Proceedings of the Sixth International Mathematics Education and Society Conference

Volume 1

Uwe Gellert, Eva Jablonka and Candia Morgan (Editors)

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MES 6 Conference Logo

The conference logo is a composition based on a photograph of the Philological Library of the Freie Universität Berlin, designed by the British architect Lord Norman Foster. For its unusual structure, it has been named colloquially “The Berlin Brain”.

Acknowledgements

The conference organisers would like to acknowledge the financial support of the Department of Education and Psychology, Freie Universität Berlin.

Table of contents

PREFACE TO THE SECOND EDITION

This second edition of the proceedings of the Sixth International Mathematics Education and Society Conference (MES 6) includes some of the contributions that made part of the conference programme but that were not available at the time of publication of the previous edition. Those contributions are some of the reactions to plenary papers.

We are looking forward to meeting again at MES 7 in year 2012.

Berlin, Luleå and London, June 1st, 2010

Uwe Gellert, Eva Jablonka and Candia Morgan

INTRODUCTION

The First International Conference on Mathematics Education and Society took place in Nottingham, Great Britain, in September 1998. The Second Conference was held in Montechoro, Portugal, in March 2000. The Third Conference took place in Helsingør, Denmark, in March 2002. The Fourth Conference was held in Queensland, Australia, in July 2005. The Fifth Conference took place in Albufeira, Portugal, in February 2008. On all occasions, people from around the world had the opportunity of sharing their ideas, perspectives and reflections concerning the social, political, cultural and ethical dimensions of mathematics education and mathematics education research that take place in diverse contexts. As a result of the success of these five meetings, it was decided to have a sixth conference in Berlin, Germany. As an international and cross-institutional collaboration in conference organisation, an international organising team of Uwe Gellert (Freie Universität Berlin, Germany), Eva Jablonka (Luleå Tekniska Universitet, Sweden) and Candia Morgan (University of London, United Kingdom), took the lead in the planning of the conference. Together with a Local Organising Team with members from the Freie Universität Berlin, and with the International Advisory Board, it was possible to set up the Sixth International Conference on Mathematics Education and Society in the city of Berlin, in March 2010.

The conference has been promoted and sponsored by the Fachbereich Erziehungswissenschaft und Psychologie of the Freie Universität Berlin.

AIMS OF MES 6

Education is becoming more and more politicised throughout the world. Mathematics education is a key focus in the politics of education. Mathematics qualifications remain an accepted gatekeeper to further education and employment opportunities. Thus, defining success in mathematics becomes a way of controlling people's pathways in work and life generally. Mathematics education has also tended to contribute to the reproduction of an inequitable society through undemocratic and exclusive pedagogical practices, which portray mathematics and mathematics education as absolute, authoritarian disciplines. The fact that particular mathematics education and research practices can have such significant impact on the type of society we live in suggests that different mathematics education and research practices could have equally significant but more socially just impact on society. There is a need for uncovering and examining the social, cultural and political dimensions of mathematics education; for disseminating research that explores those dimensions; for addressing methodological issues of that type of research; for planning international co-operation in the area; and for developing a strong activist research community interested in transforming mathematics education as an agent and practice for, rather than against, social justice.

The MES 6 Conference aims to bring together mathematics educators around the world to provide such a forum as well as to offer a platform on which to build future collaborative activity.

CONFERENCE PROGRAMME

The conference was organised bearing in mind the importance of generating a continuing dialogue and reflection among the participants. There was a range of activities directed towards the aim of generating this sustained discussion:

Opening plenary panel: Ways of acting politically in mathematics education

The Mathematics Education and Society (MES) conferences bring together a community dedicated to carrying out politically sensitive research and to engage in a discussion of political, social, cultural and ethical dimensions of mathematics education. In line with these aims, the panelists were invited to share and discuss ways for doing politics and acting politically in the arena of mathematics education, be it as teacher, researcher, teacher educator, consultant or as school official.

Panelists: Tony Cotton (Leeds Metropolitan University, UK)
Marilyn Frankenstein (University of Massachusetts/Boston, USA)
Christine Keitel (Freie Universität Berlin, Germany)
Alexandre Pais (Aalborg Universitet, Denmark)

Moderator: Mamokgethi Setati (University of South Africa)

Plenary addresses and reactions

The four invited keynote speakers were asked to address a topic of relevance to the conference, building on their current research. They held 50-minutes presentations. Each presentation was followed by 10-minutes reactions by two mathematics educators.

The four plenaries were:

- *At the sharp end of education for an ethical, equitable and numerate society: Mathematics in safety-critical context* by Diana Coben (King’s College London, UK).

  Reactors: Marta Civil (University of Arizona, USA)
  Tine Wedege (Malmö Högskola, Sweden).

- *Ideological roots and uncontrolled flowering of alternative curriculum conceptions* by Eva Jablonka (Luleå Tekniska Universitet, Sweden) and Uwe Gellert (Freie Universität Berlin, Germany).

  Reactors: Eric Gutstein (University of Illinois at Chicago, USA)
  Kate Le Roux (University of Cape Town, South Africa)

- *Not-so-strange bedfellows: Racial projects and the mathematics education enterprise* by Danny Martin (University of Illinois at Chicago, USA).

  Reactors: Anna Chronaki (University of Thessaly, Greece)
  Tamsin Meaney (Charles Sturt University, Australia)

- *Mathematics education for a better life? – Voices from MES6 participants* by João Filipe Matos (Universidade de Lisboa, Portugal)

Working groups

Groups, set at the beginning of the conference, discussed the plenary lecture and the reactions. Each discussion group produced a brief report detailing key questions or issues to be addressed by the speaker and reactors in a plenary response session.

Group Moderators: Karin Brodie (University of Witwatersrand, South Africa)
Brian Greer (Portland State University, USA)
Mônica Mesquita (Universidade de Lisboa, Portugal)
Paola Valero (Aalborg Universitet, Denmark)

Plenary response session

In these sessions, one during each day of the conference, there was an opportunity to bring back to the whole conference group the questions and concerns of each working group, and to have a further comment by the plenary speaker and reactors.

Symposia

Four symposia proposals were accepted after review of the organising committee. The symposia had two to four hours in total to engage participants in a reflection of a particular topic of interest for the conference.

The symposia were:

A.	*Same question different countries: Use of multiple languages in mathematics learning and teaching*, by Anna Chronaki, Núria Planas, Mamokgethi Setati and Marta Civil.
B.	*Analysing the uses of "critique" and "politics" in mathematics education research*, by Alexandre Pais and Mônica Mesquita.
C.	*Telling choices: Mathematics, Identity and Social Justice*, by Laura Black, Anna Chronaki, Stephen Lerman, Heather Mendick and Yvette Solomon.
D.	*New perspectives on mathematics pedagogy*, by Margaret Walshaw and Kathleen Nolan.

Paper discussion sessions

After peer review of all paper submissions, the organising committee accepted 37 papers for presentation and discussion during the conference. The full text of accepted papers was posted on the conference website and published in these conference proceedings. These projects generally centred around the presenters' current research work.

Project discussion sessions

After peer review of project submissions, there were eight accepted project discussion sessions. Discussion papers were posted in the conference's website and published in the conference proceedings.

Agora

Inspired on the Greek tradition of a "popular political assembly" taking place in a public, open space such as the market place, it was decided to have two informal, evening discussion sessions about the future of MES.

Networking

Within the programme there were slots dedicated to informal networking among participants.

Concluding panel

This time the conference organisers proposed to have a last, concluding panel with all the plenary speakers in order to discuss dilemmas and questions that have emerged during the whole conference.

	Sat 20	**Sun 21**	**Mon 22**	**Tue 23**	**Wed 24**	**Thu 25**
9:00		Plenary 1 *D. Martin*	Plenary 2 *D. Coben*	Plenary 3 *J.F. Matos*	Plenary 4 *Jablonka / Gellert*	Papers / Proj. discussion
11:00		W. Groups Plenary response	W. Groups Plenary response	W. Groups Discussion	W. Groups Plenary response	Plenary Panel Closing
13:00	Registration	Lunch	Lunch	Lunch	Lunch	Lunch
14:30		Papers / Project discussion	Papers / Project discussion		Papers / Project discussion	Departure
17:00	Opening & Open. Panel	Symposia *A, B*	Symposia *C, D*		Symposia *C, D*	
20:00	Dinner	Dinner	Dinner Agora 1		Dinner Agora 2	

THE REVIEW PROCESS AND PROCEEDINGS

All of the papers published in these Proceedings were peer reviewed by two experienced mathematics education researchers before publication. These researchers are:

Margarida Belchior, Christer Bergsten, Karin Brodie, Dimitris Chassapis, Anna Chronaki, Tone Dalvang, Elizabeth de Freitas, Marilyn Frankenstein, Uwe Gellert, Eric Gutstein, Eva Jablonka, Clive Kanes, Christine Knipping, Kate Le Roux, Anna Llewellin, João Filipe Matos, Tamsin Meaney, Candia Morgan, Balbina Mutemba, Alexandre Pais, Nathalie Sinclair, Cathy Smith, Hauke Straehler-Pohl, Paola Valero, David Wagner, Margaret Walshaw, Peter Winbourne, Keiko Yasukawa and Betina Zolkower.

Strict guidelines were followed to ensure that the papers had a significant contribution to make to the field, and were based on sound literature review and methodology. The production of the Proceedings was possible through the cooperation of many of the conference participants who offered their time to peer review papers. The challenges faced by some of our conference participants from language backgrounds other than English to write their paper in English are acknowledged and appreciated, as well as the time of some generous reviewers who provided support for language correction.

PARTICIPANTS

In this occasion there were 84 participants from 20 countries: Australia, Belgium, Brazil, Canada, Colombia, Denmark, Germany, Greece, Ireland, Mozambique, New Zealand, Norway, Portugal, South Africa, Spain, Sweden, The Netherlands, United Kingdom, United States of America, Venezuela.

ACKNOLWEDGEMENTS

Finally, we would like to especially thank the enormous support of the local organising team – Birgit Abel, Markus Kammermeier, Nicole Marschner, Sara Specker, Hauke Straehler-Pohl and, particularly, Nils Richter – who worked with us in the preparation and realisation of this conference, and Theresa Nalewalski who managed registration.

An electronic file of all individual papers as well as of the whole proceedings is available at http://www.ewi-psy.fu-berlin.de/mes6

Berlin, Luleå and London, March 1st, 2010

Uwe Gellert, Eva Jablonka and Candia Morgan

PLENARY PAPERS AND REACTIONS

AT THE SHARP END OF EDUCATION FOR AN ETHICAL, EQUITABLE AND NUMERATE SOCIETY: WORKING IN A SAFETY-CRITICAL CONTEXT – NUMERACY FOR NURSING

Diana Coben

King's College London

In this paper I draw on my ongoing interdisciplinary research on numeracy for nursing. I argue that education for such a safety-critical context is at the sharp end of education for an ethical, equitable and numerate society.

INTRODUCTION

My contention in this paper is that education for a safety-critical context such as nursing is at the sharp end of education for an ethical, equitable and numerate society. By this I mean that it occupies a place where judgements about professional competence have serious implications for the safety of others and for the professional. Somebody has to say: your numeracy is adequate for this context and yours is not. If there is no consensus on the nature and scope of numeracy in the context in question, different people will come to different judgements about the evidence required to prove adequacy. Even where a standard is set and judgements made against it, unless the standard is evidence-based it may bear little relation to the numeracy demands of the work. If such judgements are to be made, and in my view they must be made in relation to work in safety-critical contexts, then it behoves us to ensure they are based on transparent and defensible criteria and open to democratic challenge and periodic review.

In exploring this issue I shall first outline my vision of an ethical, equitable and numerate society before going on to explain why numeracy matters for individuals and for society and then focusing specifically on numeracy for nursing.

WHAT WOULD AN ETHICAL, EQUITABLE AND NUMERATE SOCIETY LOOK LIKE?

My vision of an ethical and equitable society would be one which is sustainable, with equal economic, political and social rights and opportunities for all and fair distribution of resources. It would exemplify the ethical values of honesty, openness, social responsibility, social justice and caring for others in all aspects of public policy and private endeavour. But what would such a society look like in order to deserve to be called numerate?

In a paper given at the first MES conference, Keiko Yasukawa notes the pervasiveness of mathematical models in socio-political spheres and suggests that numeracy ought to be seen as part of a broader critical technological literacy. Numeracy education for such a society accordingly entails building numerate practices across different communities of practice with numeracy educators active participants in this process. She contends that this kind of education would enable people across different socio-cultural groups to develop and participate in more numerate discourses (Yasukawa, 1998).

We have a long way to go to achieve such an ethical, equitable and numerate society in the UK. With regard to equality, overall income inequality has increased, albeit slightly, since the New Labour government came to power in 1997, the links between average health outcomes and income inequality appear strong and disturbing, and the UK still ranks equal bottom of EU15 countries in terms of child poverty (Hills, Sefton, & Stewart, 2009). With regard to ethics one need only consider the furore over MPs' expenses (http://www.telegraph.co.uk/news/newstopics/mps-expenses/) and bankers' bonuses (http://www.independent.co.uk/news/uk/politics/backlash-over-bankers-bonuses-1604034.html) or review the proceedings of the Iraq Inquiry (http://www.iraqinquiry.org.uk/) to feel that there is room for improvement there also. Meanwhile, with respect to numeracy, a survey of adults in England found that 47 per cent of the sample (equivalent to 15 million people) were classified at Entry level 3 or below (the level expected of the average 11 year old), including 21 per cent (equivalent to 6.8 million) at Entry level 2 or below (Williams, Clemens, Oleinikova, & Tarvin, 2003, p. 19). It is hard to see how people with such low levels of numeracy could "develop and participate in more numerate discourses" as Yasukawa contends they should, without the help of numeracy educators.

This raises an uncomfortable question: should participation in civil society be contingent on achieving a certain level of numeracy? It raises the spectre of numeracy tests for voting or doing jury service. This is not such a fanciful idea: many applicants for British citizenship are already required to take a test to show that they know about life in the UK (http://www.lifeintheuktest.gov.uk/htmlsite/about_10.html) and to prove that they have sufficient knowledge of English, Welsh or Gaelic. Most non-EU migrants coming to Britain to do skilled or highly skilled jobs also have to pass an English language test. What if such tests were to include numeracy? If we feel that this would infringe civil liberties, are we as educators in the paradoxical position of defending the right to ignorance just as we insist on the right to education?

NUMERACY MATTERS

The case for supporting people to become (more) numerate is fairly self-evident since we know that poor numeracy has a detrimental effect on an individual's life chances. For example, research on members of two major longitudinal studies of the British population [1] concludes that "Poor numeracy skills make it difficult to function effectively in all areas of modern life, particularly for women" (Parsons & Bynner, 2005, p. 7).

Nor are individuals themselves unaware of the importance of mathematics in their lives, however much some may appear dismissive (the oft-heard cry of "I'm no good at maths" may sometimes be a defensive rather than a celebratory statement). When adults from various backgrounds were asked about their mathematics life histories, what were scheduled as one-hour interviews usually over-ran considerably, with many speaking with real passion (both positively and negatively). The following themes emerged in many of the interviews, attesting to the importance of mathematics in their lives:

- The brick wall – the point (usually in childhood) at which mathematics stopped making sense; for some people it was long division, for others fractions or algebra, while others never hit the brick wall. For those who did, the impact was often traumatic and long-lasting.
- The 'significant other' – someone perceived as a major influence on the person's maths life history. The influence might be positive or negative, past or present. Significant others included, for example, a parent who tried to help with maths homework; a teacher who made the person feel stupid; a partner who undermined the person's confidence in their mathematical abilities.
- The door – marked 'Mathematics', locked or unlocked, which people have to go through to enter or get on in a chosen line of work or study.
- Invisible maths – the mathematics someone can do, but which they may not think of as maths at all, 'just common sense'. (adapted from Coben & Thumpston, 1996, p. 288)

In the public domain, also, numeracy really matters. For example, a major US space mission foundered on a numeracy issue:

> In September 1999, the Mars Climate Orbiter spacecraft failed to enter orbit around Mars.
> Review teams found that a contractor had used English, rather than metric, units of measurement in a navigation software program. Outputs from this program were used to compute the spacecraft's trajectory, causing a navigation error. (NASA, 2001)

In politics also, mathematics plays a role in shaping perceptions and informing public policy, potentially with far-reaching effects. The following news item shows what can go wrong when (presumably) nobody checked a crucial figure:

> *Tories criticized over teenage pregnancy figure error*
> *BBC News Channel, 15 February 2010*
>
> The Tories have been attacked as "out of touch" for wrongly claiming more than half of girls in the most deprived areas get pregnant before they turn 18.
> The party said the conception rate for this age group in the 10 most disadvantaged areas of England was 54%, while the real figure was 5.4%.
> Labour accused the Conservatives of using "smears and distortions".
> But the Tories said the misplacing of a decimal point made "no difference" to claims Labour had let down the poor.

> The pregnancy figure was given in a 20-page dossier, published on Sunday, attacking the government for allowing the creation of "two nations" - the wealthy and the impoverished.
>
> *'Deception'*
> In response, Labour said the correct figure of 5.4% represented a fall from 6% in 1998.
> (BBC News Channel, 2010)

Mathematics plays a role also in judgements of risk in healthcare. As the authors of a recent article noted:

> One of the many challenges to risk communication with the public is the difficulty in expressing quantitative information in an easily comprehensible form. Universal cognitive limitations cause biases in interpreting numerical probabilities (Cosmides & Tooby, 1996; Tversky & Kahneman, 1974). Small probabilities are particularly difficult to interpret; under some conditions people overestimate them, and under others they 'round down' to zero (Cosmides & Tooby, 1996; Tversky & Kahneman, 1974). For many consumers, these difficulties in interpreting probabilities are compounded by limited numeracy skills (Lipkus, Samsa, & Rimer, 2001; Schwartz, Woloshin, Black, & Welch, 1997) and by discomfort with numerical expressions of risk (Anon, 1998). Understanding numerical information can be even more difficult when analytic reasoning processes are impaired by age, stress, or other factors (Slovic, Peters, Finucane, & MacGregor, 2005). (Ancker, Senathirajah, Kukafka, & Starren, 2006, p. 608)

Similar considerations apply in relation to personal finance (Atkinson, McKay, Kempson, & Collard, 2006) and in people's working lives – of which more later.

But how realistic is it to think that more and better numeracy would necessarily improve this situation? Might it not be that people at all levels of numeracy get by through what Gerd Gigerenzer and his colleagues call "fast and frugal heuristics", "simple rules in the mind's adaptive toolbox for making decisions with realistic mental resources" (Gigerenzer, Todd, & ABC Research Group, 1999). Such heuristics may be at the heart of numeracy if, as I believe,

> To be numerate means to be competent, confident, and comfortable with one's judgements on **whether** to use mathematics in a particular situation and if so, **what** mathematics to use, **how** to do it, what **degree of accuracy** is appropriate, and what the answer **means** in relation to the context. (Coben, 2000, p. 35, emphasis in the original)

In their book *Simple Heuristics That Make Us Smart*, Gigerenzer and his colleagues ask:

> How can anyone be rational in a world where knowledge is limited, time is pressing, and deep thought is often an unattainable luxury? Traditional models of unbounded rationality and optimization in cognitive science, economics, and animal behavior have tended to view decision-makers as possessing supernatural powers of reason, limitless knowledge, and endless time. But understanding decisions in the real world requires a more psychologically plausible notion of bounded rationality. (Gigerenzer, et al., 1999)

The workplace is a site for fast and frugal heuristics in numeracy precisely because "supernatural powers of reason, limitless knowledge, and endless time" are usually in short supply. When the workplace is the site of safety-critical judgements by professionals, education for numeracy is at the sharp end.

The educationalist Michael Eraut has analyzed different types of knowledge and know-how used by practising professionals in their work and examined the ways in which these are acquired by a combination of learning from books, learning from people and learning from personal experience. Eraut considers to what extent professional knowledge is based on intuition, understanding and learning, including the way theory changes and is personalized in practice, and how individuals form generalizations out of their practice. He considers the issue of competence versus knowledge and the effect of lifelong learning on the quality of practice. He points out that "Given the demands and pace of professional practice, professionals learn to use routinised practices devoid of problematisation" (Eraut, 1994). Could some of these "routinised practices" be the outward and visible signs of Gigerenzer et al's "fast and frugal heuristics", i.e., practices that appear routine because they are the expression of the internalized rules to which Gigerenzer refers?

If we allow that Gigerenzer et al's argument logically includes numeracy, we could ask: can a notion of bounded rationality and fast and frugal heuristics democratize our understanding of numeracy in contexts where being numerate manifestly matters, for example, in safety-critical work contexts?

I want to explore this question in the remainder of this paper, focussing on my research, with colleagues in two interdisciplinary teams, on numeracy for nursing.

NUMERACY FOR NURSING

First some background on numeracy in and for nursing, until recently a neglected area, despite its importance. Nursing has what the sociologist Peter Nokes has called a "manifest disaster criterion" (Nokes, 1967) since errors may have serious consequences. There is a growing literature revealing a lack of proficiency amongst both students and registered nurses (Sabin, 2001) revealed every so often in alarming headlines (e.g., Hall, 5th August, 2006). The development of appropriate competence in numeracy by healthcare staff and students is a key area for concern but there is no consensus on the nature and scope of numeracy for nursing, which is still poorly-understood (Coben, Hall, et al., 2008), nor on ways of improving the situation. The need for fundamental analysis and reflection on strategies for the education and training of students is made more urgent by the safety-critical nature of nursing generally (Cooke, 2009), and in particular those aspects of nursing involving numeracy (e.g., ISMP, 2008). For example, nurses need to be able to calculate drug dosages, estimate a patient's fluid balance and nutritional status and interpret and act appropriately on data shown by equipment used to monitor a patient's condition or dispense treatment: a mistake in any of these could be life-threatening for the patient and end the nurse's career.

Nowadays numeracy is taught and assessed in a variety of modes in pre-registration nursing programmes in the UK – face-to-face, online, in simulated practice and on the ward. One might think that the latter should be preferred as the method closest to practice but real-world practice has several limitations as an arena for the teaching, learning and assessment of numeracy for nursing.

Firstly, any given instance of nursing practice may be rich or poor in numeracy terms, depending on the exigencies of the situation. Students may not be exposed to the full range of complexity of numeracy for nursing, either mathematically or in terms of nursing content, on a particular day. For example, dosage calculations involving sub-, multiple- and unit-dose may not all be called for, but a nurse needs to be able to handle all of these as required.

Secondly, teaching, learning and assessment of numeracy for nursing need to be authentic, as studies in various vocational contexts including nursing have shown (viz. FitzSimons, Mlcek, Hull, & Wright, 2005; Forman & Steen, 2000; K.W. Weeks & Woolley, 2007).

Thirdly, the quality of teaching and mentoring in any mode is dependent on the skills, knowledge and understanding of the teacher or mentor and his or her ability to communicate these to the student. Since the literature indicates a lack of proficiency amongst some qualified nurses it would not be surprising if some of those teaching or supporting nursing students had an inadequate grasp of numeracy or were unable to communicate their knowledge to novices even if they themselves understand what is required.

The following scenario (Fig. 1) shows what can happen when communication breaks down and the experienced nurse is unaware that the student has not understood what she has done. An experienced nurse is talking a student through the calculation of a medication dose to be given to a patient:

> We need Aminophylline 200 milligrams… It comes as 250 milligrams in 10ml. Therefore we need to give 8ml… OK?

The student is baffled but too embarrassed to reveal her ignorance, so a learning opportunity is missed precisely because of the "routinised practices devoid of problematisation" – or the fast and frugal heuristics - of the experienced nurse. Ironically, these very practices are the mark of her competence.

Against this background I am investigating aspects of numeracy for nursing as a member of two interdisciplinary teams, outlined here in relation to the focus of this paper.

In the first project, based in Scotland, funded by NHS Education for Scotland (NES) and here called 'the NES study', we are seeking to establish a benchmark in numeracy for nursing, focussing initially on a high risk area of nursing: medication dosage calculation' [2] (Coben, et al., 2010).

Figure 1. Numeracy in the workplace: nursing (K.W. Weeks & Woolley, 2008)

The background to both projects, and in particular the NES study, is that, in response to growing concern about nurses' numeracy, from September 2008 the body regulating the nursing profession in the UK, the Nursing and Midwifery Council (NMC), requires nursing students to achieve 100% in a test of numerical competence in the practice setting before being allowed to register as nurses (NMC, 2007). However, there are currently no national standards for teaching or assessment of numeracy during pre-registration nurse education, and, in the absence of a robust evidence-based standard (a benchmark), a multiplicity of tests, processes and criteria are being developed and deployed in pre-registration nursing programmes throughout the UK, including the university in the second study outlined below. Amidst concern that some newly qualified and experienced nurses may not have the numeracy skills required for safe practice some employers are imposing their own tests of numerical competency when selecting people for nursing posts; however, these tests may be neither reliable nor valid. Without a benchmark assessment it is difficult to determine which skills require development, or to ascertain when competence has been achieved since any measure of numerical competence is:

> ... in the eye of the recipient of evidence of that competence, be it higher education institutions, regulators, employers or service users. (Hutton, 2004)

Our work on the NES project provides a real opportunity to establish a UK benchmark for competence in nursing numeracy at the point of registration, the point at which students become qualified nurses.

As a first step towards the establishment of such a benchmark, in the first phase of the study we developed an evidence-based numeracy benchmark assessment tool utilising interactive computer simulations that approximate to real world nursing practice. The assessment tool was based on the following criteria, which we established following our analysis of the literature and a Scotland-wide consultation and strategy (Sabin, 2006). Such an assessment tool should be:

Realistic:

- Evidence-based literature in the field of nursing numeracy (Hutton, 1997; Keith W. Weeks, Lyne, Mosely, & Torrance, 2001) strongly supports a realistic approach to the teaching and learning of calculation skills, which in turn deserve to be tested in an authentic environment. Questions should be derived from authentic settings. A computer based programme of simulated practice in drug calculations, formative testing, with feedback on the nature of errors made, has been shown to develop competency in medication dosage calculation, which can be also demonstrated in the clinical areas (Keith W. Weeks, Lyne, & Torrance, 2000). Exposure of students to real-world situations is recommended (Keith W. Weeks, 2001).

Appropriate:

- The assessment tool should determine competence in the key elements of the required competence (OECD, 2005; Sabin, 2001).

Differentiated:

- There should be an element of differentiation between the requirements for each of the branches of nursing (Hutton, 1997).

Consistent with adult numeracy principles:

- The assessment should be consistent with the principles of adult numeracy learning teaching and assessment, having an enablement focus (Coben, 2000).

Diagnostic:

- The assessment tool should provide a diagnostic element, identifying which area of competence has been achieved, and which requires further intervention (Black & Wiliam, 1998). Thus it should "provide information to be used by students and teachers that is used to modify the teaching and learning activities in which they are engaged in order better to meet student needs. In other words, assessment is used to 'keep learning on track'" (Wiliam, 2007).

Transparent:

- The assessment should be able to demonstrate a clear relationship between 'test' achievement and performance in the practice context (Keith W. Weeks, et al., 2001).

Well-structured:

- The tool should provide:
  - a unique set of questions with a consistent level of difficulty;
  - a structured range of complexity; and
  - the assessment should take place within a defined framework, at points by which students can be effectively prepared, while allowing time for supportive remediation. (Hodgen & Wiliam, 2006)

Easy to administer:

- the assessment should provide the opportunity for rapid collation of results, error determination, diagnosis and feedback (Black & Wiliam, 1998).

(Coben, Hall, et al., 2008, pp. 96-97)

Having produced a computer-based learning and assessment tool based on these criteria and building on a literature review, previous research by members of the team and development work by Authentic World® http://www.authenticworld.co.uk/, we evaluated empirical evidence of the tool's reliability and convergent validity by comparing its outcomes with the outcomes of a practical activity requiring the same medicine dosage calculations. We also aimed to gauge the acceptability to learners of the assessment tools in terms of their authenticity, relevance, fidelity and value. We did this because a robust, authentic computer-based assessment tool could facilitate large-scale assessment of numeracy for nursing against the proposed benchmark.

The results of the study support the criterion-related validity of the computer simulation format, both in terms of ranking participants in a similar order of competence and in terms of participants obtaining similar absolute results (getting the same number of questions correct on the computer simulation as they would on the practical simulation). However, we noted that computer simulation does not test certain elements of the real-world dosage calculation problem (e.g., technical competency); also, we stress that these conclusions should only be applied to similar situations, populations, and constructs. A full report of the study is given in the project report (Coben, et al., 2010).

In the second project a separate interdisciplinary team (though with two members, myself and Meriel Hutton, who are also on the NES team) investigated the assessment of numeracy for nursing in a university in England [3], one of many in the UK which have produced tests, processes and criteria in order to meet the NMC's numeracy requirement. The study reveals the dangers of high stakes testing with a 100% pass mark in the absence of a reliable and valid assessment instrument set to an agreed standard and reflecting the scope of numeracy for nursing. Our analysis shows that the test evaluated in the study is neither reliable nor valid and it is not authentic; it does not indicate mastery of numeracy for nursing. Given the high stakes nature of the assessment, potential nurses whose numeracy might be adequate for the profession may be lost and others with inadequate numeracy may be pronounced safe to practice (Coben, Hodgen, Hutton, & Ogston-Tuck, 2008). Thus the findings of the second project bear out the need for the benchmark to be developed from the NES project.

My work on both these projects has led me to reflect on whether Gigerenzer et al's ideas of bounded rationality and fast and frugal heuristics could offer a way forward in democratizing approaches to education for numeracy in safety-critical work contexts such as nursing. I conclude that they could, for the following reasons.

Gigerenzer et al's ideas help us to focus on the requirements and exigencies of the context. With respect to numeracy, they help us to see that being good at mathematics is not sufficient because what is required is the ability to see through to the context-specific mathematics to appreciate the scale and scope of problems and produce and evaluate possible solutions - to make sensible judgements on "**whether** to use mathematics in a particular situation and if so, **what** mathematics to use, **how** to do it, what **degree of accuracy** is appropriate, and what the answer **means** in relation to the context" (Coben, 2000, p. 35, emphasis in the original). This shifts the focus away from simplistic notions of competence expressed in 'can do' lists of tasks divorced from the complexities of the contexts in which they are required to be undertaken towards a more holistic notion of competence which we are currently developing in the NES project. Awareness of the heuristics of numeracy *in* nursing should encourage authentic teaching, learning and assessment of numeracy *for* nursing.

Authenticity is important in numeracy education for work, as Gail FitzSimons shows in her study of the chemical spraying industry (FitzSimons, et al., 2005) and as others have argued with respect to mathematics education more generally (Forman & Steen, 2000) and to adult literacy education (Purcell-Gates, Degener, Jacobson, & Soler, 2002). Meriel Hutton and I have noted in a paper on numeracy for nursing as an example of the interface between mathematics education and industry that:

> Where mathematics is situated in professional/vocational practice it should be taught, learned and assessed in relation to that practice, both directly in practice and through authentic and comprehensive simulation of practice; the latter enables individuals to be exposed to the full range of problems associated with the use of mathematics in their

professional practice, something which may be impossible to do safely, comprehensively and effectively in real world, real time contexts. (Coben & Hutton, forthcoming)

Authenticity requires a recognition of the contingencies of real world nursing practice as encompassing often stressful situations where “knowledge is limited, time is pressing, and deep thought is often an unattainable luxury” (Gigerenzer, et al., 1999). In such contexts a notion of bounded rationality and fast and frugal heuristics can and should democratize our understanding of numeracy, allowing us to move beyond reductive notions of professional competence and inauthentic approaches to numeracy education towards a more open, democratic holistic approach that recognizes the strengths of capable, experienced professionals and the potential of novices to develop expertise and experience through an appropriate programme of teaching and learning founded on a deep understanding of the requirements of the work in question. Numeracy for nursing, as an example of work at the sharp end of education for an ethical, equitable and numerate society, supplies plenty of food for thought in this endeavour.

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ENDNOTES

1 The 1958 National Child Development Study (NCDS) and the 1970 British Cohort Study (BCS70). For further information, see http://www.cls.ioe.ac.uk/text.asp?section=000100010002.

2 The NES project report (which I draw on in this paper), details of the project team and associated materials are online at http://www.nursingnumeracy.info/index.html.

3 The project is entitled ‘Numeracy for Nurses’, Principal Investigators: Diana Coben and Jeremy Hodgen, with Meriel Hutton and Sherri Ogston-Tuck, funded by King’s College London.

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COMMENTS ON "AT THE SHARP END OF EDUCATION FOR AN ETHICAL, EQUITABLE AND NUMERATE SOCIETY: WORKING IN A SAFETY-CRITICAL CONTEXT—NUMERACY FOR NURSING"

Marta Civil

The University of Arizona

Coben's (2010) paper was thought provoking at several levels, as it combines issues related to adult education in mathematics for specific professions, such as nursing, with considerations about equity. I will focus my remarks on two themes—the concept of equity and the proportional reasoning example presented in the paper.

About Equity

I would like to know more about Coben's approach to equity in adult mathematics education. She relates issues of equity with "equal economic, political and social rights and opportunities for all and fair distribution of resources" (p. 9). But I wonder, how does this notion relate to the many efforts towards defining equity in mathematics education? For example, Gutiérrez (2007) talks about four dimensions in her definition of equity: access, achievement, identity, and power. And Leonard and Ramirez (2009) write, "Equity increases when marginalized students use mathematics for their own purposes, which include making decisions, changing the status quo, and learning mathematics that provide access to higher education and job opportunities" (p. 2). Coben talks about "democratizing approaches to education for numeracy in safety-critical work contexts such as nursing" (p. 18). Is there any potential relationship between this notion of democratizing and approaches to defining equity in mathematics education?

What does equity mean in the context of mathematics for nursing practice? Or more generally, what does equity mean in adult mathematics education? Coben raises a provocative question when she writes, "should participation in civil society be contingent on achieving a certain level of numeracy?" (p. 10) While I am more familiar with equity implications of decisions such as tracking (or setting) for school age children, the question remains the same: who decides what counts as "a certain level of numeracy" (or an appropriate level of mathematics education)? And what are the equity implications of this decision? Will some adults be left out from participating in civil society because they have not reached this externally set level? Coben compellingly calls our attention to the importance of numeracy with an example from politics when she describes the event of giving a colossal figure for the conception rate in disadvantaged areas as being 54% when it really was 5.4%. The implications of the misplacement of the decimal point are multiple yet at the same time, I could not help but wonder, did most people "catch it"? And if the answer is no, what does this mean in terms of people's numeracy or people's awareness of social context in, in this case, disadvantaged areas?

Numeracy for Nursing

This paper reminded me of the studies by Hoyles, Noss and Pozzi on the mathematical practices of nurses (Hoyles, Noss, & Pozzi, 2001; Noss, Hoyles, & Pozzi, 2002). In their work these authors present ethnographic analyses of practices of several nurses with an eye on uncovering the mathematics in such practices. Although their emphasis is more on situatedness of this practice and not on issues of equity and democratization, they do bring the notion of assessment with an argument along similar lines as what Coben pursues in her paper: “we therefore questioned whether results on written tests are valid for judging either the accuracy of nurses’ drug calculations or the methods they would use to carry them out” (Hoyles, Noss, & Pozzi, 2001, p. 11). These authors then point out that they have found little work in alternative assessment methods. Coben’s description of the criteria for an assessment tool for nursing practice seems to be a step towards addressing this gap. I wonder, what may be possible extensions or applications of the proposed assessment tool (tied to practice) to other areas of mathematics education for adults or children in school?

As a mathematics educator I was intrigued by the example presented in Figure 1 (page 15) where an experienced nurse determines that to get 200 milligrams of Aminophylline when it comes at 250mg in 10ml, one will need 8ml. There is no information on how she reaches this conclusion and the nursing student is unsure about how she worked it out but remains quiet. What did the experienced nurse do to get the answer? Hoyles, Noss, and Pozzi (2001) analyze this type of situation and describe the different strategies used: rule of three, scalar strategies, and the nursing rule (what you want divided by what you got and multiplied by the amount it comes in; so in this case: (200/250) x 10 = 8). I do not know what strategy the nurse in Coben’s case used, but my question is: if the nursing student has “only” school mathematics, will she be able to make sense of the problem? Will she have to set it up as a proportional reasoning situation, and use procedures traditionally taught in school (e.g., cross multiply), hopefully coming up with 8, but what about her understanding? And, as Coben writes, what about the need for “fast and frugal heuristics” in the nursing practice?

Proportional reasoning seems critical in nursing practice, yet it is an area that is considered as difficult in school mathematics. The transition from additive to multiplicative thinking is not always straightforward. If to this we add that often in school mathematics, “common sense approaches” seem to be less valued than formal ones, I wonder about the implications for practices such as nursing. In the particular example of the 250 mg and 10 ml, one approach could be to see that for every 50 mg, you need 2 ml; so, if you need 200 mg, you will need 4 x 2, which is 8 ml.

Several years ago, I gave the following problem to a group of prospective elementary teachers: If you need 1 1/3 cups of sugar and 4 cups of flour to bake a cake, how many cups of sugar will you need if you want to use 7 cups of flour?

One of the students drew the cups of flour and sugar and immediately saw that for every 1/3 cup of sugar she needed 1 cup of flour. So, she concluded that for 7 cups of flour, she would need 7/3 or 2 1/3 cups of sugar. Another student used an additive reasoning and two other students who had set it up along the lines of a typical school procedure (1 1/3 : 4 = x : 7; solve for x) tried to explain to this student why an additive strategy was not going to work. Their explanation was primarily based on their going through the algebra or their own approach, not addressing the meaning behind an additive versus a multiplicative reasoning. They never referred to the work of the student who drew the cups of flour and sugar, which clearly showed the relationship. Furthermore, this student although relieved that she could use "her own methods" often expressed a feeling that her methods were not as good as her peers' who could use algebra.

At least in my context, the use of cross multiplying, when presented with a proportional reasoning situation, seems to be the strategy of choice, whether conceptual understanding accompanies it or not. In fact, based on my experience, I would argue that meaning is usually not attached to this procedure. For example, in Civil and Bernier (2006), we present the case from a mathematics workshop for adults (mostly parents from children in the school district where our project was located, but there were also some teachers attending these workshops) in which a team dominated by teachers used an incorrect proportional reasoning approach (setting up a ratio between areas as equal to a ratio between lengths). One of the teachers solved this "proportion" by cross-multiplying. The following exchange took place between the teacher (R) and one of the mothers (J) in the workshop:

J: How did you know to cross…how did you do that, why did you do that?

R: We kind of have degrees in Elementary education so I guess somewhere along the line I guess we learned when we set up our proportion in order to get (pause), after we set up our proportion the next thing you do is you cross multiply.

J: All the time? Forever? Always and for every situation?

R: Always. (p. 320)

This excerpt is problematic at many levels, but in terms of the relevance to Coben's presentation I highlight the following: What kind of explanation did this teacher give? Hopefully, this is not the kind of understanding that the experienced nurse had. To me this exchange, as well as the cups of sugar and flour episode, raise issues of power and equity, in terms of what methods get validated by whom and why. And these issues, I argue, should be considered in Coben's work on nursing numeracy and its assessment.

In Closing

Coben calls for a numeracy in and for nursing (and its corresponding specific assessment). Reading this made me think of the idea of mathematics knowledge for teaching developed by Ball, Hill and colleagues (Hill, Ball, & Schilling, 2008) and makes me wonder, should this be the direction we move towards? That is, should we be thinking about mathematics in and for specific practices? What would that look like? Are there some practices for which it would make more sense to do this than others? Should the notion of "safety-critical context" be one of the considerations in deciding which practices? These are some of the questions that come to my mind as I think about implication from Coben's paper.

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REACTION TO: AT THE SHARP END OF EDUCATION FOR AN ETHICAL, EQUITABLE AND NUMERATE SOCIETY

Tine Wedege

Malmö University

ADULT NUMERACY

The international research forum Adults Learning Mathematics (ALM) was formed in 1994 and a new research field has been cultivated in the borderland between mathematics education and adult education. As a promoter and the first chair of the research forum, Diana Coben was and still is a key person in this field. That is also why I was happy to see her as a plenary speaker at the Sixth Mathematics Education and Society (MES) conference in Berlin and to have the possibility of reacting to her paper entitled “At the sharp end of education for an ethical, equitable and numerate society: Working in a safety-critical context – numeracy for nursing“.

It is obvious that the problem field of adults learning mathematics is situated within the domain of MES. In an overview presentation, at the 16th International conference on Adults Learning Mathematics in London last summer, I claimed that the key concept is numeracy and that the problem field is related to adults, mathematics and lifelong education in a societal context (Wedege, 2009). The concept of numeracy is contested among educational researchers and politicians (see for example Coben et al., 2003), but the idea of building bridges between mathematics and society is common in a long series of concept constructions. For the overview of the field in London, I had chosen three sub areas related to adults engaged in specific social practices and exactly the leading researchers in these areas were present at the MES conference: Marta Civil (parents), Gelsa Knijnik (landless peasants) and nurses (Diana Coben). The purpose of the research and development project lead by Coben is to create a benchmark for numeracy for nursing and I find that the real challenge here is to combine ALM principles taking the learner in focus with requirements for standards and assessment.

In any study of adult numeracy, two different lines of approach are possible and intertwined: a *subjective approach* starting with people's competences and subjective needs, and an *general approach* starting with societal and labour market qualification demands and/or with requirements from "school mathematics" (Wedege, 2000). The general approach is obviously to be found in international surveys on adult literacy and numeracy like OECD (2000, 2005) and in national surveys like Williams et al. (2003) with a focus on the poor numeracy in the population. As a representative of the ALM spirit, Roseanne Benn published a book in 1997 entitled “Adults count too”. Her study also examines numeracy in society, but the approach is subjective, starting with the adults. She argued that mathematics is not a value-free construct, but is imbued with elitist notions which exclude and mystify. Similarly, she rejects the approach where any problem with mathematics is located within the learner rather than the system (Benn, 1997). In Coben's definition of numeracy from 2000 it is obvious that her point of departure is an adult being competent in a societal context and not a competence or qualification pre-defined with reference to requirements from society or mathematics:

> To be numerate means to be competent, confident, and comfortable with one's judgements on **whether** to use mathematics in a particular situation and if so, **what** mathematics to use, **how** to do it, what **degree of accuracy** is appropriate, and what the answer **means** in relation to the context. (After Coben et al., 2003, p. 10, emphasis in the original)

The focus in my reaction to Coben's plenary is to challenge her reasons for establishing a benchmark in numeracy for nursing.

NUMERACY IN A SAFETY-CRITICAL CONTEXT

Coben presents nursing as a safety-critical context for numeracy. In adults' working and everyday life, one can find many contexts and situations where poor numeracy may have serious implications for the safety of others: In a large Danish electronics factory producing aircraft components, I have observed an experienced semi-skilled worker in the department where blanks from a subcontractor are subjected to quality control. She demonstrates her consciousness about the work context being safety-critical by saying: "There is a difference between the consequence of a mistake in an air plane and a television set. It could be a matter of life and death." (Wedege, 2002). In the first Swedish report on the problem field of adults and mathematics, the subtitle is precisely "a vital subject" (Gustafsson, 2004).

Numeracy is recognized as a key skill for professional practice in nursing and Hutton (1997) argued that poor numeracy can be life-threatening for the patient. Calculating fluid balance, drug dosages and intravenour drip rates are examples where numeracy is needed. Coben (2010) and her colleagues reformulate the problem as they see it and they state that "there is a growing literature revealing a lack of proficiency amongst both students and registered nurses" (p. 5). Moreover, they point to another problem that there is no recognized standard for numeracy for nursing and "without a benchmark assessment it is difficult to determine which skills require development or to ascertain when competence has been achieved (p. 8). Hence, they argue, a multiplicity of tests, processes and criteria, which may be neither reliable nor valid, are being developed and deployed in pre-registration nursing programmes throughout the UK.

> **If such judgements are to be made**, and in my view they must be made in relation to work in safety-critical contexts, then it behoves us to ensure they are based on transparent and defensible criteria and open to democratic challenge and periodic review. (Coben, 2010, p.9, my emphasis)

I agree with Coben when she requires transparency and validity if testing of adults' competences are to be made. However, she argues for the necessity of standards with reference to the "Skills for Life" national survey of adult numeracy in England:

> … with respect to numeracy, a survey of adults in England found that 47 per cent of the sample (equivalent to 15 million people) were classified at Entry level 3 or below (the level expected of the average 11 year old) … (Williams et al. 2003, p. 19). (Coben, 2010 p.10)

But this particular national survey and the other international surveys mentioned above have been object of critical analysis by the members of ALM. Here Gillespie is for example summing up on the "Skills for Life" survey:

> The findings confirm that for many, being 'at a given level' is not meaningful for the individual, as levels embody predetermined assumptions about progression and relative difficulty. (Gillespie, 2004, p. 1)

As mentioned above, Coben also argues by referring to the literature. It is unsurprising that the nursing literature shows that students and even experienced nurses make many errors on paper-and-pencil tests of drug calculations in a school context. However, in a qualitative study in the context of nursing, which is not among Coben's references, there was not found any errors:

> In our study on the ward [30 episodes], we found drug administration to be routine and error free. It was characterized by effective and flexible use of a range of proportional reasoning strategies … (Hoyles, Noss & Pozzi, 2001, p. 22, my parentheses).

Coben (2010) also refers to alarming headlines in the newspapers "about lack of proficiency amongst both students and registered nurses". We find also this kind of headings in the Nordic newspapers when they report results from the OECD surveys on adult literacy and numeracy. For example in Norway:

> 1,2 million Norwegians have problems with numbers
>
> A new OECD survey (ALL) has shown that almost 40% of the adult population in Norway "have such bad understanding of numbers that they have problems tackling daily life." (Dagbladet, 4 September, 2005)

In this article, which presented the poor results as an argument for offering numeracy courses to adults, there was a subsection, "Many people die", presenting the fact that at least 10-15 deaths every year in Norway were caused by mistakes in the handling of drugs in nursing and caring service. However, if one goes to the report from the National Helse Supervision, which had provided this information it is evident that the mistakes in medication are caused by system errors and not by nurses' calculation errors (Helsetilsynet, 2002).

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IDEOLOGICAL ROOTS AND UNCONTROLLED FLOWERING OF ALTERNATIVE CURRICULUM CONCEPTIONS

Eva Jablonka, Uwe Gellert

Luleå University of Technology, Freie Universität Berlin

With this contribution, we intend to initiate a discussion of alternative curriculum conceptions in terms of how these might facilitate or restrict access to valued forms of mathematical knowledge. For this purpose, we characterise conceptions of school mathematics as realisations of a process of dual recontextualization. As we will argue, different alternative ways of recontextualizing practices of professional mathematicians as well as everyday practices, implicate different potentials, pitfalls, (dis-)advantages and discriminations for different social groups. We will attempt to link the discussion to the political bases of the alternatives we have chosen to discuss.

INTRODUCTION

Curriculum conceptions for mathematics education are the product of a social process, including ideological struggles between stakeholders pursuing diverse economic and political goals. In many cases, the result represents a compromise between different or differently nuanced social positions and agendas. As an example, consider the curricular transformations initiated in many countries after the Programme of International Student Assessment (PISA) and the Third International Mathematics and Science Study (TIMSS) have been launched.

As curriculum conceptions often represent ideological hybrids, the consequences of mathematics curricula for different student groups in terms of their access to mathematical knowledge, their formation of mathematical identities and their positioning in the 'knowledge society' are rarely directly visible. However, these consequences are not simply more or less accepted side effects of the practice of schooling. They reflect a differential distribution of legitimate and valued forms of knowledge and position intended to reproduce or develop social structures.

Mainstream curriculum and positions of resistance

One can take the standardised curriculum versions that are manifested in official curriculum prescriptions, textbooks and test-designs as representing the mainstream in a given context. To the extent to which curriculum documents are results of compromises, they leave more or less space for alternative readings by teachers and students. Identifying these spaces requires an analysis of its own.

The students are the 'consumers' of the privileged meanings established in the curriculum, and if they successfully acquire the intended interpretations, the resulting certificate and/or the mathematical knowledge is of symbolic value and eases access to further education. As curriculum conceptions construct their ideal readers, with distinct dispositions for mastering its explicit and implicit demands, differences in "orientations to meanings" (Bernstein, 1990) generate patterns of achievement in line with social differences (such as gender, ethnicity, social class).

Alternative curriculum conceptions aim at redistributing access to privileged discourses. This can be achieved at different levels. Protagonists might be concerned with expanding the repertoire of individual students with a focus on marginalised groups and their orientations to meanings, without challenging the available reservoir of cultural meanings (such as "mathematics as thinking and problem solving" or "mathematics as a universally applicable technology"). On the other hand, more radical alternatives challenge the available reservoir of meanings. The first option might be described as a form of tactical resistance, whereas the second is aiming at deconstruction of culturally inherited meanings. In interpreting mathematics curriculum conceptions as texts in a social context that position their readers, one could attempt to classify alternative conceptions according to their position towards mainstream conceptions in a similar way as Martin (1993) differentiates resistant reading positions in the context of research on literacy: as tactical resistance versus deconstructive resistance and oppositional versus subversive deconstructions (see figure 1).

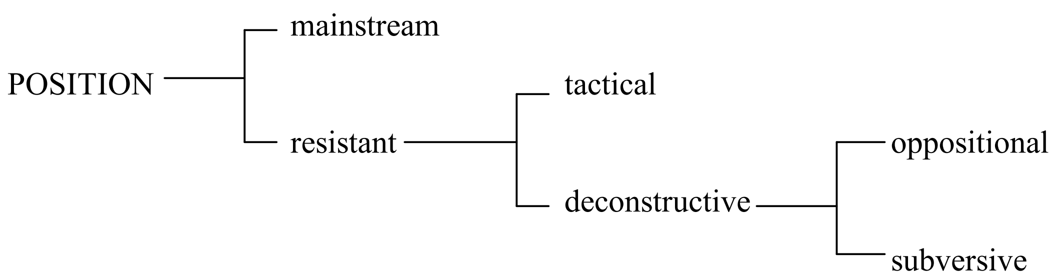

Figure 1. Dimensions of position (modified from Martin, 1993, p. 159)

It can be argued that informed opposition and dissent to mainstream curriculum conceptions (and their concomitant distributions of mathematical discourses and position) requires insight into the discourses that are the focus of critique. Apparently, there is a tension between a pedagogy of access and a pedagogy of dissent: Is access to valued forms of mathematical knowledge a precondition for a critique of social mathematical practices and their constituting discourses, or is access to valued forms of mathematical knowledge possible by critiquing mainstream discourses? How can these two poles be balanced?

In a given context, conceptions that are alternative to the curricular mainstream might be classified according to this scheme. However, what counts as the curriculum mainstream is different in different social and political settings. What currently is mainstream in one place might resemble, for instance, a tactical resistance position in other places, or a short-lived reform that has been followed by a counter-reform. The world of school mathematics curricula is not (yet) fully uniform.

Moreover, tactically resistant positions tend to aim at becoming the mainstream, hence the label 'tactical'. As a consequence, it is often difficult to identify what the mainstream position exactly consists of, even in a rather local setting. Practices of mathematics instruction are constantly (even if only slightly) changing, integrating aspects of tactical resistant positions into the mainstream. Bernstein (1996, p. 48) distinguishes between an "*official recontextualizing field* (ORF) created and dominated by the state and its selected agents and ministries, and a *pedagogic recontextualizing field* (PRF)". The PRF consists of teachers, researchers, private research foundations etc. What is considered as mainstream might be different in these recontextualizing fields.

Mathematics curricula as a product of dual recontextualization

Curriculum conceptions for mathematics education can be described as the specific product of a dual recontextualization. On the one hand, school mathematics can be seen as the result of a subordination of the practices of generating new mathematical knowledge (exploration, systematisation, proof) to the pedagogic and didactic principles of the transmission of knowledge. On the other hand, school mathematics recontextualizes vocational, domestic and leisure time activities by subordinating them to a mathematical gaze. There is a variety of ways in which this dual recontextualization can be realised in the mathematics curriculum. Some common versions of the mathematics curriculum in place, in which this dual recontextualization constructs a hybrid between domestic and mathematical knowledge, have been shown to be socially biased and self-referential.

Different alternative ways (focus on investigations and problem solving, ethnomathematics, mathematical modelling, critical mathematical literacy) implicate different potentials, pitfalls, (dis-)advantages and discriminations for different social groups. They differ in what knowledge is accessed in classrooms and in how this knowledge is made accessible. In an elaboration of Bernstein's sociology of education (Bernstein, 1996), the underlying principles can be termed *classification* and *framing*:

> I will now proceed to define two concepts, one for the translation of power, of power relations, and the other for the translation of control relations, which I hope will provide the means of understanding the process of symbolic control regulated by different modalities of pedagogic discourse. …
> I shall start first with power. We have said that dominant power relations establish boundaries, that is, relationships between boundaries, relationships between categories. The concept to translate power at the level of the individual must deal with relationships between boundaries and the category representations of these boundaries. I am going to use the concept of *classification* to examine relations between categories, whether these categories are between agencies, between agents, between discourses, between practices. (Bernstein, 1996, pp. 19-20)

In the context of mathematics education, classification refers to categorizing areas of knowledge within the mathematics curriculum. Strong internal classification means that clear boundaries between mathematical areas are maintained. Strong external classification indicates that few connections are made to other disciplines or everyday practice.

Framing draws on the nature of the control over the selection of the communication, its sequencing, its pacing, the evaluation criteria, and the hierarchical rules as the social base which makes access to knowledge possible (p. 27):

> I am going to look at the form of control which regulates and legitimizes communication in pedagogic relations: the nature of the talk and the kinds of spaces constructed. I shall use the concept of framing to analyse the different forms of legitimate communication realized in any pedagogic practice. (p. 26)

The concepts of classification and framing are useful to describe the kind of knowledge emphasised in alternative curriculum conceptions as well as the way in which this knowledge is assessed.

The following selection of alternative curriculum conceptions is made on the grounds that some of these conceptions were positioned as non-mainstream when they emerged, even though they might in the meantime have become mainstream in some places, while others still represent resistant construals of mathematical meaning. Examples and references are exemplary and not representative of the conceptions.

INQUIRY-BASED MATHEMATICS EDUCATION

Inquiry-based mathematics education starts from the assumption that young learners can be regarded as miniature scholar-specialists whose mathematical activity is not qualitatively different from that of a mathematician. Academic mathematics, often described as "the science of patterns" is mirrored in the mathematics classroom where students are engaged in discovering and exploring regularities, identifying relationships and applying their mathematical knowledge in new mathematical situations. The general idea behind has been summarised by Bruner (1960, p. 14): "Intellectual activity anywhere is the same, whether at the frontier of knowledge or in a third-grade classroom." More provocatively: "In teaching from kindergarten to graduate school, I have been amazed at the intellectual similarity of human beings at all ages, although children are perhaps more spontaneous, creative, and energetic than adults" (p. 40). This view has been criticized as "romantic" (Tanner & Tanner, 1980, p. 535) as it neglects the fundamental differences between the production of knowledge and its reproduction in schools, as witnessed in the following quote:

> The pedagogical tradition calls for transmittal of the 'given'. It is a tradition of the transmittal of certainty, not of doubt. But doubt is precisely the quality of the scholar. The scholar, taken as an intellectual, is one 'who makes the given problematic.' Our pedagogical tradition does not deal with problematic material. If we obey our tradition, we take what is problematic and make it into sets of certainties, which we then call upon

> the students to 'master'. In too many instances, our sets of certainties come dissociated from the fields of knowledge out of which they originally grew. In some cases, the contrast between the school subject and its underlying field of knowledge is ludicrous (Foshay, 1961, p. 32-33).

An inquiry-based mathematics curriculum can be understood as an attempt to overcome this pedagogical tradition by reconciling content and method: to find material that can be made problematic in order to develop knowledge both about how material is to be made problematic, and about the mathematical generalizations. Inquiry-based mathematics education is thus working in a combination of the inductive and the deductive mode.

The inductive part of inquiry-based mathematics education has been characterized by Dowling (2009) as involving skills and, moreover, tricks. By drawing on a commonly found example of school mathematical investigations he shows how the 'investigative' approach to school mathematics is actually introducing new areas of weakly classified strategies – skills, tricks – in a discipline that is apparently strongly classified. This might be misleading for some students, as the latter is generally preferred in mathematics. What makes a skill or a trick mathematically meaningful can tacitly be decided on the grounds of previously acquired mathematical knowledge. In most cases however, this decision is made through the mathematical authority of the teacher in the face of the standards of mathematical knowledge to be acquired – the abovementioned sets of certainties.

The construction of mathematical meaning through generalization of weakly classified activity and idiosyncratic notation of findings is a crucial component of inquiry-based mathematics instruction. For establishing generalized mathematical meanings when students are engaged in such activity in the mathematics classroom, two conditions (at least) have to be fulfilled. First, there have to be students who have already acquired the sufficient mathematical skills and tacit knowledge about what to look for and what to strive for when confronted with an open investigative mathematics problem; otherwise no valued generalization can be made at all. Second, only highly qualified teachers will be able to develop mathematical generalizations from the students' idiosyncratic and often not fully developed problem solutions. In many places of the world, these conditions are only partly met, and the inquiry-based curricular approach to mathematics education appears as a rather elitist option.

Inquiry-based mathematics education is problem centred and characterized by strong external classification. It has been legitimised as a contrast to the conception of the 'core curriculum':

> In the past, we saw a reality that the problems of life do not come in 'disciplined' packages. For example, a good many of the public problems we must deal with – housing, crime, transportation, and the like – go beyond the boundaries of any one discipline and must be studied on a multi-disciplinary basis. The most notable of the

> curriculum reforms intended to deal with this reality was the core curriculum, a problem-centered approach to learning, in which the mode of inquiry was to be dictated by the nature of the problem itself. We don't want our students ill-prepared for the practical problems of life, but there is another reality which we have tended to overlook. This second reality is that each of the disciplines, as they are organized, contains within its domain and methodology the best thought about reality in its own field. For example, one who knows how a chemist thinks can see more deeply into what is 'chemical' about an industrial problem than one who does not know how a chemist thinks (Foshay, 1961, pp. 33-34).

If argued like this, the conception resembles a tactical resistance position as it tries to point out why a conception with a focus on mathematical modes of inquiry is better suited for engaging with the same public reality as a curriculum stressing factual knowledge, that is, pursuing the same educational ends by different means. The details of this public reality given in the quote above – e.g. industry, economy, crime – and the claims suggest a prospective neo-conservative ideology.

In the course of the reforms and counter-reforms of the mathematics curriculum in Victoria, Autralia, the inquiry-based curriculum seemed to have a different ideological base, the main focus being on offering access for all through a conception that overcomes the levelled hierarchical nature of the traditional mathematics curriculum: "It had the potential to generalize the social reach of mathematics and to place school curricula on a new basis" (Teese, 2000, p. 169). However, the results on the "investigative project 1992" turned out to be disastrous for working class girls: 43% percent received the lowest possible grade or could not even master the minimum criteria for getting a grade (Teese, 2000, p. 171). But it was not the concern for exclusion of disadvantaged groups but the judgement by academic mathematicians that students would not learn enough and that the most talented students would be "punished" that marked the end of these reform efforts.

ETHNOMATHEMATICS

Ethnomathematics as a programme emerged in opposition to mainstream discourse in mathematics education. A Eurocentristic bias of mathematics education is most salient in curricula and textbooks developed in industrialised states and imported into former colonies. Vithal and Skovsmose (1997) interpret the emergence of ethnomathematics as a reaction to naïve modernisation theory and the cultural imperialism implied by it. By uncovering the cultural bias in historical accounts of mathematics and by documenting and analysing local mathematical practices, ethnomathematics set out to deconstruct mainstream discourse and offer new views on what counts as mathematics. Earlier work was often carried out from the perspective of cognitive anthropology, as witnessed in the reference list "Ethnomathematics: A Preliminary Bibliography" provided by Scott (1985) in the first Newsletter of the International Study Group on Ethnomathematics. The term „ethnomathematics" suggests a broad interpretation of both mathematics and “ethno”. The latter encompasses “identifiable cultural groups, such as national-tribal societies, labor groups, children of certain age brackets, professional classes, and so on" (D'Ambrosio, 1985, p. 45).

In line with this agenda, a base for the development of an ethnomathematical curriculum consists of uncovering and describing the mathematical concepts and procedures that are more or less implicit in practices of sub-ordinated and oppressed people and marginalised groups. This type of research can be described as ethnographic. Ethnographic work can be done from different positions, as for example from a dominant position of racial classification, such as the “White-on-black” research, witnessed in several studies carried out in South Africa (see Khuzwayo, 2005). Ethnographic ethnomathematical research finds itself in a difficult position because there remains the issue in whose terms the ethnomathematical practices are to be described. When incorporated into the curriculum, there is a related problem. For local practices that might be of interest to the students and are identified to contain some mathematics, there is a risk that incorporation into classroom discourse amounts to a recontextualisation for the purpose of exploitation in terms of traditional school mathematical topics. Fantinato (2008) points to the difficulties that might be faced at the level of classroom interaction:

> However, it is important to keep in mind that the mathematics teacher stands for the official mathematics image in the classroom. This person holds a knowledge considered superior to students daily knowledge due to its privileged social position in our society. This uneven status position interferes in the relations among different types of knowledges, which take part in the classroom cultural dynamics. When voicing students’ knowledges, the dialogic attitude of the teacher entails an awareness of the mythical status of his math and the depreciation of other math as an effort to reverse this difference (pp. 2-3).

Curriculum alternatives more closely linked to the original conception of ethnomathematics include the use of (historical) examples of culturally relevant practices as a springboard for developing mathematical notions (e.g. Jama Musse, 1999) or mathematical analysis of traditional artefacts, as for example decorative pattern designs (e.g. Gerdes, 1990).

The first alternative might assist in overcoming cultural alienation, but faces the same problématique as developing school mathematics on the basis of recontextuaised domestic practices. The recontextualisation of everyday domestic practices, which amounts to a collection of their traces in the form of contextualised tasks, generally has a tendency to amount to an implicit pedagogy with weakly classified content that disadvantages marginalised groups (e.g. Chouliaraki, 1996; Cooper & Dunne, 2000; Gellert & Jablonka, 2009; Hasan, 2001; Lubienski, 2000; Morais & Miranda, 1996). A similar pitfall is inherent in some versions of a mathematical modelling conception (see below).

The following task (see figure 2) provides an example of the second alternative, used in a teacher education course (Gerdes, 1999):

How many possible band patterns of the *sipatsi* type of given dimensions p and d do exist, whereby p denotes the period of the respective decorative motif and d its diagonal height? Figure 2 shows the possible patterns of dimensions 2x4. The images on the left side display the generating motives.

Figure 2. *Sipatsi* patterns and generating motives (Gerdes, 1998, p. 44)

As to the classificatory principle, this task (if posed without an initial introduction into the mathematical description of the pattern dimension) resembles an inquiry mathematics task. If already mathematised, the criteria for the inquiry become more explicit. It is then a mathematical task on a comparatively advanced level. The prospect of producing computer-generated imitations of pattern designs, based on such mathematical explorations, might for some amount to a disenchantment of the wisdom and skills of traditional crafts. Kaplan (2003), for example, presents a process for creating computer-generated Islamic Star Patterns on a web-page on which one can play around with a Taprats Applet. If the complexity of the mathematical algorithm provides an argument for the complexity of the skills involved in traditional crafts, then this value judgment privileges Western mathematics. The incorporation of local practices through their (school) mathematical recontextualization in order to ease access represents a tactical position.

D'Ambrosio (2007) locates ethnomathematics within a wider project of social change that points to the responsibility of mathematicians and mathematics educators in offering venues for Peace (p. 26). He proposes a curriculum that is conceptualised as a modern trivium, including Literacy, Matheracy and Technoracy, that aims at providing "in a critical way, the communicative, analytical and technological instruments necessary for life in the twenty-first century" (p. 28). Matheracy is connected to the capability of inferring, proposing hypotheses, and drawing conclusions, that is, to classical academic virtues associated with mathematical thinking access to which has been restricted to an elite. This conception indicates a critical stance towards teaching mathematical modelling and applied mathematics and also a departure from earlier envisaged forms of ethnomathematics. He also stresses that teaching “ethnomathematics of other cultures, for example, the mathematics of ancient Egypt, the mathematics of the Mayas, the mathematics of basket weavers of Mozambique, the mathematics of Jequitinhona ceramists, in Minas Gerais, Brazil, and so and so, it is not because it is important for children to learn any of these ethnomathematics” (p. 33). The main reasons for doing so include to “de-mystify a form of knowledge [mathematics] as being final, permanent, absolute, unique.” This is to overcome the damaging misperception “that those who perform well in mathematics are more intelligent, indeed ‘superior’ to others, and to illustrate intellectual achievement of various civilizations, cultures, peoples, professions, gender” (pp. 33-34).

Knijnik (2000) provides an example from her work with settlers of the Landless People’s Movement (MST) in Brazil where the practices of production and sale of melon crops were "naturally" changed through the process of confrontation and translation of different forms of knowledge. She argues that if the pedagogical process were limited to the recovery of the native knowledge, this would restrict access to useful knowledge and as a consequence reinforce social inequalities. Identifying practices, which could profitably be transformed by a mathematical recontextualisation remains a major and continuous task for overcoming problems of discontinuity and disjuncture between different mathematical practices and school mathematics. Which out-of-school practices are to be selected as representative of the students’ cultures remains a political issue.

MATHEMATICAL MODELLING

“Mathematical modelling” is a rather vaguely defined term for a curriculum conception that comprises many different classroom practices. Modelling conceptions can be distinguished by the strength of the internal and external classification of the respective knowledge domains as well as by the value attributed to the different knowledge domains in classroom modelling practice. A version of school mathematical modelling that stresses that the external classification remains as strong as in mainstream curriculum conceptions, is provided by Zbiek and Conner (2006):

> The primary goal of including mathematical modeling activities in students' mathematics experiences within our schools typically is to provide an alternative - and supposedly engaging - setting in which students learn mathematics without the primary goal of becoming proficient modelers. We refer to the mathematics to be learned in these classrooms as 'curricular mathematics' to emphasize that this mathematics is the mathematics valued in these schools and does not include mathematical modeling as an explicit area of study … we recognize that extensive student engagement in classroom

> modeling activities is essential in mathematics instruction only if modeling provides our students with significant opportunities to develop deeper and stronger understanding of curricular mathematics. (pp. 89-90)

Such a version is reflected in the approach of the *Realistic Mathematics Education*, where models are seen as vehicles to support 'progressive mathematization' (Treffers & Goffree, 1985), as van den Heuvel-Panhuizen (2003) points to:

> Within RME, models are seen as representations of problem situations, which necessarily reflect essential aspects of mathematical concepts and structures that are relevant for the problem situation, but that can have various manifestations. (p. 13)

A version of school mathematical modelling in which the external classification is weakened considerably constructs modelling as new (but vague) content. This version is sometimes referred to as *emergent modelling*:

> This second perspective [RME is the first one], which we favour, does not view applications and modelling primarily as a means of achieving some other mathematical learning end, although at times this is valuable additional benefit. Rather this view is motivated by the desire to develop skills appropriate to obtaining a mathematically productive outcome for a problem with genuine real-world connections ... Here the solution to a problem must take seriously the context outside the mathematics classroom, within which the problem is located, in evaluating its appropriateness and value … While the above approaches differ in the emphases they afford modelling in terms of its contribution to student learning, they generally agree that modelling involves some total process that encompasses formulation, solution, interpretation, and evaluation as essential components. (Galbraith, Stillman & Brown, 2006, p. 237)

Given the diversity of agendas and examples, the unifying principle of the modelling discourse in mathematics education can be seen in the differences constructed in relation to mainstream school mathematics without applications or in the differences to other forms of insertions of non-mathematical practices (such as word problems). There are some characteristic knowledge claims reflected in mathematical modelling: an ontological realism that acknowledges an independently existing reality that is the object of knowledge and the properties of which provide objective limits to how we can know it. However, these are seen as open to revision: a fallibility principle is acknowledged. This is a difference in comparison to school mathematics with a focus on both procedures and algorithms as well as on mathematical relationships and proof.

Julie (2002) summarises the differences as follows: a change of criteria towards acceptance of different non-equivalent answers, unrestricted time, acceptance of the provisional status of the outcome, and presentation in a format chosen by the students. The social base changes from individualistic to working in collaborative teams. Texts are not objects to be mastered, but used as resources. In a classroom such a shift would indicate a shift in the authority relationship between teachers, texts and students. Underpinned by learning theories that stress the agency of the learners, school mathematical modelling activities are also intended to encourage students to communicate their own ideas and to scrutinise the ideas of others (English, 2006).

The situation chosen as a starting point for modelling might be selected because of mathematical reasons or because of social reasons (Julie, 2002). In the first case the context is arbitrary and the mathematical concepts, procedures etc. are those specified in the curriculum; in the latter, the context is given (or selected by the students) and the mathematics is arbitrary. But any mathematics curriculum in the end prescribes a set of valued forms of mathematical knowledge. It also specifies the contexts in which this knowledge has to be applied, but only implicitly (Dowling, 1998), if it is not a critical mathematical literacy curriculum (see below).

Different versions of mathematical modelling in the classroom imply variations of classification. If the situation chosen to be modelled is selected because of mathematical reasons, the external classification might still be strong whereas the internal classification becomes weaker as a mix of different mathematical topics and procedures is legitimate. If, in contrast, the situation chosen for a modelling activity is selected because of social reasons, then the external (as well as the internal) classification might be rather weak.

It can be argued that the two modes of school mathematical modelling, which are different in terms of the relationships between the knowledge domains involved, relate to differential access to mathematical knowledge. If modelling is not subordinated to the principles of school mathematics, then the question arises to the principles of which discourse it refers. As mathematical modelling is not a uniform practice, but a set of interrelated activities in different domains, there is no set of uniform criteria for performing mathematical modelling. Consequently, the discourse of school mathematical modelling, if it is not subordinated to accessing mathematical knowledge, leaves an open space for promoting different agendas, such as developing human capital by channelling students into an engineering career pipeline, expressing and rethinking cultural identity, educating critical consumers or promoting social change.

When mathematical modelling is seen as a way to promote ‘curricular mathematics’ (cf., Zbiek & Conner, 2006), then it hardly can be regarded as a resistance position towards mainstream. In fact, the implementation of conceptions like RME demonstrates how well established the focus on mathematical models and progressive mathematization already is. In contrast, the focus on skills appropriate to obtaining a mathematically productive outcome for a problem with genuine real-world connections defines a resistant position to a curriculum structured by mathematical domains. This resistance is tactical when it (1) aims to complement rather than to overcome mainstream mathematical education practice and, simultaneously, (2) does not question the mathematical structure of the mathematics curriculum by imposing an order that takes the out-of-school problems to be modelled into account. There is no serious intention to deconstruct or subvert the mainstream mathematics curriculum.

The conceptualisation of modelling as a set of generic competencies that could be provided by mathematics education seemingly transcends the difficulties arising from cultural differences and economic inequalities because the activity of constructing mathematical models, through which these competencies are to be developed, is not seen as culture-bound and value-driven. Such a conception masks the fact that the construction of mathematical models depends on the perception of what the problem to be solved with the help of mathematics consists of and what counts as a solution. But depending on the subject position of the "modeller" in a practice, there are different models of the same problematic situation:

> For example, if the problem of a bank employee, who has to advise a client (aided by a software package), is the comparison of financing offers for a mortgage, for the manager of the bank this is a problem of profitability, and for the customer it is one of planning her personal finances. (Jablonka, 2007, p. 193)

This is not to suggest that mathematical models should be scrutinised exclusively in terms of the values connected with the underlying interests. But the discourse of mathematical modelling as providing individuals with generic competencies that enable them to become adaptive to the conditions of technological development, to overcome the limitations of specialised knowledge, to gain competitive advantage on the labour market and become critical consumers and democratic citizens, is mythologizing mathematical modelling because the causality between participating in mathematical modelling activities and the diverse educational potentials attributed to this experience is mythical. The myth embodies the claim of the ethical neutrality of mathematical modelling practices.

The popularity of modelling can be explained by the fact that it achieves a fictitious marriage between two strands of critique of a strongly classified mathematics curriculum. Such critique is on the one hand an outcome of an attack on a neo-conservative defence of canons of disciplinary specialised knowledge, which (at least historically) comes together with the reproduction of inequality of access to such specialised knowledge. On the other hand, the critique of strongly classified curricular knowledge comes from the side of those called "technical instrumentalists" by Moore and Young (2001) who advertise economic goals. Preparation for the "knowledge-based economy" is a major concern. Moore and Young observe that the scope of instrumentalism has extended from vocational training to general education under the guise of promoting the employability of all students. There is a danger that the myth of the neutrality of generic modelling skills discards the tension between neo-liberal ideology with a focus on human capital preparation and a conception of education for social change.

CRITICAL MATHEMATICS LITERACY

Critical mathematics literacy aims at identifying and analysing critical features of social realities and at contributing to the development of social justice. One strategy of pursuing these goals is sensitizing students to social problems and helping them to articulate their interests as citizens. These social problems include the particular hidden injustice students face because of their race, social class, cultural origin etc. A second strategy is directed towards the analysis of mathematics itself because of its function as part of technology, including social technology. A third strategy is concerned with the mostly discriminative practice of mathematics education itself: How does mathematics education reproduce or reinforce social inequalities? (For a discussion of different strategies see Jablonka, 2003; Skovsmose & Nielsen, 1996).

Published experience with critical mathematical literacy in (most often) secondary schooling has mainly focused on two features: On the one hand, critical mathematics literacy is strongly connected to the construction and use of data and statistical diagrams. Examples from the previous MES conference include a discussion of a "race & recess chart" (Powell & Brantlinger, 2008) and "supposedly random traffic stops" (Gutstein, 2008). On the other hand, critical mathematics literacy is directed at the official use and interpretation of socially relevant data in form of quantitative arguments. Examples from the previous MES conference include analysing the "discounting of Iraqi deaths" (Greer, 2008) and the ways numerical information can be presented in order to augment or reduce its comprehensibility (Frankenstein, 2008).

The subversive rather than oppositional deconstructive resistant position of critical mathematical literacy is apparent as critical mathematical literacy explicitly aims at demolishing the correlation between social class, race and academic achievement by demystifying the "naturalness" of this relation (Martin, 2010). It is subversive because it aims at eroding and undermining hidden principles of school mathematics instruction and social stratification. These principles serve to perpetuate the hierarchical structure of society and societies. Critical mathematics literacy scrutinizes the mechanisms by which race and social class structures are reinforced.

The examples from previous MES conferences point to a common problématique: Critical mathematics literacy intends to be simultaneously a pedagogy of access and a pedagogy of dissent (McLaren, 1997; Morrell, 2007). This includes access to higher education, to rewarding professional employment and to civic life particularly for marginalized populations, though access might also be understood in terms of personal and social emancipation. However, advanced mathematical literacy does not automatically translate into power, and it does not translate into power equally for everyone who possesses it. In a pedagogy of dissent students develop a language of critique of systems of social reproduction and of inequitable power relations in society. They critically analyse the role that mathematics and mathematics education play in legitimating and perpetuating these conditions. Is this *simultaneously* possible?

Eric Gutstein has worked in a setting characterised by a separation of pedagogies of access and dissent:

> The class I refer to here has intermittently completed social justice mathematics projects since the week they started school. Although we have only spent perhaps 15% of our total time, on three or four projects a year, they have been evidently been sufficient meaningful and memorable to students that none reported it as unusual to hear that particular framing of mathematics. (Gutstein, 2008, p.15)

Though the intention is to reverse the proportion of "standards-based mathematics" and "social justice mathematics" (p. 18), what students learn and develop in both cases is structurally different. The discourse of a mathematics pedagogy of dissent is necessarily weakly classified. Within the Freirean students' generative themes, which are the focus within a pedagogy of dissent, the mathematics might play a crucial role yet not in a very simple and visible way (Jablonka & Gellert, 2007). For the prominent case of "random traffic stops", Dowling (2010, p. 4) argues:

> One might suppose that police are often not able to estimate the ethnicity of a driver until after they have made the stop. This would seem to suggest that, if there is a correlation between ethnicity and the probability of being stopped, then we might look for the presence of intervening variables for an explanation; a correlation between ethnicity and relative poverty and the association of the latter with the use of elderly and poorly maintained vehicles having visible defects, for example.

The traffic stop problem is apparently much more difficult and not easy to grasp mathematically. As a consequence, the relation between the weakly classified social justice issue and the strongly classified relevant mathematical knowledge is obscured. In fact, students may get used to handle a piece of legitimate school mathematics – expected value in random experiments –, but this at the risk of misjudging the relationship between mathematics, social structure and social technology. In terms of a pedagogy of dissent, the dissent is only constructed towards a critique of societal power relations, but not towards the role mathematics plays in formatting these power relations. In terms of access, access is given to applications of the mathematical concept of expected value, though this in a context only marginally relevant for professional promotion or academic success.

TOWARDS A "RADICAL CONSERVATIVE PEDAGOGY" IN MATHEMATICS EDUCATION?

According to Bernstein's (1990) characterisation of a conservative pedagogy, a traditional strongly classified mathematics curriculum that establishes an explicit hierarchical relationship between teacher and students and includes explicit sequencing rules as well as explicit specific criteria is an example of a realisation of such pedagogy. It is underpinned by a theory of instruction that focuses on intra-individual changes in terms of individual's competences or performances rather than on changes in the relation between social groups. Consequently it does neither highlight shared competencies nor the sharing of experiences.

Inquiry-based mathematics and mathematical modelling do not solve the problématique of providing alternative conceptions that relate to social justice for a socially marginalized student population. As Bernstein argues, this is particularly due to focussing on the mathematical capability and development of the individual student. He also deconstructs “progressive” pedagogy because of its differential effect stemming from the implicitness of the recontextualisation principle, which makes invisible the classificatory principle of the knowledge to be acquired.

Ethnomathematics and critical mathematics literacy explicitly focus on groups of marginalized students and look for the empowerment of social groups. However, the tension between, on the one hand, the students’ generative themes or cultural heritage, and, on the other hand, an institutionally valorised mathematics can only partly be mitigated by these curriculum conceptions.

A further position of subversive resistance has been theoretically argued by Martin (1993) and outlined by Bourne (2004). Both draw on Bernstein (1990) who sketches “an apparently conservative pedagogy yet to be realized“ (p. 214). In a radical realisation of a conservative pedagogy the emphasis is on “the *explicit* effective ordering of the discourse to be acquired” (p. 214). As Bourne (2004) demonstrates in a case of literacy teaching, by establishing an overtly highly regulated discourse the teacher successfully inducts students to valued and powerful new discursive opportunities and, at the same time, coordinates the everyday discourse that students are familiar with. By managing changes in place, pace and deportment, the teacher makes the strong classification of school and community knowledge visible. As Bourne (p. 65) remarks: “Visible pedagogy is explicit in acknowledging responsibility for taking up a position of authority; invisible pedagogy (whether progressive or ‘emancipatory’) simply masks the inescapable authority of the teacher.” Bernstein (1990) characterises the logic of a radical conservative pedagogy as a logic of transmission in which the teacher is explicitly responsible for the ordering of the discourse. This is contrary to a logic of acquisition, on which progressive pedagogies as well as revolutionary pedagogies (Freire, 1971) are based. A radical realisation of a conservative pedagogy highlights shared competences and stresses that the acquirer is *active* in decoding and regulating a necessarily recontextualised practice. In a radical conservative pedagogy the students collectively access and participate in academically valued social practices and get introduced and used to the discourses by which academically valued practices are constituted. This would lead to acquire insights into the discourses that are the focus of critique and has the potential to reconcile a pedagogy of dissent with a pedagogy of access.

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RESPONSE TO JABLONKA AND GELLERT: IDEOLOGICAL ROOTS AND UNCONTROLLED FLOWERING OF ALTERNATIVE CURRICULUM CONCEPTIONS

Kate Le Roux
University of Cape Town

INTRODUCTION

My response to this paper focuses on Jablonka and Gellert's adapted model of *dimensions of position,* which they use to classify alternative curriculum conceptions according to their position in relation to mainstream conceptions. I divide this response into two parts. Firstly, I use the model and Jablonka and Gellert's presentation of the alternative curriculum conceptions as a lens to view the school mathematics curriculum in South Africa. I argue for the usefulness of this model as a theoretical tool for viewing curriculum. Secondly, on a more theoretical level, I problematise certain aspects of the model of dimensions of position and draw on work in critical literacy education to suggest an alternative.

A REMARKABLE "FLOWERING OF ALTERNATIVE CURRICULUM CONCEPTIONS": THE SOUTH AFRICAN CONTEXT

In responding to Jablonka and Gellert's paper, I have chosen to discuss the South African experience of school curriculum reform in the period since our first democratic elections in 1994 [1]. This is the context with which I am most familiar, but it is also a particularly rich context for reflecting on the usefulness of Jablonka and Gellert's work.

Harley and Wedekind (2004, p.195) argue that the reform of the school curriculum in South Africa "was of a scale arguably unparalleled in the history of curriculum change". This very rapid and large-scale change took place in a socio-political space characterised by an urgent need to redress the inequalities of the past and to cater for the pressing social, economic and scientific needs of the new nation. The product was the result of contestation and ideological struggle. Although the process involved the reform of both the primary and secondary school curricula as a whole, I focus on the mathematics curriculum in the final three years of schooling [2].

The previous school mathematics curriculum at this level could be described using Jablonka and Gellert's model of dimensions of position as *mainstream* in the sense that it displays, in Bernstein's (1996) terms, strong internal and external classification. Before the post-1994 curriculum reform process, studying mathematics in the final three years of schooling was not compulsory. However, the new school curriculum requires all students to take some form of mathematics up to the final year of schooling. On the one hand we have the subject *Mathematics* which is aimed at students who would traditionally have studied mathematics at this level, but with an explicit intention to widen access to students who may not have studied the subject in the past. Mathematics is required for access to the study of science and engineering in higher education. On the other hand we have a new school subject, *Mathematical Literacy*. I will discuss this subject briefly at the end of this section.

The *dimensions of position* as a model for viewing the school subject Mathematics

In their paper, Jablonka and Gellert focus on one possible alternative curriculum conception at a time; such is the nature of a written paper. However using their model of dimensions of position to revisit the official curriculum document for the South African school subject Mathematics, I was struck by the remarkable "flowering of alternative curriculum conceptions" that this model suggests. The definition of the subject Mathematics in the official curriculum documentation (Department of Education, 2003a) is as follows:

> Mathematics enables creative and logical reasoning about problems in the physical and social world and in the context of Mathematics itself. It is a distinctly human activity practised by all cultures. Knowledge in the mathematical sciences is constructed through the establishment of descriptive, numerical and symbolic relationships. Mathematics is based on observing patterns; with rigorous logical thinking, this leads to theories of abstract relations. Mathematical problem solving enables us to understand the world and make use of that understanding in our daily lives. Mathematics is developed and contested over time through both language and symbols by social interaction and is thus open to challenge (p.9).

Using Jablonka and Gellert's description of the various alternative curriculum conceptions, it is possible to identify in this text traces of inquiry-based and problem-based conceptions, both the approaches to mathematical modelling described in the paper, a form of ethnomathematics in which culturally relevant examples are included, and an element of critical mathematics literacy aimed at using mathematics as a tool for social justice.

On the surface, this remarkable hybrid of alternative curriculum conceptions may appear to adopt a mix of what Jablonka and Gellert refer to as *tactical* and *deconstructive* positions in relation to the old mainstream curriculum. However, a reading of the rest of the official curriculum documents and my knowledge of how this curriculum is playing out in practice leads me to argue that this curriculum largely assumes a tactical position, with the aim of providing access to a privileged mathematical discourse. Certainly, the national examination papers for the final year of schooling, the production of which was eagerly awaited in 2008 as an exemplar of how the curriculum documents should be interpreted, suggests that this is the case. My reading is that these examination papers do not differ much from those used when the old, mainstream curriculum for mathematics was in use.

Discussing the different alternative curriculum conceptions, Jablonka and Gellert refer to the possible consequences for different social groups, for example, they refer to the unintended consequences for working class girls of the implementation of an inquiry-based curriculum in Victoria, Australia. My question in the South African context is, what are the consequences for different groups of the hybrid Mathematics curriculum that I have described? Our new school curriculum was phased in gradually from 1997 to 2008. While more children are gaining physical access to schools, this does not necessarily translate into meaningful and equitable access in terms of success and progression through the schooling system. Epistemological access to Mathematics continues to be defined by the overlapping constructs of race, language and social class (Organisation for Economic Co-operation and Development, 2008; Reddy, 2006).

My concern as a mathematics education researcher in South Africa is that, while we may draw on research that is conducted elsewhere on how different curriculum conceptions affect different social groups, we do not as yet have a nuanced view of the effects for different groups in our context, a context in which there are interesting class shifts in education (see for example, Soudien, 2004). In the absence of contributions of this nature to the debate around the new curriculum, there is a risk that decisions about the future of this school reform will be based on other competing interests, as was the case in the Victoria, Australia example quoted by Jablonka and Gellert.

The *dimensions of position* as a model for viewing the school subject Mathematical Literacy

In closing this discussion of Jablonka and Gellert's model of "dimensions of position" in relation to the South African school curriculum, I refer briefly to the curriculum for the new subject, Mathematical Literacy. Students who choose not to take the subject Mathematics in the final three years of study are required to do Mathematical Literacy (this is the subject that Martin (2010, p.54) refers to in his plenary address). Mathematical Literacy has an explicit focus on using quantitative knowledge and skills in the service of everyday and workplace practice and democratic citizenship (Department of Education, 2003b). I would argue that, in its conceptualisation in the written curriculum document, this subject has the potential to adopt an oppositional position in relation to the mainstream mathematics curriculum. However, since this subject seems to hold little symbolic power in practice, this potential is not being realized.

A CRITIQUE OF THE MODEL OF *DIMENSIONS OF POSITION* AND INITIAL IDEAS ON AN ALTERNATIVE

In the previous section I have argued for the usefulness of the model of dimensions of position as a lens for viewing curriculum reform in my own context of South Africa. Yet the model needs to be problematised in two respects.

Firstly, this model has a binary construction in the sense that a curriculum is either *mainstream* or *resistant*, a *resistant curriculum* is either *tactical* or *deconstructive* etc. Such a construction encourages an "either-or" conceptualisation of curriculum models. This "either-or" conceptualisation can be seen in debates around the school mathematics curriculum in South Africa and public calls to replace the new reform curriculum with the old mainstream curriculum. In addition, with reference to what Jablonka and Gellert refer to as a *pedagogy of dissent* and a *pedagogy of access*, it may be that the use of a binary model, in fact, leads us to construct these pedagogies as being in tension.

Secondly, and related to my first criticism of the model, the alternative curriculum conceptions are explicitly defined in terms of the position they assume in relation to the mainstream. This can be seen in the naming of the alternative curriculum conceptions which sets up these conceptions as in opposition to the *mainstream*, that is, as *non-mainstream*. A curriculum which assumes an *oppositional* position can only be oppositional in relation to another curriculum. A curriculum that is in a *deconstructive* position only exists in relation to the curriculum that it deconstructs. Furthermore, the term *deconstructive* does not suggest the possibility of agency in the reconstruction of an alternative curriculum.

Having problematised the model of dimensions of position, I would also like to be constructive myself by suggesting an alternative model for conceptualising curriculum. For this, I draw on Janks' (2004, 2010) work in critical literacy education. In presenting these ideas I note that they were developed for the field of critical literacy and that the possibilities in terms of school mathematics curriculum are not yet fully formed in my own mind. Nonetheless, I do feel that this theory has the potential to address the binary "either-or" that I have identified as weakness of Jablonka and Gellert's model, and could possibly go some way to addressing the perceived tension between a *pedagogy of dissent* and a *pedagogy of access*.

Janks (2004, 2010) has developed a model for a critical literacy curriculum in South Africa, a context in which the English language has symbolic power, yet many other languages are in use. In presenting her model I refer to *literacy*, but would like us to reflect on what this conceptualisation might mean for school mathematics curriculum, and in particular to how the ideas may be useful in attending to the differential privileging of different discourses within school mathematics.

Janks (2004) argues that the aim of critical literacy is to teach students "to understand and manage the relationship between language and power" (p.4). She draws on the work of Simon (1992) to propose that,

> … critical literacy has to take seriously the ways in which meaning systems are implicated in reproducing domination and it has to provide access to dominant languages, literacies and genres while simultaneously using diversity as a productive resource for designing social futures and for changing the horizon of possibility. (quoted in Janks, 2010, p.27)

Janks (2010) argues that a critical literacy curriculum requires four orientations towards the relationship between language and power, all of which are mentioned in the above quote. Firstly, a theory of *domination* or an understanding of how some discourses are privileged over others. Secondly, *access* to these privileged discourses (in describing this orientation, Janks refers to Bernstein's explicit pedagogy, which is proposed by Jablonka and Gellert in their conclusion). Thirdly, *diversity* and an acknowledgement that the differences between discourses can be productive (in the sense that they provide the space for interrogating taken-for-granted ways of acting in particular discourses). Lastly, *design* or reconstruction and the possibility of using resources to reconstruct and transform.

Janks (2010) argues that all four orientations are important to a critical literacy curriculum. Certainly, all of the alternative curriculum conceptions explored by Jablonka and Gellert seem to display one or more of these orientations. Yet where I feel that Janks' work is potentially productive for conceptualising curriculum in school mathematics is her argument that all four orientations are not only necessary, but also "crucially interdependent" (Janks, 2010, p.26). She provides a matrix which can be used to consider the consequences of having some and not other orientations in a critical literacy curriculum, and I quote selected examples below (see Janks, 2010, p.26 for the full list).

Access without domination	Access without a theory of domination leads to the naturalisation of powerful discourses without an understanding of how these powerful forms came to be powerful.
Access without diversity	This fails to recognise that difference fundamentally affects pathways to access and involves issues of history, identity and value.
Diversity without access	Diversity without access to powerful discourses ghettoises students
Design without domination	Design, without an understanding of how dominant discourses/practices perpetuate themselves, runs the risk of an unconscious reproduction of these forms.
Diversity without design	Diversity provides the means, the ideas, the alternative perspectives for reconstruction and transformation. Without design, the potential that diversity offers is not realised.

As noted, the ideas I have put forward here as an alternative to the model of dimensions of position are new to me, and certainly require more thought in terms of how they can be applied in mathematics education. Nonetheless, my sense is that Janks' (2004, 2010) work provides a way (a) to attend to the differential privileging of different discourses within school mathematics, and (b) to conceptualise the relationship between a pedagogy of dissent and a pedagogy of access not as a tension, but rather as a productive interdependency.

NOTES:

1. I use the elections in 1994 as the starting point for the reform process. However, during the early 1990s curriculum reform formed part of broader education policy debates within the ruling apartheid government and the recently unbanned African National Congress (Harley & Wedekind, 2004; Vithal & Volmink, 2005).

2. I use the term *mathematics* when referring generally to the concept of school mathematics. This can be distinguished from my use of *Mathematics* with a capital "M" when I discuss the school subject Mathematics for Grades 10 to 12 in the South African school curriculum.

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NOT-SO-STRANGE BEDFELLOWS: RACIAL PROJECTS AND THE MATHEMATICS EDUCATION ENTERPRISE[1]

Danny Bernard Martin

University of Illinois at Chicago

Critical scholars have argued the dangers of mathematics education becoming increasingly influenced by and aligned with neoliberal and neoconservative market-focused projects. While powerful, there are often peculiar responses to issues of race and racism in these analyses. These responses are characterized by what I see as an unfortunate backgrounding of these issues, on one hand, or a conceptually flawed foregrounding, on the other. Viewing mathematics education as an instantiation of white institutional space partly accounts for these responses. Also, because mathematics education research and policy can be deeply implicated in the production and reproduction of racial meanings, hierarchies, and identities, the enterprise of mathematics education is, itself, a type of racial project.

INTRODUCTION

In her analysis of the increased corporate influence on the affairs of Canadian universities, sociologist Janice Newson (1998) suggested that these external pressures have caused a fundamental shift in the way that the university functions, including matters of day-to-day operations, the production of knowledge, and the ability of the University to serve the broader public interest. According to Newson, there has been a shift in the university from a *social project* to a *market force*. She argues:

> these changes in university practices constitute a potentially, if not realized, significant transformation in the *raison d'être* of the university: from existing in the world as a publicly funded institution oriented toward creating and disseminating knowledge as a public resource—social knowledge—into an institution which, although continuing to be supported by public funds, is increasingly oriented toward a privatized conception of knowledge—market knowledge.

To support her argument, Newson examined the expansion of the post-World War II University in terms of its initial, and evolving, relationships to democratic and economic projects:

> the expansion of higher education in the late 1950s and 1960s was justified primarily in terms of two societal needs. On the one hand, massive financial investment of public funds was premised on the need for a highly skilled and well-educated work force to contribute to the economic health of the country. On the other hand, it was also emphasized that universities should play a democratizing role, not only by promoting opportunities for social, political, and economic mobility in society at large but also by providing an example of a public institution whose structures and practices conformed to

democratic principles of governance. In fact, some commentators of that period refer to the university as a democratic social movement…. the university of the 1960s and 1970s could be viewed as having staged a contest between the two objectives of serving the needs of the economy, on one hand, and contributing to the political project of advancing democratic sensibilities and practices on the other. If anything, the democratic project of the university held a degree of pre-eminence over the purely economic project, at least in the interplay of political and cultural struggles that were taking place on campus…. And I am referring to related struggles concerning the independence of the academy from ‘external’ social, political, and economic pressures. Expressions of these struggles were reflected, for example, in… the insistence that the university must exist at arm’s length from the ‘military-industrial-complex,’ which is to also say that the university should be wary of being tied to the market…. However, the salience in university affairs of the democratizing project and its apparent equality with the economic project of the university no longer describes the political and cultural situation of and within the academy. Something has changed…in the relative balance between these two projects.

Despite this shift from the democratic project to the market project, Newson made the keen observation that the relationship between the University and external, corporate influences is not a one-way relationship; the University has not been pulled unwillingly in the market direction. Newson pointed out the limitations of the one-way perspective by noting:

Such a representation of the university's relation to its ‘outside’ is both disempowering and mystifying. It is disempowering because, in a practical sense, adapting to external pressures rarely offers much if any room for challenging the pressures themselves. It is mystifying because it camouflages the extent to which the university itself is implicated in the very social, political, and economic forces to which it then ‘must’ accommodate.

WHAT KIND OF PROJECT IS MATHEMATICS EDUCATION?

Cued by Newson’s analysis, and realizing that the word ‘university’ could appropriately be replaced by ‘mathematics education’ in the excerpts presented above, I raise two questions relative to the enterprise in which we do our work. The first question asks, *what kind of project is mathematics education*? The first question, of course, necessitates the second question, which asks, *whose interests are served by this project*?

To be sure, my two questions are not new. Over the last two decades, a number of critical scholars have offered their own assessments of mathematics education (e.g., Apple, 1992, 2000; D’Ambrosio, 1985; Dowling, 1998; Ernest, 1991; Gutstein, 2008a, 2008b; Lerman, 2000; Powell & Frankenstein, 1997; Skovsmose, 1994; Skovsmose & Valero, 2001, 2002; Tate & Rousseau, 2002; Valero & Zevenbergen, 2004). Nielsen (2003), for example, in his analysis of university mathematics education, also invoked the idea of competing projects—he highlighted critical and conservative projects in his analysis—and pointed out that such projects are all involved in a fundamental struggle to:

> dominate society and to that end give different interpretations of what is important in society. They all try to make their descriptions look neutral and objective—to look like the truth about our society…. In the words of discourse theory, these efforts are called *hegemonic projects* …. [the] point is that these struggles also extend to the arena of university mathematics education, and that this arena is both used as a resource and as a stake in the struggles. (p. 35)

Moreover, a number of scholars have also engaged in critical analyses of mathematics education in relation to market forces, market-driven goals, and increased globalization (e.g., Apple, 1992, 2000; Atweh & Clarkson, 2001; Atweh, Calabrese Barton, Borba, Gough, Keitel, Vistro-Yu, & Vithal, 2007; Gutstein, 2008a, 2008b). These scholars have provided compelling evidence that mathematics education and mathematics knowledge have increasingly been put in service to neoliberal and neoconservative projects and agendas. This has manifested itself, for example, in the prioritizing of mathematics knowledge in the development of military and national security technology as well as the commodification of learners as potential workers in these sectors (e.g. Domestic Policy Council, 2006; National Science Board, 2003; U.S. Department of Education, 1997, 2008).[2] Recently, we have witnessed the use of mathematics knowledge via financial engineering (i.e. mortgage-based securities, collateralized debt obligations, credit default swaps) to manipulate financial markets and the flow of global capital in ways that have benefited a few and devastated the lives of millions of others (e.g., Case, 2009).

Many of the scholars referenced above have extended their own analyses of the first question that I raised above to suggest the kind of project that mathematics education *should* be (e.g., D'Ambrosio, 1985; Frankenstein, 1995; Malloy, 2002; Skovsmose, 1994; Sriraman, 2008; Skovsmose & Valero, 2001, 2002; Tate & Rousseau, 2002). For example, based on his work with Latino youth in Chicago, Gutstein (2003, 2006, 2007) has argued that mathematics education should be a social justice project that resists neoliberal and neoconservative agendas and empowers students to understand and confront class-based oppressions created by differentials in wealth and power. According to Gutstein, students should do this by developing and integrating what he calls classical, critical, and community knowledge.[3]

As an outgrowth of his long history of activism in the American South, Bob Moses works with Black adolescents in the United States in the context of the Algebra Project (Moses & Cobb, 2001). Moses has argued for conceptualizing mathematics education as a civil rights project. Other scholars have made arguments supporting mathematics education as a broader democratic project (e.g., Malloy, 2002; Skovsmose, 1998; Skovsmose & Valero, 2002; Tate & Rousseau, 2002).

It is clear, depending on how the aims and goals of mathematics education are conceptualized and framed, that the enterprise simultaneously represents and serves a host of competing projects, each of which calls for a preferred structuring of mathematics teaching, learning, curriculum, assessment, research, policy, and reform.

Retreating from Race?

In this paper, I would like to argue that although much of the research cited above has linked mathematics education to globalization and market-focused neoconservative and neoliberal projects—either as complicit in or as resistant to their oppressions—there are often peculiar responses to issues of race and racism in these critical analyses.[4] These responses are characterized by what I have come to see as an unfortunate backgrounding of these issues, on one hand, or a conceptually flawed foregrounding, on the other.

These responses are particularly true for analyses of mathematics education in the United States despite the salience of race and racism in almost every aspect of American life. These responses are even more curious given the scholarly attention that race and racism have received outside of mathematics education. This research suggest that racism is a *global* phenomenon, with geopolitical variations being found, for example, in South Africa, Brazil, India, Australia, New Zealand, and throughout the European Union (e.g., Macedo & Gounari, 2006; Winant, 2004). This ubiquity suggests that the meanings for race and racial categories are politically contested and re-created in any given sociohistorical and sociopolitical context through a process called *racial formation*[5] (Omi & Winant, 1994).

My comments are not meant to suggest that there are no references to race or discussions of the plights of various racial groups in mathematics education. This is clearly not the case, as reflected in numerous studies and reports that refer, for example, to "underrepresented" and "minority" students and so-called racial achievement gaps. However, racism, especially white supremacy (and colonialism), are rarely centered in the analyses, rarely theorized for conceptual clarity (see Martin, 2009a for a more detailed critique), and rarely theorized in relation to the market-driven goals of globalization and the neoliberal and neoconservative projects that mathematics education is said to increasingly serve.

In his discussion of mathematics education reform, markets, and educational inequality, Michael Apple (2000) only briefly mentioned deep structural racism and other processes of racialization (Miles, 1988) in his analysis. It was through a single footnote that he directed readers elsewhere for a more thorough discussion of the racial state. In a much earlier paper devoted to analyzing standards-based reform, Apple (1992) did entertain race, class, and gender intersections in his analysis. However, the word 'racism' appears nowhere in the text of his arguments. The text, *Internationalism and Globalisation in Mathematics and Science Education* (Atweh, Calabrese-Barton, Borba, Gough, Keitel, Vistro-Yu, & Vithal, 2008), contains twenty-seven chapters spread over more than 500 pages. A word search of the index revealed zero instances of the words *race* and *racism*.

Moreover, few of the most visible and most referenced research and policy documents in mainstream mathematics education address race as more than a categorical variable in reference to differences in achievement (e.g., National

Research Counci1, 2001; U.S. Department of Education, 2008). The *Handbook of Research on Mathematics Teaching and Learning* (Grouws, 1992) and *Second Handbook of Research on Mathematics Teaching and Learning* (Lester, 2007) confine their discussions to a single chapter in the former case and a just few chapters in the latter, largely disconnected from the other chapters focused on teaching, learning, curriculum, and assessment. I would argue, based on my own work (Martin 2006, 2007, 2009a, 2009b), that mathematics teaching and learning, for example, can be conceptualized as *racialized forms of experience* and that this is true for all students. By this, I mean that the meanings for race in a given sociohistorical and sociopolitical context are highly salient in structuring the ways that mathematical experiences and opportunities unfold and just as salient in shaping beliefs about who is perceived to be competent in mathematics.

Without discounting the great importance of the work, even the math-literacy-as-a-civil-right perspective of Moses is tempered by the fact that mathematics literacy is deemed the key to participation in the very same technology-based opportunity structure critiqued by many critical mathematics educators. Moses' message about Black participation in this structure, as well as the prioritizing of Algebra in the mathematics curriculum and experiences of students, also shares much with the rhetoric found *in Final Report of the National Mathematics Panel* (U.S. Department of Education, 2008), which was convened by former Republican President Bush George Bush. The lack of a deeper racial analysis limits discussion of the fact that the access granted to Blacks and envisioned by Moses and others, rather than being democratic in nature, is likely to be selective and partial, in protection of white male privilege. My own view is that even if larger numbers of Black workers were to find themselves in the mathematics and engineering pipeline, they would only be absorbed into the workforce up to the point of not threatening the status of white workers. Examination of the public debate reveals the angst, resistance, and cries of racial preference that are often associated with the introduction of just one qualified Black person into a given context even when that context has been previously dominated by Whites (e.g., Berry & Bonilla-Silva, 2008; Bonilla-Silva, 2001, 2003).

Moreover, Moses' consideration of racism faced by Blacks in the United States is primarily historical, not accounting for the contemporary evolving, politically expedient forms of everyday, institutional, and structural racism in the post-Civil Rights era, including neoliberal racism and neoconservative color-blind racism. Nor does Moses interrogate the increasingly nationalist, nativist, and racist tones associated with reform rhetoric linking mathematics education, national security, and U.S. international competitiveness (e.g., Domestic Policy Council, 2006; U.S. Department of Mathematics Education, 2008). Analyses linking mathematics education to democracy and citizenship, in some idealized forms, would be strengthened by pointing out the contradictions with democracy and citizenship as they are actually experienced in fundamentally racist societies (Du Bois, 1998/1935). Much in the same way that Critical Race Theory scholar Tara Yosso (2005) challenged Bourdieu's notions of cultural capital by asking, *whose culture has capital*? it is important to ask, *whose democracy*? and *democracy for whom*? Similarly, profound analyses of democracy and freedom cannot take place without equally profound analyses of racism and slavery (Patterson, 1991; Winant, 2004). As noted by Winant (2004):

> Racism has always been an issue of democracy, an indicator—the most reliable one we have— of democracy's limitations. Just as race and racism were central o the creation of modernity, the development of capitalism, and the elaboration of Enlightenment culture, they were also key to the evolution of modern forms of democracy…. It is not often recognized that democracy in the modern era was conceptualized as the opposite of slavery, that citizenship and social identity were for many centuries conceived in racial terms… (p. 111)

Furthermore, an explicitly racialized characterization of globalization by critical mathematics educators would seem to be warranted given sociological analyses, which suggest that:

> Globalization is a re-racialization of the world. What have come to be called "North-South" issues are also deeply racial issues. The disparities in status and "life chances" between the world's rich and poor regions, between the (largely white and wealthy) global North and the (largely dark-skinned and poor) global South have always possessed a racial character…. globalization is a racialized social structure…. It is a system of transnational social stratification under which corporations and states based in the global North dominate the global South…. [through] a worldwide pattern of employment discrimination, violence, morbidity, impoverishment, pollution, and unequal exchange that shares a great deal with its colonial antecedents. This global system of stratification correlates very well with racial criteria: the darker your skin is, the less you earn; the shorter your life span, the poorer your health and nutrition, the less education you can get. (Winant, 2004, p. 131-134)

Equally true, an explicitly racialized characterization of neoliberal policies and practices would acknowledge that these policies and practices are:

> predicated on the wholesale exclusion of most of the world population from partaking equitably in the world's resources, including education and health care, accelerating a downward shift toward unconscionable poverty and human misery. This form of blatant exclusion cannot be viewed as anything other than poster racism. The permanent status of underdevelopment affects mostly countries the dominant racialized discourse characterizes as '"nonwhite" and "other." In addition to the characterization of otherness in order to devalue other human beings, neoliberal policies implement racist practices by largely excluding millions of people from equal participation in the economic world (dis)order it imposes. (Macedo & Gounari, 2006, p. 12)

Turning the Gaze Inward

My comments thus far have focused on what I perceive to be limitations in analyzing the racialized nature of the external forces to which mathematics education must respond. In my view, there is even less evidence in the scholarly record—in both U.S. and international contexts—that critical scholars, particularly critical white scholars, have turned their analyses inward to examine the internal structure of the mathematics education to expose its own contributions, enactments, and validations of racial hierarchies and inequalities (e.g., Anderson, 1990; Powell, 2002). With respect to this last point, I raise the additional question, *how do race and racism structure the very nature of the mathematics education enterprise*?

On one hand, there is the possibility that mathematics education is a purely anti-racist domain, free from racial contestation, stratification, and hierarchies, and fundamentally different in character than all other racialized societal contexts. Under this assumption, there is no need to turn the gaze inward since the norms, ideologies, and institutional practices and arrangements are, in the best sense of the word, democratic in nature and all the actors in the domain exist free from oppression and are uninvolved in the racial oppression of others.

On the other hand, I suggest that a race-critical structural analysis would show, for example, that configurations of power and privilege in the domain are not simply the result of democratic principles, practices, norms, and access. In terms of knowledge production, a great deal of mainstream mathematics education research and policy, particularly in the United States, can be deeply implicated in the production and reproduction of racial meanings, disparities, hierarchies, and identities. For example, not only do scholarly interpretations of children's mathematical behaviors serve to inform societal beliefs about race, racial categories, abilities, and competence, I would argue that race-based societal beliefs about children from various social groups also serve to inform the ways that mathematics education research, policy, and practice are conceptualized and configured in relation to these children (Martin, 2009a, 2009b).

As I have noted elsewhere (Martin, 2009a), despite mathematics education research and policy feeding the public's common sense understandings of racial hierarchy and difference, *race* still remains under-theorized in mathematics education. While race is characterized in the sociological and critical theory literatures as sociohistorically and politically constructed with structural expressions, most studies of differential outcomes in mathematics education begin and end their analyses with static racial categories and group labels for the sole purpose of disaggregating data. One consequence is a widely accepted, and largely uncontested, *racial hierarchy of mathematical ability* that, in the U.S. context, locates children who are identified as Black, Latino, and Native American at the bottom and children who are identified as Asian and White at the top. Beliefs in so-called *racial* achievement gaps and subsequent attempts to close such gaps by raising Black, Latino, and Native American children up to the level of white and Asian children help to perpetuate this hierarchy. Rather than challenging and deconstructing this hierarchy, many math educators take it as the natural starting point in their analyses. Disparities in achievement and persistence are then inadequately framed as reflecting race effects rather than as consequences of the *racialized* nature of students' mathematical experiences.

A cursory examination of the ways Black children in the U.S have been researched and represented in mainstream mathematics education research and policy further shows very clearly how mathematics education research is implicated in the production and reproduction of racial meanings, disparities, hierarchies, and identities (see Martin, 2009a, 2009b, 2009c for more details).

The dominant story line, or masternarrative, about Black children in both research and policy contexts is one that normalizes failure, ignores success, and uses white children's mathematical behavior and performance as the benchmark for competence and ability. This masternarrative has helped to support negative social constructions of these children. Mathematics education policy reports dating back 25 years have explicitly labeled Black children as mathematically illiterate (e.g., National Research Council, 1989). More recently, Black 12th graders have been told, in a very public fashion, that they are only as skilled and demonstrate math abilities at the level of white 8th graders (Thernstrom & Thernstrom, 1997). After their comprehensive review of over 16,000 studies, the members of the National Mathematics Advisory Panel reduced their research recommendation for Black children to issues of motivation, task engagement, and self-efficacy. These areas are important but they focus attention on Black children as though they are unmotivated, inclined to disengagement, and lacking in agency. Institutional and structural barriers inside and outside of school, including racism, that affect student mathematics achievement, engagement, and motivation received no attention in the report (Martin, 2008). Resistance and disengagement among some students may, in fact, be rational responses to oppressive and racist schooling practices.

In other research contexts, it has been claimed that poor (Black) children enter school with only *pre*-mathematical knowledge and lack the ability to mathematize their experiences, engage in abstraction and elaboration, and use mathematical ideas and symbols to create models of their everyday lives (e.g., Clements & Sarama, 2007). Left unanswered is whether researchers who report these findings understand, even partially, the "everyday lives" of Black children. As I state elsewhere (Martin, 2009b):

> Because the tasks, assessments, and standards for competence used to draw these conclusions are typically not normed on African American children's cultural and life experiences, once could argue that the … preferred ways of abstracting, representing, and elaboration called for in these studies and reports are based on the white, middle-class and upper-class children…. very little consideration is given to exploring patterns in the ways that low-income and African American children do engage in abstraction, representation, and elaboration to determine if these ways are mediated by their cultural experiences in out-of-school settings and whether preferred ways of engaging in these processes serve useful functions relative to those experiences. (p. 15)

In the U.S., it is only in the last decade or so that studies of mathematics learning and participation among Black children has focused on these children *as* Black children, situating their learning and participatory experiences within the network of meanings for race and the consequences of their racial group membership.

White Institutional Space

I contend that it is only within certain kinds of ideological and material spaces—contexts that sociologists have called *white institutional spaces*—that the peculiar responses to race described above and widespread beliefs in so-called *racial* achievement gaps can co-exist. The term *white institutional space* comes from the work of sociologists Joe Feagin (1996) and Wendy Moore, who, in her book *Reproducing Racism: White Space, Elite Law Schools, and Racial Inequality* (2008), examined the white space of law schools and how the ideologies and practices in these schools serve to privilege white perspectives, white ideological frames, white power, and white dominance all the while purporting to represent law as neutral and objective.

White institutional spaces are characterized by (1) numerical domination by whites and the exclusion of people of color from positions of power in institutional contexts, (2) the development of a white frame that organizes the logic of the institution or discipline, (3) the historical construction of curricular models based upon the thinking of white elites, and (4) the assertion of knowledge production as neutral and impartial unconnected to power relations.

In Martin (2008), I provide a more detailed discussion of how I believe mainstream mathematics education research and policy contexts in the U.S. represent instantiations of white institutional space. But I will say there that a structural analysis reveals that the pervasiveness whiteness—represented numerically, ideologically, epistemologically, and in material power—which characterizes U.S. mainstream mathematics education research and policy contexts bears a strong family resemblance to the manifestations of whiteness found in other societal contexts (Martin, 2008, 2009a). In Martin (2009b), I distinguish *mainstream* mathematics education research and policy as that which has relied on traditional theories and models of teaching and learning (e.g., information processing, constructivism, situated cognition) and research approaches (race-neutral analyses, race-comparative analyses) developed primarily by white researchers and policy makers to normalize the mathematical behavior of white children. Simultaneous to their use for normalization and generalization, these models have generated and validated certain conventional wisdoms about Black children and mathematics.

My characterization is not meant to imply that all mainstream mathematics education research and policy is detrimental to Black children. Meaningful and insightful research findings have sometimes led to the creation and implementation of policies that have had beneficial effects for these children. Nor do I suggest that white scholars have not, and cannot, work in the best interest of children who are not white.

However, the numerical dominance of white scholars, whatever their ideological and epistemological orientations, may insure that the perspectives of white scholars become the *only* perspectives that matter. In addition, it is quite possible that the critical stance taken by many liberal white scholars escapes self-interrogation. As noted by Macedo and Gounari (2006):

> many white liberals (and some black liberals as well) fail to understand how they can embody white supremacist values and beliefs, even thought they may not embrace racism as prejudice or domination (especially domination that involves coercive control). They cannot recognize how their actions support and affirm the very structure of racist domination and oppression they profess to wish to see eradicated…. By not understanding their complicity with white supremacist ideology, many white liberals reproduce a colonialist and assimilationist value system that gives rise to a form of tokenism parading under the rubric of diversity. (p. 32)

These sentiments were echoed by Liz Appel (2003) in her focused critique of liberal white participants in the movement against the prison industrial complex:

> many well-intentioned white folks wish to incorporate an anti-racist approach in their work. Seeking a quick resolve, the problem of racism is often superficially addressed, however. Focusing on tangible and visible solutions, they tokenize individual people of color... in an attempt to demonstrate the "diverse" nature of the struggle and those that make up the fight. This is not to say that every attempt to incorporate people of color is inherently racist and self-serving…. [But does] not the fact that whites are able to select people of color for inclusion… reaffirm our power and privilege? (p. 84)

In a field that increasingly purports to be committed to equity for all children, I am left to wonder why there are no explicit discussions of the pervasive whiteness in mathematics education research and policy contexts or of the fact that the norms and values of these white institutional spaces are increasingly being applied to populations of other people's children. Why are there no discussions of how mainstream mathematics education continues to socially blacken some children by producing research that implies their inferiority? Is it that the power and privilege characterizing white institutional spaces are so strong that they lead us to believe this state of affairs is normal and acceptable?

Why am I levelling these critiques of mathematics education and what is the relationship of my critique to the three questions that I have raised thus far in this paper: *What kind of project is mathematics education? Whose interests are served by this project? How do race and racism structure the very nature of the mathematics education enterprise?* My intent is not to implicate particular individuals. The individual psychology of this scholar or that one is not my concern. Rather, my goal is to examine, from a structural point of view, how mathematics education as an enterprise contributes to larger racial dynamics in society, locally and with respect to global racial hegemony.

In short, I wish to argue that the enterprise of mathematics education, examined in relation to well-known hegemonic projects; and examined for the ways in which it backgrounds and foregrounds race and racism can be conceptualized as a type *racial project*.

What is a Racial Project?

According to the sociological literature, a racial project is "simultaneously an interpretation, representation, or explanation of racial dynamics and an effort to reorganize or redistribute resources along particular racial lines. Racial projects connect what race *means* in a particular discursive practice and the ways in which both social structures and everyday experiences are racially *organized*, based upon that meaning" (Omi & Winant, 1994, p. 56). Moreover, there are competing racial projects such that the "discord and conflict among various racial projects construct the racial order visible at any given moment; over time they produce a deeply racialized society, as preexisting themes are reworked and social institutions reformed time and time again" (Winant, 2004, p. 53). As noted by Macedo and Gounari (2006), not all racial projects are racist. Those that are can be are characterized by their attempts to create or reproduce hierarchal social structures based on essentialized racial categories (p. 45). Sociologists have characterized several white racial projects that have figured prominently in the evolution of white supremacy and white identity in the U.S. These include the *far right*, *new right*, *neoconservative*, *liberal*, *neoliberal*, and *new abolitionist* racial projects. Defining characteristics of each are summarized below (Giroux, 2006; Omi & Winant, 1994; Winant, 2004):

> *Far right racial project*: Belief in an ineluctable, unalterable racialized difference between white and nonwhites. This belief is biologically grounded. Fascist elements maintain an insurrectionary posture vis-à-vis the state and openly admire Nazi race thinking, advocate racial genocide, and advocate establishment of an all-white North American nation. (Winant, 2004)
>
> *New right racial project*: Has its origins in resistance to the black movement of 1950s and 1960s. Has employed anticommunism, racism, southern chauvinism, states' rights doctrines, agrarian populism, nativism, and America First isolationism. Argues that white supremacy is not an excrescence on the democratic "American creed" but a fundamental component of U.S. society. Revives anti-immigration hysteria, targeting Latinos. Associates whiteness with capitalist virtues. Presents itself as the tribune of disenfranchised whites. Rather than espouse racism and white supremacy, espouses familiar "code-word" phenomenon to manipulate white fear. Accepts a measure of non-white social and political participation. Political success depends on its ability to interpret white identity in positive political terms. (Winant, 2004)
>
> *Neoconservative racial project*: Seeks to preserve white advantages through denial of racial difference. Racial difference is something to be overcome, a blight on the core U.S. values of universalism and individualism. Casts doubt on the tractability of racial

> equality, arguing that the state cannot ameliorate poverty through social policy but in fact only exacerbates it. Argues that every invocation of racial significance manifests 'racial thinking' and is thus suspect amounting to a defense of the racial status quo. Defends the political and cultural canons of Western culture. Argues for 'color-blind' racial politics. Has served to organize and rationalize white working-class and minority middle-class resentments. Seeks to label Asian Americans and some Latinos as 'model minorities' and extend 'honorary white' status to distinguish them from the black underclass and to simultaneously exempt them from affirmative action. (Winant, 2004)
>
> *Neoliberal racial project*: Rather than operating as a discourse of denial regarding how power and politics promote racial discrimination and exclusion, neoliberal racism is about the privatization of racial discourse. Asserts the insignificance of race as a category at odds with an individualistic embrace of formal legal rights. Dismisses the concept of institutional racism or maintains that it has no merit. Asserts that since American society is now a meritocracy, government should be race neutral, affirmative action programs dismantled, civil rights laws discarded, and the welfare state be eliminated. (Giroux, 2006)

Moreover, consider this partial accounting of how the *neoliberal* racial project evolved in the 1990s in the context of American politics:

> In order to win the [1992] election and reinvigorate the once-powerful Democratic coalition, Bill Clinton believed he needed to attract white working class voters—the "Reagan Democrats." His appeal was based on lessons learned from the right, lessons about race. Pragmatic liberals in the Democratic camp proposed a more activist social policy emphasizing greater state investment in job creation, education, and infrastructure development. But they conspicuously avoided discussing racial matters such as residential segregation or discrimination…. Thus the surprising shift in U.S. racial politics was not… the Republican analysis which placed blame on the racially defined minority poor and the welfare policies which has supposedly taught them irresponsibility and dependency. The "surprise" was rather the Democratic retreat from race and the party's limited but real adoption of Republican racial politics, with their support for "universalism" and their rejection of "race-specific" policies.… This developing neoliberal project seeks to rearticulate the neoconservative and new right racial projects of the Reagan-Bush years in a centrist framework of moderate redistribution and cultural universalism. Neoliberals deliberately try to avoid racial themes, both because they fear the divisiveness and polarization which characterized the racial reaction, and because they mistrust the "identity politics" whose origins lie in the 1960s…. Unlike the neoconservative project… racial neoliberalism… does not claim to be colorblind; indeed it argues that any effort to reduce overall inequality in employment, income, education, health care access, etc., will disproportionately benefit those concentrated at the bottom of the socioeconomic ladder, where racial discrimination has its most damaging effects. In its *signifying* or representational dimension, the neoliberal project avoids (as far as possible) framing issues or identities racially. Neoliberals argue that addressing social policy or political discourse overtly to matters of race simply serves to distract, or even hinder, the kinds of reforms which could most directly benefit racially defined minorities. To focus too much attention on

> race tends to fuel demagogy and separatism, and this exacerbates the very difficulties which much racial discourse has ostensibly been intended to solve. To speak of race is to enter a terrain where racism is hard to avoid. Better to address racism by ignoring race, at least publicly (Omi & Winant, 1994, pp. 146-148)

Now, consider the proposition that contemporary mathematics education reforms have been aligned with, and can be implicated in, New Right, neoconservative, liberal, and neoliberal racial projects that continue to shape larger racial dynamics. How might one shed light on the racialized character of mathematics education reforms? Internationally, there are some interesting cases ripe for further critical analysis, including the introduction of Mathematical Literacy, *vis à vis* Mathematics, in post-apartheid South Africa (Julie, 2006) and the policies put in place to assist Ethiopian Jews in Israel (Mulat & Arcavi, 2008).

In the U.S. context, consider three major math reform efforts covering the last 50 years: the new math movement ushered in by U.S. reaction to the launching of Sputnik on October 4, 1957; the *Mathematics for All* movement of the late 1980s and 1990s; and the formation of the National Mathematics Advisory Panel by former Republican President Bush.

New Math in the Civil Rights Era

Although Cold War politics are put at the forefront of explaining the U.S. reaction to Sputnik, a number of race-based considerations are in order. First, the push to educate a generation of students who would help protect the U.S. from the Soviet intellectual threat did not include Blacks. Just over a decade earlier, African Americans were largely excluded from taking advantage of the GI Bill that helped many white males enroll in colleges and universities.

It is true that in 1954, just three years prior to Sputnik, the U.S. Supreme Court announced its decision in the case of *Brown v. Board of Education of Topeka*, ruling that separate educational facilities are inherently unequal, thus overturning its previous ruling in the 1896 case of *Plessey v. Ferguson* and paving the way for school integration. However, as pointed out by Derrick Bell (1980), it was *interest convergence* rather than moral compunction that explained this landmark decision. Interest convergence suggests that "gains for blacks coincide with white self interest and materialize at times when elite groups need a breakthrough for African Americans usually for the sake of world appearances or the imperatives of international competition" (Delgado, 2002, p. 371). As explained by Delgado (2002):

> the NAACP Legal Defense Fund had been litigating school funding and desegregation cases for decades throughout the South, generally losing or winning, at most narrow victories. Then, in 1954, the skies opened—the Court declared, for the first time in a school desegregation case, that separate was no longer equal. Why then? Bell pointed out that the country had just celebrated the end of a bloody world war against Germany and Japan, during which many black men and women had served gallantly. Having risked their lives for the cause of freedom, they were unlikely to return meekly to the former

> regime of menial jobs and segregated facilities. For the first time in decades, the prospect of serious racial unrest loomed. At the same time, the United States was in the early stages of a Cold War against the forces of monolithic, atheist communism, competing for the loyalties of the uncommitted Third World, most of which was black, brown, or Asian. Incidents like the murder of Emmett Till and the death sentence of handyman Jimmy Wilson splashed across the pages of the world news, reflecting poorly on America. The balance of interests shifted; elite whites now saw a powerful reason to advance blacks' cause. For Bell, the Brown decision came about when it did, not because of altruism or advancing notions of social morality. Rather, elite whites on the Supreme Court, in the State Department, and in other circles of power simply perceived that America's self-interest lay in publicly supporting blacks so as to gain an edge in the Cold War with Russia. (p. 372)

Of course, the desegregation ruling did not end racism or quell the racial climate. In August of 1955, fourteen-year-old Emmett Till was kidnapped, beaten, shot, and dumped in the Tallahatchie River allegedly for whistling at a white woman. In December of 1955 Rosa Parks, a Montgomery, Alabama seamstress, refused to give up her seat on the bus to a white passenger and is subsequently arrested and fined, giving rise to the Montgomery bus boycotts. And, on September 4, 1957, just one month before Sputnik, the Governor of Arkansas deployed National Guard troops to block nine Black children from integrating Central High School. It was not until 1964 that the 24th amendment abolished the poll tax and the Civil Rights Act increased Black access to voting.

An extended chronology of Civil Rights history in the post-Sputnik era, culminating in the death of Dr. Martin Luther King Jr. in 1968, shows that the new math reform movement was not an anti-racist vessel in the sea of racial discord characterizing that time. In fact, with its emphasis on the "best and the brightest," it was just another, although short-lived, mechanism for maintaining white privilege. If the nation was not willing to integrate Black children into their schools and other public institutions, it was certainly not willing to integrate them into the mathematics education reforms of the day.

Mathematics for All?

More recently, *Mathematics for All*, as one of most egalitarian movements in the field, seeks to reorganize and redistribute access and opportunity in mathematics (National Council of Teachers of Mathematics, 1989, 2000; RAND Mathematics Study Panel, 2003). In my view, it does so, and does so seductively, by appealing to liberal, neoliberal, and neoconservative racial projects.

In the liberal racial project, there is an underlying appeal to white middle- and upper-class consciousness to convince them that others must now share in the opportunities that they have long enjoyed; that is "their needs—for more and better jobs, access to education and health care…can be linked to those of the minority poor if the 'wedge issue' of race can be blunted" (Winant, 2004, p. 60). However, as noted by

Schoenfeld and Pearson (in press), the appeal to white consciousness early in the *Mathematics for All* was sometimes met by resistance, revealing the racial dynamics at play in public and political negotiations of democratic access. This was particularly true in California, where a number of other public initiatives invoked similar, race-based reactions:

> Simply put, the anti-reform forces in reading and mathematics grew strong at a time of the resurgence of the right wing in California politics. San Diego politician Pete Wilson had ridden "wedge politics" (appeals to the fears of the White middle-class voting majority regarding the rising populations and rights of minorities) to become mayor of San Diego. Wilson was a strong supporter of Proposition 187, a 1994 ballot initiative designed to deny illegal immigrants social services, healthcare, and public education. (The proposition won at the ballot box, with non-Latino Whites being the largest voting block in favor; it was later declared unconstitutional.) In 1996, California voters passed Proposition 209, which abolished affirmative action programs in public institutions (Office of Legislative Analysis, State of California, 1996). In 1998, voters passed Proposition 227, which "requires all public school instruction be conducted in English" (California Voter's Guide, 1998) and severely curtailed bilingual education. The [NCTM] *Standards* represented a clear tilt toward the "democratic access" view of education. Advocates of reform believed in "mathematics for all"—in particular that it was possible to achieve excellence *and* equity, without sacrificing one for the other. There are many who believe that the goals of equity and excellence are in tension, and that making mathematics accessible to many more students necessarily entails "dumbing down" the mathematics. If one believes this, then two consequences of the democratization of mathematics as proposed by reform are (a) a weakening of the mathematical preparation of our best students, and a concomitant weakening of the nation's base of mathematically and scientifically prepared elite and (b) a different demographic mix of those who are considered to be prepared for entry into elite institutions and professions. (p. 573)

Mathematics for All also aligns well with the neoliberal and neoconservative racial projects in that universal programs (i.e. Algebra for All) that supposedly work for all students are promoted in lieu of group-specific efforts and objectives (Winant, 2004). Merit and individual effort will determine success and failure and race-conscious interventions are frowned upon. Even the *Equity Principle* of the most recent NCTM standards document (NCTM, 2000) contains no explicit references to African American, Latino, Native American, or poor students. It is in these ways that the subtext of *Mathematics for All* rhetoric is about assimilation. In classical assimilation theory, assimilation is defined as "the decline, and at its endpoint the disappearance, of an ethnic/racial distinction and the cultural and social differences that express it" (Alba & Nee, 1997, p. 863).

Viewed more critically, *Mathematics for All* is also about nationalism because it appeals to U.S. international competitiveness and calls for strengthening of the scientific and technical (i.e. national defense) workforce in relation to real and perceived foreign threats (Gutstein, 2008a, 2008b; Martin, 2008). Like assimilation, nationalism seeks to erase meaningful cultural differences among social groups and to silence internal racial identity politics in favor of collectivism. Moreover, some scholars suggest that racism and nationalism are intimately linked (e.g., Mosse, 1995). According to Miles and Brown (2003), "racism is implicitly defined as an excess of nationalism, therefore dependent on nationalism for existence-as-such" (p. 10).

So, while *Mathematics for All* in the U.S. has an equity-oriented veneer, it would appear that there are other ideologies at play that are not based exclusively on moral and humanistic concern for those who are marginalized in mathematics. In my view, it is inconceivable that the real goal of *Mathematics for All* is to contribute to the reconstruction of the opportunity structure in such a way that we move from an arrangement that has long served white males and the wealthy to an arrangement where Blacks, Latinos, and Native Americans share equitably in material benefits and power.[6] Very rarely, if ever, has it materialized that these groups have collectively enjoyed access to the best learning opportunities, best teachers, best curriculum, most funding, and greatest levels of social and economic reward. In view of these limitations, efforts like *Mathematics for All*, must be analyzed for their deeper racial content, racial signification, and hidden agendas despite their rhetoric about equity and access (Martin, 2003).

Mathematics Education and Nationalism

Similarly, a critical analysis of the *Final Report of the National Mathematics Advisory Panel* (U.S. Department of Education, 2008) report reveals how it, too, contributes to racial projects. The fact that former President Bush was able to successfully extend new right and neoconservative politics—characterized by nationalism, nativisim, security concerns, and anti-Muslim sentiments—into mathematics education with the formation of the National Mathematics Advisory Panel further reveals the connection between mathematics education reform and the larger racial politics of the day (Martin, in press).

In this report, the learning of mathematics in U.S. schools is linked directly to the preservation of national security. The third paragraph of the Panel's *Executive Summary* is very clear in making this link:

> Much of the commentary on mathematics and science in the United States focuses on national economic competitiveness and the economic well-being of citizens and enterprises. There is reason enough for concern about these matters, but it is yet more fundamental to recognize that the safety of the nation and the quality of life—not just the prosperity of the nation—are at issue. (p. xi)

Two key questions can be asked about the excerpt presented above. First, what threats to national security and quality of life in the United States is the report referring? Second, how is the identification of these threats related to "the organizing principles that generate, shape, and sustain white supremacy designed to exclude other human beings by virtue of their race, language, culture, and ethnicity so that they can be exploited" (Macedo & Gounari, 2006, p. 3)? Macedo and Gounari's (2006) cogent analysis of the racialized nature of the "threat" is particularly helpful:

> The dichotomy [between "us" and "them"] has been astutely used by the Bush administration to conduct its war on terror and expand its imperial ambitions unimpeded by a domestic opposition. By constructing a terrorist enemy that encompassed all Muslims (a "group" that amounts roughly to 1.2 billion people worldwide and comprises numerous countries, societies, traditions, languages and lived experiences), the Bush administration, aided by a compliant media, exacerbated the racism present in U.S. society so that all Muslims became suspected terrorists. And it legitimized racist treatment of Muslims, as when "Muslim-looking" individuals are deplaned by major airlines because white folks fear of flying in their company. However, the same racial profiling was never applied to white males resembling Timothy McVeigh after the terrorist bombing of the federal building in Oklahoma City, where more than one hundred fifty people died, including women and children. The us-versus-them dichotomy … produces the "reality" of what it means to have different races." (p. 5)

Moreover, while *Mathematics for All* may promote assimilation and nationalism in more subtle ways, the discourse associated with the National Math Panel's final report is much more explicit. A word search of the document produced 21 instances of the word *American* (with repetition of some sentences), 11 instances (with repetition of some sentences) of the word *citizen*, only two non-repeated references to the word *minority*, and only one mention of the word *resident*. Moreover, while a search produced 98 instances of the word *quality* (i.e. excellence), the document contains zero instances of the word *equity*. Such references, according to van Dijk (2000), contribute to the discursive construction of the Other that is needed in nationalist and racist ideologies. This implicit distinction between citizens and non-citizen, American and non-American, despite the rhetoric about "all our people" is more clearly understood in the context of anti-immigrant policies and sentiments flowing from former President Bush's Republican Administration. This includes, as an example, the passing of the Secure Fence Act of 2006 (Pub.L. 109-367), which:

> allows for over 700 miles (1,100 km) of double-reinforced fence to be built along the border with Mexico, across cities and deserts alike, in the U.S. states of California, Arizona, New Mexico, and Texas in areas that have experienced illegal drug trafficking and illegal immigration. It authorizes the installation of more lighting, vehicle barriers, and border checkpoints, while putting in place more advanced equipment like sensors, cameras, satellites and unmanned aerial vehicles in an attempt to watch and control illegal immigration into the United States.[7]

In his official statement to the press following passage of the bill, former President Bush stated the following:

> This bill will help protect the American people. This bill will make our borders more secure…. We must face the reality that millions of illegal immigrants are already here. They should not be given an automatic path to citizenship; that is amnesty. I oppose amnesty.[8]

To the degree that mathematics education reform policies and rhetoric embrace and appropriate these nationalist sentiments, it is insufficient to focus on the market-focused goals of neoliberal and neoconservative projects. Simply put, race and racism matter.

CONCLUSION

Earlier in this paper, I raised three questions: *What kind of project is mathematics education? Whose interests are served by this project?* and *How do race and racism structure the very nature of the mathematics education enterprise?* A deeper structural analysis of the domain shows that it is an instantiation of white institutional space. An examination of both mainstream and critical research shows that there are often unfortunate backgroundings or conceptually flawed foregroundings of race and racism. An examination of mathematics education reforms shows that they have been aligned not only with neoliberal and neoconservative market-focused projects but these reforms have also been aligned with new right, liberal, neoconservative, and neoconservative racial projects. As a result, I claim that the enterprise of mathematics education is deeply implicated in the production and reproduction of racial meanings, hierarchies, and identities, making it a type of racial project.

NOTES

[1] This paper draws heavily from Martin (2008, 2009b, 2009c, in press).
[2] Efforts to shift the structure, ideology, and content of mathematics education toward or away from one project or another have not happened without contestation on many different levels (Schoenfeld, 2004; Schoenfeld & Pearson, in press).
[3] Similarly, Ernest (2002) has suggested empowerment for learners along three dimensions: epistemological, social, and mathematical.
[4] Research by senior scholars William Tate and Arthur Powell are notable exceptions along with the work of a number of emerging African American scholars and white scholars like Stinson and Jackson. See Martin (2009b) for recent work by these scholars.
[5] Omi & Winant (1994, p. 55) define *racial formation* as the sociohistorical process by which racial categories are created, inhabited, transformed, and destroyed.
[6] I am not suggesting that one form of racial hierarchy be substituted for another.
[7] Retrieved December 1, 2009: http://en.wikipedia.org/wiki/Secure_Fence_Act_of_2006)
[8] Retrieved on December 1, 2009 from http://georgewbush-whitehouse.archives.gov/news/releases/2006/10/20061026.html

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REACTION TO: NOT-SO-STRANGE BEDFELLOWS: RACIAL PROJECTS AND THE MATHEMATICS EDUCATION ENTERPRISE

Tamsin Meaney

Charles Sturt University

Danny Martin's paper raises a number of points about the way that mathematics education constructs identity, including the sense of belonging to specific groups. He states that mathematics education is a type of racial project because it reinforces racial inequalities in the way it is researched, taught and learnt. As a whitefella or Päkeha who works mostly with Indigenous communities, it made me think about how ideas around 'racial projects' connect with the theoretical constructs that my colleagues and I work with.

In order to identify points of connection between our respective research orientations, it seems valuable to discuss what is meant by race and racism which are key terms in Danny's paper. Differences in varieties of English and conceptions of the key issues complicate any discussion and thus they need to be clarified.

There are many definitions of race and racism. My computer dictionary defined 'race' as:

> • each of the major divisions of humankind, having distinct physical characteristics: *people of all races, colors, and creeds.*
>
> • a group of people sharing the same culture, history, language, etc.; an ethnic group: *we Scots were a bloodthirsty race then.*
>
> • the fact or condition of belonging to such a division or group; the qualities or characteristics associated with this: *people of mixed race.*
>
> • a group or set of people or things with a common feature or features: *some male firefighters still regarded women as **a race apart**.*

Racism is defined as:

> • the belief that all members of each race possess characteristics or abilities specific to that race, esp. so as to distinguish it as inferior or superior to another race or races.
>
> • prejudice, discrimination, or antagonism directed against someone of a different race based on such a belief *: a program to combat racism.*

As an Australian who has worked in many places with different groups of people, I tend to talk about issues of ethnicity, inclusion and exclusion, social justice and equity. Generally, I would not talk about race nor about racism. Why is this the case? In Australian English the most common definitions, such as the ones above, about race and racism would be those that are based on physical characteristics that are genetically determined. In the same way that twenty years ago we stopped talking about sex differences and began using the term gender differences, I choose to use other terms than race, such as ethnicity. For me, ethnicity like gender emphasises that these differences are socially constructed rather than genetically determined.

So what does it mean to talk about ethnicity as a social construction? With my Mäori colleagues Colleen McMurchy-Pilkington and Tony Trinick, we conducted a literature review on mathematics education research done in regard to Indigenous students in Australasia across the period on 2004-2007. In order to conceptualise how Indigenous learners were constructed as knowers, doers and learners of mathematics by the stories told about them in research, we developed Figure 1. There are four main groups who tell stories about Indigenous learners: societies; communities including parents; teachers; and children.

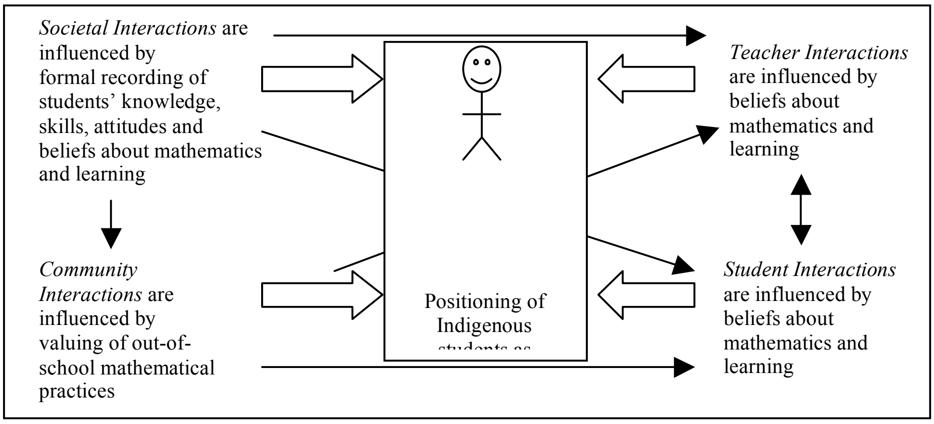

Figure 1: The influence of different kinds of interactions on the positioning of Indigenous students (from Meaney, McMurchy-Pilkington & Trinick, 2008)

Although stories about the poor performance of Indigenous learners were frequent, especially from societal interactions with Indigenous students and their communities, they were not the only stories. Irrespective of the stories told in research, what seemed important was the way that the stories that came from different interactions were woven together to influence each other and to contribute to how Indigenous students were perceived and perceived themselves as knowers, doers and learners of mathematics. No individual story was solely responsible for the labelling of some students as disadvantaged because of their Indigenous ethnicity. It was the pattern made from the weaving together of the research stories that continually reinforced the perception that Indigenous students were disadvantaged. However, there was little research into the affect that this labelling had on the type of research that was being undertaken. This would correspond with Danny Martin's complaints about the lack of discussion about the how children get blackened by research stories. In our literature review, more recent research focusing on how students' indigenous background contributes to the successful learning of mathematics alters the pattern of the cloth being woven and thus how Indigenous students are perceived and perceive themselves.

Although it is clear in Danny's argument that he sees race as something that is socially constructed, he has chosen for good reasons, connected to the US situation to continue to use the terms 'race' and 'racism'. Certainly, there is no clear equivalent to racism when one uses the term ethnicity. This makes it very difficult to discuss what racism might look like in mathematics education. However, it is unlikely that just looking for blatant acts of racism, such as suggested in the definition of racism above, will contribute in a meaningful way to an understanding of why these students become labelled as disadvantaged.

Let me give an example. In recent research that I have undertaken with Uenuku Fairhall and Tony Trinick in New Zealand, we have been looking at the bilingual exams that are provided to students who complete high school in a Mäori immersion school. Mäori set up schools that taught in the Mäori language because the language was dying and because Mäori children's academic achievement was so poor that they had nothing to loose by moving them out of the system. As time went on, the schools were accepted and then funded by the system with a lot of trumpet blowing about how the students at these schools were out performing their Mäori peers at the mainstream schools. Figure 2 comes from a government report into the performance of Mäori students in Mäori immersion and English medium schools. It shows that in the three final years of high school that students in Mäori-medium schools gain more qualifications at a higher level in 2004, 2005 and 2006.

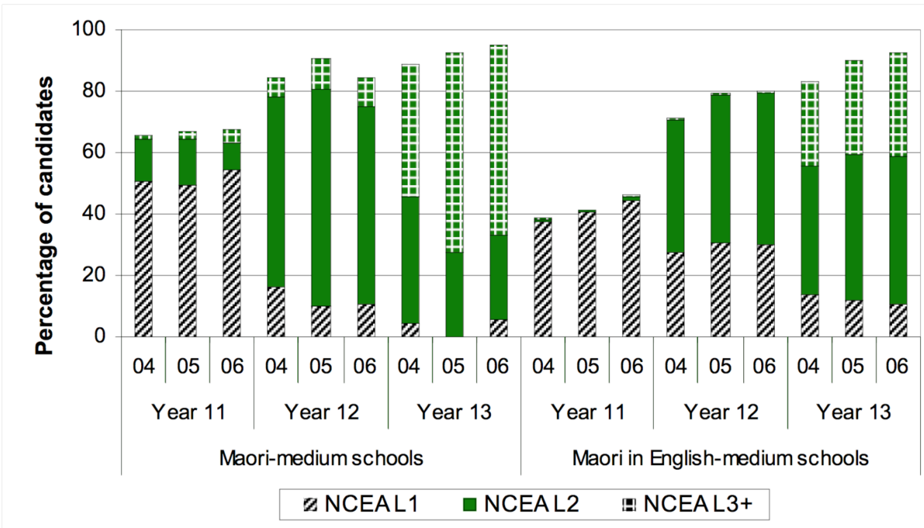

Figure 2: Highest NCEA qualifications gained by Years 11 - 13 candidates at Mäori-medium schools and by Mäori at English-medium schools, 2004 - 2006 (from Wang & Harkess, 2007, p. 3)

Thus, the system went from blocking the establishment of the schools to linking themselves to their success. Part of the system's support for this process has been the provision of bilingual exams at the end of high school so that students who learnt in *te reo Mäori,* the Mäori language, did not have to do exams in English which had been the case up to ten years ago. Many students are second language learners of Mäori, so the original questions in English are also provided.

However, no extra time is given to the students if they need to move between the languages. As part of our ongoing research, we interviewed students who sat these exams to find out about their experiences and found that they did use both languages, moving between them continually. Uenuku then looked at the te reo Mäori in one set of exams, only to find that the translation from English to Mäori had been done so appallingly that students were unlikely to be able to just use the Mäori to understand the question even if they were first language speakers of Mäori. Therefore, the lack of exams in *te reo Mäori* could be seen as a blatant example of racism whilst the provision of bilingual exams could be seen as an example of subtle racism.

So although on the surface it appears that the New Zealand Qualifications Authority (NZQA) has moved beyond its open resistance to Mäori taking control of the schooling for their children by providing bilingual exams, subtle obstacles are put in the way. The outcome of this ensures that the students recognise the pre-eminence of English and the fact that Mäori cannot do mathematics. These obstacles are likely to have contributed to the very small numbers of students who achieve at more than the standard level. Work by Stewart (2007) on the first three years of the new examination process in New Zealand show just how appallingly the results were in mathematics achievements. Table 1 shows these results.

Table 1: Number of mathematics exam papers completed by Mäori immersion students between 2002 and 2004 (from Stewart, 2007).

	Number of papers	Not achieved	Achieved	Merit	Excellence
All Pangarau [mathematics]	941	693 (73.5)	210 (22.3)	38 (4.0)	0 (0)

So where does racism appear in this story? And who exerts this racism? Although *te reo Mäori* is recognised as an official language of New Zealand because most non-Mäori do not speak it, the difficulties of doing translations are not well understood. oes this make those in charge of organising the translations racist? Or is it the Mäori organisations who do the translations for NZQA who are complicit in this racism? The issue is full of complexity. However, if we conceive that the outcome of continuing poor academic results in mathematics for Mäori students is “an effort to reorganize or redistribute resources along particular racial lines” then this situation is one of a ‘racial project’ as Danny Martin has described it.

One of the points that Danny Martin makes is that race can be back-grounded in discussions about globalization. I would also contend that ethnicity as a social construction can also be backgrounded whenever we try to simplify complex issues. Theoretical considerations need to help unpack the processes by which some stories become more valued than others are other stories are never told. I would contend that there are some white theorists who can contribute to this unpacking and given white people's contribution to the establishment of privilege it is essential that they take on this theorisation. Elizabeth De Freitas and Alexander McAuley (2007) show how the discomfort that just reflecting on the advantages that this privilege brings needs to be embraced if teachers are to see the diversity of their students in positive ways.

Foucault's ideas about the connection between power and knowledge can provide some insight into the processes that contribute to the complexity of situations such as that of the bilingual exams. At the present time, it is difficult to have a discussion about changes that could contribute to improvements because too many educators, European and Mäori, are invested in the stories of Mäori immersion schools being successful. For the educational system as a whole, the stories of success of these schools mean that they are not required to do anything more and for the Mäori community is a way of ensuring that children continue to be sent to these schools and that the language becomes strong. Mathematics achievement for these students in secondary in both considerations and may be this is how it should be?

The bilingual exam story illustrates the complexity of the situation. Racism comes in both blatant and subtle forms; these latter ones can often be difficult to identify and overcome. In one part of the paper, Danny describes the *interest convergence* that resulted in African Americans receiving better educational opportunities. Moral compunction is unlikely to ever be enough to overcome racism because its impact on the research that is undertaken and given credence is likely to be negligible. Research after all is supposed to be achieved scientifically. Without a change to the way that we conceptualise what research is and how it is done, there always will be too many other stories that are woven together that reinforce the acceptability of the differentiation of results by ethnicity. Therefore, there is a need to better understand how practices such as education are confined by the societal configurations that affect what research is funded and what information is fed back into the system. Unless the complexity in which we as researcher work is understood in relationship to the complexity of schooling systems and students' outside school lives then racism is likely to be perpetuated. Danny's paper ensures reflection on the educational research enterprise but more work is needed if we are to move from identification of problematic nature of the racial project of mathematics education to doing something about it.

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MATHEMATICS EDUCATION FOR A BETTER LIFE? – VOICES FROM MES6 PARTICIPANTS

João Filipe Matos

University of Lisbon

Does critical mathematics education embody an obsolete line of thought? Is it just a leftover from an outdated leftist educational movement? If not, what could critical mathematics education mean today and for the future? This is the type of questions that Ole Skovsmose raises in his most recent book. Inspired by the voice of such a senior critical mathematics educator, I decided to survey MES6 participants in order to put their voices in tone for the conference. It is my understanding that this may result in a powerful exercise to start establishing a redefined form of conceptual harmony where minor and major tones frame mathematics education priorities for a better life.

SYMPOSIA

TELLING CHOICES: MATHEMATICS, IDENTITY AND SOCIAL JUSTICE

Laura Black, Anna Chronaki, Stephen Lerman, Heather Mendick, Yvette Solomon

University of Manchester, University of Thessaly, London South Bank University, Goldsmiths, University of London, Manchester Metropolitan University

In this symposium we are interested in exploring ways of conceptualising identity in mathematics education and the social justice implications of different approaches. Each presentation analyses the choices in relation to mathematics of one or two individuals, drawing on a range of theoretical tools across sociocultural, discursive and psychoanalytic perspectives. We aim to challenge the dominant neoliberal constructions of choice as an unproblematic and individual act and to open out discussion of the possibilities and constraints of different understandings of choice.

RATIONALE, AIMS AND PLAN OF THE SYMPOSIUM

In this symposium we will explore the use of different theoretical lenses to understand why some people choose to study mathematics and others choose not to, and how they make sense of these choices. In the current neoliberal context, choices are not simply acts of consumption; they are a means of making one's-self: 'Individuals are to become, as it were, entrepreneurs of themselves, shaping their own lives through the choices they make among the forms of life available to them' (Rose, 1999: 230). However, although choices are presented as individual acts, they are constrained by conditions of social class, 'race'/ethnicity and gender. We want to understand choices in relation to mathematics so as to disrupt the neoliberal idealization and individualization of choice. We thus have three aims:

- To explore the nature of choice generally and in relation to mathematics.
- To reflect on the role of theories of identity in our understandings of choice, and the possibilities and constraints of particular approaches.
- To examine the social justice implications indicated by taking different theoretical approaches to choice and identity.

We will use a mixture of presentation and discussion in this symposium. Each of the five presentations will draw on a range of theoretical approaches to identity that can be broadly classified as socio-cultural, discursive and psychoanalytic (Black et al., 2009), using them as toolkits with which to unpack the complexities of particular individuals' choices as they are constructed within interview data. The symposium will be spread across two conference sessions. The first will involve three presentations followed by collective discussion of the issues raised. Two further presentations follow in the second session, with an extended discussion of how we can theorise and understand choice in mathematical relationships. The presentations are outlined below.

SESSION ONE: NARRATIVES OF CHOICE

'Whose choice? Self-positioning in the interface between discourses of value/ability in mathematics and family narratives' by Yvette Solomon

Drawing on ideas from Sfard and Prusak (2005) and Holland et al (1998), this paper *explores how two young women position themselves with respect to the* discourses of value and ability that pervade school mathematics as they explain their choices (in Becca's case) to keep on studying mathematics and (in Gerry's case) to stop doing so. It appears that Becca's choice to take mathematics at A-level is underpinned by her subscription to these discourses: she expounds the value of mathematics as an important marker of her intelligence and her future employability. Gerry's choice, on the other hand, is based on resistance to these dominant discourses: instead she takes up a position of autonomous 'real' choice in her creative 'project of the self'. While both are at pains to say that they have made their choices independently of their parents, their accounts indicate that family narratives play an important role in their positioning of self. A closer inspection raises questions about the interweaving forces of parent-child identification and reparation narratives and the respective parts played by parents and children in them. It also raises issues about the role of cultural and economic capital in 'choice' and its filtering through the family narrative.

'The role of post hoc rationalisation in narrating choice across time' by Laura Black & Valerie Farnsworth

In this paper we utilise narrative inquiry to analyse the 'choices' made by two students, David and Christopher, in relation to mathematics and their future aspirations. We draw on socio-cultural theory which argues that narrative is central to the way in which we understand ourselves in the world and the reality in which we operate (Bruner 1996, Sfard & Prusak 2005, Gee 1999). Using longitudinal interview data, we look for evidence of post hoc rationalisations where the same 'choice' or event is told differently by the student at different points in time. We have interviewed both students on five occasions ranging from the start of their post compulsory studies in college (aged 16) to the end of their first year at university (aged 19). Evidence of 'post hoc rationalisation' appears in both students' accounts – David changes the way he describes an early aspiration to study Physics at university and subsequently, his motive for studying mathematics, whereas Christopher changes his description of the role of family influence in his 'choice' to study mathematics. The paper will explore what motivates these students to re-frame their 'choices' in this way and how this then affects their relationship with mathematics.

'Social class and identity: how free is anyone to choose?' by Steve Lerman

Marxist sociologists of education demonstrate how the distribution of social goods is strongly determined by social class. Bernstein (2004) shows how disadvantage, in relation to children from middle classes, has its origins in the home, in restricted as compared to elaborated language, and is reproduced at an early stage of primary schooling. Thus the option to choose to study mathematics at the upper levels of schooling appears to have been taken away from students from working classes. But this is not the whole story. Students can and do find ways to resist these restrictions and make choices that may appear surprising to teachers and researchers, and indeed the students' families. Identity is multiple and a student apparently powerless in one discourse may see themselves as powerful in another. As researchers we need rich descriptive tools for analysing identity and the resources potentially available through shifts in discourses. I will exemplify this through the retrospective account of Jane, a woman successful in mathematics from a working class background.

SESSION TWO: DISCOURSES OF CHOICE

'Choosing mathematics for choosing life: identity-work through discursive rationales' by Anna Chronaki

Alexis, a male undergraduate, studies architecture and has an interest into programming for engineering design. This interest confronted him with the need to learn more advanced mathematics – an option that is not formally offered in his course. His choice for learning mathematics is enveloped with his choice for professional involvement with programming and architectural design – opening up for him a novel career path. Contrary to predominant stereotypic images of mathematics as a 'boring' subject 'disconnected from real life' – a view shared by many young male and female students, Alexis invests in mathematics and gets passionately involved in it. His choice is rationalised by encountering the discourse of mathematics as a tool-kit for solving engineering problems. Further, his choice for mathematics is also enveloped within the need for gaining control and autonomy over his future job and life as architect/programmer. Walkerdine (2003) and Rose (1999), amongst others, explain that discourses related to an impetus for governing modern life are based on the virtue of self-reliance (autonomy, self-regulation, self-efficacy, and so on) and reflect mainstream and conservative psychology (i.e. cognitivism) or sociology (i.e. neoliberalism). Rose (1999), in particular, argues that the burden of 'choice' conceals the broader social context in which jobs for life have disappeared leaving instead the fiction of life-long learning. Simultaneously, inability to choose signifies inability to perform as an 'autonomous subject of choice' which then results in indecision and lack of success.

'Alice Through a Psychosocial Looking Glass: gender, control and mathematics' by Heather Mendick

The focus of this presentation will be Alice, a Turkish woman, who took part in a focus group and interview as part of a wider study of the role of popular culture in learners' relationships with mathematics (Mendick et al., 2008). She originally studied history of art but, at the time of the focus group was nearing the end of a mathematics degree at a London university, and at the time of the interview was waiting to begin a one year postgraduate course of teacher training. The presentation will explore psychosocial approaches by using these to make sense of Alice's choices. A psychosocial account understands her relationship with mathematics as psychic *and* social, without reducing one to the other, and explores how she 'invest[s] in discourses when these offer positions which provide protections against anxiety and therefore supports to identity'(Hollway and Jefferson, 2000: 21). In particular, the presentation will look at how Alice's desire for control over her life leads her to invest in mathematics and its discourses of mastery and at some of the gendered implications of this that play out in her rejection of a position as a productive mathematician and her take-up of a supportive role as a reproductive teacher of mathematics (Walkerdine, 1988).

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SAME QUESTION DIFFERENT COUNTRIES: USE OF MULTIPLE LANGUAGES IN MATHEMATICS LEARNING AND TEACHING

Anna Chronaki, Núria Planas, Mamokgethi Setati, Marta Civil

University of Thessaly, Universitat Autònoma de Barcelona, University of South Africa, and University of Arizona

Awareness about cultural and linguistic diversity in school mathematical practices has led some researchers towards exploring the use of multiple language use as a resource (rather than a constrain). The proposed symposium aims to discuss how research; a) analyses the complexities of multiple-language use in school mathematics and highlights how students (and teachers) are caught in dilemmas of language choice, and b) problematises the taken for granted dichotomies of language choice and explores possibilities for theorising how multilingual students and their teachers can challenge hegemonic discourses in school mathematics practices and engage with(in) mathematical learning identity work.

SYMPOSIUM RATIONALE, AIM AND PLAN

Our concern for how children in bilingual and multilingual classrooms learn mathematics best, has led us to develop research with children from marginal communities in Greece, South Africa, Spain, and USA. These are children of immigrant or non-dominant ethnic background, as well as, previously disadvantaged African students in South Africa. In our work, we move toward a discourse-based critical perspective that takes into account both the socio-political role of language and the backgrounds of the students to explore the conditions of participation and identity change of bilingual and multilingual learners in mathematics classrooms. In this symposium we draw on our local data to offer a joint view on school mathematical practices and multiple-language-use in varied sociocultural contexts. Our aim is to comment on questions concerning how certain theoretical perspectives have an influence on the analysis of our data, how certain discourses and policies serve to perpetuate marginalisations and inequalities in a variety of school practices, and how certain theorisations can work towards encouraging teachers and students to imagine alternative possibilities. The symposium will take place in one session (90 minutes). A combination of presentations and discussion will be used to debate the above issues. Each one of the presentations will last 15 minutes, allowing approximately 30 minutes for a collective discussion to take place.

Troubling language priority: Reversing identity fortune: Opening up mathematics learning? by Anna Chronaki

'*Mathematics runs in our blood!'* says 12 year old Panagiotis when asked to calculate mentally 3 digit numbers. Panagiotis has considerable experience of what we call 'real life mathematics' as he, from an early age, participates actively at his family business. But, when Panagiotis has to cope with formal arithmetic at school he confronts difficulties in expressing himself. Although his use of Greek is moderate, he struggles over finding the proper words in explaining problem solving processes. '*Our language is forbidden here'* says Panagiotis, explaining that *'[...] when we talk Romani during breaks they think we curse'*. As the Greek school curricula practices are still based on monolingualism, unavoidably Romani language-use along with its users –Greek Gypsy children- remain marginalised, oppressed, silenced and become 'other'. No-language literally results into no-voice for them. The complex relation amongst political-ideological factors and language use in bilingual communities has been highlighted (e.g. Bourdieu, 1991) and has been argued that the hegemonic imposition of monolingualism has negative effects on academic, economic and political arenas of individuals and communities alike. The present paper, is based on earlier work (e.g. Chronaki, 2005), and aims to discuss a politics of possibility. In other words: Can we create, by means of a 'teaching experiment', a stage that 'troubles', in Judith Butler's[1] words, hegemonic discourses about who is constituted able to do school mathematics? Can we 'trouble' hegemonic discourses concerning who sets the rules and who leads the language-game(s) for performing mathematical rituals by deliberatively shifting roles in the course of the everyday classroom life? Can we use this on-stage performed 'troubling' metaphor for engaging learners from varied cultural and social backgrounds in dialogicality?

Mathematics in multilingual classrooms: from understanding the problem to exploring possibilities by Mamokgethi Setati

Research on multilingualism in mathematics education all over the world has sought to understand the problem of access and success of students who learn mathematics in a language that is not their home language. In its exploration this research has explored the different ways in which languages are used in bilingual and multilingual mathematics classrooms, the range of discourses and language practices that teachers and students draw on as well as the nature of the mathematics that students are exposed to (e.g., Adler 2001; Moschkovich 2002; Setati, 2005,). In this presentation I will give a brief description of what we know now as a result of this research. I will argue that while the questions that research in this area of study has sought to understand have to do with mathematics, language as well as pedagogy, they are all concerned with the uneven distribution of knowledge and success in mathematics. I will further argue that while there is agreement on what shapes the research, this area of study is plagued by dichotomies of language choices and theoretical perspectives, which in my view contribute to the slow growth of knowledge and limited theorising. I will then describe a theoretical stance that I have adopted in a quest to explore possibilities for teaching and learning mathematics in multilingual classrooms. Drawing on data collected in South African classrooms over the past ten years, I will open up a debate and discussion on what this theoretical stance enables us as researchers to 'see' theoretically and multilingual students and teachers to 'do' mathematically.

Bilingual students acting as monolingual in the mathematics classroom by Núria Planas

Catalonia, North-Eastern Spain, is an autonomous region of more than seven million people with considerable immigration. Recent data indicates that immigrants or children of immigrant parents represent sixteen per cent of the total population. This percentage is even higher in the urban area of Barcelona, where twenty-eight per cent of the population has recent immigrant origins, including a fifteen per cent of first and second generation immigrants from South-America. In this context, the achievement in mathematics of immigrant students is much lower than that of local students whose first language, Catalan, is the language of teaching. This low achievement can be attributed to a variety of reasons including the knowledge of the local language. However, it is needed to pay special attention to social issues related to the use of the languages represented by the groups of students in the classroom. In my recent studies (e.g., Planas, Iranzo & Setati, 2009) I explore when, how and why students from a non Catalan speaking background prevent from participating with other groups of students that have the language of teaching as their first language. I draw on social theories to argue for the ways talking can be used to reduce opportunities for them to engage with classroom mathematical practices. My findings show that the classroom as a culture and the students as individuals send conflicting messages about the values of bilingualism through the separate use of languages. The use of Catalan, the official language of teaching, in the whole group and the development of a consciousness about the appropriate language in well-defined situations (small group vs. whole group) seem to have an influence on the production of monolingual strategies and monolingualism discourses.

Language policy and participation in the mathematics classroom by Marta Civil

In 2000 the voters in Arizona passed Proposition 203 that limits bilingual education in the schools in that state. More recently those students who are classified as English Language Learners (ELLs) have to spend four hours per day learning English (usually for one year), thus leaving little time for other academic subjects, as well as creating in many cases a school within a school (Valdés, 2001), where the ELLs are kept separate from the non-ELLs for most of the school day. In this presentation I will focus primarily on a mathematics class of eight seventh-graders (12 year-olds) during the first year of the implementation of the 4-hour separation model. These eight students were all of Mexican origin and classified as ELLs; the teacher was also Mexican but due to the language policy in place, she used English most of the time. I worked with the teacher and this group of students from February till May. During that time we implemented an approach in which students worked in small groups (where they spoke practically always in Spanish), wrote (in English) about their solution processes and from time to time also presented their approaches to the problems to the whole class (in English, switching to Spanish). We videotaped 30 hours of class and my analysis centers on issues of participation, role of language, and quality of mathematical discourse (explanations, reasoning). My focus in the presentation will be on these students' high level of competence as mathematicians once they felt free to use their home language to express themselves. The situation, however, is not as simple as suggesting the use of their home language; interviews with these students show a deep awareness of their being in a different section of the school and in most cases their desire to leave that section and be with the other students, which means becoming proficient in English as soon as possible. This creates a dilemma in terms of language choice and use in the classroom. My goal is to engage in a discussion around this issue during the symposium.

NOTES

[1] Judith Butler is an American post-structuralist philosopher, who has contributed to the fields of feminism, queer theory, political philosophy and ethics.

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ANALYSING THE USES OF "CRITIQUE" AND "POLITICS" IN MATHEMATICS EDUCATION RESEARCH [1]

Alexandre Pais, Mônica Mesquita
University of Lisbon

In a conference like MES, which explicitly tries to contribute for a discussion of the social, ethical and political dimensions of mathematics education, we wish to organize a space where the research on these dimensions is putted under critical scrutiny. We intend to do that by exploring the meaning behind two notions that have been used very often in the research discourse: the notions of critique and politics. It is our concern to understand the way researchers conceive these notions and how they put in motion in their research powerful ideas without loosing all the emancipatory potential of them.

AIMS

Our aim is to create a space of discussion where we can put aside for a moment our more immediate research concerns and critically reflect upon our research itself. We assume that people who participate in the MES conference are one way or another concerned with understanding how their research findings have a political impact in the social discourses that fuel educational practices. Therefore, we aim to develop a discussion that lead us to critically analyze how is that our research on the social, cultural and political dimensions of mathematics is actually making the emancipatory societal change that we so much desire.

RELEVANCE

During the last two decades there has been an increasing concern with political issues in mathematics education research. New trends of research have been emphasizing the critical aspects of mathematics, and the importance of doing research that takes into account the broader social and political contexts. Terms as critical mathematics education, socio-political perspectives, ethnomathematics, social justice, and others, have been encompassing research that tries to move beyond the didactical perspective which characterizes the majority of the research in the field. Although we are akin of such research, we suggest a moment of pause, by engaging on a theoretical discussion on how notions as "political" or "critical" are being used in mathematics education research. We strongly believe that sometimes the best way to act is to stop "acting" - in the sense of doing research that immediately implies some kind of action - and chew over. We suggest that in the case of mathematics education research, more than repeating research it could be a good idea to stop and ponder. Nevertheless this has been the call of well renowned researchers as (Niss, 2007) [2]: "It appears to be one of the weaknesses of our profession that many of us, myself included, tend to write and speak to much and read and contemplate too little" (p. 1311). Although the appeal of Niss, this appetence for reflection, that is not immediately concerned with action in the sense of providing solutions or strategies for improving the teaching and learning of mathematics, is usually not well accepted in the educational sphere, particularly in mathematics education. However we believe that without a deep reflection about our own actions we take the risk of moving blindly.

Our suggestion is to focus our reflection on the way the word “critique” and “political” is being used in mathematics education research. Our capitalist society in order to reproduce itself demands for perpetual reforms by means of integrating what could be new and potential emancipatory acts into well established social structures. The word “critique” has become a common place among educational research and curricular documents, being used as a signifier implicitly conveying different ideologies about what it means to be critical. Today we can find notions of “critique” in a variety of contexts such as school curriculums (“educate people to become critical citizens”[3]), in teacher education (“Tips for teaching critical thinking skills” [4]), professional education (“Education and Knowledge in Safety-Critical Software”[5]), online education (“Role of critical thinking in online education”[6]), and so on. One consequence of this extensive use is the lost of meaning. That is, words begin to function as empty signifiers, representing no more than a way of symbolizing some assumed shared meaning. Very often, the use of these words lacks a deeper concern for understanding what could be the ideologies filling the empty space conveyed by these words.

Finally, we share the idea of Valero (2009) that mathematics education as a research field needs to develop research where its own principles and practices are putted under scrutiny. She argues that “developing awareness on the research perspectives that I adopt has, therefore, been as central to me as generating particular understandings and interpretations of the practices of teaching and learning in mathematics classrooms” (p. 2). Therefore we claim for the need of a constant critical analysis of the way we engage on research and how we understand its results. This kind of analysis demands looking at research from a socio-political perspective (Valero, 2004) that explicitly searches for connecting the role of research – in particular in mathematics education – to the discourses and ideologies that fuel our current society. In order to understand the dynamics of the teaching and learning of mathematics and the way research results influence what is happening in mathematics classrooms, we need to contextualize these practices and the social modes of living that characterizes the world today.

PLAN

Since our intention is to promote an open discussion, the plan for the symposium will depend on the way people engage on such discussion. We will start by presenting the concerns which motivated us to propose this symposium, and then pose some crucial questions to generate discussion. Examples of those questions are:

1. How do we understand and use the notions of “critique” and “political” in our research?

2. How do we conceive “change” in an educational system structured around values that most of the times conflict with the transformations we want to carry on?
3. Why do we need schools to perform the role of systematically posing people in a social network of value, therefore creating exclusion?
4. Why is there a persistence of failure in mathematics education?
5. Do we desire our desire for change?

We will need one session to develop the symposium.

NOTES

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2. But also Shlomo Vinner, when in a regular lecture in ICME9 he appealed for other approaches in mathematics education research: “I would like to use this opportunity to reflect on mathematics education from some angles which generally are ignored while we are so busy with investigating particular mathematical concepts, problem solving processes, the use of computers and internet” (Vinner, 2000, p.1).

3. Portuguese, Colombian, South African curriculums.

4.http://www.modernghana.com/news/203119/1/tips-for-teaching-critical-thinking-skills.html

5. http://ercim-news.ercim.org/content/view/446/699/

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NEW PERSPECTIVES ON MATHEMATICS PEDAGOGY

Symposium Coordinators: Margaret Walshaw, Kathleen Nolan

Massey University, University of Regina

Symposium Contributors: Tony Brown, Tony Cotton, Brent Davis, Elizabeth de Freitas, Moshe Renert, David Stinson, Fiona Walls

"New Perspectives on Mathematics Pedagogy" represents a serious attempt to understand pedagogy within mathematics classrooms. To that end, this symposium will address the key questions and issues surrounding mathematics pedagogy presently confronting vast numbers of researchers, as well as educators, and policy makers. Organised around presentations, responses, discussion and debate, the symposium is intended not only to enhance understanding but also to stimulate fresh thinking and initiate ongoing critical dialogue about the practice of mathematics pedagogy within teaching and learning settings.

AIMS OF SYMPOSIUM

This symposium aims to engage the audience in a critical discussion on a new and provocative book in the field of mathematics education. *Unpacking Pedagogy: New Perspectives for Mathematics* is a forthcoming (December 2009) publication by Information Age Publishing. Based on the chapters of this edited collection, this symposium will address the key questions and issues surrounding mathematics pedagogy presently confronting vast numbers of researchers, as well as educators, and policy makers. By pedagogy we mean the elements of practice characterised not only by the regularities of teaching but also the uncertainties of practice. If pedagogy is about the production of mathematical knowledge and the construction of mathematical identities, it is also about social relations and values. Pedagogy takes into account ways of knowing and thinking, language, emotion, and the discourses made available and generated within the physical, social, cultural, historical, and economic community of practice in which mathematics teaching is embedded.

The symposium directly involves nine chapter authors who will present their research and/or act as respondents. Their presentations are not intended to provide analytic consensus in their attempts to understand what it is that structures the pedagogical experience. Highly influential in informing the analyses will be Foucault's understanding of how practices are produced within discourses and within power configurations; Lacan's notion of subjectivity; evolutionary frameworks of complexity science to rethink mathematics pedagogy; and Bourdieu's notion of habitus to explain the teaching/learning nexus.

RELEVANCE OF SYMPOSIUM

As educators and researchers, we believe that mathematics pedagogy is at a crossroads. The harsh reality is that many students do not succeed with mathematics; they are disaffected and continually confront obstacles to engaging with the subject.

Recent analyses of international mathematics test data have revealed patterns of social inequity that provide a sobering counterpoint to claims of an equitable pedagogical experience for all our students. Many settings, where teachers have been trained to deal with undifferentiated learner populations, now require teachers to contend with diverse learner cohorts, as well as differing behavioural and epistemic responses from students of mathematics.

These difficulties and challenges are nested within a much larger complex of social, cultural, technological, and economic phenomenon. Within this context, the policy response to poor student performance has commonly been the classic deficit response: to put the blame on teachers. As a result, teachers and teaching have become objects of scrutiny and critique, resulting in heavy workloads and new curricular policy mandates. Increased surveillance, set within a new audit culture, along with demands for evidence-based practices and scientific pedagogical methods and testing are the order of the day. For example, in the US *No Child Left Behind* initiatives, funding for schools is linked to heightened standardised performance measures. Attempts have also been made to standardise teacher-student interactions, instructional approach, and the kind of mathematics constructed within the learning context.

The postmodern vocabulary for talking about mathematics pedagogy within these contexts and conditions is more relevant than ever. "New Perspectives on Mathematics Pedagogy" takes that vocabulary seriously and engages symposium participants in responses to key issues through theory development. Working from the premise that new ideas are too important and complex to be ignored, presenters will speak about pedagogy in ways that participants may not have imagined possible. The responses to theory, highlighting a direct application to practice in mathematics education, will allow participants to build new knowledge about mathematics pedagogy and its situatedness within institutions, as well as within historical, cultural and social fields. As a result, the symposium will be a key medium for interrogating and understanding teaching. In that role, the symposium will raise thorny questions about the generalised discourse of mathematics pedagogy by theorising the contradictory realities of teachers and the complexity and complicity of their work.

The symposium contributors believe that negotiating through the epistemological indeterminacy of the postmodern moment can be facilitated by closely examining the concrete, material and human specificities of the mathematics pedagogical experience. Approaches that draw on postmodern ideas are now widespread across many disciplines. However, postmodern ideas have not yet been given any significant platform within the discipline of mathematics education. Until now, students and scholars alike in mathematics education have tended to present their findings and their claims from the standpoint of more traditional thinking and have often based their analyses of pedagogy on examples from 'sanitised' classrooms. In the process many crucial aspects of the pedagogical relation have remained unquestioned. In particular, discussions of classroom practice have tended to gloss over intersubjective negotiations that take place in the development of teacher identity and that take place in the construction of mathematical knowledge.

The symposium presentations will pay careful attention to these crucial aspects. This is new territory for researchers of mathematics education classroom life and, because of this new ground, the presentations are designed to stimulate thinking and to question the way we think about pedagogical work in mathematics classrooms. Presenters will do this by offering conceptual resources to develop a new sensitivity to everyday pedagogical practice. Relational, contextualised, and in some ways provocative, the presentations will provide, above all, an opportunity to explore what drives mathematics teaching practice and to examine pedagogy's effects.

SYMPOSIUM PLAN

The symposium will represent a coherent set of theoretical, narrative, empirical and practical applications of postmodern concepts to the field of mathematics pedagogy. Two main objectives structure this set of applications. One objective is from a theoretical perspective that involves examining the issue of teacher subjectivity and exploring how intersubjective negotiations shape the production of classroom practice. A second objective is to apply these theoretical understandings to the construction of both mathematical knowledge and teacher identities in the contexts of actual mathematics teaching and learning settings. To that extent, we plan a two-session symposium (i) a focus on theory that involves examining key concepts and thinking, and (ii) a focus on practice that applies those theoretical concepts to pedagogy within specific historical, cultural, and social contexts.

The first objective will be met during Day 1:

- Introduction to the key issues concerning mathematics pedagogy
- Three presentations on theory relevant to understanding and researching mathematics pedagogy
- Questions, discussion and debate [both peer and symposium-wide]
- Closing and Day 2 preview

Contributors to Day 1: Margaret Walshaw (Introduction/closing); Brent Davis and Moshe Renert; Fiona Walls; Tony Brown.

The second objective will be met during Day 2:

- Short overview of theoretical presentations from Day 1
- Three responses to the theories; each response discusses the relevance of the 3 theories to a particular issue/setting and applies one theory in particular to a setting/issue
- Questions, discussion and debate [both peer and symposium-wide]
- Closing comments

Contributors to Day 2: Kathleen Nolan (overview/closing); David Stinson; Tony Cotton; Elizabeth de Freitas.

All contributors are deeply involved in working with new ideas in their research in mathematics education. They represent a range of geographical regions and countries: Canada, Australia, United States, England, and New Zealand. As stated previously, their presentations will be based on the chapters in the volume *Unpacking Pedagogy: New Perspectives for Mathematics.* This collection is edited by one of the symposium coordinators, Margaret Walshaw, and contains a chapter contribution from the other coordinator, Kathleen Nolan. Participants attending the two symposium sessions will have opportunities during both days to ask questions, to discuss, to debate and to critically interrogate the content of the presentations. That opportunity has specifically been scheduled into the timetable.

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PROJECT PRESENTATIONS

MATHEMATICS FROM THE PERSPECTIVE OF CRITICAL SOCIOLOGY [1]

Sikunder Ali Baber

Autonomous University of Barcelona

The promotion of a conception of mathematical literacy is related to the promotion of a particular social practice in the context of modern societies. This raises the importance of viewing mathematics with the lens of critical sociology. Looking at mathematics from critical sociology can help us to pay attention to the particular cultural practices that produce mathematics. This approach can bring about implications for the education of mathematics for schooling.

This contribution presents an argument in support of looking at mathematics with the lens of critical sociology. Further, it shows how this argument can help citizens to become critical of the practices of mathematics surrounding their life. Moreover, I will present ways of approaching mathematics as a cultural practice and of citizens to become critical about the "static nature of mathematics". This type of particular practice of mathematics might not necessarily lead citizens to become critical on the structures based on mathematics. Instead this particular culture of mathematics can uphold the interest and power structures of particular groups of people.

BACKGROUND

Often, the learning of mathematics has been associated with socially constructed fear of mathematics - learners at different levels conceive of mathematics as a boring, a dry and an uninteresting subject; the subject one just needs to pass in the examination. This socially constructed attitude towards mathematics has put many people in a situation where they distance themselves from learning mathematics and ultimately are devaluing mathematics. Subsequently they put themselves into the disadvantageous position vis-à-vis their needs of understanding, interpreting, creating their meanings and actions within the complex world, where practices of mathematics have colonized this world. Critical mathematics education can help in linking mathematics with society in order to understand and resolve issues of society, such as the issue of equity in distributing the benefits of mathematics to all (Skovsmose, 1994; D'Ambrosio, 1996). Furthermore, the research on critical mathematics education is suggesting that understanding mathematical principles is very critical for a person in order to remain an active participant of this increasingly globalized world. And these principles are not devoid of value judgments. That is, there would remain strong chances that experts with mathematical knowledge may misguide us and create many troubles in our lives. This brings our focus on the banality of expertise. And there also emerges another relevant question: how much autonomy has a learner in formulating knowledge which addresses not only her/his individual needs but also her/his collective needs as a citizen within the society? In this respect, the role of mathematics in the context of the learners has vital importance for the preparation of citizens to become critical citizens.

CRITICAL CITIZENSHIP

Critical citizenship is an important aspect of a critical sociology of knowledge. Some of the aspects associated with the concept of citizenship with relevance to mathematics education are: Social Justice: fairness, equity, responsible action; Inquiry into issues: racism, inequality in different dimensions (human rights violation etc), poverty etc; Democracy: proportionate representations, polling, and population; and a need to understand the complex interplay of the situations, which necessitates the need to realize the goal of creating critical citizenship. These situations may include free flow of information through various media, which may necessitate the need for critical appraisal and evaluation of the available information and of actively taking part in the production and disseminating of the information to a wider public. For example, Ole Skovsmose (1998) elaborates the relationship of citizenship with mathematics education by using the term *Mündigkeit*:

> ... the *Mündigkeit* can be given specific interpretation, such as the students being able to participate in political discussions taking place in a local community. And, most important, *Mündigkeit* also includes competence in investigating decisions with mathematically formulated arguments (Skovsmose, 1998, pp.196-199)

That is, in order to face the fast changing world and to become actively engaged not only the preparation to become critically literate is decisive, but also mathematics plays an important role. For example, (a) Morgan (1997) has brought our focus on the importance of critical literacy:

> Critical Literacy encourages students to challenge taken-for-granted meanings and 'truth' about a way of thinking, reading and writing the world. It works against the notion that meaning is transparent, neutral and unproblematic. Critical literacy also questions the neutrality of power relations within the discourses. In pedagogic terms, students should be encouraged to develop enquiring minds that question the cultural and ideological assumptions underwriting any text. They also learn to investigate the politics of representation in the discourse, interrogate the unequal power relations embedded in texts and become astute readers of the ways texts position speakers and readers within discourse (Morgan, 1997, p. 259).

And (b) mathematics can be conceived of as a human invention, and as a tool it can help to provide a critique of different social structures of the society. Normally mathematics has been conceived of as a neutral discipline having nothing to do with the social activities of people, despite of the fact that it is one of the products of the human enterprise. This characterization of mathematics as infallible has on the one hand led mathematicians to invent complicated symbolic languages to handle complex models of human thoughts; on the other hand, it has created space for conceiving of mathematics as being only for gifted people. In consequence, a socially constructed fear was created and sustained. This raises the importance of viewing mathematics education in relation to its role within wider society and how this relationship can play a role in distributing power among different sections of society. One way to conceive of mathematics as a cultural practice can be related with the efforts of looking at mathematics as a part of the political project with conflicting interests and ideologies. This way the citizens could have possibilities to appraise cultural practices that generate mathematics (Jablonca, 2003).

CHALLENGES TO EDUCATIONAL PRACTICES OF MATHEMATICS

Mathematics education faces varied challenges especially when it comes to prepare future citizens who are ready to be critical to face the challenges of the increasingly globalized world. These challenges are, for example, reflected through the reform efforts of the National Council of Teachers of Mathematics (NCTM) of the USA. The major focus of NCTM is to pay attention on students to become problem solver and encouraging students to be engaged in the process where they can be led to see the generalizations of mathematical statements and see the potential of these generalizations in creating abstract systems. However, there is an ideological basis for these reform efforts and also there are political interests that the NCTM is trying to achieve through these reforms. One of the political objectives is to introduce mathematics as part of "mathematical sciences". This designation leads one to conceive of mathematics as having an essence and it can conceal the traditions and networks of associations responsible for the production of mathematics as a discipline. In this way mathematics has been reduced to a field which is stable and not changing. That is, it gives an illusion that the future is certain. On the contrary, the future is uncertain. In other words, through the illusion of problem solving the participation of learners is regulated by the expertise with an idea that problem solving increases flexibility. In this way, the human agency is restricted and circumscribed by the expertise whose political objective is to stabilize and harmonise the world of participation. Here expertise retains its power of defining the domain and practices of mathematics education. In this vein, it is important to consider mathematics as a field of a cultural practice that is constituted by an amalgamation of institutions, authority relations, analogies, memories and images that are assembled together in different junctures of time and places to arrange and categorize objects of reflection and action (Popkewitz, 2004). For example, Ian Hacking (2002) argues that mathematics embodies different ways of thinking about and creating new objects. Each style of reasoning in mathematics opens up different objects of scrutiny and provides a classificatory scheme by which lives are experienced, truths authenticated, and futures chosen. He compares algorithmic and combinatorial styles of reasoning with special styles that are "self-authenticating." That is, each style "introduces its own criteria of proof and demonstration, and… it determines the truth conditions appropriate to the domains to which it can be applied" (Hacking 2002, p.4). Thinking of mathematics in this way can direct our attention to its practices and it could be considered as a group of techniques for bringing new kinds of facts to our awareness. This is one of the challenges that future mathematics education has to confront when one wishes to pay attention to critical features of the society.

CONCLUSION

Here I have made an argument that looking at mathematics with the perspective of critical sociology is an important aspect of the education of mathematics for schooling. This perspective requires us to take the notion of critical citizenship seriously so that citizens can retain their ability to give an input to authority in order to bring a critical eye on mathematically formatted arguments. Moreover, it can also lead us to equating mathematics with a tool like language which can be used to interrogate a variety of claims that can be made through mathematics - in this way mathematics can make the familiar strange.

NOTES

1 An expanded version of this work will appear in Alrø, H., Ravn, O., & Valero, P. (2010) (Eds.). Critical mathematics education: Past, present and future. Rotterdam: Sense.

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COLLECTIVE MATHEMATICAL REASONING IN CLASSROOMS WITH A MULTILINGUAL BODY OF PUPILS

Birgit Brandt, Marcus Schütte

Goethe-University Frankfurt am Main

The students of German schools are affected by multilingualism and various cultural backgrounds due to continual immigration in the recent decades. Currently, almost one-third of the students in German schools have a migration background. This circumstance would not be worth considering if each student had equal prospects for a successful school career. However, this is not the case (cf. Beauftragte der Bundesregierung für Migration, Flüchtlinge und Integration 2005 PISA)[1]. Against this socially relevant background, we will present a research project in progress, which ought to investigate possibilities to tie in with the adaptive potential of language within teaching mathematics in primary schools. Thereby, we will combine two different theoretical considerations concerning a) learning by participating in collective mathematical reasoning (Krummheuer, 2007; Brandt & Tatsis 2009) and b) the linguistic accomplishment of mathematic learning processes in multilingual classroom settings (Bernstein 1996; Schütte 2006).

The basic idea is that especially in classrooms with a multilingual body of pupils, the teaching of mathematics must focus on encouraging collective mathematical argumentations and supporting mathematical expressiveness (which is beyond learning mathematical vocabulary). It is crucial that the pupils verbalize their ideas and thoughts and address dialogue partners. There is a close connection between cooperative learning opportunities and the significance of verbalization within mathematical learning processes. Thus, in our project we will systematically realize different types of collaboration in classes with a multilingual body of pupils (3rd and 4th grade classes). Thereby, the question of adaptive effectiveness of different collaborative scripts is examined with respect to the pupils' language-related participation in collective mathematical reasoning. Given the fact that this empirical design-based study occurs during everyday teaching, instantaneous moments of teaching methods can be attained which render the possibility to support a multilingual body of pupils. But in this paper we will focus on our theoretical backgrounds and the combination of them for theoretical purposes.

PARTICIPATING IN COLLECTIVE MATHEMATICAL REASONINGS

Our perspective on the classroom processes is an interactionistic one (f.e. Cobb & Bauersfeld 1995). In this approach, the interaction serves as a place for joint negotiation. From this perspective, we have developed a model for learning mathematics in everyday classroom situations (Krummheuer, 2007; Brandt & Tatsis 2009). According to this model, the way students are involved in explaining, reasoning and justifying content-related actions is crucial to their learning. Within our conception, tuition is a meshing of "smooth periods of interaction" (SPI)

(Krummheuer, 2007, 64), and “condensed periods of interaction” (CPI) (ibid., 68). Whereas SPI are undemanding and superficial and reduce the risk of conflict, the CPI optimise the conditions for content-oriented learning by deepening the requirements for participation in the productive aspects as well as in the receptive aspects.

In particular, CPI differ from SPI with regard to the following modes of interaction: (a) in terms of the complexity and explicitness of the argumentation, (b) in terms of the chaining of the utterances of different speakers, (c) in terms of the involvement of the listeners in this argumentation and (d) the requirements for a change from a listening to a speaking form of participation. Thus, we see the CPI as optimization of social conditions for content-related learning by participating: Hence, relatively elaborated forms of argumentation were produced with distributed responsibilities amongst the participants. Beyond this optimization for content related learning through an active participation, on the one hand CPI can enhance the opportunity for learning through listening. The speaking participants must thereby take the “audience” into consideration by the choice of words and the degree of contextualization and indexicality (s. a. Brandt & Tatsis, 2009). On the other hand, the speaking persons of a CPI could exclude listening persons by using an inaccessible language code and thus hinder learning by listening.

MATHEMATICAL LEARNING PROCESSES UNDER THE TERMS OF LINGUISTIC AND CULTURAL PLURALITY

Tracing back to concepts of Bernstein (1996) and Gogolin (2006), Schütte (2009) analyzed the linguistic accomplishment of mathematic lessons in primary city-schools (4^{th} grade classes) that contain a high percentage of pupils from educationally disadvantaged families with a low socioeconomic status and/or migration background. As a result of his analyses, Schütte reconstructed that tuition in these classes is predominated by a language usage that significantly depends on colloquial everyday language and in spite of introducing new mathematical terms does not achieve a formal linguistic status. Furthermore, the interaction structure of instruction is characterised by the phenomenon of implicitness concerning statements and proceedings of the teacher during the introduction of new mathematical terms. Concerning the linguistic realisation of instruction, Bernstein (1996) develops a differentiation between two forms of discourse. The common knowledge is expressed by “horizontal discourse”, whereas the communication about specialised knowledge happens in terms of “vertical discourse” (ibid., p. 171). With regard to this distinction the analyzed discourse features characteristics of a horizontal discourse.

According to Gogolin (2006), pupils in German schools are submitted to the normative standard, that they are receptively and productively in command of the cultivated linguistic variations in class. This language of higher school education – described by Gogolin as “Bildungssprache” *(formal educational language)* (ibid, p.82 ff.) – has on a structural level more in common with the rules of written linguistic communication. It is in large part inconsistent with the characteristics of the everyday verbal communication of many pupils (cf. "CALP" by Cummins 2000, p. 57 ff.).

Hence, the children who require a linguistic introduction to the *formal educational language* of a vertical instructional language within class are not satisfied by a kind of linguistic accomplishment and interaction structure which resembles a horizontal discourse. Consequently, especially those children are disadvantaged by primary school classes in terms of their future educational success within secondary schools who require an introduction to a formal educational language.

THE COMBINATION OF BOTH THEORETICAL APPROACHES

Concerning learning by participating in collective mathematical reasoning, the reconstructed linguistic accomplishment seems to hinder the emergence of CPI sequences because of its orientation towards a horizontal discourse. In fact, the implicit instruction of new terms in connection with an everyday language is rather consistent with the SPI that stays at an argumentative surface with low chaining of the utterances. With regard to the interaction theory of mathematical learning processes, the reconstructed teaching praxis in classrooms with a multilingual body of pupils does consequently not benefit sequences of interaction which provide optimal requirements of facilitation for learning mathematics.

Pupils from educationally advantaged families apparently possess the abilities to compensate the deficits that are located in the linguistic accomplishment of instruction due to the competences that they acquired at their homes. They not only possess greater competences in the domain of a formal educational language but they are also already familiar with interaction patterns of teaching due to their family environment. This might effectuate that they are more likely involved actively into the moments of CPI, that selectively occur during lessons. In opposition, the opportunities to learn about new technical terminology seem to be limited for those pupils who do not possess these abilities by virtue of family socialization. They are reduced to the prevalent form of argumentative-superficial as well as conceptual-informal discourse of the SPI where mathematical terms can be acquired as "vocable-knowledge" but the potential of formal linguistic elements is neither accessible nor identifiable in terms of a profound as well as argumentative consideration. They therefore seem to be excluded from the optimised moments which make learning possible within CPI-sequences. This kind of teaching method has a double selective effect due to the early selection within the German school system as well as the interaction during lessons simultaneously representing a "learning area" just as it is applied to appraise the pupils' performances.

DISCUSSION

The question that arises is how can the instruction support all pupils in order to learn a *formal educational language* and how can it suitably be applied to collective argumentations for the purpose of learning mathematics via an (active) participation. We believe that an approach is given by the use of special collaborative modes of learning which focus on equal inclusion of pupils with differing linguistic and professional knowledge. According to our learning theory that was posed above, the initial priority is to increase the emergence of CPI-sequences within the interaction processes. Furthermore, we will provide linguistic support to the pupils that functions as frames structuring the interaction. The linguistic support is oriented towards the approaches of second language acquisition and aims at the subjacent aspects of argumentation. Thus, the design of learning environments within our study targeted on the deficits that we analysed in the linguistic accomplishment.

Concluding, we expect an improvement of the possibilities to participate in CPI-sequences for all pupils and consequently optimised facilitational requirements to professional learning. We assume that the adaptive effect accumulates if the pupils' scope of participation simultaneously grows along. It will be crucial that they use the expansion of scope options not only to upgrade their formal linguistic competences but also to make cumulative contributions to the collective mathematical argumentation.

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[1] Commissioner of the federal government for migration, refugees and integration

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CONSIDERATIONS ON BASIC ISSUES CONCERNING RESEARCH ON "CONTENT KNOWLEDGE IN TEACHER EDUCATION"

Reinhard Hochmuth

University of Kassel

Research on the optimisation of subject matter related teacher education could potentially benefit from objections concerning basic issues that were discussed in the 70s and 80s. This these reflects in particular the observation that actual research in mathematics education applies often directly methods and agendas from psychology and with that take over their deficits that were figured-out during that time. Issues that will be touched in the following cover so different topics as the general embedding of actual educational research, categorical concepts for grasping the mediation of individual and societal reproduction and didactical reconstruction.

INTRODUCTION

As a result of PISA the education of educational students attracts new research interest. One expects that higher teacher competences will lead to higher student competences. In contrast to the 70s and 80s, when research on university education was mainly considered as general didactic and sociological, the actual efforts focus on the respective subject, for example on mathematics. Whereas in G.B. and in the U.S.A. research on higher education in mathematics is well-established this is not the case for Germany. As a mathematician with interests in research on mathematics education I am involved in several projects working on a change of this unsatisfactory situation. In the following I will briefly discuss a few basic issues, which are relevant in this field, but are as such located beyond those projects.

GENERAL EMBEDDING AND ORIENTATION

Actual educational research its organizational circumstances like funding for example and its goals should be seen in the context of GATS (1994) and BOLOGNA. A crucial point regarding BOLOGNA is whether intrastate standards and guidelines, if they exist at all, are seen to ensure important societal functions and allow democratic participation or are regarded as constraints for economical prosperity. These poles do not represent a general contradiction but are contradictory under certain societal "boundary conditions". This remark does not mean that projects like PISA and its follow ups has only to be seen as a function of problematic societal processes, but they have **also** to be seen in relation to them. From my point of view there is too little research on those aspects and nearly no research on a "critical" explication of the gain of knowledge obtained by this research.

That the educational system is not only the result of efforts optimizing learning processes is, of course, not a fundamental new insight. It is well-known that comprehensive scientific analyses of self-contradictorily phenomena emerging in the educational system demand approaches, which are able to take into account their dialectic character. Moreover an understanding of learning requires a remedy of the "subject"-problem: Accepting the assertion that all learning has to be seen as arising from and situated in a socially and culturally structured world the "subject"-problem leads to the "challenging problem … to address the structural character of that world at the level at which it is lived." (Lave & Wenger, 2008, p. 123) This task lies in the heart of Critical Theory, see already (Horkheimer, 1937): One of the crucial goals in Critical Theory considering society was to show, that and how forms of social life, which appear as "natural", are in fact produced and reproduced in historical and societal processes. The corresponding analytical categories reflecting those processes present abstract assignments of relations in the "real" world and claim to cover the emerging "forms" of social life in their historical specificity. That Critical Theory remained critical and could not become "constructive" arises to some extent from their denunciation of scientific-analytic forms of knowledge as "logic of dominance", see (Furth, 1980).

CATOGORICAL CONCEPTS

Critical Psychology claims to present a scientific discussable elaboration of basic concepts (categories) for grasping the mediation of individual and societal reproduction, which allow the integration of "mainstream" theories and their empirical results. The central category worked out in (Holzkamp, 1985) is action potence, which is the potence to ensure the disposal about "my" individual living conditions together with others. Crucial is the "possibility relation" with respect to "reality", which characterizes to some extent the actual form of subjectivity and is in particular related to the basic experience of intentionality. Therefore a very basic aspect of action potence is given by the relation between possibilities and restrictions. Its historic-specific concretization with respect to bourgeois society is described as the relation between restricted and generalized action potence. Regarding learning this area of conflict is concretized by the distinction between defensive and expansive learning actions, see (Holzkamp, 1993). This distinction expresses a categorical caused problem that might be taken over by the student and/or the teacher for their self-understanding.

In the face of the specific modus of subjective action experience world conditions are given in terms of meanings, which are understood as generalized societal action possibilities. Meanings that are action relevant for "me" become premises. Therefore psychological considerations are essentially given by premises-reasons-relations. "Premises", "reasons" and their relationship are not obvious. Besides the fact, that psychoanalysis and critical psychology have in common the subject-scientific level of categories and procedures, the specific significance of psychoanalysis with respect to the historic-specific form of action potence consists in its accentuation and analysis of unconscious processes: It "must be understood that, owing to the ineradicable contradiction between immediate experience and the societal mediated nature of individual existence, unconscious aspects of subjective experience of self and the world play a necessary role in the struggle for a conscious mode of living." (Holzkamp, 1991, p. 99)

Obviously, the main critical psychological and psychoanalytical concepts are meaningless in a variable psychological context. Moreover, their categorical concepts, with respect to which they consider real world phenomena and work out empirical theses, cannot be justified by a variable-psychological approach. On the other hand the historical and societal nature of psychological phenomena cannot be grasped by treating the subject abstractly and by an identification of variables and studying their relations. This impossibility can be seen as a main source for fundamental methodological problems of the "mainstream" approaches regarding for example validity, relevance, indeterminacy and partisanship, see (Markard, 2009). With respect to validity consider for example competence models, (Schaper, 2009): Until now they show severe problems with respect to criteria validity and there is only little progress with respect to content validity. Typically in projects that evaluate competence models "only" construct validity is considered, which reproduces more or less well-established views of experts.

DIDACTICAL RECONSTRUCTION

Didactical reconstructions fit basically to a subject-scientific approach, since they consider mathematics in terms of meanings. Concepts like "fundamental ideas" or "Grundvorstellungen" can be understood as pragmatically determined concepts describing meanings in the context of mathematical learning problems. Didactical reconstructions take also into account the importance of different contexts. In principal "all the various spheres of practice (academic mathematics is one of them) in which mathematics is used are, in principle, relevant sources of meaning." (Biehler, 2005, p. 61) But the elements of reconstructed mathematical subjects are a priori not on par in the "world" as well as in "premises-structures". In (Skovsmose, 2005, pp. 83-85) the author emphasized that meaning "can also be described in relations to social structures, which requires that the whole educational process be taken into consideration" and "it is also possible to ask about the meaning of a (mathematical) task as part of an educational practice." For example one has to take into account that the educational situation at university might corrupt meanings (i.e. possibilities), which were a priori described in reconstructions of a mathematical concept. Taking not only into account social aspects of meaning but also the specific modality of subjective experience could prevent didactical reconstructions to turn into something that was criticized as "conceptualism", see (Skovsmose, 2005). In the end it remains an actual empiric question, whether a specific meaning of a mathematical concept becomes realized and is suitable for a successful learning.

CONCLUSION

Whereas general objections concerning "traditional" scientific approaches were already discussed for a long time, see for example (Horkheimer, 1937), the significance as well as the fruitfulness of "critical" approaches in actual research on subject matter related teacher education seems to be rather limited. Recognizing general deficits of actual dominating approaches could serve as a motivator for further research but cannot replace a thorough analysis and own developments. In the end the practical relevance of "critical" objections can only be proven in actual empirical research processes, which however require suitable basic concepts that are waiting to be worked out in detail regarding the considered field.

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ETHICAL AND/OR POLITICAL ISSUES IN CLASSROOM BASED RESEARCH: IGNORING THE EXCLUDED

Christine Knipping, David Reid

Acadia University

As researchers we have a political aim to understand the exclusion of students from school mathematics so as to be better able to promote changes to schooling that would mitigate the social factors related to that exclusion. But we find that ethical standards we follow as university researchers undermine our political aims and exclude students in our research. In our essay we address this problématique and propose it as a theme of discussion for the conference.

Joe sits at the back of the room, to one side, where our cameras are least likely to record him. We do not speak to him, nor to his mother when she visits in the classroom. Our interest is the way that classroom interactions translate social differences into disparity of achievement in school mathematics, but we do not inquire into Joe's family background and we do not interview him to see how he perceives his place in the class or the world. Officially, we do not observe Joe at all, but we cannot help to observe that he is one of the ones who does not behave in a way that suggests to us or his teacher that he will be successful at mathematics. He is one of the ones who will be at the losing end of the disparity that emerges quickly in his mathematics class. But we may not ask why in his case, and so we do not.

For a university researcher to conduct research in a public school in Canada, the research must be approved by the school superintendent and the university's Research Ethics Board (REB). The Research Ethics Board is guided by the principles set out in the Tri-Council Policy Statement: Ethical Conduct for Research Involving Humans (TCPS). Central to these is the principle of informed consent: "Ethical research involving humans requires free and informed consent" which "refers to the dialogue, information sharing and general process through which prospective subjects choose to participate in research involving themselves." (TCPS, commentary to article 2.1). For school based research the normal procedure is that a permission letter be signed by the students and her/his guardian prior to research beginning.

When a student or her/his guardian does not sign the permission letter the research may proceed, with every effort being made to avoid including data from a non-participating student. This means the student is excluded from being observed, interviewed, taped, etc. Joe did not sign the permission letter, so he is not a participant in the research.

Why didn't Joe sign? It is difficult to ask without either involving him in the research or interfering with his freedom to make the choice he did. In fact, Joe is necessarily a fiction as we cannot report on the behaviour of the non-participants in our research.

Occasionally we get some insight into the situations of students who have excluded themselves from our research by not signing their permission letters. Keith did not return the letter he was given on the first day of school and so he was excluded from the research until we had the opportunity at a parent-teacher night to ask his father if he had any questions about the letter. Keith's father was interested to hear more about the research and signed the permission letter without hesitation after the research was described to him and the terms of the letter explained. Hence, we can report on Keith's situation. Keith's mother is illiterate and so was not able to sign the letter when it came home. Keith's father works long hours and may not have had a chance to read the letter carefully, if he even saw it when it first came home.

The particular case of Keith, which is the only one we have where we can say something about a non-participant, suggests the interpretation of "informed" as "provided with a written letter describing the research" may result in the exclusion from our research of students whose social backgrounds might make them vulnerable to being excluded from school mathematics.

Another path of exclusion occurs through the power relations inherent in schooling. For some students the first time a figure in authority offers them a genuine choice to participate or not in an activity comes when a researcher asks if they are willing to participate in a research project. Some say "no" before hearing the nature of the research, simply to test whether the choice is real or not. When they have tested choices offered by parents or teachers in the past, the reaction to their saying "no" may have indicated to them that the choice was not a genuine one, and so it comes as a surprise when the researcher then proceeds to exclude them from video taping and interviews. We have been asked by students "Why can't I participate?" when the answer is simply that they said (to us and to their parents) that they did not want to. Their decision in this case was made not on the basis of being informed, but simply to test the genuineness of the choice.

It could be said that informed consent is not possible when social factors such as literacy, contact with parents, and power come into play. Participants are required to make a decision before they have a real opportunity to understand the research. Decisions would certainly be more informed if they were made after the research had been conducted. While there are circumstances in which the TCPS allows the normal procedure of seeking consent before the research commences to be altered, providing participants the opportunity to make a more informed decision after the fact is not one of them. This suggests an implicit exclusionary aspect to the standard procedures for securing informed consent, perhaps revealing a particular set of social assumptions held by those preparing and implementing policies of ethical conduct. While following the TCPS seems from that perspective to result in ethical behaviour, the exclusion of some from participating in research on the basis of social background might seem unethical from other perspectives.

Researchers with an interest in social issues have recognised that there are limits to working within organisations that do not have explicitly social agendas. This has led, for example, to the formation of groups like MES to provide a forum for exploring social issues that are marginalised in mainstream groups. But seeking to develop separate granting agencies and ethics policies for research with a social agenda is not a viable option. Is it possible instead to work within granting agencies and university Research Ethics Boards to raise awareness of the political and social implications of existing policies and to work toward acceptable modifications? If it is possible is it worth the diversion of energy from research?

Some might argue that our concerns about the ways that existing ethics policies and practices exclude students who might benefit from our research are misplaced. Observational, non-interventionist research that seeks to better understand that status quo rather than to change it sets up an explicit hierarchy between the observer and the observed. Our "participants" are so called to be politically correct, but it could be said that the traditional term "subjects" better expresses the reality of our research. In contrast, research that begins in a different context, for example action research in which the researcher has a prior and stronger role as teacher, treats the participants as true participants, and in such a context the research comes in the reflections, at a point where truly informed consent is possible.

As researchers we have a political aim to understand the exclusion of such students from school mathematics so as to be better able to promote changes to schooling that would mitigate the social factors related to that exclusion. But we find ourselves in our research excluding those students ourselves. The ethical standards we follow as university researchers undermine our political aims. This leaves us with many questions:

- Is it possible and worthwhile to attempt to shift ethics policies through participation in organisations (e.g., university REBs) that make and implement these polices?
- One circumstance in which it is possible for the requirement for informed consent to be waived is if "The research could not practicably be carried out without the waiver" (TCPS, article 2.1(c)iii). Would the circumstances we have described above qualify?
- Is expending effort on research observing the status quo worthwhile, given our role as outside observers, the methodological compromises required by ethics policies, and critiques from those who would take a more interventionist approach?

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VIRTUALLY THERE: INTRODUCING THE INTERNSHIP E-ADVISOR IN MATHEMATICS TEACHER EDUCATION

Kathleen Nolan

University of Regina

The purpose of the research project described in this paper is to create and sustain a model of professional development for a faculty advisor and her secondary mathematics student teachers during their four-month internship field experience in schools. In the research, the design and use of desktop video conferencing as part of the faculty mentor/advisor role introduces and explores the notion of an e-advisor. This paper highlights the promises of virtual mentoring in the development of mathematics teachers, while also acknowledging that the process of e-advising is not without its real limitations.

INTRODUCTION

In a recent study with secondary mathematics student teachers (Nolan, 2008), attempts were made to mentor student teachers during their field experience (four-month internship) as they negotiated theory-practice transitions from university courses to school classrooms. The mentoring fell short, however, in that it focused on the student teachers' individual experiences rather than recognizing the benefits of participating in reform-oriented mathematics communities. In response to this realization, I initiated a research project to explore the possibilities of a virtual community of practice during internship. Research on the design and use of desktop video conferencing in the mentoring of secondary mathematics teachers is being conducted through a participatory case study approach with three secondary mathematics interns. In this brief paper, I report on several key aspects of this ongoing research project, including a discussion of preliminary outcomes of two recent video conferences as well as a discussion of new directions and possibilities for developing a model of 'e-advisor' internship supervision.

BACKGROUND AND CONTEXT

In the Faculty of Education at the University of Regina, faculty members are assigned to the supervision and advising of several student teachers (interns) during their four-month practicum semester (internship) in various schools in the province of Saskatchewan, Canada. As part of the professional development process during internship, faculty advisors are expected to visit, observe and conference with their interns 3-5 times per semester. While this may be an acceptable approach under some circumstances, such a 'limited contact' approach is generally not conducive to creating and sustaining a relationship with the intern that is supportive of the intern's professional growth. The visits are too infrequent and generally too short in duration for the faculty advisor to make a difference in the theory-practice transitions of mathematics teachers.

The theory-practice transitions of secondary mathematics teachers are an issue that requires special research and attention in order to realize a desired change in teaching practices. From my perspective as a teacher educator, encouraging prospective teachers to reflect and act on new, inquiry-based pedagogical strategies in mathematics is a challenging task. In spite of introducing new strategies during curriculum classes in the teacher education program, traditional textbook and teacher-directed approaches still prevail in many secondary mathematics classrooms (Jaworski, 2001; Lerman, 2001). Through this research project, I am introducing 'virtual' visits with interns in such a manner that the faculty-student conferencing process is ongoing, synchronous, and without geographical boundaries, expanding into the realm of individual office and classroom spaces.

PURPOSE AND OBJECTIVES

Overall, the key objective of the project is to work toward the development of a working model for creating and sustaining an ongoing, synchronous dialogue between faculty advisors and their assigned interns during the four-month internship field experience in schools. Instead of the current model of visiting interns in schools 3-5 times during the semester, this working model would encourage faculty advisors to supplement these 'real' visits with additional 'virtual' visits; in other words, to introduce and explore the potential role of an e-advisor. It is anticipated that such a *virtual* approach will result in two *real* outcomes: (1) a continuum between pre-internship university courses and the internship field experience can be created and fostered to encourage and stimulate professional reflection on becoming a mathematics teacher and, (2) this new model for intern supervision could result in a reduced burden of travel and labour costs associated with supporting faculty advisor travel between the University of Regina and schools throughout Saskatchewan.

In working toward the key objective, a preliminary aspect of the research project focused on researching the strengths and shortcoming of several available products/applications that support a shared desktop and video conferencing. While the findings of this aspect of the research project are reported elsewhere (Nolan & Exner, 2009), it is worth noting that *Adobe Macromedia Breeze* (now called *Adobe Connect*) was selected as the desktop video conferencing tool for use in this project.

PRELIMINARY FINDINGS

Thus far, I have held two web conferences with the three project interns using *Breeze* software. The first conference session barely got off the ground because one of the interns forgot the time and arrived online quite late; another intern forgot her microphone and the built-in computer microphone resulted in too much feedback to understand her clearly; and the third intern did not seem to be in an environment which had a strong and consistent internet signal. In comparison, the second conferencing session progressed quite smoothly, with a goal of exploring the use of web conferencing for collaborative brainstorming on an algebraic functions and graphing lesson. In addition to this content-based focus, the plan was also to conduct a general discussion on the internship thus far and if, as a small community of learners, we could use this virtual time to share some of the successes and challenges experienced. Since my desire was to engage in a flowing discussion, where interruptions and "technical difficulties" (like those of the first conference) could be reduced, I made a point of posting the material to be used on several whiteboards, in the chat pod, on the discussion screen, and on my own desktop. I used this variety of tools within *Breeze* so that if one form/pod failed to work well, we could direct our attention to a different one. While our preparation for, and initial engagement in, this second conference seemed much improved over the first, the overall result was still disappointing. The following quote is taken from my researcher reflections (digital diary), written post-conference.

> In general, I found that there were relatively long delay times between actions and the visual representations of them. In addition, we found that the audio feature kept malfunctioning on us. We each tried to be sure that we held down or locked the talk button when we wanted to share something, but for some reason the audio still cut in and out without any of us having a sense of how to fix it. We tried writing more to compensate for the audio problems, but even the chat tool was slow, making the flow of conversation quite a challenge. We tried collaborating on the white board—I would ask the interns to use the text tool to contribute their ideas on how to use a given mathematics problem to teach students about non-linear functions, but even textboxes were not consistent in format or in delay time; some interns could not even find their whiteboard tools (but they did not experience this problem in our training session!).
>
> At times, it was almost comical. I would ask a question and there would be a long period of silence. I thought perhaps the interns were scribbling on a piece of paper, thinking about and planning how to answer my question. From their perspective, however, they were waiting patiently for me to get on with things, wondering what they were supposed to be doing. As it turned out, I was waiting on a response to a question they did not even hear me ask. The *virtual* environment was becoming a *real* problem!

NEXT STEPS

The two web conferences in this study would not be considered successful in terms of working toward the desired project outcome; that is, to develop a working model for creating and sustaining an ongoing, synchronous dialogue between a faculty advisor and her interns. It is feasible, however, to conclude that there is strong potential for developing such a working model with more time devoted to addressing the limitations.

One limitation of this research project is the issue of student and faculty training in effective use of the technologies. It seems absolutely necessary to either invest considerable time and money into training everyone involved in the use of these more advanced technologies (such as *Breeze*) so that the technology is less of a barrier in working toward the research goals or, alternatively, to use more basic and accessible technologies that the interns are already quite familiar with through other facets of their lives. These more familiar technologies include wiki spaces, web blogs, Google docs, Moodle, tinychat.com, etc. Thus, the 'next steps' of this research project (underway at time of writing) uses these familiar forms of ICT, hoping that the technology tool itself will become more invisible and we will be able to focus our attention on the pedagogical and professional goals of the research.

CONCLUDING THOUGHTS

Student teachers' negotiations of theory-practice transitions from university courses to school classrooms requires an exploration of multiple modes and models for mentoring and professional development. The use of ICTs in education presents possibilities for creating an ongoing, feedback-oriented conversational approach, which could help establish more of a continuum between university courses and internship field experience. By introducing *virtual* visits with the interns, *real* possibilities exist for taking the faculty-intern mentoring process in a new direction— one that encourages student teachers to discuss and grapple with the many theory-practice transitions facing them at their schools and with their students.

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IDENTITY IN A BILINGUAL MATHEMATICS CLASSROOM – A SWEDISH EXAMPLE

Eva Norén

Stockholm University

INTRODUCTION

In this paper I report shortly from a project were some of the complex relationships between school mathematics and discursive practises in one bilingual mathematics classrooms in a Swedish context were explored. My aim was to *explore how bilingual students' identities are constituted in a bilingual mathematics classroom*.

Mathematical teaching and learning in bilingual and multicultural classrooms has often had a focus on language aspects though from different perspectives, such as psycholinguistics, socio linguistics and sociocultural (see for example: Moschkovich, 1999). Research in bilingual mathematics classrooms (among others: Adler, 2001; Moschkovich, 2007) has claimed that the use of students' first language(s) is a resource for teaching and learning mathematics. In addition researchers in bilingualism have argued for bilingual teaching to support students' learning of a second language as well as subject content learning (Thomas & Collier 1997).

These research findings had a huge impact on a number of Stockholm politicians, principals and official school clerks some years ago. They initiated a bilingual teaching of mathematics project in five schools in the suburban areas of Stockholm. The project was running from August 2004 until December 2006. I evaluated the project and also did additional ethnographic fieldwork in two of the schools for one and a half more years. While evaluating the project I found that multicultural issues in mathematics education couldn't be reduced to just multilingual or cultural issues (Norén, 2008; see also Gorgorió & Prat, 2009). Also bilingual students' identity formation and positioning as mathematics learners called for further study.

THEORETICAL CONTEXT

Identity formation can be seen as a constantly ongoing process of becoming; in this case a "school" mathematician, and as constituted by political and institutional processes, which means neither as an individual nor as a social process (Foucault, 1984). According to Lerman (2000) identities are produced in discursive practices and "[d]iscourse carries with it notions of regulation, of the power/knowledge duality of Foucault, /…/" (2009, p 13). In exploring students' identity formation my research considers the influence of dominant discourses in the mathematics practices, but also students' agency.

DOMINANT DEFICIENCY DISCOURSES

Underlying deficit theories tend to be applied to immigrant students who don't succeed in school mathematics even though most researchers today have moved beyond the thinking that it is within the students themselves or their families and culture that are at fault (Gutiérrez, 2002). In Sweden immigrant students in school frequently are constructed as deficient. The lack of competence in the dominant language; Swedish, is an explanatory factor for unsuccessful immigrant students and their low achievement in school mathematics. Research studies in Sweden, though not specifically focusing mathematics education, have shown that deficit theories are applied to minority students' failures in school (among others Parszyk, 1999 and Runfors, 2003). In Parszyk's study students identified themselves as "immigrants" and perceived school was not for them, but for the Swedish students. In Runfors study students were continually defined by teachers *as* the "Other" – the immigrant student – not the ordinary and "normal" "Swedish student. In the defining and categorizing process teachers identified themselves as "well-meaning" and "passionate" about "giving" the students "equal chances". Runfors says the interaction in school circumscribed the children's freedom of action. It is not possible to say how students in Runfors study identified themselves as she did not investigate those aspects. But it is possible to say that there seemed to be little space for the students to identify or position themselves.

OPPOSING DISCOURSES ON BILINGUALISM

According to Lindberg (2002) there are many myths about bilingualism; I call them public discourses on bilingualism as they are what Gee (2008) refer to as people's taken for granted models often reflected through media. Such discourse is that researcher do not agree on the advantages of using students' mother tongue in educational situations in a second language learning environment and that the use of mother tongue should have a negative influence on the learning of a second language. In Swedish research it is reported that Swedish teachers often are adopting such a discourse on bilingualism (Parszyk, 1999; Runfors, 2003). A normalizing discourse that works towards Swedishness, including language, culture, values and habits, is operating within the borders of the institution; the school (Sjögren, 2001).

Contrary, researchers agree on the significance of mother tongue for second language learning and the importance of mother tongue for bilingual students' achievement in school (Lindberg, 2002). The Swedish National Agency for Education (2002) and official policies in general promote bilingualism and multiculturalism.

SOCIAL CLASSROOM PRACTICES

In the project shortly reported in this paper the bilingual mathematics teacher, exercised a reform-oriented (Boaler, 2000) classroom practice where social relationships with students were important. The teacher listened to her students as they, during lessons from time to time brought up experiences from their daily life. It was a social practice, to speak about what happened outside the mathematics classroom and to speak about other things than mathematics in this particular classroom. One example of this is when one of the students, a boy that I call Amir, enters the classroom a minute after they all have started working with mathematics.

The teacher does not ignore comments from Amir, about his physical education teacher being a racist not giving him a pass grade, though he is late and the lesson is supposed to be about mathematics. The teacher challenges his statement and discusses it with him. When they have discussed it for a while, in both Arabic and Swedish, Amir comes to the conclusion that the PE teacher can not give him a pass grade, as he has not attended the lessons that he should. The teacher asks Amir why he calls the PE teacher a racist when the grade or not has to do with Amir's own decisions, not participating in the sports classes. Amir then says he hopes he can talk with the PE teacher about it and that he really not is a racist. He goes on (Arabic in italics):

> I was just so mad when I understood I was not going to get a grade that I called him a racist! I will talk with him and maybe we can agree on me doing additional work. /…/ [saying something to himself, but not hear able,] *I know I will get a good grade in mathematics though* /…/ what are we doing today?

The narrative shows Amir's aspirations and his ability to handle a problematic situation he has put himself into by giving a suggestion to solve the situation. The last utterance gives an indication of Amir identifying himself as an engaged mathematics learner and becoming a good mathematician, as he does at other occasions as well.

Social practices in this particular mathematics classroom may position students regarding their attitudes towards their learning of mathematics. Amir didn't loose his face and he felt comfortable in the mathematics classroom showing an awareness of his future possibilities, positioning himself with good grades in mathematics and an eagerness to go on working with mathematics and to learn more.

The teacher's use of the word *habibi* [the Arabic word for my friend, beloved or darling] when calling for individual students' attention on mathematical matters, and when acclaiming a kind of bonding between herself and the students' as they use the same mother tongue in their homes and vernacular life, is an example of how Arabic filled a social function within school mathematics practice. The students performed out of identities as bilingual. Contrary to earlier findings in Sweden neither deficiency discourses on bilingualism nor normalizing discourses towards Swedishness affected Amir's identity formation in this particular mathematics classroom.

CONCLUDING REMARKS

In this short presentation I have tried to elaborate on how bilingual students' identities may be formatted trough bilingual school mathematics practices. Students seem to gain self-confidence and take responsibility for their learning of mathematics in a school mathematics practice where in teachers and students can use mother tongues on a regular basis to facilitate meaning making of mathematics. And in which bilingual students have the space of becoming mathematics learners on the same terms as Swedish speaking students, formatting identities as responsible mathematics learners.

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RACIST BEAUTY CANON, NATURAL BEAUTY AND CRITICAL MATHEMATICAL EDUCATION

Norberto Jesús Reaño Ondarroa

Universidad Pedagógica Experimental Libertador

This work is characterized by a claim from a perspective of critical mathematics education, to address a problematic situation that is deteriorating, because what has been conceived as Natural Beauty possibly leads men and women to a strong inability to see their natural essence. Therefore, it is important to generate proposals that allow humans to learn to value and appreciate their selves and their environment. All this with the aim that through the review of mathematical education, it will contribute to society, in a revolutionary way, to set up a language that gives meaning to the creation of the natural elements that make up the universe.

RACIST BEAUTY CANON

A problem that has boomed since the late fifties and has been exacerbated in the last ten years, both in Venezuela and Latin America, has been the promotion of a female image that attaches too much importance to physical appearance, evidenced visibly in the Latin American obsession to fulfill their dreams of reaching the (90-69-90) body measurements, according to a report prepared by Hare (2009) in Montevideo. Finally, the idea of selling human beauty and all that it represents, enables the birth of a new culture, which leads to see the man and woman like industrial products, as evidenced by the mobilization of millions of dollars used by women, for body changes through cosmetic surgery, a practice which, according to Hare (2009), increased over 200% in the region over the past ten years, which has generated great concern, since it goes against Latin American multiracial reality, it states that "the model of beauty that prevails in the region that comes to mind is the North: white skin, light eyes and blond hair(phenotypic characteristics totally alien to our Venezuelan and Latin American reality).

CRITICISM OF MATHEMATICAL EDUCATION

The problem described above, which is a first approach to the proposed theme, shows the importance of reflecting on the contributions that mathematics and science educators can develop in achieving Latin American and cultural welfare. But, referring to education as useful to society, requires educational proposals, which according to Moschen (1975) are not exclusive to these, as the problem of the incorporation of the person(s) in a particular organization, but rather prioritize the realization of being. Therefore a fundamental educational proposal may be to foster a critical and emancipatory comprehensive training for all citizens, whose primary purpose is "to achieve personal development in everything that improves their social dimension". It is thus important that mathematical education explains, understands and transforms the social facts, such as the case of the racist standards of beauty among others, taking into account the students as individuals and the historical and social context associated with the individuals' world, whose role should be oriented towards the total emancipation of the people. Because of this I discuss an example of how to enter criticism from mathematical education to the problem of a racist beauty canon.

THE CIPHERED NATURE OF RECONCILIATION: THE NATURAL BEAUTY AND A FIGURE OF RECONCILIATION

The study of nature and all beauty that it implies, leads to the search for patterns, which can be developed from mathematics. That is why the golden number is considered (or the number phi), which serves as a standard in this case, to narrow the gaps between women, men and nature to which they belong, rediscovering the existence of the harmony and balance of power in order to be valued as human beings and to value everything that surrounds them. That is why I show an example of three tasks done about this, which seek to recognise the link between natural beauty, symmetry and proportion. Having inserted the photo of the student at the computer, with the assistance of Cabri Geometry, we proceeded to calculate the distance from the foot (point I) to the head (point G), $GI = 10{,}45$ and the distance from the navel (point H) to the head (point G), $HG = 6{,}43$. Following the principle of Euclid (larger segment / lower segment), finding the ratio between these lengths we obtain: $\frac{GI}{HG} = 1{,}625194....$ this result approaches the golden number ($ö = 1{,}61803398 87.....$).

Similarly we proceeded to find the ratio for the distance between the tip of the middle finger of either of your hands, in this case the left hand (B) and left shoulder (A), the result was, $BA = 4{,}27$ and the distance from the tip of the same middle finger (B) to the respective elbow (C), we have $BC = 2{,}63$.

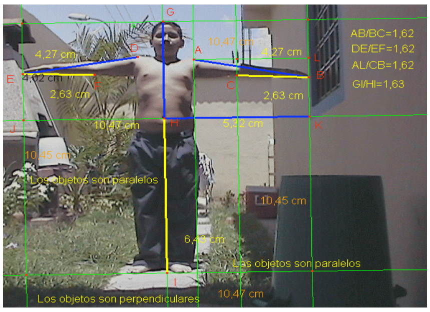

Figure 1. Obtaining the golden number through the medium proportion and the extreme proportion in the human body

The reault of finding the ratios between these lengths was: $\frac{BA}{BC} = 1{,}623574...$ noting that this result also approximates the golden ratio ($ö = 1{,}6180339887.....$). When carrying out the same procedure with the other arm we found similar results, fulfilling the mathematical parameters of symmetry and proportionality. It is funny but important to note, that the participant is inscribed in a rectangle whose side lengths are determined by its height and distance from the tips of his fingers (arms outstretched), which are approximately the same. In other studies, we examined the human body (basic education student), in order to obtain the balance and harmony through the relationship between the parts in all, starting with the body position that was intended by the student, and using similar procedures as Leonardo da Vinci did in the Renaissance age, who considered the man as the measure of all things and represented the human body with the five-pointed star (symbol of the Geometric man of Vitruvio). This work with the human body and the mathematical elements, allowed a glimpse of the presence of phi in the expressions of nature. Finally, for purposes of this work, phi becomes the pattern that gives meaning to the creation of the natural elements that make up the universe, making it clear that men and women are naturally beautiful.

CONCLUSIONS

From the universe around us (to which we belong) and the study of mathematics, a language emerges and gives meaning to the creation of the elements that compose it, prevailing characteristics of men and women who make up, promoting, the rescue of human beauty. The release of the peoples of the southern hemisphere should be the primary point of departure in education and the goal to which mathematical education can contribute. Mathematical education is to counteract the series of events that operate against the essence of being human and social construction. Men and women must learn to value themselves as natural beings in order to assess their environment, not wanting to deform it in the way mercantilist prescriptions suggest. We have to move away from the natural beauty canons and assume the standards that are typical in nature, to which men and women belong, linked to symmetry and proportion, and contribute to the care of these, considering them as beautiful reservoirs. An important aspect, according to Moya (2008), is that teachers see the interdisciplinary nature of mathematical education and its relation to different areas of knowledge and that these are linked to different aspects of knowledge, such as ontological-epistemological, technological and sociological, which allows them to improve learning and then obtain a product in different areas in which mathematical education takes place, such as information technology and communication, education and health, among others.

In conclusion, education in mathematics is to serve society in a revolutionary sense, develop critical thinking and foster democratic dialogue with all citizens who are members of that society, which requires the cooperative and collaborative work of other sciences.

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INTENTIONS FOR LEARNING MATHEMATICS

Henning Westphael

Aalborg University Denmark

The aim of this presentation is to discuss a theoretical framework for researching student's intentions for learning mathematics for education from a social cultural perspective. I will combine the theory of identity by Sfard & Prusack and the theoretical notions of foreground, intention and learning by Skovsmose.

INTRODUCTION

How do student teachers go about the business of learning about learning and teaching mathematics? Research has shown that often teacher education has not had the impact on the students that is expected (Ponte & Chapman, 2008; Gellert, 2009). In my Ph.D. project I am focusing on the students intentions for learning by participating in the mathematics teacher education. The learning actions the students perform, at the teacher training college, I see best described as learning by participation in the discourses of teaching mathematics, thereby highlighting the link between the education and the practices. This approach also embraces the unique feature of the teacher education that "*what they are learning is also how they are learning"* (Liljedahl et al., 2009, p. 29). Student teachers communicate about mathematics and the learning and teaching of mathematics, and thereby they participate in the use of objects, mediators and rules specific of the mathematics teacher discourse (Sfard, 2006).

For students to benefit from their learning actions according to the intentions behind the learning activities, students' intentions for learning need to intersect with the intentions behind the teaching. Whereas the intentions behind the teaching activities are traceable along the way from the official curriculum to the enactment of the educator in the teacher education, the students' intentions are a more difficult matter to research. My aim for this paper is to combine the notion of intentions based on the work of Skovsmose (1994) and a discursive approach based on the work of Sfard and Prusack (2005) to build a theoretical framework for researching the mathematics teacher students' intentions for learning.

INTENTIONS AND IDENTITIES

In my attempt to characterize student teachers' intentions for learning my starting point will be (Skovsmose, 1994) where the notion of *intentions for learning* is developed as a part of a theoretical setup around learning in action. The basic idea is that the intentions for learning, connected to a learning action, spring out of a person's dispositions, characterized by foreground and background (Alrø, Skovsmose & Valero, 2009; Skovsmose, 1994).

Sfard and Prusack (2005) see identities as narratives about individuals that are reifying, endorsable and significant, and divided into actual– and designated identities. Learning in this context is the bridge between the actual identity and the designated identity seen as being able to participate in a (mathematical/educational) discourse (Sfard, 2006). I would like to argue that dispositions can be understood as a certain kind of identities (narratives of first kind – see later) with foreground equivalent to designated identities and in similar way background equivalent to actual identities. This makes a connection between identities as narratives and intentions for (learning) actions. Such a connection has already been suggested in (Stentoft & Valero, 2009), but I would like to elaborate it further focusing on the notion of intention.

Connecting foreground/background and identity

Let us consider the following small extract from an interview [1] about participation in the teacher education.

Interviewer:	Why did you choose mathematics as your first main subject?
Student:	I have always been good at mathematics. I like that there is one right answer to the questions. Not like it is in Danish, where you can discus everything.

Here we see a student, who tells a story about herself being good at mathematics. It is clearly a part of her background, and a reason for choosing mathematics as her first main subject. It is also a narrative told by her about herself to the interviewer. It is reified by the fact that it is a story that stems from a number of actions, in which she has shown to be able to do the mathematical tasks asked of her. It is significant as it is given as part of a reason for choosing mathematics. It is only partly to be in coherence with the rest of the answer, but it is sufficient for this example.

My argument is in general, that foreground/background always will be narratives of the first kind (aAb) (Sfard & Prusack, 2005) since foreground is defined as a person's interpretation of her own future possibilities. Similar is background defined as a person's history made of socially constructed network of relationships and meanings. Whenever a person (a) relates to narratives of second or third kind, it will happen by retelling the story. This will make it a narrative of the first kind (aXb), where (X) is a narrative in itself of either type (bAa) or (bAc), and as such a part of that persons dispositions (foreground/background).

When we look at the above statement, we can see that she in this interview setting presents an expectation of liking mathematics at the teacher training college more than Danish, based on her understanding of what mathematics is. The student choosing mathematics because she is good at it, which relate to her background, can both be seen as the student intending to do well at the study to become a mathematics teacher, or the student intending getting through the study as easy as possible. That is two different intentions relating to the same statement, that we will have to explore further, looking at other parts of the interview. This is shows that researching students' intentions is not a strait forward task, and that there will always be an element of interpretation in creating a picture of the students' intentions for learning about learning. This brings me to a discussion of how to research intentions of learning.

RESEARCHING INTENTIONS FOR LEARNING

The intentions come to exist through the learning action —the participation in the discourses— and are as such related only to that particular situation with no special representation that exists prior to the action (Sfard & Keiran, 2001). An account (in retrospect) of the intentions behind a given action will always be a reconstruction relative to the context in which it is told. By virtue of that new context the intentions will be different, but bear a 'family resemblance' to each other (Lerman, 1998). Focusing on the participation I will, Inspired by Anscombe (2000, 1957) divide intentions into three categories: 1) Intentions of a given act of participation, 2) Intentions of obtaining B by doing A and 3) Intention stated without followed by an act of participation.

Another way to look at intentions is to focus at the expectations (aims and goals) characterised by different dimensions:

- Expectations narrated in different timeframes in the sense that they can relate to a) expectations to be fulfilled here and now or b) expectations that reaches in to the future.
- Expectations relate to different narratives within either foreground or background.
- Expectations are always relative to the context in which they are formulated and the intension behind the words (Lerman, 1998).

In the above example we saw the student having expectations for the future building on her background, and we saw her making a choice of subject to obtain something else, even thou it is indecisive what it is she wants to obtain. This indicates that these two ways of looking at intentions can be a fruit full start in looking at the empirical material recognizing, that evaluating the material will most likely produce yet other ways of describing students' intentions for participating.

CONCLUSION

In this paper, framed in a discursive view on learning, I have presented a notion of intentions, which opens up the possibilities of researching students' intentions for learning by participating in mathematics teacher education. By focusing on students' dispositions as narratives and the relations to intentions, I am developing tools helping me to interpret students' intentions relative to these narratives, and thereby enabling me to look for the needed intersections of students' intentions for participating and the intentions behind the curricula taught.

NOTES

1. This interview was conducted as a pilot study in my Ph.D. project. I had group of three students who commented on results from a small survey. The statement is translated from Danish by the author.

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RESEARCH PAPERS

ACTION-RESEARCH IN THE VENEZUELAN CLASSROOMS

Rosa Becerra Hernández

Universidad Pedagógica Experimental Libertador

This paper proposes an alternative framework for educational research in a critical and emancipator way in Venezuelan classrooms, which raises a number of elements that enable a different form of the relationship between theory and practice to emerge, in which knowledge is seen as a process marked by dialogue between equals. This way of conceiving research leads to the formation of democratic values and, consequently, a new conception of citizenship. We have developed participative action-research in our university, in which reflexivity is critical and essential, and which can be a support in the necessary process of transformation and emancipation as a legitimate concern of a society like that of Venezuela, struggling to be the protagonist of its own destiny.

RESEARCH IN TEACHER TRAINING

The teacher training prevailing in Latin America has been marked by changes originating from realities other than ours. Each new policy or project starts from scratch and the knowledge and experience gained in previous attempts made in each country or region is unknown. (Torres, 1996). In addition to the above, Messina (1999) in her research about the state of the art of teacher training, in the nineties, in Latin America shows as one of the most important findings the fact that the region remains under the aegis of knowledge transfer as a paradigm in teacher education, where research is not a structural element of pedagogical praxis. This traditional training school completes its philosophy by treating students and their teachers as beings incapable of building knowledge intersubjectively and transforming the social environment of education. The author also shows the "poor relation of training institutions with the surrounding sociocultural reality" (p.2).

Venezuela does not escape the reality of the region and this poses a challenge which is deeply rooted in the thinking of the Venezuelan educationalist Prieto Figueroa (1968), who, a few decades ago, stated the fundamental characteristics of a true educator to be:

- confidence in education as a force for human life transformation and as a tool for changing social structures;
- faith in the Future, which is projected towards his/her educational work;
- confidence in educational opportunities, the possibility of change being received by the education and the society where it operates, and
- ability to put all material and spiritual resources at the service of educational work.

These characteristics expressed by Prieto Figueroa concur with our development focus, the human being in different social contexts. He also asserts the human being’s ability to transform the environment. Despite admitting the undeniable power of the school as a transmitter of the prevailing ideology, the author believes in the potential of students to transform their social surrounding and to convert the school into an alternative place of creation.

From this way of thinking, the teacher is one who favors the construction of knowledge and takes an investigative and critical perspective on his training. The teacher education model that we propose meets “a complex perspective, critical and constructive...” which “...involves a strategic goal, an investigative conception of teachers' work” (Porlán, R., Martín del P., R. Martin, J., Rivero, A., 2001, p.15), where knowledge is not by any means neutral.

However, these approaches are far detached from Latin America reality, and research and knowledge accumulation have not converted into an improvement in the lives of its inhabitants. Here, knowledge has been appropriating by a

minority, suiting it to their particular interests, and this has allowed the persistence of situations such as school dropout and exclusion, illiteracy, unemployment, asymmetric relations in the distribution of income, among many other problems we face.

This paper proposes a break with a conception of research that has addressed only the mere accumulation of knowledge. It posits a form of research linked to our educational practice, education governed by a profoundly human condition that includes a thorough understanding of the society in which we are engaged and able to build a critical consciousness, together with the understanding and interpretation situations, leading to the implementation of action plans that allow a real transformation of a reality that has been imposed by various mechanisms of power.

We agree with Freire (1990) that to "replace just a naive perception of reality by another criticism is not sufficient to liberate the oppressed". Rather, it is an inescapable duty of those who believe that a better world is possible to create the space needed to advance research that is committed to knowing and doing, with participation and action, and with the development of a critical consciousness that leads to transformation processes.

A CRITICAL AND EMANCIPATORY VISION OF RESEARCH

As teacher educators and researchers in the field of education, we consider it essential to orient readers to what we conceive as our educational practice, seen as a social fact determined over time. For this educational practice in a particular social context to reach its highest level and become dynamic and fruitful it should include research (Ruiz and Rojas Soriano, 2001). This indissoluble union of teaching and research is supported by Freire (1974) who assures us that "Education and research theme in the conception of problem-posing education, become moments of the same process" (pp. 131-132). Therefore, we conceive research coupled with educational practice, adding to this a valuable tool for reflection and action that will allow researchers to improve their teaching-educational intervention. This type of educational practice leads, as proposed by Ruiz and Rojas Soriano (2001), to "allow individuals to form critical of its historical reality and interested in the construction of knowledge through their involvement in specific research" (p. 118). Thus, a first element that characterizes our idea of research will be its role to recreate and transform the teaching task.

A second consideration is the recognition of the concept of the theory-practice relationship, and the attempt to form a single dialogic unit, where the educational account of the theory is determined by how it relates to practice and the way this practice changes our theoretical references. We must become aware of the alleged dichotomy of theory versus practice; this is a false dilemma to be unveiled (Becerra and Moya, 2008a).

From the relationship of theory and practice, we consider a third element that relates to the fact that all research involves a knowing, a wanting to know about something, so it is necessary to make explicit our considerations of what is meant by a deep understanding of the subject is being addressed. In the first instance, we assume that knowing is always a process that does not end with the completion of an investigation. Successive approximations are shaping truths that can be temporary and shared. This leads to a demystification of knowledge from something static and unchanging, that is done, to a process, in transit, en route (Bigott, 1992).

Based on the foregoing, we understand knowledge as a dialectical process, where "my vision" does not prevail over the "vision of the other," where my beliefs are not more valid than those of others. This would lead to the formation of a fourth element. Therefore, dialogue is an essential tool of research, understood as something more than a simple conversation or a lively exchange of ideas. This dialogue involves the confrontation of different views around common interests, not with the intention to impose an idea or to consider others less successful, but with the intent to understand, to know and to advance the search for truth that is shared with others (Fierro, Fortoul and Rosas, 1999).

A fifth fundamental element of our research work is that the reflection and construction are not done alone; man is a social being, a historical being. For us in the ontological, epistemological and axiological dimensions that mark our work as researchers, the pursuit of knowledge is a social fact, which is nourished by my views and the visions of others. The construction of knowledge in our classrooms makes sense within its real possibility of social relevance.

A sixth element is the relationship, not always respected for methodological pluralism, between epistemology and methodology in the context of education; we seek to make it more constructive and critical.

We tried in our investigative journey to understand and explain how we gain knowledge of reality and to unravel the interpretations and understandings that make it up (Becerra, 2003, 2006).

Against this background we believe it is our duty to make explicit the rationale we sustain. We assume a perspective where, from the interaction of individuals with reality and the dialogue among themselves, meanings emerge. Understanding that individuals can construct different understandings of the same reality, but if we promote dialogue and sincere arguments between them, they can construct knowledge in their relevant social interaction and their own reality, thus overcoming distorted individual understanding. Therefore, we move away from the false dichotomy of subject and object, as objectivity and subjectivity are, from our epistemological perspective, mutually constitutive.

A seventh element that guides our work is linked to the inalienable right to participate actively and consciously in the construction of a new citizenship. To increase such participation and that of our students as part of a research and education agenda committed to the development of man/woman as a social being (Becerra and Moya, 2008b). The scope of citizenship compels us to build an ideological framework in which a citizen should mature fully committed to their society. Thus, we oriented the research for the development of critical educators that, as Martin claims (1997, pp. 24-25):

a. perceive “the interdependence of seemingly unrelated facts and phenomena”;

b. expand their responsibilities and bear the consequences of their actions, showing a shift to perceive that “the effects it causes in others are not desirable”;

c. argue for their views, not impose them;

d. accept the reasoning of others and question their own;

e. recognize Individuals as mediated by society, their training and community of practice;

f. recognized themselves as assets that can influence the collective improvements;

g. confront reality with what should be, realize the injustice of certain situations and put forward ways to overcome them;

h. make use of dialectical thinking including the “consequences of an act or phenomenon, think in terms of possibilities of a sign (which generates benefits and to whom) and opposite (which causes damage and to whom)”;

i. ask for arguments which are open to examination.

As the eighth and last but not least important element, according to the socio-critical paradigm research cannot be considered a neutral field, because we all, consciously or unconsciously, will choose the rules that guide it and no researcher escapes them. We share the very appropriate idea of the research process in education presented by Bigott in his book Alternative Research and Popular Education in Latin America (1992). This Venezuelan teacher conceives research as "...a process of knowledge production that is being socialized and produces cracks in the monopoly of knowledge" (p. 106).

The combination of these eight elements leads us to propose Emancipatory Action-Research as an alternative for the development of our educational practice, a choice that confronts entrenched conceptions of knowing and learning, struggling to break a status quo that has masked realities and that has led to domestication processes. We are committed to research that interprets and comprehends facts, but dares to go beyond, that transcends the necessary understanding to progress towards the transformation of that reality.

PROCESSES OF EMANCIPATORY ACTION-RESEARCH

The process that characterizes action-research differs in several respects form other researches. We therefore considered as an option the sequence developed by Venezuelan educator Carlos Lanz (1994):

a. Framing the issue: this concerns the approach to the participants through open discussions, conducting presentations on critical issues affecting the group or practice established.

b. Object required: targets are designed and action plans are initially developed.

c. Delineating the object of study: We answer questions like What, Who, Where and When and delimit social action, social subjects and the spatial and temporal dimension.

d. Reconstruction of the object of study: We favor the synthesis and the placement of some aspects of the object and the measurement of knowledge are combined.

e. Theoretical and methodological perspective: The theoretical and philosophical perspective are examined and discussed, outlining the premises of action-research and defining theoretical keys.

f. Directionality of research: The proposed change from analysis and reflection of collective praxis is defined.

g. Operational Design: We define techniques and instruments for collecting information that take into account the characteristics of the object of study. The information is classified in thematic units, such information is categorized and the theoretical development is done using a comprehensive-explicative approach.

h. Conclusions and Results: The results are obtained by crossing different sources of information gathering and different actors.

A VENEZUELAN EXPERIENCE IN CLASSROOMS

In correspondence with all previous approaches are two experiences in the Universidad Pedagógica Experimental Libertador in Venezuela, where we tried to approach a reality that is constituted not only by external events but also by the variety of meanings, symbols and interpretations issued by the subject himself interacting with others.

In both studies we followed the guidelines of Action-Research as a methodological option. The categories for the analysis and interpretation of information emerged from a critical documentary study of in-depth interviews conducted with students and the information obtained from participant observation. Interpretation and organization of the collected information, also called data encoding, was performed, following the approaches of Strauss and Corbin (2002, p. 13), through three types of procedures: a) conceptualizing and reducing data, b) developing categories based on their properties and c) correlating them. The process of Triangulation (Martinez, 2000) was used to compare the collected information, the verification of interpretations and the processing of the results outlined.

The first study was conducted with students from the programme in Integral Education (teachers from grades 1 to 6) and we proposed, through action-research, to build a participative methodology strategy for the course of Geometry. Flexible action plans were designed, which could be modified depending on the work done during the course.

As an example we show information classified under the category "Theoretical Assumptions". This was developed by students in small groups and concerned mathematical theorems and relations corresponding to the contents of units on Triangles and Quadrilaterals. Workshops were held to resolve problematized situations, to draw conclusions on each group and compare the arguments in plenary, where part of the work was done. At the end, written work was submitted, including the assumptions and justifications prepared to sustain each group's position. In reviewing this, perhaps the most interesting observation was the domain the students had for their justifications. The theoretical assumptions made in most cases were adequate, even though difficulties may have appeared in the problem solving process.

Some of the information provided by students during the course development are reported below.

a. Julmi a key informant, responding to the request for description of the process of theoretical assumptions in her group:

   Julmi: At first it cost us a lot, but then little by little, we were integrating and taking postulates and theorems and it was something we didn't have to read and memorize, but we used many strategies to see if what we were telling each group would verify, trying everything, it cost us a little, but later we were seeing and checking.

b. Interview with another student of the course:

   Interviewer: How was the process of developing the theoretical assumptions?

   Betty: To develop the theoretical assumptions proved to be very difficult, but at the end we draw conclusions, the teacher asked us to clarify and so it went around and we realized that we had in the group.

Interviewer: How did you perceive the process?

Betty: At first I didn't believe it, then I reviewed my books and saw that really what we draw in class as the conclusion was what was written in the book.

Both students realize how difficult such strategies were for students and their complexity. The extracts also indicate the students' lack of experience in solving various problems and trying to draw conclusions common to a variety of them. Similarly, the student shows disbelief in her capacity, and that of her peers, to build knowledge.

The approaches outlined in this research show the reflection and transformation that take place when in the act of education the students are the true protagonists of their learning and become aware of their potential. This is not to leave students alone, but to prepare the educational ground so they are the protagonists and owners of knowledge, with the appropriate involvement and illumination, but not the imperative, of their teachers.

The second study was conducted with students from the programme of Teachers of Mathematics (high school teachers). The students, organized in small groups, should design projects related to high school content and having relevance for the local community or for Venezuelan society in general. Once the draft projects were designed, workshops were held in secondary schools near the university to present and exchange views with teachers of these institutions.

Below we illustrate part of one interview with student number 2:

Interviewer: What was the subject addressed in your project and how did you pick it?

Student 2: Water, for its importance, and also a participant of the group was in the "technical working groups of water" that were being organized at that time in the barrios of Caracas, when we began the project and he told us that there was something interesting to do with the water and from that we began, read the materials and made the choice.

Interviewer: How did your project relate to mathematics and society?

Student 2: We focus on the problems of drinking water, water for human consumption, in fact we worked with some chart from some neighborhoods, with a water consumption graphic in a residential area we could see at what time they consumed more water, if at noon, or night, ...at dawn it decreased, from that graph we constructed the function concept, worked what was the domain, range, slope of a line, we really worked much about it.

The responses of this student permit us to visualize one strategy that promotes the study of mathematics through current social problems, allowing students to be aware of the reality and to act on these problems. In this project the students found substantial differences between the thickness of the tubes that carry water to the different neighborhoods depending on the socioeconomic status of its inhabitants. These findings promoted discussion around the concept of equality that lay hidden in the distribution of the drinking water to the population.

Both studies are part of what we intended to develop knowledge, to socialize it and to make students aware of their reality and the role they should play in its transformation.

REFLECTIONS ON A PROCESS UNDER CONSTRUCTION

In the first place, we cannot forget that we live in a capitalist society and, although we think that the economic relations and social class dynamics can explain everything that is of particular importance to the investigation, we cannot ignore, as Apple claims, that "Its influence means to set aside some of the most insightful analytical tools that we possess" (1997, p. 177). Therefore, it is our duty to train future researchers within organizations that reproduce unequal class relations in our society and to prepare them to make more democratic and egalitarian institutions. In this approach the proposal of an emancipatory action-research has an exemplary role to play.

A research proposal such as the one we have been building, characterized by criticism, reflexivity and respect for man and woman, cannot and should not be forced. The sustainability of the strategy involves changes in attitude, performance and organization. Although these have started in some areas, the fact remains that the most profound changes take time to permeate organizations and break the rigid structures imposed from various fields of power.

This transformation occurs as a cluster of inescapable uncertainty and doubt. Nevertheless, we believe this is the way to constitute for ourselves a theoretical and methodological benchmark with real ethical and political aspirations within the framework of dialogical reasoning.

The characteristics of any emancipation-research-action process such as the one we propose cannot be develop in a hasty manner. Changes and transformations that begin to loom will go deeper and permeate the various organizations, in both formal and informal ways, to the extent where each student is ready to assimilate them and the rest of the group provides sufficient support and encouragement to move forward.

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REPRODUCTION AND DISTRIBUTION OF MATHEMATICAL KNOWLEDGE IN HIGHER EDUCATION: CONSTRUCTING INSIDERS AND OUTSIDERS

Christer Bergsten[1], Eva Jablonka[2], Anna Klisinska[2]

[1]Linköping University, [2]Luleå University of Technology

The design and conduct of calculus courses has been and is an object of curricular debates and reforms. By the reconstruction of the establishment of the mathematical sub-area now called 'calculus' and its fundamental theorem as a piece of institutionalized mathematical knowledge for the purpose of its reproduction, we reconsider the notion of knowledge recontextualization within the field of knowledge production by showing that standardization of knowledge evolves in a dynamic relationship with its production. Interviews with mathematicians suggest that they, as teachers, create different recontextualization principles. In undergraduate teaching calculus they suggest to include the criteria of the field of knowledge production (e.g. proof) for the future 'insiders', while for those who will not pursue a career within this field, the 'outsiders', the criteria change towards computational efficiency.

INTRODUCTION

Calculus courses have a prominent position in undergraduate teaching in a diversity of academic areas. Steen (1988, p. xi) even claims that "calculus is a dominating presence in a number of vitally important educational and social systems". The design and conduct of such courses has been and is an object of curricular debates and reforms. As an example, in the 1980's there was a public debate in the U.S.A. about the need to improve a situation described as problematic with far reaching social and economic consequences:

> Nearly one million students study calculus each year in the United States, yet fewer than 25% of these students survive to enter the science and engineering pipeline. Calculus is the critical filter in this pipeline, [...] . The elite who survive are too poorly motivated to fill our graduate schools; too few in number to sustain the needs of American business, academe, and industry; too uniformly white, male, and middle class; and too ill-suited to meet the mathematical challenges of the next century (ibid.).

The outcome of the debate was the 'calculus reform', based on the insertion of applications and use of technology, which amounts to a weakening of the classification of the content. Similar debates are currently taking place in several countries. These debates bring to the foreground the issue of the institutionalization of knowledge for the teaching of mathematics in higher education: how is the classification established and who gains access?

By our case study of the emergence of the delineated sub-area that came to be called 'calculus' we show that the standardization of this mathematical sub-area along with its concomitant knowledge claims evolves in a dynamic relationship with its production. The historical study also points to the emergence of an independent recontextualization field for higher education out of the field of production. Interviews with researching mathematicians, some of whom are also teaching calculus courses, display that they suggest different recontextualization principles for different groups of students. This differentiation also emerges in calculus textbooks.

RECONTEXTUALIZATION IN TERTIARY EDUCATION

When discussing the structuring of pedagogic discourses, Bernstein (1990) describes a recontextualizing context operating "between" a primary context where the production of knowledge takes place, and a secondary context of knowledge reproduction. The latter is divided into four levels, i.e. pre-school, primary, secondary, and tertiary. Fields structured by the recontextualization context are defined by positions, agents, and practices, whose function is to "regulate the circulation of texts between the primary and secondary contexts" (Bernstein, 1990, p. 60). By a "principle of decontextualizing", this process of recontextualization changes the text through a delocation followed by a relocation subordinated to the rules of the field of the relocation. Once the decontextualizing principle has regulated the new ideological position of the text, a second transformation is taking place within the field of reproduction in the pedagogic process of teaching and learning.

Bernstein's theory of the pedagogic "device" is mainly concerned with the recontextualization of knowledge into the *school* curriculum, its functioning in the production, distribution, and reproduction of official knowledge and its relationship with structurally determined power relations. For the school curriculum, the discourse from the field of knowledge production (mainly universities) is recontextualized by agents in the Official Pedagogic Recontextualizing Field (OPRF). Other discourses, in particular those in the Unofficial Pedagogic Recontextualizing Field (UPRF) are also recontextualized (Bernstein, 1996). In the process ideology comes to play.

At institutions of higher education it is hard to identify a distinct unofficial recontextualizing field in times where there are no reform movements, and a weak influence of an official recontextualizing field on the curriculum specification; usually there are only general rules for carrying out examinations at universities. There are no clearly delineated recontextualizing fields, but individual recontextualizing agents, who are sometimes at the same time researchers. This situation has been observed by Bernstein (1990, pp. 196-198):

> The recontextualizing field brings together discourses from fields which are usually strongly classified, but rarely brings together the agents. On the whole, although there are exceptions, those who produce the original discourse, the effectors of the discourse to be recontextualized, are not agents of its recontextualization. It is important to study those cases where the producers or effectors of the discourse are also its recontextualizers.

A question to ask is therefore how the process of institutionalization of knowledge in a specific field of knowledge production is affected in cases where its members act as its recontextualizing agents. How does this situation affect the recontextualizing principles? We start with the suggestion that the discourse in undergraduate mathematics teaching is a pedagogic discourse. That is, there operate criteria of evaluation that differ from the ones operating in the activities of researching mathematicians. These criteria, together with the principles of selection, organisation, sequencing and pacing also contain a model of the learner, the teacher, and their relationship. For example, it is demonstrated by Bergqvist (2006) how success on exams in calculus courses in Sweden requires only memorizing of rules and examples. The pedagogic discourse may differ for students aiming at a career within the field of knowledge production and students enrolled in less specialised study programs, but at many places students from different programs are put in the same courses for economical reasons.

RECONTEXTUALIZING THE CALCULUS

The development of the Fundamental Theorem of Calculus (the FTC) changed the classification of sub-areas as it linked integration and differentiation. We are interested whether the recontextualization of the calculus keeps the classification or not and whether there is a dynamic between recontextualization and production in its historical development. In addition, we ask how members of the field of knowledge production see this area when they act as agents of recontextualization. For this purpose, we re-address the set of interview data and the outcomes of the historical investigation from Klisinska's (2009) study about the didactic transposition of proof.

The standardization of calculus and its fundamental theorem

The development of the statements connected with the FTC, which gave rise to a new classification of knowledge that became institutionalized, was studied with reference to original works of researchers and classical works about the history of calculus. As indicators of institutionalization we considered the reference to a sub-area or to a proposition with a common name as well as textbook or handbook appearances. Our study of the propositions related to the FTC and of the names used for basic concepts in calculus covered well-known early textbooks. As there was no independent recontextualizing field, we separated 'textbooks' from research publications by their intention to address an audience with less specialized knowledge in the area of knowledge to which the sub-area under consideration belongs. The use of a shared name (in some variation) for the theorem was a criterion for the selection of later textbooks. For the more recent textbooks, the choice resembles a "longitudinal cut" with some examples from different decades and from different countries. Only the formulation of the FTC and its proof were investigated.

Classical outlines of the history of mathematics commonly trace the ideas of calculus back to mathematicians in ancient Greece [1]. It is also common to refer to Leibniz and Newton as "inventors" of the modern calculus in a personalised history of mathematical “heroes”. However, Leibniz and Newton did not invent the same calculus, and did not set out calculus as a well-defined sub-area of mathematics as they differed in problems studied, approaches taken, and methods and notations used (Boyer, 1959; Baron, 1987).

Calculus changed a lot before developing into the form that is presented in introductory courses, especially in terms of the changing criteria for the field of knowledge production (formalization and standards of proof). The early development of the limit concept was crucial for the standardization of the calculus. By using limits as the basis for definitions, Cauchy’s work established new criteria. The collection of useful methods was integrated by definitions and proofs. These were complemented later in the 19^{th} century with the formal ε-δ definition of limit by Weierstrass, the definition of the Riemann-integral and a set theoretic definition of function. The recontextualized versions of the elements from these different areas make up the calculus in today’s undergraduate teaching. In this development, standardization of knowledge for the purpose of teaching was one motive for the changing criteria. In the historical development, it is not easy to differentiate between criteria for the field of knowledge production and for its reproduction. Universities became the centers of both mathematical training and research, which led to the development of pure mathematics as an independent field (Jahnke, 2003), with internally socially shared knowledge codes for legitimate productions.

While the first developments in calculus were communicated entirely within the field of knowledge production through personal communication, soon textbooks for the wider distribution of calculus appeared. The first printed textbook in differential calculus appeared in Paris in 1696, by de l’Hospital with the help of Johann Bernoulli. From the introduction (1716 edition) it becomes clear that the name ‘Integral Calculus’ [“Calcul integral”] was already in use. Thus, by having a specific name it had gained an ‘official’ status as a classified part of knowledge to which one could easily refer. However, what was signified by this name changed considerably.

The *École Polytechnique* in Paris was established to increase the number of engineers needed to maintain the new French Republic. The school was kept after the counterrevolution to serve the military. That mathematical knowledge was considered important for these purposes, is apparent from the admittance rules:

> Les élèves n’ayant obtenu leurs admission qu’après avoir satisfait à un examen sur l’arithmétique, les éléments de la géométrie et ceux de l’algèbre, c’est état de leur instruction dut être pris pour point de départ, et il fut établi que les connaissances mathématiques enseignées a l’Ècole comprendraient l’*analyse* et la *description graphique des objets* (Fourcy, 1828, p. 42).

Cauchy’s *Cours d’analyse* from 1821 and *Résumé* from 1823, written for The *École Polytechnique*, were the first textbooks in which calculus appeared as an integrated body of knowledge with clear borders towards other mathematical areas. It included a proof of a proposition that looks like what now is called the FTC. We interpret the textbook as an attempt to provide access to a knowledge code promoted in the field of mathematical knowledge production. There was no different code for the initiation into the fields in which the knowledge was supposed to be applied.

Also in other early textbooks the propositions related to what is now called FTC are not named, but in the *Course d'analyse mathématiques* from 1902 by Goursat, translated from French into English already in 1904 and widely spread, the "fundamental theorem" refers to the fact that "every continuous function f(x) is the derivative of some other function". Later in the book this name is used also in the context of complex analysis. In the textbook *An introduction to the summation of differences of a function* by Groat, printed in 1902, the expression "the fundamental theorem of the integral calculus" appears, as well as the more short "fundamental theorem". In *The theory of functions of a real variable & the theory of Fourier series*, published in 1907 by Hobson, one chapter has the title "The fundamental theorem of the integral calculus for the Lebesgue integral". That the name of the theorem serves as a chapter title as well as extended to a more general application indicates a strong level of institutionalization. Wiener refers several times to "the fundamental theorem of the calculus" in *Fourier transforms in the complex domain* from 1934. That this name became standardized is evident from the classical book *What is mathematics?* from 1941, where Courant and Robbins use the chapter title "The fundamental theorem of the calculus", and in a simplification of the history write (p. 436):

> There is no separate differential calculus and integral calculus, but only one *calculus*. It was the great achievement of Leibniz and Newton to have first clearly recognized and exploited this *fundamental theorem of the calculus*.

The textbooks mentioned all contain a proof of the FTC as criteria for its status as legitimate knowledge, but there were also textbooks published at the beginning of the 20th century written for an apparently growing body of non-academic readers, as for example *Calculus made easy* by Silvano Thompson from 1910, which recontextualizes computational algorithms and mathematical notation in everyday discourse.

The development of the calculus shows that the process of institutionalization of a body of knowledge has to be seen in relation to the practices of its circulation and reproduction; there is a dynamic between the fields of production and reproduction. For example, reference to Cauchy's scholarly work is commonly made by drawing on his textbooks. That his textbooks became popular and his exposition of the calculus was generally adopted (Boyer, 1959) can be explained by the combination of its influence on the field of knowledge production through applying a knowledge code that became internally socially shared, and the relative autonomy of teaching that accounts for its circulation. In this case the fact that a producer is at the same time a recontextualizer affects both, the unmediated and the pedagogic discourse.

An interview study: producers as recontextualizing agents

For the interview study, the principle of selecting extreme or atypical cases was used in the selection of eleven mathematicians from universities in Canada and Sweden. They have diverse backgrounds in education in different countries and work in different, mostly unrelated, areas of research in mathematics, and have varying teaching experiences. In the interviews the first question aimed to reveal their most spontaneous conceptions of the FTC. Then they were presented eleven examples of formulations of the FTC from textbooks from different countries and periods in history, and asked which they considered closest to the FTC as they understood it. Other questions raised issues of significance, meaning and reference of the FTC, as well as issues linked to the teaching of the theorem.

When answering the first question, six of the interviewees used the word ‘inverse’ when describing the FTC, saying that differentiation and integration are ‘inverse processes’, ‘inverse operations’, or simply ‘inverses’. In only one interview it was pointed out that the integral should be defined as a limit of a sum, and not simply as an antiderivative, to make the FTC interesting. Three persons mentioned versions of the FTC for multivariable functions or with weaker assumptions. In general, the interviewees had a quite informal approach to the formulation of the theorem without providing assumptions or a remark about possible parts of the theorem. Only those with the experience in teaching calculus, were more careful. The criteria for the pedagogic discourse seem to be more strict than those used in these conversations.

The big variation of the answers to the question where eleven formulations of the theorem were given reveals substantial differences in how the mathematicians evaluated the given formulations as matching their own views. Reasons given for accepting or not accepting a formulation often indicated that a personal view was expressed.

When discussing issues related to teaching, different ‘versions’ of the calculus and of the FTC for different groups of students were mentioned by nine interviewees. Some point to a difference for students in different tracks, others differentiate between levels of calculus courses, and some refer to different needs and dispositions of students in the same course:

> I think you have to adapt it [teaching] to the kind of students you are working with. If you have an honours class [ ] you can go a bit further. But really, if they had as good understanding of the basic principles as Newton and Leibniz did, that would be extraordinary (I6).
>
> We do it [the FTC] for honours class [...] one teaches the FTC in a way by attempting to I don’t want to say trivialize it so much as reduce its scope and make it manageable (I4).
>
> I am saying that it is different how I am teaching at the first year calculus and it is different when I have a real analysis, where I am proving the theorems. Very big difference. Completely different kind and way of thinking about the theorem (I10).

> It depends. We are talking about the first course, I guess. If they understand in an intuitive way that will be enough for them (I8).

In the first course the FTC is presented as "a recipe" for how to find the function that satisfies some requirements, followed by a number of almost the same exercises. Then, "nothing is beautiful with fundamental theorems of calculus":

> Why we should explain for first year students that this theorem is beautiful? Maybe we should not? Maybe it is a recipe and that's all. And why we should, why we should force people outside the university to understand that some theorems are beautiful? Should we? I don't believe so (I10).

Engineering students are by some interviewees seen as a group who do not have to gain access to the criteria of the field of knowledge production:

> And for engineering students: It's just engineers. They would never think that something is beautiful in mathematics (I10).
>
> Here at the University of Technology we usually don't spend much time on the theoretical parts of the foundation of the subject (I11).

A distinction between students who might grasp the FTC and its proof and those who would not was commonly made in terms of their working habits and intellectual dispositions:

> I feel I could do more [...] if there was some very good student who gets A+ no matter what, if there was some students who don't work and get F no matter what. But there are some students in between then I could help (I3).
>
> With a bright bunch of students, I would like to go into it a bit more hoping that some of it will stick so that when they come back to it later on they'll have something to build on (I6).
>
> And then I would prove it but not according to Newton [...] I would use a straight forward proof with whatever; the limits and you know the mean value theorem [...] And maybe, maybe some people will understand the proof, maybe (I2).

Thus, in the interviews a distinction was made between different students, which was crucial for the way they should or can be taught. Most beginning students and engineering students, as well as those described as not having the right dispositions, were seen as needing mainly access to the computational aspects of the FTC and of calculus. In contrast, for special groups of students and for the more advanced course, the proposed discourse could be described by a knowledge code belonging to the field of production. In their reasoning the interviewees rely on common sense discourses about the needs and dispositions of the students rather than on a recontextualised version of a discourse from pedagogy or psychology. In an interview question about how they would attempt to explain the FTC to beginners (where a situation was outlined with initial work on a problem using a velocity graph), the uniformity of the answers shows that the proposed pedagogic discourse remains at an informal level, relying on visual impression, their "intuition", and on approximations not described by means of specialised mathematical language. This can be seen as an expression of a view that the students imagined here, need not be invited to take part in the internal code for mathematical knowledge production. These answers, as well as views expressed in the previous quotes, represent an insider-outsider perspective where only a small community is seen to be able to take part in and get something out of theoretical work in mathematics, while only technical skills need to be taught to the 'outsiders'.

OUTCOMES AND DISCUSSION

According to Bernstein (1990, p. 60), there are at least four recontextualizing fields involved in the shaping of the school curriculum: official educational authorities, university departments of education, specialized media of education, and fields not specialized in educational discourse but able to influence it. The data presented in this paper suggest that when the distance between producers and transmitters of knowledge is reduced the influence form these fields is also reduced.

However, an important outcome of our findings is that the fact whether an agent from the field of knowledge production at the same time is a recontextualizing agent as such does not make a difference for the criteria that are transmitted in pedagogic discourse. There is a difference in this scenario in the historical and in the present context of higher education. While in the historical context the criteria for the two discourses (the unmediated and the mediated) match or are developed in dynamic relationship between producers and transmitters, in the present context the mathematicians suggest a switch as soon as pedagogic discourse is at issue. That is, there seems to be a recontextualizing principle that constitutes insiders and outsiders.

Another outcome in relation to the process of knowledge recontextualization is the fact that as textbook writers for higher resp. tertiary education, researchers contributed and still contribute to the institutionalization of the mathematical sub-area of calculus. In the historical account some examples were given where a person relocates outcomes of her/ his own knowledge production into the field of reproduction. When producing a textbook, the text was by other scholars brought back to the field of knowledge production, contributing both to the standardization of knowledge and to further developing the criteria for knowledge production. At present time when persons from the field of production of mathematical knowledge teach undergraduate calculus courses, the knowledge to be transmitted is not resulting from their own knowledge production. As recontextualizing agents for tertiary mathematics education, their recontextualizing principles seem to construct 'insiders' and 'outsiders', based on a metaphor of participation for the former, responding to the interests internal to the field of knowledge production, and on the alleged applicability for the latter, responding to perceived external interests (from study programmes in which mathematics is a service-subject) only in the utility of mathematics. Some rely on a discourse that naturalises the distinctiveness and inaccessibility of mathematical knowledge.

It is not evident why a massification of higher education should lead to the abandonment of transmitting the code for knowledge production (by leaving out the proof, for example). This might as well be a coincidence of two separate developments. However, the move is vindicated by an ideology of the exclusiveness of the code (by mathematicians) and fed by the demand for applicability of academic knowledge on the side of technical instrumentalists who become more influential in tertiary education. Arguments of applicability of the knowledge for the outsiders can be interpreted as a means to make the students complicent of the strategy. On the other hand, the computational version of undergraduate calculus courses can also be seen as an outcome of a progressive agenda attempting to make it more accessible and relevant. In the 'calculus reform' in the U.S.A. that made calculus more applied and computer based, reference was commonly made to its use value. "Calculus now is more important than ever", because:

> the most serious reality we face today is the need to harness science and technology for economic growth. And harnessing science and technology for economic growth means harnessing calculus. (Steen, 1988, p. 6)

It is worth considering that the power relations in the structure of academic fields related to mathematics are not aligned with the simple distinction between pure and applied mathematical sciences. The distinction has always been a battleground for ideology. It is not clear that the cultural capital of being educated as a theoretical mathematician is easily exchangeable into economic capital. From this perspective, there is no need to restrict access to studies in pure mathematics. Keeping this classificatory principle and the symbolic capital of being a theoretical mathematician still remains possible by means of policing, most prominent with strategies that draw on knower characteristics and pointing to the distinctiveness of the objects of interest in the field of knowledge production. Some of the interviewee's comments might be interpreted in this light.

What is transmitted to the insiders and also is reflected in the classical books about the history of calculus, is based on a view that the acceptance and rejection of mathematical theorems is primarily, or even usually, a matter of evidence or reason that follow an internal logic of rational evolution of knowledge towards more "exactness". The concomitant view of the development of mathematical knowledge corresponds to 'Science as rational knowledge' in Callon's (1995) classification of models that account for the dynamics of science. In this model the actors are the researchers themselves restricted to their role as researchers. According to Callon, this model inherits a "tragic beauty" in that "it is the scientists and scientists alone who have to choose which statements to preserve and which to discard" (p. 36). In relation to the account above, the tragic beauty is its ignorance of the social history of mathematical knowledge.

A CONCLUDING REMARK

The study and our discussion make it obvious that the calculus and the FTC as part of a modern undergraduate programme can be analysed as being purely a social construction. The above discussion points to the space left for ideology in this construction. However, the knowledge transmitted in such a curriculum is not fully arbitrary either. One could exchange the example from Boyle's Law to the FTC in Young's (2005) observation (and perhaps also exchange "the Chinese" into another cultural group):

> However, what is distinct about the formal knowledge that can be acquired through schooling and that therefore needs to be the basis of the curriculum in any country is (a) the conceptual capacities it offers to those who acquire it, (b) its autonomy from the contexts in which it is developed (the Chinese are interested in Boyle's Law but not in the gentry culture of which Boyle was a part). (p. 14)

NOTES

1. See Juschkewitsch and Rosenfeld (1963) for alternative interpretations of the historical roots of calculus.

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DISCOURSES OF ASSESSMENT ACTIONS IN MATHEMATICS CLASSROOMS

Lisa Björklund Boistrup

Stockholm University

This paper presents some tentative results of a study on assessment actions in mathematics classrooms, as they appear in the interaction between teacher and student. The study has four research objectives and in this paper results concerning one objective, discourses of assessment in mathematics, are presented. The theoretical frame-work is an institutional/discursive perspective coordinated with a social semiotic perspective. For the analysis additional theories are included. When it comes to discourses of assessment and/or of mathematics education, a dichotomous picture is often described in literature. The results in this paper broaden this picture, including drawing attention to aspects concerning students' agency in assessment in mathematics.

This paper presents some preliminary results from an ongoing classroom study on assessment actions in mathematics classrooms. In this sense, this paper is an empirical paper. At the same time, the paper is an example of what a coordination into one theoretical framework of two theories, institutional/discursive (Foucault 1969/2002, 1971/1993) and social semiotics (Hodge & Kress 1988; van Leeuwen 2005), can contribute. In this sense, the paper can be considered as a theoretical one.

CLASSROOM ASSESSMENT IN MATHEMATICS

The concept of classroom assessment in this paper is taken to be a concept with broad boundaries. Obviously, assessment takes place explicitly when students are given their mathematics test results. But often, assessment is implicit during teacher-student interaction in learning sequences. One example is the following: a student asks the teacher about a certain mathematical "rule" and wonders where it comes from. The teacher's answer, by way of different communicational modes, shows that this particular student does not have to bother about such a question. S/he is just asked to follow the rule. When another student asks the same question, the teacher engages in a discussion about the historical development of this particular rule. The first student in this example learns, through this implicit assessment, that the teacher does not consider her/him capable enough to understand this kind of reasoning. My assumption is that both the explicit assessments and the implicit assessments in mathematics classrooms play a key role for students' learning.

The purpose of the ongoing study is to describe and understand aspects of classroom assessment that have potential to afford possibilities and restrictions for students' learning. Since at least Black & Wiliam (1998), there has been a call for classroom studies in depth in this area, and there still is a need (e.g. Hattie & Timperley, 2007).

I address four research objectives concerning: kinds of assessment actions; aspects of mathematical competence; roles of different communicational and representational modes; and institutional discourses of assessment in mathematics. The results presented in this paper are primarily related to the fourth research objective, but they are also connected to the first three,

In the ongoing study the interactions between teacher and students in five classrooms of 10-year olds (fourth grade in Sweden) are analysed. Two students in each class are (for ethical reasons) randomly chosen and the analysis is focused on the interaction between these two students and the teacher. The data material, from which the examples in this paper come, consists of video recordings and written material.

THEORETICAL AND ANALYTICAL FRAMEWORK

The two main perspectives are briefly described here and additional theoretical structures that are operationalised in the analysis are presented.

Institutional/discursive perspective

Drawing mainly on Foucault (1969/2002, 1971/1993), one of the two main perspectives in this paper is an institutional/discursive perspective. Assessment in mathematics education is taking place in school where there are institutional aspects present. Institutional aspects have both direct and indirect effects. Decisions may be made at different "levels" in the school system, which have a direct impact on the classroom work. There are also indirect aspects, such as classificatory systems, norms and dominant discourses (traditions) developed over time.

Discourse according to Foucault (e.g. 1969/2002, 1971/1993) is a broad notion which incorporates not only all 'statements' but also the rules that affect the formation of the possible statements in the discourse. By this, the discourse is more than everything that is communicated and the way it is communicated. The discourse is also present in what is not communicated, or what is communicated through gestures, attitudes, ways of being, patterns of actions, and the rooms and furniture. According to Foucault, discourses contain a limited numbers of 'statements', that is, discourses are finite. Other features are that they have a history, they have social distribution and they can be realised in different ways (Foucault, 1969/2002, 1971/1993; van Leeuwen, 2005). Discourses are materialised into discursive practices where the discourses are maintained by the ones that participate in the practice. My understanding of the term discourse is to be seen as being in line with a dynamic view, where "the thinking and meaning-making of individuals is not simply set within a social context but actually arises through social involvement in exchanging meanings" (Morgan 2006, p 221). This dynamic view involves a stronger position for the individual, and *agency* is another concept that is operationalised in the analysis described in this paper (see also Mellin Olsen, 1993).

Social semiotic perspective

The other main theory in the used theoretical framework is a social semiotic perspective with a multimodal approach (Hodge & Kress, 1988; van Leeuwen, 2005). In a multimodal approach, all modes of communication are recognised and have to be taken into consideration for example in research on assessment in mathematics. In O'Halloran (2000) there is an interest in three semiotic resources/modes: mathematical symbolism, visual display and language, and the author addresses the impact that the multisemiotic nature of mathematics has on classroom discourse. In this paper, and in relation to assessment in mathematics, the range of possible modes is considered broader, including modes such as gestures and gazes, pictorial elements and moving images, sound. Modes according to e.g. van Leeuwen (2005) are seen as socially and culturally designed in different processes of meaning-making, so their meaning changes over time. Kress (2009) argue for the importance of understanding multimodal communication to be able to fully understand a phenomenon as assessment. Language, in the sense of communication, "may serve as a crucial window for researchers on to the process of teaching, learning and doing mathematics" (Morgan 2006, p 219).

Assessment of learning from this perspective is about acting on signs of learning, as shown by different communicative modes (see Kress 2009, see also Pettersson 2007). This perspective is based on an understanding of learning as an increased engagement in the world, and as an increased capacity to use signs, modes and artefacts for meaningful communication and actions (Selander 2008).

Inspired by Halliday (2004), social semioticians usually talk about three communicative *meta-functions*: the ideational, the inter-personal and the textual. In Morgan (2006), these functions are used with a focus on the construction of the nature of school mathematics activity. In this paper the three meta-functions contribute to the focus of the construed discourses. The *interpersonal* meta-function is about how language (used in a broad sense in this paper) enacts "our personal and social relationships with the other people around us" (Halliday 2004, p 29). In this paper it concerns what kind of assessment in the form of feedback is taking place in the interaction between teacher and student. The *ideational* meta-function is related to human experience and representations of the world (Halliday 2004). In this paper it concerns what aspects of mathematical competence are represented and communicated in the assessment actions. The *textual* meta-function is related to the construction of a "text", and this refers to the formation of whole entities which are communicatively meaningful (Halliday 2004), Here the focus is on what roles different modes play in assessment in mathematics classrooms as well as on how modes are accepted by teacher and students.

Discourses of assessment in mathematics education

When it comes to institutional aspects of Swedish mathematics education, a dichotomous picture is often noticed (e.g. Persson 2009). On the one hand, the discourse of mathematics education is seen as “traditional”, whereby students are expected to spend a good deal of time solely on solving all the problems in a textbook. On the other hand, the “wanted” discourse of mathematics education emphasises a joint exploration in which, for example, students are invited to be active participants in problem-solving. These two discourses of mathematics education in Sweden are similar to the discourses described in the literature on assessment in general. For example Broadfoot and Pollard (2000), drawing on Bernstein, present two discourses of assessment: A ‘performance’ model and a ‘competence model’. The two discourses of assessment in mathematics that are a starting point for the analysis in this paper can be summarised in the following way:

“Traditional” discourse	**“Active participant” discourse**
The teacher is the only one who assesses,	The student is also part of the assessment
Focus on teacher’s guidance	Focus on the teacher promoting thinking
Focus on the correct answer, the product	Focus also on processes
Focus on the number of finished tasks in the textbook in mathematics	Focus on the quality of the mathematical accomplishments
Focus only on the aspects of mathematical competence the student shows on her/his own	Focus also on the aspects of mathematical competence the student shows when working with peers
Focus only on written tests in mathematics	Focus also on documentation of the learning in mathematics

Table 1: Assessment discourses, with inspiration from Lindström & Lindberg (2005)

In Björklund Boistrup & Selander (2009), we kept to these dichotomous discourses. In this paper, I broaden the scope of discourses in relation to the findings of the study. Another discourse that will be related to the results is Walkerdine’s (1988) test-discourse. In this discourse the teacher poses “unreal” questions, questions to which the teacher already knows the answer of.

Additional concepts used in the analysis

In the ongoing study, each of the three meta-functions, ideational, interpersonal and textual, is a base for a respective research objective (described in Björklund Boistrup & Selander, 2009). To make the description and construal of the discourses as “thick” and elaborated as possible the three meta-functions also serve as inspiration and structure here, in relation to the fourth institutional/discursive research objective. There are thus some additional concepts in use. These are briefly described here:

For the interpersonal meta-function three kinds of feedback (Hattie & Timperley, 2007) are operationalised: *feed-back* – what aspects of competence has the student shown?; *feed-up* – how can the aspects shown, and future learning and teaching, be related to stated goals?; and *feed-forward* – what aspects of competence might it be best to focus on in future teaching and learning? How to “go about it”? These three kinds of feedback can go in two directions, from teacher to student and vice versa. The latter can be the student giving feedback to the teacher or the teacher using the students' shown learning in mathematics as feedback for the teaching. For the ideational meta-function, four kinds of possible focus for the assessment are operationalised: *self* – the student as a person, e.g. "You are (I am) good in mathematics."; *task* – the product/result, e.g. the answer to the task or what the student should do (instead of learn); *process* – what is required e.g. to solve a task (there is a clear focus on the (shown) knowing and/or learning); and *self-regulating* – the student as the driving force of her/his learning. There are also aspects of mathematical competence (Skovsmose, 2005) in use here: *Mathematical knowing itself*; *Practical knowing* (knowing of how to use mathematical knowing); and *Reflective/critical knowing* (a meta-knowing for discussing the nature of mathematical constructions, applications and evaluations). For the textual meta-function the focus is on which *modes* and *artefacts* are used and what role they play. This includes e.g. to what extent there is an acknowledgement for the student to use any mode s/he wants and also when there is a restriction of modes, why this is the case. The *time*-mode has appeared to play an essential role for some of the discourses. These concepts along with the concepts derived from the main theoretical framework, especially *agency*, are present in the descriptions of the discourses that are found in the visited classrooms.

ANALYSIS AND RESULTS

Following a social semiotic perspective with a multimodal approach the transcription of the video material has been performed multimodally. The transcription along with the coding is done in the software Videograph. For all categories there are elaborated criteria. The categories are based on the concepts presented in the earlier section of this paper. A similar process is done with the written material. The process of construing the discourses has so far included these steps: (a) using the dichotomous discourses in an early attempt of interpreting discourses in the material; (b) broadening the first two discourses through capturing diversions from the two 'starting-discourses'; (c) choosing the most solid ones among the first suggestions; and (d) elaborating on the discourses when using the meta-functions as a base as well as bringing in a few new features. Four preliminary discourses are construed (drawing on a method described in Foucault, 1969/2002, p36).

"Do it right and do it quick" (1):

In this discourse *feedback* is mostly from teacher to student. Questions posed by the teacher are "unreal" questions (Walkerdine, 1988) and there are rarely follow-up questions. *Feed-forward* concerns what to *do* next (as opposed to what to learn). There is not much *feed-up*. The focus is on *task/product* and mainly whether an answer is right or wrong. The focus can also be on doing instead of learning when the teacher emphasises practical issues. Occasionally there is a focus on student's *self*. The used *modes* and *artefacts* are the ones that are stated by the text-book. When it comes to *time*-aspects, both teacher and student talk in short sentences and there are rarely longer silences. The main agent in this discourse is the teacher and the student's possibility for active *agency* in the discourse is not high. One exception might be a student who takes on the teacher's role.

"Anything goes" (2):

There is not so much articulated *feedback* in this discourse, apart for a lot of *approval*. Also here the feedback is mainly from teacher to student, but the student is encouraged to contribute to the discussion. Both real and "un-real" questions are posed. There are few critical discussions about students' solutions, and wrong answers can be left unquestioned. Focus is mainly on *task*, but there is also some focus on *process*. Both *mathematical knowing* and *practical knowing* are focused. Different *modes* and *artefacts* are welcomed, and *additional* modes and artefacts, e.g. manipulatives, apart from those mentioned in e.g. the textbook, are introduced occasionally by the teacher and/or the student. Modes are never excluded. Teacher and students use short sentences and there is not often silence. Also in this discourse, the teacher is the most active agent. There seems to be a high possibility for the student to also take part as an active agent, since there is so much "positive" approval going on. My interpretation is that this is, in fact, not the case. When the teacher values the student's performances so often, the teacher simultaneously takes the role as the main agent, "the one that is judging".

"Anything can be up for a discussion" (3):

There is a lot of feedback (*feed-back* and *feed-forward,* and sometimes *feed-up*) going on in this discourse, both in direction from teacher to student and the other way around. Mostly real questions are asked, and teacher and student often show interest in each other's reasoning. The focus is mostly on *process* and *self-regulation* and on *mathematical knowing* and *practical knowing*. "Wrong" answers are also starting-points for discussion, but there is always, in the end, clarity about what can be counted as mathematically correct. Different modes are acknowledged. Sometimes the teacher *restricts* the use of some *modes*, and this seems to be for promoting a process. There is not much silence. Teacher and students communicate in longer utterances (e.g. sentences), but not more than a few utterances at each time. In this discourse the possibility for the student to take active agency seems quite high. This is especially clear when there is feed-up as a "neutral" comparison between the students shown knowing and stated goals.

"Reasoning takes time" (4):

Also in this discourse the *three kinds of feedback* are present and in *both directions* between teacher and student. Sometimes the feedback is shown by *silence*. The posed questions are real ones, and there are signs of interest, sometimes mutual, between teacher and student. The focus is mainly on *process* and *self-regulation*. All three aspects of *mathematical competence* can be present including *reflective/critical* knowing. Different *modes* and *artefacts* are acknowledged and the use of modes/artefacts can also be restricted, when promoting a certain process. In this discourse *silence* is common and the possibility (for both teacher and student) to be silent seems to promote mathematical reasoning. Teacher and student can both be active for a *longer time-period*. The possibility for the student to take active agency again seems high. The possibility to be quiet and think for a while seems to promote this possible agency along with the extent to which there is a "neutral" comparison between students' shown knowing and stated goals

Examples of two of the four discourses

In relation to the first discourse, *Do it right and do it quick*, the example is from a lesson where the students are working by themselves in the textbook. The student Catrin is waiting for Cecilia, the teacher, to come and check her finished diagnosis. In the first line of the transcript, the students' speech (SS) and the teacher's speech (TS) are noted. In the next line, we find the students' and teacher's gestures (SG and TG), and in the bottom line the students' and teacher's body movements and gazes (SB and TB). The actions that occur simultaneously are written above each other. Cecilia comes to Catrin's desk and both look at her work:

Time

SS: *B*

TS: *"1. Which angles are straight?"* *A and* *Yes, good.*

SG:

TG: In one hand red pencil, ready to write in notebook. Writes *R* in Catrin's notebook.

Other hand pointing at task in text-book.

SB: Looks at notebook and text.-book back and forth Looks at angles in text-book.

TB: Turns to Catrin. Looks at notebook and text.-book back and forth

Is standing behind Catrin leaning over her head.

The same pattern continues for two more questions: Cecilia reads the question and Catrin answers the same as she has written in her notebook. Cecilia marks *R* with her red pencil. Suddenly Cecilia addresses how Catrin is writing in her notebook: *What big numbers you have done!* Cecilia writes the number of the task in the margin of the page and tells Catrin to do the same in the future. During the sequence there are no longer silences and the utterances are short.

The reasons why this is considered to be an example of the first discourse are: (a) The only feedback is in the direction from teacher to student; (b) There is a focus on the correct answers to the tasks (signalled already at the beginning of the sequence by the red pencil) and there are no follow-up questions. Later, the focus is not on mathematics, but on the correct way to write and draw in the notebook; (c) The only modes are the ones used in the text-book and there are few silences and short utterances; (d) Possibilities for the student to take active agency seem few.

For the third discourse, *Anything can be up for a discussion*, written material may serve as example. In this case it is a document from the school used for parent/teacher/student meetings. The same structure is used for all these meetings in all classes at this school. First there are two pages where the student is asked questions. These are expected to be answered before the meeting. The student Ali has answered yes to the question whether it is *important to gain knowledge at school* and no to the question whether he *takes own responsibility*. One can read that he thinks that *I am good at* a few things in mathematics and that *I want to get better at* a few things. Then there are pages for the teacher to fill in before the meeting. One can see that for mathematics Anna, the teacher, considers *the knowledge status* for Ali to be "G?". G is explained as Good in relation to the goals. When it comes to *Working concentrated and goal oriented* Anna has marked "G–" (minus) and when it comes to *Exercising and accounting for home-work and assignments* she has marked "MbI" (Must be improved). The final document is filled in during the meeting itself. There are spaces for comments on both short-term and long-term goals. For long-term goals we can read this:

Content	**School's contribution**	**Student's contr.**	**The home's contr.**
Strengthen your math knowledge.	*Provide assignments suitable for Ali.*	*Work good. Concentrate*	*Help Ali with home-work and remind him.*

The reasons why this document is considered to be an example of the third discourse are: (a) There are possibilities also for feed-up. Feed-forward concerns student's as well as teacher's courses of action; (b) There is a focus on the learning process and the student's self-regulating; (c) There are several possibilities for the student to take active agency.

DISCUSSION

These institutional discourses can be seen as part of "traditions" developed over time. I mean that each of the four discourses have similarities with (at least) one of the discourses in the dichotomous picture described earlier in this paper, and thus, I argue that they have a *history* (van Leeuwen, 2005, referring to Foucault). All four discourses are found in several of the visited classrooms, which mean that they seem to have a *social distribution* (*ibid*). It is also clear that the discourses are *realised in different ways* (*ibid)* in the interaction between teacher and student, e.g. in different kinds of educational situations, in the video material as well as in written material. All these aspects can be viewed as indirect, but nevertheless they seem quite clear. Institutions are present in these indirect aspects, on one hand since they take place in the institution of school, and on the other hand since the institutional facts (Foucault, 1969/2002) in the discourse can be perceived to be as concrete for the people involved as other, more easily observed and experienced, "facts". The presence of the institutions is considered more direct when it comes to "frames" such as e.g. documents from the municipalities or schools. In these documents it is possible to find one or more of the presented discourses. Since the participants are expected to follow these documents during a parent/teacher/student meeting these discourses have direct impact at least during this meeting.

Students' and teachers' interactions are part of different discourses and this is really obvious in the visited classrooms. As I see it a student (or teacher) always takes agency in some discourses, sometimes in an "assessment in mathematics education discourse" and sometimes in totally different discourses (Mellin-Olsen 1993). This means that a discourse is steering the individual regarding what is considered "good" and who has the authority to act and so on. The individual, on the other hand, has the possibility to take part in another discourse instead. This dynamic view offers possibilities for teachers and students to take active part in the teaching and learning through participation in possibly alternative discourses. The student's possibilities to take active agency in one of the discourses presented in this paper are to a high extent dependent on the discourse itself. However, it is also a matter of the interplay of discourses in the classroom. For example, if the discourse of "Anything goes", with a lot of approval, is common in one particular classroom the student might not be empowered to take active agency when the discourse of "Reasoning takes time" is suddenly introduced by the teacher. The student may be in one discourse, "Anything goes", while the teacher is in another.

CONCLUSIONS

In this paper, some preliminary findings concerning assessment in mathematics classrooms are presented. The combination of an institutional/discursive perspective coordinated with social semiotics has proven to be fruitful. Drawing on the three meta-functions for the construal of the discourses has contributed to more elaborated and focused descriptions than would be the case without them. Moreover, the multimodal approach has shed light on the role of different modes. Here the mode of time is especially mentioned. Silence, and absence of silence, play an essential role in several of the discourses, as does the length of utterances. When adding a focus on agency as well, the roles of teacher and students have been emphasised.

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DILEMMAS OF STREAMING IN THE NEW CURRICULA IN NORWAY

Hans Jørgen Braathe

Oslo University College

The paper describes aspects and dilemmas concerning streaming of students according to ability in an ongoing developmental project in a small town in Norway. The project is part of a governmental effort to implement the New Norwegian curricula plan "Kunnskapsløftet[1]". This new plan introduces streaming of students, which is new in the Norwegian school context. The data presented are from focus groups involving teachers in which ideological conflicts are identified.

INTRODUCTION

Norwegian schools are strongly influenced by ideologies associated with the principles of collective teaching and learning and equal rights for education. This is partly a curricular effect of 60-70 years of social democratic politics and striving for social and economic levelling, including the equal right members of society should have to obtain positions in society regardless of their parents' socio-economic status (Telhaug 2005). Hence at the start of the new millennium nearly 100% of Norwegian students are in public schools, all written plans (curricula) for all levels and for all disciplines all over Norway have the same form and function, there are no marks given before year 8, until 2003 permanent and structural streaming based on marks or ability was not allowed, and there is no choice for specialisation in disciplines or branches before year 12 (of 13). Sources of this strong and nationally shared egalitarianism can be traced in the political and cultural history of Norway.

Braathe and Ongstad (2001) have located and problematized some major ideologies in mathematics education such as rationalism, activism, competitivism and 'autodidaktism' on the one hand and egalitarianism on the other, and asked how these are challenging the Norwegian egalitarian mathematics classroom. In 2001 Norway got a right wing government. One of the main tasks this government took on was to revise the curriculum plan for the school system from 1st to 13th grade. This new curriculum "Kunnskapsløftet", implemented from 2006 on, represent to some extent the same challenges, where solidarity and egalitarianism are in essence challenged by competition and inequality. This process can be traced to international trends connected to OECD's use of education as indicator for economic status for participating countries. This again has resulted in revised curricular plans in various countries (Grek, 2009). School mathematics is one of the subjects that have been used as such an indicator and is strongly influenced by these trends. The teachers of mathematics experience this in their everyday teaching as there has been a focus on the quality of their job. This focus has so far in Norway been negatively charged, especially for teachers in the primary schools.

The right-wing government decided to allow schools to organize the children into groups according to perceived ability, which had been forbidden by law from 1978 on. This was a major break with the egalitarian ideologies widely accepted in the Norwegian social democratic school system. It initiated many discussions among teachers and school leaders. This reform, together with a focus on the contemporary push in education for outcomes-based learning where students' progress is mapped against levels, led many schools all over Norway to leave the old system with heterogeneous classes as the organizing unit and to stream the students into ability groups. These groups are organized according to perceived ability in mathematics following standardized tests given to students at the beginning of the school year. The same groups are often kept in other subjects as well.

This breaks with earlier principles introduced in M74, the national curriculum for compulsory education from 1974. This provided a curricular framework ('rammeplan') in general and for each subject, a means for differentiation within a common, supposedly shared culture. In mathematics the quite open plans gave general guidelines for the teaching which enabled different students to work on different topics and levels, and at different paces and depths, within the same subject area. At the same time, all students took the same final exams at the end of grade 10 (grade 9 before 1997). A general critique of the framework plans from 1974, 1984 and 1987 by educators in mathematics, was that they were too open and non-specific. This criticism contributed to the more specific guidelines provided in 1997 for each school year as well as the definition of minimum levels of subject matter for the common final exam at the end of grade 10. However the framework ideals were kept in the sense that it gave room for individual differentiation, since all goals were formulated as areas that the students are supposed to work with.

Kunnskapsløftet breaks with this tradition and is the first curricular plan to be goal-oriented. It formulates the goals as "students shall know…". Differentiation becomes adjusted education, and allows for streaming of students from first grade on. The expressed intentions are still that all students shall reach the same knowledge goals and meet the same final exams at the end of grade 10.

CHANGE OF PRACTICE

The Norwegian government initiated and financed a program for developmental projects to support schools and communities in implementing Kunnskapsløftet. The program is called "Kunnskapsløftet – from words to action". Oslo University College took on such a project to strengthen the teaching and learning of mathematics in a small city near Oslo. The ongoing project has, through observing and working with teachers and groups of teachers in their classrooms, emphasised a communicative and exploratory pedagogy. Through activities and reflections, the teachers' awareness of students' learning and mathematical meaning formation are problematised. These reflections are intended as elements in strengthening and possibly changing practices of teaching and learning of mathematics in schools and classrooms.

All teachers involved were asked in spring 2009 to write a short report on their experiences in the project. These reports pointed at three elements that have influenced their practice in the classroom:

- More use of practical material (as illustrations, as representations, ...)
- More practical mathematics (as relevance, connecting to students' experiences, ...)
- Speaking more mathematically (both between teachers and students and between students, to put words to concepts and to give concepts meaning, ...)

The teachers also reported more self-confidence and freedom to improvise more and to let go of the textbook at times.

THEORETICAL ORIENTATION

The point of departure for understanding teaching and learning of mathematics in this paper is connected to how teachers create their own and collective understandings and mathematical meanings. Identification as a teacher of mathematics, through acting, or performing, as a teacher in mathematics, is closely associated with meaning making in mathematical contexts. Conceiving teachers' knowledge as part of a complex set of interactions involving *action, cognition and affect*, places teaching as a complex practice. A main perspective then is a view of teaching and learning as communication (Braathe, 2007; 2009; Ongstad, 2006; Sfard, 2008). Seeing mathematics and mathematics education as kind of communication will be to see mathematics and mathematics education as genres. I take the perspective that teachers involved in development projects like this are participating in different genres, kinds of communication, including mathematical genres, and are potentially experiencing different ways to act as a teacher. It is helpful to call this process 'learning'. This will be connected theoretically to seeing learning as semiosis in the field of teaching mathematics. This is consistent with seeing learning as communication. This shifts seeing development from a psychological to a semiotic perspective, thus locating developmental principles in the making of meanings. As I see learning, or developing of identities, as being positioned in communicational genres, I locate identities as dialogically situated in, negotiated and formed by genres, and so they can have many expressions dependent on the context. Identity can then be seen as dynamically combining the personal, the cultural and the social (Braathe, 2007).

Such an assumption can be researched by considering how constructions of meaning and understanding of knowledge of mathematics and teaching and learning of mathematics connect to historical, social and cultural frames. Developmental projects like this can open up possibilities for questioning dominant discourses on knowledge formation and forms of knowledge.

METHOD - FOCUS GROUPS

During communicative interactions, people use narratives to make their words and actions meaningful to themselves and others. They can be thought of as presenting themselves as actors in a drama, with different parts or "positions" assigned to the various participants. Positions made available in this way are not fixed, but fluid, and may change from one moment to the next, depending on the storylines through which the various participants make meaning of the interaction. Focus groups provide possibilities for exploring how knowledge, language and storylines emerge in given cultural and social situations. Barbour and Kitzinger (1999) argue that focus groups are an ideal method for exploring people's experiences, meanings, wishes and worries, and the method is well suited for exploring beliefs. When it comes to beliefs Putcha and Potter (2004) holds that beliefs are not some independent ideas that a person expresses in specific contexts, but are produced continuously and especially in situations where there are opportunities for discussions.

In June 2009 we conducted nine focus groups with teachers involved in the first two years of the project. The theme for these focus groups was the teachers' experience of change of practice during the two years. The focus group discussions have been analysed to identify discourses and positions. Our focus in the analysis is to identify constructions of meaning. We have been aware of power relations in the groups, both our own, and internal power relations within the groups of teachers. We have asked follow-up and contradictory questions in order to produce contradictions and disagreement, but also built on aspects that stood out as collective stories from several teachers. We have identified that some teachers had similar reflections on being part of a mathematics project like this, independent of their individual positions in the different schools. Some of these collective arguments are what we argue may be identified as aspects of dominant discourses. Hidden or tacit discourses allow some dominant aspects to create and legitimise particular ideologies in mathematics education. We have identified some of these as dichotomies, as nodal points in the creation of meaning (Braathe and Otterstad, in press). In this paper I will focus on one of these, the tension between differentiation in heterogeneous classes and adjusted education in ability groups.

THE UNITY OF THE CLASS VS. STREAMING

When the project started in August 2007 all the schools involved had already organised ability groups in mathematics in all grades. We were a bit surprised that this practise had been established so quickly, but the project took this as a premise and had no intention to interfere.

Research literature gives little support for the practice of streaming, yet this is a wide spread practice all over the world. Zevenbergen (2002) has used a Bourdieuian analysis, using field and habitus to understand why this practice is still so wide spread and why teachers so easily accept it. She raises questions as to why and how the field of mathematics education supports the practice of streaming. Such support may not be overt, but can also be ideological, and hence less open to criticism.

> One reason why there is this support can be found in the dominant ideology in mathematics education where it is widely accepted that mathematics is hierarchical in structure (Ruthven, 1987). If it is perceived that there is a hierarchy in the complexity and demands of the discipline, then it would be logical that students can be mapped against this hierarchy. When this is coupled with the contemporary push in education for outcomes-based learning where students' progress is mapped against levels, there is a congruency between the teachers' beliefs about curriculum organisation, student learning and assessment. This enables teachers to justify streaming on the basis that students can be exposed to content that matches their levels of understanding. The hierarchy of learning is further supported through the belief that appropriate learning activities and scaffolding can be developed to move the students on to greater levels of understanding and competence (Slavin, 1990). While streaming may be seen as an anathema to good teaching practice, the ideology of mathematics being hierarchical, in concert with the levelling ideology of outcomes-based education reforms, creates an environment that reifies a learning hierarchy. In so doing, this supports the use of grouping students according to their achievement levels. This ideology provides the structuring practices through which teachers are able to organise curriculum and learning under the guise that the practices that they develop support student learning (Zevenbergen, 2002, p. 3).

I will argue that the field of mathematics teaching and learning in Norway to some extent adheres to the above described beliefs. Zevenbergen further uses the reflexivity of field and habitus to explain why this practice of streaming has become part of mathematics education habitus in many countries all over the world. Streaming however has not been part of the Norwegian habitus. This should create some tension in this reflexivity, and should be possible to trace in the focus groups.

The inputs during the project has been with a focus on communicative and exploring aspects of teaching and learning mathematics, and have problematised how the organization of teaching influences students' learning and meaning formation. This shifts focus from the abilities of individual students to the social and environmental importance for learning. This also challenges the beliefs described above, which to a great extent are underpinned by a "belief in the notion of an innate ability whereby the students' abilities in mathematics is the major reason for the performance in mathematics" (Zevenbergen, 2002, p. 4). In this way underachievement is seen as the fault of the student due to their innate propensity for mathematics rather than of social conditions or other factors.

Since Kunnskapsløftet has the same knowledge goals for all students, there is an underlying idea that all students in all ability groups should work with all topic areas. Therefore we asked about this.

> Teacher 1: Not all, like in the weakest group we don't do that

Teacher 2: That is the case in our team too. The weakest group is a little bit on the side .. works differently.

Teacher 1: In the next weakest group I try to follow some of the subject areas, but very selectively in the sense that I skip things like probability and things like that. I will not use too much time on that, it is much more important that we use more time on the fundamental so it becomes that I select what we have to learn.

The two teachers express the beliefs quoted above, and the result is that they select the mathematics for the student according to perceived ability. These teachers are working with students from 4th to 7th grade. This belief is challenged when they reflect on what kind of students they prepare for the lower secondary school where they will meet other teachers and also been given marks.

Teacher 3: In the lower secondary they are focused on "reality orienting[2]" their students because they will all meet the same examination. They probably mean that we pamper too much with them, that is what they say. … They have very short time and they shall through so much stuff and they are obliged to come through with all before the final exam. They are so extremely occupied by the exam and that the marks shall be fair, then all must have had the possibility to have learned. Then I think there must be an enormous gap for the weakest students. … They will fall off … will be all black in their faces and just fall through and experience a, I think, terrible feeling.

Teacher 2: Yes, but I think that is not only for the weakest.

Teacher 3: No, there is probably many, but of course for the weakest, they will not understand anything. They are good at streaming at the lower secondary, it is not that I am saying anything else, but they have to get through, they have a absurd demand on them, so much to cover.. they talk all the time of this reality orienting of the students.

Teacher 1: And I think for some of them, for some have been to some of the courses [referring to courses given by The University College during the project] with basic teaching and learning of the four arithmetic operations. So some of the teachers at the lower secondary that took part, there and then I think they saw the points and thought it was exciting, but then I think when they get back to their teams and starts discussing and … they get caught in..

Teacher 4: And then they hear from the upper secondary that their students are not up to standard and ..

We read a distinct division between us and them, the teachers in primary school and in the lower secondary, despite the fact that they are working in the same school which is a school for children from 1st to 10th grade. The dilemma created by the final common exam for all students and the practice in the ability groups of selecting only parts of the subject area for some students to be prepared in comes to the surface.

According to Zevenbergen (2002) this is part of the habitus established around streaming in mathematics education, and concerns the problematic of assessment. As these teachers communicate about the subject, their communicative positioning expresses tensions and uncertainty about the normative rights towards the students that this practice offers. It breaks with the egalitarian ideology that is part of the Norwegian habitus.

In another of the schools in the project these tensions have resulted in a retreat back to heterogeneous groups.

Teacher: We have decided to go back to heterogeneous groups from the ability groups we have had for some years. That was a major change…

Braathe: Can you say some more about that, why did you go back?

Teacher: It was..we did not think it gave any results and it destroyed the oral aspects of the mathematics that we were used to earlier. The groups became too homogenous and it..it was more students that fell down from the best group than it was the other way so..I felt that I missed very much someone to play up against, someone in the middle. [The weakest] were not used to be orally active in mathematics and I could not get a word out of them if I asked questions, they were not used to be in the lead so I found that it functions much better now when there are students from diverse levels in the group because there is always someone who dares to ask questions and say that they do not understand so it's much easier to start dialogs with the students and between the students.

Braathe: Is it consensus between you teachers at the lower secondary about this, or have there been some disagreements?

Teacher: There has more or less been full agreement, but it was perhaps worse to let go of the system for the one who had the best students. If I had had that group then it could also have been that I would fight hard for that it is a brilliant system, but after struggling with the middle and the lowest groups then..

Braathe: Hm..

Teacher: You only got the ones who did not function socially.. the classroom is supposed to be some social balance, that was totally crashed when you got all the ones who thought that mathematics were boring and did not bother to work with math to sit in the same classroom..

This teacher expressed himself on behalf of colleagues in this school's lower secondary department and he got full support in the focus group. We can hear his emphasis on the social importance of the classroom for creating good situations for learning and construction of meaning. He underlines the communicative and dialogic aspects of teaching and learning as a support for all students' learning. In the above utterance it is first of all the weakest students' learning that is emphasized, and the situations for the teachers having these groups. The teachers at this school, through this consensus, reveal beliefs that see teaching and learning situations to be equally important as individual students' innate abilities. So the long tradition of not streaming in Norway, with the underlying ideologies of egalitarianism (Braathe and Ongstad, 2001), have so far established a habitus that brings tensions to the Norwegian mathematics classroom when it comes to ability grouping.

SOLIDARITY VS. COMPETITION

The examples presented from the focus groups can be read as if the project, with its focus on communication and reflection on classroom situations, has vitalised the collective discourses with strong ideologies associated with the principles of collective teaching and learning and equal rights for education. The tensions that the new curriculum has started have deep roots in the egalitarian ideology of Norwegian society. New right wing neoliberal challenges of streaming in the Norwegian school system, as well as the globalisation effect of international comparisons in the name of strengthening competitive ideologies, have met ethical resistance from the well-established habitus with its collective values of solidarity that most teachers themselves have experienced both as students and as student teachers. In this way the reflexivity of the field of mathematics education and habitus show tensions on both ideological and ethical levels among teachers of mathematics in Norway.

NOTES

[1] The official translation of the Norwegian name, Kunnskapsløftet, of this new Curriculum plan is "Knowledge Promotion". The name indicates a will to focus knowledge, indirectly criticizing former plans for focusing democratic and social aspects of schooling in Norway.

[2] This is a translation of the Norwegian word "realitetsorientere". This was frequently used by the teachers in the focus groups and is a signal of the break from the primary school to lower secondary where the students meets the marking system for the first time. There is a belief among teachers in the lower secondary that many students for the first time gets an explicit negative feedback on their work in mathematics at this moment in their school career.

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CALLED TO ACCOUNT: CRITERIA IN MATHEMATICS TEACHER EDUCATION

Karin Brodie, Lynne Slonimsky, Yael Shalem

University of the Witwatersrand

In this paper we present the conceptual underpinnings of a teacher education project where we attempt to hold the social and mathematical together. We argue that this is done through conceiving of mathematics and mathematics teaching as practices, and the key aspects of a practice are to account for what counts: the criteria of the practice. We show how our programme tries to support teachers' accountability to each other and to the practices of mathematics and teaching. We do this through the artefact of an international test and a series of structured activities, which focus on learner errors in mathematics. We thus subvert the status quo of assessment and accountability – using them as vehicles for teacher development, rather than teacher regulation and denigration.

INTRODUCTION

Across the globe, in the name of accountability, standardised tests are being used to monitor and regulate teachers' practices, to reward and sanction, and "shame and blame" schools and whole countries. Research has shown that these testing practices at best provide more and more data, and at worst lead to resentment and compliance but not to improvement of learning and teaching (Earl & Fullan, 2003; Fuhrman & Elmore, 2004; McNeil, 2000; Walls, 2008). To counter the disempowerment that tests usually produce, we have developed a teacher-education project, using a standardised test together with other forms of data from practice. We argue that standardised tests, if used appropriately, can provide a mechanism for teacher growth and empowerment. In this paper we describe some of the conceptual underpinnings of the project.

In conceptualising our project, we find ourselves firmly in the grasp of the dilemma of mathematical specificity (Valero & Matos, 2000), which questions the extent to which researchers can manage to keep our gaze on both the mathematical and the social and political aspects of teaching and learning. If we argue that both are important, then we need to find ways to keep both in focus, rather than losing one at the expense of the other. Our project draws on notions of accountability to and in practice to argue that the mathematical and the social go hand-in-hand and are inseparable if we want to truly empower teachers through teacher education.

In developing a teacher education programme that does empower teachers, a key principle is that we should not expect dramatic teacher change unconstrained by teachers' current positions and practices and we should strive to work from where teachers are, rather than from some ideal of where they should be. In researching the programme, we would have to try to understand teacher change in more nuanced and textured ways than whether they take on reform practices in the ways in which we want to see them.

THE PROJECT

Our argument is strongly informed by our context – the mathematical experiences and achievements of South African learners. As with all aspects of life in South Africa, the education system is characterized by large disparities between rich and poor, and most of our schools and learners are of very low socio-economic status. Most teachers in South Africa teach big classes in very poorly resourced schools. Disaffection and alienation are rife (Motala & Dieltiens, 2008) and failure rates are high, particularly in mathematics, where failure begins as early as grade 3. Reviewing the research, Taylor, Muller and Vinjevold (2003) conclude that "studies conducted in South Africa from 1998 to 2002 suggest that learners' scores are far below what is expected at all levels of the schooling system, both in relation to other countries, including other African and developing countries and in relation to the expectation of the South African curriculum". Many grade 3 learners struggle with basic skills such as adding and subtracting two-digit numbers that require 'carrying' or 'borrowing'. Learner failure and alienation is compounded through the years of schooling, culminating in very low pass rates in mathematics in the final grade 12 examinations, particularly for black learners.

Any teacher education programme working in South Africa needs to take seriously the mathematical empowerment of teachers and learners. This raises the question of what mathematics might be empowering. One candidate is critical mathematics (Gutstein, 2008). Another is a notion of mathematics as a practice or set of practices (Ball, 2003), which are both reasoned and reasonable (Ball & Bass, 2003). The practice of mathematics includes: symbolising, generalising, solving problems, justifying, explaining and communicating mathematical ideas. Just as mathematics as a knowledge system is a practice, so is mathematics teaching, and a key task for teachers is to work across these two practices to give access to the practice of mathematics to their learners (Ball & Bass, 2003; Brown, Collins, & Duguid, 1989). Our choice of mathematics as a practice allows us to recognise that changes in teaching and learning are not simply about changes in consciousness but are also about extending repertoires of knowledgeably skilled identity (Wenger, 1998).

Our project uses a range of data to help teachers develop their knowledgeably skilled identities within the practices of mathematics and teaching. The data includes test results, curriculum documents, academic papers written for teachers, lesson plans and videotapes of lessons. The project structures a set of activities for teachers over three years:

1. Analyses of learner results on an international standardized, multiple-choice test[1], through an analysis of the distractors on the test;
2. Mapping of the test in relation to the South African mathematics curriculum;

3. Reading and discussions of texts in relation to a learner errors on a concept (our first concept was the equal sign and its meanings);

4. Developing lesson plans drawing on these analyses and discussions, which aim to engage learner errors and misconceptions in relation to the concept;

5. Reflections on videotaped lessons of some teachers teaching from the lesson plans.

The project participants are Grade 3-9 teachers from a number of schools in different socio-economic contexts in Johannesburg. The teachers come with different histories and different taken-for-granted conceptions of mathematics and of teaching mathematics. They meet once a week at our university where they work in small grade-level groups of 3-4 teachers per group, with a team-leader, who is either a member of staff or post-graduate student at our university. Part of activity 4 and activity 5 are conducted in larger groups across grade-levels.

ACCOUNTABILITY IN PRACTICE

The project draws on a number of key understandings of what teacher learning and empowerment might mean. In working with an international standardized test, we consciously move from the use of test data for benchmarking and monitoring teachers' and learners' performance to the use of test data as a vehicle for teacher development. Here we use Earl and Katz' (2005) distinction between "accounting", which is the practice of gathering and organising of data and "accountability", which refers to educational conversations about what the information means and how it can inform teaching and learning. For Earl and Katz, internal accountability is where teachers are "constantly engaged in careful analysis of their beliefs and their practices, to help them do things that they don't yet know how to do" (2005, p.63). This implies that accountability conversations can give participants imaginations for possibilities that they do not yet see. The question for us is: how does this work?

There are two key elements in any practice: the criteria for what counts as appropriate within that practice, and how the community that constitutes the practice defines what counts and holds people to account to the criteria of the practice. As Ford and Forman argue (2006), "In any academic discipline, the aim of the practice is to build knowledge, in other words, to decide what claims "count" as knowledge, distinguishing them from those that do not" (p. 3). Explicitly articulating what counts as knowledge means that boundaries are delineated (Bernstein, 2000), within which people can learn to act within the bounds of the practice and hence begin to gain access to the practice. Through using language in activity in practice, people communicate to each other what counts as that practice and hold each other to account. In other words, the construction of meaning happens symbolically in practice.

Across communities of mathematics teachers, there are different criteria for what counts as mathematics and as teaching. We know that internationally and in South

Africa, many teachers work with a relatively narrow version of mathematics and teaching. This means that teachers have limited possibilities for appropriate action and it is difficult for them to imagine other possibilities. As Bourdieu (Bourdieu & Eagleton, 1994) points out, the symbolic constitution of our universe is "something you absorb like air, something you don't feel pressurized by, it is everywhere and nowhere and to escape from that is very difficult" (p. 270). This implies that unless there is something to disrupt taken-for-granted assumption in practice, our practices are extremely resistant to change.

In our project, the structured activities and artefacts support teachers to talk in and across differences in their taken-for-granted criteria, articulating what counts for them in relation to mathematics teaching and learning. In the process, their own criteria become objects for conversation and reflection for themselves and others, thus opening up new conditions of possibility for action. What is a key issue here is that the teachers account to each other through practices that are mathematical. As they analyse the test, map the test to the curriculum, plan and teach lessons and reflect on their lessons in public, they account for their actions in practice. In this way, the notion of accountability bridges the social and mathematical in that it positions teachers to both tell and listen to different views on mathematics. As teachers give accounts of their practices, they are able to distinguish commonalities and differences in their contexts that are different to what they imagined. Teachers talk about resources, the curriculum and their challenges with learners in ways that help them to see different ways of seeing. But most importantly, they talk about mathematics and are coming to see different ways to see learners through the mathematics of the curriculum.

MATHEMATICAL KNOWLEDGE

We start with Michael Young's (2008) notion that there is powerful knowledge, and that empowering learners means providing access to this knowledge. Mathematics as a discipline is identified as such powerful knowledge. While this knowledge is socially constructed, Young's argument goes beyond identifying mathematics as merely knowledge of the powerful, i.e. as a convenient filter to keep most people out of power, even though it is often used in this way. Rather he argues for a sociology of knowledge that understands how and why the structures of different kinds of knowledge provide more powerful ways of seeing and living in the world. Although Young points to the structure of knowledge, he does not explain sufficiently what the power of this powerful knowledge is. What he does say is that powerful knowledge creates symbolic relationships, which support re-visioning the world from a more distanced perspective, thus providing a means of escape from Bourdieu's symbolic prison.

We have argued elsewhere (Slonimsky & Brodie, 2006) that developing powerful knowledge allows people to impose new grammars or orders of being on the world. This happens through two interacting processes: differentiation, which opens up established constructs making more textured understanding possible; and integration, which enables the construction of more powerful and economical concepts on the basis of what is previously seen as unrelated. So the process of learning is a transformation of relationships among current knowledge into ever more powerful, differentiated and integrated accounts of practice. We have shown previously, in relation to the curriculum mapping activity, that teachers have begun to see conceptual linkages between different parts of the curriculum they previously saw as distinct (integration), that they can distinguish different meanings for one assessment criterion rather than making a quick association it with a mathematical topic (differentiation), and that they are able to articulate points of alignment and misalignment between the official curriculum and their own teaching (Brodie, Shalem, Manson, & Sapire, 2008).

In the case of teacher knowledge and practice, differentiation and integration of practice occur in relation to what Bernstein (2000) calls the pedagogic device. The pedagogic device, which structures both the medium and the messages of schooling, consists of three message structures: distributive rules – which determine what is taught, i.e. the curriculum; recontextualising rules, which structure how teaching happens, i.e. pedagogy; and evaluative rules, which structure how what counts as learning and as knowledge are communicated. Evaluative rules are what make for accountability in practice. For Bernstein, evaluation coordinates the workings of distribution and recontextualising, thus condensing a range of messages through its most powerful message. Our program tries to help teachers to both differentiate the three message systems and bring them together through a focus on evaluation. We maintain our focus on evaluation through the use of the test, and also through a focus on learner errors.

LEARNER ERRORS

A focus on learner errors may seem strange. We have been asked why we do not focus on teaching goals and the strategies to reach them. Our focus on learner errors has one, obvious source. The results on the international test were very poor, as are the results on all comparative tests for South African learners. Making the test data available to the teachers immediately raises the question: why do our learners do so badly? While we aim to get away from the “shaming” that such results often produce, we do want teachers to seriously reflect on the fact that the vast majority of our learners are well-below grade level. But we want them to reflect on learners’ performance in ways that do not blame learners or themselves and which provide ways for them to work with learner errors in order to transform them. We have shown elsewhere that this awareness is beginning to develop in relation to cognitive challenge and progression across the curriculum (Brodie, et al., 2008).

In terms of our argument above, errors are two-fold. On the one hand, labelling something an error invokes the criteria of the mathematical practice. On the other hand, errors are an important part of any practice, because they illuminate what mechanisms need to be put in place to give access to the practice. So errors point to the demands of the practice, while at the same time are the point of leverage for opening access to the practice. Errors give us a way to help teachers to see learners as reasoning and reasonable thinkers and the practice as reasoned and reasonable, and bring these two into a relationship. If teachers search for ways to understand why learners may have made errors, they may come to value their thinking and find ways to work it into classroom conversations. Errors are also a key area of evaluation for teachers – so talking about why learners make errors, and how teachers respond to these, brings together Bernstein's three message systems through a focus on evaluation.

A key theoretical understanding of our work is that learner errors are a normal part of the learning process (Smith, DiSessa, & Roschelle, 1993), are reasonable and make sense to the learners. Everyone makes errors in mathematics, even "good" students and teachers, and they provide for points of engagement with current knowledge. The constructivist view is that errors are produced by misconceptions (Confrey, 1990; Smith, et al., 1993), which make sense to learners in terms of their current conceptual structures.

One of the key characterisations of misconceptions (Confrey, 1990; Smith, et al., 1993) is that they are remarkably similar across a range of contexts and resistant to instruction, because they are so firmly part of the learners' conceptual structures. When the teachers worked together to analyse why significant numbers of learners may have chosen particular distractors on the tests, they were somewhat surprised to find that across schools in a diversity of contexts, their learners often make the same or similar errors. When the teachers discussed readings about common errors with the equal sign (how operational meanings of the equal sign, i.e. that the equal sign signifies "find the answer", interfere with the relational meaning of equivalence) they were able to further place their learners' difficulties in a broader context because learners the across the world share the same struggles. So these activities provide the teachers with a way to differentiate and integrate their own experience with those of their colleagues, both in the immediate context of the project and more broadly.

The readings about common errors with the equal sign also supported the teachers to articulate some of their implicit understandings of why learners struggle so much with the concept of equality and some of the many errors that they see in learners' work. For some teachers this was the equivalent of learning new mathematical knowledge, a subtlety of mathematics as a discipline that they had not understood previously. For others, this was an articulation of what they knew previously, in a nascent way. Thus reading the accounts of researchers in relation to their own practice, allowed these teachers to account more fully for their own practice and knowledge.

Errors also provide a useful focus because teachers orient towards errors in different ways. In more traditionally-oriented teaching, errors are either to be avoided or corrected, in the pursuit of correct mathematical knowledge. There are also concerns that a focus on learner errors suggests inappropriate evaluations and judgements about learners (hence preference for the term “alternate conceptions” rather than “misconceptions” in much of the literature). Thus errors might be avoided to prevent “shaming” of learners. Other reasons for avoiding a focus on learner errors is a fear that bringing them into the public realm will support a “spread” of errors among learners and create more obstacles and stumbling blocks, or that teachers will be distracted from their focus on their teaching goals and strategies (which often does happen). In more reform-oriented teaching, errors are to be embraced, as point of contact with learners’ thinking, or as points of conversation to generate discussions about mathematical ideas. Thinking about their own responses to errors in developing lessons plans and reflecting on teaching, supports teachers to see how different systems of evaluation constrain and support different teaching approaches.

Yet another moment of possibility for more texture and nuance in teachers’ thinking about their work relates to the role and responsibility of teachers in producing errors. An important point is that misconceptions are seldom taught directly by teachers. All learners develop them at some point, even in the most “reform-like” of classrooms (Ball, 1993; 1997). However, teachers sometimes exacerbate errors through “thoughtless”, i.e. taken-for-granted use of language and concepts, and, at another level, through not making them public and dealing with them. At yet another level of complexity, a deeper understanding suggests that teachers cannot deal with errors quickly or easily because they are firmly held by learners and often resistant to instruction. So a focus on errors allows teachers to develop extremely nuanced understandings of the nature of mathematics, teaching and learning. Many teachers are starting to articulate some of these nuanced understandings to us in project sessions and as they speak to us about the project in interviews. We are also starting to see some of these understandings make their way into teachers’ practices.

SOME INITIAL DATA

Although this is primarily a conceptual paper, we present some data here to illustrate our framework and indicate how data analysis will progress. In the lesson presentations, the teachers were asked to present two episodes: one where they dealt well with learner errors and one where they did not deal so well with a learner error. As an example of the first, a Grade 8 teacher presented an episode where he had acknowledged a learner contribution that was correct, but looked incorrect if learners were working with an operational conception of the equal sign. The learner wrote $2=x$ rather than $x=2$ and was greeted with shouts from his classmates that it was incorrect. The teacher calmed the class down and asked learners to justify their positions, as to why $2=x$ is correct or not. There was a short conversation where justifications for both views were discussed. In other words, the teacher asked learners to account for their criteria of what counted as an answer. After this the teacher explained why both are correct, i.e. he accounted for what counts for him and in mathematics.

In reflecting on the episode, the teacher told us that ordinarily he would have evaluated the answer 2=x as incorrect, and merely told the learners that they should write x=2. Given the work he had done in the project, he realised firstly that the learner was in fact correct, and secondly, that the learners who disagreed did so because they were working with an operational notion of the equal sign. So he was working with a more textured understanding of learner errors and how they might be evaluated. His conception of what counts as the meaning of the equal sign has opened, his conception of pedagogy has opened and his conception of learning and learners' meanings has opened. So his own conditions of possibility for practice as a mathematics teacher have been expanded. He was able to translate this understanding into a practice of working with the learners' ideas, giving them space to justify their thinking and then explaining why in fact both expressions were correct. So he could transform his understandings of learner errors into working with them in practice. Just as he has had to account for his meanings in practice in the project, so he supported his learners to do so.

It is notable that this was the only episode in this teacher's three lessons where the teacher was open to learner errors. In many other instances he ignored learner errors, or told the students that they were incorrect without listening to the reasoning behind the errors. But in trying to work with more textured shifts in teachers' practices, we need to acknowledge the small, but significant step that he did make.

A key part of our project is ongoing conversations about how criteria of practice and practices themselves are changing. When we asked this teacher why he was able to act differently in this instance, he said that as he was thinking about the learner's contribution, he looked up at the video camera and it reminded him about the project. While we all laughed at this comment, it is very significant. When he was back in his customary community of practice in his school and classroom, within the saturated atmosphere or symbolic "air" of the conditions of his usual practice and criteria, the video camera provided an interrupter, a mechanism to remind him of the project and his accountability to his colleagues. This reminds us, that the path from other-regulation to self-regulation in a community of practice is slow and uneven and contingent on ongoing accountability in practices in context. While teacher education may open up increasingly powerful and new possibilities for action, the next challenge is, as we have pointed out elsewhere (Slonimsky & Brodie, 2006), how to support teachers to make these changes in their customary communities of practices, so that these expanded criteria and resources of practice become a taken for granted part of the local community.

NOTES

[1] The results on the test were very poor as is to be expected given the context described above. The average percentage correct in the tests were as follows: Grade 3: 38%; Grade 4: 37%; Grade 5: 35%; Grade 6: 30%; Grade 7: 30%; Grade 8: 25%; Grade 9: 25%. In most cases, except for two or three items in each test, the majority of learners got the item incorrect.

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EXPERIENCING THE SPACE WE SHARE

Tony Brown

Manchester Metropolitan University

This paper offers some theoretical and practical reflection on how we share geometry and make it part of our lives and in so doing link to a shared heritage. It draws on Husserl's speculations on how geometry originated but how then it increasingly became seduced by language as a result of human attempts to capture and share its concepts. After discussing work by undergraduate students engaged in body movement exercises and other geometry it considers more generally how the truth of mathematics relates to its representation in cultural forms.

INTRODUCTION

In geometrical study we are confronted with ideal mathematical objects that nevertheless in some respects, very often, are also a function of their cultural heritage, that is, of their human construction, with respect to configurations observed in the physical world. Husserl (in Derrida, 1989, p. 173) argues that to understand geometry or any other cultural fact is to be conscious of its historicity, albeit implicitly. I take this to mean that ideal objects can only ever be accessed through technology or perceptual filters that are both time and culture specific where those technologies or filters display some historical continuity, revelatory of how they emerged from earlier manifestations. Yet our very selves have been created in a world that has a physical organisation and analytical heritage consequential to a long history of geometrical awareness. How do I fit in to the social world through participation in shared ways of organising the world? Our perceptions of the world are inevitably processed through aspects of this heritage. We cannot be geometrically naïve insofar as our subjectivity results from identifications with this shared heritage. Our physical experiences are processed through that vocabulary of set moves and analytical strategies. We have learnt some of these things in school, or through everyday life experiences, but in a fundamental sense they are also part of us, contributory as they were to our very formation. This paper offers some theoretical and practical reflection on how we share geometry and make it part of our lives and in so doing link to a shared cultural heritage.

THE ALGEBRAICISATION OF GEOMETRY

An early part of my work in mathematics education revolved around an interest in the work of Caleb Gattegno (e.g. 1988) who I had the pleasure of meeting on a few occasions. An aspect I remember particularly well is Gattegno's notion of the *algebraicisation* of geometry, or how geometrical experience is transformed, perhaps compromised, by an insistence on it being converted to symbolic form. I remember Gattegno talking about a baby pointing to a fly walking across the ceiling. Each fly position on a continuous path was associated with a particular (discrete) arm position. But a key concern was that in school, geometrical experience generally gets converted into algebraic experience and that this results in a loss. Whilst not in anyway detracting from the importance of algebra in emergent mathematical understanding, Gattegno was keen to educate the "whole brain" where experiences of the continuity of geometry were more often fore-grounded in classroom geometry. This notion of geometry being compromised through its algebraicisation will underpin the discussion that follows. Before moving to some theoretical discussion I shall describe some practical work with students.

SHAPING UP

To explore these issues I shall recount some fun that I had with a group of students recently as a result of pursuing my interest in how we apprehend geometric phenomena. I have a weekly session with a group of first year undergraduate students (aged from 19 to undeclared middle age) preparing to be teachers of mathematics in British secondary schools. In one session we tried out various activities in which various instructions were followed that resulted in the students walking the loci of certain geometric objects: *Walk so that you are always equidistant from your partner who is standing still* (circle). *Walk so that you are at all times equidistant from your stationary partner and a wall* (parabola). *Now get in to groups of three where there are two people each standing still at some distance apart: Walk so that you remain equidistant from both partners. Walk so that you remain twice as far from one partner as you do to the other. Walk so that you can still touch a piece of loose string held firmly at each end by your two partners*. (Photographs will be available for the presentation.) In setting the task on the first occasion for some time I had some expectations, based on my own hazy memories, of some of the figures that would be generated. But given the zest and determination of this particular group of students, explorations went further than expected with some very familiar figures emerging from unexpected directions. And for the students there appeared to be a very real sense of acting out shapes and feeling them before recognising them as more or less familiar, yet perhaps now being understood differently given the novelty of the approach. The ideality of any given object cannot be apprehended in an instant, or rather, that ideality can give forth its properties in many ways, such that there comes into being a perceptual architecture that supplements the ideality with a necessarily cultural layer.

In steering a particular course a student had to stay twice the distance from one partner as she was from the other. As I observed I had some vague memory that a hyperbola might be the result. Yet it eventually became clear to those present that there was just one curve and that it seemed to be closed. Yet the relative imprecision of the body movements resisted anyone achieving complete certainty as to whether it was closed and if so if its regularity suggested a circle or an ellipse. We all experienced glimpses of possibilities but remained unsure if our conjectures could be confirmed without more sustained analysis using drawings or calculations. A conceptual layer was needed to confirm intuitive assessments. But these initial moments provided exciting insights into emergent understandings, all the more intense for the person attempting to walk the path of the curve, experiencing the mathematical rules through actual bodily movements. For others there was the challenge to assume some specific perspective on the emerging locus. For the other partners this was from a fixed point.

As all of these activities involved walking on the floor the shapes constructed were all two-dimensional. Yet I was firmly caught out by one interpretation that both surprised and delighted me. With an instruction where the moving player was required to be equidistant from two stationary partners I had anticipated a straight line but surprisingly to me the moving partner, Sally, decided to stand on a chair and then on a table between her two partners. A third dimension was brought in to play where for any given distance a circle in the third dimension could be imagined. This radical departure led to an unexpected exploration later on for all of the other erstwhile two-dimensional shapes.

Together such activities provided the students with experiences of moving in space according to more or less precise instructions, more or less drawing on conventional geometrical terminology, such that continuous movement was associated with a sequence of discrete instructions. Yet like empirical science geometry comprises objects idealised by humans where the technology productive of those idealisms can never be fully separated except at the limit of our conceptualisation.

Back in the regular classroom and later at home subsequent attempts were made to capture the bodily movements in drawings and reflective writing and a new world of geometric figures were generated. Much work was carried out on the two-dimensional shapes. The mathematical objects were generally familiar once encapsulated but the routes to them made them seem somehow new, as though they were being encountered in a fresh way that made them seem different. And following the ascent of the chair and table, later developments considered how the various tasks could be extended in to three dimensions. Ellipses became eggs. Circles became balls. Lines became walls. And various bowls and saddles of infinite dimension and curious orientation also emerged. And in certain circumstances eggs could become balls or even walls.

WHAT OR WHEN IS A CIRCLE?

In a separate session several weeks later I asked the members of the same group to each write answers to the following questions without, in the first instance, sharing their thoughts with others: What is a circle? How do you imagine that circles were invented? (cf. Bradford & Brown, 2005) They then read out their thoughts for everyone to hear. Here are some of the results for the first question:

> A circle is a 2D shape, which starts and finishes at some point it is a continuous curve and has 360 degrees. Clockwise from the centre point to the curve is called the radius and the radius is the same distance to the curve all the way around the circle. We use the

> radius to calculate the area and the diameter, which is twice the radius gives us the circumference when multiplied by π.
>
> A circle is a regular 2D shape, which has no straight sides. Every point on the circle is an equal distance from its centre point. This distance is called its radius. The distance around the outside (circumference) is known from the formula $2\pi r$ and the area from πr^2
>
> Lots of coordinates plotted on a graph and when joined with a line it makes a circle shape. It has a centre point, and from the centre point to the edge is called the radius of the circle. Double the radius = diameter. The points can form an equation in the form $(x-a)^2 + (y-b)^2 = c^2$ where a, b is the centre point and c is the radius.

The students then speculated on how were circles invented:

> Circles were first invented by the Aztecs. They are widely regarded as the first astronomers of our time. They saw the shape of the moon and the sun and recreated the same image on the ground with sticks in the mud, which later became marks on walls like Egyptian hieroglyphs.
>
> By God when he made the human eye – Ask him!
>
> In the days of caveman they decided it was easier than carrying certain objects to put them on a sledge type thing and pull them along. ... But when they travelled over gravely ground they realised the ground was assisting the movement. This gave them the idea of raising the sledge up off the ground and attaching large bits of gravel to the bottom. Over time they developed the axle helping the stones move and again over time the stones wore down to a circular shape.
>
> After Allah created the moon and sun they were observed by man and copied.
>
> Circles were invented when a man cut down a tree and noticed the shape of the stump was of a different shape and the logs it created were a different shape. He also noticed that it rolled easily enough and he realised this may be a good template for a new shape…

Quite apart from the humour these stories suggest some interesting social constructions. In particular, some curious historical perspectives are apparent. The definitions of circle are occasionally dependent on words or ideas derivative of circles. Indeed one of the descriptions cannot avoid using the word circle in the description of a circle. How might we have imagined circles without this linguistic and symbolic apparatus that is seemingly consequential to the supposed existence of circles? How in the present day might we see ourselves engaging with Husserl's quest to understand how geometrical configurations originated? Where and when could we possibly start? We could envisage extending the search to other mathematical objects, or indeed any empirically derived scientific object. And such an attempt would alert us to the cultural nature of each and every mathematical idea encountered in our mathematics educational quest, and of the cultural derivation of the framework that produces those ideas. Or do we encounter the situation in which some mathematicians suppose they can identify mathematical objectivity beyond culture and its history? And if we do encounter that situation how would it impact on our understandings of how humans apprehend mathematical phenomena? To what extent could one suppose a clear historical perspective on such concerns and how are such perspectives functions of particular linguistic constructions? History and our collective understandings of time are both linguistic constructions. Time is a function of the stories we tell about it (Ricoeur, 1984; 2006). People in earlier times did not understand history better than we do today. As an example, during a visit to an art galley in Venice my then seven-year old daughter Imogen was rather taken by Tintoretto's 16th century painting entitled *Creation of the animals*. But she was alarmed by an apparent omission: 'Where are the dinosaurs?' Her awareness of cultural history could detect the limits of Tintoretto's worldview that had been shaped by assumptions that have been revised in more recent years. After all dinosaurs, a twentieth-century human construction, were unknown to our earlier ancestors. Her brother Elliot, meanwhile, chipped in with a comment that he had not realised that God was a man. I speculated on the many ways in which cultural histories have been revised since the painting was created and thus on how individuals understand themselves fitting in to the world we inhabit. History and histories are revisable, for individuals and for cultures, yet residues of previous eras, and earlier conceptions of those eras, remain locked in to the genesis of later formulations. Circles are now a function of contemporary thinking and perhaps cannot any longer be understood independently of that cultural baggage. But was that ever possible? And if so, in which ways could this be possible? We have also changed as humans, such that those earlier humans could not have known circles in contemporary terms, and those earlier humans and their apprehensions could not be processed in contemporary terms. And so many other mathematical constructs would have histories and meanings rooted in different, more or less recent, intellectual circumstances. The growth of mathematical knowledge for example has much to do with market forces and how universities and individual mathematicians get funded to focus on different types of mathematical knowledge such that new and existing mathematical phenomena derive their meanings from how they now relate to this ever-expanding mathematical knowledge.

And as with the group of impressive but maybe fairly typical trainee teachers introduced above, specialising in secondary mathematics, we might speculate on how other mathematical constructions are held in place by incomplete memories of school learning and how those areas or gaps are manifested by teachers in schools working with children who, like all of us, will have specific and restricted historical and mathematical conceptions in some areas of their knowledge.

THE SEDUCTION OF LANGUAGE

Husserl sought to enquire how geometry came into being and concluded that without the anchorage of words (that is, culturally specific constructs) it was quite difficult to conceptualise.

> It is easy to see that even in [ordinary] human life, and first of all in every individual life from childhood up to maturity, the originally intuitive life, which creates its originally self-evident structures through activities on the basis of sense experience very quickly and in increasing measure falls victim to the *seduction of language*. Greater and greater segments of life lapse into a kind of talking and reading that is dominated purely by association; and often enough, in respect to the validities arrived at in this way, it is disappointed by subsequent experience (Husserl, in Derrida, 1989, p. 165, Husserl's emphasis).

Husserl saw geometrical understanding as being linked to an implicit awareness of its historicity, which I see as pointing to the understanding being formed through the subject's constitution with respect to the historically derived, yet still forming, discursive environment. I sit on chairs, climb stairs, wash round dishes, ride on ferris wheels, travel on trains and fly in planes. Our bodies have learnt to function and know themselves in physical environments that result from culturally situated conceptions of geometry. Derrida himself posits *the* geometric or mathematical science, whose unity is yet to come, where "The ground of this unity is the world itself … the infinite totality of possible experiences in space in general ... To pose the question of this traditional unity is to ask oneself: how, *historically,* have all geometries been, or will they be, geometries?" (p. 52). The sum total of cultural knowledge about geometry remains incomplete, but "the infinite totality of possible experiences in space in general" could never be completed. And we cannot yet know, and never will know, how reliable an indicator current knowledges are of knowledges to come. Or more prosaically, we do not know how much school knowledge as currently defined prepares the pupil for the knowledge required in later life. Geometry as an ideal field is held in place by its cultural technology which doubles as a mode of access for those learning the subject. But this technology is culture and time dependent implying a two fold task for students - learning the culturality of mathematics for social participation in that era and also access to the ideality so often seen as key in mathematical understanding.

The stories we have learnt to tell of the world often sediment into fixities that have departed from the truth they sought to capture. The stories lose their zest. And as a result truth always escapes our grasp. This can be readily understood in the context of mathematics. The geometry of Galileo is still largely true, in a sense, but its present coexistence with string theory and other contemporary geometry redefines its relationship to mathematical universality, and how we understand it fitting in as it were, and how we ourselves relate to it. But at the same time Galileo was surely formalising much that had previously been known intuitively. He could not have been the first person to notice the phenomena that he described, but perhaps his encapsulation enabled alternative modes of noticing, that shaped later thinking. History has a tendency to organise previously intuited stuff – looking back on the past or the current to project into the future. Geometry, like much knowledge from the empirical sciences, comprises human constructions but these constructions do have a ring of truth about them.

Geometry gets converted (and perhaps compromised) into particular linguistic forms for accountancy purposes or formal recognition, such as tests/exams, but so too do we as students and teachers, since, for example, we are not teachers in ourselves but teachers subject to particular cultural specifications that restrict how others read our actions and indeed how we assess our own practice.

- What is lost and what is gained by maths being forced into descriptive categories?

(And in turn, a question asked less often in mathematics education research),

- how is the learner/teacher lost (or gained) in being read through descriptive categories?

And those descriptive categories cannot come clean.

- Mathematics is always polluted in its interface with humans as a result of a human need to mediate mathematical experience for the cultural existence of mathematics to be acknowledged, whether in humans theorising, as a manifestation in the physical world or as explicitly pedagogical form (Brown, 2001).
- And we as learners, teachers and researchers are also polluted since we similarly read each other and ourselves through descriptive categories that take us away from truth (Brown & McNamara, 2005).

DISCUSSION

How might a mathematical object be understood given its changing relations with the social apparatus that locates it? What alternatives might we have? What or how or when does a mathematical object signify? How do we understand the apprehension of such signification? Yet how different are mathematical objects to other objects? And how is it decided that certain objects are defined mathematically? Such questions are central to the task of mathematics education research. I have speculated on how notions of the circle, as an example of a mathematical concept, are developed, transmitted and transformed through the need to traverse cultural and historical perspectives. The objectivity of this mathematical concept was shown to be far from stable, although it would be difficult to achieve clear consensus on how mathematical objectivity is understood. If we take a circle as an example of a mathematical object, how might we understand its original conception as an object and how have apprehensions of circles evolved as circles acquired so much historical and cultural baggage as they have been progressively used more as elements in building constructions of the world around us? The original coining of the term circle to capture some apprehended aspect of the world has now become a common primitive in shaping the world thereafter. In Badiou's philosophy the term circle would originally have been "counted as a one" (e.g. the set of points obeying the relation obeying the relation $x^2+y^2=1$, or the points passed through by a boy on a roundabout) but thereafter became a member of other sets of objects (e.g. regular shapes {triangles, ellipses, squares, circles etc}) seen as making up the world and utilised in organising our apprehension of the world (e.g. Badiou, 2009). As proposed by one of my students, perhaps early man looked at the moon or the sun and saw the two objects displaying similar characteristics, characteristics that may have also been seen in other naturally occurring objects (for example, berries, oranges, eyes, etc). The similarity was eventually given a name, circle, or sphere. Yet uptake of such terms would be different across cultural groups according to how the terms intervened in everyday living or were included in the intellectual life of the cultures. And as different aggregations of such objects shape our wider apprehensions of life the formative impact of "circle" continues to evolve and operate in diverse ways. Yet increasingly such usage conceals its original historical contingency as an arbitrary construction from the past, more or less motivated by empirical observation, against which we could perhaps understand aspects of the wider world in a different way. As a wider example I am sure that many aspects of the mathematics produced by the ancient Egyptians retain validity today, yet the meaning of these valid elements now need to be put alongside more sophisticated or contemporary mathematics such as that produced by Newton, Einstein or Hawking. Any supposed universality of the Egyptian conceptions would be disrupted by later developments.

This later concern opens the wider question of how do we define the limits of mathematics and how does the assumption of any frame result in an adjustment to the meaning of the constituent terms? It would indeed be difficult to achieve consensus on how such limits could be drawn. And in the analysis so far mathematical meaning has been considered as though this could be decided by being clear about definitions of what constitutes mathematics. Yet the meaning also depends on how it is apprehended. People are diverse in character and any individual can be understood through a variety of social filters. And we need to make a further decision as to whether we privilege the individual or the social filter as the frame of analysis. Moreover this decision introduces yet a further layer whereby we ask the question as to where the meaning is located, in the object, in the apprehension (however that is located) or somewhere between.

And is a circle a good example of mathematical objects more generally? Most people can immediately apprehend a circle. It is a widely recognised cultural object. Yet there would be a considerable variety of meanings brought to it as indicated. But many mathematical objects or entities or definitions require rather more specialist training to even apprehend their existence, let alone their finer qualities. Depending on how we make sense of the mathematical field the conception of a mathematical object could be understood as being represented in many entities; writing a quadratic function, producing a set of axioms, following a statistical procedure, demonstrating rotational symmetry, showing topological equivalence, etc. As an example, a mathematical generalisation reached through some investigation could be thought of as a mathematical object. Mathematics education research, especially where it is conceived of as a corrective to a fault in the system that has produced hordes of failing students, is in the business of enabling students to better apprehend and use socially derived mathematical apparatus and draws on social interactive processes that locate but also transform the objects concerned. Given this focus mathematical objects are recast as pedagogical objects that result in the specifically mathematical definitions becoming implicated in socially governed processes. The meaning of the mathematical objects is necessarily a function of the relationships within such social settings. The truth of mathematics is constructed, preserved and disseminated through apparatus that is necessarily cultural and hence temporal. The truth of mathematics is never substantial except in its cultural manifestations (Badiou, 2009), manifestations that derive from and feed history but never fully locate truth. Geometry as a field comprising ideal objects is held in place in the collective memory through the technologies that have been developed to access it, or perhaps in the school context, those technologies used to formally assess understanding of it. Truth, in any eternal sense, is beyond that technology, yet accessed through this continually evolving technology. Truth is located and to some extent preserved thorough its crude indicators but potentially at a cost to the profundity of the understanding achieved.

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THE IMPORTANCE OF THE RELATION BETWEEN THE SOCIO-POLITICAL CONTEXT, INTERDISCIPLINARITY AND THE LEARNING OF THE MATHEMATICS

Francisco Camelo, Gabriel Mancera, Julio Romero, Gloria García, Paola Valero

Universidad Distrital Francisco José de Caldas, Universidad Pedagógica Nacional, Aalborg University

We analyze the design and implementation of a learning milieu around the topic of nutrition. This local curricular design was part of a larger project aiming at developing a curriculum for 7th grade students, inspired by the tenets of critical mathematics education. The design propitiated interdisciplinary learning in mathematics, natural sciences and computer science. It also involved in a direct way the students' social, cultural and political context, as a way to contributing to the education of democratic and critical citizens.

INTRODUCTION

In 2008, a curriculum development and research project was carried out at a public school called Federico García Lorca (FGL) in Bogotá, Colombia. This project was sponsored by the Institute for Educational Research and Pedagogical Development (IDEP, Colombia), the National Pedagogical University (UPN, Colombia), and Aalborg University (AAU, Denmark), in collaboration with Francisco José de Caldas Distrital University (UD, Colombia). The project intended to contribute to the improvement of mathematics teaching for 7th grade students in an area of economic, social and political conflict. The project was inspired by some key ideas of critical mathematics education (Skovsmose, 1994). One of the central concepts in this approach, as we understand it, is to find a relationship among mathematical meaning, students' activities and students' socio-political lives. The project involved a team of teachers from the school and a team of teacher educators and researchers from the universities mentioned above. The curricular development and research team worked collaboratively in proposing alternative ways of teaching mathematics for the 7th graders (Mancera, Carreño & Camelo, 2008; Peñaloza & Segura, 2008), as well as in theorizing the experience in relation to the possibilities of critical mathematics education inspired approaches for contexts such as the one FGL's school represent (Camelo, García, Mancera & Romero, 2008; Mancera, Camelo, Romero, García & Valero, 2009).

In this paper it is our intention to present one of the learning milieus designed and implemented and comment on its effect in terms of the relationship between the students' socio-political context, the need for interdisciplinarity and the learning of mathematics. In order to do so, we start with a contextualisation of the locality, the students and the school's curricular organisation. Secondly, the collective interaction of the team involved in setting and performing the project is illustrated. Thirdly, the activities concerning the specific learning milieu where students took part are presented, supported by the theoretical referents that framed this proposal. Finally, a couple of conclusions about the three aspects of socio-political context, interdiciplinarity and learning are presented.

CONSIDERING THE CONTEXT

Following Valero (2004), one of the important elements in developing this project was paying attention to the way in which the series of layers of context surrounding the mathematics classroom play a role in the possibilities for teachers and students to engage in mathematics education. Therefore, for researchers and teachers it was important to inquire about the important features of the setting of the school and of the students.

FGL is a government school, located in Usme (Bogotá, Colombia), where economic issues and social problems arise in large numbers. Such difficulties reflect the permanent contradictions present in the way its inhabitants tend to solve their problems (Camelo & Martinez, 2006). According to Galvis and Soler (2006, p. 2) this is an area of rural traditions, which used to be a native village, nowadays in the process of joining the Capital District of Bogotá due to its urban expansion. The way in which people come to populate the area is determined by land trade on the side of former land owners and also, more recently, because this area offers one of the biggest possibilities of urban expansion for Bogotá. Hence, in this district, problems such as inappropriate uses of land, illegal land occupation and disputes, and water-source environmental issues come together. All these conflicts bring about different forms of violence that are associated with a high-rate urban growth and the poor conditions in which people from the area live. As far as the population is concerned, many of the inhabitants of the area come from conflict zones of the many different "wars" in Colombia. Displaced populations from other parts of the countries move to this locality in search for a safer life and for a possibility of doing a living in the capital city. As a result, coexistence and security problems are the rule in the locality.

The locality has seven Zone Planning Units (UPZ). Five of these Zones are being used as residential areas for poor people of the lowest socio-economic groups, who live in either squatter settlements or social interest buildings. Most of these five UPZs have not been totally urbanised and are known as "non-consolidated peripheral areas". They also show deficiencies in infrastructure, accessibility, community facilities and public space. One of these UPZs is a scarcely developed area with its lands largely occupied; another UPZ is a big area aimed at the production of urban and metropolitan construction supplies —sand and stones— known as "The Quarry's Zone", which exploitation must be dealt with under special regulations due to its large size compared to the whole urban structure of the locality.

FGL is one of the big public schools servicing the population in this locality. After many years of discussion about the pedagogical mission of the school, the school community decided to construct a pedagogical profile making of formative moral values as well as a cultural historical approach (Camelo & Martinez, 2006) the core of its Institutional Pedagogical Project. This means that in the school teachers should strive for creating "learning situations" based on: i) considering that students are immerse in a particular historical-social-cultural context and; ii) the fact that everyone's beliefs and conceptions must be considered by teachers when planning such situations. The conjunction of these two conditions would make possible for teachers to lead students to develop more complex knowledge, capabilities and attitudes building on their previous conditions and eventually helping them reaching more advanced development stages.

Within this general approach, the mathematics teachers have been working together in constructing a pedagogical proposal in mathematics that would be in accordance with the general institutional proposal. Therefore, they set themselves the task of finding and implementing teaching methodologies that allow bringing together three aspects simultaneously: an emphasis on formative values, a cultural-historical approach, and a focus on conceptual change (Camelo & Mancera, 2005). The focus on conceptual change is based on considering problem-posing-and-solving as the most significant way to encourage students to develop desired levels of complex mathematical thinking. Four aspects are taken into account: i) the relationship between individuals and every single meaningful aspect of a proposed situation; ii) the history or "tissue of situations" upon which students have built up their knowledge iii) the implicit models students associate with this knowledge and; iv) the kind of conceptions students will reject when solving the problem, the kind of mistakes they will avoid, the "savings" they will seek, the new formulations they will re-take, etc.

In 2007 an interdisiciplinary group of teachers, including the mathematics teachers, at FGL started implementing their pedagogical ideas with a group of 120 students from 4 different grades, all of them aged between 11 and 13 years old. This work made it possible to accurately condense the social, academic, cultural and political problems that characterized FGL, and it also evidenced some of the difficulties related to classroom management and teaching practice itself (Camelo, Carreño & Mancera, 2008). From that work emerged a characterization of the students in the school. The following excerpts from teachers' assertions illustrate the image that teachers in FGL used to have about their students:

> Students who have little formative values, since their gestures as well as their oral, written and graphic expressions show that they move around a world wherein "normality" appears to be represented by verbal and physical aggression.
>
> Students who show little or no interest in their own learning, particularly in learning maths.
>
> Those who lack a well-defined attention focus and therefore bring about awkward dynamics in the classroom, overwhelming other students who otherwise would be willing to get involved in the proposed activities.

Students who in spite of being relatively close to public libraries, have not form a habit of using this kind of facilities nor of devoting some time to reading.

Having very few community centres that promote artistic and cultural activities in the district, and also the fact that they are mostly unknown to children and youngsters, limits the intellectual, sports, social and artistic development of the activities conducted at school.

In 2008, a group of teachers in the school looked for collaboration with researchers in order to tackle some of the problems mentioned above. The project resulting from the collaboration generated great expectations among groups of students in seventh grade who were —as mentioned before— representative of youngsters and their problems in the locality.

FIRST STAGES OF THE PROJECT

The project began by forming teams of teachers intended to perform "collaborative" tasks where teachers from FGL contributed their practical knowledge and teachers from UPN, UD and UAA added their theoretical mastery. It is important to mention that every team had been already working (on their own) on the different teaching-and-learning phenomena in the classroom, especially on the lack of interest shown by students in education in general and in the learning of mathematics in particular. In this way, the group of teachers at FGL focused their efforts on developing projects aimed at raising critical awareness among students, particularly of the role they play in society regarding topics such as the role played by women in a community where sexual inequality and male control has prevailed. Students had already become interested and critical awareness was being built among them (Camelo, Avila, Carreño & Peñaloza, 2008).

Meanwhile, teachers at UPN working in a network of rural mathematics teachers and with teachers in other depressed areas of Bogotá were conducting some research. They were mainly concerned about the lack of interest shown by students during maths lessons and the lack of support given to primary and secondary school teachers, especially to maths teachers when it comes to facing the large amount of problems related to their job at school. The purpose of the project was to identify the contribution of doing collaborative practice among participants of different academic disciplines and views, and holding research-exploratory lessons in order to transform the curriculum and contribute to the professional development and qualification of the math teachers involved in such collaborative groups (García, 2006).

The SMERG group at Aalborg University (UAA) has develop a perspective on critical mathematical education, characterised by issues such as: i) the relation between maths, society and power; ii) the relation between school maths and social-political, inclusion-exclusion processes of different groups of people; iii) the relation between school maths and other fields of knowledge; iv) the development of inclusive-dialogue pedagogical practices; v) the contribution of maths teaching to social democratisation and vi) cooperation between researchers and teachers aimed at curricular development and production of relevant knowledge about it (Skovsmose, 1999; Valero, 2004).

Sponsored by IDEP and COLCIENCIAS, these three teams of researchers and teachers got together with the purpose of, among others, exploring a curriculum development strategy inspired by some of the tenets of critical mathematics education and of a socio-political approach to mathematics education. Central points of inspiration were ideas such as:

1. The creation of learning milieus (Skovsmose, 2000) that allow teachers moving from the paradigm of exercises to a landscape of investigation for an active learning of mathematics.
2. The organization of learning around problemic areas and the realization of collaborative projects among students.
3. The consideration of students as full socio-political beings and not just cognitive agents (Valero, 2002) and therefore the need of addressing students' intentionality for learning in terms of the relationship between their backgrounds and foregrounds as an important source for the creation of meaning (Skovsmose, 2005).

Given that it was necessary to get to know students so as to think about the most appropriate topics for the design of significant learning milieus, it was decided that the project should begin by broadly contextualising the different groups of students, their interests and concerns (Peñaloza, 2008). As a result, three possible themes emerged as a possible topics for the project-based work that was intended for students to do: free time activities, design and nutrition. In a pilot stage, every maths teacher involved in any of the teams was in charge of making and implementing a design with his students. All findings had to be reported to the whole team on a regular basis in order to reflect on what had happened and to find a new target for the students' activities. As it has been mentioned, this paper concentrates on the milieu about nutrition in grade 701.

NUTRITION AS A MILIEU FOR LEARNING MATHEMATICS

A "learning milieu" (Skovsmose, 2000) was constructed and applied in the first six months of the research project, based on the results from the discussions and actions taken within the context of the project. This "milieu", in this particular case designed by teachers and university researchers jointly, must be understood as "a scenario" that allows teaching and learning practices wherein mathematical contents are connected with situations and activities that students find significant. The process of designing and implementing the milieu served as a first entry for identifying some relevant aspects that were to be considered when conducting the main activities of the project.

In order to design and develop learning environments that regarded student-interest problematic areas, we attempted to identify social-real situations and their relationship with mathematical modeling processes. Following ideas from Valero (2008), the first step was to acknowledge that: i) mathematics is not neutral knowledge, but it is knowledge used by human beings in many different social-life situations to promote a particular view of the world; ii) there are different kinds of mathematical knowledge associated to various social and cultural practices, and iii) mathematical learning practices can not be exclusively defined in terms of individual thinking processes.

The creation of the learning milieu about nutrition began when suggesting that students should reflect on and analyse the impact of the media on family decisions about what they eat every day. The aim was to discuss and identify the nutritional contribution that daily diets offer to school students.

The second activity allowed making connections between biology, mathematics and the social-political-cultural context, since items such as calories intake and nutritious information of some of the food students eat on a daily basis were studied and analysed. This kind of activity involved an immediate context that makes interdisciplinary work possible with the aim of allowing students exploring mathematical contents in order to shape the situation.

Alimentación y bienestar

Discutiendo la alimentación

Lean detenidamente cada una de las afirmaciones sobre hechos relacionados con la alimentación:

- Los medios de comunicación, principalmente por medio de los anuncios publicitarios, influencian a muchas personas en la escogencia de los alimentos.
- Es fundamental estar atento a las citadas propagandas, ya que aumentar el consumo de determinados alimentos o medicamentos es el principal objetivo.
- Una alimentación inadecuada trae prejuicios a la salud, lo que puede ser verificado con estudios que indican un elevado índice de colesterol en niños, problema que en general se debe a los adultos.
- La alimentación es una necesidad biológica vital, común a todos los seres vivos. Tenemos la necesidad de consumir diariamente substancias fundamentales para la construcción y el funcionamiento del cuerpo. Sin embargo, la alimentación de muchas personas es pobre en vitaminas y proteínas.

Discuta las afirmaciones con sus compañeros. Después respondan.

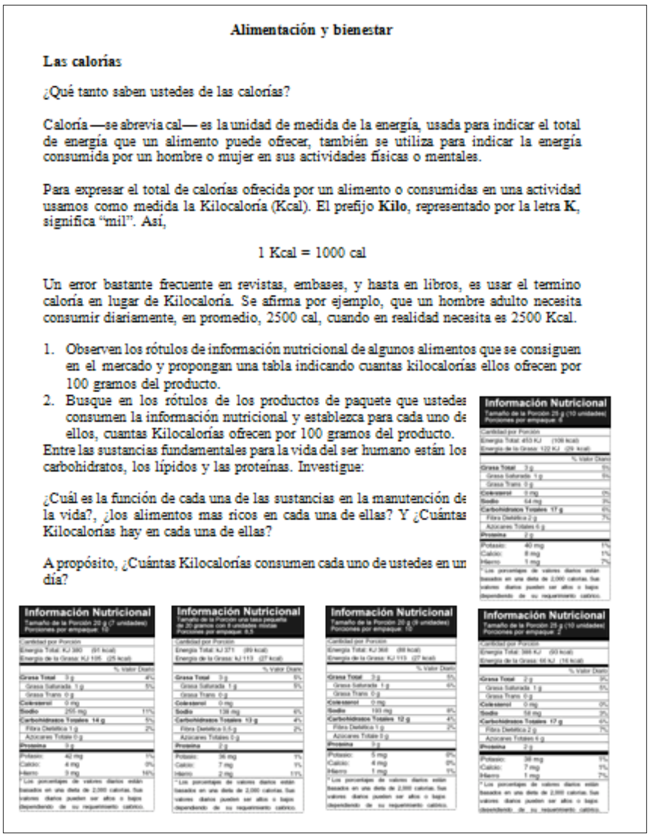
1. ¿Esta de acuerdo con cada una de las afirmaciones anteriores? Justifique su respuesta.
2. De tres ejemplos para ilustrar cada una de las ideas (use dibujos, tablas, ilustraciones…)

Activity 1

Given that, at a first stage, students had to analyse the nutritional habits of the rural-Usme inhabitants compared to the Food and Agriculture Organisation of the United Nations (FAO)' recommendations found on the internet, the third activity attempted to link both the context and the most recent reflections in a direct way. At a second stage, students had to devote some time to analysing their home nutritional habits.

An interesting part of the discussion with students in the first activity focused on the approach one of the groups adopted regarding the fact that although the media has an impact on what they consume, consumption habits are also influenced by the products themselves and the way they are displayed in local shops and supermarkets. Every one agreed that when it comes to do the shopping, most mothers tend to buy cheap products in small amounts; otherwise money would not be enough to cover their everyday needs.

The situation allowed some initial change in the classroom environment, as students did not limit themselves to follow the instructions given on the teacher's worksheet, but instead, they queried some of the written statements on the sheet itself broadening the point of view that was to be discussed. A critical position began to appear as one of the groups stated that they could not afford to buy the kind of products advertised on TV, since the products sold in their neighbourhood were cheap, low-quality, and normally packed in small quantities by the resellers in local shops .



Alimentación y bienestar

Las calorías

¿Qué tanto saben ustedes de las calorías?

Caloría —se abrevia cal— es la unidad de medida de la energía, usada para indicar el total de energía que un alimento puede ofrecer, también se utiliza para indicar la energía consumida por un hombre o mujer en sus actividades físicas o mentales.

Para expresar el total de calorías ofrecida por un alimento o consumidas en una actividad usamos como medida la Kilocaloría (Kcal). El prefijo **Kilo**, representado por la letra **K**, significa "mil". Así,

1 Kcal = 1000 cal

Un error bastante frecuente en revistas, embases, y hasta en libros, es usar el termino caloría en lugar de Kilocaloría. Se afirma por ejemplo, que un hombre adulto necesita consumir diariamente, en promedio, 2500 cal, cuando en realidad necesita es 2500 Kcal.

1. Observen los rótulos de información nutricional de algunos alimentos que se consiguen en el mercado y propongan una tabla indicando cuantas kilocalorías ellos ofrecen por 100 gramos del producto.
2. Busque en los rótulos de los productos de paquete que ustedes consumen la información nutricional y establezca para cada uno de ellos, cuantas Kilocalorías ofrecen por 100 gramos del producto.

Entre las sustancias fundamentales para la vida del ser humano están los carbohidratos, los lípidos y las proteínas. Investigue:

¿Cuál es la función de cada una de las sustancias en la manutención de la vida?, ¿los alimentos mas ricos en cada una de ellas? Y ¿Cuántas Kilocalorías hay en cada una de ellas?

A propósito, ¿Cuántas Kilocalorías consumen cada uno de ustedes en un día?

Información Nutricional

Información Nutricional

Información Nutricional

Información Nutricional

Información Nutricional

As a result, in the second activity, it was decided to analyse some of the nutritional information labels attached to students' daily consumption products. This was intended to challenge whether students' nutrition was appropriate or not. As the analysis seemed endless and based on the descriptions given by students, it was decided to limit this exercise only to the analysis of the amount of calorie-intake. Then, it was possible to bring together two subjects like natural science and maths. It was also evident from the discussion that mathematical content appeared as a tool to understand one's own situation by solving direct proportional problems that involved multiplication of decimal numbers.

In the third activity students got to relate their own context directly with their eagerness and interest in doing the activities. Some of them came up with expressions like "country dwellers never eat meat in their meals! It can't be possible!" One group of students even decided to find out more on the internet in order to validate the information given in the lesson. As a result, the need for students to study their own nutrition arose out of a suggestion made by one of the groups. Then, students and teacher conducted a survey on the members of their families at home so that they could determine the amount of proteins, nutrients and vitamins each person was consuming in a single day. This activity allowed the maths lesson to go far beyond the walls of a classroom, since information was being gathered form external sources. Additionally, all mathematical contents were learned due to a genuine need to know whether nutritional conditions were appropriate compared to the recommendations posted on the internet by official international organisations such as FAO.

Unfortunately, the amount of information gathered by students made them repeat the same sort of calculations several times. As a result, students lost the interest they had already taken. Having this situation, an attempt was made to use a spreadsheet for the calculations, but the organisation of the school did not allow quick, effective access to the computer room, which led to the end of the activity with each group presenting their results in front of their classmates.

CONCLUSIONS

Taking into account that critical mathematical education intends to enhance the political, cultural, social and mathematical skills of an individual towards forming democratic citizens, it is remarkable that the learning milieu described made it possible for students to begin to understand the problems, their context and the mathematical tools to be used when querying their living conditions.

Firstly, students realised that, for example, the media does not allow reflection about the quality and convenience of food regarding the needs of the district inhabitants so as to supply them with nutrients appropriate for the jobs they do, the weather of the region, etc.

Secondly, students found that their consumption habits were also influenced by financial matters, since they must be fed on cheap food in order to buy enough supplies for "the whole" family. Given the social conditions of the district the most important thing is to be "filled up" rather than being "well nourished".

Thirdly, such reflections made it possible to tackle the lack of interest in learning mathematical contents previously observed on students, since students conceptualized ideas about science (food components), mathematics (table and chart interpretation, proportions, survey design and data collection) and computer science internet information search); all this thanks to the creation of a learning milieu that surpassed the shared-awareness of contents and engaged students by giving them "a role" different from the one of just listening to and repeating what the teacher says and writes on the blackboard.

Fourthly, it is important to highlight that the organisation of the institution has a strong influence upon any initiative that may arise towards alternative teaching and learning proposals. In fact, such organisation at the beginning allowed teachers teamwork, which led to successful student learning. However, this did not happen at the end of the experience, leading students to lose interest as the amount of information involved required database use and access to the school computers was not granted. Last but not least, the whole experience shows that it is possible to make proposals, inspired on critical mathematics education, aimed at democratic citizen education that allow us to dream about a society where justice and equity may come true.

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A FRAMING OF THE WORLD BY MATHEMATICS: A STUDY OF WORD PROBLEMS IN GREEK PRIMARY SCHOOL MATHEMATICS TEXTBOOKS

Dimitris Chassapis

University of Athens

Assuming that mathematical concepts, beyond their mathematical meaning, acquire multiple referential meanings whenever applied to different real-world situations, it is claimed in this paper that the word problems included in mathematics textbooks convey specific social values and patterns of thought, assigning relevant contextual meanings to the mathematical concepts employed. Evidence to support this claim is briefly reported, drawn from an analysis of the word problems included in two sets of primary school mathematics textbooks which have been used in Greece in the last twenty-five years.

INTRODUCTION

Mathematics textbooks play an important role in mathematics education as they not only identify and organise the mathematical content of classroom teaching but also actually structure classroom lessons with examples, exercises, problems and activities. They, therefore, may be considered to provide a particular interpretation of mathematics to teachers, students and their parents.

In Greece, as possibly in many other countries, the teaching of mathematics in primary and secondary schools is almost exclusively based on the use of textbooks. Therefore, it may be claimed that textbooks define that is called "school mathematics" as well as determine what is known as the "learning path" for the majority of students.

A typical organisation of content found in most mathematics textbooks involves three parts: exposition, examples and exercises, the latter in many forms (Love & Pimm, 1996). The exposition part is intended to support students' learning of mathematics concepts and techniques which are taught by the teacher in the classroom. The examples either pave the way to mathematics concepts within a rationale of 'guided discovery' or follow the exposition part as prototypes to be copied by the students answering exercises and solving problems posed in the third part. The third part accommodates exercises and problem solving tasks, sometimes graded in order of difficulty. Exercises and problem solving tasks are quantitatively dominant in many mathematics textbooks (Pepin & Haggarty, 2001) and among them word problems represent a common way in which "real world" contexts are introduced into school mathematics. A word problem is defined as any "verbal description of problem situations wherein one or more questions are raised the answer to which can be obtained by application of mathematical operations to numerical data available in the problem statement. In their most typical form, problems take the form of brief texts describing the essentials of some situation wherein some quantities are explicitly given and others are not and wherein the solver - typically a student who is confronted with the problem in the context of a mathematics lesson or a mathematics test – is required to give a numerical answer to a specific question by making explicit and exclusive use of the quantities given in the test and mathematical relationships between those quantities inferred from the text." (Verschaffel et al, 2000, p. ix).

Word problems may also combine a written text with other kinds of information, e.g. a table, a picture, a drawing.

Word problems have been extensively problematised and their many dimensions have been analysed from a variety of perspectives, including mathematical (e.g. Verschaffel et al, 2000), linguistic (e.g. Gerofsky, 1993), psychological (e.g. Lave, 1992) and sociological (e.g. Cooper and Dunne, 2000). Considering word problems as the main vehicle for introducing real world contexts into school mathematics, then the type and kind of real life situations selected and used to frame the mathematical questions posed by these problems is of crucial importance, particularly as a topic for perceiving the ideological influences that mathematics teaching exerts on children conveying specific social values and patterns of thought. At the same time, however, mathematics is considered as a politically and ideologically neutral discipline.

As an example, it is claimed in this paper that the word problems included in primary school mathematics textbooks used in Greece assign value-laden meanings to the mathematical concepts employed; research evidence for supporting this claim is briefly reported.

CONCEPTUAL AND REFERENTIAL MEANING OF MATHEMATICS

As has been elsewhere analysed (Chassapis, 1997), each mathematical concept acquires its meaning by a particular mathematical theory in which it is embedded. This is its conceptual meaning, assigned by the propositions of a particular mathematical theory. For example, the concept of addition for natural numbers is defined recursively by the Peano axioms (i) $a + 0 = a$, and (ii) $a + Sb = S(a + b)$, where Sa is the successor of a. Accordingly in set theory addition is defined by the cardinality of the disjoint union and in any other kind of mathematical structure is defined in terms of its propositions. Mathematical concepts, however, are used to describe or are "applied" to the real world on the basis of a mapping between them and real world situations. Every such mapping is indispensably mediated by a class of non-mathematical concepts that circumscribes the real world situations to which the mathematical concept is applied as well as by a set of pertinent linguistic or more generally symbolic expressions that signify these concepts. These non-mathematical concepts and their symbolic expressions assign another non-mathematical meaning and simultaneously specify a reference of the mathematical concept to a particular description or application. This meaning is the contextual or the referential meaning of a mathematical concept. Number addition, for example, may be used to describe or may be applied to a class of real world situations circumscribed by concepts of change, combine or compare. Particular instances of these concepts (e.g., the growth of a quantity as an instance of change, the union of two quantities as an instance of combination or the difference of two quantities as an instance of comparison) together with their signifying linguistic expressions attribute to the mathematical concept of addition various referential meanings, according to the case, which are not in their every aspect identical.

The referential meaning of a mathematical concept beyond its practical sources is related to the values associated by the people employing it in their everyday activities and/or by various communities using mathematics applied to their practices. This second aspect is socially and historically determined, since one mathematical concept can be valued in one context and de-valued in another, while its value in the same context can change over time due to social changes. Adopting the view that real world situations acquire their meanings by the implicated human activities which are always meaningful since intentional, it may be claimed that any real world situation and its representations bear meanings that are never value-free nor ideologically neutral. One step further, it may be claimed that the selected real world situations and consequently the associated references of the mathematical concepts to specific aspects of the real world used as examples, applications, questions or problems-to-be-solved in the teaching of school mathematics are never value-free and bear - in any feasible case - a more or less definite, even if not clear, ideological orientation. They thus assign to the mathematical concepts analogous, ideologically oriented, referential meanings. The ideological orientations of the referential meanings assigned to number addition, for instance, when applied to, and interpreted as describing, a growth process of profit in a situation of commercial dealings or a growth process of nuclear waste in a situation of environmental pollution are not identical. The two situations highlight different aspects of human activity, and implicitly emphasise different attitudes and patterns of thinking towards human activities, support different life values and ultimately transmit different social ideologies.

From this point of view, school mathematics, just as many other school subjects, may not be considered as an ideologically and hence socially neutral subject of knowledge, derived from a similarly neutral scientific mathematical activity. It has to be conceived as a school subject composed of selected mathematical topics bearing ideologically oriented referential meanings assigned to mathematical concepts and tools by their selected mappings in selected applications to selected real world situations.

AN ANALYSIS OF EXAMPLES AND WORD PROBLEMS INCLUDED IN GREEK PRIMARY SCHOOL MATHEMATICS TEXTBOOKS

Two sets of textbooks used in Greek primary schools for teaching mathematics were analysed. The first set was adopted and used from the school year 1982-83 until 2005-6 (hereinafter referred to as set A) and the second from the school year 2006-7 to the present day (hereinafter referred to as set B). Each set includes one textbook for each of the six primary school grades being the unique textbook for every primary school in Greece, distributed free of charge to the students according to relevant legal provisions. These textbooks follow the mathematics curriculum which is centrally prescribed in detail by the Ministry of Education and includes the study of natural numbers and fractions, their fundamental operations and associated algorithms, basics of measurement, basic geometric relations and shapes as well as handling and presenting numerical data. Exercises and problems are emphasised for each topic while references to everyday activities are dominant in both sets of mathematics textbooks. Every mathematics textbook is accompanied by a teacher's manual dictating in detail every teaching unit, its content, teaching method and learning tools. Since the curriculum, the teaching materials and methods are centrally designated and controlled, mathematics teaching in Greek primary schools may be considered as uniform in any of its main aspects.

The related question encountered in this paper is twofold. First, which are the prominent characteristics of the real world situations prevailing in the Greek primary mathematics textbooks as references of mathematical concepts to real world and second, which is the social ideology, if any, advocated by the meanings of these references? In other words, which aspects of the real world are selected and in which patterns are they structured and nominated by the Greek primary mathematics textbooks as the prominent real world objects of mathematical activity.

For the analysis of textbooks a technique of textual analysis was employed, comprising two steps. First, all worked examples and problems included in mathematics textbooks were sorted out and classified into categories according to their contexts and the activities described or referred to. The outcome of this first step of quantitative analysis is outlined in Table I.

Second, the subjects referred to as actors in word problems and examples as well as the material objects manipulated by these subjects or simply involved in calculations were spotted and counted. Due to space limitations in this paper only the most significant data from this analysis will be reported.

The findings of these analyses suggest that in the word problems included in mathematics textbooks three classes of situations are privileged as references of mathematical concepts to the real world, in almost equal share in both sets of textbooks:

Table I: Context of examples and problems by textbook set and school grade

Context	Textbook set	School grade					
		1st	2nd	3rd	4th	5th	6th
Counting -Measuring	A	22 73%	38 53%	72 40%	38 15%	87 25%	27 7%
	B	48 63%	47 27%	31 27%	60 30%	58 31%	101 31%
Buying - Selling	A	6 20%	22 31%	58 33%	92 36%	37 10%	110 30%
	B	14 18%	45 25%	28 25%	53 27%	45 24%	83 26%
Production-Consumption	A				13 5%	30 8%	39 10%
	B			5 4%	18 9%	2 1%	15 5%
Income-Expenses-Salaries-Taxes-Deposits-Insurance	A				8 3%	1 1%	7 2%
	B						
Sharing	A		9 12%	7 4%		12 3%	
	B	5 7%	10 6%				
Craft - Manufacturing-Construction- Household-calculations	A		3 4%	35 20%	51 20%	84 24%	92 25%
	B		9 5%	25 22%	31 16%	36 19%	79 24%
Trip & travel expenses	A				32 13%	31 9%	21 6%
	B		7 4%	9 8%	11 6%	30 16%	38 12&
Game scores	A			6 3%	19 8%	11 3%	7 2%
	B	5 7%	15 8%	15 13%	14 7%	18 10%	9 3%
Drawing-Graphing-Geometry	A	2 7%				60 17%	68 18%
	B	4 5%	44 25%		12 6%		
Total of problems & examples	A	30	72	178	253	353	371
	B	76	177	113	199	189	325
Total of problems & examples out of any context	A	24 80%	38 53%	72 40%	38 15%	147 42%	95 26%
	B	52 68%	91 51%	31 27%	72 36%	58 31%	101 31%
Total of problems & examples framed in a real world context	A	6 20%	34 47%	106 60%	215 85%	206 58%	276 74%
	B	24 32%	86 49%	82 73%	127 64%	131 69%	224 69%

(1) First is a class of financial and, especially, commercial situations devoid of any pertinent social relationships. Buying and selling of commodities, money returns

and payments, business profit accounts, individual incomes and expenses, consumption bills, funds debit and credit, tax payments, insurance and related transactions compose the prevailing references of the mathematical concepts.
For example:

> *"You have 20 Euro. How many will be left over if you buy a T-shirt costing 15 Euro?"* (1st grade, textbook set A)

> *"Mark wants to buy balloons for his birthday costing 3 Euro each. He has 25 Euro. How many balloons may he buy? Will he have any money left over?"* (3rd grade, textbook set B)

> *"The nearby stationary shop sells notebooks for 30c each. George paid 90c and John 60c for buying some of them. How many notebooks did each one buy?"* (3rd grade textbook set A)

> *"One chair constructed by a cabinet-maker cost 35.22 Euro and it sold with a profit of 8.8 Euro. How much was it sold for?"* (6th grade textbook set A)

Even calculations of interest on capital deposits and loans are included as a distinct teaching unit for the 6th grade in the set A of mathematics textbooks, containing problems as the following:

> *"A farmer took a loan from a bank which is payable in 10 months at a rate of 10%. What amount ought he to pay back to the bank?"* (6th grade, textbook set A)

> *"Mrs George has deposited today at a bank an amount of 293.5 Euro at a rate of 22% for 18 months. The interest is added to the capital at the end of every six-month period. How much will she have after 18 months and will it suffice to buy an appliance which is on sale for 836.4 Euros?"* (6th grade, textbook set A)

Many of the financial transactions described in mathematics textbooks derive from a socially abstract material production and distribution, which is introduced in the word problems by statements of the type "a factory constructs", "a farm produces", "a store sells", or "bottles are packed", "apples are sold", "drinks are canned" etc. All these appear as of being on their own, in the absence of any human agent, out of any spatial, temporal or social structure. On the other hand, whenever any person is participating in such a situation, it is indicated by an occupational identity (e.g. a bookseller, a grocer, a confectioner, a craftsman), by a position in a division of labour (e.g., an employer, an employee, a manager, a producer) or is presented as a socially indefinable agent (e.g., a man, a woman, a buyer, a seller). In any other case, the student is assigned the role of an actor in financial transactions being addressed by the personal pronoun "you", a speech act inciting participation both in mathematical and even in imaginary financial activities.

Furthermore, an interesting feature of many problems of this class of situations is the confusing use of economic terms, especially those concerning "values", such as "cost" and "price" of goods or services, which are always used interchangeably, thus obscuring fundamental aspects of commercial reality. For instance:

"You have 20 Euro and you buy a t-shirt that costs 15 Euro. How much money will you have left?" (1st grade, textbook set A) and *"Apples were sold for 78c per kg last year and this year their price increased by 12c per Kg. How much are they sold per kg?*" (3rd grade, textbook set A)

(2) Second, mathematical concepts are referred to a class of situations, similar in its main characteristics to the previous class, however requiring calculations concerning various construction, manufacturing or household activities. Coins constitute an indispensable element of this as well as the previous class of situations and they are indirectly ascribed an almost natural existence. Their manipulation is introduced as a prominent reference of numerical activities from the first teaching units of the primary school mathematics onwards, as may be concluded from the data shown in Table II.

Table II: Problems including manipulation of coins

textbook set	School grade					
A	1 4%	14 19%	70 33%	108 38%	16 6%	115 30%
B	9 13%	41 26%	30 31%	44 27%	46 29%	75 32%

Coins are used even in quite unrealistic cases as in the following problem posed as an application of decimal fractions.:

"You had 8/10 of a ten-drachma coin and spent 5/10 of it. How many tenths of the ten-drachma coin have you got left?" (4th grade, textbook set A).

A marked division of activities by sex is also clear in the problems classified in this type of situations. Men are crafting, constructing, building, while women are cooking, sewing, knitting, typing and sometimes calculating expenses either within or outside their home. For instance

"The 15 girls of the 5th grade of a school bought 9.75m of fabric in order to make embroideries of the same size. How much fabric did each girl use?" (5th grade, textbook set A).

(3) Third, word problems refer to socially indefinite or artificial situations coming out of an ostensibly abstract, and thus neutral, world. A world made up of material objects, (playthings, clothing, pieces of furniture or everyday objects), plants (flowers, vegetables or trees), fruits (apples, oranges or bananas) and animals (birds, dogs or cats), which being as a rule out of any meaningful context are unmediated objects of mathematical manipulations, usually incited by the question "How many?" or "How much?". For instance, the problems included in textbooks for the 1st grade refer mainly to toys (43% of total problems in set A and 33% in set B) and foods and fruits (26% and 28% respectively).

The persons, if any, involved in this class of situations, required to count, measure, compare or share objects are referred to by personal names (e.g., George, Helen), family relations (e.g., mother, brother, sister), sex identity (e.g., a girl, a boy), membership of a group (e.g., students, passengers, spectator) or personal pronouns (me, you, we). All these activities are, as mentioned, detached from any social setting and their exclusive aim of exercising makes their context obviously superfluous. For example

A window had 8 panes but 3 of them broke. How many panes were left? (1st grade textbook set A)

Summing up, it may be claimed that the commercial market is directly endorsed by the Greek primary school mathematics textbooks as the prominent field of mathematical activity and indirectly as a dominant aspect of the social world. In addition commercial transactions are nominated as the dominant mode of human activity while commodity production and distribution are abstractly presented as impersonal and socially neutral, therefore as essentially technical, activities. Such options are not directly imposed by any learning requirement of primary mathematics per se and therefore these may not be considered as ideologically neutral options. Intentionally or not, the authors of both sets of the analysed textbooks, using the aforementioned real world applications of mathematics, reinforce and promote a particular image of worthy modes of human activity and thus a particular conception of society. The following problem, albeit ridiculous, is an exemplary culmination of this point of view:

"Bill Gates, the owner of Microsoft Company, had during the year 2003 an income of 5c for every second of the time. How much money did he make in one minute, one hour, one day, one month and one year?" (5th grade, textbook set B).

CONCLUDING COMMENTS: FRAMING THE WORLD THROUGH MATHEMATICS

School mathematics word problems framed in real world contexts play a mediating role between mathematics per se and real world situations, suggesting and in most cases creating templates for "reading" mathematically the objects and events of the world. In such a context the role and function of mathematics word problems may be understood from the viewpoint of what Goffman (1986) calls the "frame" of a (social) situation. Frame is primarily a psychological concept that refers to the cognitive process wherein people bring to bear background knowledge to interpret an event or circumstance and to locate it in a large system of meaning (Oliver & Johnston, 2000). In Goffman's perspective, the concept "frame" implies that there is a definition of a situation which the participants share and most of them take for granted. A frame can be seen as the participants' shared response to the question "what is going on here" (Goffman, 1986, p. 18), which means that they have construed events, actions or utterances in line with the frame which they perceive as relevant.

Frames are basic, individual, cognitive structures, which guide the perception and representation of reality or, put in other words, frames structure which parts of reality become noticed. Frames select and organise information drawn from real experiences and about people and objects and which are actually in the world, therefore they orient and guide interpretation of individual experience, that is "enable individuals to locate, perceive, identify and label occurrences" (Snow et al. 1986, p. 464). A distinction between the concepts “frame” and “framing” is rather helpful. “Frame” is a mental structure. “Framing” is a behaviour by which people make sense of both daily life and the grievances that confront them. Frame theory, therefore, as developed after Goffman’s founding contribution, embraces both cognitive structures whose contents can be elicited, inferred, and plotted in a rough approximation of the algorithms by which people come to decisions about how to act and what to say and the interactive processes of talk, persuasion, arguing, contestation, interpersonal influence, subtle rhetorical posturing, outright marketing that modify—indeed, continually modify—the contents of interpretative frames (Oliver & Johnston, 2000).

Conveying frames for reading the real world mathematically, textbook word problems infuse in children a practical relationship with mathematical knowledge, a relationship of usage. Such a relationship of usage without doubt contributes to the construction of mathematics knowledge. This knowledge however is primarily a practical knowledge of using mathematics and alongside it is a scientific knowledge of the mathematics subject matter. This fact may, in my view, be considered as the essential meaning of the concept of “mathematical enculturation”; a mathematical knowledge invested in a practical knowledge of its usage. For this reason among others, mathematics traditionally constitutes a fundamental component of the socio-cultural indoctrination of children. Mathematics trains children towards “the correct” modes of thinking, “the correct” modes of deducing, “the correct” modes of decision making. A relation of this type between school mathematics and its subject matter may not in any case be considered as an exclusively learning relation. Rather, it is a relation of ideological indoctrination of children, which, by using mathematics – a subject matter commonly agreed to be valuable - habituates them in particular standpoints and specific patterns of behaviour and directs them through mathematics towards the prevailing social values (Althusser, 1970).

In conclusion, school mathematics - as well as the teaching of many other school subjects - incorporates a double relation with its subject matter: a scientific relation as a means for theoretical knowledge of mathematics concepts and tools and an ideological relation as a vehicle for practical knowledge about the use of mathematics. This practical knowledge concerns particular patterns of behaviour towards the theoretical and social function of mathematics. In this sense, the teaching of mathematics aims both at learning mathematics concepts and tools and at appropriating an ideology for mathematical activities and their outcomes, that is, an ideology concerning mathematics, based on a specific conception of place and function of mathematical activity, its outcomes and applications in the present-day dominant social reality.

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DESIRING / RESISTING IDENTITY CHANGE POLITICS: MATHEMATICS, TECHNOLOGY AND TEACHER NARRATIVES

Anna Chronaki with the contribution of Anastasios Matos

University of Thessaly

This paper discusses how secondary mathematics teachers invest in discourses of 'change' as part of their professional development trajectories. The paper is based on an ethnographic study and focuses on a small group of mathematics teachers who are trained to become teacher trainers for technology mediated mathematics. Based on an analysis of 'teacher narratives' through interviewing and extensive participant observation, we claim that their professional development paths envelope both desiring and resisting identity change. The micro-physics of the everyday life in their school simultaneously requires appropriation and resistance of computer-based regulatory discourses about 'change' that in turn leads to unending, fluid and fractured identity formulations.

Teacher A: You shouldn't ask me, because you see me. At least myself, I was at a different phase at the beginning and I have now developed. I understood what you wanted, then I questioned it, then I was convinced and I, now, hold on to what I personally want from what you wanted.

MATHS TEACHERS AND TECHNOLOGY: INTENSITY FOR CHANGE

During the last day of a three month long training course for technology-based mathematics teaching and learning, a small team of mathematics teacher trainees and their tutors (the authors of this paper) discuss – as a way of collective reflection - the significance of the particular course for themselves as professionals. The focus soon shifted to the kind of '*changes'* they had experienced in this particular training course. 'Change' -the desired outcome of the course- was almost felt as a pressure inscribed at different levels. At one level, the training course documentation itself – following dominant mathematics curricula reform recommendations (e.g. NCTM, 2000; Hershkowtiz, et al., 2002; Marioti, 2002; Kaput, 1992) - emphasizes 'change' in the sense of mathematics teachers developing certain 'new' skills, competences and attitudes through the use of appropriate software tools. On another level, mathematics teachers themselves anticipate, by means of participating in this training course, that learning about technology in mathematics would immediately result in significant 'changes', not only in ways they use and appreciate technology for mathematics teaching and learning, but also in ways they practice and value mathematics education in a broader sense.

Teacher A, as seen above, during that group discussion, argues that he has indeed experienced through the course a kind of 'change' – a change that has transformed him personally. In a surprising tone he says: '*You shouldn't ask me, because you see* *me*' - meaning that the aforementioned changes are obvious through behavioral changes. His experience of change is described as a turbulent cycle where he moves from trying to interpret what the tutors demand (via the documentation and the practices that are modeling in the course) towards problematising those demands and choosing what he appreciates as useful for his teaching. Teacher A seems to appropriate the underlying discourse of 'change' through computer-based mathematics as he narrates an almost smooth developing path that highlights an identity transformation towards a 'new' mathematics teacher identity. But, is it really so? And is it so for any teacher?

A few weeks after the training course was completed, and as teachers go back to their daily school duties and routines, the discussion continues at a personal level of interviewing. The issue of 'change' comes up again. The teachers now talk about 'change' as a more complex experience that cannot be detected at a behavioural level and they talk about 'change' as something that cannot be really achieved. Humour helps them to cope with the uncertainty of having to deal with this utopian experience of 'change':

> Teacher B: When Axxx (a tutor) was asking during the last day of the course how 'technology' has changed us, he (Teacher A) said: '*Why do you ask? Don't you see me?*' We now have used this as our slogan. When we talk with other colleagues, we ask '*How do you see me? Have I changed?*' (laughter)

Contrary to initial views (see what teacher A says above), teacher B resists the discourse of a smooth 'change' simply by means of technology-use. Along with colleagues teacher B jokes about the possibility of any noticeable 'change'. Humour is a significant characteristic of cultural communities (even animal communities) and jokes, in particular, constitute powerful metaphoric forms of communication. Currently, anthropological studies draw attention to biosocial and evolutionary aspects of humour as intrinsic aspects of social life and relations (Apte, 1985). Handelman and Kapferer (1972) argue that a 'joking license' must be allocated for jokes to be communicated amongst friends and companions. This 'license' constructs a context in which participants are issued the freedom – within boundaries - to insult or abuse one another without damaging the relationship between the parties. Hence, events of humour can even be approached as important conceptual and methodological tools because they provide insights into behavioural and cultural patterns of societal relations and support to express, to describe and to evaluate taken for granted institutional symbols, relations and values.

In our case, teacher B and his colleagues seem to have this 'joking license' to talk about 'change' and laugh about the incomplete process of changing towards a 'new' mathematics teacher identity. The significance of this humorous event points out a shared concern amongst teachers that aims to disturb the stereotypical image of computer as a 'mythical' and 'heroic' mediator for enhancing change in mathematical school life and culture. It further indicates their experience of 'identity change' as a complex and slippery process with ups and downs, risks and ambivalences - a process that envelops strain, fear, and uncertainty. Their humour, thus, highlights the 'gap' between the actual and designated phases of their becoming as computer-maths teacher trainers and as such indicates also the reinforcement of a specific desire. Brown and McNamara (2005) resort on Lacan who suggests that '[...] *the distance between life and its supposed symbolisation must not be obliterated. This very gap creates the desire and shapes life itself*' (p. 35).

COMPUTER-BASED MATHEMATICS AS A 'CHANGE' MACHINE

The rapid pace of technological change, characterising the end of the twentieth century and the beginning of the twenty-first, leads to the rise of what Lister et al (2003) call 'upgrade' culture. Computer related hardware and software products become parts of continuous 'upgrading practices' and, as such, the notions of 'computer' or 'media' become reconstructed as technologies in flux. In this way the computer, along with varied forms of digital media, are easily seen as non-fixed, non-achieved or non-stable pieces of technology. Instead, the very notion of 'computer' and 'technology' pinpoints to continuous cycles of innovation and renewing.

Simulation technologies developed within computer science in the 1960's have challenged traditional visual culture through new visual media and imaging technologies (e.g. virtual reality and digital cinema). Central issues for theories of photography, film and cinema have been their realism and the nature of visual representation. Such deeply embedded traditions in Western visual representation forms become now reconsidered in the light of practices such as virtual reality simulation, computer-generated animation, modelling applications etc. New visual media create new visual cultures and pave the path for new social-semiotic cultures. They enable possible re-readings of cultural relations and promote varied representations and imaginations of both historical and future worlds. The late 1980's brought significant technological changes in the educational and popular culture(s) terrain with 'tools' that emphasize, amongst others, computer-mediated communication, new ways of distributing and consuming media texts characterised by interactivity and hypertext forms, virtual reality - from simulated environments to fully immersive representational spaces, and a whole range of transformations and dislocations of established media such as photography, film and television.

In mathematics education, the emphasis on 'new media' choice continues to be placed on specific software environments that support the learning of particular mathematical content. As such, digital tools for dynamic geometry, computer algebra, data handling, statistics, programming and modelling aim to encourage the development of specific mathematical skills and competences within the boundaries of certain curriculum areas (see Hershkowitz et al., 2002; Ruthven et al., 2009). However, we recently witness an expansion towards web-based communicative contexts such as *TeleCabri* - the web version of Cabri-geometer - that represents real time teaching at a distance (Balacheff & Kaput, 1996). In addition, numerous websites are now designed and maintained aiming to enhance instruction, to provide tutoring, to serve as resources to teachers and learners, and to become the platform for developing projects and collaborative activities between schools and classrooms cross-nationally (Chronaki, 2000).

At the very least, we face on the one hand, a rapidly changing set of technological experiments and, on the other hand, a complex set of interactions between new technological possibilities and established media forms. New mathematics education technologies constitute social institutions of mathematics education practices that are simultaneously independent and inseparable from established commercial and educational cultures on which they draw to advance and support their agendas for design and implementation frameworks. These 'new' media become disseminated, received and consumed by and through their various audiences (teachers, pupils, parents, the state, the market). The politics of the hidden curriculum (via subject content and assessment procedures) acting as 'ideological state apparatus' (see Althusser, 1971) guide and control the shaping and distribution of 'new' technology products for mathematics teaching and learning. One of the teachers, as part of her observations of teachers' authoring activities for 'new' technologies in mathematics teaching, observes how teachers become trained to appropriate certain discourses about software use:

> Teacher V: [...] Reading the work of last semester (teacher) trainees referred to Fxxxxxx software I get the impression that these people, without being aware of it, worked for a serious advertisement for this particular software. It is as if they were paid to advertise it.

Teachers, at large, experience the imperative for change being mediated by the 'newness' offered by computer hardware and software. The sense of the 'new' becomes a reference to the most glamorous and recent, and this in turn carries the ideological sense that new equals better. The 'new' signifies 'the cutting edge', the avant-garde, the place for forward-thinking people to be as designers, producers and practitioners. Discourses of 'change' as connotations of the 'new' are related with the long-lasting modernist belief in social progress and development as smoothly delivered by technology. Most of the teachers, unlike teacher V above, appropriate varied discourses of professional 'change' towards the 'new' with romantic enthusiasm.

Investment in discourses related to the revolutionary impact of technology (and new media) assumes that there are profound transformations of everyday life that are taking place in terms of both structural organisations and evolving relationships amongst humans and computers. The transformative nature of 'new' media is expected to influence identity sense, habits of consumption and communication, politics of gender and age, as well as local/global geopolitics.

The transformative impact of 'new' media in mathematics education has been mainly discussed in terms of teachers' epistemological, pedagogic and didactic potential for change through a number of experimental case studies that exemplify beneficial potentials. But, technology integration in mathematical classrooms remains a challenge for most maths teachers (see Ruthven, Hennessy, & Brindley, 2004). Moreover, teachers, as also our study reports, rarely narrate experiences of any transformative learning as far as their professional practices are discussed. However, they engage with discourses of 'change' as they talk about the potential uses of technology in mathematics education. Change embodiment can be incorporated into an understanding of teacher-self as part of broader 'identity change' politics. Our intention in this paper is to explore how teachers use notions of 'change' discursively when they talk about technology-mediated mathematics. Our interest in this paper is the political dimension of how desired and/or resisted 'change' constitutes identities and how these identities are assigned to maths teachers through discourses, narratives and performances.

THE STUDY: CONTEXT, SUBJECTS AND METHODOLOGY

The present study is part of a larger ethnography (in-progress) concerning how a small group of 11 teachers (7 maths teachers and 4 language teachers) narrate their professional 'development' as they learn to become teacher trainers for computer-based mathematics. The research study took place in the context of implementing a specific teacher training course for technology in mathematics education (see PAKE, 2007). Because the Greek Ministry of Education had directed the nationwide initiative of training teachers to be ICT trainers, the training course had rather a centralised character. Specifically, there were central 'national' guidelines to be followed by the university departments that undertook the course implementation, including common tools to be used, and common methods of implementation and assessment. As a result creative initiatives were prevented from growing and emerging. Within this context, we aimed to capture how teachers appropriate and resist certain discourses of 'identity change' in terms of disciplinary cultures, curricula and possibilities for technology use. A preliminary analysis suggests that discourses of 'change' embrace talking about experiences with technology as 'fluid shifts' from established disciplinary norms towards 'new' routines, rituals and politics of representing, communicating and producing mathematical knowledge. Discourses of identity change politics as 'content-aesthetic', 'power relations' and 'cultural/discursive' shifts are discussed in the section below.

CHANGE AS CONTENT-AESTHETICS SHIFT

Amongst the most commonly used arguments concerning the valuing of 'new' technologies in mathematics teaching is that specific software and tasks enable an aesthetic change of mathematical content representation on the computer screen and enhances human-computer interaction. Teachers contrast the imagined enhanced aesthetics (e.g. via exemplary cases) with a harsh reality of an established mathematics culture. Talking about their experiences of current mathematics education localities (i.e. emphasis on content, paper-pencil and teacher-chalk, national exams for university entry, private tutorials etc.) teachers claimed that the system serves to place emphasis on training students to solve difficult (but not necessarily challenging) mathematical problems and on making them competitive problem solvers. Teachers seem to agree that the situation is almost tragic and claim that 'something must be done', or 'we cannot continue like this'.

At the very beginning of the course, teachers were asked to write down a few verbs that exemplify what children do in a mathematics classroom. Jokingly, they referred to verbs such as 'sleeping', 'moaning', 'doing exercises', 'waiting for time to pass by', 'sitting still', 'solving problems, 'sketching figures'. Verbs referring to 'having fun', imagining, being creative, sharing ideas or experimenting were completely absent from their list. Thus, given the above context, technology – as also other innovative initiatives (see Thale's friends: a popular community for mathematical literature reading in Greece) - becomes a savior or an easy solution to a long-term problem. When mathematics teachers were asked how technology might support their teaching, they referred to learners being enthused, attracted, motivated and engaged participants.

Teacher C: Children become enthused. To start with, they become enthused due to the fact that this change [from paper-pencil to computer use] takes place. And the children ….absorb what you say to them.

Teacher A: This medium is more attractive, for sure. It [means the computer] will place the learner …It will make him …in simple words… not bored from the endless bla, bla…even from the dialogue [means talk] during the lesson. It is different. It is more attractive. It will support pupils' engagement..

Teacher P: [..] In this technology lesson. You must see them [implies the pupils]… all of them. Focused. Ah, do you believe it?! This thing happened! This thing happened in a vocational school.

Enhancing the variety and appeal of classroom activity was amongst the emerging themes identified by Ruthven et al. (2004) in a study concerning teacher representations of successful use of computer-based tools and resources. Teachers, as in our study, referred to activities involving technology as 'something different', 'making a change', 'a change from the routine of the classroom'. We can add, based on our data, that the 'novelty' and 'appeal' associated with technology-use is coupled with mathematics being approached as a commodity - a commodity that needs immediate change. This change, in the teachers' words, will need to take pupils from inertia to activity, from boredom to creativity and from a disciplined reading of mathematical content to free expression. They indicate that the 'new' screen aesthetics (e.g. multiple representations, the dynamic image, simulations, modeling of real life situations etc.) happen as part of human-computer interactions and can capture pupils' attention and imagination. These 'new' content-aesthetics are inextricably linked to teachers' and pupils' increased awareness of a mathematics culture that has ceased to motivate and inspire teachers and pupils alike. This rise in consciousness is supported by increased tensions with a post-industrial information age shift (see Castells, 1996) that stresses a shift in employment, skill and production of material goods in which new media seem to epitomize.

CHANGE AS POWER RELATIONS SHIFT

Teachers have come to see the implication of their 'new' role as users of computer-based mathematics as having to change from being a transmitter of knowledge to a facilitator of knowledge construction (Chronaki, 2000). In the computer-mediated classroom, the teacher as a transmitter of information and controller of knowledge is becoming redundant and is being replaced by a co-worker, co-learner, facilitator or supporter to pupils' learning (Schofield, 1995). Teachers in this study seem to embrace this discourse with some anticipation:

> Teacher A: […] for pupils, if we can create this move for pupils. To talk. To try and try. To explain why we did such and such. They will feel it as theirs..[..] the knowledge that will come later on. They will feel it belongs to them. In other words, it [mathematical knowledge] does not come from the teacher. Or, if you like, it has been validated by the machine (means the software). I think, in this way, we win the students. We win them…

Reading through the above extract we realize, at first, a disappointment of an established culture of human interactions amongst teacher-pupils with unequal power dynamics. Teacher A relies on the computer as a 'power mediator' that might disrupt established power dynamics. Power is thus becoming distributed more evenly amongst pupils, teachers and computer. The teacher, thus, does not need to directly control their teaching activity because the 'control' is regulated via the machine. The computer-in-use, acts as another type of bureaucracy (as a panopticon machine or a technology of power) disciplines and contributes to a process of disindividualisation – a tendency to think that power resides in the machine itself rather on those who use and operate the machine (see Foucault, 1977). This form of 'disindividualised power' rests on the view that power resides in the machine itself. The computer becomes, for teacher A, a hidden mediator of teacher and curriculum power over the process of constructing knowledge. This discourse, in some way, entices learners making them believe that teachers are not controlling their activity and that they are free to choose their learning path.

CHANGE AS CULTURAL/DISCURSIVE SHIFT

As mathematics teachers experience the move towards becoming computer-based teacher trainers the intensity for change relates to their intention to enrol within both a 'youth' and a 'scientific' community. Through the training course, they are required not only to learn using specific 'new' media, but also to read academic papers that justify theoretically and empirically the choice of specific software and to provide examples of design and evaluation of computer-based mathematical activity.

Involvement with technology was seen by some mathematics teachers as a way to connect with pupils and bridge an age generated cultural gap:

Teacher V: [..] If we are disconnected from pupils […] they wouldn't be interested at all for all we try to pass on […]. We will cease be convincing. We will belong to the Paleolithic age.

Whilst, new media enable them to bridge the generation gap between themselves and pupils, becoming trainers stresses the need to develop a theoretical language that supports a move to becoming 'scientific'. Laughter helps them again to cope with the complexities of this becoming process.

Teacher P: [..] I do not know what these theories talk about […]. For activities-scenario description I saw pages and pages [..}. And I said, I am not going to read and learn all these. I quitted. And instead, I turned to read Einstein's theory (laughter)….//

(later on)

Tutor: (refers to the notion of 'didactic outcome' and explains it)

Teacher P: I do not understand such scientific jargon […]. Ok?!. I am asking now my colleagues. To see who has realized the meaning of this term…

(later on)

Tutor: […] please, have a look now what I can do with this tool (refers to a specific software). It is a simple thing.

Teacher P: (using irony). Ok. It is possible. It has positive effects of learning … according to Vygotsky. (laughter from all)

BY WAY OF CONCLUSION

This study aimed to identify how mathematics teachers who experience the complexity of becoming technology-use trainers invest in discourses of 'change' as part of a complex identity-work process. Teachers experience the intensity of 'change' and 'identity change' as they are introduced to course material (theoretical resources and practical classroom activity design) and to the demands of technology integration in mathematics classrooms. It became obvious through our study that teachers' investment in discourses of 'change' is not an individual issue. Instead, they resort collectively and politically on past experiences and future aspirations about mathematics curriculum reforms taking into account the constraints and demands of their everyday realities in school classrooms and communities. They discuss 'change' as 'fluid shifts' to something 'new' referring to content-aesthetic, power relations and cultural/discursive developments. 'Change' refers to contextual issues and 'identity change' is inscribed as a continuous move amongst possible acts and potential imageries about how these acts could adopt or resist 'change' in a material sense (i.e. content representation/simulation on screen, communicative rituals, routines and politics).

Whereas a modernist approach to understanding social life would view the individual as an agent having an authentic core of essential identity and being responsible for social transformation, the present study comes closer to a post-structural conception of the self as involved in a continuous production of identity in historical, discursive and material contexts. Imagined discursive practices of ‘change’ as they are related to technology-use by mathematics teacher-trainers operate towards identity-politics that stress individuals binding to certain subject positions that involve the development of ‘new’ teacher/learner roles by means of using ‘new’ media.

Stressing the transformative role of ‘new’ media is an old concern that reflects broader socio-economic politics (see Castells, 1996). As far as ‘new’ media are concerned with mathematics education their transformative role, while a main concern amongst many stakeholders, is not widely experienced by teachers in the everyday experiences at school. Even the teachers in our study who obtain an intrinsic motivation for technology integration and a desire for change, experience ‘change’ not only as a complex and turbulent process but also as unachievable. Through humour they disrupt predominant notions of a smooth ‘identity change’ by means of ‘new’ technology use. At the same time, they invest in discourses of ‘change’ such as content-aesthetic, power relations and cultural/discursive shifts. Mathematics educators seem to adopt collectively those claims hoping that broader ‘changes’ in mathematics education can be materialised. Technology-based mathematics education becomes a heavy political arena that serves to regulate teachers, learners and curriculum designers toward a particular collective identity change in the name of the ‘new’ mathematics teacher.

Acknowledgment: We would like to thank all teachers who participated in our study and shared with us a number of issues related with technology-in-mathematics-education.

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DISCURSIVE AUTHORITY IN THE MATHEMATICS CLASSROOM: DEVELOPING TEACHER CAPACITY TO ANALYZE INTERACTIONS IN TERMS OF MODALITY AND MODULATION

Elizabeth de Freitas, Betina Zolkower

Adelphi University, Brooklyn College

This paper discusses discourse analytic tools used to develop teacher capacity in analyzing classroom interaction. We examine the linguistic tools of modulation and modality (used to express degrees of obligation, inclination, probability and usuality) as markers of epistemic authority and deontic agency. We then discuss the first year results from a research project using these tools with beginning middle school mathematics teachers, and show how they developed skills at analyzing transcripts for evidence of discursive authority.

INTRODUCTION

This paper reports on a research project focusing on the social semiotics of whole-class interaction in mathematics classrooms. The ongoing project engages 12 middle school mathematics teachers who work in urban high needs schools in New York City. Teachers meet seven times per semester to collaboratively work on developing their understanding of the linguistic and semiotic challenges of teaching and learning mathematics. Session activities consist of a variety of investigations into the challenges of orchestrating meaningful whole-class conversations about mathematics problems. In this paper, we focus on the use of classroom transcripts in teacher development. In particular, we discuss one transcript that was analyzed and interpreted on two different occasions throughout the first year of the project, and we show how the two different sets of teachers' responses to the transcript indicate how their attention to correlations between language use and authority changed their interpretation of the given interaction, and increased their understanding of how grammatical modality and modulation are related to student agency.

Session activities throughout the first year were designed using a social semiotics framework. In this case, social semiotics is defined as a framework which focuses on the function of multiple semiotic systems (symbolic notation, oral and written language, graphs and visual displays, gestures and the use of material objects) and grammatical patterns (technical vocabulary, dense noun phrases, "being" and "having" verbs, logical conjunctions, visual codes, canonical gestures) in spoken, written and performed mathematical texts. The "social" part of social semiotics aims to unpack the complex use of multiple semiotic tools in positioning participants in terms of power, agency and authority. We draw on critical discourse analysis to help explore the manner in which classroom discourse constitutes and is constituted by power/knowledge relations, focusing on the use of language as a tool for negotiating subject positions through interaction in particular contexts. This approach proposes that we interpret and analyze transcripts and other mathematics texts in socio-cultural terms, and attend more carefully to the ways that power relations are constituted through language use. For instance, critical discourse analysis examines the linguistic features of texts as a means of understanding the enactments of identity through inculcation of cultural norms, submission or resistance to authority, and positioning and agency between speakers (Fairclough, 2003).

In any mathematics text – be it spoken or written or gestured – one can identify an array of grammatical forms that imply different kinds of authority and agency. One can say that authority and agency are "realized" in particular grammatical forms, and in turn, that grammatical forms position participants, assign authority, and re-inscribe power relations between participants. For instance, while most mathematics texts employ a form of address that minimizes agency on multiple levels, as in *"What is the probability that a rolled die will be a 1?"*, where the "rolled die" occurs without a causal subject or agent, and the question demands a statement of fact, the same question can be re-written to convey *authoring* agency, that is, as a statement that recognizes the reader as uniquely inventive, as in *"How would you decide whether a 1 is likely to occur when you roll a die?"* Learning how to decode mathematics texts for implied forms of address that locate the reader in terms of agency and authority strengthens the capacity of teachers to modify resources so as to better engage students (de Freitas & Zolkower, 2009; Dowling, 2001; Morgan, 2006; O'Halloran, 2005).

MODULATION AND MODALITY

The concepts of modulation and modality offer insight into how authority is managed and marshalled during classroom interaction. Halliday (1985) examines modality within propositions (statements and questions) and modulation within proposals (offers and commands). In propositions, modality expresses the degree of usuality (sometimes, always) or probability (possibly, definitely), whereas in proposals, modulation expresses the degree of obligation (supposed to, must) and inclination (might, determined to). Halliday (1985) uses the term "modulation" for obligation and inclination, and uses "modality" for usuality and probability. Modality is often considered the domain of epistemic variation and modulation the domain of deontic variation, although it is evident that in certain cases the line between these two becomes fuzzy [1]. The focus on modulation and modality allows teachers to study the way that action (or imagined action) is built into particular linguistic functions. Prospective teachers can begin to decode classroom conversations in terms of the subject positions implied by the grammar ("Which number would (could, can) you try?", "The cube would (could, should) then have edges of length 12"). The focus on modality and modulation also reveals the crucial role of grammar in constituting the border between necessary and contingent truths ("This number must (could) be prime"), and thereby introduces teachers to the grammatical forms attached to logical implication. Discussing modality and use of pronouns also helps teachers examine the ways in which their students are invited to participate. This speaks directly to issues of agency and authority in mathematical discourse, and reveals the complex relationship between language use and subjectivity.

Speakers use modality when operating between the polarity of yes and no. Polarity, according to Halliday (1985), is what makes something arguable: "Modality means the speaker's judgment of the probabilities, or the obligations, involved in what he is saying. A proposition may become arguable by being presented as likely or unlikely, desirable or undesirable – in other words, its relevance specified in modal terms." (p. 75). Modality is the means of mapping varying or intermediate degrees between the two polar extremes in various speech functions. According to Halliday (1985), "yes/no" utterances should be considered within the textual metafunction, because they relate the polarity to what has gone before, and play a huge role in sustaining the textual coherence of the conversation (p. 85). They are "intertextual" in an important way. Modality can be expressed via "finite verbal operators" such as: can, may, could, might (low modality), will, would, should, is to, was to (median modality), must, ought to, need, has to, had to (high modality).

Halliday (1985) considers other ways of expressing modality, such as "modal adjuncts" (usually, already, inclined, unfortunately, happily), which can express opinion, assertion, evaluation, prediction, validation, and desire. Examples are: "in my opinion, to my mind, personally" (opinion) or "I assure you, frankly, honestly, believe me, to tell you the truth" (assertion) or "as expected, by chance, to my surprise" (prediction) or "broadly speaking, on the whole, strictly speaking" (validation). In terms of its "arguable" status, and thus in terms of the authority and agency implied and construed by the utterance, the subject of the clause functions as that which is "responsible" for the modal claim; the subject is "something by reference to which the proposition can be affirmed or denied" (p. 76). It is the subject in whom is vested the success or failure of the proposition – that being the "functioning of the clause as an interactive event" (p. 76). The subject is not always the actor, but often the two correspond (in "I'll draw the graph" they coincide, but they don't in "I'll follow the instructions.")

Halliday (1985) argues that many instances of opinion ("I think the answer is two") are actually examples of interpersonal metaphor, since the "I think" stands in for the more congruent statement, "It might be two". The latter is considered more congruent (less metaphoric) because the low epistemic modality of "might be" better captures what the sentence is about (that being the measurement and its accuracy). "I think" is therefore metaphorical since thinking is not the theme or focus of the sentence. The "I think" is functioning as modal operator. This becomes clear when one considers the "tag" that might clarify the meaning of the statement, a tag defined in this case as the question posed to identify the subject responsible for the modal claim. For the statement, "I think the answer is two", the tag would be "isn't it?" not "don't I", thereby pointing out that it is not the belief that is up for question, but the validity of the assertion [2]. In the context of the mathematics classroom, this interpretation of “I think” statements sheds light on the complex linguistic practices by which authority and agency are negotiated. Statements such as “I think” can be considered a form of “hedging” and an attempt to “manage” the affective consequences of participating in high risk and high modality mathematics discourse (Rowland, 2000). In studying students’ language use as they grappled with mathematical tasks focused on generalizing, Rowland found that “I think” was the most common hedge used by students in their attempt to create “plausibility shields” (p. 138). These plausibility shields allowed them to move away from unqualified propositional statements, which were subject to truth or falsity judgments, and towards conjectural speech acts, for which less was at stake. Such plausibility shields, which also include adverbial prefaces of various kinds (“probably” or “apparently”) do not effect the truth conditions of the proposition, unlike other hedges – such as approximators (about, around) – which modify the set of arguable options entailed.

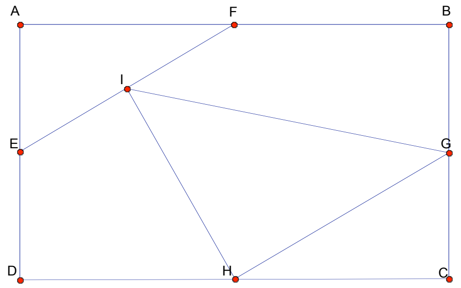
Halliday (1985) further classifies modality into implicit/explicit and objective/ subjective distinctions as in: I think Tamir knows (subjective/ explicit) and Tamir’ll know (subjective/implicit) and Tamir probably knows (objective/implicit) and it’s likely Tamir knows (objective/explicit). The subjective/objective distinction identifies to what extent the assertion seems to emanate from the person. Within the subjective/objective distinction, the explicit case involves a projection of fact (“it is”) or subjectivity (“I think”) that alternately identifies the subject as responsible or erases responsibility from the clause. Negation in these instances is also interesting in how it maps onto agency: “I don’t think Tamir knows” or “it isn’t likely Tamir knows”. In these, the modality is what gets negated, despite that being obviously not the intent. As Halliday (1985) suggests, the modality takes on the burden of the negation because it is so strongly centered as theme. This transfer of the negation between the modality and the proposition itself occurs most often in the case of median modality (not high/low).

Finally, it is worth noting that there is a paradox in the modal system. We only say we are certain when we are not. Whenever we introduce modal operators like “I’m certain it’s seven” we are actually acknowledging an element of doubt. If there weren’t any doubt, we would simply say “It’s seven.”

TEACHERS ANALYZING TRANSCRIPTS

The transcript under discussion in this paper is a one page excerpt from a grade 8 classroom in which a problem and diagram were introduced to the whole class. This was the first transcript that the teachers in our study discussed as a group. We selected this transcript because it concerns a good non-routine problem and it appears on the surface to be an exciting discussion using an interesting problem. We wanted to use a transcript that was seemingly a strong example of rich classroom interaction, with classic examples of good teacher questions, such as “Can someone say that in a different way?”, so as to elicit the teachers’ first positive reading and then direct the teachers' attention to certain silences in the transcript that indicated serious problems in terms of meaningful interaction. Below is the transcript (de Freitas & Zolkower, 2009):

The following problem is written on the blackboard



If E, F, G, H, and I are all midpoints, what is the relationship between the area of triangle GHI and the area of rectangle ABCD?

1	Teacher (T):	(Reading aloud.) What is the relationship between the area of triangle GHI and the area of rectangle ABCD?
2	Stud. 1 (S1):	Wait! You have to give us some numbers!
3	S2:	We don't have any measurements!
4	T:	No measurements
5	S2:	I don't get it. What are we supposed to do?
6	T:	Let's look closely at the statement written in here. What is this about?
7	S3:	A triangle and a rectangle.
8	S4:	One is inside the other.
9	S2:	It's about the areas of those shapes.
10	T:	Do we have to find the areas?
11	S5:	No, we have to find the relationship.
12	S1:	What do you mean by relationship?
13	T:	Can someone say that in a different way?
14	S6:	It asks how triangle GHI and rectangle ABCD relate to each other.
15	T:	That sounds like the same thing, right?
16	S2:	Oh, I get it! We have to figure out what part of the rectangle is occupied by the triangle.
17	S3:	It's a fraction… like a half or a third or… I don't know!
18	S4:	It has to be less than ½!
19	T:	How do you know that it's smaller than ½?
20	S4:	You can tell by just looking at the picture.
21	S2:	This reminds me of a problem we did about a garden covered with grass.
22	T:	Ok. So, if we put the problem in that context, what would we be looking at?
23	S7:	How much of this rectangular garden has grass in it.

24	S5:	Yes but we don't have to find how much it is. The question asks us to compare the two areas. It's like what S2 said before.
25	S1:	So I think that what we have to do is first find the area of the rectangle, then find the area of the triangle, and then see what fraction is one from the other.
26	S2:	But how are we going to find those areas if we don't know the lengths and stuff?

The teachers first focused on the student contributions. They noted (1) the high frequency of student contributions, (2) the students were "fixated" on measurement and didn't attend to qualitative aspects of the problem, (3) the word "relationship", which was in the problem, was causing confusion, and (4) the students' demonstrated "accountable talk". One teacher then pointed out that she liked the way the students were "talking about what a fraction is before they use the word fraction: 'What part of the whole is something?'" and this observation shifted the conversation to the place in the transcript where a student introduces the word fraction with the statement "It's a fraction … like a half or a third or … I don't know!" Another teacher then suggested that the students were pushing back until this moment, and that this "big leap" is where the "lesson took off". The moderator then asked the teachers: "How does the teacher use the diagram?" and one replied that she didn't because she was focusing on the language of the problem. When asked by the moderator "Are there any points where you think she could go to the diagram?" the teachers then debated the teacher moves in lines 16-24. One teacher suggested that after contribution 20 "You can tell by just looking at the picture", an alternative teacher contribution might have been "Let's look at the picture and think about why we might know that?" The grammar of this phrase stands in stark contrast to the question that was actually asked, "How do you know that it's smaller than ½ ?" Comparing the two, in terms of grammar, reveals that the proposed alternative:

(1) Commands the students to perform a perceptual act - to "look". This emphasizes the central role of material actions in doing mathematics, and the importance of interacting with the diagram on the perceptual plane.

(2) Uses an inclusive command "Let's look at …" instead of the interrogative "How do you …" The former commands the class as a collective, while the latter isolates the speaker.

(3) Uses the low epistemic modality mental process ("think about") instead of the high epistemic modality mental process ("know");

(4) Uses a low modality verbal operator "might" in "we might know".

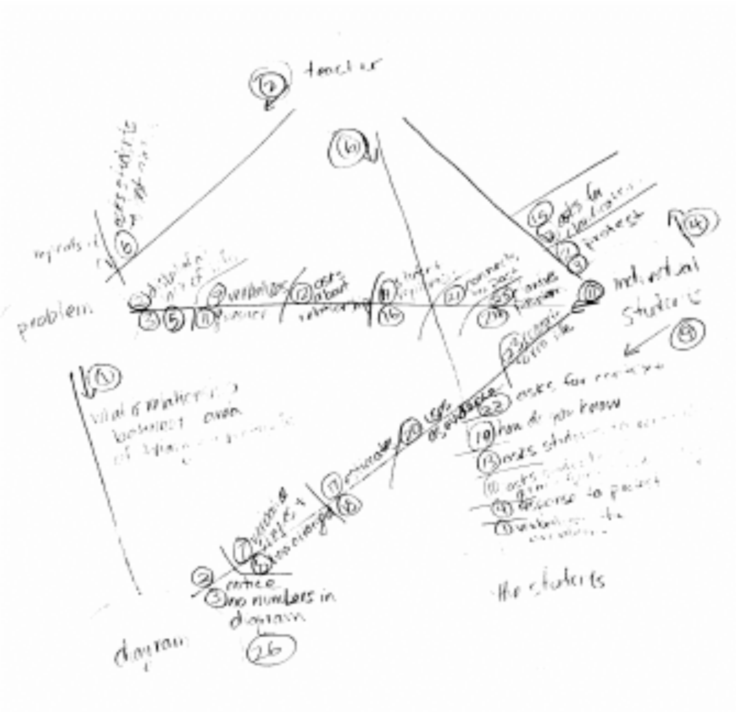
The proposed alternative highlights some of the key issues regarding modality and modulation in classroom discourse. These key issues became the focus of many subsequent discussions of classroom transcripts. Our aim was to help the teachers – especially those who initially disagreed with the proposed alternative – to begin to think more explicitly about the linguistic choices they were making during whole-class interaction.

During the next few months, teachers studied other transcripts and discussed modality and modulation, as well as other semiotic and linguistic patterns in classroom conversations, often focusing on how particular grammatical choices functioned to position students and teachers in particular ways. Teachers examined transcripts to see how different teacher moves changed the texture of the conversation, and in particular how changing the modality and modulation of the statements, questions, commands and offers seemed to impact on the kind of agency the students enacted during the interaction. Re-examining the important moves of students S2, S3 and S4 in lines 16-24 in the above transcript, for instance, one can see that the participating students move from low modulation in "we have to figure out" to a medium modality, made medium by the use of the explicit subjective "I don't know" to the high epistemic modality in "It has to be less than ½!".

As a result of these discussions, teachers became more sensitive to the impact these small changes in language use had on classroom interaction. During the end of Year One of the project, teachers examined the original transcript that they had discussed the previous semester, and they were asked to draw an interaction map that represented the whole-class interaction, and then explain their map to the others. They were given the following assignment to do individually:

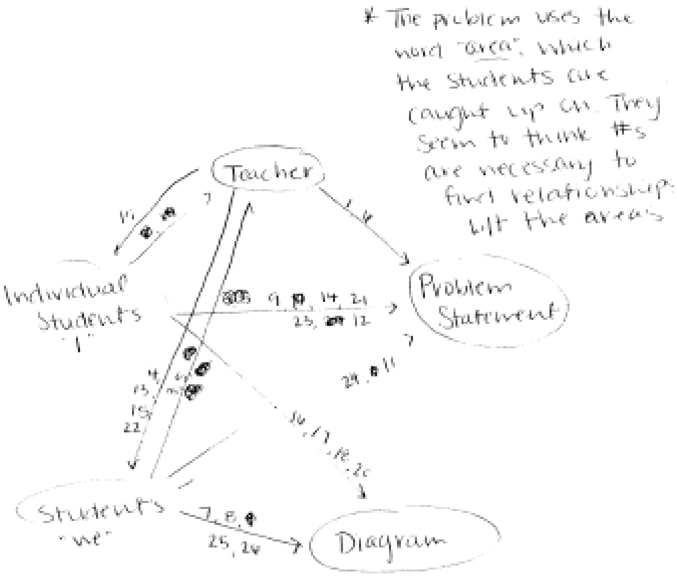
> Consider the five participants in this interaction: (1) the teacher, (2) the problem statement, (3) the diagram, (4) the students (realized grammatically via ''we''), and (5) individual students (realized via ''I''). Use the transcript to draw an interaction map that visualizes the number and nature of interactions between these participants. Take note of the use of grammatical choices in student responses such as "you have to give us some numbers" and "The question asks us to compare the two areas" to help draw your interaction map. How does your interaction map represent the agency or authority (or lack thereof) for each participant in the classroom discourse?

In this paper, we discuss four participant responses to this task, and include two of the diagrams. Bonnie, having counted interactions and looked for the ranking of frequencies, concluded that most students interacted with the problem statement, and that "the problem [statement] has the most authority". She pointed to "We have to find …" and "It asks how …" and "The question asks us to compare the …" as evidence that students and teachers interacted most with the problem statement, and that the nature of these interactions inscribed a certain authority onto the statement itself. When asked to explain what sort of authority, she claimed that its authority lay in it being the target of the students' questions, that they were "trying to get at it", and she pointed to particular pronouns, as in "The question asks …" and "It asks …" as support of her claim. The use of "it" as a linguistic pointer is an important part of mathematics classroom discourse when students are grappling with concepts they have yet to name (Rowland, 2000; Pimm, 1987). This deictic use of "it" is effectively leveraged by students as they answer vague questions such as "what do you notice?" or "What is the relationship between A and B?" In her interaction map, Bonnie connects the problem statement to the diagram because she felt that the former implicitly referred to the latter. She also maps student-diagram interactions, revealing that the students were indeed interacting with the diagram, despite the fact that the teacher was directing their attention away from it and towards the language in the problem statement.



Annette and Lada focused on the use of I/you/we pronouns, and concluded that most of the students who used “I/you” were asking the teacher for help, whereas those using “we” engaged the diagram. “You” frequently functions in mathematics classroom discourse as a generalizing pronoun to designate a form of abstract agency, as in “Then you subtract 4 from both sides”. Rowland (2000) notes that students often switch from “I” to “You” when grasping and communicating the generality of a pattern, and that “you” in such instances indicates a detachment from the strategy or actions described (p. 112). These instances contrast with the use of “you” as a form of address, which is considered a high-stakes enactment of a power relation. The two uses of “you” are found in the given transcript: the first in “You can just tell by looking at the picture” and the second “Wait! You have to give us some numbers!” Annette felt that the students who used “I/you” were tentative in their engagement with the conversation. She decided that students who used “we” were positioned in terms of strong agency because they interacted with the diagram, and that they were able to use “we” effectively precisely because they were interacting with the diagram. During previous discussions, we had debated the way in which “we” is operationalized in classroom discourse, pointing out that teachers often use “we” in strategic ways that tacitly enlist the listener into complicity. According to Wills (1977), “we” is highly imprecise in terms of its referent, and for that reason the pronoun is regularly exploited in manipulating conversations. Furthermore, Pimm (1987) suggests that teacher use of “we” is sometimes used to bolster the authority of a particular utterance, by implicitly citing an absent (expert) collective. Lada, who focused on the same use of pronouns, disagreed with Annette, and stated that the students who used “I” were more confident and more engaged, and that the less confident students hid behind the “we”. She pointed to the statements with high obligation modulation as evidence of limited agency, as in “Do we have to find the areas?”, but then pointed to statements that used explicit subjective modality, as in “So I think that what we have to do …” as evidence of how confidence mapped onto the use of pronouns.

Cameron stated that the problem statement dominated the classroom interaction because the word “area” was a key word that caught their attention, and that the students became fixated on their measurement associations with the word area. Area, according to Cameron, was a “huge word” which directed the conversation. When asked if the concept of area should be considered a participant in the interaction, Cameron thought it wasn’t that significant. Again, one can see in Cameron’s interaction map that the teacher fails to interact with the diagram. Her map also indicates the different students that used “we” and “I” and how these addressed either the diagram or the problem statement.



CONCLUSION

During the first year of the project, teachers developed skills at attending carefully to different patterns in transcript data. Focus on modality and modulation allowed them to look for grammatical patterns that might easily be associated with authority and agency. The interaction maps offered an opportunity to trace the complex network of exchanges in an alternative format, and to visualize the relationships between participants. They justified their maps and their understanding of the distribution of authority by reference to the degrees of modality and modulation found in the transcript. Teachers were asked to consider the problem statement as a participant in the interaction, and were able to see how particular grammatical constructions assigned authority to it. They were also asked to consider the problem diagram as a participant, in order to raise their consciousness about how the diagram was an important but neglected "agent" in the making of a meaningful interaction. Their attention to the use of pronouns in conjunction with modality and modulation helped the teachers trace the agency of the students during the interaction. Although these results don't yet speak to how their teaching practice was affected by participation in the lesson study, they do indicate that the teachers have developed an increased awareness of the connections between language use, agency and the distribution of authority.

NOTES

1. In other domains, such as modal logic, the term modality is used for both cases.

2. Consider "John thinks the answer is two, doesn't he?" where "thinks" is no longer metaphorical.

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PHILOSOPHY OF MATHEMATICS IN THE MATHEMATICS CURRICULUM. QUESTIONS AND PROBLEMS RAISED BY A CASE STUDY OF SECONDARY EDUCATION IN FLANDERS.

Karen François, Jean Paul Van Bendegem

Vrije Universiteit Brussel

This paper presents the results of an empirical investigation into the mathematics curriculum of secondary education in Flanders. The research question asks whether there is room for philosophy of mathematics within the curriculum. The method used was a screening of the curriculum with a focus on the philosophical parts. As a result we can present some initial philosophical concepts which are formulated in the general objectives of the curriculum. First we want to give an insight into the rather complex structure of the educational system in Flanders. Secondly, we want to clarify the different levels at which the mathematics curriculum is described and set out. Thirdly, we shall present the initial findings of our research. Finally we shall formulate some suggestions for a philosophy of mathematics and we will raise some questions.

INTRODUCTION

This research takes place in an inter-university research project (IUAP) in which we are looking for the relations between sciences, society, politics and the democratic constitutional state. The project has the title: "*The loyalties of knowledge. The positions and responsibilities of the sciences and of scientists in a democratic constitutional state*." Within this project, one of the key questions is the place of mathematics in this overarching alliance.

The first question is how mathematical knowledge is reproduced in our society, how mathematics is handed down from generation to generation. Obviously, education is an important way, if not one of the most important ways, to reproduce knowledge in our society. So, we transform the first question to ask whether there is room for a kind of philosophical reflection within the mathematics course. The question is not concerned with the implicit philosophy of mathematics, which is of course embedded in the curriculum, but with the way in which there is explicit room made for a philosophy of mathematics. The two questions are bound together and obviously, an answer to the second question gives us a partial answer to the first question. We shall return to this theme at the end of this paper but first, we need to explain the organisation and the structure of the Flemish education and school system.

ORGANISATION OF THE FLEMISH EDUCATIONAL SYSTEM

We are speaking of Flemish education, not only because of the difference in language in which Flemish and Walloon pupils are taught, but because of the completely separated school systems. Belgium is a federal state with three communities (the Flemish, the Walloon and the very small German community). Educational matters are under the control of these communities. Each community has the authority to decide on its own educational system and structures. It is the Flemish Community and more specifically, The Organisation for the Development of Education, which develops the curriculum that will be enforced by the Flemish parliament.

Moreover, educational freedom is provided for by the constitution. This means that we have the provision of differing schools, namely public schools, subsidized private schools and subsidized community schools which are provided by the local government. Private schools, which are mostly Catholic schools, receive public grants for as long as they are able to meet the community standards. Private schools are extremely popular in Flanders and take up to 75,5% of the pupils. The French community has a more balanced position, where 49% of the pupils attend public schools, and some 51% chose private institutions. This is a typical situation in Belgium where Flanders is largely Catholic and Walloon is primarily secular.

The differing schools	**Public** schools	**Subsidized official** schools (at the level of the local government)	**Subsidized private** schools: **Catholic** schools
Ratio of the pupils	16.3 %	8.2 %	75.5 %

Table 1: Flanders Secondary Education

It is at the level of the community that curricula are developed and these are compulsory for all schools. This is rather new in Flanders where educational freedom is limited by the law of 1990. Due to the provision of educational freedom by the constitution, the three school organisations retained some freedom in the sense that they had the potential to develop their own curriculum, which is based on the curriculum of the Flemish government. This freedom can be used in the formation of their own pedagogical methods and didactics. They also have the potential to add extra objectives and attainment targets. The curricula developed by the various school organisations must be at the level determined by law. These developments are regulated by strict inspection by the government.

Due to this double system of freedom on the one hand and compulsion on the other, we have two levels of curricula: 1) the level of the community - the curriculum as strictly enforced by law and 2) the level of the authorities of the various school systems. Schools have to integrate the attainment targets in developing their own curricula. Within education, we have three levels: 1) nursery and primary schools where basic instruction of mathematics are taught; 2) secondary education where pupils are taught mathematics ranging from the basic skills which enable them to survive in our society up to (and in the higher levels) mathematics as the purest scientific discipline; 3) high schools and universities where students are educated so that they can become mathematicians and teachers in mathematics.

The development of the content of the teaching of mathematics is positioned on four levels: 1) the level of the community - the curriculum as strictly enforced by law; 2) the level of the authorities of the different school systems - the curriculum as accepted by the government; 3) textbooks which are based on the curriculum; 4) the teacher in the classroom who has a constrained freedom. Before a teacher enters his or her classroom to teach mathematics, much about the teaching of mathematics has been debated and negotiated, written by diverse commissions and voted on in parliament.

METHOD

In our research, we have concentrated on the curriculum of secondary education. Secondary education has four forms: general, technical, art and vocational secondary education. The four forms of education are not organised separately in the first stage. From the second stage, they are organised separately. In the first grade, there is an A class which gives access to the general, technical and art secondary education. There is also a B class, which only gives access to vocational secondary education.

At the level of the development of the content of the teaching of mathematics, we focused on the curriculum as developed by the community because this level of the curriculum is strictly enforced by law for every school (system).

The central research question is whether there is room for philosophy of mathematics within the curriculum of secondary education. The method used is a screening of the text of the complete curriculum. We placed "philosophy" in inverted commas because we needed to use a very broad interpretation of philosophy (in fact all non-technical aspects of the math curriculum) so as to have some paragraphs in the curriculum. We need to point out that there are two parts of the curriculum where "philosophical" issues can be found. Part one is the view on mathematics in education in general; part two is the attainment targets.

After screening the curriculum, "philosophical" fragments are listed according to 1) part within the curriculum (general view versus attainment targets) 2) grade within secondary education (grade I is 12-14 years old, grade II is 14-16 years old and grade III is 16-18 years old) and 3) type of education.

FINDINGS

In a first table (Table 2) we present the results for the first part of the curriculum (the general view) for all grades and for all types of education.

Grade	Type of education			
I	A type General			B type Vocational
II	ASO General	TSO Technical	KSO Art	BSO Vocational
III	ASO General	TSO Technical	KSO Art	BSO Vocational

Table 2: General View in the Curriculum: Overview

A first result we can present is the fact that we only find philosophical issues in the general type of education (marked in grey) and not at the level of vocational education (or the B type in the first grade).

In the following table (Table 3) we present the detailed results of the screening for the first part of the curriculum (the general view) for all grades and for the general types of education (since there are no issues at the level of vocational education).

Grade	Type of education
I	A type
	Ontological proposition: The proposition that mathematics is abstract and formal and that mathematics has no connection with reality, up to a certain degree. **Appreciation**: Pupils must be encouraged to see the beauty and the perfection of a geometric figure, the clarity of a well reasoned argument and the elegance of a formula. **The cultural and dynamic meanings of mathematics:** The pupils should experience that mathematics has a practical use, and that it has an educative and aesthetic value. The history of mathematics helps pupils to understand that mathematics is an important aspect and component of culture, both in the past and the present. Mathematics in the past developed via many cultures. Due to the emphasis on this development, pupils will gain the knowledge that mathematics is a dynamic process. **The fundamental goals are:** Pupils will have the experience of mathematics as a dynamic science Pupils will have the experience of mathematics as an important cultural component.

II	**ASO: general**
	The ontological proposition: is absent **Appreciation:** In addition: when the commission determined the selection of the goals, they took into account, the effect of the development of a relationship with mathematics. **The cultural and dynamic meanings of mathematics:** (more abstract) The pupils should experience that mathematics has a practical use, and that it has an educative and aesthetic value. Attention to the development of mathematics helps pupils to understand that mathematics is an important aspect and component of culture, both in the past and the present. In this manner pupils will gain the knowledge that mathematics is a dynamic process. **The fundamental goals are:** Pupils will have the experience of mathematics as a dynamic science Pupils will have the experience of mathematics as an important cultural component.
II	**TSO en KSO: technical and art**
	Idem II ASO
III	**ASO: general**
	Idem II ASO; in addition to the previous goals: Pupils can gain an insight into the contribution that mathematics has: in the development of the exact and human sciences, and of art, critical thinking, and technique.
III	**TSO en KSO: technical and art** The text marked in grey is omitted at this level.
	Idem II ASO **The cultural and dynamic meanings of mathematics:** (a partial interpretation) The pupils should have an experience that mathematics has a practical use, and that it has an educative and aesthetic value. Attention to the development of mathematics helps pupils to understand that mathematics is an important aspect and component of culture, both in the past and the present. In this manner pupils will gain the knowledge that mathematics is a dynamic process. **The fundamental goals are**: (one goal has been dropped) Pupils will have the experience of mathematics as a dynamic science Pupils will have the experience of mathematics as an important cultural component.

Table 3: General View in the Curriculum: Details

Now we will move on to the second part of the curriculum: the attainment targets. In Table 4 we first present a general overview of possible locations for "philosophical" issues.

Grade	Type of education			
I	A type General			B type vocational
II	ASO General	TSO technical	KSO art	BSO vocational
III	ASO general	TSO technical	KSO art	BSO vocational

Table 4: Attainment Targets: Overview

As one will see there are no philosophical (historical or cultural) goals formulated, either for the B type, or for the first grade. The philosophical issues (marked in grey) are reserved only for the second and third grades of general education.

In Table 5 we present the results in detail for the second part of the curriculum, namely the attainment targets, for grade II and III (since there are no "philosophical" issues in grade I) and for the general type of education (since there are no issues at the level of vocational education).

Grade	Type of education
II	**ASO: general**
	Pupils can give examples of the contribution of mathematics to art.[1]
II	**TSO en KSO: technical and art**
	Pupils will gain appreciation for mathematics (possibilities and limitations) in confrontation with the cultural, historical and scientific aspects of mathematics.[2]
III	**ASO: general**
	Idem II ASO; in addition to the previous goals: Pupils can gain an insight into the contribution that mathematics has: in the development of the exact and human sciences, and of art, critical thinking, and technique.[3]
III	**TSO en KSO: technical and art**
	Pupils can give examples of the application of mathematics in other courses and in society in general.[4]

Table 5: Attainment Targets: Detail

CONCLUSIONS

As a first general remark we have to conclude that the first part of the curriculum, the general overview, contains more philosophical issues than the second part which contains the attainment targets. As teachers are more focused on part two, because the attainment targets are the criteria for the evaluation of pupils, we have to conclude that there is little room for a philosophy of mathematics within the curriculum of math education. There are some initial formulations at the level of the general overview in the curriculum which are not completely translated into the attainment targets.

A second general remark is the fact that there is no room for "philosophy" of mathematics within the vocational type of education. Here we want to remark the difference between vocational and general education. On one hand, we can say that mathematics in vocational education is completely embedded in a modular system and attention is paid to core skills. On the other hand we must say that pupils are prepared for specific (professional) occupations, for personal and social functioning, in order to survive in our society. Access to higher education is theoretically possible but in practice impossible. Mathematics in general education is an independent course. General education provides a strong base for higher education (e.g., university.)

Using the distinction Alan Bishop (1988) has introduced between the small m and the large M of mathematics, where the small m stands for basic mathematical competence such as: counting, locating, measuring, designing, playing and explaining, and the large M stands for mathematics as the Western scientific discipline, we can conclude that pupils in vocational mathematics are taught the small m and pupils in general education are taught the large M. The more general the education, the larger the M, and the higher the status in society.

A third general remark is the fact that –at the level of the attainment targets- there is no "philosophy" of mathematics included in the first grade of general education and there is little room for it when we look at technical and art education. Also here we have to conclude that the more general the education is, the larger the M, and the higher the status in society is.

Maintaining the difference between vocational and general education, we can conclude that, for an explicit philosophy, there is very little space in general education and there is none at all in vocational education.

In as far as an implicit philosophy can be identified, it seems to us that it is mostly a rather absolutist view that is present, seeing mathematical truth as absolute and certain, and connected with some humanistic values.

In support of our claim that the curriculum presents the absolutist view, we want to refer to the following arguments:

- there is no room to discuss the status of mathematics,

- the status is very clear and rather static,
- there is no philosophy at all in vocational education,
- the larger the M, the higher the status in society,
- the appreciation for mathematics that pupils are encouraged to gain is seen as the highest form of motivation,
- experience-based learning is only used to gain the interest and to motivate disinterested pupils, to help them to gain appreciation for mathematics with the truly large M.

As to the humanistic values, we observed the following:

- there is only a small space for philosophy in education in general,
- there is some limited attention given to “the possibilities and the limitations of mathematics”, although in the curriculum it is placed between brackets,
- some attention is given to the applications of mathematics,
- there is some limited attention to historical and cultural components (where in addition most of the space is filled with art).

The challenge we wish to propose (and at the same time the source for the questions we would like to raise) is to show how an implicit philosophy can be made explicit and how the implicit philosophy can be modified. In short, what philosophical topics could have a place in the curriculum compared to the present implicit philosophy?

We would like to end with five questions for further research.

(1) Is there room for an explicit philosophy of mathematics in higher education, at the university, and in teacher training? (see, e.g. the work of Ernest (1994), Ernest (1998)).

(2) If so, what kind of philosophical approach? Should one stress the fallibility of mathematical knowledge, should one stress the social nature of mathematics, or should one stress the curious mechanisms that have led to such a strong consensus among mathematics (see, e.g., the work of Heintz (2000)).

(3) Related to (2), what should be the role of ethnomathematics in western school curricula? Is the distinction between big M mathematics and small m mathematics productive, interesting, provocative, necessary?

(4) In the confrontation with culture at large, how can we move beyond the stereotypical associations between mathematics and the arts. Do we always need to refer to Escher? Are there really no other possibilities? Is there no mathematics in the work of Jackson Pollock, to give but one possible alternative?

(5) Given that all the above questions can to some extent be answered, what should the teacher do in the classroom? How should these ideas, views and confrontations be implemented? In short, what are good practices for teachers?

NOTES

[1] Attainment target 8 of the I. General attainment targets.
[2] Attainment target 11* of the I. General attainment targets. The * says for 'attitude'.
[3] Attainment target 19 formulated in 'Mathematics and Culture'.
[4] Attainment target 7 of the I. General attainment targets.

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DEVELOPING A CRITICALMATHEMATICAL NUMERACY THROUGH *REAL* REAL-LIFE WORD PROBLEMS [1]

Marilyn Frankenstein

University of Massachusetts/Boston

INTRODUCTION: ALL MATHEMATICAL WORD PROBLEMS ARE NON-NEUTRAL

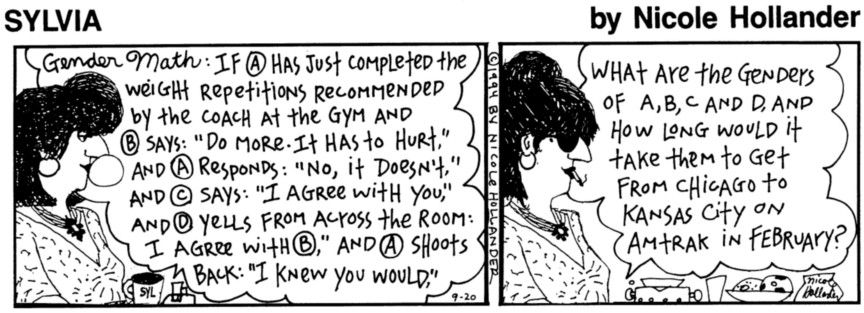
A great honour was conferred on me a few years ago when right-wing conservative Lynne V. Cheney (1998), former USA Vice-President Dick Cheney's wife, trashed my work because I stated that in mathematical texts, "A trivial application like totalling a grocery bill carries the non-neutral message that paying for food is natural" (Frankenstein, 1983, p. 328). Contrary to Cheney's claim that I would not want students to solve problems totalling grocery bills, I certainly would want students to solve such problems – comparing grocery bills in poor neighbourhoods with those in rich neighbourhoods, for example, or countless other mathematical investigations that could relate to issues of hunger and capitalism where tens of millions of tons of surplus food rot for the profit of a few (Mittal, 2002) while approximately 40 million people die from hunger and hunger-related illness every year and "available evidence indicates that up to 20,000,000 citizens [living in the USA] may be hungry at least some period of time each month." [2] (National Council of Churches, 2007). I argue that all real-life mathematical word problems contain non-numerical "hidden" messages, and that, if those problems are presented as neutral, they can stifle creative thought and questioning, by increasing the aspects of our society that people take for granted.

In this paper, I'll discuss some political concerns about other aspects of the non-neutral "hidden" curriculum that result from particular selections of real-life data used to create contrived and/or context-narrow word problems. Then, I'll suggest various categories of *real* real-life mathematical word problems, problems that are presented in a broad enough context for students to appreciate how understanding numbers and doing calculations can illuminate meaning in real life. In conclusion, I will discuss some pedagogical and political questions about the real-life use of *real* real-life mathematical applications, returning to the issue of the non-neutrality of knowledge, and addressing the question of teaching difficult, pessimistic perspectives.

The main goal of a criticalmathematical literacy is not to understand mathematical concepts better, although that is needed to achieve the goal. Rather it is to understand how to use mathematical ideas in struggles to make the world better. In other words, the question to be investigated about my criticalmathematical literacy curriculum is not "Do the *real* real-life mathematical word problems make the mathematics more clear?" The key research questions are "Do the *real* real-life mathematical word problems make the social justice issues more clear?” and, “Does that clarity lead to actions for social justice?”

PROBLEMS WITH REAL-LIFE MATHEMATICAL WORD PROBLEMS

Real-Life Mathematical Word Problems Without Real Meaning



In a French study (IREM de Grenoble, 1980), a 7-year-old was asked the following question: “You have 10 red pencils in your left pocket and 10 blue pencils in your right pocket. How old are you?” When he answered: “20 years old,” it was not because he didn’t know that he was 7 in real life, or because he did not understand the relevant mathematical concepts. Rather it was, as Pulchalska and Semadeni (1987, p. 15) conclude, because the unwritten social contract between mathematics students and teachers stipulates that “when you solve a mathematical problem... you use the numbers given in the story... Perhaps the most important single reason why students give illogical answers to problems with irrelevant questions or irrelevant data is that those students believe mathematics does not make any sense”.

Clearly, “educating” people to accept nonsensical statements uncritically in order to “fit in” is a political problem. Moreover, it is also politically problematic even when mathematical word problems do not ask nonsensical questions, but use real-life numerical data without *real* meaning, but only as “window dressing” to practice a particular mathematical skill. First, when assumptions about what are the “natural” conditions of real life (e.g., heterosexual families) are used as the “window dressing” context for mathematical problems, students who do not fit those “natural” categories are disrespected and/or made invisible. Further, the “hidden curriculum” about what is “natural” gets reinforced, making it less likely that students will question these taken-for-granted assumptions. Second, the *real* significance of the “window-dressed” real-life data is also hidden. When no better understanding of the data is gleaned through solving the mathematics problem created from the data, using real-life data masks how other mathematical operations, as well as other non-mathematical investigations, could be performed that would illuminate those same data. It gives a “hidden curriculum” message that using mathematics is not useful in understanding the world—mathematics is just pushing around numbers, writing them in different ways depending on what the teacher wants.

Real-Life Mathematical Word Problems Without *Real* Real Context

There are, of course, curricula that contain real-life mathematical word problems that involve using numbers to gain more information to help make real-life decisions. How-ever, often these problems assume everyone's real-life context is the same. Underlining this point, Apple (1992, pp. 424-425) concludes that the NCTM Standards (1989) do not address "the question of *whose problem* ... by focusing on the reform of mathematics education for 'everyone', the specific problems and situations of students from groups who are in the most oppressed conditions can tend to be marginalized or largely ignored (see Secada, 1989, p. 25)." The Standards do not contain, for example, suggestions for mathematical investigations that would illustrate how the current US government's real-life de-funding of public education, through funding formulas based on property taxes, creates conditions in which the real-life implementation of the NCTM student-centered pedagogy is virtually impossible except in wealthy communities (Kozol, 1991).

Real-Life Mathematical Problems Without Enough Real Context

Those "neutral" real-life mathematical word problems that do include a real-life context like totalling grocery bills still omit the larger contexts of individual economic differences within a system where a 1997 report from the US Department of Agriculture declared that 11 million citizens, including 4 million children, "live in households categorized as moderately or severely hungry." (Sarasohn, 1997, p. 14).

Other "neutral" real-life mathematical word problems involve numerical descriptions that omit the larger contexts that created the reality of those descriptions. For example, *Multiplying People, Dividing Resources* (Zero Population Growth, 1994) contains a worksheet of real-life mathematical word problems designed to help students conceptualize large numbers. In the section on "Explanations/Applications," there is their "neutral" comment that:

> When Columbus arrived in the Americas in 1492, there were probably 5 million Native Americans living in the area of the United States, and 57 million in the two American continents. World population at that time was about 425 million, and did not reach one billion until approximately 1810. . . . In 1994, the United States has approximately 260 million people within its borders . . .

Hidden in this real-life context is the larger context of what happened to those Native Americans. Although there is some academic debate about the number of people living North of Mexico in 1492 (ranging from about 7 million to 18 million),

> There is no doubt, however, that by the close of the nineteenth century the indigenous population of the United States and Canada totalled around 250,000. In sum, during the years separating the first arrival of Europeans in the sixteenth century and the infamous massacre at Wounded Knee in the winter of 1890, between 97 and 99 percent of North America's native people were killed (Stannard, 1992, p. 432) .

REAL REAL-LIFE MATHEMATICAL WORD PROBLEMS [3]

Real real-life mathematical problems occur in broad contexts, integrated with other knowledge of the world. I (Frankenstein, 1983) contend, along with Freire (1970; Freire & Macedo, 1987) that the underlying context for critical adult education, in this case criticalmathematical literacy, is “to read and re-write the world.” In that case, mathematical skills and concepts are learned in order to understand the institutional structures of our society.

Below are various categories of problems that, of course, overlap in different ways. The overarching activity is gaining a better analysis of the issue through understanding the meaning of the numbers, and gaining more knowledge about the issues through performing relevant calculations. The purpose of discussing the examples in this manner is to show many types of situations in which numbers can be used to make sense of the world, and then to make justice in the world.

Understanding the Meaning of Numbers

The *real* real-life mathematical word problems whose solutions involve understanding the meaning of numbers focus on using different kinds and arrangements of numbers (e.g., fractions, percents, graphs) to:

- describe the world
- reveal more accurate descriptions of the world
- understand the meaning of the sizes of numbers that describe the world
- understand the meanings that numbers can hide in descriptions of the world
- understand the meanings that numbers cannot convey in descriptions of the world

Understanding the meaning of the numbers is needed to understand the meaning of these situations, situations that illuminate the way our world is structured.

Using Numbers to Describe the World

Example:

> Although Helen Keller was blind and deaf, she fought with her spirit and her pen. When she became an active socialist, a newspaper wrote that "her mistakes spring out of the… limits of her development.” This newspaper had treated her as a hero before she was openly socialist.
>
> In 1911, Helen Keller wrote to a suffragist in England: “You ask for votes for women. What good can votes do when ten-elevenths of the land of Great Britain belongs to 200,000 people and only one-eleventh of the land belongs to the other 40,000,000 people? Have your men with three millions of votes freed themselves from this injustice?” (Zinn, 1980, p. 337).

Students are asked to discuss how numbers support Helen Keller’s main point and to reflect on why she sometimes uses fractions and other times uses whole numbers. Information about the politics of knowledge is presented as a context in which to set her views, including class discussions about Keller’s militant answer to the editor of the *Brooklyn Eagle* (Zinn, 1980, p. 338) and about why so many children’s books ignore her socialist activism (Hubbard, 2002).

Using Numbers to Reveal More Accurate Descriptions of the World

Example: Students are asked to read articles that present numbers that counter taken-for-granted assumptions that many view as "natural" facts about the world. For example, an article which shows that in spite of widespread belief that "illegal" [4] immigrants are robbing tax payers through their use of hospital emergency rooms and public education, not only do "illegal" immigrants pay sales and other such taxes, but they also pay over $6 billion in Social Security and about $1.5 billion in Medicare taxes, without collecting any of the benefits from those taxes (Porter, 2005).

Understanding the Meanings that Numbers Cannot Convey in Descriptions of the World

Example: Following this is an example of art encoding quantitative information. The numbers are the data of our world—our wars; the art allows us to understand the quantities in ways we could not understand from the numbers alone. As Toni Morrison states: "Data is not wisdom, is not knowledge" (quoted in Caiani, 1996, p. 3).

The famous memorial in Washington, D.C. by artist Maya Lin lists the names of 57,939 Americans killed during the Vietnam War. In "The other Vietnam Memorial" (Museum of Contemporary Art in Chicago, IL), Chris Burden etched 3,000,000 names onto a Rolodex-type structure, standing on its end, that fills the entire room in which it is displayed. The names represent the approximate number of Vietnamese people killed during the US war on Vietnam. Since many of their names are unknown, Burden created variations of 4000 names taken from Vietnamese telephone books. Also, the museum notes comment that by using the form of a common desktop object that functions to organize professional and social contacts, Burden underlines the unrecognized loss of Vietnamese lives in US memory.

Understanding the Calculations

The *real* real-life mathematical word problems whose calculations are an integral part of understanding a situation focus on:

- verifying/following the logic of an argument
- understanding how numerical descriptions originate
- using calculations to restate information
- using calculations to explain information
- using calculations to reveal the unstated information

The purpose underlying all the calculations is to understand better the information and the arguments, and to be able to question the decisions that were involved in choosing which numbers to use and which calculations to perform.

Understanding how Numerical Descriptions Originate (Seeing how Raw Data are Collected, Transformed, and Summarized into Numerical Descriptions of the World)

Example: Students are asked to read the excerpt below so that they are thinking about issues of how to teach and how people learn mathematics at the same time that they are learning the mathematics. Then, they are asked to: describe the study's methodology (i.e., what procedures were followed in the study, what the "raw" data consisted of, and how the raw data were transformed and summarized); re-write the findings described by creating a chart; discuss which presentation of the data is clearest, and why; list conclusions they can and cannot draw from the data; and indicate what other information they would want in order to clarify the data or strengthen and/or change their conclusions.

> Sixty-six student teachers were told to teach a math concept to four pupils - two White and two Black. All the pupils were of equal, average intelligence. The student teachers were told that in each set of four, one White and one Black student was intellectually gifted, the others were labelled as average. The student teachers were monitored through a one-way mirror to see how they reinforced their students' efforts. The "superior" White pupils received two positive reinforcements for every negative one. The "average" White students received one positive reinforcement for every negative reinforcement. The "average" Black student received 1.5 negative reinforcements for each positive reinforcement, while the "superior" Black students received one positive response for every 3.5 negative ones. (Sklar, 1993, p. 53)

Using Calculations to Restate Information (Changing the Quantitative Form)

Example: Students study a letter I wrote (Frankenstein, 2002) responding to an article by Howard Zinn (2002) in which he argues that the numerical descriptions of the deaths from the US war on Afghanistan can obscure those horrors. To dramatize my argument that numbers can illuminate the meaning of data and deepen connections to our humanity, I conclude that the 12 million children who die every year from hunger "are dying faster than we can speak their names." (Frankenstein, 2002, p. 23).

Using Calculations to State the Unstated Information

Example: Students learn about percents while analyzing the following political poster in the context of the politics of language where people who constitute a majority of the world's population are referred to as "minorities." Students also see that numbers are "behind" many economic, political, and/or social issues even if there are no numbers "visible" in the picture (Figure 1).

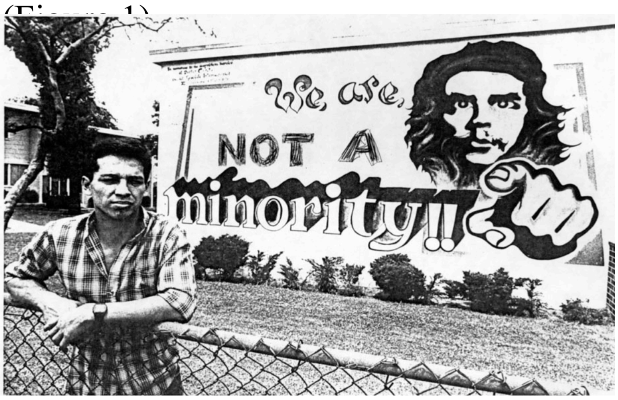


Figure 1. Los Angeles Hispanics and other recent immigrants are demanding their piece of the pie (Guardian, 1978, Mario Torero, with Zapilote, Rocky, El Lton, and Zade)

CONCLUSION: PEDAGOGICAL AND POLITICAL DIMENSIONS OF TEACHING THROUGH REAL REAL-LIFE MATHEMATICAL WORD PROBLEMS

Pedagogical Dimensions

Following ABC's 1983 airing of a film about *The Day After* a nuclear war, the network presented a panel discussion, chaired by Ted Koppel, of mostly conservative government officials and Carl Sagan, a liberal scientist. At one point, Sagan refuted the then Secretary of State Schultz's contention that the Administration was already disarming, pointing out that "its current build-up calls for an increase in the number of strategic warheads, from 9,000 to 14,000." Koppel turned to Sagan and said "… I must confess statistics leave my mind reeling and, I suspect, everybody else's too." (Manoff, 1983, p. 589)

Certainly, students need enough mathematics so that their heads do not reel from comparing the size of two numbers! As a prerequisite to accomplishing the goal of a Freirean "reading and re-writing of the world" using a criticalmathematical literacy, students need confidence that they can learn enough mathematics to use as part of understanding public and community issues. When students realize that their teacher has confidence in them and expects, with studying, that they will learn the mathematics, they can begin to let go of the negative expectations many have internalized from past mathematics learning experiences. Also, confidence is gained from analysis of the politics of language, where the label "mathematically anxious" can have contradictory effects. Naming that situation can initially reassure students that their feelings about mathematics are so common that educators have a name for them. However, the label can also focus the problem inward, "blaming the victims" and encouraging solutions directed solely *at* them. The label can direct attention away from the broader social context of how their learning got mystified, and what interests might be served by widespread mathematics "anxiety" and avoidance. And, confidence is gained from understanding the politics of knowledge that have discounted some people's knowledge and privileged others' knowledge. For example, I ask students to reflect on Freire's (Freire & Macedo, 1987) insistence that "the intellectual activity of those without power is always characterized as non-intellectual." (p. 122)

Once students are confident in their ability to learn mathematics, and motivated to reason quantitatively about public and community issues, then the question is: How much of the structure of mathematics must be demystified in order for students to be able to use numerical data for demystifying the structure of society? It is important for students to understand enough concepts behind the basic algorithms to be able to use those rules comfortably in many different situations. However, as Lange and Lange (1984) found, although mathematics education can be empowering in a more general way, it is not necessarily the best approach in working with people on specific empowerment issues. The piece-rate workers they were organizing in the textile industry in the southern United States were struggling with a pay system made intentionally obscure. The Langes' experience was that teaching the concepts of ratios and fractions behind that rate system was not the most effective way to empower the workers in their struggle for decent pay. It was more empowering to create a slide-rule distributed by the union that did the pay calculations for the workers, making the mathematical problem disappear, so that the workers could "focus on the social and economic relations underlying the way they are treated and paid" (p. 14).

In my context, my curriculum is loosely organized by a linear thread of underlying mathematical concepts (i.e., the meaning of whole numbers, then fractions, later percents, and so on). But, the lessons also involve non-linear explorations of *real* real-life public and community issues and much interdisciplinary content. However, in thinking about what numeracy citizens need to solve *real* real-life problems, I am not advocating getting rid of college preparatory mathematics. As Powell and Brantlinger (2008) argue, teaching "traditional" mathematics with understanding to students who have been marginalized from college or certain professions is another form of criticalmathematics education appropriate to that context. I would argue that all citizens need the criticalmathematics I am describing, but it does not need to replace more "traditional" mathematics.

Political Dimensions

I suspected trouble when, at a 1981 National Council of Teachers of Mathematics (NCTM) Conference, the president of the organization opened the meeting by stating that Ronald Reagan's election was great for mathematics teachers. But, I did not suspect how outraged the teachers would be by the biases in my real-life word problems. They did not accept my argument that no mathematical word problems are neutral.

A few years after my NCTM audience was furious at my biased word problems, the NCTM journal, *The Mathematics Teacher*, (March 1984, December 1984) was running multi-page spreads advertising a US Navy slide show "Math and Science: START NOW!" Toll-free phone numbers to arrange for a class presentation by a Navy representative were included. They published one critical letter that focused on the inappropriateness of the Navy starting recruiting drives in junior high school and questioned why there were no ads from government groups "whose mandate is more closely tied to social and environmental problems" (Milne, 1984). The editor answered that the Navy paid for the ad and any government agency could do likewise. He did not publish my strong critique that accepting an ad from the Navy implied:

> … a certain level of support—especially since the NCTM's Executive Director is quoted in the ad as saying "Without hesitation, we endorse the project"!! In addition, your ad policy will be skewed towards those governmental agencies with the largest advertising budgets—therefore, those agencies, such as the military, which are favored by the current administration, will also be

favored by NCTM ad policy. Finally, we did pay for the ad—not through our NCTM dues as you stated—but certainly, through our tax dollars.

One final point: the *real* real-life context illuminated by the *real* real-life mathematical word problems in my adult criticalmathematical literacy curriculum are outrageously horrible. How can these topics be taught without discouraging people and thereby stopping resistance? The context of my students' lives is such that many have been involved in our struggle to change this situation. And different groups of us have experienced some victories. However, given the resources of those in power to regroup, we wind up fighting the same battles over and over and often initial victories are overturned or co-opted. Nevertheless, those of us who are committed to the struggle for a just liberatory world keep fighting.

Audre Lorde (1988) reminds us in *A Burst of Light* that:

> … hope [is] a living state that propels us, open-eyed and fearful, into all the battles of our lives. And some of those battles we do not win. But some of them we do. (p. 80)

NOTES

1. A much longer version of this paper is found in Chapter 6 "Developing a Criticalmathematical Numeracy through Real Real-life Word Problems," pp. 111-130 in Verschaffel, L., Greer, B., Van Dooren, W. & Mukhopadhyay, S. (Eds.) (2009). Words and Worlds: Modelling Verbal Descriptions of Situations. Rotterdam, The Netherlands: Sense.
2. Related to the politics of language, The Progressive (2007, p.11) cites a Washington Post article indicating that the United States Department of Agriculture will no longer use the word "hunger" to describe people who cannot get enough food to eat; instead these people will be described in official government documents as having "very low food security."
3. Due to space limitations the examples are presented in an abbreviated form, and I am not giving examples for each category. I am developing these and others into a collection of columns for various websites and newsletters. Since June 2008, they have been appearing in Numeracy Briefing, edited by Europe Singh. For more information contact them at numeracy@basicskillsbulletin.co.uk. If any reader is interested in syndicating these columns, free of charge, contact me at marilyn.frankenstein@umb.edu
4. I use quotes around illegal to draw attention to who gets to make the laws that determine who is "legal" and who is "illegal".

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