the course; conduction of pedagogical monitoring such as sending messages to all participants and participation in the discussion forums; and technical support such students and tutors access reports in the VLE. According to this perspective, students' engagement with a sociocultural environment helps them to be involved in meaningful and complex activities. It is through social interaction among teachers and students from distinct cultural groups that learning is initiated and established.

The Importance of the Virtual Learning Environment (VLE)

In the mathematical modeling process, the social environment also influences cognition in ways that are related to cultural contexts. In this context, collaborative work through the VLE between teachers, tutors, and students makes learning more effective as it generates levels of mathematical thinking through the use of socially and culturally relevant activities Thus context allows the use of a *dialogical constructivism* because the source of knowledge is based on social interactions between students and environments in which cognition is the result of cultural artifacts in these interactions (Rosa & Orey, 2007).

For example, the results from the study conducted by Orey and Rosa (2014) showed that mathematical modeling became a teaching methodology that focused on the development of a critical and reflexive efficacy that engaged students in a contextualized teaching and learning process, which allowed them to become involved in the construction of solutions to problems of social significance. This critical dimension of mathematical modeling is based on the comprehension and understanding of reality, in which students were able to learn, reflect, analyze, and take actions on their reality. Students explored examples and problems taken from their own reality, which helped them to study the symbolic, systematic, analytical, and critical contexts through the use of technological tools provided in a VLE.

According to this context, critical and reflexive mathematical modeling provides concrete opportunities for students to discuss the role of mathematics as well as the nature of their models as they study systems taken from reality through the use of technological tools in the VLE. It can be understood as a language to study, understand, and comprehend problems faced by community. For example, mathematical modeling is used to analyze, simplify, and solve daily phenomena in order to predict results or modify the characteristics of these phenomena.

The purpose of this modeling process becomes the ability to develop critical and reflexive skills that enable teachers and students to analyze and interpret data, to formulate and test hypotheses, and to develop and verify the effectiveness of mathematical models. In so doing, the reflections become a transforming action, seeking to reduce the degree of complexity of reality by choosing a system that can represent it (Rosa & Orey, 2007).

By developing strategies through technological tools provided by VLE encourage students to explain, understand, manage, analyze, and critically reflect on all parts of this system. This approach optimizes pedagogical conditions for teaching and learning so that students understand a particular phenomenon in order to act effectively and transform it according to the needs of the community. In order to lead students towards the understanding of the critical and reflexive dimension of mathematical modeling is to expose them to a wide variety of themes (Rosa & Orey, 2007). As part of this process, questionings in the VLE are used to explain or make predictions about the phenomena under study through the elaboration of models that represent these situations.

Final Considerations

The study of new methodological proposals becomes relevant because it originates with the ideas regarding social changes resulting from ongoing continuous contemporary scientific and technological developments. In order to enable teaching methods using structured learning materials and existing technological resources, it was developed the long distance learning, which refers to planned learning that normally occurs outside of school (Moore & Kearsley, 2005).

In the last three decades, critical mathematical modeling as a teaching and learning methodology has been one of the central themes in mathematics education in Brazil and has come to offer a way to rebuild or restore what has become for many, a fragmented and meaningless mathematical knowledge. This approach appears to encourage them to develop more informed and research-based opinions in their real life.

Long distance education contributes and can assist students to overcome difficulties regarding the adoption of critical and reflexive mathematical modeling courses because technological tools offered by the platforms such as Moodle are simple and funcional. Through the use of discussion forums and videoconferences, professors and tutors are able to critically analyze interactions enabled by these tools, which contributed to the reflexive development of the elaboration of mathematical models in the VLE.

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Balancing Students' Valuing And Mathematical Values

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The students' influence and responsibilities on content and learning processes are important objectives emphasised in all steering documents for Swedish education. However, results from a large-scale survey exploring what students find important for learning mathematics show that students may not value such openness in mathematics teaching and learning. We found that aligning teaching to students' valuing would rather conserve a tradition of teacher authority. In this discussion essay, these results will be related to the obstacles teachers may experience when fulfilling educational objectives of students' responsibility, participation, and influence on the planning of teaching and learning mathematics.

Conflicting Values in Swedish Mathematics Education

The two most important things for learning mathematics, due to Swedish students, responding to the WiFi-survey (see below), are:

1. Explaining by the teacher
2. Knowing the times tables

These results left us with questions about the importance teachers should assign to students' valuing of activities for learning mathematics. With the result that Swedish students value teacher explanations and times tables as the two most important activities when learning mathematics, how do these answers relate to values? And how will such values allow students' influence on the planning and evaluation of teaching, as stipulated in the curriculum?

We will, in this paper, describe how mathematical values were used as an analytical tool for better understanding tensions between students' valuing when learning mathematics and statements in the

Swedish mathematics education steering documents. Values/valuing act on different levels in the networking practises of mathematics education (Valero, 2004). In order to learn more about students' values in the Swedish mathematics classroom, we participated in an international values-survey, the WiFi-survey (What I Find Important, when learning mathematics) (Seah & Wong, 2012), which will be used as a background of the coming discussion. WiFi is a quantitative web survey, in Sweden distributed to 750 respondents in grade five and eight (eleven and fifteen-year old students). The participating students were asked to value 64 items, consisting of activities from the mathematics classroom, by marking their importance for learning mathematics on a scale, from absolutely unimportant to absolutely important (Andersson & Österling, 2013). However, having used mathematical values (Bishop, 1988) as an analytical tool, we realized that values in different parts of the network were in conflict. Our aim in this paper is to discuss how teachers' possibilities to fulfil curricular goals of student influence on teaching in mathematics are affected by such value conflicts.

Values in Curriculum and Classroom Culture

The Swedish educational system is not value-neutral. It grants students a large amount of influence and responsibility, as being part of civil democratic obligations. Students' right to influence and their responsibilities are clearly stated in the recent Swedish steering documents (Skolverket, 2011, p. 17):

Teachers should:

- take as their starting point that the pupils are able and willing to take personal responsibility for their learning and work in school,
- be responsible for ensuring that all pupils can exercise real influence over working methods, forms and contents of education, and ensure that this influence grows with increasing age and maturity…
- together with the pupils plan and evaluate the teaching, and
- prepare pupils for participating and taking responsibility, and applying the rights and obligations that characterise a democratic society.

Swedish mathematics classrooms can be regarded as cultures consisting of stable values. The steering documents emphasize the importance of general values. Here we focus on influence, participation, and responsibility. Inculcation of the values stated in the objectives, and values of mathematics, is part of entering a mathematics community, and thereby part of learning mathematics. However, since values often are obscure for the participant in a cultural group (Hofstede, Hofstede, & Minkov, 2010), it is difficult to relate events in the classroom to values. Thus, we did not regard values as something that could be derived from direct observations of classroom practices. Instead, we accepted Bishop's mathematical values (1988) as an analytical tool that allowed us to categorise our observations. Together with knowledge of cultural practices in Swedish community (c.f. Andersson & Österling, 2013) and in the Swedish mathematics classroom, this analysis allowed us to discuss underlying factors in order to explain teachers' and students' choices. Bishop (1988) has introduced the concept of "mathematical values" with the purpose to understand the choices of teachers and students in the mathematics classroom. Three pairs of opposing values describe three dimensions of values in mathematics, and the table below is an attempt to briefly present those mathematical values:

Table 1
Mathematical values (Bishop, 1988)

DIMENSION	PAIRS OF OPPOSING VALUES	
Ideological values: the ideology of mathematics	*Rationalism* – reasoning and argument is valued	*Objectism* – symbolising and applying ideas of mathematics
Sociological values: who can do mathematics	*Openness* – mathematics is democratically open for anyone to use and explain	*Mystery* – fascination and mystique of mathematical ideas and their origin
Sentimental values: What sensations mathematics can bring	*Control* - a sense of certainty and power through mastery of rules	*Progress* – the sense of ideas growing through questioning

These categories were used as an analytical tool for describing values in the different networking practices, from steering documents and classroom culture, to the view of the child in Swedish culture.

The aim of the steering documents, to allow and underline the importance of students' influence on the planning and evaluation of mathematics teaching, can be expected to align well with the valuing of *openness* in mathematics teaching. Information about mathematics is easily accessible for students to make use of, since no one "owns" the Pythagorean theorem or other mathematical objects. Thereby, students should be able to use and chose mathematics content from a number of sources, if openness is valued in the mathematics classroom.

Thus, to what extent does mathematics lend itself to such negotiation? The sentimental mathematical value of control focuses certainty and power through the mastery of rules (Bishop, 1988). Thereby, having access to the formula of the Pythagorean theorem will not automatically render the correct use of it. The authoritative nature of mathematics through its "interest in certainty" is central (Wagner & Herbel-Eisenmann, 2013). This authoritative nature is related to the valuing of control (Bishop, 1988). In this way, mathematics is epistemologically different from other subjects in school, since deductive reasoning based on already stated axioms, rather than empirical explanations and students own initiatives, are valued. Thereby, arriving at students' influence on the planning of teaching may be challenging in mathematics. We present three examples from Swedish research, which demonstrate teachers' efforts to meet the expectations of students' influence and responsibilities.

First, a study made by Hansson (2011) shows that the responsibility for learning in Swedish classrooms too often is passed on from teachers to students and their textbooks (Johansson, 2006), by expecting students to work on their own in textbooks. Teachers, in the name of individualized or student-centred learning, too often abandon students who need their guidance. Hansson (2011) showed how especially second language learners suffered in this type of teaching. Instead of improving inclusion in mathematics education, this way of making students "responsible" for their learning widens the gap between students who are familiar with the expectations and discourses within the mathematics educational culture, and students who do not have access to the particular classroom culture. However, there is nothing in the way these teachers chose mathematical activities that can be related to the valuing of openness in mathematics. It could be the case that students are expected to take responsibility in a classroom culture of mystery, where the origins of mathematical ideas and procedures are hidden for students.

Our second example is a study by Lingefjärd and Maier (2010), where interactions between secondary students and their teachers in a mathematical modelling process are compared. A modelling process can be expected to be an open activity, where students develop mathematical models for describing real world phenomena. In one classroom, a group of students wants to discuss their formula with their teacher, who responds: "Well, if it is your formula, then go ahead and explain it!" (p. 103). When researchers asked further about this interaction, the teacher quoted the Swedish steering document (Skolverket, 1994), which states that students should learn to "work on their own". However, it became obvious that these students neither benefitted, learned mathematics, or to work independently from their interactions with their teacher. They did not have access to the knowledge required for taking responsibility for learning; it rather became evident that allowing students to take responsibility will not take all responsibility away from the teacher. Wagner & Herbel-Eisenmann (2013) discuss both the authoritative nature of mathematics, and how teachers, as being the more knowledgeable, need to handle authority at different classroom levels:

> It is important to be *an* authority in mathematics and to be in authority to some extent as a teacher, but it is also important to establish a routine in which each student sees him/herself as *in authority* of his/her own learning so that s/he too could become *an authority* in mathematics" (Wagner & Herbel-Eisenmann, 2013, p. 8).

In the examples above, the teachers resigned as authorities before the routine of students being authorities of their learning was established.

In our third example, we discuss a research project where Swedish upper secondary students became agents for their learning through taking responsibilities for the planning of mathematical projects and assessment, and thereby fulfilled objectives in Swedish curriculum (Skolverket, 1994, 2011), again concerning students' influence and participation. Andersson (2011) worked as a researcher together with Elin, a mathematics teacher. The valuing of openness regarding the mathematics content aligned with the aim of allocating students responsibilities for planning and implementations of the projects. Through working consciously in this particular way, initially not recognized by students as part of the mathematics classroom culture, a change developed over time. Recognition is a prerequisite for change and formation of a new discursive practice (Gee, 2011). Thereby, until this personal responsibility is recognised as part of the mathematics classroom, it must be acknowledged that preparing students to participate and take their responsibility needs both time and practice.

The three examples above illustrate how students' influence on teaching might be challenging for mathematics teachers. Students do not automatically benefit and learn more mathematics when responsibilities are negotiated. Students' possibilities for achieving influence and acting in a responsible way are facilitated if students understand and agree on the stated objectives. Mathematics learning objectives may focus the mathematical content, but it may also focus mathematical values. Among the mathematical values, in our examples above, valuing openness seemed important when the teachers' intention was to allow students' responsibility for learning.

Students' Values: Some Results from the Wifi-Survey

The underlying assumption in the WiFi-study is that values guide students' decisions about what is important for learning mathematics. Such values may be mathematical values, general educational values, or cultural values. Mathematical values can be defined as "the deep affective qualities which education fosters through the school subject of mathematics" (Bishop, 1999, p. 2). Values being "deep" suggest that there may be an unawareness of the values held by a person, or values in one's own culture. It is easier to become aware of values in a foreign culture, than to see your own culture's values (Hofstede, Hofstede, & Minkov, 2010). When "education fosters" values, there therefore may be an unawareness of possible alternatives. The fostering of mathematical values hence becomes part of learning mathematics, as enculturation. In grade five or eight, the enculturation and fostering of mathematical values has been going on for five or eight years respectively. Students' answers must therefore be understood as reflecting values of Swedish mathematics classrooms, rather than a metacognitive valuing of the importance of an item for learning mathematics (Andersson & Österling, 2013). Considering the results we got, aligning teaching to what students value as important seems to render a rather "traditional way", or in Skovsmose's (2001) words, teaching within an "exercise paradigm" with a teaching based mainly on teacher instructions, teacher authority and drill learning. Thus, the result we received in the WiFi-study rather raises questions about how students' valuing may improve more inclusive and democratic forms of teaching.

The data analysis from the WiFi-study was made through calculating means and standard deviations. The respondents valued each item on a scale from "Absolutely important" to "Absolutely unimportant". This scale was transferred to a Likert-scale, where "Absolutely Important" was assigned the value 1, and "Absolutely Unimportant" the value 5. "Explaining by the teacher" was most important, with a mean of 1.33 and a standard deviation of 0.70. The second most important item was "Knowing the times tables", with the mean of 1.43 and standard deviation of 0.77. These two results are used as illustrative examples for a discussion of how students' responses can be related to mathematical values. We posed three questions relating to each of the items:

1. How do students understand the question?
2. What can we say about the mathematical knowledge related to this question?
3. How does the question relate to mathematical values?

We start with the second most important item, "Knowing the times tables". First, how do students understand the question? Mathematics teaching does not exist in a vacuum. It is affected by, and at the same time affects, cultural expressions. For example, the Swedish children's books about Pippi Longstocking, a nine-year old very strong girl (first published in 1945) may serve as an illustrative example. Pippi lives on her own without any caretaker. However, her two (well-behaving) friends tell her she needs to attend school. This scene describes the *very first* (and only) time Pippi ever enters a classroom:

> "Hey, everybody," hollered Pippi, swinging her big hat. "Am I in time for pluttification?" (Lindgren, 2007, p. 60)

Pluttification tables, or the multiplication tables, as a properly fostered student would put it, are well known by Swedish students. Swedish children's literature contains a theme of children challenging authoritative structures (Aronsson & Sandin, 2014), and even though Pippi is a fiction she captures the importance of the multiplication tables, and at the same time challenges the motives for school and school mathematics. The Pippi-books are known by (almost) all children in Sweden, and this example illustrates how Swedish children recognise times-tables as being part of the school mathematics. Therefore, we have concluded that students answer according to what they usually do in mathematics, rather than how important it is with respect to their mathematical learning.

Second, what can we say about the mathematical knowledge related to this question? The predominant way of treating times tables in Swedish mathematics classrooms is to automatize the knowledge. Hence teachers may ask children to "Practice until you know the answer even when someone wakes you up in the middle of the night". One explanation might be found in Swedish schooling history. Lundin (2008) outlines the history of Swedish mathematics education, where there were two curricula until 1968. One school curriculum was intended for working-class children, expected to learn mathematics as a skill useful in their future profession. Bourgeois children attended an academic mathematics curriculum, where for example Euclidean Geometry had an important role. Even nowadays, after the school has been united for 45 years, there are still tensions between the utilitarianism of mathematics and the academic view in the steering documents. Calculating skills, as knowing the times tables, still seems to be valued mathematical knowledge.

Third, how do "knowing the times tables" relate to mathematical values? Depending on how the teaching of times tables is enacted, different values will be in focus. Valuing rationalism would be, for example, to focus explaining why $7 \times 0 \neq 7$, whereas $7 + 0 = 7$. Valuing the opposite, objectism, is focusing to learn times tables by heart, as a mathematical object. Another value that could relate to times tables is control, the mastery of rules. From the discussion above, we will consider "knowing the times tables" as related to objectism and control, rather than the opposing values of rationalism or progress.

We now pose the same three questions to the most valued item: "Explaining by the teacher". First, how do students interpret the question? We cannot know if students value the explaining of the teacher when someone raises the hand, individually at the desk, or if it is the teacher explaining in a whole-class discussion. In either case, the teacher is expected to possess mathematical knowledge that can be transmitted to students by explaining.

Second, what can we say about the mathematical knowledge related to this question? Mathematics seems fixed, as always containing a correct explanation and a correct answer, and what education does is to allow students to take part of this fixed knowledge. Questions that would allow students to question or explore mathematics, such as "Investigations" or "Making up my own math questions", are valued less important than "Explaining by the teacher" (the differences in means are significant at the 5% level).

Third, how does "Explaining by the teacher" relate to mathematical values? Striving for a correct explanation or answer relates to the value of control. It could be a way of valuing rationalism over objectism, assuming that the explanation contains questions that asks students to reason, or that the explanation provides examples of reasoning. Both forms can be found in Swedish classrooms (Björklund Boistrup, 2010). It may also relate to the value of mystery over openness, since the origin or explanation of mathematics remains obscure. The teacher might know, and must be the one explaining to students rather than opening up mathematics through questioning.

To sum up, the results presented here demonstrate how Swedish students primarily value control and objectism in mathematics learning. Openness is not valued as equally important. Now we ask ourselves, how can values be part of students' influence and participation in mathematics education?

Values and Fostering

It seems unlikely that students' valuing of "knowing the times tables" would reflect a valuing of the importance of times tables for their learning of mathematics. The activities students' value due to the WiFi-survey seems instead to be in line with the teaching they have experienced from prior school years. Therefore, students valuing of "teacher explaining" and "knowing the times tables" is interpreted rather as a reflection of how mathematics teaching in Sweden often has been conducted. Using mathematical values (Bishop, 1988) as our analytical tool allows us to uncover how values at different levels of the networking practices of mathematics education sometimes are at conflict. We could relate students' valuing to the values of control and objectism, and we have shown that these results align with what is common practice in Swedish mathematics classrooms and in Swedish culture (Aronsson & Sandin, 2014; Lindgren, 2007; Lundin, 2008). Hence, students answering the WiFi-survey have been told by Pippi, siblings, parents, and teachers that times-tables are important for learning mathematics.

The steering documents (Skolverket, 2011) focus student influence, responsibilities, and participation in planning of teaching and their learning, which align with the mathematical valuing of openness. However, students value items that relates to openness as less important according to our results from the WiFi-survey. A contradiction occurs where openness in our curriculum invites students to influence teaching, and students chose not to value openness. The WiFi-survey reveals that aligning students valuing of activities for learning mathematics results in conserving a traditional way of teaching mathematics, focusing objectism and control, or the mastery of rules and right answers. Therefore, teachers who intend to foster another view of mathematical knowledge, such as rationalism or openness, will encounter difficulties, when students on the one hand are valuing the opposite, and on the other hand are allowed to "exercise real influence over working methods, forms, and contents of education" (Skolverket, 2011, p. 19).

As presented above, there seem to be at lest two strategies teachers use to obtain students' influence; they either abandon their authority and expect students to be responsible for their learning (Hansson, 2011; Lingefjärd & Maier, 2010,), or they try to empower students by offering learning experiences where students have agency for deciding on context, evaluation, and assessment (Andersson, 2011). The fostering of mathematical values takes place in the mathematics classroom, in contrast to general values, that are fostered in different cultural contexts (Bishop 1988). It is the mathematics teacher who introduces the students to the culture of mathematics, and thereby to the mathematical values. Two of the examples discussed (Andersson, 2011; Lingefjärd & Maier, 2010) deal with modelling and project work, a mathematical content that easily lend itself to students' influence on planning, and offers possibilities for openness. However, in Lingefjärd & Maier (2010), students seem to have too little mathematical knowledge to be able to solve the task. Thus, working independently and being responsible for learning does not exclude the importance of a knowledgeable teacher. Thereby, teachers earn their authority as being more knowledgeable in mathematics. Students may be invited to be an authority of their learning (Wagner & Herbel-Eisenmann, 2013). However, this process urges a recognition of mathematics as not being an authoritative subject, but an open subject, allowing learners to participate by posing questions and choosing how to use or explore mathematics.

In the research accounted for in this paper, the concept of values has been useful as an analytical tool for understanding conflicts between steering documents and the culture and traditions in the mathematics classroom. We argue that before expecting students to take responsibility for learning, the mathematical values needs to be addressed and negotiated by teachers and students. The valuing of mathematical openness seems important for this purpose. Discussing values in mathematics education brings us all the way back to fundamental questions like what is the nature of mathematics, and what is the purpose of teaching mathematics in school. Indeed, a lot is yet to be explored about the role of values, in the process of fostering students' to become participants of the mathematics culture, as well as for obtaining student influence and responsibility in learning mathematics.

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Ethnomathematical Barters

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An assumption embedded in characterizations of ethnomathematical research, that is widely held by both practitioners and critics of this academic field, is identified. This assumption concerns the relationship between mathematics and culture. We argue that many developments and theoretical conflicts within the field can be traced to that assumption. An alternative approach is proposed to explain ethnomathematical practice, trying to respond to some theoretical critiques and prompting new horizons for the academic field.

Now ethnomathematics….

Since its beginning, ethnomathematics has been trying to find a proper theorization and definition. Almost every researcher has attempted to give his/her personal comprehension about what the subfield is and intends. Although this is proper for a new and growing area of research, sometimes the diversity in methods and approaches is seen as a signal of disorganization, non-cohesion, or absence of a shared horizon. The practitioners declare the need to share the initial conception of ethnomathematics as a research program, but develop it in disparate directions. Even D'Ambrosio has been attempting over the last 15 years to propose a more holistic, transdisciplinary theorization of ethnomathematics, rewriting parts of his seminal statements, proposing not only to consider mathematics, but also knowledge in general (D'Ambrosio, 2001).

In the last five years, new attempts have been made to find a theorization. Miarka and Bicudo (2012) want to consider and to reflect upon the relation between mathematics and ethnomathematics through a phenomenological explanation for academic fieldwork. Rohrer and Schubring (2013) proposed a conceptualization of a theory, by claiming that this theory "needs to be regarded as an interdisciplinary discipline that covers theories from both the exact and social sciences" (p. 78).

Together with such theorizations, since its early attempts, parallel and periodically several critiques of ethnomathematics have been published, pointing to assumed meanings of terms like "culture" and "mathematics". Some of the critiques warn about the inconvenient uses of ethnomathematics in mathematics education (Dowling, 1993; Rowlands & Carson, 2002). Another group of critiques points to the effectiveness of achieving the intended political goals (Pais, 2011; Vithal & Skovsmose, 1997). The second group presents critiques in more constructive and proactive ways than the first one. Both share an external view, insofar as none of them carry out research in ethnomathematics. Only a few internal critiques have been made, for example by Alangui (2010), when he warns about the very old fashioned concept of culture that is commonly used. Knijnik, Wanderer, Giongo, and Duarte (2012) discussed assumptions on the importance of using students' reality for classroom activities.

…has an "intersection approach"…

Without wanting to reduce the diversity of approaches and purposes, the common feature of inner trends and external critiques is the intention of addressing the existence/absence of common things between mathematics and culture (despite the diverse definitions of those terms). By considering the particular culture of a group as one set and mathematics as another set, ethnomathematics as an academic field might be concerned to examine the *intersection* of those two sets. Such intersection can be called the ethnomathematics of that group, or even the mathematics of that group. Whatever the chosen name, and without repairing the possible methodological procedures to perform such examination, or the theoretical considerations that would make it impossible to compare those sets, the underlying assumption is that intersection matters.

It is not difficult to show the presence of that assumption in the growth of ethnomathematics as an academic field. To find how ethnomathematicians have considered such an intersection set as the main part (but not the only one) of their object of study, it is enough to read any PhD thesis that has made an historical account of the field, e.g. Alangui (2010), Miarka (2011) or Rohrer (2010).

It is very illustrative that the journal sponsored by the North

American Study Group on Ethnomathematics states: "The journal's contents examine the intersections between mathematics and culture in both western and non-western societies, and among both math professionals…and non-professionals." It is important to acknowledge that this journal has very broad definitions of culture and mathematics.

Also, in research articles it is common to find expressions like "Every culture has mathematics" (Selin & D'Ambrosio, 2001, p. xvii), "ethnomathematics seeks to revive mathematics *living* in different traditions and cultures, not by considering them to be exotic, but by including them in the new historiography of mathematics" (Rohrer & Schubring, 2013; emphasis added) or even in the dilemma pointed out by Bishop: "Is there one mathematics *appearing in* different manifestations and symbolizations, or *are there* different mathematics being practiced which have certain similarities?" (Bishop, 1994, p. 15; emphasis added). So, there is something in the intersection.

Even historically, we can find a continuity: from early approaches, like Marcia Ascher defining ethnomathematics as mathematical practices of non-literate people, through one reconceptualization of D'Ambrosio saying "Ethnomathematics *is* the mathematics *practiced* by cultural groups" (D'Ambrosio, 2001, p. 9; emphasis added), to contemporary works like Furuto (2014), assuming ethnomathematics defined as "the intersection of culture, historical traditions, sociocultural roots and mathematics" (p. 112). Bill Barton (1996, p. 213) explicitly pointed to the intersection in a diagram when he was trying to analyze the comprehension of the field by three representative researchers over time.

…problem…

There are different ways in which critics and followers of ethnomathematics position their arguments according to the "intersection dilemma", as well as to their personal alignment with the ethnomathematics program. In this section, possibilities and their consequences are considered.

The first possible situation is to subscribe to the ethnomathematical program and also claim that such intersection is not empty, therefore one will try to show within it cultural artifacts and practices as some mathematical notion, skill, or concept that "lies therein". The ethnomathematician's duty becomes one of uncovering that presence and mathematical modeling appears as a natural complement. Therefore, there is a necessity to develop proper methodological tools to do that uncovering, as Ferreira (1994) and more recently Albertí Palmer (2007) intended. If ethnomathematics is theorized using these arguments, the paradox of Millroy quickly arises as problematic. This paradox points out that it is not possible to find any knowledge other than the academic, because researchers will be acting with their mathematical gaze. This is an echo of the anthropological "reflexivity problem" (Woolgar, 1988), resulting in considering ethnography as the proper methodology for ethnomathematics.

A second possibility is to agree with ethnomathematics as a program to investigate knowledge and education, but consider empty the mentioned intersection, due to nonexistence of the category "mathematics" in some cultures. This posture focuses on how knowledge is developed in different cultural groups, by recognizing how it affects and is affected by educational discourses, asking as Lizcano did, "what can we see if, instead of looking at popular practices through 'mathematics', we look at mathematics through popular practices?" (Lizcano, 2002). This positioning puts in doubt a pre-eminence of mathematics as a superior knowledge over the others. Conversely, efforts to embed holistic knowledge into the restricted boundaries of mathematical discipline are rejected. This position seems to be a subtle essentialist view of culture because it leaves unexplained why a culture that does not have a category of mathematics would not be able to understand the existence of such a category in other cultures, or at least to create an inner explanation about mathematics.

A third option presents similar reasons to consider the intersection empty, but differs from the second option because it does not follow ethnomathematics as a program to expand the social understanding of mathematics as part of culture. In this posture it is common to use an argument of authority: cultural practices are not mathematics because they are not developed within a scholarly context. They have not been legitimized by one particular institution (i.e., the academic community of teachers, mathematicians, universities, journals). And, because of that, they lack a "warranty" certificate. This attempt to create an essence for mathematical knowledge using the hegemony of one particular group, was summarized by Rômulo Lins with irony:

"mathematics is the thing made by mathematicians when they said that they are doing mathematics" (Lins, 2004). Although such a definition cannot be contested by its circularity, its self-sufficiency is at the same time its big weakness, as it implies an unacceptable omission: those institutions are cultural and historical. Lins reminds us how the professionalization of mathematics appears only in the 19th century and most things recognized as mathematics before that time hardly satisfy current standards.

The last possible position is less radical, as far as it recognizes that mathematics can be present in several cultures (i.e. non-empty intersections). This account of the development of the discipline is driven by a hierarchical model, believing that the world has adopted conventions of mathematics, "because they have been sifted and tested and refined within the crucible of practical experience, which yields neither to passion nor to ideological persuasion" (Rowlands & Carson, 2002, p. 86). In such a model, mathematical knowledge has been evolving constantly in a universal process of improvement that transcends civilizations. If such an approach is accepted, any strong review of the history and epistemology of mathematics is impossible and ethnomathematics has nothing worthy to do. Its duty would be, then, to fill in minor details of how the one and only possible rationality was improved across space and time until now, as an inevitable fate.

Whatever the case, either for or against an ethnomathematical program, for all four positions it is natural to operate in terms of intersections between mathematics and culture, because they refer mainly to what is (or has been) mathematical, instead of what could be.

We consider the intersection problem a mistaken dilemma, responsible for the criticism received and also for the growing "domestication" from which ethnomathematics has suffered in the last decade, as Pais (2012) attests.

…that can be changed…

We want to develop an alternative approach to theorize ethnomathematics that goes beyond the "intersection" problem, by taking into account several processes of multilingualism (Barton, 2008; Caicedo et al., 2009; Cauty, 2001; Meaney, Trinick, & Fairhall, 2011) like an impulse to reflect upon the possibilities of developing dialogic processes within cultural groups around the very concept of mathematics, as well as its educational implications and political uses. We also consider the political views of Knijnik et al. (2012) and Alangui's methodological contribution about Mutual Interrogation (Alangui, 2010), because they give a central role to the interactions, despite not working directly with linguistic issues.

The task is not to discover or find elements within the intersection of mathematics and culture, but to create links between them. When the Māori Language Commission proposed terms to be used within schools (Meaney et al., 2011), by taking care of sensible features of the Māori language and their cultural heritage, they were creating a new knowledge that did not exist before. Those terms expanded boundaries in mathematics, culture, and language simultaneously.

Even if a more classic ethnomathematical work is considered, we can find in the work of Gerdes (2007) the same type of movement, when he proposed new mathematical ideas inspired by African cultural practices. Eglash recognizes it:

> Ethnomathematics of indigenous societies is not limited to direct translations of western forms, but rather can be open to any mathematical pattern discernable to the researcher. In fact, even that description might to be too restrictive: previous to Gerdes' study there was no western category of "recursively generated Eulerian paths"; it was only in the act of applying a western analysis to the Lusona that Gerdes (and the Tchokwe) created that hybrid. (Eglash, 2000, p. 17)

The basic idea is to provide an interpretation for the practice of ethnomathematical research, like intentional and deliberate processes, that generate connections between mathematics and culture, in a strongly non-essentialist understanding applied to both constructs.

This approach assumes a different role of the researcher, from one who *looks for* something hidden and pre-established, to one who *creates* representations and meanings. With such consideration researchers can be found on both sides, not only the academic one. In the same way that Chambers (1996) proposed in post-colonial studies and Cauty (2001) explored in ethnomathematics, practitioners and knowledge-holders become researchers as well, with their own agenda. As a consequence, the intended links are not only one way, creating mathematical interpretations of cultural practices, but also providing culturally grounded explanations of mathematical practices. This last part puts forward a relationship between academic researcher and communities far from the realm of ethnography.

We will explain the idea of "creating links", using an example that Alexandre Pais proposed to criticize ethnomathematics. He imagined a group of indigenous people observing students in a mathematics classroom where the topic of the day was Pythagoras' theorem. After some time watching the students:

> They realise that what the students are doing while seated at tables with pens in their hands solving exercises on a sheet of paper *is actually* the construction of a house. Why does this sound absurd? Why is the direction of research always one of going to the local communities to recognize as mathematics what these people are doing? (Pais, 2013, p. 3 emphasis added)

As we argued, it is irrelevant if that mathematical practice "is actually" the construction of a house or not. Certainly, it could be less problematic if that group says something like "this exercise *looks like* the way that we build a house" to put it with Gelsa Knijnik´s proposal of the "family resemblances" metaphor of Wittgenstein (Knijnik, 2012). Nonetheless, the important thing is the act of the group claiming a connection between one system and the other. Pais found his story absurd because he maintains an unchanged colonial relationship on which ethnography relies. In his story, facts have no consequences and there are no interactions between people. He, like many followers and critics, does not conceive of ethnomathematics as a form of barter.

Let us imagine a continuation for the story, decreasing that colonial bias. Someone says that those equations on the chalkboard remind them of the building of an indigenous house. Another replies, "why?" A third says, "well, because we always try to guarantee that those pillars fit into…" and so on. A discussion starts involving different worldviews, with explanations from multiple sources. This conversation is a process requiring collaboration among agents. Contrasting, criticizing, and appropriating ideas about the practice observed, is a barter of insights. This interaction is in itself an educational process that does not intend to arrive at a happy shared end by destroying differences in a common unified knowledge.

This second part of the story does not sound absurd to us, for a simple reason: it already happens. Meaney et al. (2011) have related the challenging process of one Mãori community in New Zealand trying to educate their children in a Mãori-immersion school, highlighting how collaboration becomes central to confront the different challenges that arise in every stage. Gelsa Knijnik related how Brazilian farmers in a settlement discuss the different ways in which the land is measured, contrasting farmers' techniques with official ones used by banks and the state (Knijnik, 1996). Caicedo et al. (2009) reported the experience of an indigenous community in Colombia trying to appropriate mathematical knowledge for their political process of cultural resistance, applying their own idea of research collectively along the way.

Although this new story is very close to the idea of mutual interrogation proposed by Alangui (2010), we prefer to consider that the groups involved are building a relation that creates new knowledge. Coming back to the initial image of mathematics and culture as sets, instead of studying their intersection, it is possible to study the mappings between the sets. We only require knowers of those sets working together. It is important to notice that although such mappings do not belong to any of the previous sets, their existence affects them.

By adopting a posture towards interactions, the theoretical positions that criticize ethnomathematics paradoxically help to cast light on knowledge as a social and historical practice that is able to be changed. Indeed, the definition provided by Lins (2004) entails an invitation to challenge authoritarian efforts, as far as the uses that people give to mathematics cannot be controlled. Every particular appropriation of a concept expands its limits, transforming its meaning with the unavoidable presence of social facts. By using a non-colonial perspective, one that understands power and knowledge as imbricated things, we cannot be passive with arguments of authority.

It is fair to say that this idea is not completely new, since in his breakthrough paper D'Ambrosio (1985) stated:

> We are collecting examples and data on the practices of culturally differentiated groups which *are identifiable* as mathematical practices, hence ethnomathematics, *and trying to link* these practices into a pattern of reasoning, a mode of thought. (p. 47)

Also, Barton cited something similar, when he conceived of ethnomathematics as "a process of the social construction of knowledge at a cultural level" (Barton, 1996, p. 217) and claimed:

> Ethnomathematics does create a bridge between mathematics and the ideas (and concepts and practices) of other cultures. Part of an ethnomathematical study will elucidate why those other ideas are regarded as mathematical, and therefore why they might be of interest to mathematicians. Such a study creates the possibility both of mathematics providing a new perspective on the concepts or practices for those within the other culture, and of mathematicians gaining a new perspective on, (and possibly new material for), their own subject. (Barton, 1996, p. 216)

The difference stresses the central role that is proposed in the awareness and political intentionality in making such connections, the requirement to involve different voices to make possible the links, and also to notice that such processes can be considered educational beyond school or curricular boundaries.

…to a nicer problem!

The aim of expanding the social understanding of concepts like mathematics or knowledge becomes clearer, as far as the object of study of ethnomathematics is no longer the intersection, but the connection that can be built. Instead of previous and pre-established things to be uncovered, we might look upon the multiple and unexpected possibilities to be developed.

If, with this approach, ethnomathematics can solve the "reflexivity problem" posed by the Millroy paradox, then a "symmetry problem" arises, by considering that those negotiations and new meanings imply active participation of different stakeholders, within long term processes. How can such a thing be guaranteed? How can cultural groups be interested in establishing such processes? All the examples provided here are embedded in broad political projects of organized communities, pre-existing the particular research project. How can such dialogue processes be accomplished with non-organized groups? Can that happen in the limited space of a school system?

More considerations can be made around the features of such interactions. How much time does a process of dialogue require? What types of instances are demanded? By following the path of a mutual interrogation, how can the other research on its own terms? How are its questions actually from "it" without mimicking a traditional researcher? What kind of results can we expect? How can these be validated? Naturally, this demands a new role for ethnomathematicians. How could it be?

Many of these questions require an analysis that goes beyond the scope of this text (the approach presented is being developed in the doctoral thesis that Aldo is doing under the supervision of Paola), or cannot be answered directly. Nevertheless, they configure a promising landscape for the approach; as long as they emphasize a condition of inherent uncertainty for every ethnomathematical piece of research. This vision of ethnomathematics as a process could restore the seminal impulse to reveal the historical and cultural grounding of mathematics, as well as reinforce its critical positioning in the relationship between power and mathematical knowledge.

Conclusion

Ethnomathematics should not observe only the past, but look towards the future. It does not need to be concerned with how others build their knowledge, to better understand western knowledge, but to be engaged in changing the accepted body of knowledge. That "broader vision of knowledge", that D'Ambrosio claims, cannot be static, but dynamic.

This paper retakes an old idea that considers mathematical discipline as central to ethnomathematics. Certainly, we could have made more or less the same argument by substituting "mathematics" for "western/academic knowledge", but we preferred this way because it is precisely with mathematics that comparisons and links are more heretical. If mathematics is left out of the focus of debate, the ethnomathematical field is subsumed in a general discussion, losing its strong point of criticizing epistemology and the education of mathematics.

Ethnomathematics is not intended to empower people because their culture is now one step up, closer to the divine conventions of mathematics. Instead, mathematics becomes demystified when it is moved one step down, closer to mundane affairs. For that reason ethnomathematics can change mathematics in an epistemological dimension.

This paper argued: *currently ethnomathematics has an "intersection approach" problem that can be changed to a nicer problem!* This content is a way to invite interplay with the dynamic condition of culture and mathematics, instead of merely watching it. Mathematics is the thing that our efforts sculpt. So this can clarify as much as possible, that the problem is not to perceive the difference, but what to do with it. How can we live with, and through, the difference?

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Soundings: Toward Mathematics and Peace

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This paper explores why and how the use of mathematics has contributed toward peace and violence, and how we, as educators, can resist the negative tendencies, and, in our teaching, work toward peace.

Introduction

From the beginning of the modern scientific project, the measurement of the world, and its analysis through mathematics, have been central to the scientific project. Descartes developed Cartesian geometry as a tool for scientific knowledge. Brahe's empirical research was conducted through careful enumeration of the positions of the planets. Kepler systematized this research into his three mathematical laws. Newton then invented calculus in part, to rigorously prove and explain, Kepler's Laws.

Because of this close connection between mathematics and the other STEM fields, science, technology, and engineering, mathematics educators contribute to this scientific project by training future scientists and engineers in mathematics—particularly when we teach at a university level. And even when we teach students who will not themselves be scientists, we introduce our students into a society that values mathematics in large part because of its importance in the other STEM fields, and inspire them to share those values. To ask then what it is for mathematics educators to attend to the ethical dimension of teaching mathematics, particularly, to the uses of mathematics for peace, and for breaking peace, is to ask how this larger scientific project is tied to peace.

The ethical responsibilities of mathematicians and mathematics education have been stated with utmost clarity by Ubiratan D'Ambrosio:

> It is not sufficient to say, as it is common in our profession – — indeed, in every profession – — that we are fulfilling our commitment and responsibility to mankind "By doing good Mathematics" or "By being a good Mathematics teacher" But doing good mathematics should be complemented with the question "What will be done with the Mathematics I am helping to develop?" and a good mathematics teacher must always be asking "How will my students perform? Will they be conscious of their moral commitment in their professional life?" Our responsibilities include the uses society makes of our intellectual production and what is the influence we have in the behavior of future generations. (http://en.wikiversity.org/wiki/Ethics/Nonkilling/Mathematics)

In his *Discourse on Method,* Descartes set the program for modern science. "It is possible to arrive at knowledge highly useful in life… by means of which, knowing the force and the action of fire, water, air, the stars, heavens and all other bodies that surround us, as distinctly as we know the different crafts of our artisans, we might also apply them in the same way to all the uses to which they are adapted, and thus render ourselves the lords and possessors of nature." (Part VI). He goes on to call for people to sacrifice their own personal agendas for the sake of the good of the human race, particularly through medical research.

This quote clearly articulates the scientific project, and the goals and hopes entailed by the project. Most prominent is the wholly admirable attempt to be of true service to the neighbor, and to sacrifice personal interests for the interests of the whole of humanity. That is to say, to pursue universal peace.

And many of the positive goals of science and mathematics have been realized. Infant and maternal mortality are significantly down. Many ancestral diseases have been turned back and destroyed. Smallpox is no more. Leprosy is curable, and, in much of the world, is not a scourge. Plague is, in much of the world, no longer a threat. Rabies has a vaccination, and, in much of the world is hardly a threat. And though it is indeed unjust that gains have inequitably favored the wealthy nations, there is some hope that the progress can and will be extended to the rest of the world.

However, the quote also brings to light the negative aspects of the scientific project: First, and this will become clearer through interaction with Arendt, the relationship between science and craftsmanship. Second, a selfish domination of "them"—nature—for our good.

In the twenty-first century, this second problem is clear even when "they" are nature: The ecological destructions wrought by our scientific, technological society defy imagination, and through global warming, civilization itself—perhaps, if scarcer resources bring nuclear war, life itself—is threatened. Nor have "they" always been nature. Science has equally been used to help: We Americans through the domination of the rest of the world—thus the U.S. Military Industrial Academic complex (Giroux, 2007), Hiroshima, etc. We Germans, through the domination of the Jews. We White People, through scientific racism, and the domination of darker peoples.

And, even when "we" have attempted to help "them", we have often done so with the assurance that we know better than they do, and so have imposed ourselves on them, for instance, in the destruction of traditional Native Farming practices through the mathematization and rationalization of farming (Brandt, 2009; c.f. Eason & Robbins, 2011) or of Palestinian culture (Fasheh, 2012). Or, for that matter, imposing the vision of the elite intellectual Europeans on the European laborers and workers (Sturt, 1963). Indeed, Bishop (1990) claims that since "Western" mathematics is a particular culture's product, but claims universality, "'Western mathematics'…[is] one of the most powerful weapons in the imposition of western culture."

These beneficial and negative poles of the scientific project raise the important question: How can we, as educators, continue to initiate students into what is good in the culture of mathematics and science, while resisting the bad. That is, how can we, "fulfill, as Mathematicians and Mathematics Educators, our commitments to humankind?" (D'Ambrosio, 2011, p. 125).

This question is particularly pressing for teachers in the United States, where the education agenda is ultimately set by the desire to out-compete our rivals. A desire even President Obama seems complicit in, for instance, in his speech introducing "Race to the Top", the Federal initiative behind Common Core, he summons us to educate well, because "countries that out educate us today will out compete us tomorrow."

To begin to answer the question of our ethical duties, this paper examines more closely the nature of the domination of nature inherent in the scientific project, so that the root of the evils can be seen more clearly. After exploring the nature of science in a little more depth, this paper asks what sources there are for remedying the problem. To that end, non-“Western” philosophies are inquired, first, in hopes that they might have solutions to the problems “Westerners” have created, and second, in hopes that by the very act of, as teachers, attending to the other, and opening ourselves up to new perspectives, we might encourage our students to do the same.

Science and Conflict

While there is much disagreement about the nature of science, one theme is particularly common, occurring in Henry, Arendt, Heidegger, American philosopher of technology, Albert Borgmann, and mathematicians Davis and Hersh. As Henry (2012) argues, science externalizes things, abstracting them away from the felt and experienced life-world, and thereby, transforms them into items for manipulation.

This theme is explored from a very different perspective, but in considerable depth, in Arendt’s *The Human Condition* (1998). She distinguishes between three different types of human activity: labor, work, and action. Labor is the production of food and the other necessities of existence, “man’s metabolism with nature” (p. 98, quoting Marx). Especially important, in labor, the distinction between ends and means doesn’t make sense: Does the laborer work so he can eat, or does he eat so he can be strong and work (p. 145)? This is in contrast to work, or craftsmanship. Here, the distinction between ends and means comes to the fore: The legs of a table are made with the goal of a table in mind. Work is good, and necessary, when a part of human society, particularly (and this is me, not Arendt), when it is intimately connected to beauty. However, when it becomes the driving force of society, as it is in our scientific, Capitalist society, abstract and arbitrary value, for consumption, replaces the intrinsic worth of items (p. 166)—and dangerously, this principle is then applied also to society, that is, to other humans, instrumentalizing and colonizing them.

Arendt’s claim that a society based on making transforms the worth of objects into external, arbitrary exchange value is similar to the conclusion reached by Albert Borgmann (1987) in his analysis of technology, and its benefits and pitfalls. According to Borgmann, technology transforms what used to be a location of communal life into an external commodity for consumption. Thus, for instance, the hearth, formerly the locus of family life, both as a family huddles around it to escape from the cold, and as they order their various activities toward it, is replaced by the commodity "heating". This, obviously, has advantages, but nevertheless, it serves to sever people from nature, transforming a locus of integration into an external, abstract, commodity, subject to human domination.

Finally, turning more directly to Mathematics, mathematicians Davis and Hersh (1986) argue that when mathematical abstraction is applied to humans and to human processes "dehumanization is intrinsic to the fundamental intellectual processes that are inherent in mathematics" (p. 283) precisely because it abstracts away from the individuals and the groups, reducing them to abstract, impersonal numbers. I would add that the same is true regarding nature: Mathematics reduces our animal and plant neighbors to numbers. Thus, in the act of doing mathematics, the mathematician or mathematical society is cut off from them, and them from our community. This is not to say that all such attempts are illicit, but that it becomes problematic when the useful abstraction inherent in mathematics is not treated as an intellectual exercise, to be passed through, but as a final statement about reality.

In all these explanations of the modern scientific condition, as in Descartes' quote, the power of science exists in the separation of us humans from "them", from nature. If we, as educators, wish to address the ambiguous nature of science, particularly in our teaching, we should perhaps begin by attempting to unify us and them—by making them, once again a part of the community that includes, but is not coextensive with, humans, and inspiring our students towards the same. For this task, "non-Western" metaphysics are particularly helpful, particularly, indigenous sciences, and non-dualistic philosophies like Daoism.

Though it would be wrong to suggest that there are no resources within the "Western" traditions, looking abroad for cross pollination is important for several reasons. First, listening to "them" is an important aspect of overcoming the dualism between us and them that I have argued is important to the doing of science. Second, throughout the European tradition, helpful and damaging strands are entangled, and by looking away from Europe, we do not need to spend quite so much time disentangling. Finally, one of the problems that mathematics currently faces is that though it claims universal jurisdiction, it is the work of a particular people, and so, Eurocentric. By listening to non-European traditions, we can hope to begin to overcome that Eurocentrism, and to inspire our students to follow that same course.

It also needs emphasized that this should not be taken as dividing the world into Westerners and non-Westerners, as if there were two different peoples. Rather, this movement outside is motivated by a recognition that though each people has their inheritance of wisdom, the current intellectual map a little too closely resembles a Mercator Projection, in its overemphasis on Europe and North America. And we can all gain by listening to the wisdom of people from the corner of our global map, and thus, shifting toward a more just projection. However, just as, in either case the map is a map of the world, so too here, it is the human community that stands to benefit from hearing the inheritance of the "corners".

Before beginning to sketch the nature of a potential solution, a more thorough investigation of the nature of the competition and desire is required. For that, I turn to Girard (Girard, 2009; Palaver, 2013) and Dupuy (2013).

Drawing heavily off literature, particularly Dostoevsky, Girard has developed an anthropology that attempts to describe the roots of violence in society. According to Girard, humans desire deeply, but we do not know the object of our desires. We learn the object of our desires by imitating the desires of others, particularly by those close to us. This inevitably leads to conflict and violence, as imitation of a friend's desires puts one into direct rivalry with the friend, since both cannot possess the object of desire. Girard's claim is then that this mimetic rivalry drives individuals, and groups, into deep, irresolvable conflict: For instance, on his analysis, World War One was the result of a mimetic crisis, as Prussia attempted to imitate Napoleon's military success, and France, in turn, attempted to imitate Germany's nascent success, propelling the two nations into inevitable conflict. Or, in U.S. international policy, a perceived threat of out-competition from other nations, is met by a call for further competition—even by President Obama, as his speech introducing Race to the Top indicates.

Various mechanisms have been developed by different cultures to contain this violence, notably in our society, money. As Dupuy

(2013) argues, we each imitate our neighbor's pursuit of success and monetary accumulation, thus fueling the drive for consumption, and for the unrestrained growth of the national economy. People and nations follow their neighbor in projecting forward a more prosperous future, and then act, collectively and individually, to pursue that goal. This imitative pursuit of money inevitably will bring conflict since resources are scarce—and disappearing, due to global warming. Furthermore, in teaching science, we inspire students to pursue science, and thus to participate in this problematic pursuit of national and individual "success".

Recommendations for Education

This is enough to begin to give outline to the sort of strategy that educators could use in overcoming the negative, violent tendencies in mathematics and science, as we educate our students. We need to find a way to treat nature, or other peoples, not as "them", but to view us all as members of one community—or, following Derrida (1999), to distinguish between "us" and "them", but to recognize that we are fundamentally the guests of nature, and nature the host, and we are called to be hospitable. However, to do this merely as private individuals, is to play into the hands of the competition: The problem is corporate, and what is needed is not an individual response, but a corporate response stronger than the corporate drive for competition. One aspect of the corporate nature of this project particularly important to educators, is the inspiration of students to participate.

In order teach our students to desire both to learn from "them", from colonized peoples, rather than mastering them, and to treat nature as part of the community that Includes, but is not limited to humans, mathematics educators can turn to several sources: First, to the knowledges and sciences of indigenous and neo-indigenous peoples. ("Neo-Indigenous" is a term for "non-Western" civilizations coined by Aikenhead and Ogawa (2007) to emphasize the many commonalities that sciences have in these civilizations and in indigenous communities). Second, to appreciation of the beauty inherent in mathematics. Finally, to "non-Western" philosophies, particularly, Daoism.

Indigenous and neo-Indigenous sciences are important for two reasons: First in attending to other peoples' understandings of nature, we model the inclusion of a people often treated as "them" among "us"—and, following Girard, inspiring them to desire to join in our attempts. Second, these Indigenous peoples have created a rigorous, empirical, form of knowledge, which is sustainable, and treats nature as part of a community with humanity, not as a separate "they" (Aikenhead & Ogawa, 2007).

Concrete work on how to integrate Indigenous understandings of nature, or more accurately, indigenous ways of living in nature, into the science curriculum, has been carried out and educators could begin to draw on that research. For instance, Waiti & Hipkins (2002) suggest teachers contrast "Western" Science with other forms of knowing nature, and offer three suggestions for doing so: School curricula could be broadened to include indigenous ecological knowledge; students could use insights from other peoples to critique "Western" Science; and indigenous ways of knowing nature could be valorized separately from "Western" Science and the importance of "Western" Science correspondingly downplayed. And Belczewski (2009) describes her own process of and struggles with decolonizing herself as she taught science to First Nations students in New Brunswick.

There does not seem to be as much work on bringing indigenous and neo-indigenous mathematics into the "Western" classroom—particularly if not only the mathematics but the connections between mathematics and the indigenous ways of living in nature are emphasized. This is even more pronounced at the college level. I offer here three concrete suggestions. First, our curricula could incorporate Indian infinite series (Raju, 2007), contrasting the forms they take in Indian and European cultures, and the different values reflected in the two mathematical traditions. Second, rather than presenting mathematics as an abstract, universally true system, the stories and histories of European mathematics could be more present, with the goal of making the lack of "non-Western" perspectives at least noticeable, and the silence of "non-Westerners" audible. Third, Magrebi symbolic algebra (Schubring, 2008) could be incorporated into the algebra curriculum, and the values of Islamic mathematics contrasted with the values of European mathematics.

Second, and related, (Eason & Robbins, 2011) educators could draw attention to the beauties of science and mathematics and of the natural realities investigated in science (Winston, 2010), and model appreciation of the beauty for students. Beauty is a major component of much European science (Lang, 1985), however, it is often lacking in our curricula. An emphasis on beauty is important because in our recognition of something's beauty, we unite ourselves with it, rather than treating it as external.

However, important as these suggestions are, they don't have the corporate power to oppose the corporate system of competition. For that, we need something that is both powerful, and yet, paradoxically, devoid of power. A power that could control the world, yet does so, precisely, by undertaking nothing—since otherwise, we merely enter into the system of competition. As far as I know, the philosophy which has been both powerful, and yet, has advocated the power of emptiness and nothingness, as it were, is Daoism (Laozi, 2010). And Daoism does offer a power, but a power based on "undertaking nothing" (Dao de Jing, 48), whose method is based on weakness (Dao de Jing, 40), and, particularly relevant for the competitive drive for acquisition, claims "There is no calamity greater than now knowing what is sufficient, there is no fault greater than wishing to acquire" (Dao de Jing, 46). So finally, educators can attempt to find ways to incorporate Daoist perspectives in teaching, perhaps through the Contemplative Education movement (Gunnlaugson et al., 2014), and the incorporation of silence into the classroom (Acheson, 2007; Senechal, 2014).

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Ethnomathematics: Connecting Cultural Aspects of Mathematics through Culturally Relevant Pedagogy

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The implementation of Culturally Relevant Pedagogy helps to develop student intellectual, social, and political learning by using cultural referents to acquire knowledge. It uses prior experiences of minority students to make learning more relevant and effective in order to strengthen connectedness with schools. Presently, there is a need to examine the embeddedness of culture in mathematics, which takes on the cultural nature of knowledge production into the mathematics curriculum. Ethnomathematics and culturally relevant pedagogy-based approaches to mathematics curriculum are intended to make mathematical content relevant to students. The objective of this theoretical article is to discuss the principles of culturally relevant education according to an ethnomathematical perspective.

Introduction

Major demographic shifts in most countries have led to increasing numbers of culturally, linguistically, socio-economically diverse, and minority students in the educational systems. For example, in the United States, the passage of the *No Child Left Behind Act* (NCLB, 2001) and the resulting requirement that all schools report disaggregated data have brought increased attention to achievement gaps that have persisted for years between minority students and their mainstream peers (Gándara, Maxwell-Jolly, & Rumberger, 2008). According to a wide range of educational indicators including grades, significant inequities continue to exist for these students' scores on standardized tests, dropout and graduation rates, and enrollment in higher education (Education Trust, 2004).

One possible explanation for these gaps may be that the disparities in achievement that stem in part from a lack of fit between traditional schools, in which practice is derived almost exclusively from Western cultures, and the home cultures of minority students. Learners whose cultural backgrounds are rooted in Western ways of thinking possess an innate educational advantage as compared to students from alternative social cases and other cultural backgrounds. In this regard; minority students are required to learn through cultural ways of thinking and practices other than their own (Rosa, 2010).

In the last three decades, theories of culturally relevant pedagogy and ethnomathematics were developed in order to ease sociocultural concerns. The terms culturally relevant pedagogy (Ladson-Billings, 1995) and culturally responsive pedagogy (Gay, 2000) are often used interchangeably. However, we prefer to apply the term *culturally relevant* as it combines an examination of the cultural and socioeconomic influences on the process of teaching and learning mathematics. As well, it includes knowledge along with a commitment to the challenging of social injustices and reflections upon educational challenges by identifying obvious and subtle individual, institutional, and cultural actions that perpetuate social structures. The overall goal of these theories is to empower students through learning activities that help them to develop their literacy, numeracy, technological, social, and political skills in order to be active participants in a democratic society. It is also important to emphasize that a culturally relevant pedagogy studies the cultural congruence between the cultural backgrounds of students, communities, and schools, which form one the principals of an ethnomathematics program. In relation to the pedagogical work in schools, mathematical curricular activities must be relevant to the students' cultural backgrounds.

The views of pedagogy within the literature on ethnomathematics are compatible with work on culturally relevant pedagogies (Hart, 2003) because they examine the cultural congruencies between the community and school. This means that cultural congruence indicates the teachers' respect for social, cultural, and linguistic backgrounds of their students. However, it is equally necessary that school leadership and teachers acquire the knowledge of and respect for the various cultural traditions, languages, and mathematical knowledge of their students so they are able to implement the principle of cultural congruence in schools and classrooms (D'Ambrosio, 1990). On the other hand, since mathematics usually tends to be presented as a set of objective and universal facts and rules, it is viewed as *culture free* and not considered a socially and culturally constructed discipline.

To change this perception, it is necessary that curriculum developers and teachers take into account what counts as mathematics and how this knowledge itself may be related to the norms and values of diverse cultures (Rosa, 2010). If as educators we come to integrate the diverse cultures we encounter in our school communities, then there is a need to create a conceptual framework to make coherent decisions regarding these curricular activities concerning the mathematics curriculum.

In this regard, the aim of this paper is to show the need to examine the ethnomathematical perspective where-in the embeddedness of mathematics in all cultures takes on the cultural nature of knowledge production into a mathematics curriculum. The argumentation is that culturally relevant pedagogies may be considered as an ethnomathematical approach to the development of a mathematics curriculum because they intend to make school mathematics relevant and meaningful regarding the promotion of the overall quality of students' educational experience.

Culturally Relevant Pedagogy and Ethnomathematics: Curricular Implications

An important change in mathematical instruction needs to take place in order to accommodate continuous and ongoing changes in the demographics of students in mathematics classrooms. It is necessary to integrate a culturally relevant pedagogy into the existing mathematics curriculum because it proposes that teachers contextualize mathematics learning by relating mathematical content to students' real life-experiences.

The guidelines of both the National Council of Teacher of Mathematics (NCTM, 1991) and the Brazilian Ministry of Education and Culture (Brasil, 1997) highlight the importance of building connections between mathematics and personal lives and cultures of students. Along with this line, when practical or culturally-based problems are examined in a proper social context, the practical mathematics of social groups is not trivial because they reflect themes that are profoundly linked to the daily lives of students (Rosa & Orey, 2007). In this perspective, students may be successful in mathematics when their understanding of it is linked to meaningful cultural referents, and when the instruction assumes that all students are capable of mastering the subject matter (Ladson-Billings, 1995) such as mathematics.

Curricular activities developed according to principles of culturally relevant pedagogy focus on the role of mathematics in sociocultural contexts that involve ideas and procedures associated with ethnomathematical perspectives to solve problems. The mathematics knowledge found in a culturally relevant pedagogy is perceived as a version of ethnomathematics because *ethno* is defined as culturally identifiable groups with their jargons, codes, symbols, myths, and even specific ways of reasoning and inferring; *mathema* is defined as categories of analysis; and *tics* is defined as methods or techniques for solving problems faced daily. In a culturally relevant mathematics classroom, teachers build from students' previous knowledge (*ethno*) and direct the lessons toward their culture and experiences (*mathema*) while developing critical thinking skills (*tics*) (Rosa, 2010).

The inclusion of cultural aspects in a mathematics curriculum has long-term benefits for student mathematical attainment because these aspects contribute to recognizing that mathematics is part of our daily lives and deepen the understanding of its nature by enhancing students' ability to make meaningful connections. Thus, pedagogical work towards an ethnomathematics perspective allows for a broader analysis of school contexts in which pedagogical practices transcend classroom environments (Rosa, 2010).

Therefore, ethnomathematics presents possibilities for educational initiatives and new curriculum objectives because it is a research program that guides educational pedagogical practices. However, the incorporation of the objectives of a pedagogical action of an ethnomathematics program is a recent field of study that is developing its own identity in mathematics education (Rosa & Orey, 2007). Thus, a dilemma regarding this issue is how to prepare teachers to elaborate curriculum activities based on culturally relevant pedagogy and ethnomathematics (Greer, 2013).

The trend towards ethnomathematical approaches in mathematics curriculum development and culturally relevant pedagogy initiatives reflect a comprehensive development in mathematics education. Ethnomathematical approaches are intended to make school mathematics more relevant and meaningful to students in order to promote the overall quality of education; and to plead for more culturally relevant views of mathematics. For example, it is important to elaborate mathematics curricula that are based on the previous knowledge of students, and which allows teachers to have more freedom and creativity (Powell & Frankenstein, 1997). In this context, it is necessary that teachers value the diverse home cultures of students by explicitly addressing curricular connections to the previous knowledge they bring to school. In this approach, teachers need to teach mathematics in a culturally appropriate manner situated within students' funds of knowledge in order to underscore connections to their home culture and use them to scaffold learning (Moll, Amanti, Neff, & Gonzalez, 1992).

The application of culturally relevant pedagogy and ethnomathematical perspectives in classroom validates and incorporates students' cultural background, ethnic history, and current societal interests into teachers' daily instruction. It addresses students' socio-emotional needs and uses ethnically and culturally diverse materials for its pedagogical action in classrooms (Gay, 2000). In this regard, culturally relevant pedagogy is an educational approach that empowers students intellectually, socially, emotionally, and politically through the use of sociocultural and historical references to convey knowledge, imparts academic skills, and change students' attitudes towards academic instruction (Ladson-Billings, 1994).

This pedagogical approach is achieved through dialogue when community members, teachers, and students discuss mathematical themes that help them to reflect about problems that are directly relevant to their community. In this context, students investigate conceptions, traditions, and mathematical practices developed by members of distinct cultural groups in order to incorporate them into the mathematics curriculum. Teachers learn to engage students in critical analysis of the dominant culture as well as the analysis of their own culture.

We would like now to share one example of how teachers might investigate a common object and integrate this investigation into the school curriculum. We chose quilts here, as they are common folk-art traditions in both Brazil and the United States and because both countries have sizeable Afro-American populations who have made many contributions to cultural developments and in both countries.

Symmetrical Freedom Quilts: A Culturally Relevant Activity Based on an Ethnomathematical Perspective

Throughout time, quilts have been created as a vehicle for sharing family history, a moral message, or as a reflection of important historical and cultural events. Quilts may be considered as cultural, artistic, and mathematical expressions and manifestations of mental models that represent a specific cultural activity. One example of this representation was related to the life of slaves in the United States and Brazil. In this activity we explore the symmetrical patterns found in specific quilt patterns in the United States known as *freedom quilts,* as well, the connections between culturally relevant pedagogy, ethnomathematics, and the tactile craft and art of quilting (Rosa & Orey, 2012).

The study of quilts made by people who endured slavery in the US provides an opportunity to study its history from perspectives that are not well represented in history books. Fabrics used, designs constructed, and stitches made tell stories about the terrible oppression, suffering, and resilience of African-Americans living in that time period. Symmetrical freedom quilts were the physical traces (cultural artifacts) of a people who made a community around the creation of the quilts that expressed their shared hopes, values, and beliefs. In this sense, quilt making was a collective response to their human experience and suffering (Rosa & Orey, 2012).

The story of symmetrical freedom quilts offers a mixture of fact and myth. Its oral tradition may not give us accurate information, but it reflects a greater truth inherent in the pride of the members of this specific cultural group (former slaves). It could be that there was no real role of symmetrical freedom quilts in the Underground Railroad during slavery in the US, as there are numerous debates related to if quilts were really used as directional codes in helping slaves to escape. According to Rosa and Orey (2012), whether or not the story of the symmetrical freedom quilts is true, or it is myth, it is an appealing story and has touched the hearts of many, and presents us with a story with mathematical potential.

The focus of this project is on one important form of communication as used on the *Underground Railroad* by African-Americans who were escaping slavery. This term has come to us from a story of a farmer chasing a runaway who testified that this slave vanished as if they had left on some kind of *underground railroad*, which was used to describe the network of abolitionists and safe houses that helped slaves escape north to Ohio and Canada. Safe houses along the way were known as s*tations*, and those who guided the escapees were called conductors and the runaways themselves were called *passengers* (Burns & Bouchard, 2003).

What it is known is that the *Underground Railroad* was organized by former slaves, freed blacks, and sympathetic whites for the slaves to find shelter, food, drinking water, safe hiding places, and safe paths to follow as they moved to the free states of the north and into Canada. The quilts are referred to as *Freedom Quilts* and they were often hung over a clothes lines, on porches, windowsills, and balconies to signal what to do or where to go by using different designs that indicated directions, safety, danger, clues, and landmarks to guide the slaves to freedom.

The quilts were sewn to serve as coded messages for runaway slaves to memorize. Since most slaves were not taught to read or write in English, they developed an intricate system of secret codes, signs, and signals to communicate with one another along the escape routes. In order to memorize the whole code, a sampler quilt was used, which included all the necessary patterns arranged in the order of the codes they needed to know. Freed slaves traveled from one plantation to another to teach other slaves the patterns and designs and the translation of codes in the sampler quilt patterns (Wilson, 2002).

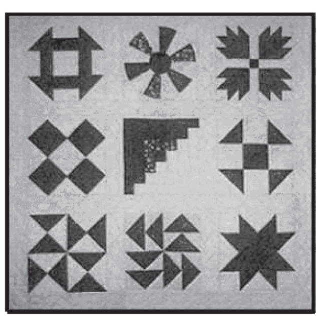

Figure 1: Ozella's Underground Railroad
Symmetrical Freedom Sampler Quilt

Knot-making was also a practice that has an interesting historical background from Africa. Knots were tied in the quilts to encode objects with meaning, messages, and protective power. It is possible then that symmetrical freedom quilts contained ties with knots that were often used to indicate the dates and times for slaves to run away or meet from their working plantation. For example, five knots in the cord meant that they should escape on the 5th hour of the 5th day of the 5th month. As well, if a quilt showed a house with smoke coming out of the chimney it meant that the house was safe (Wilson, 2002) for shelter.

This method of communication was very effective because bounty hunters apparently never caught onto the quilts and their messages (Rosa & Orey, 2012). Symmetrical freedom quilts presented an ingenious, indeed highly creative, and complex way in which to communicate between slaves and safe houses because they did not show any overt connection to slavery (Rosa & Orey, 2012). These quilts may have played a key role in the ending of slavery in the United States, however we do understand that there is little to no corroborating scientific and historical evidence that support these ideas.

The main objective of an ethnomathematical perspective of this context is to understand and comprehend mathematical practices developed by the members of this specific cultural group in the course of dealing with problems faced in their daily lives (D'Ambrosio, 1990). The quilt codes may be considered as mathematical techniques (tics) used by the slaves (ethno) who were trying to manage the problems and activities that arose in their own social and political environments (mathema). These codes were transmitted to relatives of the slaves to their ancestors and across generations (Rosa & Orey, 2012).

The use of culturally relevant pedagogy values the previous knowledge of the members of a given cultural group such as former slaves by developing the process of elaborating mathematical procedures in its different contexts such as political, social, economic, and environmental. In this regard, the mathematics practiced and elaborated by the members of distinct cultural groups, and involves the mathematical practices that are present in diverse situations in the daily lives of members of these diverse groups.

Mathematizing ideas involves connecting the informal mathematics developed in a given cultural group to formal mathematical concepts by using ideas, procedures, and mathematical practices that are used by the member of specific cultural groups. In this context, symmetrical freedom quilt designs such as Shoo Fly quilt block contain geometric concepts like symmetry, similarity, congruence, translations, rotations, and reflections (Rosa & Orey, 2012).

In the context of culturally relevant pedagogy, students can be successful in mathematics when their understanding of it is linked to meaningful cultural referents (Ladson-Billings, 1995). According to this perspective, Shoo Fly is one the simplest traditional patterns. Although Shoo Fly is a basic pattern, its versatility provides quilters with opportunities for creative use of colors, fabrics, and stitching.

Figure 2: Shoo Fly symmetrical quilt block

For example, students mathematize a point of reflection of the Shoo Fly quilt block, which is determined when a figure is built around a single point called its center. For every point in the figure, there is another point that is found directly opposite on the other side of the figure. While any point in the x-y coordinate system may be used as a point of reflection, the most common point used is the origin. In the *Shoo Fly* quilt block, the point of reflection is at the origin of the x-y coordinate system.

By applying the general mapping of transformations in the three points of reflection in the triangle below it is possible to find their images, which are . In this specific case, triangle A'B'C' is the image of triangle ABC after a reflection on the origin of the Cartesian coordinate system. Figure 6 shows the point of reflection of the Shoo Fly quilt block at the origin of the *x-y* coordinate system.

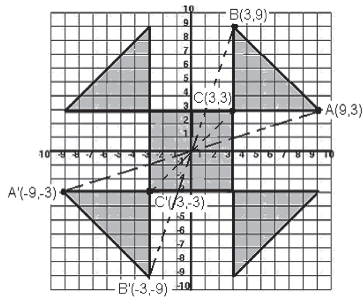

Figure 3: Point of Reflection of the Shoo Fly quilt block
at the origin of the x-y coordinate system

The point of reflection is also called the point of symmetry. In a point of symmetry, the center point is a midpoint to every segment formed by joining a point to its image. The three straight dashed lines that connect *A* to *A'*, *B* to *B'*, and *C* to *C'* pass through the origin, which is the midpoint of each line segment. A figure that has point symmetry is unchanged in appearance after a 180°rotation.

Final Considerations

It important to emphasize that this kind of curriculum may motivate some students to recognize mathematics as part of their everyday life and can enhance students' ability to make meaningful mathematical connections by deepening their understanding of all forms of mathematics. For example, Duarte (2004) investigated the uniqueness of mathematical knowledge produced by workers in home construction industry through the study of mathematical ideas and practices they developed in construction sites. In this study, there was a reflection on the mathematical knowledge possessed by the members of this working class to academically legitimate their knowledge in order to determine the pedagogical and curricular implications that were inferred in the process of production of this knowledge.

The objective of developing an ethnomathematical curriculum model for classrooms is to assist students to become aware of how people mathematize and think mathematically in their culture, to use this awareness to learn about formal mathematics, and to increase their ability to mathematize in any context in the future. This kind of curriculum leads to the development of a sequence of instructional cultural activities that enable students to become aware of potential practices in the mathematics in their culture so that they are able to understand the nature, development, and origins of academic mathematics. Students also value and appreciate their previous mathematical knowledge, which allows them to understand and experience these cultural activities from a mathematical point of view, thereby, allowing them to make the link between school mathematics and the real world. An ethnomathematical curriculum helps students understand the nature of mathematics because it presents us an effective tool that can contribute the learning of mathematics of students (Rosa & Orey, 2007).

The integration of ethnomathematics and culturally relevant pedagogy into the mathematics curriculum focuses on the development of this research area as a process, rather than a collection of facts because it is based on the idea that mathematics is a human creation that emerges as people attempt to understand and comprehend the world around them. Therefore, mathematics can be seen as a process as well as a human activity rather than just as a set of academic content (Rosa, 2010). The implication of this kind of curriculum is not just about the application of relevant contexts in learning and teaching mathematics, but is also about generating formal mathematics from cultural ideas.

Mathematics knowledge in the context of culturally relevant pedagogy can be perceived as an ethnomathematical perspective because teachers build from the students' informal mathematics and orients the lesson toward their culture and experiences, while developing their critical thinking skills. In this context, students are considered as a culturally identifiable group with their own jargons, codes, symbols, myths, and specific ways of reasoning and inferring (ethno) who develop their own categories of analysis (mathema) and apply specific methods or techniques to solve problems faced daily.

Since ethnomathematics studies the cultural aspects of mathematics and presents the mathematical ideas, procedures, and practices of the curriculum in a way that is related to student cultural backgrounds by enhancing their ability to make meaningful connections and deepening their understanding of mathematics. This perspective matches teaching styles to the culture and home backgrounds of their students, which is one of the most important principles of culturally relevant pedagogy.

Her we have offered a brief example of how ethnomathematics can be used to link diverse ways of knowing and learning and culturally embedded knowledge with academic mathematics. Quilt exploration allows learners to explore the rich academic and culturally rich traditions diverse populations. Teaching mathematics through cultural relevant and ethnomathematical perspective helps students to know more about reality, culture, society, environmental issues, and see mathematics in the world around them. By providing students with mathematical content and approaches that enable them to successfully master academic mathematics, an ethnomathematics approach to the mathematics curriculum is considered a pedagogical vehicle for achieving such a goal.

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Critical Pedagogy of Place in Mathematics: Texts, Tools, and Talk

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This paper explores the potential of integrating critical and place-based perspectives supported by mobile, digital technologies in secondary mathematics. The paper describes two curricular modules: Local Lotto, a mathematical investigation of the state lottery, and Cash City, a mathematical investigation about pawn shops, which were piloted in a high school in an underserved neighborhood in New York City. The analysis focuses on texts, tools, and talk (Philip & Garcia, 2013) introduced by a critical pedagogy of place in mathematics.

Introduction

Critical pedagogy of place (CPoP) in mathematics connects "the real world" and mathematics by integrating themes of critical mathematics education (Gutstein, 2005; Skovsmose, 1994) with an orientation of place (Gruenewald, 2003a, 2003b). Mathematics can be used to question, illuminate, elucidate, and communicate issues of power, fairness, and social justice, and such explorations can be considered in geo- and socio-spatial terms. In other words, mathematics can be essential to understanding spatial justice by illuminating not only how a social phenomenon works but where it takes place, who participates and with what frequency, who is impacted, and if it is fair. Essential characteristics of urban spaces–their density and traversability by foot–make them especially conducive for CPoP. This paper explores the potential of CPoP with a specific focus on youth in high schools in underserved, urban neighborhoods in the United States.

Perspectives

Space and Place

Notions of place and space are distinct but interrelated. A space becomes a place as an individual develops familiarity with it (Tuan, 1977). Place is not static and exists at multiple levels of scale, from the level of home as a place to the scale of a neighborhood as a place, which "acquires visibility through an effort of the mind" (Tuan, 1977, p .170). In this paper and in the project it describes, we take up high school students' places/spaces at various levels of scale.

Place in teaching and learning has been articulated in terms of place-based education (PBE, Gruenewald, 2003a, 2003b), with theoretical groundings in Dewey (1959). PBE is oriented around contextualizing learning in the learner's physical environment. PBE is typically used in rural contexts, since rural spaces offer accessibility to environmental phenomena that can generate data productive for statistical investigations, mathematical modeling, and follow-up advocacy.

Although "justice has a geography" (Soja, 2010) and despite the ubiquity of space in all that is social, critical pedagogy adopts historical and social perspectives and leaves spatial components unaddressed. For example, critical mathematics education (CME, Frankenstein, 1990; Gutstein, 2005; Skovsmose, 1994; Vithal, 2003) typically focuses on social or historical aspects of cultural activities in urban spaces, often without a lens on the spatial context in which those activities are practiced. CME supports students in developing critical and functional mathematical literacies (Apple, 1992 as cited by Gutstein, 2005); students use mathematics to challenge an oppressive status quo and, in so doing, learn more mathematics. CME could be extended to include spatial reasoning in functional and critical capacities. Critical spatial reasoning "challenges discriminatory geographies" (Soja, 2010), such as patterns in residential segregation or unequal distributions of resources across neighborhoods.

Analogous to the way that CME often ignores issues of place and space, PBE typically ignores issues of diversity, race, social class and power. Integrating the two frameworks in a critical pedagogy of place (CPoP, Gruenewald 2003a, 2003b), specifically in mathematics, opens up possibilities for "reading and writing the world with mathematics" (Freire, 1998; Gutstein, 2005) with a spatial justice perspective. CPoP takes up critical and spatial perspectives, since "everything that is social (justice included) is simultaneously and inherently spatial, just as everything spatial, at least with regard to the human world, is simultaneously and inherently socialized" (Soja, 2010, p. 24).

Mathematics, Technology, and CPoP

School mathematics, with data literacy, quantitative reasoning, and mathematical modeling, is a natural disciplinary fit for analyzing "discriminatory geographies" (Soja, 2010) or spatial aspects of experience. Most students study geometry in high school, an entire course that is focused on space, but space is usually not contextualized and, if contextualized, it is rarely in terms relevant to youth's places. Algebraic concepts of proportional reasoning or relationships between quantities link arguments about justice with geometric concepts like distance or area. For example, Brantlinger (2005) describes a "geometry of inequality" project in which high school geometry students compared the relative distribution of community spaces to liquor stores in South Los Angeles, relying on concepts of proportion and area to make arguments about spatial justice.

New technologies support CPoP in mathematics and allow for the overlaying of representations of data on to representations of place and space. Mobile data gathering devices (c.f., Quintana, 2012; Trouche & Drijvers, 2010) or global positioning devices and mapping software that can readily represent geospatial data (c.f., Enyedy & Mukhopadhyay, 2007) make spatial perspectives especially conducive in urban school settings. The diversity of peoples and practices in a city, comingled with a high population density and associated geospatial patterns, provide a rich context to examine social phenomena in spatial contexts. A wide range of questions can be posed about a spatial arrangement; and geospatial technologies enable data gathering and representation of that data in spatial terms, relying on fundamental systems of geometric measurement, concepts of ratio and proportion, and mathematical representations. Learning environments and curriculum enabled by critical, spatial perspectives and supported by mobile and geospatial technologies have the potential to disrupt the notion of "classroom as container" (Leander, Phillips, & Headrick Taylor, 2010) by facilitating the integration of the world outside of school into students' in-school learning.

CPoP for Mathematics in Action: City Digits

Learning Mathematics of the City in the City is a design and research project[1] to develop and study innovative "City Digits" resources for high school students' learning of mathematics with a CPoP perspective. Four principles guide the development of a curricular arc and corresponding set of geospatial tools. The first principle is that mathematics is essential for understanding the phenomenon (how does it work?), beyond descriptive statistics or proportional reasoning. Second, the phenomenon must have a spatial (where does it occur?), quantitative component (how much/ how often?). Third, the phenomenon must also include subject and cultural dimensions (who is involved and how do they participate?). Finally, the fourth principle is that there must be potential to explore sociopolitical and social justice ramifications (who is impacted, who makes the decisions, and is it fair?). These design principles can be applied to a range of social themes or phenomena and can further guide the creation of new curriculum and spatial tools.

City Digits has used these principles to design and study innovative curriculum and integrated digital tools that allow students to learn mathematics by investigating spatial perspectives of various phenomena. The design team selected the phenomena of lotteries and pawn shops as topics that readily correspond to the design principles. The first module, *Local Lotto*, focused on the state lottery in terms of how one plays, how likely it is to win, where in New York City people buy lottery tickets or win prizes, and with what relative frequency. The second module, *Cash City*, takes a similar spatial perspective and focuses on understanding how a pawn shop works in terms of the mathematics and practical considerations of a transaction, relative to borrowing money in other ways. The understanding of how a pawn shop loan works is set in the context of an analysis of the spatial distribution of mainstream and alternative financial institutions across New York City; where are banks, check cashers, pawn shops and wire transfer outlets located, what is the relative density of these categories of businesses, and what are relationships between these densities and other variables like population density, household poverty rates, density of new immigrants, or household income. Both curricular modules prompt questions about fairness, social justice, and space in that they can be viewed as systems that cater to, or prey on, the needs of people in low-income neighborhoods of a large metropolitan city.

In the City Digits modules, students conduct field research with mobile technologies in their local streetscapes to gather quantitative and qualitative information. The mobile tools enable students to take photographs, conduct and gather audio interviews with citizens, and record data about locations of interest. The GPS functionality of the tools instantly publishes all of these types of media by location on a shareable map. Back in the classroom, or even at home, students can view their own and their peers' findings to further contextualize their study of the phenomenon. By opening up opportunities for spatial analysis of a social phenomenon using mathematics, we invite youth to investigate the "geography of opportunity" (Jacobs, 1992) so to be able to "to take greater control over how the unjust spaces in which they live are socially produced" (Lefebvre, 1991 as cited by Soja, 2010, p.95). Neighborhoods must be aware of their own challenges so that they can defeat–and not be defeated by–them (Jacobs 1992). Since youth have unique insights into space and place, the City Digits tools allow youth to access spatial perspectives and to contribute their own voices and ideas about spatial justice.

Methods

This study is grounded in design-based research methodology (Brown, 1992), comprised of iterative cycles of design, testing, and revision, in collaboration with classroom teachers, of two CPoP mathematics modules with associated technologies. Each cycle of the design-based research includes classroom implementation by classroom teachers in a high school in an underserved neighborhood in New York City. Findings from classroom implementation lead to new insights and questions, and this fuels additional iterations of cycle. We have undergone two iterations of this design-based research cycle with *Local Lotto* and with *Cash City*.

Technology is often assumed to be a "quick fix" for schools in underserved neighborhoods, based on assumptions about student interest but can detract attention away from pedagogy and result in greater inequities (Philip & Garcia, 2013). In this paper, we analyze affordances of CPoP in mathematics enhanced with technologies according to Philip & Garcia's (2013) notions of texts, tools, and talk. We examine technology in terms of the affordances it provides relative to pedagogy: how technology offers teachers, classrooms, and students new forms of text (sources and modes of information), tools (ways to interact with texts), and talk (means of communication).

Our research question is: What text, tools, and talk does technology-infused CPoP introduce for mathematics teaching and learning? We draw on the following data sources: 1) design artifacts, including documentation of the curricular arc, individual lesson plans and their revisions, drafts of digital tools and their revisions, data and data representations used in the design of lessons and tools for *Local Lotto* and *Cash City*; and 2) data from two rounds of implementation of *Local Lotto*, including audio and field notes of classroom sessions and debrief sessions with teachers, video of student focus group interviews, and artifacts including student work.

Results

Our analysis is organized according to the texts, tools, and talks prompted by technology-infused CPoP. We note a distinction between opportunities to reason about data at individual/local and aggregate/global scales, because students are typically adept at reasoning about data at the individual/local level of scale but extending reasoning to the aggregate/global scale is known to be challenging (Ben-Zvi & Arcavi, 2001). In particular, our analysis demonstrates how the texts, tools, and talk prompted by CPoP in mathematics can bridge these levels of scale.

Texts

CPoP in mathematics compels new types of texts for teaching and learning, at multiple levels of scale. At the individual level of scale, students'

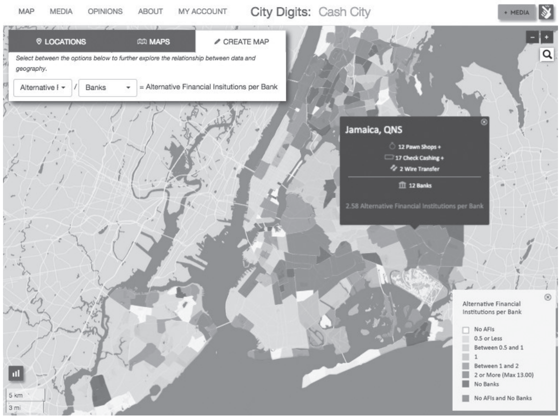

Figure 1: Alternative financial institutions per bank (City Digits, 2014a)

out-of-school knowledge and experiences are the cornerstone texts. *Local Lotto* and *Cash City* begin with students sharing what they know in a classroom word-wall activity organized to elicit and share students' own daily, lived experiences to the given topic. This deliberate dedication to draw out students' prior knowledge enables students to effectively "take up space" (Hand, 2014) at the classroom level with their ideas, experiences, and questions. The aggregated visual display of students' ideas is a text that frames and guides the subsequent teaching and learning.

To expand students' understanding of a spatial theme beyond the individual level of scale, students use mobile technologies to collect novel types of text in the form of interviews with people in their neighborhoods, photographs of streetscapes, and other qualitative and quantitative data of interest. These texts help students to learn more about how the topic affects their neighbors and how it is spatially organized. Interviews with pedestrians and shopkeepers give students a sense of who and how many participate in the phenomena of the lottery and pawn shops as well as why they do so. Photographs of streetscapes shed light on the prevalence of the phenomenon in the local landscape in context.

CPoP in mathematics emphasizes cartographic representations, or maps, that are the primary forms of texts that provide students information on the city by neighborhood scale. *Local Lotto* and *Cash City* present New York City choropleth maps by neighborhood to represent data and illustrate patterns across the city. For example, a *Cash City* map shows the relative availability of various financial institutions (Figure 1). The series of maps

Figure 2: Lottery net gains and loss per store (City Digits, 2014b)

enable students to investigate relationships between relative densities of financial institutions and other variables, such as median household income or percentage of population born outside of the United States. Patterns in and across these relationships highlight issues of equity and justice.

The *Local Lotto* and *Cash City* maps facilitate connections between texts at the local/individual and aggregate/global scales. For example, in *Local Lotto*, students can zoom in to view lottery sales and prize data at individual retail stores (Figure 2). Similarly, in *Cash City*, students gather data in field research and their findings are aggregated on a map, creating a micro-neighborhood scale representation of a spatial distribution.

Tools

Tools associated with CPoP offer students new ways to participate in learning. CPoP's commitment to teaching students to understand the mathematics behind how local phenomena work invites the use of tools that aid toward reaching this goal. For example, in *Local Lotto*, students run trial simulations of various lottery games and then work together to create a physical tree diagram modelling all the possible outcomes of a small-scale lottery game.

City Digits map tools enable students to use and remake an authorable map space (see Figure 3) as a text that works for them in understanding the organization of space where they live. Maps

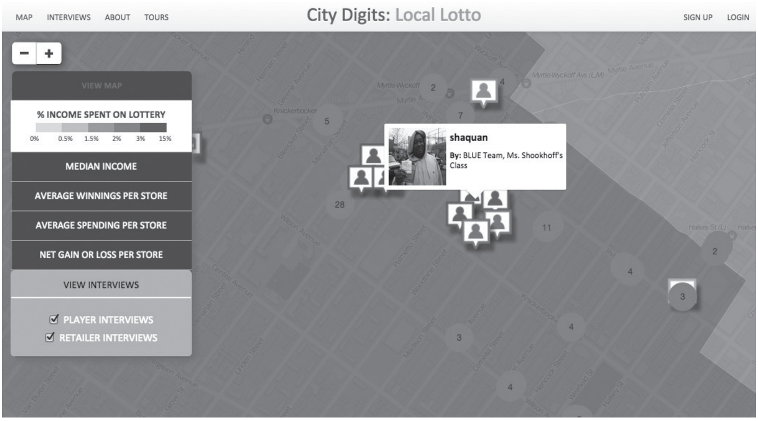

Figure 3: Text linkage: interview linked to map (City Digits, 2014c)

possess a traditional status as scientific/objective reproductions of reality but can be redefined as subjective examples of argumentation (Wood, 1992). City Digits map tools operate as a mediator between students' local knowledge ("connaissance") and the institutionalized knowledge ("savoir") about their neighborhoods and enables one to speak back to the other as students both come to understand and then criticize geographic/social boundaries as artificially imposed by structures of power (Lefebvre, 1991). In this sense, the City Digits mapping tools work like crowd-sourced reviewer apps, with the end product being an authored environment populated by narrative accounts and reviews by local youth. The mapping tool allows students to toggle between their own authored map and larger city level maps.

Digital, authoring tools associated with CPoP offer opportunities for justification and reasoning with data. Students select and organize texts (i.e., "snapshots" of spatial representations, data points, interviews, classroom artifacts, etc.) to develop their own opinions and then substantiate and justify those opinions through a web-published, digital "tour." The automated spatial tagging of data students gather situates local texts within a global representation, and the "tour" tool encourages students to synthesize their analyses from across local and global texts to form a coherent story.

Talk

New forms of mathematical communication are introduced through CPoP's texts and tools among students and between students and the wider public. Data that students collect is aggregated online, and students' digital tours are published on the web-based tool as well, allowing students to see each other's work. The digital platform includes a comment feature for both student collected data and student created tours, further enabling students to communicate with each other or with teacher, in or out of school. On a broader scale, students talk to an authentic general audience through their web-published tours. In this way, the mathematical dialogue transcends the classroom both in terms of space and participants. Students have a new interface for creating a multi-scaled understanding of the phenomenon they are studying.

Conclusions

Critical pedagogy of place offers new forms of texts, tools, and talk that engage students in mathematical analyses that focus on a local phenomenon. These new modes of texts, tools, and talk span local and global scales; a critical, place-based study encourages defining and probing relationships between variables at local scales and detecting patterns at aggregate/global scales. This is especially important and promising for the teaching of mathematics in urban high schools. At a basic level, CPoP supports students in developing or improving functional mathematical literacies in terms of core concepts like ratio and proportion, area, percent, rate, or measures of central tendency. However, by using mathematics to pose and answer questions about spatial justice, CPoP also provides rigorous opportunities for students to develop critical mathematical literacies (Apple, 1992; Gutstein, 2005). Finally, by centering mathematical analyses of complex phenomena on students' local environments, essential connections are forged between the in school learning and out of school knowledge and experiences.

City Digits encourages students to "reinhabit" and "decolonize" their neighborhoods (Gruenewald, 2003b). City Digits encourages "decolonization" by revealing mathematically the inequities of the lottery or pawn shop systems and revealing spatially the distribution of these services across economic/social/geographic boundary-lines. Students come to understand not only the injustice of the astronomical chances of winning a state-promoted lottery or how a pawn-shop loan accrues interest but also how these systems specifically target low-income people in neighborhoods like their own. Using this knowledge informed by mathematics, students engage in "reinhabitation" through interaction with shop owners and exploration in their neighborhoods. These explorations lead students to develop a sense of how they are inhabitants of their neighborhood with a stake in the way in which it is organized as opposed to residents of their neighborhood who simply traverse it on their way to and from school (Orr, 1992 cited by Gruenewald, 2003b). Both "reinhabitation" and "decolonization" co-constitute the development of a mathematically informed critical perspective on place and the ways in which space is organized in everyday lives.

Notes

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Socio-Political Aspects of Personal Experiences in University Teacher Education Programmes

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This paper presents and applies the subject-scientific approach to learning, which integrates a socio-political dimension to learning on a theoretical level and provides analytical categories for research. An ongoing qualitative study about learning experiences of pre-service mathematics' teachers throughout their university studies is presented. The analytic categories are exemplified by the case of the student Helena. She is splitting her learning in various institutional contexts, which realise different functions in her learning trajectory that could be interpreted in their socio-political dimension within the subject-scientific approach.

Introduction

In Germany dissatisfaction with the current institutional arrangements of teacher training are expressed by teacher students as well as lecturers. Research, addressing German teacher training, mainly follows a deficit approach, excluding voices of the teacher students. The presented study focuses on the limits, obstructions and opportunities from students' standpoints of institutional arrangements of teacher education in university settings in Germany.

Pais and Valero (2011) demand a dialectical approach in considering the socio-political dimension in mathematics education research. Mathematics education practices are embedded in social, economic, political and historical practices and discourses, which establish mathematical rationalities. Furthermore mathematics education research practices also construct naturalised discourses on how mathematical rationalities in the social world are formed. A reduction of mathematics education research to learning theories, that only focus on the individual learner and deny the societal and political magnitude of education, is strongly criticised. Learning cannot be seen disconnected from socio-political practices that also constitute mathematics education. Mathematics education practices are given meaning by social, political, economical and historical configurations.

Yet, how can learning be conceptualised in mathematics education research without leaving out the socio-political dimension? In the following, a theoretical framing of learning – the subject-scientific approach – based on German Critical Psychology and the communities of practice approach, is introduced. The subject-scientific approach integrates social, political, economical and historical configurations already on a theoretical level and not just as additional variables (Holzkamp, 2013b). The analysis starts from the subject's standpoint and proceeds to relate the findings to societal arrangements.

How Is Society Taken into Account in Critical Psychology?

The subject-scientific approach takes the subject standpoint as starting point for further analyses. Holzkamp (1985) proclaims a paradigm-change considering meaning-reasoning relations, instead of behaviour-determination. He argues that humans recognise the world from their own perspective and with purpose; the reality is interpreted by the subject in connection with her or his experiences and intentions as well as in view of their *life interests.*

Subjects are conceptualised within societal and political structures, able to consciously react to their environment, according to what is reasonable from their standpoint (Holzkamp, 1985). What seems reasonable depends on the subject's *personal situatedness* (*Personale Situiertheit*), a concept that contains the past and future prospects of the subject's unique biography.

Human beings can only be understood in their relation to the societal system. The subject stands in a two-sided relation to society (Holzkamp, 2013a). On the one hand, human beings are producers of their life conditions and reproduce their life conditions; on the other hand, they are subjects to their life conditions. Holzkamp summarises the idea of the subject-scientific approach as follows:

> … we are attempting to elaborate this two-sided relation as an interrelationship, i.e. to analyse human beings as producers of the life conditions to which they are simultaneously subject, and to conceptualise the mediation between the vital necessities of sustaining the societal system as a whole and these necessities on the subjective level of the discrete individuals (p. 20).

Analytical Lense

The analytical concepts presented here are designed dialectically; they are not confined to the individual, but all are defined as mediating the two-sided relation between subject and society (Holzkamp, 1985).

Capacity to Act

The core-category *capacity to act* (*Handlungsfähigkeit*)[1] describes the mediation between the individual and societal life-sustaining activities (Holzkamp, 2013a). In cooperation with others, the individual is capable of gaining more control over her or his own life conditions. It includes both the individual's opportunities and constraints to act. The (subjectively perceived) quality – opportunities to act and constraints of those opportunities – of the own capacity to act is reflected in the individual's *existential orientation* (*Befindlichkeit*). Overcoming the dependency on current situations and gain of influence over one's own life prospects is seen as the central moment in prospectively developing one's own individual life quality (Holzkamp).

Antagonistic class conditions of capitalist society are specific for the historic conditions we are living in (Holzkamp, 1985). Hence by trying to increase one's capacity to act, in particular gaining more control over life-sustaining conditions, the subject is risking getting in conflict with agents of power (Holzkamp, 2013a), and therefore the risk of provoking restrictions has to be anticipated. Thus, the category capacity to act is a dual concept describing two analytically split alternatives, depending on how the subjects tries to resolve her or his *subjective need* to broaden their influence on her or his life-conditions.

In a variety of different situations, it can be reasonable for the subject to act within the given limits of current situations, even though, in principle, the development of the *capacity to act* by broadening one's own influence over one's life-conditions is always possible (Holzkamp, 2013a). Acting within given limits characterises the *restrictive alternative of capacity to act* (*restriktive Handungsfähigkeit*) while extending one's own capacity to act in cooperation with others describes the *generalised alternative of capacity to act* (*verallgemeinerte Handlungsfähigkeit*).[2]

Knowledge entails a critical and emancipatory potential for extending one's own *capacity to act*, but it is controlled by institutional power structures (Holzkamp, 1995). The learning process is initiated by difficulties that could not be overcome without learning. The intended outcome of any learning process is mastering those difficulties. Students' learning actions are based on their premises for action that depend on the possibilities offered by their particular societal context and how these are interpreted (Holzkamp). Learning is conceptualised as activities, reasonable from the standpoint of the subject. The analysis of learning processes aims to reveal contradictions in societal arranged learning environments.[3]

Communities of Practice

The communities of practice approach highlights that learning is situated in social practices (Lave, 1988). It criticises a perspective on learning processes emphasising isolated acts of learning that build *trees of knowledge* and favours the view of *landscape of practices*. Knowledge is not decontextualized, therefore new knowledge is located in communities of practice. The approach deepens the theorizing on social learning environments and conceptualizes relations between different learning contexts that take place at the boundaries, where diverse communities of practices can be connected (Lave & Wenger, 1991). Boundaries can provide learning opportunities, but also divide communities of practice and can be a source of separation and fragmentation of and into different communities. Some people act as brokers between different communities of practice, while others (who are potential brokers) do not provide cross-boundary connections. A crucial point in the analysis of learning processes is the participation/non-participation in (between) communities of practice.

Learning Trajectories

The social practice of learning is arranged in a variety of institutional settings, e.g. school, university, vocational training and sports clubs (Dreier, 2008). Different learning problematics are faced in different contexts in one's own personal learning trajectory. The contrasting and comparing of different learning problematics across different places might lead to new learning tasks.

Dreier (1999) acknowledges Holzkamp's concept of *capacity to act* (as mentioned above) and emphasises aspects of the social contexts of actions that the subject is part of. His analytical categories are building up on Holzkamp's concepts of capacity to act and the communities of practice approach, providing a further distinction to grasp the importance of different settings in a more specific way.[4] The categories – *aspects of situatedness, forms of cooperation, mode of participation* – guiding the analysis are presented in the following passage. These concepts cannot be seen as stable or fixed, but are changing throughout the subject's life trajectory, changing participants, societal and political situatedness.

Aspects of situatedness: The opportunities and the nature of the learning processes are affected by the specific institutional arrangements the social practice of learning is located in (Dreier, 2008). These institutional arrangements have to be taken into account by the learning subject (Dreier, 1999). The learning subject has to balance her or his own personal learning goals and the scope of the specific institution. Therefore, it is important to understand the relation between *the personal situatedness of* the subject (as mentioned before) and the situatedness of social context. These aspects of situatedness can help to reconstruct limits and obstructions to the learning process (Langemeyer, 2006).

Forms of cooperation: The quality of learning and understanding, as well as the affective relation to it, are strongly influenced by the participatory dimension of the subjects' activities (Dreier, 1999). Hence different forms of cooperation, that structure the subjects' participation in social contexts, have to be looked at in more detail. The particular position the subject occupies in the present social context affects her or his opportunities. By merging the concept of capacity to act and participation, Dreier suggests that

> "[t]he fundamental human duality between acting within the existing limits of social practice and extending its scope of possibilities is grounded in a similar duality of modes of participation, i.e. of participation in the reproduction of the current state of affairs or of contributing to change it so that participants may extend their degree of disposal over the social practice" (p. 6).

Mode of participation: Subjects take part in different social contexts and find themselves in different contexts (Dreier, 1999). Thus, it must be taken into consideration how subjects manage the participation in heterogeneous social contexts. Subjects change their personal mode of participation according to the specific local contexts. Dreier explains:

> "When subjects move from one context into another, their structure of personal relevance changes. Which particular structure of social conditions matters for them depends on their present location" (p. 12).

The subject's *stance* describes her or his standpoint in relation to the institutional settings that guides her or him in the transition between contexts (Dreier, 1999).

Research Design

The following exemplification (The story of Helena) of the prior introduced concepts are part of an ongoing qualitative study addressing learning experiences of pre-service mathematics teachers in the institutional context of university teacher education programs in Germany. Teacher training in Germany is organised in two phases. The first phase is situated at university (Bachelor and Master's degree) and the second phase is a practical training. The study only looks at the first phase of teacher education. Mathematics teacher degree programs include the study of math, math didactics and general pedagogy, provided by the particular departments.

So far nine pre-service teachers – aiming to be primary, secondary or vocational school teacher – have been interviewed. The university students had different educational backgrounds and were at different stages of their current studies. The educational programs for pre-service teachers of different school types vary in their structure and mathematical requirements.

Data is obtained from semi-structured interviews, supported by the assembly of a Mind Map done by the interviewee. The interview addresses questions about students' learning activities, the student's personal situation, social support and conditions of studying. Additionally, the student is asked to complete a Mind Map on what she or he considers being important for her or his learning process.[5]

The subjective standpoint of the students is basis of the analysis. The data is analysed using a combination of grounded methods (Strauss & Corbin, 1990) and techniques provided by objective hermeneutics (Wernet, 2009). Grounded methods are used to identify manifest topics that are perceived to be important for their learning by the students and their relation to each other. Objective hermeneutics aims at latent meaning structures and shifts the view from the subject's perspective to underlying shared structures.

The Story of Helena

In this paper, the case of Helena is discussed. The data is presented in form of a vignette (Erickson, 1986). The original German excerpts of the interview, referred to in the presentation of the case, can be found in the endnotes.

The interview with Helena took place during the fifth bachelor semester of her studies to become a primary school mathematics teacher. Throughout the interview a long learning trajectory of becoming a teacher was revealed: She had already studied in another teacher education programme, but after the first phase at university, she did not proceed to the second phase. Afterwards she worked as a substitute teacher before enrolling in the present mathematics teacher education programme. In addition to her university studies, she attends a course at another institution to get a supplementary qualification in progressive education.[6]

Even though Helena is an atypical case because of her long learning trajectory, problems of teacher education are revealed that also concern other students with a more prototypical learning trajectory.

Aspects of Situatedness

Helena's first teacher education programme (aiming at being an economics high school teacher) left her with a strong feeling of insecurity towards her future profession. Her economics studies contained university mathematics classes, which Helena characterises as "beyond good and evil, what a teacher will EVER need at all".[7] She experienced a devaluation of her skills as a teacher-student compared to economics-major-students,[8] which led to a lack of confidence in her mathematical skills. The strong focus on the subject matter courses led her to the conclusion of not being prepared to be a teacher[9] considering teaching methods. Her personal situatedness can be described by insecurity towards her skills related to her future profession. From this arises a strong subjective need of gaining confidence.

The first teacher education programme did not provide her with a secure knowledge base she can rely on in her practical phase. It reflects problems of university teacher education in general:

- The practical significance of university studies remains unclear to teacher students.
- In the hierarchy, teacher students are treated as a lower class than the students majoring in one subject.
- Especially mathematics classes seem to create a lack of confidence in mathematical skills, instead of providing students with the feeling of security and the ability of handling mathematics.

Forms of Cooperation and Modes of Participation

Her current studies of becoming a mathematics primary school teacher are split in two different institutional settings: A mathematics teacher education programme at university and an additional course on progressive education.

These two different institutions serve different functions in her learning process and therefore differ in forms of cooperation and modes of participation. In the university programme, she situates her learning of mathematical skills, while in the progressive education course she situates her learning of teaching methods.

(University) mathematics: She anticipates regaining confidence in her mathematical skills during the university programme. Helena still positions herself in a contradictory way to her mathematical skills. She refers to mathematics as a personal strength,[10] but on the other hand, she has the subjective need to gain confidence in mathematics and to develop basic mathematical skills.[11] The first topic she brings up in the interview is the need to strengthen her confidence in her own mathematical skills, again.[12] This confidence remains an issue throughout the interview, indicating the magnitude of this topic for institutionalised teacher education.

She refers to the mathematical content class that she is currently taking, as a class she "is obliged to"[13] take. The mode of participation in class context leads to knowing requirements for attaining a certified qualification, but do not necessarily imply deep engagement in mathematical activities. She openly admits that one reason she is taking a certain class is an anticipated good grade in the written exam.[14] Getting a good grade helps her to build up confidence in her mathematical skills, by stressing the "high demand"[15] of the mathematics classes, (she refers to it several times throughout the interview) she is strengthening this experience. Her preparation for a mathematics exam can be described as rote learning. Helena just receives mathematics knowledge from others. Mathematics remains a tool of power over her.

Just focusing on Helena's learning in university classes would deny here actual learning efforts to pursue her goal of becoming a mathematics teacher. Her forms of participation differ between her university communities. She participates in a learning group[16] that she strongly relies on for framing her learning activities in university context. Helena refers to her learning group meetings as one of her central learning strategies and stresses the importance of learning together for mathematics.[17] The learning group has developed binding group norms.[18] She relates the extensive preparation of homework (for university classes) to the participation in the learning group and not to the particular class contexts. The mode of participation in this learning group requires all members to be committed to their studies, actively engaging in mathematics activities and striving for high achievements. In her learning group her mathematical abilities are being recognised; here she is part of a mathematical community.

Progressive education course: During her last semester at the former teacher education programme she got to know about progressive education theories for the first time. She decided to become a teacher at a progressive education school and complete her second phase of teacher training at a progressive education school.

The progressive education course provides her with a set of teaching methods, which is theoretically related to a stage model of (mathematics) learning. These teaching methods therefore help to build up a secure basis for teaching and build up the needed confidence regarding her pedagogical content knowledge.

Consequences of Institutional Splitting

Asked for the relationship of her university studies and her progressive education qualification course, she does not perceive it to be benefitting her in her current situation, but anticipates her progressive education qualification course to be of benefit in the future.[19] Her work as a substitute teacher remains a disconnected experience in her current learning as well. Helena clearly has the stance of not relating the two institutional settings. Helena does not act as a broker between the different learning contexts. Due to not connecting both of her learning opportunities (subject knowledge and teaching methods), her personal teacher studies remain fragmented.

In her Mind Map, she does not integrate her substitute work as a teacher and just mentions progressive education approaches to be missing in university context. She is not linking these experiences to the learning in the mathematics teacher education programme. Also, in other Mind Maps, the problem of learning in university context remains detached from other experiences, indicating a specific situatedness.

The splitting of the different components of a teacher education programme seems to be a limiting boundary in teacher education. The presented case here is quite drastic because it is fragmented between different institutional settings. Also other students report difficulties of relating the modularised components – math, math didactics and general pedagogy – to each other in a holistic picture. The conjunction of these components in regard to teaching practices is crucial for teachers' development. The institutionalised teacher education in this fragmented form seems to leave the connections at the boundaries of the different components open to the students.

Due to the fragmentation the power over the knowledge for teaching remains split. This intensifies the feeling of being exposed to others that possess the knowledge that is relevant for their future profession.

Further Questions

The cross-boundary connections between the components of math, math didactics and pedagogy need to be taken a closer look at, in order to develop ways of fostering the cross-boundary learning opportunities and enabling future teachers to obtain an integrated view. Attaining an integrated view and the confidence in associated skills are crucial for teacher's professional development, but seem to be an obstruction to teacher learning so far.

- How can an integrated view be supported in an institutional teacher education setting?
- How can institutional teacher education foster the confidence skills relevant for teaching?

The affective relations at the intersections of the components seem to play a crucial role. The hypothesis emerged that challenging students' confidence in university mathematics classes is related to dissatisfaction with university teacher education programmes concerning the subjective need for more practical significance in teacher education. This possible relation and its constitution, as well as students' subjective need of more practical significance of university studies, need to be further examined. Is the demand for more practical significance for teaching an anticipated way of regaining power of the knowledge for teaching?

Notes

1. The German term *Handlungsfähigkeit* is difficult to translate into English. Different translations can be found in publications in English: *action potence, agency, power to act*, and *capacity to act*

2. Hereby it should be mentioned, that by enforcing one's own needs at the costs of others also narrows one's own possibilities in life by isolating oneself and the reduction of possibilities to alliance-formation with others

3. The subject-scientific approach of Holzkamp (1995) differs from the activity theory approach to learning, used in mathematics education research (e.g. Roth & Radford, 2011). Both approaches (Langemeyer, 2006) see contradictions as essential for the analysis of learning. But what is meant by contradictions differs (Langemeyer). In the activity theory approach, contradictions mark the starting point for learning, Holzkamp, however, defines contradictions as a hindrance to learning.

4. Klaus Holzkamp, Jean Lave and Ole Dreier worked together collaboratively. Even though their studies about learning have different foci, they shared basic conceptions and their theorizing inspired each other.

5. The students complete the Mind Map by themselves. Just the word "Learning" is given and they are asked to write down on blank paper in key terms, what they perceive to be important for their learning process. They can also relate their key terms with arrows to each other and use signs to indicate importance and the affective relation.

6 Certified progressive education teachers have to complete the two phases of general German teacher training and additionally the course in progressive education. These two institutions do not collaborate.

7. Es war jenseits von Gut und Böse, was JEMALS ein Lehrer überhaupt braucht.

8. [ ] wurden wir eigentlich NUR geprüft, ob wir irgendwie mit denen [Fachstudenten] mithalten können

9. Ich fühlte mich da auch ÜBERHAUPT nicht gut vorbereitet als Studentin auf meinen Lehrberuf.

10. Mathe war schon immer eine Stärke von mir

11. Ich muss immer noch Grundlagen [in Mathematik] auch nachholen um

12. [ ] so dass ich so Sicherheit erlangt habe durch das Studium auch wieder in diesem Fachbereich [Mathematik].

13. „Elementare Funktionen" MUSSTE ich besuchen

14. [ ] weil ich gedacht habe, dass man bei Herrn [Name des Dozenten] auch ganz gut die Klausuren bestehen kann.

15. E.g.: auf diesem HOHEN Niveau im Mathematikstudium irgendwie zu beherrschen

16. Helena collaborates with other mathematics teacher students in her semester

17. Also in Mathematik treffe ich mich mehr mit meiner Gruppe. [ ] In Mathematik bin ich auf meine Gruppe angewiesen.

18. wir sind wirklich ein Team, wir legen ALLE Wert darauf, dass wir uns treffen und die Hausaufgaben auch schon vorbereitet haben und das ist eben auch ein Bestandteil vom Lernen bei mir

19. Also fürs Studium eher weniger. [ ] für meinen Beruf als Lehrerin profitiere ich UNENDLICH.

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Upper Primary Mathematics Curriculum, the Right to Education In India, and Some Ethical Issues

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The Right To Education Act (2009) promises free and compulsory education for all children in the age group of 6 to 14 in India in schools that follow centralised curriculum. I explore the implications of this regulation for socio-economically marginalised learners, in particular their opportunities for accessing primary mathematics. By revisiting some of the questions raised in an earlier paper about the possibility of realising the existing upper primary curriculum for these learners, I ask if ethical issues are involved in designing and developing a curriculum, particularly when the state designs a uniform curriculum for all.

Introduction

The period between 2005 to 2010 saw significant changes in Indian education. A new curriculum framework (National Council of Educational Research and Training, 2005) was brought into action in 2005. It advocated "learning without burden" and proposed the following five guiding principles for curriculum development: connecting knowledge to life outside the school; ensuring that learning shifts away from rote methods; enriching the curriculum so that it goes beyond textbooks; making examinations more flexible and integrating them with classroom life; and nurturing an overriding identity informed by caring concerns within the democratic polity of the country. Along the lines of the national curriculum framework, focus groups brought out 21 position papers on subject areas and other themes. Revised curriculum documents, syllabus, and textbooks were developed by the national board and the states were required to revise their curriculum frameworks to align with the National Curriculum Framework (NCF) (National Council of Educational Research and Training, 2005).

In 2009, after much debate and deliberation at various levels, the Right to Education act (RTE) was enacted by parliament. This act mandates that the state should provide free and compulsory education to children in the age group six to fourteen. A large number of poor children, many belonging to marginalized castes and tribes, and children from the most backward regions, study in government-run schools where education is free. The RTE act also mandates that children should be admitted to grades appropriate to their age, even if they have not had any prior schooling, stating that: "no child admitted in a school shall be held back in any class or expelled from school till the completion of elementary education" (up to Grade 8), and that: "no child shall be required to pass any Board examination till the completion of elementary education". The onus of bridging the gap between what children know and what they need to know for accessing prescribed content is left to the teachers who are expected to schedule extra classes to make up for the deficit. The act also mandates that the teachers "complete the curriculum within the specified time".

The NCF of 2005 and the RTE act are seen as positive measures, as more and more parents from the socio-culturally and economically marginalised sections want to educate their children. However, it is important to see how curricular choices in elementary mathematics are made and what are the implications for these children.

Curriculum development in India happens largely at two levels. At the national level, it is done by the National Council for Education Research and Training (NCERT) for the central schools and schools affiliated to the Central Board of Secondary Education (CBSE) across the country. At the state level, it is done by the twenty-four state boards of education for the government-run schools in the state, as well as the privately run schools affiliated to the state board. Though the National Curriculum and the textbooks brought out by NCERT function as a reference point for the state level curricula, the state boards are not bound to conform to it. As disciplines like mathematics and science are viewed as neutral and objective, impervious to the socio-political values of those in power, and significant in deciding the learners' career opportunities and economic status, there is often a large overlap in the curricula prescribed by different boards, across states and nationally. At the primary level, the focus of the curriculum is on ensuring that students acquire basic arithmetic competencies, basic ideas in spatial understanding and measurement, and applications to real-life situations. At the upper primary level (Grades 6 to 8), students are expected to move beyond everyday mathematics and engage with abstraction, acquire logical thinking, ease with symbolic representation, and competence to do mathematics in the higher classes (NCERT, 2006a). Typically, the upper primary curriculum deals with integers, rational numbers and their properties, algebra, geometry, data handling and "commercial mathematics". Revisiting the issues raised in an earlier paper (Subramanian, Umar, & Verma, 2014) from the caveat of RTE, I describe the schools, the socio-economic background of the students, their competence at the upper primary level, and the implications for these students of what is taught. As a critical mathematics educator, I argue that curriculum design involves ethical issues as it has consequences for a large number of learners and that curricular choices made by the boards of education, rather than representing the needs and interests of these children, function to further marginalize them. I stress the need for an alternative vision of upper primary mathematics curriculum that is informed by the preparedness of the majority of the learners and serves their interests.

Upper Primary Mathematics in India

A significant step in curriculum development in mathematics following NCF 2005 is the stated shift in focus in the national curriculum "from mathematical content to mathematical learning environments, where a whole range of processes takes precedence: formal problem solving, use of heuristics, estimation and approximation, optimization, use of patterns, visualization" (NCERT, 2006a, p. v). The syllabus for upper primary mathematics lays emphasis on "the need to look at the upper primary stage as the stage of transition towards greater abstraction, where the child will move from using concrete materials and experiences to deal with abstract notions" (NCERT, 2006b, p. 80). Consistent with these aims, the upper primary mathematics books focus on the discipline, though they build on the spirit of the primary textbooks, while the textbooks of the Madhya Pradesh state government (to which the schools that we work with are affiliated) impart content largely in the form of rules and algorithms and encourage drill and practice, even though they claim that the national curriculum framework has been taken into consideration while developing them.

In spite of very significant differences, there is a major overlap between the two boards in the content of the upper primary curriculum. For the strand on Numbers and Number Systems, beginning with natural numbers, integers and their properties in Grade 6, the syllabus moves on to introduce decimals, rational numbers, their representation on the number line, factors, multiples, exponential notation, prime factorization of numbers, ratio, proportion, percentages, and finishes with finding square roots and cube roots, including the algorithm for finding square roots in both the boards.

Algebra begins in Grade 6 with introduction of symbols to stand for numbers. While the NCERT textbook limits itself to writing linear polynomial expressions and equations in one variable at the Grade 6 level, the state board introduces higher degree polynomial expressions in two variables and the operations of addition and subtraction on them, ending with solving linear equations in one variable in Grade 6. But by Grade 8, both boards cover the three operations on higher degree polynomials in two variables, factorization or division, and algebraic identities.

In Geometry and Measurement, the content begins with giving some idea of what geometrical objects such as points, lines, and line segments mean, then moves on to polygonal figures, the notion of angle, using the geometry kit to measure angles, construct triangles, quadrilaterals, perpendiculars and angle bisectors, properties of triangles and quadrilaterals, circles, three-dimensional objects. Formulas for finding the perimeter, area, and volume are either derived or presented. Apart from these Numbers and Number Systems, Algebra, and Geometry, there is some exposure to data handling, probability, and statistics, and commercial mathematics.

In other words, both curricula take a particular view of what constitutes mathematics and center the curriculum on it, though within the community of mathematics educators, at least, academic or research mathematics, which constitutes the discipline of mathematics, is referred to as “mathematicians’ mathematics”, and is seen as one of many kinds of mathematics. This situation is not singular to the Indian context as can be seen by glancing through upper primary curricula in other countries.

Elementary Mathematics: A Report From Classrooms

Eklavya, a reputed NGO in central India, in which the author was a fellow, carried out explorations and sustained experiments in elementary mathematics (Agnihotri, Khanna, & Shukla, 1994; Batra, 2010) over a period of about seven years and used three different settings: four private, English-medium schools catering to children coming from low-income groups; Muskaan, a school run for scrap-pickers in an urban location; and three government-run schools in a rural location, of which one is a primary school for girls where the team worked with the same set of children from Grade 3 to 5. In most of the schools we worked with, we directly taught the students.

Government schools in India cater to the poorest sections of society. This also means most of their students come from marginalized castes or tribes. In one government school, 68% of the children belonged to "Other Backward Castes", 17% were Dalits (Scheduled castes), and 10% were *Adivasi* (Scheduled Tribes). Most of the children going to a government school do not have geometry boxes and carry a single notebook in which they copy everything. Government provides one meal a day, a set of school uniforms, and the required textbooks. Parents of the students are either illiterate or barely literate and children do not get any support from the parents in their studies nor can they afford paid tutors.

Student absenteeism was a common feature in both the government schools that we worked with. Part of the reason for absenteeism lies in the fact that they are bored at school as very little teaching happens. Our regular visits to the school only confirmed this, as it was common for teachers to leave the classroom, assigning the students some writing work or asking them to memorize the tables. While part of this pattern could be attributed to the bias teachers carry against the caste/class background of the students, a significant part of it also has to do with the lack of training and continued support for the teachers to cope with teaching first-generation school learners. Other reasons for absenteeism are corporal punishment, paid labor, and domestic responsibilities.

Parental ambitions for their children, particularly boys, could be considered high as they take the first opportunity to enroll their children in private schools in which the medium of instruction is English. However, it would be difficult to believe that the children got direct help from their parents to support or supplement what they were taught at school. In the private schools, unlike in government schools, it was rare to find a classroom without a teacher. However, the teachers' own subject matter knowledge and pedagogical content knowledge, their comfort level with the language of communication and instruction in the classroom, the number of classes and hours they teach in a day, and the class size were major limitations.

A Description of the Classrooms

Referring to the physical space of the school and the classroom described by Bopape in South Africa, Skovsmose (2007, p. 84) says:

> How could it be that this hole in the roof has not been seriously addressed by mainstream research in mathematics education? Learning obstacles can be looked for in the actual situation of the children and with respect to the opportunities which society makes available for the children. The actual distribution of wealth and poverty includes a distribution of learning possibilities and learning obstacles. This distribution is a political act. Paying attention to this means re-establishing the politics of learning obstacles.

The physical ambiance and the amenities of classrooms are marked by the socio-economic class of the children. The classrooms in government schools have no furniture for children. They usually sit on "tat pattis" (mat rolls made of jute) or "dhurries" (thick woven material like carpets) spread on the floor. During winter when the floors are too cold to sit on, they huddle together. Cleaning the classrooms is left to the students. As the classrooms double up as dining hall, the students have the choice of sitting in classrooms with food lying around or sweeping the classroom for the second time in a day. Typically, the government schools have no power supply. The classrooms in private schools catering to low income groups are small, packed with benches and desks leaving very little room for the teacher to walk around. The situation becomes worse when rain-water leaks from the ceiling. In contrast, Novodaya schools and central schools run by central government and private schools catering to economically better off children have much better facilities with spacious classrooms, enough furniture for children, built-in shelves, and electricity. The differences in physical amenities are important because they make a difference to what is possible to teach. On a rainy day, with children sitting on dirty and wet floors, in winter with children too poor to afford warm clothing huddled together, in classrooms where the traffic noise can drown the din of children, well-designed curriculum, good textbooks, well worked-out lesson plans, and the beauty of mathematics seem the last things on anyone's mind.

Teaching Learning Activities in the Classrooms

Typically, the main activity that children carry out is copying from the board or from commercially available keys; problems are already worked out. On several occasions we found Grade 6 students copying how to simplify expressions like (118-{121+(11x11)-(-4)-(+3-7)}) and 4x2-[9x2-{-5x3-(2-7x2)-6x}], the question whether "the difference between 65 and 56 is zero" is true or false, or how to "subtract 13x-4y from the sum of 6x-4y and – 4x-9y", though they could not carry out simple division or subtraction on their own. In private schools, teachers may assign children homework and their notebooks may contain signs of inspection by the teacher. In the government schools, there is no evidence of any feedback being given for written work that children hand in. Given that this is what serious (and, understandably, tedious and boring) learning means, any attempt to engage children in discussions to understand and solve problems seem like "khel" (play) for them. In the private schools where "covering the syllabus" is important, children themselves sometimes express anxiety over losing precious time in khel.

A Description of Learning Levels: Oral Versus Written Mathematical Skills

In our interactions with Grade 6 children from government school, we found that many of them help their parents at the vegetable market or shops and nearly half of them could solve simple arithmetic problems orally, but when asked to write the same problem symbolically, they had difficulty. Inability to read and write numbers clearly makes it impossible for them to use algorithmic approaches. On one occasion we found that only 13% of the children in Grade 6 could write two-, three-, and four-digit numbers correctly on hearing the number words. The following year, we found only about 70% of the children in Grade 6 could do three digit addition without error, only about 55% of the children could subtract a three-digit number from 1000 or multiply a three-digit number by 3. For most of the children there was no concept of writing down the steps in a systematic manner; they would do some rough calculations and write the answer. Our experience with class 3 children was similar – most of them could not write two-digit numbers. This may also happen because number words for two-digit numbers in Hindi do not follow the order in which they are written. The Hindi equivalent of thirty-six for example would be something like "six, thirty" (Khan, 2008). Children going to private school were better, but we found that there were quite a few students in Grades 6 and 7 who would write 3110 as a successor of 319 and 70036 or 700306 instead of 736.

Similarly, most of the children in Grades 6 and 7 cannot link the fraction words they know with the fraction symbols, and think 1/3 is bigger than ½ even though the primary curriculum introduces fractions in Grade 3 and finishes all operations on fractions by Grade 5. Some of the private school children may be able to use standard algorithm for comparison, addition, and subtraction of fractions mechanically, even though the fraction symbol may not mean anything to them.

Our Design Experiments and Findings

As our objective was to evolve alternative approaches to teach mathematics at the upper primary level, we began our explorations with integers for Grade 6 students and rational numbers for Grade 7 in one of the private schools. Soon we realized we needed to develop an alternative approach for teaching fractions right from the primary school level, which we did, continuing alongside an interaction with Grade 6 students on numerical representations and the division algorithm, algebra, measurement, and geometry. A brief report from these interventions and explorations follows.

An Alternative Approach to Teach Fractions

Following our collaborative work with the Homi Bhabha Centre for Science Education team that combined the share and measure meanings of fractions, and drawing from the work of Streefland (1993), we took up a longitudinal study beginning in Grade 3 and worked with the same set of children up to Grade 5 in a government primary school for girls.

We introduce a fraction as the share that a child gets when, for example, 7 "rotis" (circular homemade bread) are equally shared among 8 children, beginning with unit fractions. Children draw 7 circles on top to represent rotis and 8 stick figures to represent children for the symbol 7/8, divide and distribute, writing the share of each child below. One distribution scheme might result in the relation 7/8 = 1/2+1/8+1/8+1/8 = 1/2 +3/8, another might result in 7/8 = 1/2+1/4+1/8 (Umar, 2010; Verma, 2010). Through this experience, they know how to compare unit fractions. Over a period of time, they also learn the equivalence of different division schemes and can move from one to another with ease. Children carried out measurement activities using a unit scale and subunits from ½ to 1/10. In the third year (Grade 5) we introduced representation of fractions on the number line, equivalence of fractions, and gave an idea of how to use equivalence to compare or add fractions.

There follow some instances of students' reasoning schemes that emerged spontaneously. Nikita (Grade 5) compares 4/5 and 7/8 as follows. First she divides each of the 4 rotis each into 5 equal parts giving each child 4/5. Then she shares out 7 rotis among 8 as 1/2+3/8, quickly changes her mind about 4/5 and writes it as ½+1/10+2/10 =1/2+3/10 and concludes that 4/5, which is equal to 1/2+3/10, is less than 7/8, which is equal to ½ +3/8 because 1/10 is less than 1/8. There are several such instances of spontaneous reasoning that children come up with because for them, over a period of three years, the fraction symbol means many things. Pooja devised a scheme to represent 1000+300/700 on the number line by declaring that "no one can divide the gap between 1000 and 1001 into 700 equal parts. So we will divide into 7 equal parts and assume that each part represents 100", though she was not sure how to write one thousand and one and hesitantly wrote 10001. We found that students were able to hold on to the meaning of the fraction symbol as a share and model many "word problems" in the language of share, and answer correctly (Umar, 2010).

Though the experiment convinced us that children can be taught fractions in a meaningful way from Grade 3, it also gave us an opportunity to understand the limitations in a government school context. Most of the children had difficulty reading, and so arguments that children came up with were oral, which the teacher documented after the class, though children certainly could write equations like 7/8=1/2+1/4+1/8.

Partial Quotients Method for Division

In continuation of the work with the same students, now at Grade 6 level, we found that many of them could not carry out the standard division algorithm as it made no sense to them. We introduced the Partial Quotients method currently used in the NCERT Grade 5 textbook that works with the whole number rather than individual digits as the standard division algorithm does. Some of the Grade 6 students needed the support of materials such as matchsticks to distribute and record the results, while many others could carry out division using the Partial Quotients method on paper and pencil; more importantly, almost all the students were able to relate to division in a meaningful way (Khemani & Subramanian, 2012).

Explorations in Negative Numbers

We attempted to introduce negative numbers through games and meaningful situations to Grade 6 and 7 students in two of the private schools we were working with. We found that, while students could carry out addition of integers, subtraction and ordering posed major challenges; also students did not relate to negative numbers in any meaningful way, though with practice they could carry out addition.

Explorations in Algebra

In our effort to assess what levels of abstraction children can engage with, we adopted a procedural approach to introduce algebra to Grade 6 children in a government school. Any trial in algebra is very challenging because children cannot write simple arithmetic expressions or even numbers properly. So we made children compute solutions for verbally stated arithmetic expressions and we wrote down on the board what they said and used these as the basis to introduce algebraic expressions. For example, they could be asked "think of a number, double it, add 5 to it, subtract the number you thought of, add another 5, subtract again the number you thought of, and tell me what you got" and report how they calculated. The whole equation was recorded by the teacher for the benefit of all to see. For example, if a child said "I thought of the number 3" and proceeded with the calculation, it would be recorded as: $(2x3)+5-3+5-3 = 6+5-3+5-3 = 10$. After a few examples they realize that the number they doubled got subtracted twice and so only 10 will remain. After much discussion and reasoning, we also arrived at the equation $2x+5-x+5-x = 10$ by saying "x represents the number in someone's mind which we do not know". Almost always they were able to notice the pattern, say what would be the result and why. In one rare instance, a student even made a purely mathematical remark by saying that "the result of $2x-5+3-1$ would always be an odd number" meaning if we substitute natural numbers for x we will only get odd numbers. While students participated in these exercises eagerly, it was clear that barring three or four students in the class, the rest would not be able to write these expressions on their own. In other words, our attempt to make them see algebra as generalized arithmetic and work with symbols rather than numbers did not succeed.

A Call for Re-visioning the Upper Primary Curriculum

According to the annual statewide surveys, 50% of children in Grade 7 cannot divide a 3-digit number by a single-digit number. By contrast, children from marginalized backgrounds who engage in economic activity to supplement family income bring oral arithmetic skills to the classroom from their workplace (Khan, 2004).

However, if we refer back to the upper primary curriculum in force, we see that there is very little that it has to offer these children, irrespective of whether they study in government schools or private schools catering to the poor. From the point of view of the curriculum, these children are disposable. Any curriculum design begins with many assumptions that include normative notions about childhood, about the nature of the disciplinary knowledge that it plans to impart and its relevance to the learner, about the learners' prior knowledge, cognitive capabilities, their interest in what the curriculum proposes to deliver, about teachers' competence and conviction to teach the content. Since a curriculum does not exist in a vacuum but is realized in practice, one way to know what these assumptions are is to inspect whom it turns out as successful at the end and what purpose the curriculum serves them. If the upper primary mathematics curriculum turns out predominantly urban and semi-urban middle-class children as successful, then it would be fair to say that the curriculum is premised on their prior knowledge and support structure and that it functions to meet their aspirations. In other words, the curriculum would not thrust on the normative students the kind of mathematics that they cannot cope with, as it does with children from the marginalized backgrounds. On the other hand, if the curriculum had a vision for those from the marginalized who constitute the bulk of students who drop out of school at various stages, then it would attempt to incorporate mathematics that will add value to their lives. It is impossible to conceive of an upper primary mathematics curriculum that will engage learners (whatever their socio-cultural and economic background might be) with operations on polynomial expressions, if the learners we have in mind are those whom we described in the previous section – children who have the same reasoning skills but have not been trained in written mathematics. Oral arithmetic competence, while necessary and important, cannot serve as a sufficient platform to launch teaching of algebra as generalized arithmetic but the cognitive and computational skills involved in oral arithmetic can be productively used in mathematical projects designed to understand some of socioeconomic realities. Children for whom school learning is synonymous with copying meaningless symbols, and whose reading, writing, and arithmetic skills at Grade 6 level are at the level of what is expected of a Grade 2 child, cannot be the children the upper primary curriculum has in mind if the content of the curriculum were to remain what it is. Curriculum design, as any other human endeavor, involves ethical as well as political considerations. However, adopting a specific understanding of what constitutes mathematics, and where it is useful, allows us to believe that curriculum design in mathematics is neutral, linear, and cumulative.

The RTE act could be interpreted to mean schooling as contributing to the development of the deprived children in other ways even if they cannot access subject knowledge. However, such an interpretation could apply to children from the dominant class as well, and in fact that would allow a re-visioning of mathematics curriculum that goes beyond preparing the learner to engage with higher mathematics. More specifically, upper primary mathematics curriculum could also contribute to critical understanding of one's social reality and provide a scope for empowerment. Incorporating such content would allow for collaborative and meaningful learning for all children, including those who may have oral competency but are not well trained in primary school mathematics. Skovsmose (2011) gives some examples how this can be done even in a small way, and that could be a starting point for re-visioning the curriculum.

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Intergenerational Analysis of Mathematical Cultural Tool Appropriation in Transnational Families

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Sociocultural theory maintains that human learning is mediated by cultural tools and also highlights the relationships between agents and cultural tools, which are embedded in social and historical contexts. This paper investigates whether and how transnational families engage in the appropriation of a mathematical cultural tool, a finger multiplication strategy, while contending with the pressures to conform to the mainstream practices of their host country. This study is based on ethnographic interviews with transnational parents and children in Japan. This paper offers insights into the process of out-of-school mathematics learning across generations and how identities are tied with this process. Findings also suggest how broad systems of power impact out-of-school mathematics learning.

Background and Conceptual Framework

Global mobility of populations has contributed to a growth in the number of transnational students and parents in schools internationally. Thus, there is a need for research examining curriculum and pedagogy responsive to these individuals' needs. In this study, I will examine transnational students' and parents' mathematical learning by focusing on one particular type of mathematical cultural tool. The term, transnational, is used to refer to the state of individuals who live beyond and across national boundaries. It describes the state of participants in this study who were living beyond and across the borders of Japan and the Philippines.

One of the central tenets of sociocultural theory is that human learning is mediated by technical and psychological tools including languages and culturally specific computation strategies (Vygotsky, 1978). These tools are cultural; they are created, shared, and used in collective practices. Vygotsky (1978) maintains that the use of these cultural tools is developed as an interpsychological process between people first and later internalized into an intrapsychological process within a person. This idea of internalization was later elaborated by Wertsch (1998) with careful attention to a tension between learners as active agents and cultural tools. Wertsch distinguishes mastery, appropriation, and resistance of cultural tools. Mastery is described as knowing how to use cultural tools, whereas appropriation is defined as "making something one's own." Resistance is considered to occur when agents distance themselves from particular cultural tools or when they perform certain actions with cultural tools under the forced circumstance. For example, in the case of Estonians in Wertsch's study, they mastered the official account of Estonia's history learned in the school and exhibited their understandings in the forced situation such as exams. However, they resisted to appropriate the official historical account and took it as someone else's history.

As seen in this example, mastery, appropriation and resistance of cultural tools are strongly connected with an agent's identity development. Polman (2006) further elaborated on the aspect of identity development and maintains that agents are taking up certain cultural tools while simultaneously creating possibilities for what kind of person one can be in the future. This study thus describes appropriation of cultural tools in relation to identities. Identity in this study is described aligned with Holland, Skinner, Lachicotte Jr, and Cain's (1998) account. First, identities are the social products and "imaginings of self in worlds of action" (Holland et al., 1998, p.5) which are developed in social practices. Second, identities are "psychohistorical formations that develop over a person's lifetime, populating intimate terrain and motivating social life" (Holland et al., 1998, p.5). Holland et al.'s case studies illustrate how cultural tools can afford and also constrain agents' possibilities of becoming. These conceptual frameworks led to my examination of transnational students' and parents' appropriation and resistance of a particular mathematical cultural tool in relation to identities they developed socially and projected for the future.

Literature Review on Non-dominant Students' Mathematics Practice

This paper highlights a relationship between agents and a mathematical cultural tool with the focus of transnational families living in Japan. Previous studies on social practices involving mathematical tools help frame the current study. Overall, research in this area has highlighted non-dominant students' and communities' competence in collective practices that involve mathematical tools (Abreu & Cline, 2007; Gonzalez, Andrade, Civil, & Moll, 2001; Nasir & Hand, 2008; Saxe, 1988, 2012; Saxe & Esmonde, 2005; Taylor, 2009). These studies altogether foregrounded mathematical competence that has traditionally been neglected in the school contexts. For example, in the study of low-income children's candy selling practices in Brazil, Saxe (1988) revealed the mathematical understanding that these children constructed in order to achieve the goals linked to the practices in which they participated. Taylor's (2009) analysis of low-income children's purchasing practices in the United States demonstrated the influences of the macro system (such as concentration of poverty and taxation policies) on these children's mathematics learning. Nasir and Hand's (2008) comparison of students' participation in mathematics classrooms and basketball games demonstrates significance in attending to students' opportunities for taking up integral roles and for developing identities of competence in a particular practice. Examination of students' practices outside school has been offering significant insights to non-dominant students' mathematical understanding and identity development. The current study builds on these works and examines transnational families' practices living in Japan, which have been understudied.

In this paper, I focus on one cultural tool (i.e., a finger multiplication strategy), which was observed among Filipino families living in Japan. The details of the finger multiplication strategy will be explained in a subsequent section. Saxe's (2012) seminal work on Oksapmin people's 27-body-part counting system illuminates how the counting system has been reproduced and altered along with the shift in political, economical and educational macro systems in Papua New Guinea. While the longitudinal historical analyses are beyond the scope of this study, I examined how a particular finger multiplication strategy was appropriated and (not) maintained among transnational parents and children. In examining appropriation of cultural tools, Polman (2006) called for further research on this topic across time and across locations. This case study will offer insights into the adaptation and rejection of cultural tools, along with these families' cross-national movement and their pressure to fit into the mainstream. In doing so, this paper addresses the following research questions: 1) Whether and how do Filipino parents in Japan appropriate and pass on the finger multiplication strategy to their children? How do their narrated identities relate to this process? 2)Whether and how do children of the parent participants appropriate the finger multiplication strategy? How do their narrated identities relate to this process?

Methodology

The main data source for this paper was obtained through a set of semi-structured interviews with 12 women who self-identified as Filipino and lived in Japan as well as semi-structured interviews with 10 of their school-aged children. All the interviews were conducted between November 2012 and February 2013. Child participants were either born in Japan or in other countries. In the interview, I elicited their transnational experiences, their language and mathematics practices in and outside the school. Both parent and child interviews included computation problems and word problems and they were asked to explain their strategies. The computation problems were presented as similar to school quizzes (10-15 problems for each participant). Word problems included both textbook problems and original problems that reflected the context in which parents used mathematics in daily lives. The majority of the interviews were conducted once, for approximately 1.5 hours. Depending on the preference of each participant, I conducted the interviews in either English or Japanese and later translated all quotes into English. Based on the findings from the interview study, I facilitated a series of workshops and addressed topics such as exploring the application and extension of the finger multiplication strategy. For the purposes of this paper, I focus on the findings from the parent and child interviews.

Analysis

This paper presents an intergenerational analysis of cultural tool appropriation among transnational families. In other words, I analyzed perspectives of both parents and children with respect to their use of a particular cultural tool, finger multiplication strategy. After descriptive coding, I wrote an analytic memo for common themes appeared across participants and generated themes for focused coding. The focused coding highlighted for this paper was on identity and mathematical cultural tools. In this paper, the analysis of identity was limited to the aspects that were narrated in relation to mathematics. Under each coding, I identified common themes that appeared in more than half of the participants' interviews. For the analysis of child participants' appropriation of finger multiplication strategy, I identified parents who indicated they had taught the strategy to their child and then focused my analysis on those three children. My analysis investigated whether they used the finger multiplication strategy solving a computation task and also how they described the use of the strategy in school.

Findings

Finger Multiplication Strategy

In this paper, I draw on the use of the finger multiplication strategy as a window to examine power and tensions that arise in the discourse of transnational families and their use of cultural tools. In the process of computing school mathematics problems, some parent participants and child participants used a multiplication strategy using their fingers. This strategy was observed among five of the participants when they were solving a single-digit multiplication of numbers between six and nine. For example, this strategy was used when participants were solving the problem, 9 x 9. Using two hands, the strategy was described as follows by participants (each hand represents one number): Five is represented by the closed hand and any number above five is represented by the number of fingered opened (e.g., nine is four fingers open) (Figure 1 is a participant's representation

Figure 1. A parent participant's finger multiplication strategy (9 x 9)

of 9 x 9). Then, participants counted the number of open fingers and multiply ten to the number (product A) (e.g., in the case of 9 x 9, participants computed 8 x 10). Then, participants multiplied numbers of closed fingers in each hand (product B) (e.g., in the case of 9 x 9, participants computed 1x 1). The final multiplication product was calculated by adding the above products A and B.

Participants explained that they used this strategy mainly for completing computation problems quickly in the school. For example, Irene said, "when we were at school and then we were doing some tests - that needs to be quick." None of the participants reported other contexts where they would use this strategy in their everyday lives. Participants acquired this strategy from various places: from their family members, friends, and at school. Child participants acquired this strategy exclusively from their parents. Neither the parent nor the child participants could identify the historical origin of this strategy. Some parent participants stated that the strategy was wide spread in their communities in the Philippines.

Parent Participants' Appropriation and Resistance of Cultural Tools and Identity

In the interview, parent participants associated their cultural tools including the above-introduced finger multiplication strategy with their perceived national identities. Specifically, they described their use of cultural tools as "Filipino ways" that would significantly differ from the ways in which their children were learning at school in Japan. Parent participants stated that mathematics and English were the only subjects they could academically support their children. However, they also expressed difficulties with teaching children and attributed the difficulties to the perceived differences between Filipino ways and Japanese ways. Karen's narrative illustrates this point. When asked whether she would help children learn mathematics at home, Karen said, "We don't know really exactly what kinds, how you solve the mathematics. We don't understand your ways (…) Japanese schools, you have your own system." In this narrative, Karen's use of pronoun indicates how she set up a boundary between Japanese ways (e.g., your ways, your own system) and Filipino ways and she distanced herself from Japanese ways. In the interview, parent participants commonly described how the cultural boundary restricted them from communicating on school mathematics with their children.

Furthermore, in the interview, parent participants undervalued their cultural tools compared to the cultural tools their children were acquiring at Japanese schools. For example, in her interview, Janice repeatedly said Japanese mathematics teaching was higher as represented in the following quote: "Japan is at a high level…what to say, I think mathematics is also at higher level. Filipino children learn division at around Grade 5 and Grade 6 but Japanese children learn it at a higher level." This perceived hierarchy-influenced participants' recognized roles as parents. Parent participants associated their national identities with helplessness to participate in children's school education. They expressed this powerlessness in relation to their backgrounds that they recognized to be different from the mainstream in Japan. For example, Michelle said, "I'm Filipina and I can't offer anything as a parent because I'm a foreigner. So, I always tell my child, 'you don't have to be number one, two or three but you have to follow what everyone else is doing.'" This quote resonates with what other participants described regarding their identities. Parent participants adapted an assimilative discourse summarized as follows, where they hoped their children would exclusively master the mainstream cultural tools.

These parent participants' identities shaped family practices surrounding the finger multiplication. All the parent participants positioned this finger multiplication strategy or the use of finger as secondary to strategies taught at school in Japan. When the finger multiplication strategy was taught at home, it was as a "secret" and parent participants emphasized that children should acquire the mainstream cultural tools taught at school. In her interview, Karen described a main function of this strategy as "only for an emergency purpose," as using fingers would be considered to be illegitimate in Japanese schools. When teaching this strategy to their children, Karen said they also encouraged their children to memorize a multiplication table. These participants' accounts are similar to Wertsch's (1998) account of resistance to the extent that parents taught their children their cultural tool for emergency purposes while also teaching to perform the mainstream way at school. The parent participants' story in this study is more assimilative in the sense that they positioned their cultural tools to be secondary and "secret tools" compared to mainstream mathematics practices.

Furthermore, parent participants connected the finger multiplication strategy with perceived children's computation fluency and memorization skills. There was a parent participant who selectively taught the strategy to the child who were struggling with computation and memorization but not to the child who were doing well in mathematics. Regarding this point, Irene, Ryan's mother, said, "I'm telling Ryan this because his memory is a little bit… slow. So, I told him, 'you can use this one' and he said, 'mommy, I think this is harder – I can't. I have to remember.'" As such, teaching or not teaching of informal knowledge was connected to how parents envisioned their children's identities. In the following section, I describe how child participants appropriated or did not appropriate the finger multiplication strategy.

Child Participants' Appropriation of Cultural Tool and Identity

Parents of the following three child participants indicated they taught the finger multiplication strategy to their children. May was a Grade 6 girl who came from the Philippines to Japan when she was in Grade 4 and repeated Grade 3 in Japan due to her limited Japanese language proficiency. At the time of study, May's parents were considering whether they should send her back to the Philippines for junior high school. Eric was a Grade 4 boy, who came from the Philippines to

Japan after finishing Grade 1. His parents described him as an excellent student in the Philippines but that he was academically behind in Japan, receiving extra support from his classroom teacher after school. Ryan was a Grade 4 boy, who was born and raised in Japan. Because neither of his parents were Japanese citizens, their residency had to be updated annually and thus the family was not guaranteed for permanent residency.

The Japanese mathematics curriculum covers multiplication of single digit numbers in Grade 2 and requires students to recite the entire multiplication table between one and nine, using a culturally specific recitation strategy called *kuku.* For May who came to Japan after Grade 3, she did not have a chance to learn the *kuku* strategy. Both Eric and Ryan indicated that they learned the *kuku* strategy at school. Ryan stated in the interview: "I was placed in the slowest group for computation in Grade 2 and was forced to practice *kuku* repeatedly." Thus, multiplication learning at school was a locus for constructing students' academic identities.

In order to analyze how child participants appropriated or did not appropriate the finger multiplication strategy, I first examined how child participants solved single and two digits multiplication problems. May and Eric both used the strategy for single digit multiplication problems of numbers between six and nine. They both explained that they learned the finger multiplication strategy from their parents. Ryan did not use finger multiplication strategy but used the *kuku* strategy. May was able to reach the correct answers for both single and two digits multiplication problems but Ryan and Eric both could reach the correct answers for single digit multiplication but could not reach the correct answers for two-digit multiplication.

How did child participant use the finger multiplication strategy in school, where the use of finger was not encouraged and acquisition of *kuku* strategy was expected? May and Eric both mentioned that they would not use the strategy openly at school. Eric said he never showed the strategy to school teachers. May also said she would not use this strategy during mathematics quizzes but would use computation strategies that the teacher showed. She provided an example of 7 + 8. Outside the school, she said she would usually compute this addition problem by decomposing the number by 5 (as 5+5+2+3). But in the mathematics quizzes at school, she would follow the way the teacher taught without decomposing numbers. This example shows how both

Eric and May were aware of what is considered to be legitimate and correct at school and they would strategically hide computation strategies different from the ways in which they learned at school.

The following quote from May provides a further account on how she appropriated the finger multiplication strategy to maneuver a competitive school environment. When asked in the interview if she would like teachers to acknowledge her finger multiplication strategy, May said, “No, I don’t want to (show it to the teacher). Everyone else is very fast at computation. I shared this only with those who were slow (at computation).” And she further added, “Students who go to afterschool cram schools (*juku*) are very fast (at computation).” Attending private cram schools (*juku*) during after-school hours have been common among children in Japan and even among upper grade elementary school students for previewing and reviewing school subjects and also for preparing for entrance exams. The fee for a cram school ranges between 50 dollars to 200 dollars per month. As such, there can be a social class divide between those who can attend a cram school and those who cannot. May’s interview describes how she observes a gap in computation fluency. In facing this gap, May selectively shared the finger multiplication strategy only with students like herself, namely, those who were slower at computation and did not have access to a private cram school. By doing so, May tactically used the finger multiplication strategy to create a context where she could be a competent student in mathematics.

Discussions and Implications

In sum, this paper explores the use of the finger multiplication strategy to demonstrate tensions that transnational families must contend with as they endeavor to assimilate into the dominant culture of their host country. Filipino parents hoped for their children to master the use of mainstream cultural tools and thus taught the finger multiplication strategy to be used as a secret tool. This decision was connected to the narrated hierarchy between their home country and their host country with regards to mathematics teaching. Parent participants felt distance from the mainstream, but hoped for their children to fit in this mainstream. Thus, they chose not to openly pass on their cultural tools. Child participants who appropriated the finger multiplication strategy tactically used the tool to maneuver a competitive school environment and to position themselves as competent mathematics students. By employing the intergenerational analysis, this paper offers insights as to how a cultural tool is appropriated or (not) maintained across generations and how identities are strongly tied with this process.

This finding is particularly significant to the discussion on how to meaningfully bridge out-of-school resources and in-school learning. As opposed to the deficit views toward immigrant students and households, there has been studies on mathematics teaching, which highlights immigrant families' funds of knowledge embedded in cultural practices that they engage in (e.g., Gonzalez, et al., 2001). For example, the finger multiplication strategy highlighted by this study can be useful for challenging students' "fossilized" (Vygotsky, 1978) knowledge (with a recitation method of multiplication) and for diversifying students' ways of approaching multiplication.

This paper also describes how social power imbalances can influence the use and maintenance of cultural tools. Findings from this study adds to the discussion of "social valorization" (De Abreu & Cline, 2007) that highlights the processes in which different kinds of mathematics (e.g., Brazilian peasant mathematics and school mathematics) are valorized or devalorized with reference to social classification. This paper depicted the interplay between participants' identities and the appropriation and resistance of a cultural tool, which was undervalued by parents and in the school. Also, this paper highlights child participants' agency to create a context where this socially undervalued cultural tool can be meaningfully used. Further research will be beneficial to identify school curriculum and pedagogy which can contribute to challenging the stabilized social valorization and which can support and affirm diverse ways of knowing. Such curriculum and pedagogy will eventually lead to affirming students' and parents' identities, because cultural tools and identity can be closely intertwined, as demonstrated in this paper.

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Racial and Class Tensions in Colombian Mathematics Classrooms

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Underachievement in mathematics along lines of race, class, and gender is a pervasive phenomenon in the Colombian educational system. In particular, black and poor students persistently lag behind their mestizo and wealthier peers. Explanations for the low performance of these students' population are usually grounded in essentialist perspectives that mainly attribute school failure to student backgrounds. Adopting a socio-political perspective, the study I report sought to understand the ways wherein exclusion and marginalization operate within the mathematics classroom. The study focused on analyzing the relationships between students' racial and class background, teachers' expectations, and teaching practices.

Background

Colombia is a society characterized by an unequal educational system that seems to reinforce social inequalities (Garcia, Espinosa, Jiménez, & Parra, 2013). Despite this fact, few studies have focused on investigating the ways wherein school practices contribute to the marginalization of particular groups of students. Regarding school mathematics, an important goal of the Colombian educational system is to provide support and opportunities to learn mathematics for *all* students (Ministerio de Educación Nacional [MEN], 1998). However, this is not the current situation. As the students' outcomes in national and international large-scale assessment show, the failure in school mathematics affects mainly students of marginalized communities (Instituto Colombiano para el Fomento de la Educación Superior [ICFES], 2010), which are largely inhabited by Blacks and other racial minorities. Although it appears that the intersection of race and class shapes Colombian students' opportunities to learn mathematics, little is known about how teacher expectations and teaching practices contribute to this reality. Hence, this study is aimed to analyze the mechanisms by which students' racial and social backgrounds shape teachers' expectations and practices either hindering or promoting students' opportunities to learn mathematics. The study addressed the following questions:

1. How are Colombian teachers' expectations about eighth graders' ability to learn algebra related to the students' racial and social backgrounds?
2. How are these teacher expectations expressed in their teaching practices?

My interest in issues of race, class, and mathematics education relates to my own experiences as a black, low-income student trying to succeed in the Colombian educational system. Exploring teachers' beliefs about race and poverty in relation to mathematics learning, I face my own story as a student who many times heard her teachers expressing surprise at finding a "successful" poor, black student in the classroom. Thus, my interpretations of the phenomena in this study may be influenced by these experiences and others related to my own work as a mathematics teacher and as a teacher educator in my country.

Theoretical Perspective

This study drew upon recent work in the field of mathematics education that addresses issues of power from a critical sociopolitical perspective (Apple, 1981; Valero, 2012). The paradigm that supports this type of research contends traditional approaches that introduce deficit views of marginalized students to seek explanations for their mathematics failure. Instead, this study was grounded on the premise that, in order to understand such failure, it is fundamental to investigate and focus on the school practices that unequally distribute knowledge, dispositions, and opportunities to learn (Weber, Radu, Mueller, Powell, & Maher, 2010). Beyond trying to bridge the achievement gap between minority and mainstream students, the main purpose of research conducted within the theoretical perspective assumed in this study is to uncover the structure and mechanisms that install and perpetuate inequity at school (Martin, 2009).

In this perspective, schools are not neutral spaces in relation to issues of power and in particular of racism. Indeed, schools are partially structured along racial lines. Students are located in racial structures in ways that determine their differential access to knowledge, abilities, and dispositions. Racial tensions are acutely expressed at schools and in this sense they are privileged contexts for analyzing the forms wherein racial discrimination operates to produce the outcomes and experiences of black and other racial minority students.

In the context of the sociopolitical stance assumed in this study, I introduce Bonilla-Silva's concept of *framework* or *racial ideology* to deepen into the forms in which ideological representations of black and poor students emerge at school and configure teacher-students interactions and teaching practices. Frameworks constitute the sources that nurture the representations teachers elaborate about particular group of students and allow them to explain, justify and naturalize the school failure or success of students. Such frameworks are conveyed through *discursive strategies.* Everyday conversations and institutional talks and texts are the key mechanisms in the reproduction of prejudices (Bonilla-Silva, 2010; Van Dijk, 1992) and in their naturalization. Frameworks and discursive strategies are fundamental in understanding how power operates at school.

The Study: Context and Methodology

Using a comparative method and an interpretative approach, I analyzed the expectations and teaching practices of three different eighth-grade mathematics teachers. The study was conducted in one low-income, one working-class, and one upper middle-class schools in Cali, the Colombian urban center with the highest percentage of black population (Urrea, Viáfara, Ramírez & Botero, 2007). Because of the overlap between race and class in Colombia, black students were overrepresented in the low-income school whereas they were absent among the student population at the upper middle class school. I filmed the participating teachers' lessons of algebra and four students in each classroom wore sunglass cameras to record their interactions with their peers and teachers. I also interviewed teachers and students. Finally, I conducted weekly debriefs with each teacher to discuss class episodes previously selected. The teachers identified themselves as *mestizos* and middle-class. Table 1 summarizes the racial and class compositions of the participating schools as well as the characteristics of the participating teachers.

Table 1
Racial and Class Compositions of the Three Schools

SCHOOL	TEACHER'S NAME	TEACHING EXPERIENCE		PERCENTAGE OF BLACK STUDENTS	PERCENTAGE OF POOR STUDENTS
		Mathematics	Algebra		
Low Income School	Diana	16	3	41%	100%
Working Class School	Juan	18	3	9%	23%
Upper Middle Class School	Pedro	20	4	0%	0%

I used an interpretative analysis perspective to make sense of the participating teachers' expectations and practices. The process of conducting an interpretative analysis comprised three different stages. First, the transcriptions of a small piece of the data sources were read to look for potential categories and themes. Second, as the categories emerged, the process continued going over the data and pulling them from those categories. And third, the links between the categories were identified.

Findings

The findings of this study draw attention towards the critical role of the ideological representations of poor and black students in the expectations teachers hold about the ability of these student populations to learn algebra. In my discussion, I will focus on the interplays between teachers' expectations and their instructional strategies.

Ideological Representations of Black Students: Cultural and Class Eeficit Frameworks

The results confirm a dominant view in education according to which racial minority and poor students have lower ability to learn mathematics than their wealthier and mainstream peers (Zevenbergen, 2003). The teachers in this study constantly held lower expectations for black and poor students and accordingly adjusted their teaching practice to such anticipations. Both race and class constituted the main sources of representations, ideas, and meanings that the teachers used to interpret and make sense of the students' attitudes and behaviors during mathematics instruction. *Cultural* and *class deficit* frameworks were identified as the main sources used by the participating teachers to justify and explain poor and black students' performance at school. Juan's narrative about black people exemplifies a cultural deficit framework:

> Luz: How would you explain Daniela's (black student) and Sandra's (native student) difficulties in learning algebra?
>
> Juan: There may be little interest to develop [mathematics thinking] in those communities. They might think "our educational project is not aimed to get students ready for college but to prepare them to farm and become farmers." Or they might think "our educational project is oriented towards developing handcrafted abilities" and then, learning mathematics is not a goal for them. Or they might think "we have the biotype for becoming athletes". I think [cultural differences] might affect [the students' performance]. [They affect] the majority of students although there are some exceptions.

The cultural deficit framework is also noticeable in Diana's narrative about her black students. It allows her to justify the black students' low performance at school, to naturalize their low performance, and to avoid the responsibility in her students' failure:

> Luz: Do you think black students in your classrooms need additional support to learn mathematics?
>
> Diana: And why just for learning mathematics? I think it is not just to learn mathematics; they need support to learn anything. They do not like to read and reading is fundamental for [learning] any subject matter at school; they do not like to read, so they are not going to understand the different concepts or basis

they are given in the different classes. And they do not have the culture of [complying with] homework.

Luz: What type of support do you think they would need?

Diana: I think they would need psychological and occupational therapy… Because they could take their problems out. That is something teachers cannot do; we do not have time for that, we are not trained for that. We are trained for teaching, so we do not know how to face the difficulties these students are having—displacement, drugs, gangs—all this kind of stuff. Sometimes you just do not know how to deal with them, so you ask yourself, "God! What else should I do? What else can I do? Besides we do not have time for that

Diana's narrative reveals the inferior social and cultural status in which she positions black students depicting them as lacking the dispositions needed to perform well at school. They are positioned as different and divergent from expected and accepted social behaviours and cultural differences are portrayed as pathological conditions that need to be treated. Thus, through cultural deficit views the participating teachers convey representations of black students as culturally inferior and lacking the dispositions to succeed in learning mathematics.

Regarding a *class deficit framework*, the participating teachers depicted poor students as deviant from mainstream values and dispositions needed for academic success. Poverty was seen as a condition that impeded the students' access to complex forms of algebraic knowledge and higher order algebraic thinking. The teachers perceived poverty as an obstacle that needed to be overcome before the students could reach any achievement. The following narrative exemplifies a class deficit framework:

Luz: Why do you think that idea is so widespread?

Diana: Because the context they live in? People have generalized in that way. I mean the areas inhabited by vulnerable people, that have low employment opportunities; then, you relate [the areas] with people who do not want to study and do not want to reach something better or they cannot reach something else,

> I do not know. She [another teacher] has always told me that the students, the group of students here is different, I mean, the environment they have here is different, the conditions they have here in the *distrito*, and it is not possible to do the same work here that could be done in another school [out of the *distrito*].

In contrast, references to students' backgrounds are absent in Pedro's narratives about his students' mathematics performance, as may be noticed in the following narrative:

> Pedro: All students in my class have the same chances to pass. They like algebra a lot. I think they enjoy the class and want to learn. There are a few students that might have some problems, but [their future performance] depends on you, as the teacher. If they failed is because there was a lack of teacher's effort.

The intersection of race and class in the learning of algebra constitutes a double jeopardy for the students in this study. Black students are overrepresented in the low-income population in Cali and, in this regard, discrimination and marginalization at school doubly affect them. On the one hand, black students are positioned as incapable of learning mathematics because of ideologies that have naturalized representations of black culture as inferior and its people as lacking the abilities for carrying out processes of thinking strongly associated to the dominant mathematics rationality in the Western world. The trivialization of the black culture that associates it with the dominance of sense over thinking, handwork over intellectual work, and body over mind, translates into low expectations at school. On the other hand, the marginalization of poor students comes from dominant ideologies that present middle-class values and dispositions as legitimate "while treating the cultural\communication styles of working-class people as inferior or worthless" (Lim, 2008, p. 93). This is why, in my opinion, Pedro does not use his students' backgrounds to explain their school performance: They fulfill the requirements to be successful at school. That the participating teachers are biased against black and poor students is clear when I asked Pedro to compare the mathematics performance of the students in the upper-middle class school with the students in a public school mainly attended by poor pupils and in which he teaches during the afternoons. Pedro told me:

> Pedro: What I would say. [In the public school] there are some students that assist for earning a certificate rather than for learning. [They think] "education is free, I am forced to attend, I have to comply with" [it]. Currently, I'm teaching in two ninth grades there and when I talk with my students, I know where they are going: nowhere. If you asked them, "What do you want to do in the future?", [you would know that] most of them do not have expectations.

Researchers have argued that school, as one of the main agencies of cultural reproduction, promotes and recognizes values and dispositions of the ruling class that are unfamiliar to economically disadvantaged students (Bourdieu, 2011). Wealthy students adapt and easily respond to the dynamics of the school to the extent that they possess the cultural capital to do so, and then, school practices and discourses favor this student population (Zevenbergn & Niesche, 2008). The findings in this study coincide with this trend and show that the teachers highly value some dispositions that are presented as legitimate and whose absence indicates academic deficit and inferior status. Poor students are disadvantaged by hegemonic representations that position them as "others" in contrast to dominant assumptions of middle-class learners (Archer, 2003). Stereotypes as conveyed through the cultural and class deficit frameworks are ways of perpetuating the exclusion and marginalization of poor and black students. Ideological representations about black pupils helped the participating teachers naturalize the mathematical performance of the students and present themselves as neutral participants in the configuration of their low mathematics performance.

Teaching Practices

The findings of the study also reveal the ways wherein teacher expectations shape their teaching practices. Low and high expectations resulted in different opportunities to learn for the students and then, their experiences in learning algebra were markedly different across the three social contexts.

Low expectations translate into teaching practices that, in this study, diminished the quality of the algebraic content to be taught

(Anyon, 1980; Lee, Smith, & Croninger, 1997). The teachers tried to foster the learning of algebra by adjusting their instruction to the perceived abilities of the students. Both, the language and the teaching practices displayed by the teachers were aimed to facilitate the learning process of a group of students who was perceived as low achieving. Poor and black students were exposed to teaching practices that watered down the quality of the algebraic content. To illustrate this finding, I compare the strategies used by Diana and Pedro to introduce the notion of variable for first time to their students.

The following class episode shows the strategy used by Diana:

Diana: *(Diana draws a set of five hearts on the blackboard)* Here we have a set, and because we are always in love, we have little hearts. This is a set of what?

Students: *(Students answering in chorus)* Hearts!

Diana: *(She writes "hearts" on the blackboard)* How many hearts do we see here?

Students: Five!

Diana: We have five hearts, don't we? *(She writes "five hearts" on the blackboard)*. However, writing "five hearts" takes forever, so we can express it with the number five and a little heart that it is the symbol *(She writes the number five and draws a heart)*. But, what [would happen] if instead of a heart, we had a butterfly? It would take too much time to write "butterfly," so we can replace the heart with a letter. Which letter?

Students: The [letter] ***h***! The [letter] ***m***!

Diana: Ok, let us write the [letter] ***m***. What is the meaning of that [letter] ***m***?

Students: Heart!

Diana: Right, heart *(She writes the word "heart" on the top of its drawing)*. Look, here we have started algebra. It is as simple as this. The letters are going to replace unknown and known objects, things that we may know or not. Let us observe that after having a set of five hearts, we have got "five ***m***", where the letter ***m*** is representing an object called "heart"

Rather than a quantity that varies in a numerical range, Diana introduces the variable as a letter that stands for a material object or as a *label* (Philipp, 1992). In this particular case, the status of the variable is that of a letter that depicts an object. The variable is not characterized as a mathematics object that makes sense in the context of mathematics theories but it is objectified as a letter that can be easily manipulated. The letter representing the variable is an abbreviation whose introduction avoids the students writing long sentences. In addition, the mathematical relationship represented by the algebraic expression 5***m*** is completely ignored. The process through which the expression is built does not focus on the additive relation between the coefficient and the variable but rather on an economy of words and as a result, conceptual understating is sacrificed.

In contrast, Pedro introduces the notion as exemplified in the following class episode:

Pedro: Today, we are going to start using variables. Write down this problem. Take your time solving it. Write down three consecutive numbers and add them up. Repeat this process with five different sets of three consecutive numbers.

Students: Teacher, could the numbers be forty-one, forty-two, and forty-three?

Pedro: Any three consecutive numbers. Just be sure that the numbers are consecutive, ok? Observe all your responses and write down what you observe. Do you see a pattern? Do you see a rule or a regularity? What relationships between the three consecutive numbers and its sum do you observe? How would you describe such relationships? Take your time. Write down your answers and we will go back to them later

Pedro's teaching practices expose the students to mathematics experiences that researchers have pointed out as important for developing algebraic thinking such as solving problems and generalizing patterns (Kieran, 2007; Mason, 1999). The wealthier students are invited to build meaning for algebraic objects and procedures by exploring mathematics tasks in ways more aligned to those suggested by researchers in in the field. In this sense, their chances to access meaningful algebraic knowledge needed to succeed in the school systems are higher than those of their poor peers. While Diana focuses her teaching on the syntactic aspects of algebra and appeals to context-dependent situations to help students build meaning for algebraic operations, Pedro frequently focuses on asking the students to translate from the natural language to the algebraic symbolism and vice versa. He would ask the students to express in natural language an algebraic expression or, given and algebraic expression to represent it using natural language. As held by several researchers (e.g., Kieran, 2007), by introducing this strategy Pedro engages his students in a complex process of understanding and building meaning for algebraic objects that is absent in Diana's instruction.

Summarizing, in this study, the process of turning algebraic objects into objects of the material world and focusing on the procedural aspects of algebra arose as common practices to facilitate the students' learning. By linking the meaning of algebraic objects to objects of the material world, and by focusing on the syntactic features of algebra, the teachers in this study deny black and poor students opportunities of developing more powerful levels of algebraic thinking and building mathematics bases for acquiring more advanced mathematics knowledge (Sfard & Linchevski, 1994). This is an important way wherein teaching practices lessen the quality of the algebraic knowledge and prevent already marginalized students to move forward in their educational process.

Conclusions

The findings of this study draw attention towards the critical role of the ideological representations of poor and black students in the expectations that teachers hold about the ability of these student populations to learn algebra. The findings are consistent with research that points to the significant impact of the beliefs that teachers hold about students on mathematics learning, school experiences, and outcomes. Moreover, the results confirm a dominant view in education according to which racial minority and poor students have lower ability to learn mathematics than their wealthier and mainstream peers (Zevenbergen, 2003). The teachers in this study consistently held lower expectations for black and poor students and accordingly adjusted their teaching to such anticipations. Clearly, the teachers' expectations were supported and nurtured by ideologies related to the students' social backgrounds. Both race and class constituted the main sources of representations, ideas, and meanings that the teachers used to interpret and make sense of the students' attitudes and behaviors during mathematics instruction. Likewise, dominant ideologies about poor and black pupils helped teachers naturalize the mathematical performance of the students and present themselves as neutral participants in the configuration of their low mathematics performance.

The study is particularly significant for both the Colombian mathematics education community and the Colombian society itself. First, few studies addressing issues of power within the mathematics classroom have been conducted in Colombia due to, among other reasons, the dominance of cognitive and sociocultural perspectives in mathematics education research. Second, Colombia is considered as a racial democracy in which every individual regardless race, class, and gender has the same opportunities of social mobility. However this study shows that not all Colombians are equally positioned at school. In this sense, more studies addressing these issues are needed in order to better understand power within the mathematics classroom in developing countries and "racial democracies" as Colombia is considered.

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The Construction of the Child in the New Push for Mathematics Education in Early Childhood

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This article reports on an analysis of scholarship published over the last 20 years in four journals focused on early childhood education and mathematics education, which examined the discourse around how children are constructed as learners and doers of mathematics. The analysis found that attention to the whole child was minimal, largely as a result of a dominant focus on clinical studies examining family background as a variable in determining children's mathematics performance. This review suggests a need for research on how young children engage in mathematics in their homes and communities and how preservice teachers can learn to teach early mathematics in ways that draw on those resources and meet the needs of the whole child while promoting mathematical growth.

Introduction

In the 1991 three weather systems off the coast of the northeastern United States collided to form what many in the press and popular culture referred to as "the perfect storm". We see a perfect storm emerging in the US in early childhood mathematics education as three forces are colliding to shape how young children might be taught mathematics. Those forces—policies encouraging universal prekindergarten, research on early mathematics, and standards driven practices—have turned researchers', funding agencies', and educators' attention to mathematics learning opportunities for young children. Publically funded prekindergarten in the US has expanded rapidly in the past decade with states eyeing the promise of achievement gap reductions and economic return on investment. Simultaneously, several studies have suggested that early experiences with mathematics supports later academic performance in all content areas (Duncan, et al., 2007), something policy makers and administrators care about as they compare the performance on international tests of US students to those in other countries. When we situate these in an educational environment dominated by standardized testing and focused on performance, we find ourselves in a perfect storm circling around the mathematics education of preschoolers.

Our concern is that just as the perfect storm in 1991 was a force majeure in terms of damage done to personal property, this perfect storm has also emerged as a sort of *force majeure* with unintended consequences for the education of young children and, with no one liable for the outcome. Although the term, force majeure, has been used historically in legal contracts to rid either party of liability in the event of an act of nature, we find it useful in describing what we see happening in early childhood mathematics education. Definitions of force majeure include notions such as unpreventable, external, unpredictable, and beyond the control, all of which we see as occurring today in the ways that early mathematics education is being framed and, in particular, the ways in which the child as a mathematical being is framed. Thus, while we are embracing the attention that early childhood mathematics education is receiving, we want to raise a warning flag with respect to how the mathematics educators and early childhood educators that teach mathematics methods courses are thinking about children as learners and doers of mathematics. We are particularly concerned that educators from the two paradigms bring with them different resources and understandings about teaching young children, and in the end knowing that mathematics may be privileged over knowing how to teach the whole child. One way to understand how this is playing out is to examine how the bodies of research from early childhood education and mathematics education are framing children.

Discourse as a Window

The research articles published in both mathematics education and early childhood education journals are both *products of* and *producers of* the discourses surrounding early childhood teacher educators. We are not suggesting that teacher educators necessarily read research articles in order to make decisions about their teaching (although they may), but that these articles provide an empirical window into the discourses available to teacher educators when doing their work.

This view of discourse draws on poststructural theories, which see truth as produced through continual retellings in both spoken and written texts (Foucault, 1990; Parks, 2009). From this perspective, we would expect to see dominant beliefs in the discourse – such as the idea that young children engage in mathematical practices in multiple ways in school, homes, and communities – show up in a variety of ways, such as in discussions of the child in research articles, in informal hallway conversations among university instructors about play as a site of learning, and in collections of objects such as unifix cubes in mathematics education classrooms used for early childhood methods. The more often we see an idea reiterated in the discourse, the more likely the idea is to be taken-as-true by members of the discourse community (Foucault, 1980). In addition, distinctions among discourse communities may be drawn in part by differences in the dominant ideas circulated as truth. For example, making the statement that children progress through predictable developmental stages, which ought to be used as a primary determinant of appropriate instruction would mark the speaker as an insider in some discourse communities (where this statement is taken-as-truth) and as an outsider in other communities (where the falsity of the same statement is taken-as-truth). One of our goals in performing this analysis was to hold the discourses of the early childhood and mathematics education communities up against each other to identify instances where the truth claims of the two communities overlapped and instances where assumptions about truth diverged. Our hope is that doing this not only illuminates the knowledge bases that are being drawn on in the education of early childhood practitioners, but that the analysis also invites insiders in both communities to re-examine their community's truth claims.

Mode of Inquiry

Informed by our theoretical perspective, we drew on methods of textual analysis (Prior, 2003) to examine the content of research articles published about early childhood mathematics education over the last

20 years. Our interest in the differences among scholars who situated themselves primarily in the early childhood community and scholars who situated themselves primarily in the mathematics education community, prompted us to examine two journals in each field, including the *Early Childhood Research Quarterly,* the *Journal of Early Childhood Teacher Education,* the *Journal of Research in Mathematics Education,* and the *Journal of Mathematics Teacher Education.* Although there are numerous journals we could have reviewed, we chose these as the key journals in each field that attend to education research and teacher education research.

We read the titles and abstracts of each research article (editorials, commentaries, book reviews and other short features were excluded) for each target journal published from 1994-2014 to identify articles that addressed both early childhood and mathematics. We operationalized early childhood as attending to children from birth to Age 8 or to preservice or practicing teachers who worked with this population of children. In some cases, judgments about whether to include articles in our dataset were unclear. For example, we included the articles in our dataset if they attended to an area of concern for early childhood teachers, so that a K-12 study about teacher dispositions would be included, but a study that included only a small number of 8-year-olds in the total pool and focused on advanced mathematical content would not be. Using these guidelines, we identified a total of 239 articles of the 1,993 articles we reviewed. The breakdown of these articles by journal is shown in Tables 1. These figures are important to keep in mind throughout the article as we discuss the lack of emphasis on the young child. Even by our generous definitions, about 75 percent of the articles published in mathematics education journals did not address early childhood mathematics in any way. Thus, our claims in the findings section about attention to early childhood contexts are based only on those articles that did claim to address preschool to Grade 3 classrooms or children in some way.

Table 1
Data set: Number of articles from 1994–2014

JOURNAL	TOTAL	EC MATH	PERCENT
Journal of Research in Mathematics Education	353	90	25%
Journal of Mathematics Teacher Educators	307	70	23%
Early Childhood Research Quarterly	737	61	8%
Journal for Early Childhood Teacher Education	596	18	3%
Total	1,993	239	12%

To guide our analysis, we focused on this research question: What discourses about children are circulated in the research literature?

For this analysis we used NVivo, a software program for qualitative data analysis and coded each article for a variety of features, including the specific mathematical content (e.g., number, geometry, etc.), the context of the study (e.g., university, elementary, preschool, out-of-school, clinical-like site etc.), participants of the study (e.g., practicing teachers, preservice teachers, children), and family background or practices. Using NVivo to support our analyses, we identified a variety of relationships (such as a focus on family background addressed in early childhood journals versus mathematics education journals) and identified smaller pools of articles for closer examination (such as all articles that dealt with out-of-school contexts). Our analysis of these smaller pools of articles moved beyond coding and identifying relationships and toward a more holistic analysis of the arguments and assumptions addressed. These analyses were supported by conversations between the authors and through the writing of analytic memos (Emerson, Fretz & Shaw, 1995). The analysis presented below draws on both our coding of the large data set and our close examination of selected texts.

Construction of Children in the Literature

In our analysis of the ways that children were constructed throughout the literature, we identified three important themes framing children as: participants in clinical studies, mathematical beings disconnected from the rest of the child, and members of (dis)advantaged families and communities.

Children as Study Participants

First, children were frequently seen as participants in clinical experiments or interviews, rather than as students in classrooms or as members of families or communities. Of the 239 articles about early childhood mathematics that we identified, 106 of them focused on the experiences of children, rather than preservice or practicing teachers. Of these 106, 29—or slightly less than a third—of the articles described research that took place in clinical contexts. The majority of these studies involved interviews that took place in private settings in school buildings (e.g., Curtis, Okamoto & Weckbacher, 2009; Sophian, 2004), although some were conducted in university laboratories (e.g., Dilworth-Bart, 2012). Studies relying on clinical settings were almost evenly divided between the *Journal of Research in Mathematics Education* and the *Early Childhood Research Quarterly,* suggesting no significant differences between these two communities.

However, one interesting difference pertaining to national context did emerge in relation to clinical studies. While 16 percent of the studies in our total sample took place outside of the US (with roughly the same percentage of the studies focused on children also taking place outside the US), only 6 percent of the studies situated in clinical contexts took place outside of the US This means that the knowledge base for US scholars and teacher educators is even more likely to be influenced by findings produced in clinical settings than that of scholars and educators from other countries. This finding matters because clinical settings are different from early childhood classrooms in a variety of ways: the children's relationship with the adults in the study is short-term, the setting for instruction or assessment is unfamiliar, the social context is often solitary rather than communal; and the concern of the researchers is often focused only on children's cognitive performance. For example, the clinical studies we reviewed typically were conducted by researchers who had no prior relationship with the child outside of the regular classroom setting and were focused on constructs such as "verbal ability" (Dilworth-Bart, 2012), "numeracy skills" (Domitrovich et al., 2013), "mental relationships" (Kato et al., 2002), and "computational resources" (Sherin & Fuson, 2005).

As a result of these important differences, teacher educators drawing on research produced in clinical settings must take care to think more expansively about children as learners when using clinical research as a knowledge base for teaching practice (Wager, Graue, & Harrigan, in press). Children do not always perform in similar ways in assessment interviews as they do in classroom settings, and in particular, children from non-majority cultures may be impacted by the unfamiliarity of the clinical interview context (Ginsburg & Pappas, 2004). Therefore, results produced in clinical interviews may or may not reflect what will occur in classroom settings. Similarly, the suggestion in the research that all children will follow particular learning trajectories or that certain answers or non-answers in interview settings clearly indicate mathematical understandings (or lack of understandings) could be problematic if it is taken into the teacher education classroom without acknowledgement of the way that context matters in the findings or of the ways that variations in performance are expected across diverse children in typical classrooms. Lack of attention to these important differences in context could leave new practitioners with the idea that there is something "wrong" with their children because they do not behave or answer questions as children in clinical interviews do.

Mathematical Learners in School

Second, children who were not positioned as clinical study participants were typically positioned as mathematical learners in school, less typically as children in school with engagements beyond mathematics, including interest in other content areas, in social relationships, and in physical activities, and less typically still as people with involvements outside the walls of the school. For example, as with many clinical studies, when students were discussed in classrooms, the focus of the researchers was on the cognitive domain, with studies examining the impact of various pedagogies on characteristics like "emergent literacy" (Barnett et al., 2007), "mental models" (Bofferding, 2014), and "participation in discourse" (Empson, 2003). The few studies located in classrooms that did include emotional and social characteristics in their analysis, such as "aggression," (Arnold, Kupersmidt, Voegler-Lee & Marshall, 2012) and "emotional skills" (Bell, Greenfield & Bulotsky-Shearer, 2013) were primarily located in the early childhood journals, again suggesting that mathematics educators may be less likely to consider children's social and emotional experiences as relevant to mathematical learning.

Of the 106 studies focused on the experiences of children, 21 discussed children's experiences outside of the classroom. These studies included analyses of school-based but non-classroom contexts, such as mathematics clubs (e.g., Turner, Dominguez, Malonado & Empson, 2013), descriptions of children's engagements in mathematical tasks in communities (e.g., Schliemann et al., 1998), and examinations of children's mathematical experiences in their homes (e.g., Dearing et al., 2012; Tudge & Doucet, 2004). The studies focused on children's experiences in their homes differed in their stance toward families. Some (e.g., Anderson, Anderson & Shapiro, 2004) sought to describe the kinds of mathematics that occurred during family activities such as playing games, cooking, or reading books. A slightly larger group of studies primarily sought to identify discrete characteristics of home environments in order to link them to mathematics related outcomes, such as linking the spatial skills of parents to those of their children (e.g., Carr, Jessup & Fuller, 1999; Dearing et al., 2012; Manolitsis, Georgiou, & Tziraki, 2013). These differences are important because of the ways that they contribute to how educators at all levels think about children and families. Studies that describe the mathematics that is intertwined with family practices construct a discourse where knowing the rich repertoire of potentially valuable mathematical experiences is important for those seeking to teach young children mathematics (Wager & Delaney, 2014). In contrast, studies that primarily seek to demonstrate links between variables and success in mathematics construct a discourse where the patterns of success and failure in mathematics are put into place before children enter school so that remediation for some children is the only possible intervention strategy. The dominance of studies focused on linking family characteristics to academic performance suggests a need for more research that seeks to identify family practices—within all kinds of families—that might support mathematical learning.

Members of (Dis)advantaged Families

Our third and perhaps most unsettling finding was that one third of the studies focused on children explored the mathematical understanding of children based on their family background. A search of the 31 studies that included family background as one aspect of the study found that 26 of them mentioned "income" a total of 652 times —one as many as 126 times (Starkey, Klein, & Wakeley, 2004). We find ourselves somewhat conflicted about this, on the one hand we recognize the need to unearth some of the differential learning opportunities available in schools to children from low income families; on the other we found the frequency of this reference problematic as it emphasized research on particular families and particular children. We do want to call attention to the existence of diverse families but believe that constructing diversity *as* a problem *is* a problem. More concerning was that 14 of those articles used the following phrases: disadvantaged children, disadvantaged families, and/or disadvantaged schools. This deficit view of children and families perpetuates the idea that "disadvantaged" or "poor" families do not have the necessary resources to support their children's learning and, therefore, need to be studied. Further, this idea sets up a binary with disadvantaged placed opposite of advantaged. This binary begs the questions: Who are "advantaged" families, children, and schools? Why aren't they being studied? In short, we believe this language of disadvantage has to go.

The majority of the articles attending to family background were in *Early Childhood Research Quarterly,* which may provide some insight into the types of studies the journal privileges—those from large-scale clinical trials in which various aspects of family background is set as a variable against which children's cognitive performance is measured. "Disadvantaged" in these studies was generally defined as low-income, ethnic minorities. For the most part the types of studies reported on in these articles included: comparing the academic functioning of "disadvantaged children" who attended various levels or qualities of preschool (e.g. Domitrovich et al., 2013; van Tuijl & Leseman, 1997) and comparing later learning outcomes after mathematical interventions in preschool (e.g. Starkey, Klein, & Wakeley, 2004).

To dig further into how the child's home experience was constructed in the data set we identified those articles coded for both family background and family practices. In a reading of this subset of the family background articles we found that six took a distinctively deficit view of family practices and four had an asset based view of family practices. The six articles with deficit views examined family practices from a particular perspective (an interview or survey) in which the researchers were looking for specific predetermined behaviors from families. When those behaviors or practices were not present, the families were deemed as lacking, yet in none of these studies were the culturally bound nature of the expected behaviors discussed. Of the four articles that took as asset based view of family practices, two examined the ways that school practices that built on cultural and community practices provided greater learning opportunities for students (Kisker, et al., 2012; Meaney, Trinick, & Fairhall, 2013). In the other two (Guberman, 2004; Tudge & Doucet, 2004) the researchers went into homes to identify the practices that families engaged with. For example, Guberman compared out-of-school practices of Latino/a and Korean families. His approach was to observe the practices that families *did* engage in and how that connected to school mathematics. We need more of these asset-based studies to develop a different "truth" about children, families, and communities.

Conclusion

Although there were some exceptions, broadly the research we reviewed about early childhood mathematics did not embody the early childhood community's historical concern for educating the whole child, or the equity community's concern for taking asset based views of children and families. For example, in naming important principles that should inform learning in early childhood environments, the National Association for the Education of Young Children (2009) handbook called for attention to "all domains of development and learning—physical, social and emotional, and cognitive," (p. 11), for the promotion of "secure, consistent relationships with responsive adults and opportunities for positive relationships with peers," (p. 13), and for recognition that "play is an important vehicle for developing self-regulation as well as for promoting language, cognition, and social competence" (p. 14). While we would not expect that these themes would be dominant in the research literature focused on mathematics education, we would expect that these themes would influence the kinds of research questions and analyses conducted in early childhood environments, and would therefore be more visible than they were in our review. In addition, we would suggest that those conducting mathematics-related research in early childhood contexts would benefit from attending to the ways that the instructional practices and treatment of children in their studies does or does not align with broader conceptions of children in the early childhood community.

This brings us back to the perfect storm that is circling around our youngest children. If the way that children are constructed in the literature we reviewed becomes taken as true, we are unlikely to meet the goals that are driving the perfect storm – the hope for equitable mathematical learning opportunities for all children. Further, this force majeure will mean that no one will be responsible for the perpetuation of inequities.

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Examining Relations between Teachers' Diagnoses of Sources of Students' Difficulty in Mathematics and Students' Opportunities to Learn

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We present a quantitative analysis of the relation between a large sample of middle-grades mathematics teachers' views of students' mathematical capabilities and the quality of learning opportunities they provide for their students across large, urban U.S. districts pursuing reform. Specifically, we examine the relation between teachers' diagnoses of sources of students' difficulty in mathematics and the distribution and quality of students' mathematical discourse. Our findings suggest that the relation is stronger in classrooms with higher percentages of students of color or students identified as limited in English proficiency.

Fostering rich classroom mathematical activity and discourse requires fundamentally different practices on the part of both teachers and students than are typical in mathematics classrooms. For teachers, in addition to developing the mathematical knowledge for teaching and teaching skills to enact such forms of practice, it likely requires the development of particular stances regarding what students are able to do (Lampert, 2001). For example, if teachers view their students as incapable of explicating their mathematical reasoning, it is unlikely that they will press or support students to explain their reasoning in classroom instruction. Several small-scale qualitative studies support this supposition (e.g., Diamond, Randolph, & Spillane, 2004; Horn, 2007; Jackson, 2009), but we know of no study that has investigated on a large scale how teachers' views of their students' mathematical capabilities might relate to the quality of learning opportunities they provide for their students.

In this paper, we present a quantitative analysis of the relation between a large sample of middle-grades mathematics teachers' views of their students' mathematical capabilities (specifically, their diagnoses of sources of students' difficulty) and the quality of learning opportunities they provide for their students across large, urban districts in the United States pursuing reform. Of particular interest was the extent to which that relation varies, depending on the classroom composition of students they teach.

Framing Ideas

Teachers' Views of Their Students' Mathematical Capabilities

In recent years, with our colleagues on the Middle-School Mathematics and the Institutional Setting of Teaching (MIST) project (Cobb & Jackson, 2011), we have worked to understand what it takes to improve the quality of middle-grades mathematics teaching at the scale of large, urban districts. One focus has been teachers' views of their students' mathematical capabilities. In our investigation, we were particularly influenced by empirical work that suggested that how teachers framed (Goffman, 1974) the issue of student difficulty in mathematics would be a potentially useful way to get at what they viewed their students as capable of. In general, framing refers to how a particular certain situation is understood or interpreted (e.g., Benford & Snow, 2000; Goffman, 1974). It offers a particular representation of a problem, and as such, suggests particular solutions as possible (and occludes others). Sociologists attend to at least two "framing tasks" when analyzing framing processes—diagnostic framing and prognostic framing (Benford & Snow, 2000). The former involves articulating sources of the problem, whereas the latter involves identifying potential solutions, but with the two tasks being "intertwined, in that prognostic framing often rests implicitly on the problem definition and attribution that is part of diagnostic framing" (Coburn, 2006, p. 357).

In the MIST project, we developed an interview-based assessment to elicit information regarding these two framing dimensions, which as Jackson, Gibbons, and Dunlap (under review) argued, provide considerable insight into teachers' views of their students' mathematical capabilities. In what follows, we describe in more detail what we mean by teachers' diagnostic framing regarding sources of students' difficulty in mathematics, as that is the focus of this particular paper.

Teachers' Diagnoses of Sources of Students' Difficulty in Mathematics

Building on the work of Horn (2007), we make the following distinction when assessing how teachers diagnose students' difficulty in mathematics: does the teacher describe student difficulty in terms of inherent traits of the child or due to factors outside of instruction (e.g., families, community), or does the teacher frame a problem of student difficulty in relation to the nature of instruction, or learning opportunities? We have termed the former unproductive diagnoses of student difficulty and the latter productive diagnoses to signal that if teachers frame sources of student difficulty as outside instruction, it is unlikely that they will act to examine or alter current instruction to support students facing difficulty, but if teachers frame sources of student difficulty as in relation to what happens instructionally, we conjecture that they would be amenable to examining, and even altering instruction.

Related Constructs

There exist several related, but distinct, ideas that aim to account for teachers' views of their own capability or responsibility in supporting all students' learning. Measures of *self-efficacy* generally refer to a teacher's belief that s/he "can influence learning" (Sosa & Gomez, 2012, p. 879), whereas measures of *responsibility* "[focus] more on the teachers' willingness to take responsibility for helping all students learn rather than on teachers' beliefs about their professional effectiveness" (Halvorsen et al., 2009, p. 183). However, very few studies have examined relations between teachers' self-efficacy or responsibility and the quality of mathematics classroom instruction. Those that have investigated such relations have generally been qualitative, small-scale studies (e.g., Diamond et al., 2004; Sosa & Gomez, 2012). In general, studies of teachers' self-efficacy and responsibility have found that "achievement gains are significantly higher in schools where teachers take responsibility for students' academic success or failure rather than blaming students for their own failure" (Lee & Smith, 1996, p. 103), but that such responsibility tends to be higher in schools serving greater percentages of White and economically advantaged students, as compared to schools serving more racial minority students and students from lower-income backgrounds (Diamond et al., 2004; Lee & Smith, 1996).

We located no large-scale studies focused on the relations between teachers' self-efficacy or responsibility and teachers' instructional practice. However, based on their observations of classroom instruction, Diamond et al. (2004) found that teachers they described as having low responsibility also tended to enact instruction that communicated low expectations for students. For example, in a school characterized by teachers' low sense of responsibility, they observed that a fifth grade mathematics teacher assigned tasks that focused solely on developing procedural facility, and "spoon[fed the students] correct answers," and that teachers in schools characterized by low responsibility were, in general, "reluctant to try 'new things' because they feared that students would not be able to handle more innovative practices" (p. 88).

Equity in Opportunities to Learn

The literature referenced above suggests a notable pattern: there is evidence that teachers view students from historically under-served populations in less productive terms than students from historically advantaged populations. Further, there is at least small-scale evidence that such views matter for the quality of learning opportunities that teachers then provide for their students. This evidence indicates that a reason why teachers might not enact high-quality instructional practices in classrooms is because they do not view their students as capable of engaging in such activity.

These findings map onto current discourses about students' performance in mathematics more generally. As scholars such as Flores (2007) and Martin (2009) have argued, the dominant way of understanding disparity in performance on mathematics assessments between historically under-served populations and historically advantaged populations is through a lens of the "achievement gap." This way of understanding student performance often ends up, explicitly and implicitly, as casting historically under-served populations as the problem in need of fixing (Martin, 2009); further, rarely are the root causes of the "achievement gap" discussed (Flores, 2007). An alternative way of understanding such disparity in performance is as an "opportunity gap" (Flores, 2007). An opportunity gap perspective highlights that current disparities in achievement are the product of long-standing structural inequities, such as access to highly qualified teachers, resources, and so forth (Darling-Hammond, 2007; Flores, 2007). In other words, from this perspective, the "problem" does not rest with the individual students or the communities they come from, but with the opportunities that have (or have not) been provided to students.

In this analysis, we are concerned with both the quality of the learning opportunities teachers provide to students and the distribution of those opportunities. In light of the existing literature, we conjectured that teachers' diagnoses of the problem of student difficulty in mathematics (as due to individual traits of students or their communities, or as in relation to the opportunities provided to learn in the classroom) may differentially relate to the quality and distribution of learning opportunities provided to their students, depending on the composition of students in the classroom.

Study Context

The data we analyzed were collected in the first four years of the MIST project. The research team collaborated with the leaders of four large, urban school districts located in three states. The four collaborating school districts were typical of large, urban districts in the U.S. in that they had limited resources, large numbers of historically under-served populations of students, high teacher turnover, and disparities among subgroups of students in their performance on state standardized tests (Darling-Hammond, 2007). The districts were atypical, however, in their response to high-stakes accountability pressures. They responded by focusing primarily on improving the quality of instruction rather than on focusing exclusively on raising student test scores. Namely, each district was attempting to achieve a vision of mathematics instruction in which all students would have regular opportunities to collaboratively make sense of and solve challenging mathematical tasks, and, in discussing their solutions, develop robust understandings of key mathematical ideas. Three of the four districts (which we will call Districts A, B, and D) adopted a popular U.S. reform textbook as their primary textbook resource, and the fourth (District C) encouraged teachers to at least supplement their more typical series with that same reform text. Additionally, each district had initiated comprehensive professional development plans intended to support teachers in improving their instruction (e.g., curriculum frameworks, coaching, regularly scheduled time to collaborate with colleagues on issues of instruction, and/or professional development for instructional leaders.)

In each of the four districts, the research team and district leaders selected 6 to 10 middle-grades schools that reflected variation in student performance and in capacity for improvement in the quality of instruction across the district. Within each school, up to 5 mathematics teachers were selected to participate in the study, for a total of approximately 30 teachers per district. The schools remained constant throughout the study, but, as is typical, some of the teachers changed schools or roles during study. In each case, we recruited replacements in order to maintain a representative and consistently sized sample.

The collaborating districts provided settings in which we could investigate our relations of interest within the context of four, distinct, at-scale reform efforts. Specifically, our research questions were: 1) How are teachers' diagnoses of sources of students' difficulty related to the distribution and quality of students' mathematical discourse? and 2) Does the relation between teachers' diagnoses of sources of students' difficulty and classroom discourse vary depending on student-level characteristics of the classroom?

Methods

Sample and Primary Measures

Our primary analytic sample included 165 middle-school mathematics teachers pooled over the four years of the study with multiple years of data for some of the teachers, resulting in a total of 275 (statistical) observations (9 teachers with 4 years of data, 20 teachers with 3 years, 43 teachers with 2 years, and 93 teachers with 1 year of data). Because we do not have information about student-level characteristics for some of those teachers, we use a reduced sample of 156 teachers with 238 observations to answer the second research question. In each of the four years of the study (2007-2011), we collected several types of data to test and refine a set of hypotheses and conjectures about district and school organizational arrangements, social relations, and material resources that might support mathematics teachers' development of high-quality instructional practices at scale. We drew on a number of these data sources in this analysis, including video-recordings of teachers' classroom instruction; interviews with teachers; and a written assessment of teachers' mathematical knowledge for teaching (Hill, Schilling, & Ball, 2004). Additionally, our analyses included student demographic information collected and provided by the districts. The number of students served in the four districts ranged from approximately 35,000 to 160,000 students. On average, 29% of the students were White, 33% of the students were Black, and 36% of the students were Hispanic. Approximately 20% of the students were classified as limited English proficient, and 68% of the students were eligible for free or reduced price lunch.

The two outcome variables in our analyses are measures of the distribution and quality of students' mathematical discourse. In each year of the project, teachers' instruction in two consecutive lessons with the same class was video-recorded mid-year. Each lesson was scored using the Instructional Quality Assessment (IQA; Boston, 2012), which, with its 8 rubrics, aims to assess the "academic rigor" and the quality and distribution of Accountable Talk® in instruction. For our outcome variables, we employed the three Accountable Talk® indices that focus on students' contributions to and participation in classroom discourse: *student providing* accounts of his/her reasoning, *student linking* to and building on each other's ideas, and the percentage of student *participation* in class discussion. With respect to the first two, a score of 0 represents no whole-class discussion, a score of 1 represents no student providing or linking, a score of 2 represents providing procedural accounts of reasoning (e.g., describing steps taken or calculations performed) or superficial linking, a score of 3 represents a few strong efforts to provide conceptual accounts of reasoning or link ideas, and a score of 4 represents consistent strong efforts to provide conceptual reasoning or link ideas. For participation, a score of 0 represents no whole class discussion, 1 represents at most 25% of the students participating, 2 represents 26-50% of students participating, 3 represents 51-75% of students participating, and 4 represents 76-100% of students participating. Because we were interested in the distribution and quality of students' mathematical discourse, we used the student providing and student linking scores as two indicators of quality and weighted each by the participation score, which accounted for the distribution of students' discourse. We divided each product by 4 to match the usual IQA scale.

The key independent variable of interest in our analyses was interview-based assessments of teachers' diagnoses of sources of students' difficulty in mathematics (hereafter "diagnoses"). During annual interviews, we asked questions like, "When your students don't learn as expected, what do you find are typically the reasons?" We also asked teachers to describe the challenges they face, which often provided insight into how they framed student difficulty. All interviews were transcribed and coded at the unit of a turn of talk and any relevant text we used to make sense of that particular talk turn. For each relevant passage, coders assigned a code of "productive," "unproductive," or "mixed" (wavering between the two previous kinds of explanations). In order to use these categorizations in our quantitative analyses, we assigned numerical values to each interview based on our coding. If all coded passages were categorized as unproductive, the interview was assigned a value of 0. If all coded passages were considered productive, the interview was assigned a 2. A score of 1 was assigned if either all coded passages were categorized as mixed, or if there was a combination of productive and unproductive passages.

In our models, we also included student demographics and prior achievement, and measures of (a) *cognitive demand of classroom activity* (average of IQA scores for the potential and implementation of mathematical tasks, based on the mathematical task framework of Stein, Grover and Henningsen, 1996); and teachers' (b) *mathematical knowledge for teaching* (scores from a pencil-and-paper instrument, Hill et al., 2004); (c) *visions of high-quality mathematics instruction* (an interview-based assessment of the sophistication of teachers' instructional vision across key classroom dimensions: role of the teacher, mathematical tasks, student engagement in classroom activity, and discourse, with scores generally ranging from 0 to 4, and directionality roughly mirroring that of the IQA; Munter, 2014); and (d) *years of experience teaching.*

Analyses

To answer our first research question, we employed a series of linear regression models to investigate how teachers' diagnoses are related to each of the classroom discourse outcomes described previously (student providing and student linking), each weighted by participation. In each model, we controlled for other factors that could explain the relation between teachers' diagnoses and opportunities to learn, including teachers' mathematical knowledge for teaching; instructional vision; years of experience; and district membership. We constructed multi-level models to account for the nested nature of our data: observations within teachers, within schools.

To answer our second research question, we added information about student-level characteristics of the classroom, and investigated statistical interactions between those characteristics and teachers' diagnoses. For each of the two outcomes, we estimated four different models including interactions between four different student-level characteristics and teachers' diagnoses: 1) students' prior mathematics achievement; 2) the percentage of students in a school eligible for free or reduced price lunch; 3) the percentage of students in a class classified as limited English proficient; and 4) the percentage of students of color in the class.

Results

Across the two models of the relation between teachers' diagnoses and distribution and quality of students' discourse, results suggest that on average, for this sample of teachers, teachers' diagnoses are significantly related to the distribution and quality of student providing (b=.315, p<.05) but not to student linking (b=0.083, p=0.58). In particular, student providing of reasoning was about a third of a standard deviation higher in classrooms of teachers who articulated productive diagnoses than in classrooms of teachers who articulated unproductive diagnoses. Of the control variables included, only the cognitive demand of the classroom activity was significantly related to either outcome. Teachers who chose and implemented tasks with higher cognitive demand tended to have better distribution and quality of students' mathematical discourse.

The parallel series of models testing for statistical interactions yield a number of interesting results. For student providing, we found a significant interaction between mean prior student mathematics achievement in the class and teachers' diagnoses (b=-.2999, p<.05), suggesting that the relation between teachers' diagnoses of sources of students' difficulty in mathematics and student providing varies based on the prior achievement of students in the class. In general, teachers' diagnoses are not related to differences in the distribution and quality of student providing when they teach in classrooms in which most students have previously been successful on standardized mathematics assessments. However, we did not find a similar interaction with respect to student linking (b=-.211, p=.142).

With respect to both student providing and student linking, results suggest that there is not a significant interaction between teachers' diagnoses and the percentage of students eligible for free or reduced price lunch, but that there is variation in the relation between teachers' diagnoses and outcomes based on the percentage of students classified as limited English proficient and the percentage of students of color. With respect to the first outcome, this suggests that perhaps what manifested as variation in the relation between teachers' diagnoses and student providing based on prior student achievement may have actually been attributable to these other student characteristics. For both of these characteristics, as the percentage of students in the subgroup increases, so too does the relation between teachers' diagnoses and student providing (%LEP: b=.360, p<.05; %SoC: b=.214, p<.05) and between teachers' diagnoses and student linking (%LEP: b=.390, p<.05; %SoC: b=.179, p<.05).

For example, at the mean percentage of limited English proficient students in the class (12.7%), teachers' diagnoses are not significantly related to student providing, but at 1 standard deviation above the mean (27.5%) diagnoses are significantly related to student providing. Specifically, the results suggest that, on average, in classrooms with 27.5% of their students classified as limited English proficient, student providing is just over a third of a standard deviation better for teachers who articulated productive diagnoses than for teachers who articulated unproductive diagnoses. Also, at the mean percentage of students of color in the class (78.4%), teachers' diagnoses are not significantly related to student providing, but at half a standard deviation above the mean (90.7%), teachers' diagnoses are significantly related to student providing. This finding suggests that, on average, in classrooms with 90.7% of students of color, student providing is nearly a fifth of a standard deviation better for teachers who articulated productive diagnoses than for teachers who articulated unproductive diagnoses. More generally, we find that the relation between teachers' diagnoses and both student providing and linking is stronger in classrooms with higher percentages of historically under-served students.

Discussion

Our findings suggest that whether there are more or less equitable opportunities to engage in rich discourse in mathematics classrooms is related to teachers' diagnoses of sources of students' difficulty in mathematics. On average, students in our sample were more likely to participate in discussions in which they and their peers provided their reasoning and made connections between strategies if their teacher diagnosed sources of student difficulty in mathematics as related to the nature of instruction or learning opportunities, rather than in terms of inherent traits of the students or due to factors outside of instruction (e.g., families, community). We view this finding as providing an important insight into the factors associated with teachers' enactment of practices that afford important learning opportunities to all of their students, and as large scale confirmation of what previous, smaller scale studies have asserted that teachers' views of their students' mathematical capabilities are related to the quality of learning opportunities they afford their students. It also provides potential insights into how teachers act on their sense of responsibility (Halvorsen et al., 2009) in terms of the nature of discourse that they foster through instruction.

But more than this, the results of our statistical interaction analyses suggest that the strength of this relation depends on the composition of students in the classroom with respect to race, ethnicity and/or language status. For example, students in classes composed (almost) entirely of students of color were more likely to have opportunities to participate in discussions in which students provided reasoning for their solutions if their teacher articulated productive diagnoses of sources of their difficulty. In other words, without White students in the room, teachers' views of their students' mathematical capabilities—whether productive or unproductive—were, on average, more likely to be reflected in their instructional practice.

Given that the interactions that we observed between teachers' diagnoses and both percentage of students with limited English proficiency and percentage of students of color did not hold for percentage of students who were eligible for free or reduced-price lunches, these findings imply that teachers' views of students' mathematical capabilities may be more influenced by students' race or cultural background than by their economic status or even past achievement. To the extent that this is the case, it may be that the more "visible" characteristics of race and limited English proficiency, as compared to social class and prior achievement, are more likely to trigger the kind of low responsibility/low expectations-oriented instruction identified by Diamond et al. (2004).

Our findings suggest that there is clearly much work to be done in shifting how teachers frame the problem of students' difficulty in mathematics, particularly when serving historically disadvantaged groups of students. How can teachers be supported in developing more productive views of students' mathematical capabilities and translating new diagnostic frames into productive prognostic approaches in instructional practice? Our sense is that answering those questions will require further investigation of teachers' cultural and racial competence (Milner, 2003) in discipline-specific contexts, and a better understanding of and support for teachers in building relationships with students that extend beyond knowing them from a cognitive standpoint, potentially including integrating current models of instructional practice with the affective, relational, and emotional aspects of classroom interactions.

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Critically Reading the OECD Survey of Adult Skills

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The first round of the OECD Survey of Adult Skills conducted during 20011/2012 sought to measure adults' proficiency in literacy, numeracy, and "problem solving skills in technology rich environments". This paper reports on a critical examination of the findings of the performances of two participating countries: Japan and France, as reported in the official reports of the OECD and by the media in the two countries.

Introduction

On 8 October, 2013, the Organization for Economic Co-operation and Development (OECD) released the results of the Survey of Adult Skills (SAS) for 23 of the 24 countries that participated in the survey's first round. (Results for Russia have been delayed.) There is growing scholarship around the increasing influences of international assessments, such as the PISA and the OECD adult literacy surveys, on educational policy making and curricula at the national levels (Black & Yasukawa, 2014; Hamilton, 2012; Meyer & Benavot, 2013; Tsatsaroni & Evans, 2013; Grek, 2010; Walker, 2009). Following the historical record of public reporting of the results enables us to trace what is taken up (and what is not) in related educational policy processes emerging from such survey results. While this paper focuses on only two countries, France and Japan, critical comparative studies can begin to highlight salient themes that are highlighted by the OECD and picked up by the national media to build public understanding about transnational policy processes.

Critically Examining the Survey of Adult Skills

During 2011/2012 the first round of the Survey of Adult Skills (SAS), the international survey element of OECD's Programme of International Assessment of Adult Competencies (PIAAC), was conducted in 24 OECD countries. SAS was the third in a series of international adult skills surveys that have been conducted over the last two decades: the previous two being the International Adult Literacy Survey in the 1990s and the Adult Literacy and Lifeskills Survey conducted in the 2000s. SAS was designed to measure adults' proficiencies in the three areas of (reading) literacy (L), numeracy (N) and "problem solving in technology rich environments" (PS). Proficiencies for the L and N domains were assessed on a numerical scale (0 to 500) and then ranked in groups from Below Level 1 (lowest) through to Level 5 (highest).

The survey targeted the adult population (16 to 65 year olds) in the participating countries. The three areas assessed are regarded by the OECD as representing "cross-cutting cognitive skills that provide a foundation for effective and successful participation in the social and economic life of advanced economies" (OECD, 2012, p.10). The OECD argues that SAS together with the PISA (Programme of International Student Assessment) for 15 year olds could provide insights into how the performances observed in the PISA are "maintained, reduced or increased as the cohorts pass through subsequent education and training" (p. 11).

The influence of the OECD in shaping national education policies and understandings of literacy and numeracy is complicated by existing local educational and economic debates. A critical examination of the ways that international surveys such as the SAS are received and circulated can expose the cultural and political dynamics of the policy process and in turn the popular representations of literacy and numeracy (Hamilton, 2012; Rubenson & Walker, 2014; Thomas, 2005). One of the longer term aspects of our focus is how the possible tensions between the purported dual economistic and social aims of PIAAC are played out in local contexts.

This paper draws on a study by our colleague Mary Hamilton and ourselves, on the media reporting of the first SAS results in late 2013. We chose three countries to represent the top (Japan), middle (UK) and lower (France) ends of the "league tables" that were produced from the L and N results of the Survey. Another reason for the choice of countries was that, within the research team, we had the capacity to read the press reports in the languages of publication (English, French, and Japanese).

The initial reports in the largest national newspapers in UK, France, and Japan were sourced from the internet and analysed according to the issues that they highlighted, the credibility they brought to their interpretation of the findings, and any policy implications that were identified or suggested. A comparative analysis of the reception in the three countries was then made.

Another important critical dimension relates to ways in which "numbers" are used to build stories about the L, N, and PS proficiencies of adults in the different countries. In such large-scale international surveys, the methodological challenges are enormous, and the numbers that mainstream media use to report on findings are but a very tiny proportion of what is available as "data".

"Numbers" featured prominently in the media reports and we examined how the media's emphasis and interpretation of these "numbers" compared with those of the OECD. We investigate which of these data were picked up by the media, and how the choices they made were woven into the stories about the SAS results for public consumption. We conclude with some observations about the significant role that the SAS might play in transnational policy making, and thus the heightened importance of exercising critical numeracy in the reading of the results.

The OECD Reports

Besides the overall international reports (OECD, 2013c, 2013d), for most participating countries, including France and Japan, the OECD published a Country Note (OECD, 2013a, 2013b, respectively). These reports contain general information about the SAS methodology, a summary of the proficiency results for the country, and a list of key issues identified by OECD for that country's results.

What is noticeable about these presentations is the effort to rank and compare each country's performance with those of other countries using quantitative language: *highest levels; average; lower than average; relatively equitable distribution.* We now present key aspects of the country notes for our two countries.

(a) France

In France some 7000 adults 16 to 65 were surveyed from September to November, 2012. Key findings identified in the Country Note (OECD, 2013a, p.1) include:

1. Literacy and Numeracy skills of the French are among the lowest of countries participating. But the differences between generations are rather marked, compared with other countries. (Problem solving scores were not measured in France.)
2. Literacy and Numeracy scores of the French varied noticeably, according to level of education and their social origin, and to a greater extent than in the average participating country: France displays a comparatively high level of inequality in L and N scores.
3. Differences in Literacy skills among individuals born in France and those born abroad are more pronounced than the average for the countries participating, and the improvement of skills with the length of residence in the country is very limited.
4. According to survey respondents, literacy and ICT skills are little in demand for professional staff, nor are those for resolution of complex problems. On the other hand, French employers are among those who require more the numeracy skills of their workers.
5. In France, as in all participating countries, a positive and significant relationship is detected between the level of Literacy skills and both hourly wages and the probability of being employed. However, these relationships are weaker for France than on average.

France was ranked 21st for Literacy (of 23 countries) and 20th for Numeracy. Yet there was some hope in one of the demographic differences of most interest to policy-makers and the media: the youngest SAS age group (16-24) obtained better results than the older (55-64); this was a substantial gap between age groups, similar to that seen in Finland and the Netherlands (which had high average scores overall on L and N), and Korea (middling average scores overall). This differs from the UK, where there was no such age difference; these latter results are more worrying for policy makers and others looking for an improvement in the younger generations.

The other two distinctive features of the French data that were to attract attention in the media were the differences to do with place of birth (3. above), and the extent to which workers' measured skill levels (in L and N) correspond with the extent to which their holders perceive these skills to be used in the workplace (4. above).

(b) Japan

In Japan 5200 adults aged 16 to 65 were surveyed between August 2011 and February 2012 (OECD, 2013b, p. 1). The Country Note identifies the key issues as:

1. Adults in Japan display the highest levels of proficiency in literacy and numeracy among adults in all countries participating in the survey.
2. The performance of Japanese adults in the assessment of problem solving in technology-rich environments is around average. In Japan, younger adults perform lower than the average in this domain.
3. There is a relatively equitable distribution (i.e. low dispersion) of proficiency in information processing skills (i.e. Literacy, Numeracy and PS scores) across the Japanese adult population, with only small differences between groups such as the old and the young and the better and less well educated.
4. Japanese women have high levels of L and N proficiency, but have low rates of participation in the labour force. Thus, they represent an underutilised resource of skill.
5. For these high levels of proficiency to translate into economic growth and well-being, competences must be put to their best use. Japanese employers do not appear to be making the best use of their workforce's competences. Furthermore, the returns to proficiency in terms of higher wages and employment rates are lower than in other participating countries (OECD, 2013b, p.1).

While Japan came "first" in L and N, when the proficiencies are broken down by age, the very high levels of L and N among the oldest age groups in Japan (compared with the same age groups elsewhere) made the Japanese results distinctive:

> While the average proficiency of 16-24 year olds is [high and] similar to that of cohorts of the same age in Finland, Korea and the Netherlands,… the proficiency of 45-54 year olds and 55-65 year olds in Japan is well in excess of their counterparts in other countries. (OECD, 2013b, p. 2)

Nevertheless, the 16-24 group scored higher (on average) than the 55-65 age group by an amount around the OECD average. However, other socio-demographic differences such as those related to educational levels, parents' educational levels, and occupational categories are shown graphically to be below OECD average. The Report explains that the gap in mean score for those with a tertiary qualification over those with less than a completed upper secondary qualification is among the lowest among participating countries. Furthermore, it points out that "the proficiency of Japanese adults with less than a full secondary education is the highest of all countries in the survey" (OECD, 2013b, p. 3).

The Report gives findings related to employment and labour market issues. It suggests that Japanese women, who performed highest among women in participating countries, are an "untapped supply of high quality human capital" because a significant number of them are not in the labour force (OECD, 2013b, p. 3). Further criticisms are made about the use of available human capital by employers:

> A sizeable share of Japanese workers—close to 10%—are in jobs for which their literacy competencies are higher than required. (OECD, 2013b, p. 7)

The Media Reports

Initial reactions to the SAS findings in French and Japanese newspapers published from 8 October when the results were released internationally, up to 1 December 2013, were analysed. Media attention on the SAS results in both countries declined rapidly after the first week. A crowd-sourced project produced an archive of articles relevant to understanding the reception of SAS. This was supplemented by a search using Factiva and the newspapers' websites.

(a) French Media

Analysis of media reports in the French press is based on five national papers: the three main dailies, *Le Monde* (circulation 314,000, left), *Le Figaro* (circulation 321, 500, centre-right), and *Libération* (circulation 134,800, far-left), supplemented by *Les Echos* (circulation 120,000, right) and *La Croix* (106,000, Roman Catholic); estimates of circulation and political position are according to Wikipedia[3].

The French newspapers had relatively bad news to convey; this may have led to coverage "petering out" faster over the first week than in the other countries studied. In addition, composing the headlines appears to have posed challenges; the words used officially by SAS for L and N, *littératie* and *numératie,* are not widely used in France. Thus, papers mentioned "the written (domain)" [*l'écrit*] or "reading" [*la lecture*] and "calculation", "figures" or "maths". In only three of the papers were L, N, and PS defined, or even used. Thus the terms used in France may undermine the arguments of those policy-makers, researchers and teachers who have argued for adult literacy and adult numeracy to be seen as rich and distinctive concepts.

First, considering the issues and findings that are highlighted, most newspapers reported the proportion of French respondents who scored at very low levels in L and N, as the proportion in levels 1 or below 1. *Le Monde* reported those up to level 2 versus those at level 3 and beyond, while *Le Figaro* and *Libération* reported the proportion of high scores as levels 4/5. The low percentages were compared with the OECD average, or sometimes with Spain and Italy, which scored below France in both L and N. In most reports, Japan and Finland were mentioned as "good students", reflecting a pervasive school-like metaphor. PS was largely ignored, presumably because France (like Spain and Italy) did not participate in that part of the Survey. Nonetheless, several indicators, such as the extent to which literacy, numeracy,"ICT" and "complex problem solving" are (perceived by respondents to be) in demand by French employers, were available, from the Background Questionnaire.

All five papers were focused on broadly the same demographics and their relation to L and N: namely, age, respondent's level of education, and parents' level of education. *Le Monde* went somewhat further in its interpretation: "Social origin and level of education play a more discriminating (stratifying) role in France than in many countries. Just like being born in France or not." But the main finding highlighted was that "the young (16-24 years) obtain better results than the older (55-64)". And "as in Korea and Finland, the gap between the two [age groups] is substantial in France." Glenda Quintini, an economist with OECD, is quoted in *La Croix,* as conjecturing that the difference between the two age groups is "tied to the increase in the level of education" (*Le Monde,* 15.10.2013), over time.

Le Monde's article finishes by quoting an "expert" at OECD, to the effect that "what is most problematic is the inequalities in the system", relating to age, place of birth, level of education, and parents' level of education. (This issue is taken up in Eric Charbonnier's blog ["Education Déchiffrée"] for *Le Monde,* (15.10.2013.) The inequalities mentioned in the *Libération* headline, in contrast, appear to relate to *inequalities between countries.* Only *Le Monde* indicates that some questions remain open: "OECD is unable to say if the older [lower performers] left the system with a mediocre level, or if their competences deteriorated in their professional life." This continues to be a live question in reactions to the findings in many countries.

Credibility is established, in all of these newspapers – by quoting at least one OECD official: presumably being based in the same city as the HQ of OECD facilitates such access. Most quoted was Stefano Scarpetta, Director of Employment, Labour & Social Affairs at OECD; *Le Monde* also quoted a second official, Eric Charbonnier, (who also does a blog for *Le Monde*). *La Croix* quoted Glenda Quintini, an economist with OECD. On the other hand, *Libération*'s report quoted Angel Gurria, Secretary General of OECD and Androulla Vassiliou, Commissioner for Education for the EU, speaking at a press conference in Brussels on the day of the release of results.

As for citing the survey's methodological features, the overall sample size (166 000) is mentioned in only three newspapers, and that in France (7000), in two.

Concerning policy-related conclusions, all the main articles were published in the day or two after the Survey, so they are "first reactions". *Le Monde* mentions "inequalities" (see above), and *Le Figaro* mentions the "spread" of scores; *Libération,* considers that "in too many European countries, the future of children is pre-determined by the situation of the parents" (quoting Vassiliou). When seeking factors responsible for the problem, *Les Echos* considers it to be "'first tied to the large numbers of adults whose parents did not do HE, i.e. the socioeconomic milieu." They go on to quote Scarpetta at the OECD: "'The school forms initial competencies. But these develop next at work …which implicates [economic] ministries, but also enterprises: France has a problem of skill, and of use of skills: many talents are not exploited…The OECD is concerned by the high number of fixed term and part-time contracts, which reduces the level of skill use." He is also concerned that "continuing education tends to prioritise the most skilled, and deepens the gap with the weakest". In *La Croix,* Glenda Qunitini, largely echoes Scarpetta, in citing the lower commitment in France to lifelong learning, and the idea that French workers are "less incentivised to use the ensemble of their skills at work … or to develop new know-how".

In terms of what should be done, *Le Monde* implies that inequalities within France must be tackled. *Le Figaro* quotes Scarpetta at the OECD, who emphasises "three important dimensions: access to education and training; development of competences throughout professional life; use of skills adequate for the post held". *Libération* considers "immediate measures needed at European level" (quoting Vassiliou), presumably to tackle inequality between countries. More generally, Gurria of the OECD considers the results "a wake-up call, to see what others [countries] do, and to draw lessons from that". For *Les Echos,* "France must act to better use its talents."

(b) Japanese Media

The corpus of Japanese media reports was sourced from the three largest national newspapers in Japan: Yomiuri Shimbun (circulation 10,042,075, right), Mainichi Shimbun (circulation 3,974,559, liberal/centre) and Asahi Shimbun (circulation 8,093,885, left); estimates of the circulation figures and the political positions are sourced from Wikipedia (http://en.wikipedia.org/wiki/Japanese_newspapers).

All three newspapers provided broad information about the SAS methodology: the number of participants overall and the number of participants in Japan. They also provided information about the levels used in the different assessed domains.

The *Asahi* and *Mainichi* published rankings of the top five countries in each of the three areas, showing the country's name and the mean score achieved in the relevant domain. For PS, the percentages of survey respondents in these countries scoring at the top two levels combined were also shown, and indicated that Japan came tenth.

Problems of translation and meaning in the Japanese newspaper articles are significant, though not acknowledged as such. The terms L and N in the PIAAC have been developed with considerable deliberations by international expert groups (OECD 2013d). Even in English, the language in which the expert groups' reports were first written, L and N are often associated with "basic skills", that is those skills that people might be expected to learn in primary school. In Japanese, SAS has been translated as 成人力調査 (survey of adult ability), and L as 読解力 (reading comprehension ability) and N as 数的思考力 (numerical thinking skills); and it is unclear whether literacy and N are intended to be understood as inherent abilities of individuals or whether they are understood as learned skills. Moreover, in some articles, L is used interchangeably with 学力 which can mean both "being learned or educated" and "academic ability". In some articles, examples of the types of questions in the SAS were described to illustrate what was being assessed in these domains.

For a country that came first in the international league tables, it is not surprising that the headlines in all three papers highlighted this:

> Japan's "adult skills", number 1 in 2 areas … weakness is IT (*Yomiuri*)
>
> International adult skills survey – a survey of adults' academic ability (literacy): Japan comes top, reflection of compulsory education and training (*Mainichi*)
>
> Unexpected, but proud, Japan world 1st in adult skills survey (*Asahi*)

Interpretations are added to the statistical expressions of the survey performance by citing educational experts. The *Mainichi* cites Takashi Hamano, an educational sociologist from Ochanomizu Women's University who explained how the overall favourable results pointed to the high standard of compulsory education: "compared to the West, our [curriculum is] high in the degree of consistency and density". The *Mainichi* also quotes a researcher in comparative education from the National Institute of Education Policy Research, Yasuo Saito, as saying that "the power to maintain academic ability / literacy is another explanation for the favourable results because academic ability / literacy generally falls when they are not used". Saito is quoted as saying that "in Japan, there are many adults who read the newspapers and magazines, and this makes it more difficult for ability / literacy learned in school to decline".

Mainichi reports on the OECD finding that the average scores of Japanese respondents whose highest qualification was junior high school completion (year 9) had average scores higher than those who had completed senior high school in Germany and USA.

However, the top position in the league table is examined more critically in the *Yomiuri.* It explains that while Japan's overall mean in L and N came top, the proportion of the survey respondents who scored at levels 4/5 in L was highest in Finland, then Australia, the Netherlands, Sweden, and then Japan as fifth. In N, the order was again Finland as first, and then Japan as seventh.

A focus is also placed on the "less good" outcomes in L and N in the younger age groups, compared to the older groups (i.e. when comparing both internationally). The *Asahi* reports that some attribute this to the policy change, known as *yutori kyoiku,* a move to de-intensify the curriculum. A counter view to that extolling the virtues and success of the traditional compulsory education system that enabled the older generations to perform well is voiced by Manabu Sato, an education professor from Gakushuin University (*Asahi*). He points to the significantly lower percentage of immigrants in Japan compared to many of the other OECD countries, and how a greater linguistic diversity in the population could have markedly changed the results. In response to Japan's recent "less good" outcome in PISA (eighth in literacy, ninth in mathematical literacy, out of 62 countries), Sato is cited as saying that the difference between the PIAAC and the PISA should not be explained by saying "the education that the adults had received was correct". Rather, he attributes this to a problem in senior high school and university education and an over-reliance on industry for the education of adults.

Many of the articles repeated or sought to interpret the key issues identified in the OECD country report. Thus the poorer performance shown in PS compared to L and N was also picked up in two headlines:

> International adult skills survey, success of basics focussed education, "cell phone" generation not *au fait* with PC (*Yomiuri*)

> Detection of tardiness [in] information literacy education, ... (*Yomiuri*)

Interestingly, however, the underutilisation of skills held by women identified by the OECD as a key issue was not taken up with any emphasis by the papers.

Discussion and Conclusion

Our paper examines early reports of the OECD SAS in two countries, and how the Survey findings are interpreted by the OECD, a transnational organisation, and in national media, as a way to begin to understand their effects on local policy processes and curricula. First, our analyses illustrate the heavy emphasis on international comparisons, particularly with reference to league tables. These are based on rankings according to average country scores or proportions in the 'highest levels'. This is mirrored by a corresponding relative neglect of the spread (dispersion) of scores, within the national media at least – though the OECD is clearer about pointing to these spreads as measures of *inequality!*

Second, we have pointed to concerns raised in a number of countries, represented here by Japan, when the scores of the younger (16-24) age group *seem* 'lower' than those of their elders (see above). This feeds into the existing anxiety on the part of politicians/policy makers on the impact of any curricular reforms on the competitiveness of the future population in the globalised economy.

Third, even if the opposite finding, that of the superior performance of the younger age group over the older, engenders "relief" in a country like France, there are nevertheless concerns about the "decay" of skills over the life-span. Our reading of the media in the two countries revealed a range of explanations suggested for this putative "decay" To begin with, it may be the result of a lack of on-the-job training. Or it may be the result of the lack of cognitive demands on employees in many jobs (French papers and OECD). Or again, we have the explanation from Japan that "literacy falls when it is not used", and that the Japanese avoid this by reading newspapers and magazines. Or it may be the sparse provision of lifelong opportunities as noted by one French paper, as compared, say with Finland, which came second to Japan in L and N.

Overall, although a large amount of socio-demographic information was collected in the SAS, the strong human capital discourse in the OECD reports suggests that the extent to which the findings inform workforce development and labour market policies alone, or whether implications will also be drawn for broader lifelong learning policies is a space to be watched critically.

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