PROCEEDINGS OF THE SEVENTH INTERNATIONAL MATHEMATICS EDUCATION AND SOCIETY CONFERENCE

Volume 1

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MES 7

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MES 7 Conference Logo
The MES 7 logo design is based on the engraved markings found on the Blombos ochre plaque, thought by many to be the world's first symbolic object. This artefact was found by Christopher Henshilwood in 1991 in a small cave near Still Bay on the southern Cape coast of South Africa, and has been reliably dated to about 75 000 years ago. Although there is some debate about the meaning of these markings, many believe that they are evidence for symbolic thought processes as the systematic pattern suggests that the markings represent information rather than decoration.

Acknowledgements
The conference organisers would like to acknowledge the financial support of the following institutions: Rhodes University, University of Cape Town, University of KwaZulu-Natal, University of the Witwatersrand, National Research Foundation of South Africa. We also acknowledge the role of the South African Mathematics Foundation and the Centre for Higher Education Development at the University of Cape Town in underwriting the costs of the conference.

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INTRODUCTION

The first International Conference on Mathematics Education and Society took place in Nottingham, Great Britain, in September 1998. The second conference was held in Montechoro, Portugal, in March 2000. The third conference took place in Helsingør, Denmark, in March 2002. The fourth conference was held in Queensland, Australia, in July 2005. The fifth conference took place in Albufeira, Portugal, in February 2008, and the sixth was in Berlin in March 2010. On all occasions, people from around the world had the opportunity to share their ideas, perspectives and reflections concerning the social, political, cultural and ethical dimensions of mathematics education and mathematics education research that take place in diverse contexts. As a result of the success of these six meetings, it was decided to have a seventh conference in Cape Town, South Africa.

The South African conference is a cross-institutional collaborative effort of the Universities of Cape Town, Witwatersrand, Rhodes and Kwa-Zulu Natal. The South African Mathematics Foundation and the Centre for Higher Education Development at the University of Cape Town have underwritten the costs of the conference. The conference has been sponsored by the above four universities and the National Research Foundation of South Africa.

AIMS OF MES 7

Education is becoming more and more politicised throughout the world. Mathematics education is a key focus in the politics of education. Mathematics qualifications remain an accepted gatekeeper to further education and employment opportunities. Thus, defining success in mathematics becomes a way of controlling people's pathways in work and life generally. Mathematics education has also tended to contribute to the reproduction of an inequitable society through undemocratic and exclusive pedagogical practices which portray mathematics as an absolute, authoritarian discipline. The fact that particular mathematics education and research practices can have such significant impact on the type of society we live in suggests that different mathematics education and research practices could have equally significant but a more socially just impact on society. There is a need for uncovering and examining the social, cultural and political dimensions of mathematics education; for disseminating research that explores those dimensions; for addressing methodological issues of that type of research; for planning international co-operation in the area; and for developing a strong activist research community interested in transforming mathematics education as an agent and practice for, rather than against, social justice.

Holding the conference in South Africa brings many of these issues into stark relief. Eighteen years into a democratic government, South Africa remains one of the most unequal societies in the world and social inequality both produces and is reflected in educational inequality. Mathematics achievement in the country is very low for the majority of learners, who are historically disadvantaged through the legacy of apartheid, and the transformation of society in general requires the transformation of education, including mathematics education.

The MES 7 Conference aims to bring together mathematics educators around the world to provide a forum for collaborating on these issues as well as to offer a platform on which to build future collaborative activity.

CONFERENCE PROGRAMME

The conference is organised with the importance of generating a continuing dialogue and reflection among the participants in mind. There are a range of activities directed towards the aim of generating this sustained discussion:

Opening plenary panel: Transforming society and mathematics education in South Africa

Given the location of the conference in South Africa, it was decided to focus on specific issues in South African Education as a lens through which to view broader social and political issues.

Panelists:	Jill Adler (University of the Witwatersrand, South Africa) Phadiela Cooper (Centre of Science and Technology, South Africa) Yoliswa Dwane (Equal Education, South Africa) Renuka Vithal (University of KwaZulu-Natal, South Africa)
Respondent:	Robyn Jorgensen (Griffiths University, Australia)
Chair:	Karin Brodie (University of the Witwatersrand, South Africa)

Plenary addresses and reactions

The four invited keynote speakers were asked to address a topic of relevance to the conference, building on their current research. They offer 50-minutes presentations. Each presentation is followed by 10-minutes responses by two mathematics educators.

Anna Chronaki, University of Thessaly, Volos, Greece
Title: *Identity work as a political space for change: The case of mathematics teaching through technology use.*

Respondents:	Troels Lange, Malmö University, Sweden Peter Pausigere, Rhodes University, South Africa

Zain Davis, University of Cape Town, South Africa
Title: *Constructing descriptions and analyses of mathematics constituted in pedagogic situations, with particular reference to an instance of addition over the reals.*

Respondents:	Candia Morgan, Institute of Education, University of London, UK Surgeon Xolo, University of the Witwatersrand, South Africa

Tamsin Meaney, Malmö University, Sweden
Title: *The privileging of English in mathematics education research, just a necessary evil?*
Respondents: David W. Stinson, Georgia State University, USA
Lindiwe Tshabalala, University of South Africa, South Africa

Swapna Mukhopadhyay, Portland State University, Portland, Oregon, USA
Title: *The mathematical practices of those without power.*
Respondents: Joi A. Spencer, University of San Diego, USA
Shaheeda Jaffer, University of Cape Town, South Africa

Working groups

Groups, set at the beginning of the conference, discuss the plenary lecture and the reactions. Each discussion group produces a brief report detailing key questions or issues to be addressed by the speaker and reactors in a plenary response session.

Group Moderators: Beth Herbel-Eisenmann, Michigan State University, USA
Vera Frith, University of Cape Town, South Africa
Hauke Straehler-Pohl, Freie Universität, Germany
David Wagner, University of New Brunswick, Canada

Plenary response session

In these sessions, one during each day of the conference, there is an opportunity to bring back to the whole conference group the questions and concerns of each working group, and to have a further comment by the plenary speaker and reactors.

Symposia

Four symposia proposals were accepted after review. Each symposium has one or two sessions within which to engage participants in a reflection on a particular topic of interest to the conference. The symposia are:

A. *The social function of mathematics examination questions*
Co-ordinators: Heather Mendick, Candia Morgan, Cathy Smith

B. *Exploring the relationship between in-service mathematics teacher support and retention*
Co-ordinators: Mellony Graven; Barbara Pence; Susie Hakansson; Peter Pausigere

C. *Understanding the prevalence of concrete working with number across teaching and learning in Foundation Phase*
Co-ordinators: Hamsa Venkatakrishnan, Lynn Bowie

D. *Teaching mathematics for social justice: Conversations with educators*
Coordinators: David W. Stinson, Anita A. Wager,

Paper discussion sessions

After peer review of all paper submissions, the organising committee accepted 31 papers for presentation and discussion during the conference. The full texts of accepted papers are posted on the conference website and published in these conference proceedings.

Project discussion sessions

After peer review of project submissions, 11 project presentation papers were accepted. Discussion papers are posted on the conference's website and published in the conference proceedings.

Agora

Inspired on the Greek tradition of a "popular political assembly" taking place in a public, open space such as the market place, it was decided to have two informal, evening discussion sessions about the future of MES.

Networking

Within the programme there are slots dedicated to informal networking among participants.

Concluding panel

The MES concluding panel involves all the plenary speakers and the delegates in further discussion around dilemmas and questions that have emerged during the whole conference.

Conference Programme

	Tue 2nd April	**Wed 3rd April**	**Thur 4th April**	**Fri 6th April**	**Sat 7th April**	**Sun 8th April**
9:00		Plenary 1	Plenary 2	Plenary 3	Plenary 4	Papers / Proj
11:00		W. Groups Plenary response	W. Groups Plenary response	W. Groups Discussion	W. Groups Plenary response	Plenary Panel Closing
13:00	Registration	Lunch	Lunch	Lunch	Lunch	Lunch
14:30		Papers / Project Presentations	Papers / Project Presentations		Papers / Project Presentations	Departure
17:00	Opening & Panel	Symposia *A, B*	Symposia *C, D*		Symposia *A,B*	
20:00	Dinner	Dinner	Dinner Agora 1		Dinner Agora 2	

THE REVIEW PROCESS AND PROCEEDINGS

All of the papers published in these Proceedings were peer reviewed by two experienced mathematics education researchers before publication. These reviewers are:

Lawan Abdulhamid, Jill Adler, Margarida Belchior, Margot Berger, Laura Black, Lynn Bowie, Karin Brodie, Tony Brown, Dimitri Chassipis, Anna Chronaki, Tony Cotton, Sandy Dawson, Jacques du Plessis, Regina Essack, Peter Gates, Uwe Gellert, Mellony Graven, Brian Greer, Beth Herbel-Eisenmann, Rico Gutstein, Robyn Jorgensen, Gelsa Kijnik, Troels Lange, Kate le Roux, Steve Lerman, Moeoang Leshota, Nontsikelelo Luxomo, Anna Llewellyn, Judah Makonye, Corin Mathews, Tamsin Meaney, Heather Mendick, David Merand, Monica Mesquita, Shadrack Moalosi, Nico Molefe, Candia Morgan, Eva Noren, Jarmila Novotna, Peter Pausigere, Vasen Pillay, Kim Ramatlapana, David Reid, Jessica Sherman, Ali Sikunder, Cathy Smith, Debbie Stott, Hauke Straehler-Pohl, Hamsa Venkatakrishnan, David Wagner, Margaret Walshaw, Mark Winter, Surgeon Xolo, Keiko Yasukawa.

Strict guidelines were followed to ensure that the papers had a significant contribution to make to the field, and were based on a sound literature review and methodology. The production of the Proceedings was possible through the cooperation of many participants in this and previous conferences who offered their time to peer review papers. The challenges faced by some of our conference participants from language backgrounds other than English, to write their paper in English are acknowledged and appreciated, as well as the time of some generous reviewers who provided support for language correction.

PARTICIPANTS

At this conference there are 93 participants from 16 different countries: Argentina, Australia, Brazil, Canada, Czech Republic, Germany, Greece, Iceland, India, New Zealand, Norway, South Africa, Sweden, Switzerland, United Kingdom, United States of America

An electronic file of all individual papers as well as of the whole proceedings is available at http://www.mes7.uct.ac.za.

Margot Berger, Karin Brodie, Vera Frith and Kate le Roux

April 2013

PLENARY PAPERS AND REACTIONS

IDENTITY WORK AS A POLITICAL SPACE FOR CHANGE: THE CASE OF MATHEMATICS TEACHING THROUGH TECHNOLOGY USE

Anna Chronaki

University of Thessaly

During the last two decades there has been an increasing interest for 'identity' research in the field of mathematics education. This 'turn' to identity signifies a methodological, theoretical and epistemological shift towards embracing the social, cultural, historical and political underpinnings of teaching and learning mathematics. By means of discourse theory, identity work can be viewed, here, as a struggle towards articulating meaning around hegemonic and neoliberal discourses concerning school mathematics and education. The case of mathematics teaching through technology use exemplifies how teachers negotiate engagement not only with technology but with the demands of change at the societal and pedagogic axis. As a result, the present paper discusses identity work as a potential political space for teacher change in contemporary times where both school mathematics reformation and an escape from nowadays' neoliberal crisis becomes an urgent requirement.

AN ENTRY: THE 'TURN' TO IDENTITY RESEARCH

Back in 1998, Anna Sierpinska and Jeremy Kilpatrick in their ICMI study based volume entitled *'Mathematics Education as a Research Domain: A Search for Identity'* called for the need to clarify our 'common identity'. The contribution of a number of well-known academics set up an agenda for re-considering not only goals, criteria and evaluation procedures, but also epistemologies, methodologies and ethics that designate our research and educative experiences. This ambitious endeavour concluded that despite the wish for tidying things up around 'common' grounds, identity in the field of mathematics education research needed to remain open due not only to its interdisciplinary theorising, but also to its locally determined field of practice. This event was an explicit expression for an urge to define our institutional identity using Gee's (2000) words, or, the identity of our professional community of practice in Wenger's (1998) terms. At the same time, it signified a public recognition of the inevitable impossibility in such a task when the diverse epistemological and political perspectives underpinning research in mathematics and mathematics education are seriously taken into account.

Sometime earlier, in the year 1991, Jean Lave and Etienne Wenger's book on '*Situated Learning: Legitimate Peripheral Participation'* argued for human learning as a participative phenomenon in local practices. This discussion was expanded by Etienne Wenger in his 1998 book on *'Communities of practice: Learning, Meaning, and Identity'*, where the notion of communities of practice as a professional organisation that manages systemic change was closely related to identity formation, learning and meaning. Although, Luria (1976) had already argued for learning as a long transformative identity process, it was Wenger's (1999) well cited work that captured the attention of most mathematics education researchers and mobilised the 'identity turn' (e.g., Boaler et al., 2000; Nasir and Cobb, 2007; Sfard & Prusak, 2005; Grootenboer & Zevenbergen, 2007; Chauraya, in press).

Identity is a contested term signifying diverse cultural historiographies, epistemological underpinnings, theoretical languages and political orientations. Diversity could range from representing the enlightenment subject (driven by rationality and logic), the socialised subject (acculturated via the institutions of family, church, school) or even the postmodern subject that exemplifies a hybrid 'self' (Hall, 1992). However, we could discern a mutual concern for connecting human subjectivity with knowledge and practice that have relevance for mathematics education. Questions such as; '*who are we in a mathematical classroom, community or even society at large?*', '*how do we identify with mathematical success and failure?*' or '*why do we become engaged within a mathematical practice?*' can now become addressed. At first, such questions move our gaze beyond persistence on developing competencies, cultivating learning strategies and adherence towards successful performance as assigned via national curricula and international assessment standards. At a deeper level, our gaze can be attentive to how we enact, perform and narrate our relations with/in mathematical practices, activities, objects and humans. Albeit diversity, the identity turn sensitizes us towards denoting potential perils, challenges, resistances and pleasures as we strive for connectivity and transformation in our social and cultural localities. Here, I encounter identity work as a political space for mathematics educators who try to cope with demands for change – such as technology use as an integral part of mathematical activity. Taking into account how identity can be contested with diverse significations of human subjectivity, I rely on discourse theory and post-structuralism (Laclau & Mouffe, 1985/2005; Weedon, 1987) to articulate identity work as an anti-essentialist process where fragility, fragmentation and hybridity can be recognised.

TECHNOLOGY USE, MATHEMATICS TEACHER CHANGE, IDENTITY WORK

Current reforms worldwide place a huge emphasis on teacher change towards becoming competent technology users, able to integrate information and communication technologies in curricular areas. Dominant discourses related to education and life-long learning tend to promote digital media as facilitating knowledge building, collaboration, and creative work across schools and cultures (Loveless, 2007). As such, technology use is not viewed, merely, as developing competences (e.g. technical skills, literacies, regulating strategies) but it is viewed as primarily connected to everyday routines that dissolve into habits of work and entertainment. This move is indicative of a 'new professionalism' as argued by Hargreaves (1994) where teachers' personal learning and growth is embedded within broader institutional structures. As such, teacher (and learner) development through institutional apparatuses (i.e., curriculum, classroom assessment, international evaluation programs) including the use of ICTs that regulate individual performance becomes a 'technology of self' that, in Foucauldian terms, serves to govern society by governing self (Henriques et al., 1998; Popkewitz, 2004; Pais & Valero, 2012).

During the last three decades, the transformative impact of computers in mathematics education has been mainly discussed through qualitative case studies that exemplify potential learning affordances. Emphasis on technology use in mathematics curricula has mostly favored experimentations with innovative software tools purposefully designed and implemented so that to scaffold mathematical learning and advance mathematical thinking (Noss & Hoyles, 1996; Mariotti, 2002; Hershkowitz et al., 2002). Despite such influential research, technology integration in mathematics classrooms remains a huge challenge as argued in a recent study by Ruthven et al. (2004) who observe that school teachers, by and large, still do not utilize technology to deliver the mathematics curriculum. This is, perhaps, indicative of the need to broaden the discussion over mathematics teacher change and technology use to include issues of identity work as discursively situated in professional practices of teaching and training.

Trying to address in a more profound way the complex multiplicity of teacher change, Kelchtermans (2005) highlights how teachers relate personally to the structural conditions of their profession. Reform agendas that require change, have considerable emotional impact on teachers, because of the challenges they pose on self-image, self-reflection and self-reconstruction. Recent conceptualizations in the field of teacher education pay attention to identity as a socially situated construction rooted within socio-historical, cultural, political parameters with a determinative influence on teacher formation. Beijaard et al. (2000), reviewing research on teachers' professional identity, note that inasmuch as identity is an entity of relational nature, it is unpredictable and constantly in a transition process. This implies that, while subjects construct identities collectively and in response to societal restructuring, uncertainty plays a significant role, turning identity into a shifting, unfixed, and unending entity as it involves the reconstruction of meaning over space and time. Brown and McNamara (2005), exploring how math teachers negotiate professional identity as they encounter regulative curricula reformations, denote teachers' struggle for meaning over multiple, and often, conflicting discourses.

Through a three month teacher training course focused on introducing a small group of seven experienced teachers (two women and five men) in ways of integrating technology in mathematics teaching, we had the chance to study how teachers appropriate technology and how they weave subjectivity as part of their professional growth and change by means of small scale ethnography. Teachers live in a technology-driven society and become immersed into discourses that emphasize computer literacy. At the same time, through the training course they become acculturated to refined constructivist and socio-cultural discourses concerning investigative and experimental mathematical learning mediated by 'appropriate' tool-use and pedagogical design. How do teachers engage and identify with such diverse and, at times, conflicting discourses of learning? What subject positionings do they take? How do they perform identity work as they strive to embody 'new' tools and pedagogies and how do they construe the 'old' ones? Teacher change, in this study, emerges as part of continuous efforts to reconcile personal and collective experiences and understandings of both 'mathematics teaching' and 'technology use' with societal and institutional demands of wider 'teacher identity change' politics. Taking into account the above, it is critical to consider how one could conceptualize teachers as subjects who are heavily engaged in identity work. This very concern entails the need to unravel further the notion of 'identity' itself.

IDENTITY WORK AS MEANING ARTICULATION

Identity work is not neutral. It signifies all way down historical, social, cultural, epistemological, ontological, ethical and political positions. For some, identity work is viewed as personal-social interplay and refers to the ways we narrate ourselves and how others talk about us. Personal identity is ascribed as taste, choice, belief, attitude, lifestyle or position, and is always inscribed in relation to other people, groups, communities, ethnicities, nations and sexualities. In this sense, personal or core identity in Gee's (2000) words is linked to social identity and its associated normative rights, obligations and sanctions which, within specific collectives enact behaviours, form memberships, perform rituals and generate values and emotions (Tajfel & Turner, 1986). Wenger's (1998) model of identity formation elaborates this perspective by means of encountering three distinct modes of belonging that tend to relate personal and social dimensions of identity, namely; engagement (mutually negotiating meaning), imagination (expanding images of self) and alignment (fitting self within the broader structure). Within this perspective, the individual's awareness of purpose, motives, goals and future directions is necessarily connected to his/her participation in specific social practices and representations. Such a view has been critiqued for assuming 'socialisation', 'participation' and 'engagement' as neutral processes where the individual develops rationality smoothly and where adults (parents and educators) mediate their progress and safeguard democracy (Walkerdine, 1988; Walkerdine & Lucey, 198;, Henriques et al., 1984/1998).

According to the sociologist Giddens (1991), identity needs to be ultimately seen as a project where the individual has to reflexively reconcile past experiences and future aspirations. This project is based upon self-ability to construct a narrative that represents biographical continuity, where '*self-identity is not a distinctive trait, or even a collection of traits, possessed by the individual. It is the self as reflexively understood by the person in terms of her or his biography*' (Giddens, 1991, p. 53). This perspective on identity also fails to take into account the dilemmas, the crisis and the pain involved in any attempt to account and reflect upon personal life stories as they are caught at the boundaries of 'appropriate' educational experiences. Today, in most western developed countries adult life is organised on short-term contracts, reduced social security funds, increased poverty and unemployment that undoubtedly affect an ever-growing population of young people who become more and more alienated, disconnected and marginalised from mainstream educational practices. Though productive as 'technologies of self', notions of identity as personal 'core' or reflexive self 'project' represent heavily a neoliberal politics that assumes subject agency as linked directly to a rational learner who can be successful without cost and to an independent citizen who can choose, consume and enjoy a capitalist lifestyle. Moreover, the 'reflective' individual is assumed in absolute responsibility and control as providing heroic solutions to persistent social, cultural and historical problems (Walkerdine et al., 2001).

The frequent mobilization of the fictitious image of the neoliberal free, rational, autonomous and independent agent collapses when one tries to explain 'difference' in behavioural, affective or cognitive terms. It offers little understanding when we wish to consider seriously the complex lived experience of children and teachers in relation to mathematical practices (e.g., Walkerdine, 1998; Walshaw, 1999, 2001; Stentoft, 2007; Chronaki, 2005, 2011). Empirical evidence in such studies highlights the presence of a fractured, fragile, marginalized and resistant self who is in a continuous battle to meet institutional demands for progress, development and growth. The utopian image of being able to produce coherent narratives of a trajectory that connects linearly past, present and future experiences at any time and space and, perhaps, at any cost, conceals how personal, institutional, social, cultural, racial, gendered and other subjectivities interact whilst hegemonic and marginal discourses come at play as interpretative systems. Gill (2008) has argued that we need to develop an understanding of identity in ways that do not associate individuality and subjectivity solely with 'inside' or 'interiority'. This implies that the social, cultural, political constraints upon human subjectivity should not be ignored but, instead, taken into serious consideration (Weedon, 1987).

Walkerdine was amongst the first in the field of mathematics education who opened the 'black box' of the relational biopolitics amongst mathematics, learners, teachers and educational politics in society (Walkerdine, 1988, 1989; Walkerdine & Lucey, 1989) and provided an elaborate critique of 'progressive' education (as based on mainstream notions of constructivism and educational psychology) where the 'child' is seen as progressing gradually from other-regulation to self-regulation, self-discipline and self-control. Leaning on poststructuralism, psychoanalysis, cultural studies and critical theory, Walkerdine has promoted a view of the subject (learner, teacher and parent) as relating actively with discourses and discursive practices and negotiating multiple and fluid meanings of self and other. In other words, people are not simply socialised but are involved in processes of subjectification where subject and society are interlinked (Davies, 1993; Weedon, 1987/2004). The notions of subject and subjectification are purposefully used so that to denote a move from the neoliberal notion of the 'autonomous' individual –a move that became possible through the publication of the volume *'Changing the Subject: Psychology, social regulation and subjectivity'*. In contrast to the neoliberal individual who regulates his or her behavior and adapts in the socio-cultural context, the notion of 'subjectivity' embraces the subject as fragile, fragmented and relational.

Willing to move beyond the notion of identity as 'personal core' or 'reflexive project' and to embrace identity work as a process of subjectification we turn towards Laclau and Mouffe's position of the subject as being ascribed and becoming inscribed by diverse and competing discourses. For them, subject positions that are not in visible conflict with other positions can be seen as the outcome of hegemonic regulations, whereby, alternative possibilities have been excluded and a particular discourse has been, at least temporarily, naturalized (Laclau & Mouffe, 1985/2001, pp. 47-49). Foucault's (1993) notion of 'discourses' as historically, or rather genealogically, rooted attempts to construe 'truth' in social and cultural practices is useful here to understand how discourses seem to form a consistent totality at the experienced present, but, in fact, are part of partial fixations of meaning organized in nodal points over time and space. According to Laclau and Mouffe (1985/2005) a nodal point is defined as a privileged sign (or a key signifier) around which other signs are ordered and invested with meaning through relations in chains of signification. Through this perspective, mathematics teacher change – in the course of learning to become competent users of technology – is a relational process of articulating meaning. Articulation is a temporary fixation of discursive elements in an attempt to form connections that constitute a contingent and context-specific unity (Barker, 2006). As such, our research task, as we try to interpret identity work, is to plot how the agentive subject fabricates meaning, focuses on articulations that constitute particular positions in complex interactions and accounts for their potential effects at the socio-cultural and political levels. Meeting the above, meaning articulation, in the present study, becomes evident around chains of signification where teachers personally and collectively struggle to weave connections amongst varied elements of technology effects on mathematics at the societal and pedagogical axis. These two interrelated axes will be discussed here as core chains of articulating meaning; a) *the societal:* embracing the computer as a shared commodity signifying youth digital culture and consumerism, b) *the pedagogical:* appropriating technology based learning of mathematics as an assemblage that effects in pedagogic novelty and power redistribution. In the sections below, each one will be outlined.

ARTICULATING THE SOCIETAL: YOUTH DIGITAL CULTURE, COMMODITIES AND CONSUMERISM

Teachers' involvement with technology use for mathematical learning was primarily articulated around an urge to relate with youth culture. Youth culture, affiliated with digital culture and digital youth, popularly addressed as digital natives, refers to young people who grew up and immerse into using computing technology in their everyday life (Prensky, 2001). The computer, thus, as shared commodity, becomes a way to connect with the young generation and to bridge an age generated cultural gap:

> <u>Tasos:</u> *[..] If we are disconnected from pupils [...] they wouldn't be interested at all in what we try to pass on [...]. We will cease to be convincing. We will belong to the Paleolithic age.*

Technology and youth are both seen to offer the hope and hype of society transformation – a teleological assumption of growth, progression and development (Buckingham, 2008). Young people are, often, presented, on the one hand, as the autonomous and agential ambassadors of massive cultural and technological change, and on the other hand, as uncritical consumers of digital commodities and passive users of mathematical activity. Such contradictory set of discourses denotes a wider anxiety about the changing nature of youth and childhood, the impact of technological change on social life, and, at the same time, the increasingly 'disappearing' but yet 'formatting' power of mathematics embedded in scientific and societal practices including economy and politics (James & Prout, 1990; Castells, 1998, Keitel et al., 1993; Jablonka & Gellert, 2007; Atweh et al., 2007; Chronaki, 2009, 2011).

Teachers in our study were unaware of this complexity and, instead of problematizing the presence of contradictory discourses, they resort readily to children's enthusiasm and attraction to computers and digital media as only natural. Having experienced the secondary school mathematics culture in Greece they criticize contemporary teaching practices for the over-emphasis on drill and practice of algorithms and the training in formal proof. Teachers, worryingly, argue that this situation benefits merely a few gifted or talented students in mathematics and, by and large, results in unmotivated, uninterested and marginalised learners. Their dissatisfaction was unanimously expressed when claiming that '*something must be done'*, or *'we cannot continue like this'*. Transforming school mathematics from an entirely abstract to an experiential construction accessible to all students (and not restricted to the gifted ones) was perceived as missionary obligation. Technology, at this space and time, was mythologized as a saviour that could provide heroic solutions to such persisting needs. Petros, one of the teachers, exemplifies:

> *[...] In this technology lesson [...] you must see them* [implies the pupils]… *all of them [...]. Focused [...] Ah, do you believe it?! [...] This thing happened! This thing happened in a mathematics classroom at a vocational school.*

Based on the 'dynamic' screen aesthetics, the computer in school mathematics is invested with broader hopes related to curricular reforms by taking pupils from inertia to activity, from boredom to creativity and from a disciplined reading of mathematical content to an experiential way of working.

However, computer use is part of a much wider cultural, industrial, commercial, social, educational and entertainment complex that involves people as operators, producers and consumers (Sheff, 1993; Lievrouw & Livingstone, 2004). In our study, the politics of appropriating technology in a society of consumers was mentioned by Tina, one of the two female teachers, as part of her observations on how some tutors during the training course presented particular software tools as related to mathematics learning:

> *[...] Reflecting on the work of last semester Txxxxx* (one of the tutors) *referred to Fxxxxxx* (a type of software). *I get the impression that these people, without being aware of this, worked for a serious advertisement of this particular software. It is as if they were paid to advertise it.*

Bauman (2007) has pointed out how modernity transforms a society of producers into a society of consumers and argues:

> *In this new consumer society individuals become simultaneously the promoters of commodities and the commodities they produce. They are, at one and the same time, the merchandise and the marketer, the goods and the travelling salespeople*. (p. 6)

In a similar tone, Tina problematizes the training course as a terrain for marketing educational software along with learning models (that serve to exemplify benefits and affordances for the learning of mathematics) as much needed teaching devices. Deeply concerned with identifying ways to enhance her teaching, but at the same time being cautious to an increased marketization of technology use in education, Tina turns towards deliberately questioning a neutral stance to consuming technology. Taking advantage of her expertise in informatics, Tina adopts a producer position constructing her own digital mathematics (by means of open source and free software tools) and, at the same time, she volunteers for an informal community of mathematics teachers that shares and distributes lesson plans, tools and techniques. Unlike Tina, the other teachers – claiming lack of time and expertise, experiencing restrictions due to gender and parenthood, but also acknowledging a desire to taste 'new' tools – positioned themselves, sometimes, as consumers who reuse commercial digital tools (as they were suggested during the training course or located in specialized web portals), and other times, as hybrid producers who amend or expand micro-worlds (e.g., in dynamic geometry or logo- like environments).

Consumerism is conceived a late attribute of modern society, of desiring and longing for goods but also as a social arrangement that coordinates systemic reproduction, social integration and stratification through forming individual and collective identities (Bauman, 2004, 2007). Tacit discourses involve teachers – and tutors in the training course – into the politics of marketing 'new' technologies and services for mathematical learning. Such involvement comes implicitly through engagement with theoretical and practical work in the training seminars that stress the 'newness' offered by specific computer hardware and software. A newness that resorts upon discourses of 'effective' learning design where theories, software tools and artefacts are all turned into marketable commodities. The sense of this needed 'new' becomes a reference to the most glamorous and recent, and this, in turn, carries the ideological fiction of 'new' equals 'better'. Teachers, by and large, espoused this posture and expressed, especially at the start of the training course, eagerness to learn about 'new' ideas, tools and ways of doing things. For them, 'new' signified 'the cutting edge', the avant-garde, the place for forward-thinking people to be and behave as modern designers and practitioners – perhaps a place forbidden for the so called 'traditional' mathematics teachers. Discourses of 'change' as connotations of the 'new' are related with a long-lasting modernist belief in social progress and development as smoothly delivered by technology use (Castells, 1996; Somekh, 2008).

ARTICULATING THE PEDAGOGIC: TECHNOLOGY APPEAL, POWER RE-DISTRIBUTION AND MATHEMATICAL ACTIVITY

Main discourses concerning technology use in mathematics classrooms promise potential learning gains for rigorous mathematics *only* at the provision that children are actively engaged with appropriate software and mathematical activity. Appropriateness has been discussed in terms of encouraging dynamic manipulation of mathematical entities on the computer screen, multiple representations of data in arithmetic, geometric and algebraic forms, as well as modelling and programming (Hershkowitz et al., 2002; Mariotti, 2000; Noss & Hoyles, 1996). The curriculum of our training course was nationally organised around such ideas and the group of teachers in this study worked meticulously so as to grasp the potential didactic and learning affordances of specific software (i.e., CAS, dynamic geometry and logo-like tools) by making direct relations to the school curriculum (see PAKE, 2007). In particular, teachers were geared towards constructing mathematical micro-worlds and designing their integration into pedagogical scenarios and lesson plans. In this way, the aforementioned discourses were re-contextualised through specific apparatuses (i.e., coursework and assessment tasks) and provided the 'language' for constructing and negotiating the urgent need to change current practices of mathematical teaching. Despite efforts for acculturating teachers into valuing the learning gains of specific tools they tended to prioritise technology's impetus for pedagogic novelty. They did so by considering its appeal to children, as well as, its potential to turn the mathematics classroom culture into a more talkative, collaborative and active place.

In terms of technology's appeal, teachers celebrated its visual, interactive and tangible characteristics and its attractiveness was constructed in direct comparison to the so-called traditional modes of chalk and talk or paper and pencil. For example, Andreas denotes computer's magic touch:

> *This medium is more attractive, for sure. It* [refers to the computer] *will replace the teacher. It will help the learner… It will make him… in simple words… not bored with the endless bla, bla… even with the talking* [means the need to explain using words in talk and writing] *during the lesson. It is different.*

As far as the mathematics classroom culture was concerned, technology was conceived as an assemblage (i.e., computer, software, pupils, colleagues and ways of working with knowledge) that augments classroom norms and re-allocates power over humans, tools and relations. Kostas exclaims how technology serves for a pedagogic culture where children experiment, work together and become agentive of their own learning:

> *With this software children undertake the role of a researcher and what's more this is what we need in mathematics: to activate the student in order to be able to understand ... we don't want students to continue being passive recipients […] I, finally, got enlightened as far as it concerns student collaboration and its direct relation to learning outcomes.*

Andreas, in particular, notes:

> *[…] for pupils, if we can create this move for pupils. To talk. To try and try. To explain why we did such and such. They will feel it as theirs. They can make it […]. The knowledge […] that will come later on. BUT, they will feel it belongs to them. They can make it [...] In other words, it* [mathematical knowledge] *does not come from the teacher. Or, if you like, it has been validated by the machine* [means the specific software tool]. *I think, in this way, we win the students over. We win them back....*

Fabricating the motivated, interested, engaged, active and collaborative learner of mathematics comes along with appropriating mainstream constructivist and sociocultural discourses of technology mediated mathematical learning made available through the training course. Towards materialising this much desired shift, they articulate technology as their ally. Technology was not fabricated as a teacher substitute, but, as a teacher advocate to act out pedagogic novelty at varied layers such as representing mathematical content in multimodal genres in visual, interactive and tangible terms, capturing children's attention, motivating them to actively engage in their learning, providing feedback and validating mathematical activity. Further, technology was expected to soothe power relations by loosening the demands for teacher authority and by mediating knowledge control.

Children's immersion in digital worlds was woven by teachers, almost, as a chance to proselytize them into mathematical activity – an activity that leaves the young child indifferent or feared. Pedagogic novelty by means of technology attractiveness was also, here, produced on the grounds of a brutal need to change the mathematics curriculum. Enhancing the variety and appeal of classroom pedagogy was amongst the emerging themes identified by Ruthven et al. (2004) in a study on teacher's views of computer-based teaching. Teachers referred to activities involving technology as '*something different'*, '*making a change'*, and providing '*a change from the routine of the classroom'*. It is exactly this view of 'technology mediated pedagogic novelty as a decorative gloss' that also entailed danger for undervaluing any chance to develop rigorous mathematical learning. All teachers in this study were heavily concerned to safeguard the view that the choice of tools and activity design could support the passage from *'technology as decorative gloss'* to '*technology as mediating learning'*. However, it was not easy at all times. Encountering this passage was, for some teachers like opening the 'Pandora's box' – a challenging, risky and uncertain endeavour.

Power re-distribution became further evident in ways of disrupting or conforming to essentialist approaches to mathematical knowledge. Specifically, Kostas draws on his work with dynamic geometry software and talks about how he experiences change as part of his relation to children in the context of mathematical activity and was described as a change from a fixed to a negotiable process. In his words:

> *... until now when we spoke about mathematics, we meant the fixed and hard entity that we convey as it is to the kids… we say: "that's the way mathematics is". "Why?" "There is no why" ... In this case* [means teaching with technology based tools] *we can have an open procedure where we try to make them* [pupils] *understand how this mathematics comes along. Kids can then attribute a meaning to these* [mathematical] *notions.*

Next to appreciation, Kostas draws reservations that became explicit when Andreas enthusiastically shared thoughts with the rest of the group; '*I think that technology will offer the ground to move beyond the ways we currently accept mathematical proof… for example we may not accept the classical proof any more*'. This comment upset Tasos, one of the teachers, who, almost furious, exclaimed '*I disagree! This is only your personal way to view things!*' Kostas, at that moment, although a warm supporter of technology, felt that it could not be allowed to defy established conceptions of '*mathematical proof*' and confronted Tasos by stressing:'*…we NEED proof! The education system NEEDS it. Our society needs it!*'. This event raises issues concerning identity work performed collectively by this group of teachers.

Raju (2001) points out how knowledge politics have played a serious part in how mathematics travels over time and culture and argues:

> *[h]istorically, a similar epistemological fissure between computational/practical Indian mathematics and formal/spiritual Western mathematics persisted for centuries, during a dialogue amongst civilizations, when texts on 'algorismus' and 'infinitesimal' calculus were imported into Europe, enhancing the ability to calculate*. (p. 325)

At the same time, philosophy and sociology of science discuss technology and mathematics as interrelated in implicit ways and some have agitated technology as the black-box of mathematics (Keitel et al., 1993; Bijker & Law, 1997). They denote how late modernity – primarily through advances in information technology – renders mathematical knowledge tacit due to its embodiment in processes of producing and manufacturing techno-scientific artefacts. This results in experiencing mathematics as 'hidden', 'frozen' or 'disappearing' into diverse literacies of, for example, technomathematics and ethnomathematics. Neither Kostas nor Tasos have the knowledge to critically consider technology as the black box of mathematics or as part of knowledge politics dissolved over historiographies of mathematics in action. They are both not prepared to move beyond a safe conception of 'mathematical proof'. 'Proof', for them, comprises competences related to hypothetical reasoning and logic central to mathematical thinking and culture – also connected with Greek educational culture. Maintaining this standpoint they resort to both professional and citizenship identities to articulate a fixation about technology use in mathematics teaching. For them, mathematical proof is the ultimate form of any rational reasoning and an essential literacy in democratic society – a territory that technology should not touch. In other words, they argue that technology could be used for pedagogic novelty, but not for epistemological challenge. Despite the fact that nowadays mathematicians are comfortable with uncertainty as part of scientific work, school mathematics teachers, by and large, have enormous difficulty in embracing knowledge in diverse ways (see Gutiérrez, 2012). However, by the end of the training course, and as teachers were becoming increasingly comfortable with technology-use, as well as, with the idea of mathematics as also fallible, Andreas' challenge of formal proof was re-appropriated by Tina, who commented:

> *Teaching mathematics with technology make it possible not to depend so much on classical rigid mathematical proof, as we did before. Besides we already know* [as part of this course] *that children's strategies for proof depend on various forms of argumentation […].*

This can be taken as a space where collective and discursive identity work encourages teachers to explore positions for troubling or conforming to both traditional and radical new knowledge and ideas.

CONCLUSIONARY REMARKS

Identity work, as a way to account for human subjectivity, is not neutral, but rather political. This becomes evident, when one is prepared to abandon a view of identity that casts self in a personal-social dichotomy. Embracing an anti-essentialist and anti-neoliberalist perspective encourages us to approach identity as contingent to socially and culturally specific productions. In consequence, identity cannot be seen primarily as a core or reflexive construction of a self-narrative, but needs to be considered as a deeply relational and discursive process of subjectification. As mathematics teachers try to cope with becoming experts in technology-use they increasingly realise the boundaries in being able to control its effects and outcomes. They also face the struggles over producing meaning around multiple and conflicting significations of hegemonic and marginal discourses. Could we, then, consider such struggle a path for identity work? If yes, could identity work become a space for negotiating societal change, including teacher change in ways that encourage us to redefine past and future experiences of success and failures as collective endeavours rooted in specific socio-political temporalities? And, if yes, could identity work entail the potentialities of a political space that can create radically different conditions for mathematics education and learning combatting nowadays' crisis, dilemmas and dead ends?

Contextualising identity work within the specific case of a small group of mathematics teachers as meaning makers of technology use, I have tried to capture their evolving struggles for articulating diverse discourses as they strive with the socio-material contingencies of complex professional space. Findings of this empirical study indicate how teacher articulations were woven as chains of signification around the interrelated societal and pedagogical axis. Teachers' identity work involves the production of meaning not as self-referential individual property, but as relational – a cat's cradle – towards locating status amongst diverse stakeholders and subject positions. Concern here is not to account for any 'true' meanings existing out there, but to identify how lay teachers collectively fabricate chains of meaning, and how this allows them to cope with politically grounded demands for change. I would like to stress two issues: a) intensity for change is interlinked with values and practices of a consuming society as experienced by teachers – computer, mathematics and 'effective' learning are all fabricated as commodities responsive to marketization politics, and b) change involves appropriation of certain discourses concerning 'technology use' and 'mathematical activity' that produce the need for pedagogic novelty but also have effects on redistributing power relations and troubling mathematical knowledge.

Concerning the first outcome, intensity for change in mathematics teaching through technology use is further articulated by teachers primarily on the basis of youth culture and market politics. It was a shared concern that mathematics learning and curriculum should be modified so that to satisfy and cater for contemporary young children's wants, needs and values. Within this context, teachers confronted 'technology' and 'mathematics' as commodities that enforce subjects to perform specific identities as learners and professionals. Young people and teachers become, thus, a market that is heavily targeted, so that the choice, purchase and utilisation of 'new' technologies are already implicated in broader discourses and practices where identities as well as demands for learning and life are interlinked. The 'threats' of marketization and consumerism as globalized practices to education and children's cultures, although well documented (Apple, 2004; Buckingham, 2007), are rarely considered when technology use in mathematics education is at stake. Maths teachers in this study referred to youth digital culture and questioned its potential links to a globalised marketing of educational software. Thus, technology-based mathematics education and training become heavy political arenas that serve to regulate teachers, learners and curriculum designers towards the production of 'appropriate' identity changes in the name of the 'new' mathematics teacher (see Chronaki, 2000, 2009). Vithal (2007) based on Castells (1996), argues that in the field of mathematics education, contemporary demands for technology-use can easily run into the paradox of a double process of inclusion and exclusion.

Next to these, teacher change needs to be approached as a political space for identity work performed at the core of 'technology mediated mathematical activity'. Pedagogic novelty by means of the transformative power of technology was repeatedly argued, by teachers in this study, in the hope of an urgent reformation of school mathematics. This outcome signifies how teachers experience technology not merely as a tool, but as a complex assemblage that has vital effects on change at varied levels (Latour, 2007; Bijker & Law, 1997). Here, we witness 'change' to be materialised in contextual layers of pedagogic, didactic, epistemological and ontological instances that ultimately frame mathematical activity. As such 'identity change' mobilised through technology-use is inscribed as a continuous move amongst possible acts and potential imageries on how mathematical activity could adopt or resist 'change' in concrete terms (i.e., content representation or simulation of mathematical content on screen, communicative rituals and politics of epistemology). As *change* comes with strain, *desire for change* becomes reinforced. Change, as we saw in the sections above, involves complex identity work that embodies societal and institutional demands and requires teachers to get involved in profound political choices and decisions in everyday classroom work. This becomes experienced by teachers as a fragile, fragmented, slippery and at times impossible process embodying risk and ambivalence – a process that enveloped uncertainty but also will and joy.

Addressing teacher change and identity work as a complex, multifaceted discursive process has several implications for technology integration in mathematics education. First of all, such a notion disrupts the taken for granted belief that teacher professional development in times of greater social transformation (and curricula reforms) can ever be approached as a one-size-fits-all and effective identity that teachers can easily adopt and 'wear'. Instead, teachers in transition perform identities at local borderlands of myriad discourses and enact trajectories of non-linearity and without clear outcomes. Consequently, educational policy and official training programs aiming for transforming teaching practices, should view attempts for teacher change as identity work that involves the struggle for articulating meaning as an essential space for subject position in the high density of curriculum reform politics. Based on these findings, we further suggest, that next to training teachers in 'instrumental' and 'functional' competences in technology use, there is also a need for developing critical competences that would allow teachers to encounter 'technology' within wider socio-political institutions. The above are indicative of the need to create borderland spaces in teacher training programs that encourage identity work as scaffolding and dialogue for teachers who encounter technology-use. In this way, they will broaden the 'unthinkable' and the 'yet-to-be-thought' (Bernstein, 2000) and they will denaturalize and gain awareness about teaching practices. In short, safe spaces in training for teacher change will afford experimenting and performing identity work as it is – a political space.

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WHOSE IDENTITY? RESPONSE TO ANNA CHRONAKI'S KEYNOTE ADDRESS AT MES 7

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INTRODUCTION

The genre of "response to a keynote address" is quite open for interpretation. Let me start by saying that I sympathise with the implications that Anna draws from her study. In particular, it seems important that teacher professional development programmes view teacher change as identity work, not as acquisition of functional competencies in technology use. Such a view encourages identity scaffolding and thus broadens the space for the unthinkable and yet-to-be-thought. I have chosen to focus my comments to Anna Chronaki's keynote address from a methodological perspective by looking at how she defines identity and identity work and what these concepts are doing for her. This approach springs from my own struggles with the same concepts, in particular how to give them operational definitions that allow the recognition of instances of identity and identity work in the data.

IDENTITY AND IDENTITY WORK

Chronaki did a small scale ethnographic study on a group of seven teachers doing a three month teacher training course on integrating digital technology into mathematics education. In a post-structuralist discourse analysis frame she studied "how teachers appropriate technology and how they weave subjectivity as part of their professional growth" (p. 3). Her data tell a story about how the group of teachers grapple with the different discourses they encounter in the course and which they – because they are teachers and have to act – have to reconcile into some sort of workable whole. The discourses that become actualised in the course are about consumer society; youth culture as problem and hope; digital technology in general and in particular in relation to mathematics teaching. In regards to the latter, they try to envision possibilities with regards to students' motivation and their own pedagogy, and they are confronted with questions about the function and status of proof.

Chronaki wanted to "articulate identity work as an anti-essentialist process where fragility, fragmentation and hybridity can be recognised" (p. 2). The word identity figures 77 times in the text (85 if identities are included) of which 24 are in the combination identity work. While Anna discusses a range of aspects of the notion of identity, and in strong terms distances herself from notions that involve an autonomous or individual self, she does not come up with clear-cut definition of identity. She sees identity "as a way to account for human subjectivity" (p. 12) but neither does she distinguish clearly between identity, subjectivity, subject, subject position and self, nor does she clarify their interrelationship. What is clear is that identity is collectively (re-)constructed, shifting, and unfixed:

> While subjects construct identities collectively and in response to societal restructuring, uncertainty plays a significant role, turning identity into a shifting, unfixed, and unending entity as it involves the reconstruction of meaning over space and time. (p. 3)

In terms of Brubaker and Cooper's (2000) analysis these formulations sum up to a "weak" understanding of identity. "Strong conceptions of "identity" preserve the common-sense meaning of the term – the emphasis on sameness over time or across persons" (p. 10) whereas weak understandings "by contrast break consciously with the everyday meaning of the term" (p. 10). Of the five key uses of identity as an analytical concept, that they list, Anna's use clearly belongs to the fifth.

> Understood as the evanescent product of multiple and competing discourses, "identity" is invoked to highlight the unstable, multiple, fluctuating, and fragmented nature of the contemporary "self". This usage is found especially in the literature influenced by Foucault, post-structuralism, and post-modernism. (Brubaker & Cooper, 2000, p. 8)

Among the instances of identity work in the text, are several attempts to pin down what is meant by this construct. In formulating what might be research questions, the production of meaning characterising identity work is described with the acts of *identifying* with conflicting discourses, *taking of subject positions*, and *embodying* of tools and pedagogies

> How do teachers engage and identify with such diverse and, at times, conflicting discourses? What subject positions do they take? How do they perform identity work as they strive to embody 'new' tools and pedagogies and how do they abandon any 'old' ones? (p. 4)

In a number of instances identity work is defined as articulation of meaning. The most important examples are – with my italisising –

> Identity work is viewed, here, as *a struggle towards articulating* meaning around hegemonic and neoliberal discourses concerning school mathematics and technology. (p. 1)

> Mathematics teacher change … is a relational process of articulating meaning. Articulation is a temporary fixation of discursive elements in an attempt to form connections that constitute a contingent and context-specific unity. … As such, our research task, as we try to interpret identity work, is to plot *how the agentive subject fabricates meaning*, focuses on articulations that constitute particular positions in complex interactions and accounts for their potential effects at the socio-cultural and political levels. (p. 6)

> [The teachers] also face the *struggles over producing meaning around multiple and conflicting significations of hegemonic or marginal discourses*. Could we, then, consider such process of struggling a path for identity work? (p. 12)

> Teachers' identity work involves the *production of meaning* not as self-referential individual property, but as *relational* –a cat's cradle- *towards locating status amongst diverse stakeholders and subject positions*. (p. 13)

From these examples is it clear that identity work is the production of meaning, in particular in situations where conflicting discourses compete for hegemony. The work part of identity work, that which requires an effort, the struggle in Chronaki's terms, and hence that which would qualify as work, is the act or activity of articulating, producing, fabricating meanings, or fixating discursive elements into connections of unity. It would then follow logically, that the identity part of identity work is the outcome of the work, that which is articulated, produced, etc., that is the meanings, the situated connected unities of discursive elements.

Hence, while the construct of identity mainly is defined indirectly and by what it is not, identity work has an operational definition that enables the distinguishing in the data of what is and what is not to be counted as identity work. It allows for identifying in the data temporary fixations of discursive elements that attempt to form connections that constitute a contingent and context-specific unity. As far as I can judge, this is what Chronaki does. The teachers in her study are fixating discursive elements into context-specific unities. The question for me is what is gained from labelling this production of meaning as identity work, and consequently equating identity with meanings. In the next sections, I consider these issues further.

"IDENTITY TURN"

In reviewing the growing interest in seeing mathematics education in various socio-cultural perspectives evident through the 1990s, Stephen Lerman (2000) coined the notion of "the social turn in mathematics education" for the mathematics education version of a phenomenon found in diverse academic disciplines "away from a focus on individual behaviour …toward a focus on social and cultural interaction" (Gee, 1999, p. 61). Chronaki identifies Wenger's (1998) book on *Communities of practice: Learning, meaning, and identity* as the germ of another "turn" in mathematics education research, the identity turn. Although others have noticed the increasing use of identity as a research lens in mathematics education research, as far as I know, the notion of an identity turn has been used previously in this field. The only use of the notion that I could find is in a review by Moje, Luke, Davies and Street (2009) in the context of literacy studies. They saw the "identity turn" in literacy research as developing out of the social turn in this field moving researchers attention to "literacy practices as tools or media for constructing, narrating, mediating, enacting, performing, enlisting, or exploring identities" (p. 416). Similar reasons to include the notion of identity seem to have been at play in mathematics education. Grootenboer, Smith and Lowrie (2006), for example, described identity as "a connective construct for examining the interplay between [e.g. beliefs, attitudes, emotions, cognitive capacity and life histories] and the effect such a nexus might have on mathematics teaching and learning" (p. 612).

In examining the different conceptions of identity in identity-and-literacy studies, Moje et al. (2009) found an array of conceptualisations of identity that spans a range of often rather different understandings. They note that everybody acknowledges the social, fluid and recognised nature of identity, and agrees that identity is lived out by individuals, but there are multiple positions about what exactly is the social, the fluid or the recognised.

> To acknowledge identities as social, fluid, or recognized is only part of the theoretical story; the what of identity can be represented in myriad ways, even when one accepts identity as social, fluid, and recognized. (Moje, Luke, Davies, & Street, 2009, p. 419)

Given that the intended outcome of the social practice of school mathematics also is conceived of in the form of literacy, that is, as more than a collection of skills and cognitive processes (e.g. OECD, 2003), I wonder what are the similarities and differences between identity-and-literacy studies and identity-and-mathematics studies. Would a review of the latter literature show a similar range of "the what of identity"? Paola Valero's (2009) reflections on the chapters in Black, Mendick, and Solomon's (2009) book indicates that this would be the case.

I also wonder if there would be similar concerns as to "the what of mathematical literacy" as Moje et al. (2009) express in regards to literacy-and-identities studies:

> And the what of literacy is equally problematic. More important, what do the possible ways of conceiving of identity mean for how literacy-and-identities studies are conducted? What, if any, assumptions about literacy are embedded in these different views of identity as social, fluid, and recognized? What, if any, assumptions about identity are embedded in different views of literacy? (Moje et al., 2009, p. 419).

They conclude by calling scholars to more rigorously "clarify what it means to write about and study people's identities in relation to their literate practices" (p. 432).

As noted above identity is not particularly clearly defined in Chronaki's text and its relation to 'neighbouring' constructs such as self, subject, subjectivity is not clarified. If my deduction about the definition of identity work is accepted, identity equals meaning articulations. While the archaeology of meaning certainly has its merits – and I think Chronaki demonstrates that – I do not find the equating with identity or identity work warranted.

WHOSE IDENTITY?

A definition effectively equating identity with articulated meanings raises a methodological issue of an ethical nature. This is similar to (and inspired from) Bill Atweh's (2011) reflections on identity in educational research in which he questioned whether "the identity as seen by participants coincide[s] with the identity as seen by others [the researcher]" (p. 44) and raised a concern about "the lack of clarity about what aspects of the lifeworld [the construct of identity] proposes to refer to" (p. 44).

Let me illustrate with an example. A teacher, Petros, in Chronaki's data seems to be telling a story about a teaching episode:

> Petros …: '[…] In this technology lesson […] you must see them [implies the pupils]… all of them […]. Focused […] Ah, do you believe it?! […] This thing happened! This thing happened in a mathematics classroom at a vocational school'. (p. 7)

It appears that Petros was quite enthusiastic about the episode and that he found it successful. However, to Anna his story exemplifies that

> Transforming school mathematics from an entirely abstract to an experiential construction accessible to all students (and not restricted to the gifted ones) was perceived as *missionary obligation*. Technology, at this space and time, was *mythologized* as a *saviour* that could provide *heroic* solutions to such persisting needs. (p. 7; my italics)

In this case the teachers' meaning articulations regarding their professional context is not the object of curious inquiry. Rather they are judged against the superior understandings of the researcher. Leaving that aside, another issue is that Petros might not recognise his "sayings" (Kemmis & Grootenboer, 2008) as an articulation of meaning, let alone as identity or identity work. Yet, that does not mean that Chronaki does not have a point. From a discourse analysis perspective, it could be argued that Petros was articulating situated meaning and that this meaning could be described in terms of a mythologised heroic quest with missionary obligations. The issue for me is that these sorts of insights come at a price that Petros and his fellow teachers have to pay in terms of losing control over their identities. Given that identity is "a category of social and political *practice*" (Brubaker & Cooper, 2000, p.4 italics in original) and that "semantically, 'identity' implies sameness across time or persons" (Brubaker & Cooper, 2000, p. 18) the teachers might not share Chronaki's theoretically informed construct of identity. Furthermore, this price seems unnecessary. After all, the analytical job in Anna's analysis is done by discourse analysis and not by the constructs of identity and identity work. Hence, abandoning these constructs as *analytical tools* would allow for both a respectful account of the teachers "sayings" as well as a theoretically informed analysis of these "sayings" attempting unravelling their composite discursive layers and social function.

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ON MATHS TEACHER IDENTITY: A RESPONSE TO ANNA CHRONAKI'S 'IDENTITY WORK'

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There has been an overwhelming increase in studies investigating the concept of maths teacher identity, with several reasons being given for this trend. Though it has been difficult to reach a common definition of the term the literature notes some accepted dimensions of identity. Chronaki's (2013) paper discusses some of these issues and focuses on how maths teachers change and cope with technological demands in current reform contexts. Drawing from the discourse theory, Chronaki explains how maths teachers change through engaging with technology and in the process negotiate their identity (work) within societal, institutional and curriculum reform contexts demands. This response questions some of Chronaki's arguments and also explains the implications of her unique study on maths teacher education.

THE IDENTITY (RE)TURN

There has been an overwhelming increase of studies investigating the concept of identity in education and in maths education. In maths education, this phenomenon emerged as a result of the 'social turn' in the late 1980's, with identity having had been a preserve for psychology and philosophy in the 1960s (Hall, 1991; Lerman, 2000; Sfard & Prusak, 2005). It is the borrowing and drawing from the social and the humanistic sciences that have made the concept of identity central in maths education research. There have been several notable studies focusing on maths teacher identity (Boaler, 2000; Graven, 2002; Hodgen & Askew, 2007; Lerman, 2012a; Parker, 2006; Van Zoest & Bohl, 2008; Walls, 2008, Zembylas, 2005) and currently there are a number of on-going PhD studies across local universities investigating the notion of maths teacher identity. Chronaki's (2013) paper also takes note of this increasing interest for identity research in the field of maths education calling it the 'turn to identity research'

There are several reasons why identity, and maths teacher identity in particular, has become the unit of analysis for many studies. As said earlier, the focus on identity was triggered by the maths education's research tendencies to draw from the social and humanistic sciences such as anthropology, sociology, psychology and cultural studies, these disciplines foreground the notion of identity. Chronaki (2013) also acknowledges that the concept of identity in maths education research has benefitted from interdisciplinary theorising. I also want to argue that given the local challenges, and the maths crisis in education, identity becomes a focal research point, thus in maths crisis contexts the concept of identity (*who someone is*) receives much attention. The notion of teacher identity also allows researchers to explore and investigate various aspects in education. Gee (2001) and Sfard and Prusak (2005) concur that identity can be used as an analytical and interpretive tool for studying both human conduct and important issues in education. In relation to this assertion, Chronaki's (2013) study focuses on the key and crucial issues of how maths teachers change and cope with technological demands and different maths knowledge views in current reform contexts, and within that space fashion their identity.

Chronaki (2013), like other notable studies in education (Beijaard et al., 2004; Gee, 2001) and maths education (Beauchamp & Thomas, 2009; Lerman, 2012a; Sfard & Prusak, 2005), admits the difficulty of reaching a common definition of the term 'identity'. However in the endeavour to define identity or maths teacher identity, the literature has discerned some common dimensions or features of identity which resurface in Chronaki's (2013) paper. Literature reveals that identity is dynamic and complex, constantly evolving, multifaceted, relational and is context related (Beauchamp & Thomas, 2009; Beijaard et al., 2004, Gee, 2001; Walls, 2008; Wenger, 1998). The multi-faceted, complex, relational and context related aspects of identity are discussed by Chronaki (2013) in relation to maths teacher identity change, in the context of technology and current global reforms.

INFORMING THEORIES & RESEARCH METHODS

Most studies on maths teacher identity have been informed by theories which originated from the broader modern version of the social theory (Lerman, 2000; Wenger, 1998). Generally the theoretical frame of reference that informs a study is paramount in delineating what maths teacher identity entails. Quite a substantial number of studies investigating maths teacher identity have been informed by Lave and Wenger's situated theory (Graven, 2002; Hodgen & Askew, 2007, Van Zoest & Bohl, 2008), Bernstein's sociological theory (Johansson, 2010; Lerman, 2012b; Morgan et al., 2002; Parker, 2006) or the post-structuralists (Lerman, 2012a; Zemblays, 2005). Zemblays' (2005) study compliments postructuralism with discourse theory. Similarly Chronaki's paper (2013) relies on both discourse theory and post-structuralism to investigate maths teacher identity. It also relates the personal-social interplay aspect of the discourse theory to Wenger's (1998) three modes of belonging (to communities of practice) namely; engagement, imagination and alignment. Wenger's notion of alignment relates to the macro structures which are influenced and affected by education reforms. However key within discourse theory is how narratives, stories, dialogues or discursive aspects create one's identity. Whilst this is discussed in Chronaki's paper, she however doesn't elaborate on the methodological tools which she uses in her study, how the data presented in this paper was captured, and how frequently this was done and also how the information was analysed. From a discourse perspective, which portrays identity as relational and discursive, these elements are critical aspects that readers need. Because the paper lacks a discussion on how the data was analysed one is left wondering how the two related axes, that is the societal and the pedagogical, discussed in this paper emerged, whether they were theoretically informed or arose from emerging data themes. It will be interesting to engage further with these while at the conference.

SOCIETAL AND PEDAGOGICAL AXIS

The two related axes, however, clearly illuminate how maths teachers articulate their identity changes in the process of appropriating technology to enhance maths teaching. Under the heading **'Articulating the Societal'** the sampled Maths teachers disclose how their involvement with technology for maths learning related to the youth culture and could be regarded as a result of increased marketization of technology use in education. In the section titled **'Articulating the Pedagogic'** the research participants felt that learning technology promoted learner engagement and interaction, enhanced power re-distribution in mathematics classrooms and also enabled the teachers to embrace mathematical knowledge in diverse ways. The issues raised herein, which relate to the two axes, are key and paramount in configuring Chronaki's concept of maths teacher identity. According to Beijaard et al. (2004), teacher identity relates to core teaching aspects of subject matter, didactic and pedagogical expertise. In her discussion of the two axes, Chronaki (2013) reveals how mathematics teachers' engagement in a maths-technology course improved and changed their subject matter, didactic and pedagogical approaches which ultimately impacted and influenced their identity.

THE USE OF THE TERM IDENTITY WORK

Key to Chronaki's argument is how the sampled maths teachers change through engaging in a maths-technological course and in the process articulate and negotiate their identity work within societal, institutional and curriculum reform demands. In Chronaki's work teacher change and identity work are regarded as complex, multifaceted and discursive processes. Educational policies and teacher training programmes that aim for maths teacher identity change should engage teachers in disciplinary knowledge, curriculum reform politics and most importantly improve the maths teachers' instrumental, functional and critical competences in technology (Chronaki, 2013). Chronaki's argument in this regard is quite convincing, however her use of the term identity work is not yet fully clear. The author does not justify her use of the term, neither does she trace its history or origins either in education or maths education. Could the writer have borrowed the term from Hall (1991) or from Mendick (2006) who both prefer the term in reference to being a process of identity formation or identification? In this regard identity is captured as situated, constantly evolving, relational and becomes represented in narratives (Hall, 1991; Mendick, 2006; Walls, 2008). This assertion seems to be close to Chronaki's construct of identity work, however Chronaki's paper doesn't connect her use to any of these.

My second reading and interpretation of the term identity work is that the writer might have been referring instead to work-identity. "Work identity" might be an appropriate term to describe the tension and space that maths teacher negotiate their identity as a result of the interplay between the maths discipline, societal and curriculum reform demands. The term originates from Gee's (2000) work, which Chronaki cites in the paper but fails to make a connection to, and is derived from the

Institution-Identity category which interrelates with both the Discourse and Affinity-identities (Gee, 2000). Gee's (2000) Discourse-identity coheres with the discourse theory which theoretically informs Chronaki's work. The Affinity-identity resonates with Chronaki's empirical field of study which composes of a collective social group of seven maths teachers. Given such coherence the term identity work might better be phrased as work identity.

Having cited Bernstein (2000) and Wenger's (1998) work in her discussion, I believe these writings provide opportunities for Chronaki to fully exploit the concept of maths teacher identity. The paper could have been extended by investigating and discussing how teacher-students power relations manifest in maths technological informed classes through Bernstein's (2000) concept of framing, which alongside classification is a function of pedagogical identity. The need for Bernstein's theoretical lens is mainly justified if one reads the *themes* emerging from Chronaki's axes on: **'Articulating the Pedagogic'**. Similarly Wenger's (1998) dimension of Community of Practice's shared repertoire which include artifacts, tools, discourse and concepts could have provided the analytical and explanatory tools to describe how maths teachers' identity change through their engagement with technological artifacts and tools and mathematical concepts and how the teachers articulate these. I think this is Chronaki's aim in the paper and drawing from Wenger's concept of shared repertoire might have enriched the discussion on maths teacher identity.

CONCLUSION

Besides some ideas I raised in this response, and my wish to engage further and deeper with several issues, Chronaki's study clearly contributes to the growing body of maths teacher education literature that highlights teacher change as part of identity formation, what Chronaki prefers to call identity work. The teacher change results in the transformation of their mathematical knowledge views, their pedagogical approaches and enables them to meet society and institutional demands. Chronaki's paper uniquely discusses how maths teachers in curriculum reform contexts articulate their engagement with technology and within that space negotiate and change their identity. Few studies in maths teacher education have focused on the issues raised by Chronaki and credit must be given for her unique paper and approach of investigating maths teacher identity. The suggestions and recommendations raised in this response will help strength this unique study which discusses maths teacher identity from a different perspective. May I conclude this paper with my sincere wishes for maths education to continue to learn from this work. I hope that this high concentration and explosion in 'identity' will yield viable and sustainable solutions to the challenges in maths education and will ultimately result in more effective teaching and learning of mathematics.

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CONSTRUCTING DESCRIPTIONS AND ANALYSES OF MATHEMATICS CONSTITUTED IN PEDAGOGIC SITUATIONS, WITH PARTICULAR REFERENCE TO AN INSTANCE OF ADDITION OVER THE REALS

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INTRODUCTION[1]

The general problem I am interested in pursuing at present is that of *what* comes to be constituted as mathematics in pedagogic situations, specifically the pedagogic situations that emerge in primary and secondary schools, as well as *how* such constitution is realised. My interest in the *what* and *how* of school mathematics derives chiefly from two sources. The first is my experience of teaching mathematics to high school students, most of whom appeared never to be doing exactly what I was doing, even when they solved problems correctly. The second is the work of Basil Bernstein, who repeatedly and productively employed the relation between what and how in his theorising and analyses of education policy, curriculum, pedagogy and assessment.

In his discussion of the notion of the *pedagogic device*, inspired to some extent by Chomsky's notion of the 'language device', Bernstein reminds us that any instance of school teaching always unfolds within a particular social organisation of the pedagogic context, and that pedagogy is necessarily evaluative. For Bernstein, pedagogic evaluation makes available criteria that mark out what is to count as legitimate knowledge statements in a pedagogic context, and also how such knowledge ought to be realised, in the form of *recognition* and *realisation rules* (Bernstein, 2000, p. 36-39). Bernstein's argument claiming that pedagogic evaluation is implicated in what comes to be realised as knowledge in pedagogic situations seems eminently reasonable, even if he's not all that clear on how evaluation does its work. So, when we turn our attention to the production of analytic descriptions of mathematics education as it pertains to schooling – as is the case in conducting mathematics education research – and to the training of mathematics teachers, it seems appropriate that we take Bernstein's propositions about pedagogic evaluation into account when considering the constitution of mathematics in pedagogic situations.

One practical problem that suggests itself immediately is that of what to focus on when considering pedagogic evaluation in the context of mathematics teaching with the aim of constructing analytic descriptions of the constitution of mathematics in pedagogic situations. What I propose to do in this paper is consider the issue of the construction of descriptions and analyses of mathematics teaching and learning, attuned to the productivity of pedagogic evaluation in what comes to be constituted as mathematics. I will develop my argument through the discussion of an instance of interactions between a teacher and his students on addition over the real numbers.

GENERAL METHODOLOGICAL ORIENTATION

We might describe schooling as a context in which an encounter between various fields of knowledge and learners are staged, with pedagogic evaluation as the mechanism mediating the field of knowledge-learner encounter. Pedagogic evaluation marks out what are to be taken as legitimate student responses to the recurrent demands on them to produce knowledge statements in pedagogic situations. What pedagogic evaluation thus inserts into the pedagogic situation is a demand for and assessment of, what *ought* to be the content of teacher and learner activity. In other words, the staging of an encounter between the student and a field of knowledge, like mathematics, necessarily produces a moralising of the student (and the teacher).

The *ought* of the pedagogic situation is, however, always internally split between some idea, or expectation, of specific pedagogic identity on the one hand, and particular realisations of content, on the other. In one sense, what the *ought* of pedagogic evaluation *proposes* is a correlation of a pedagogic identity and a particular realisation of content. I'll refer to this, exploiting Lacan (2006), as participating in the Imaginary dimension of pedagogic discourse. However, when we examine mathematical activity a case can be made for finding there the *production* of pedagogic identities that are not necessarily congruent with that which is proposed Imaginarily. Using Lacan again, I shall refer to the correlation of identity and content derived from mathematical activity as participating in the Symbolic dimension of pedagogic discourse. Bernstein (2000, p. 32) announces a proposition on this specific feature of what he terms *pedagogic discourse*, in a somewhat clumsy way, as the embedding of an instructional discourse in a moral discourse, where the latter is a discourse of social order and is dominant. Dowling (2009, p. 81-83) has spelt out some of the problems with Bernstein's formulation of the proposition, so I won't revisit those here.

The Lacanian distinction between the Imaginary and the Symbolic is part of a triadic relation between the categories of the Real, the Symbolic and the Imaginary, which are Lacan's names for his three *registers*, or *orders*. The Real is that which is presumed to exist outside of the order of language, meaning and the law; that is, outside of symbolic relations. The latter constitutes the Symbolic order. For Lacan, one way to get a purchase on the notion of the Real is to conceive of it as that which announces itself at the points of failure of the Symbolic. In other words, we can glimpse the Real from within the Symbolic as some or other disturbance. In Bernstein's work, we might say that one place at which the Real announces itself is in the space of the "yet-to-be-thought" that is the *discursive gap* of the pedagogic device (cf. Bernstein, 2000). Knowledge production might be understood in a Lacanian sense as a continuous attempt to symbolise the Real as it incessantly reappears as a marker of the limits of knowledge.

The Imaginary emerges from the relations that constitute the subject's identity and is bound up with the notion of the ego and its representation of the subject as a unique, unified individual. The term *imaginary* in Lacanian psychoanalysis refers directly to the productivity of the image in the constitution of the ego. The initial images of the subject within familial relations and broader social relations produce a sedimentation of ideal images constituting the ego. The Imaginary order is therefore the order in which social relations are focused on the image as it pertains to the ego; in other words, imaginary relations are relations between egos, differing from symbolic relations.

In pedagogic situations, one point at which the Real irrupts, disturbing the smooth functioning of the Symbolic is, precisely, in the encounter between the student/teacher and the field of knowledge: to the extent that the student/teacher functions as a point of resistance to reproduction of knowledge (whether deliberate or not is irrelevant), we have a point at which the Real disrupts the Symbolic.

The Real-Symbolic-Imaginary distinction enables me to talk about the regulative aspect of pedagogic discourse in a much more nuanced way than the do the resources provided by Bernstein. His notion of *classification* has been used to pick out certain regulative features of pedagogic discourse but, to my mind, most of those uses fail to deal adequately with the problem of the lack of reference in language, more of which I'll discuss later.

Žižek (2002a, p. xii) argues that the three Lacanian registers can be thought of in terms of each other, generating a fractal-like structure in which the whole triad of Real-Symbolic-Imaginary is taken to inhere in each element of the triad. This is helpful because it enables more precise descriptions of the sorts of statements and activities that we encounter in the educational contexts related to policy, curriculum and pedagogy. By using the extended set of relations between the three orders as proposed by Žižek I can begin to describe phenomena 'under the aspect' of either the Real, or the Imaginary, or the Symbolic. This is especially pertinent for us because the education system is primarily concerned with the Symbolic. The bureaucratic organisation of education, with its explicitly codified descriptions of agents, knowledge, resources and their inter-relations, all bound together in a massive system dedicated to the production of minute measurements and a consequent, differential distribution of rewards, is the quintessential Symbolic machine. However, the way in which education policy, curriculum and pedagogy respond to the student *qua* Real can have the internal mechanisms of the great educational Symbolic machine chug away under the aspect of the Imaginary, or the Symbolic. For example, in pedagogic situations today a point at which the Imaginary is evident is in the appeals to the individual pleasures of the student and in appeals to the idea of *relevance* when it is used as a means for the student to find representations of themselves in the fields of knowledge that they encounter. Such strategies are clearly exhibited in many contemporary school mathematics textbooks. The point is that, differently from responses that emerge under the aspect of the Symbolic, one way in which the student *qua* embodiment of the Real might oftentimes be dealt with in pedagogy, curriculum, policy and research, is under the aspect of the Imaginary.

Now, notwithstanding the fact that Noam Chomsky described Lacan as "conscious charlatan," who was "simply playing games with the Paris intellectual community to see how much absurdity he could produce and still be taken seriously" (Chomsky, Wilson, Rée, Osborne, & Edgley, 1989, p. 32), I'll take the perverse step of recruiting Chomsky's (1965, 1966) discussion of *adequacy* in empirical research to supplement the Lacanian and Bernsteinian ideas I've drawn on. And, despite Žižek's dismissive description of Chomsky as being too preoccupied with "facts" that tell us nothing that we did not already know about the political terrain (Žižek, 2002b, p. 77-78), Chomsky's concern with "facts" in linguistics is an exemplary instance of an ongoing series of attempts to "symbolise the Real" of language as he goes about pursuing the study of language framed as *biolinguistics*. If, for Žižek, Chomsky should be dismissed as a dupe who actually believes that he can touch the Real with his "facts", then we should invite Žižek to recall his recurrent use of the Lacanian aphorism, "only the non-duped err!" and ask: What is one doing in an attempt to "symbolise the Real" if not duping oneself into proceeding as though one can, in fact, touch the Real?

Chomsky marks out three levels of adequacy as necessary in empirical research: (1) *observational adequacy*, (2) *descriptive adequacy*, and (3) *explanatory adequacy* (Boeckx, 2006). In his later work Chomsky tends to refer to only descriptive and explanatory adequacy, appearing to take it for granted that observational adequacy will be attended to as a matter of course in the attempt to realise descriptive adequacy. Chomsky's attention to adequacy in observation, description and explanation is, precisely, a means by which he both anticipates and holds on to the disruption of his symbolising of the Real. In fact, he is quite explicit on the necessary limits to our understandings of the world (e.g., Chomsky, 2000).

Paraphrasing Chomsky (1966), and with reference to the teaching and learning of mathematics in pedagogic situations, observational adequacy is the lowest level of adequacy and is realised when our descriptive and analytic resources can generate descriptions of observed primary operational activity – data made available to students – that capture the specifics and range of apparent operational resources that emerge in pedagogic situations. What this means in practice is that one is obliged to suspend one's content expectations when describing mathematics as constituted in pedagogic situations. Rather, one attempts to get at the operational specificities of the work of teachers and students and one does not ignore, or dismiss, operational features that do not accord with what one expects of the content one associates with a topic, or which seem to one not to be 'proper' mathematics. For example, much of what emerges in pedagogic situations is often dismissed by mathematics educators as mere ‘manipulation’, as opposed to ‘conceptual’ work, yet such ‘manipulations’ are often the chief resources used to do mathematical work in very many pedagogic situations, filling out the contents associated with a topic, and are also the mathematical stuff of thought for many students and their teachers. We shall see later that much of what is referred to as ‘manipulation’ participates in an auxiliary system of operational resources that emerge alongside and in interaction with the operational resources usually recognised as ‘proper’ mathematics.

In this paper my main focus will be a rather modest concern with the production of observationally adequate descriptions of the constitution of mathematics in pedagogic situations, so I’ll forgo discussing descriptive and explanatory adequacy. The interested reader can consult Boeckx (2006) for a clear and concise discussion of Chomsky’s ideas on adequacy.

Mathematical considerations

Whatever else mathematical activity is, I believe that it’s not tendentious to claim that school mathematics is realised, in large measure, as compositions of operations. Compositions of operations are, of course, always regulated by higher level propositions and decision-making, but compositions of operations make up the stuff that is explained and written down in detail by teachers and their students. For the mathematics education researcher, students and teachers, ‘doing mathematics’ is most visibly registered in their scriptural activity (verbal, written and gestural), largely devoted to the composition of operations, and so it is there that I focus on the empirical as I seek to produce observationally adequate descriptions of the constitution of mathematics in pedagogic situations.

To realise observationally adequate data I will focus on the computational activity of teachers and students as registered at three inter-related levels, viz., levels of (1) expression, (2) syntax, and (3) semantics. Here I take some direction from Chomsky (2006, p. 111) to organise my analysis. The level of expression contains information relevant to the interpretation of expressive/lexical items (spoken, written, gestural); the level of semantics, information relevant to semantic interpretation; and the level of syntax contains information that associates expressive elements with semantic elements, thereby relating interpretations of expressions/lexical items to semantic interpretations. Studying the three levels of data demands an integrated approach. The most immediately accessible level is the expressive level; the least, the semantic level. The latter requires one to work on the data generated at the level of expression to construct an account of the syntax, from which semantic data can begin to be constructed. Given that data concerning syntax is crucial because it is the glue that binds data at the three levels into a coherent story, and the syntactic elements of great importance to my general question are those that feature in the composition of operations, it is the matter of what the operations and associated domains and codomains are that is primary to my endeavour.

The operations that populate mathematics are functions (Open University, 1970). An operation, *, is defined in general terms as a function of the form $*:D_1 \times D_2 \times \ldots \times D_k \rightarrow C$, where the sets D_j are the collections that make up the domain of the operation, the set C is the codomain of the operation; the fixed non-negative integer k, which indicates the number of arguments, is the arity of the operation. For any operation, described as a function, its elements are of the form $((d_1, d_2, \ldots, d_k),c)$ and, gathered together, constitute a subset of the cross product $(D_1 \times D_2 \times \ldots \times D_k) \times C$. In other words, the operation might be considered a particular subset of $(D_1 \times D_2 \times \ldots \times D_k) \times C$, consisting in a set of elements of the form $((d_1, d_2, \ldots, d_k),c)$. Since there is no essential difference between $((d_1, d_2, \ldots, d_k),c)$ and $(d_1, d_2, \ldots, d_k, c)$ we can use the latter expression to indicate an element of an operation. The usual basic arithmetic operations – addition, multiplication, division and subtraction –are binary, and are usually defined as functions of the form $*:A \times A \rightarrow A$. However, since I am interested in operations as well as *operation-like manipulations* including, but also those additional to, the basic arithmetic operations, I will use the more general definition of an operation.

Just as with functions in general, it is possible to replace an operation – which is the process by means of which domain and codomain elements are associated – by a rule that is composed of more than one operation but which, nevertheless, produces the same $c \in C$ for a given $(d_1, d_2, \ldots, d_k) \in D_1 \times D_2 \times \ldots \times D_k$, as does the original operation. In fact, that is precisely what often happens in the pedagogic situations of schooling. It is also not unusual to find alternate operations, or even operation-like manipulations, replacing the operations indicated by mathematical statements in the pedagogic situations of schooling. However, it is not always the case that the manipulations introduced by teachers and/or their learners can function as operations over the same domains as the operations being replaced. See Linchevski and Sfard (1991), Ma (1999), Sfard (2007, 2008), Lima and Tall (2008) and Tall (2008) for examples of instances where teachers and/or learners use alternative or operation-like manipulations.

When analysing pedagogic situations it is productive for one to adopt a stance that accepts whatever emerges in the operational unfolding of school mathematics as participating in the constitution of mathematics in the situation, so that even operations/manipulations that are not recognisable to the researcher as 'mathematical', are acknowledged and described in operational terms. Not to do so would be a case of reading the constitution of mathematics under the aspect of the Imaginary, where the image of mathematics (probably as described by the curriculum and in school texts) is taken as Mathematics. Striving for observational adequacy demands that one find a way to generate data not bound to beliefs of what ought to be the case in pedagogic situations, and focussing on the operational activity of teachers and students seems to be a reasonable way to approach such situations. I'll need to introduce a few more mathematical ideas, but I'll do so at the appropriate places in the text.

Pedagogic situations and evaluation

While Bernstein's (2000, p. 34) intuition that "regulative discourse is the dominant discourse", and that there "is no instructional discourse that is not dominated by the regulative discourse" is, broadly speaking, on the right track, he does seem to miss that there are good reasons to believe that the "recognition and realisation rules" that inform and inhere in pedagogic evaluation cannot easily generate stable criteria across participants for at least two reasons. One reason derives from the nature of knowledge contents, which is mathematics in this instance; the other, from the nature of language. We do need to ask if such "rules" can have any substantial existence, but that'll take us into a distracting extended discussion of Bernstein, so I'll leave that for another time.

As is already implied by the relation between a function and its 'rules', an insistent complication that inheres in pedagogic situations is that the criteria used by teachers and their students, and across students, need not be congruent, even when all pedagogic agents realise the same outcome from some or other point of view (like that of a marker of students' examination scripts). Substituting one rule by a different, equivalent rule, or an equivalent system of rules, is central to generating different realisations of content in pedagogic situations.

Contributing to the problem of the lack of congruence of criteria is the obvious fact that pedagogic situations are communicative situations (Bernstein, 2000), which means that pedagogy cannot avoid the consequences of the property of language that makes it clear that there is no one-to-one relation between lexical items – words and symbols – and things in the world. Mathematical entities *are* given definite descriptions and related in systems of strict denotations, but once natural language is the general medium of communication we run up against the problem of reference: it is *people* who use language to refer, and they can and do use language to refer to the world in ways that are unpredictable and novel, even when they are apparently communicating in concert with their interlocutors (Strawson, 1950).

Communication is always a more or less affair, depending for its success on the presence of a range of shared ways of looking at the world, knowledge of contextual conventions and shared uses of language. This feature of language is something that exercised Descartes, prompting him to ask how it is that we are able to use language in ways appropriate to context but in no way determined by context, and in response to which he finally conceded that the answer to that question is beyond creatures as intellectually limited as ourselves (Descartes, 2000; Chomsky, 2000). This is also one reason that gives force to the idea that pedagogy is necessarily evaluative since the creative use of language obliges us to eschew all dreams of pedagogic communication that has a priori determined effects on the learning of students.

Consequently, constant evaluation is needed to check that students and their teachers are more or less on the same page.

Content substitutions

So, criteria that emerge in pedagogic situations need not be congruent with specific content-related criteria privileged in the field of the production of mathematics and may at times even be considered to be questionable by some despite being promoted by education authorities (like teachers, curriculum designers and writers, policy makers). In fact, the particular criteria that emerge in pedagogic situations are intimately implicated in what I shall refer to as *content substitutions*. Such substitutions are instances of the replacement of content associated with a topic by different content for various reasons, which could be one or a combination of pedagogical reasons, or be derived from policy/curriculum prescriptions, or even be effects of the knowledge limitations/preferences of teachers. When one examines pedagogic situations with the purpose of asking what the mathematics contents are that come to be constituted under the name of a particular topic, one finds that content substitutions are routinely generated by criteria for the production of school mathematics (Jaffer, 2012). If it is the case that content substitutions are commonplace, then we ought to take such substitutions into account in our descriptions and analyses of the teaching and learning of school mathematics and to ask what the effects of such substitutions might be.

WHAT COMES TO BE CONSTITUTED AS MATHEMATICS?

A grade 10 teacher and his students are working on what they refer to as 'number patterns'. The teacher explains to his charges that they can think of a number pattern as generated by a rule, expressible as a formula, by which each number is determined, starting from the first number in the list -7, -2, 3, 8, … . The trick, he informs them, is to calculate the difference between successive numbers, that difference being common, and then to use the common difference to generate additional successive terms of the sequence/'pattern'. The lesson apparently proceeds smoothly up to the point where the teacher unexpectedly finds himself in confrontation with a destabilising, class-wide, irruption of what appears to be a misapplication of elementary arithmetic computational rules – rules that his students had been using since grade 8.

Teacher: Right! Now let's investigate this linear pattern. [Referring to the list -7, -2, 3, 8, … .] What is the difference between minus seven and minus two?

Students: [Chorus.] Five!

Teacher: Is it plus or minus five?

Students: [Chorus.] Minus five!

Students: [A couple of students.] Plus five.

Students: [Chorus.] Negative five!

Teacher: [Surprised.] Hey?!

Students: [Chorus.] Negative five!

Teacher: [Deliberately.] The question is, what do you do to negative seven in order for it to become minus two? Do you add five? Or do you subtract five?

Students: [Chorus.] You subtract five!

Teacher: You subtract five?

Students: [Chorus.] Yes!

Teacher: Alright. Let's explore this. We've got minus seven, and from that minus seven you subtract five. [The teacher writes the expression "-7 – 5" on the chalkboard as he speaks.] So what is the answer?

Students: [Chorus.] Minus two!

Teacher: Minus two. [The teacher completes the expression according to the students' claim, writing "-7 – 5 = -2".] <u>Nê?</u> {Right?}[2]

Students: [Chorus.] Yes!

Teacher: That's what you are telling me? <u>Nê?</u> {Right?} [Exasperated.] Minus seven, and then we subtract five. Then what do we get? [Sardonic.] We get minus two.

Student: No!

Students: [Chorus.] Minus three! [Different students call out.] No! Yes! No! Yes!

Teacher: [Silent.]

What unfolds in the scene is initially confusing. We see that most of the students very confidently assert that 5 is to be subtracted from -7 to produce -2, and that, for the most part, the class willingly agrees with the false statement: "-7 – 5 = -2". We note that the teacher's questions are answered in chorus, and that the responses of those few individuals who evidently think differently from the claque are, if not entirely drowned out, certainly not explicitly attended to.

The discussion between the teacher and his class continues thus:

Student: We subtract.

Teacher: We subtract what?

Student: We subtract two from seven.

Teacher: We subtract two from seven? Yes?

Students: [Chorus.] From seven! Yes!

Teacher: <u>Remember, the question is: ukuze sibe no-minus two senza ntoni ku-seven?</u> {Remember, the question is: in order for us to have minus two what are we doing to seven?} What have we done?

Students: [Chorus.] We subtract!

Teacher: Si-subtract bani? {What do we subtract?}

Students: [Chorus.] Five!

Teacher: This is what you are saying. [Pointing at the expression "-7 – 5 = -2" displayed on the chalkboard once again.] Nê? {Right?}

Students: [Chorus.] Yes!

How is it that the chorus chooses to pick out subtraction from seven in the speech of the student and fails to remark on the fact that he claimed that two was to be subtracted from seven when the teacher questions his calculation? Why do the students appear to ignore the teacher's apparent error when he asks them what is to be done to seven (rather than to negative seven) to produce negative two? How is it that most of the students appear to remain obstinately blind to fellow students' explicit deviations from the false computation, "-7 – 5 = -2"?

After some additional, but infuriatingly unsuccessful discussion of the required computation, the exasperated teacher reminded his students of a procedure for adding real numbers that they ought to be knowledgeable of, the significant elements of which are displayed in the following extract:

Teacher: So if the signs are the same, what do you do? You take the common sign. And then you add. If the signs are not the same, what do you do? You subtract.

Students: [Chorus.] Subtract!

Teacher: But first you take the sign of the what? The sign of the bigger number. You look at the bigger number between the two. And then you take the sign of the bigger number.

Students: [Chorus.] Yes!

Teacher: This should always be the case.

How is it that, despite the teacher's repeated questioning of them, the bulk of the students continue to confidently accept an incorrect calculation? Why do those few individuals who suggest alternatives to the solutions supported by the chorus apparently remain ignored by both the mass of their peers as well as by the teacher? How does it happen that students in their third year of high school are failing at performing elementary arithmetic calculations? These are a few of the questions that I am interested in pursuing, but to do so I have to develop descriptive and analytic resources for reading the activity that unfolds in mathematics classrooms in a systematic manner. I will not attempt to answer all the questions posed in these opening remarks because to do so requires fairly extensive work, well beyond the scope of a paper such as this. What I will focus on is what I consider to be primary to the task at hand.

I will spend some time working through the various syntactic features of the mathematics referenced in the extracts in some detail.

AN ANALYSIS OF THE TEACHER'S PROCEDURE FOR ADDITION

Recall the teacher's description of his procedure for adding pairs of real numbers: if the numbers have different signs, subtract the smaller number from the larger and use the sign of larger number as the sign in the answer; if the signs are the same, add the two numbers and use the common sign as the sign in the answer. This procedure for adding real numbers is similar to that used by other mathematics teachers at the school and at many other schools around South Africa, and even elsewhere (e.g., Sfard & Avigail, 2006; Sfard, 2007; Stephan & Akyuz, 2012). Sfard and Avigail (2006, p. 18) and Sfard (2007, p. 586), for example, describe a pedagogic situation in which some of the students studying the multiplication of real numbers use what Avigail and Sfard refer to as the *discursive template*

$$(+a)+(-b)=\begin{cases} |a-b| \text{ if } a>b \\ -|a-b| \text{ if } a\leq b \end{cases}$$, in which a and b are unsigned

as a model for a discursive template for performing computations of the type $(+a)\times(-b)$.

This discursive template for addition referred to by Avigail and Sfard is similar to elements of the procedure used by many South African teachers. For Avigail and Sfard the value of $(-7)+(+5)$ would be calculated by computing $-|5-7|=-|-2|=-2$, using their discursive template for addition: $(-7)+(+5)$ would be rendered as $(+5)+(-7)$, and since $5 \leq 7$, we use $-|5-7|$ to compute the required result. There's something not quite right about this description if it is taken to be a representation of the computational activity of students. First, if students could perform the computation $-|5-7|$ directly, there would be no need for them to use the discursive template described by Avigail and Sfard. However, if we read the expression $|a-b|$ as merely indicating something like the distance between a and b on a number line, then the description presented by the discursive template appears to be more reasonable.

Second, the implicit domain of computation for the students was probably the non-negative reals, or even the natural numbers, as is clearly recognised by Sfard (2007, p. 584), so that the *computation* $-|a-b|$, with $a<b$, would not feature for the students who are described as using the discursive template for addition. In fact, we can rewrite the discursive template in a way that might be more representative of what Avigail's and Sfard's research subjects were doing:

$$(+a)+(-b)=\begin{cases} a-b \text{ if } a>b \\ -(b-a) \text{ if } a\le b \end{cases}, \ a \text{ and } b \text{ unsigned}.$$

The difference between the two descriptions of the postulated discursive template used for addition draws attention to the issue of observational adequacy. The description of the template provided by Avigail and Sfard register the outcome of the students' computations with perfect fidelity but not the details of their computational activity. The redescription of the discursive template in a manner aligned with the domain of operation as being $\Re^{+}_{\circ}$ or N appears to be a more adequate description.

Third, there remains a question of whether the negative number outcome registered by the computation $-|a-b|$, where $0<a<b$ in the discursive template is obtained by some calculation, like $-(b-a)$, where $0<a<b$. Or, is the minus sign merely appended to the computation $b-a$, where $0<a<b$, in some way? The data provided by Avigail and Sfard indicate nowhere that $|a-b|$ (or $b-a$) is subjected to either the logical operation of negation or to the arithmetic operation of multiplication by -1, or even to any other familiar arithmetic or logical operation. So how is the result $-|a-b|$ obtained by the students? I believe that pursuing this question brings to light a domain of operation that is routinely used in school mathematics but which is not explicitly recognised by mathematics education researchers as something substantial, perhaps because it is nowhere registered in the axioms for the field of real numbers.

Getting back to our teacher and his students, if I use the criteria indicated by the procedure in the manner intended by the teacher, then the computation $-7+5$ is to be performed as follows: the signs associated with 7 and 5 are different, and 7 is bigger than 5, so I must calculate $7-5=2$; now place the sign associated with 7 in front of the answer, 2, to produce -2, because 7 is the larger of 7 and 5, and the sign of 7 is "-". The calculation $-7+(-5)$ would be realised by reasoning that, since the signs associated with 7 and 5 are the same, we calculate 7 + 5 = 12 and then attach the 'common sign', viz. "-", to 12 to produce -12 as the solution. To see that there is a shift in domain from the reals to the non-negative reals in the teacher's procedure we need merely note that the procedure is unintelligible if we do not take his use of the word 'number' to be a reference to the 'positive numbers': in the expression '-7 + 5' the biggest number is 5, so that subtracting the smaller from the bigger number and attaching the sign of the bigger number to the answer produces 5 – (-7) = +12, which is clearly not what is intended.

An object of the type $(A,*)$, where $*$ is an operation and A is a collection of objects that serves as the domain for $*$, will be referred to as a *structure*. While we would always want it to be the case that $*$ is closed over A, one occasionally has to suspend the closure requirement while trying to construct adequate descriptions of the elaboration of mathematics in pedagogic contexts. Recall that an *operation* is a function from some domain of operation to some codomain. The operations of elementary arithmetic are usually thought of as defined in a manner that has its domain and codomain being populated by the same collection of objects. For example, addition over the reals, $(\Re,+)$, is a binary operation that maps $\Re\times\Re$ to $\Re$, from which we can see that the elements populating the domain and the codomain are taken from the same collection, $\Re$. Addition over the reals is a subset of $\Re\times\Re\times\Re$, the cross product of its domain and codomain. More generally, an operation can be thought of as a subset of the cross product of its domain and codomain, precisely as one does for any function. Each structure of the type I am referring to here can be satisfactorily described in a minimal way. For example, the minimal description of the structure $(\Re,+)$ is simply constituted by the collection of axioms that describe the properties of addition over the reals:

(1) $\forall a,b\in\Re,\ \exists c\in\Re \big| a+b=c$;

(2) $\forall a,b\in\Re,\ a+b=b+a$;

(3) $\forall a,b,c\in\Re \big| a+(b+c)=(a+b)+c$;

(4) $\exists 0\in\Re \big| \forall a\in\Re,\ a+0=a$;

(5) $\forall a\in\Re,\ \exists(-a)\in\Re \big| a+(-a)=0$.

The properties of $(\Re,+)$ are not explicitly referred to in the procedure promoted by the teacher. The series of intermediate calculations performed in the teacher's calculation, -7 + 5 = -2 entails a shift from $(\Re,+)$ to $(\Re^+_\circ,-)$ and then back to $(\Re,+)$. That is, for part of the procedure the calculation shifts from addition over the reals to subtraction over the non-negative reals.

$(\Re^+_\circ,-)$ has the following operatory properties:

(1) $\forall a,b\in\Re^+_\circ,\ a-b\in\Re^+_\circ$ only if $a\geq b$;

(2) $\forall a,b,c\in\Re^+_\circ,\ (a-b)-c\neq a-(b-c)$;

(3) $\forall a\in\Re^+_\circ,\ a-0=a$, but $(0-a)\notin\Re^+_\circ$ if $a\neq 0$;

(4) $\forall a\in\Re^+_\circ,\ a-a=0$;

(5) $\forall a,b\in\Re^+_\circ$, unless $a=b,\ a-b\neq b-a$.

$(\Re^+_\circ,-)$ is very familiar to students from their elementary schooling, but it is rather impoverished when compared with $(\Re,+)$. $(\Re^+_\circ,-)$ is non-associative and non-commutative, it requires the imposition of a special condition to ensure closure, and it does not have distinct inverses. $\Re^+_\circ$ lacks the nice symmetry of $\Re$ that facilitates the ease with which the required calculation can be performed in $(\Re,+)$.

The expression '-7 + 5' appears to be treated in a manner enabling the teacher and his students to select and operate directly on smaller strings of symbols. That is, rather than focus attention on the expression '-7 + 5' as a statement of the relations between the fundamental mathematical objects to which it refers, the procedure requires the calculator to consider '-7 + 5' as a string of symbols, or characters, variously available for direct manipulation. Next, in order to derive elements of $\Re^+_\circ$ from the real numbers that are given in a sum, the signs and numerals are split apart and treated separately. Then, in order to select the correct sign to attach to the result of the calculation in $\Re^+_\circ$ the calculator needs to be able to recover the sign of the original integer associated with the number considered the "biggest number" in the calculation in $\Re^+_\circ$. Finally, producing the required answer to the integer calculation requires manipulations that produce a concatenation of the sign and the answer obtained from the 'counting number' calculation. These features of the procedure are summarised in Figure 1.

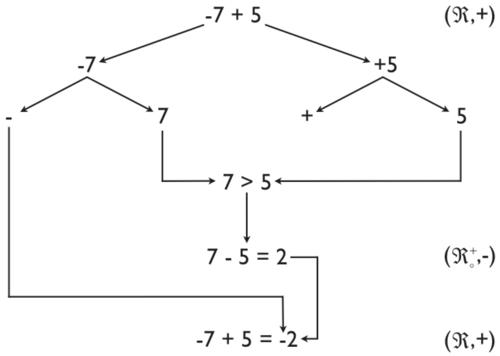

Figure 1: A schematic description of features of the teacher's procedure for performing the computation -7 + 5 = -2.

A number of the operations and operation-like manipulations just described are not found among the operatory properties of $(\Re,+)$ and $(\Re^+_\circ,-)$, yet it is clear that such operations and operation-like manipulations are central constituents of the procedure for adding real numbers. For example, separating "+" and "-" signs from numerals is something that can be done to strings of alphanumeric symbols, but we cannot change a real number from negative to positive, or vice versa, by altering their signs. A real number is just whatever it is. Most of the auxiliary operations and operation-like manipulations required by the teacher's procedure used to add real numbers can be considered as being string operations, like the string operations used in computer programming languages. String operations take strings of alphanumeric symbols as arguments and values, and can be thought of as different from the use of symbols as a means for describing collections of objects and related operations indexed by the symbols. The need to define the auxiliary operations and operation-like manipulations as various string operations is motivated by the observation that the mathematical activity of the teacher and his students is such that it includes operating directly on the alphanumeric characters that populate expressions, rather than treating such expressions as lexical records of mathematical work that is not quite the same thing as those records. In fact, an examination of the language used by teachers and students to refer to collections of objects and related operations is very revealing of the ways in which they think of alphanumeric characters as objects that can be moved about and displaced spatially, and of operations as particular kinds of spatial displacements (cf. Linchevski & Sfard, 1991; Lima & Tall, 2008).

In order to construct observationally adequate descriptions of the mathematical work of the teacher and his students, and to do so in a way that is oriented toward the Symbolic rather than the Imaginary, I choose to specify the details of the operations and operation-like manipulations used in much finer detail, and include in my description those processes that can't be directly accounted for by appealing to the properties of the arithmetic operations defined over the reals.

I indicate each of the operations/operation-like manipulations by symbols consisting of three capitalised letters followed immediately by input markers listed between a pair of brackets. I won't bother to define addition and subtraction over the reals here. The input and output collections of objects of each of the operations/operation-like manipulations are indicated in the descriptions that accompany them.

(1) STR(μ) returns an alphanumeric string, /μ/, derived from an expression, μ. For example, the word "dog" can be considered as a sequence of letters (alphanumeric characters) "d", "o", "g". Such strings are indicated here as enclosed in a pair of forward slashes: /μ/. STR(-7 + 5) returns the alphanumeric string /-7 + 5/. The spaces between the characters listed here would be recognised as alphanumeric characters in certain other contexts, like computer programming, but they are excluded as characters here. Once an expression is rendered as a character string, each of the individual characters, or combinations of them, are available to operations or operation-like manipulations that can take them as arguments or values.

(2) SUN(/λ/) sunders an alphanumeric string, /λ/, into a list of two or more alphanumeric strings (/λ_1/,…,/λ_n/), $n \in \mathrm{N}$, $n \geq 2$. So, SUN(/-7/) returns the list of alphanumeric strings (/-/,/7/), while SUN(/-7 + 5/) could return the list (/-/,/7 + 5/), or the list (/-/,/7/,/+ 5/), or even the list (/-/,/7/,/+/,/5/), or any other combination of alphanumeric strings derivable from /-7 + 5/. Clearly the result of SUN(/λ/) is not unique, its output being contingent on the decision of the

agent effecting the sundering.

(3) NUM(/λ/,A), where /λ/ is a alphanumeric string and $A \subseteq \Re$ returns the value $\lambda \in A$. This restricts the composition of /λ/ to concatenations of certain combinations of elements of, at least, the list

$$\{/\text{-}/,/+/,/,/0/,/1/,/2/,/3/,/4/,/5/,/6/,/7/,/8/,/9/,/,/./,/\cdot/\}.$$

If additional alphanumeric characters are needed to generate elements of $\Re$ they can be included in the list.

(4) MAX($x_1,\ldots,x_n$) and MIN($x_1,\ldots,x_n$), respectively, return the largest and smallest of a list of real numbers. MAX(7,5) = 7 and MIN(7,5) = 5, for example. If we have $x_1 = \ldots = x_n$, then MAX($x_1,\ldots,x_n$) = MIN($x_1,\ldots,x_n$).

(5) CON(/λ_1/, … ,/λ_n/) returns the concatenation of a list of strings (/λ_1/, … ,/λ_n/) to produce the alphanumeric string /$\lambda_1\lambda_2\ldots\lambda_n$/. For example, CON(/-/,/5/) returns /-5/.

STR and NUM change the type of object being dealt with but maintain the expression in use. This is possible because expressions can be used by teachers and students to refer in different ways. As we saw, STR activates the collection of alphanumeric objects, which I shall refer to as the set X. SUN and CON are defined over X, taking elements of X as arguments and as values. NUM takes certain strings as arguments and activates some or other subset of the reals. MAX and MIN are used to decide on the correct order of the arguments for subtraction over $\Re^+_\circ$.

I can now redescribe the teacher's procedure diagrammatically (see Figure 2), in a way that shows the composition of the operations and operation-like manipulations along with the domains and codomains that are activated as the procedure is worked through. I've chosen to indicate $\Re$ and $\Re^+_\circ$ as separate spaces in Figure 2 in order to render the details more readable.

Appealing to the computational details described in Figure 2, I argue that the teacher's procedure for adding real numbers having different signs achieves the required result indirectly, by using operations and operation-like manipulations over X and $\Re^+_\circ$ rather than using only operations defined over $\Re$. A similar description can be produced for the case where the real numbers have the same signs.

The idea that many teachers and students confronted with operations over $\Re$ actually employ operations over X and $\Re^+_\circ$ is not implausible. In fact, it may well be the case that arithmetic statements that include so-called 'signed numbers' are immediately read as character strings from which elements of $\Re^+_\circ$ are to be extracted before the usual arithmetic operations can be employed. Here we might have the interesting result that addition over the reals, while a topic for study implied by the curriculum for school mathematics, is studied indirectly and in a very limited way.

Content substitutions and compatibility

It would seem that some of the content we might commonly associate with the topic of addition over the reals has been substituted in the teacher's procedure, and the question of how it is possible for such substitutions to work productively for teachers and their students now emerges. The general issue can be framed as one of the *preservation of the effects of structure* as teachers go about elaborating mathematics in a manner that has them replacing one bit of content with another, as in the case of the particular treatment of the addition of real numbers referred to (cf. Adámek, 1983). Such substitutions of structures can be conceived of as *mappings* from one structure to another, and we can exploit the notion of a *morphism* to describe such mappings (Krause, 1969; Open University, 1970; Baker, Bruckheimer & Flegg, 1971).

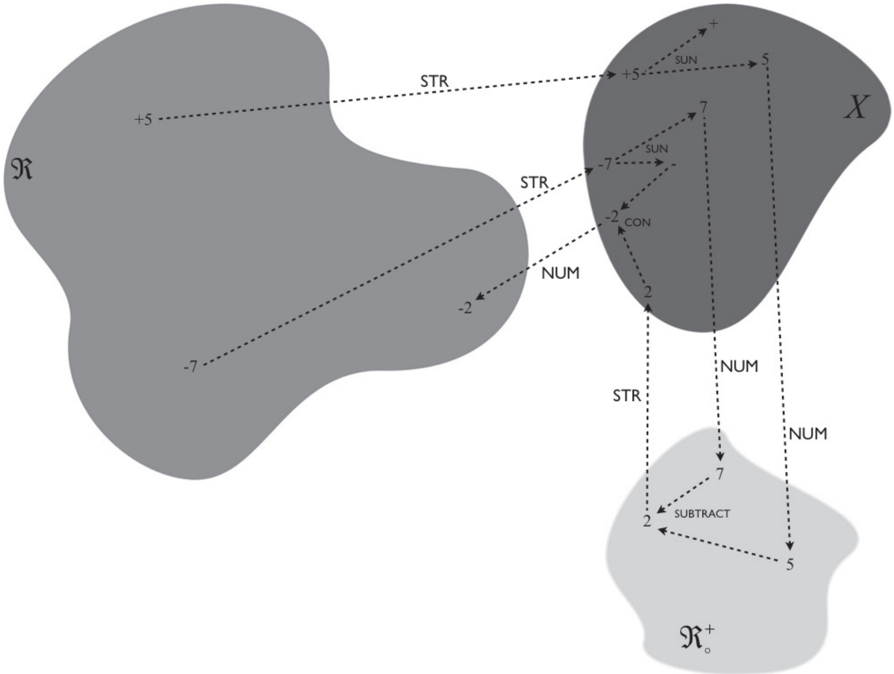

Figure 2: A map of some of the operations that emerge in the teacher's procedure for the computation: -7 + 5 = -2.

Suppose that we have two structures, $(A,*)$ and $(B,\circ)$, as well as a mapping, *f*, that takes $(A,*)$ to $(B,\circ)$. If $\forall a_1, a_2 \in A$, there exist elements in $B, f(a_1), f(a_2) \in B$ such that $f(a_1 * a_2) = f(a_1) \circ f(a_2)$, then we will say that $f:(A,*) \to (B,\circ)$ describes a morphism that maps $(A,*)$ to $(B,\circ)$. The pertinent relations of the morphism are illustrated diagrammatically in the external diagram shown in Figure 3. In Figure 3 the computation $*:(a_1,a_2)\to a_1*a_2$ is an effect of the structure $(A,*)$, while $\circ:(f(a_1),f(a_2))\to f(a_1)\circ f(a_2)$ is an effect of $(B,\circ)$. For the relation between $(A,*)$ and $(B,\circ)$ as mediated by f to be a morphism, we must have $f(a_1*a_2)=f(a_1)\circ f(a_2)$. Looking at the diagram, we can see that when we have a morphism we can get from the top-left to the bottom-right along two paths: starting with the pair (a_1,a_2), we can first apply the operation $*$ and then *f*; alternately, we apply f followed by the operation $\circ$. Since we have a morphism, the path that culminates in $f(a_1*a_2)$ and the path that produces $f(a_1)\circ f(a_2)$ generate the same outcome because $f(a_1*a_2)=f(a_1)\circ f(a_2)$.

$$\begin{tikzcd}
a_1, a_2 \arrow[r, "*"] \arrow[d, "f"'] & a_1 * a_2 \arrow[d, "f"] \\
f(a_1), f(a_2) \arrow[r, "\circ"'] & f(a_1 * a_2) = f(a_1)\circ f(a_2)
\end{tikzcd}$$

Figure 3: A diagram illustrating the morphism $f:(A,*)\to(B,\circ)$.

In what follows my principal concern is with the relation between addition over the reals and the transformation that produces a substitution of $(\Re,+)$ by some other structure. We have seen that the transformation implicated in the teacher's procedure for addition over the reals in effect behaves like the modulus function: elements of $\Re_\circ^+$ are derived from elements of $\Re$ by way of a circuitous route consisting of operations/operation-like manipulations over the set *X*.

If we ask whether or not it's possible to make the substitutions needed by the teacher's procedure by means of a transformation that behaves like the modulus function we get an interesting answer. Consider the diagram shown in Figure 4, in which the modulus function maps elements of $\Re$ to elements of $\Re_\circ^+$. To complete the morphism, we ask the question: what operation is needed to map $(|a_1|,|a_2|)$ to $|a_1+a_2|$ for $a_1,a_2\in\Re$? That is, we need to find an operation, $\circ$, such that $|a_1|\circ|a_2|=|a_1+a_2|$. We know from the familiar triangle inequality for real numbers that $|a_1|+|a_2|\geq|a_1+a_2|$, so addition certainly can't be used as the required operation. In general, there does not exist an operation, $\circ$, that will enable $(\Re,+)$ to be mapped to $(\Re_\circ^+,\circ)$ as required.

$$
\begin{tikzcd}
a_1, a_2 \arrow[r, "+"] \arrow[d, "f(x)=|x|"'] & a_1 + a_2 \arrow[d, "f(x)=|x|"] \\
|a_1|, |a_2| \arrow[r, "\circ"'] & |a_1 + a_2|
\end{tikzcd}
$$

Figure 4: A diagram of a possible morphism $f:(\Re,+)\rightarrow(\Re_{\circ}^{+},\circ)$, where $f(x)=|x|$.

Exploring the situation a bit further, consider two specific elements of the domain for addition over the reals, $a_1=-7$, $a_2=7$, $a_3=5$ and $a_4=5$. Here I've deliberately chosen $a_3=a_4=5$. Let the operation of interest be addition, and once again, let *f* be the modulus function, $f(x)=|x|$. Then $f(a_1)=7$, $f(a_2)=7$, $f(a_3)=5$ and $f(a_4)=5$. We also have $f(a_1+a_3)=2$ and $f(a_2+a_4)=12$. So, despite the fact that we have $f(a_1)=f(a_2)$ and $f(a_3)=f(a_4)$, we find that it is the case that $f(a_1+a_3)\neq f(a_2+a_4)$. This means that the modulus function fails to map the structure of addition over the reals to its range, which is $f(\Re)$. When this sort of thing happens we say that the operation (addition over the reals in this case) and the function (the modulus function) are *not compatible*.

In more general terms, suppose we have a domain of operation, *A*, along with an operation, $*$, defined over the domain, and a function, *f*, also defined on *A*. If $f(a_1)=f(a_2)$ and $f(a_3)=f(a_4)$, and we also have $f(a_1*a_3)=f(a_2*a_4)$, then $*$ and *f* are compatible; if $f(a_1+a_3)\neq f(a_2+a_4)$ then $*$ and *f* are not compatible (Open University 1970). So, $f(x)=|x|$, defined over $\Re$, is not compatible with addition over $\Re$, yet we saw that the teacher's procedure nevertheless does use a series of transformations that has the same effect as $f(x)=|x|$. The question of how the teacher manages to use such a mapping can be approached by noting that he sometimes uses addition over $\Re_{\circ}^{+}$ and at other times he uses subtraction over $\Re_{\circ}^{+}$. This suggests that the teacher's procedure restricts the conditions in a manner that enables the desired substitution of structure to be realised despite the non-compatibility of addition over the reals and any mapping that behaves like the modulus function. I'll discuss this further a bit later.

Suppose, once again, we have a domain of operation, as well as an operation and a function defined over the domain. Let's name these entities *A*, $*$ and *f*, respectively, and also suppose that $*$ and *f* are compatible. To construct a morphism using *A*, $*$

and *f*, we need to define an operation, $\circ$, over $f(A)$ such that $f(a_1)\circ f(a_2)=f(a_1 * a_2)$, where $a_1,a_2 \in A$ and $f(a_1),f(a_2)\in f(A)$. The operation we seek is referred to as the *induced operation*. It is the compatibility of $*$ and *f* that guarantees the existence of a morphism and so of the existence of an induced operation (Open University, 1970, p.34). Recall that addition and the modulus function defined over the reals are not compatible, so there is no operation that can be induced from $(\Re,+)$ and $f(x)=|x|, x\in\Re$ to construct an appropriate morphism.

The practical pedagogic problem encountered by a teacher who implicitly or explicitly attempts to perform a content substitution, where the substitution also requires a substitution of structures of the form $(A,*)$ defined earlier, is one of inducing appropriate operations to be defined over the target domain of operation. In order to achieve that goal, however, the teacher needs to define, in each such case, a suitable mapping on *A* that copes with the compatibility requirement.

Dealing with non-compatibility in the pedagogic treatment of the addition

The teacher's procedure does present a way of dealing with the non-compatibility of addition and transformations that behave like the modulus function. In this section I complete my analysis of the procedure and thereby exemplify the use of the idea of teachers and students exploiting implicitly redefined domains of operation to overcome non-compatibility, at least in a local sense.

As before, I'll indicate addition over the reals by the expression $(\Re,+)$, addition over the non-negative reals by $(\Re_o^+,+)$, and subtraction over the non-negative reals by $(\Re_o^+,-)$. For the structure substitutions to work fully or in part, as we saw with the teacher's procedure, there must be some echoing of the behaviour of $(\Re,+)$ in that of $(\Re_o^+,+)$ and $(\Re_o^+,-)$, and there must be some way of getting from $(\Re,+)$ to $(\Re_o^+,+)$ in the one instance, and from $(\Re,+)$ to $(\Re_o^+,-)$ in the other. The two substitutions that inhere in the teacher's procedure for adding real numbers can be thought of as facilitated by a mapping from an initial structure to its substitute. In a first approach, we can represent the two mappings as $f:(\Re,+)\to(\Re_o^+,+)$ and $g:(\Re,+)\to(\Re_o^+,-)$. In each instance the mapping is one that has the same effect on elements of $\mathbb{R}$ as does the modulus function.

Recall that addition over the reals is a function from $\Re\times\Re$ to $\Re$, from which it should be clear that the elements of the domain are ordered pairs, $(x,y)\in\Re\times\Re$. For example, the sum $-7+(-5)$ can be thought of as the mapping $+:(-7,-5)\to-12$. When, however, we use a procedure for addition like the one taught by the teacher, it includes addition over the non-negative reals, viz., $+:(7,5)\to 12$. The modulus function is used to produce a restriction of the domain, shrinking it from $\Re\times\Re$ to $\Re_\circ^+\times\Re_\circ^+$, a shaded portion of which is shown in Figure 5.

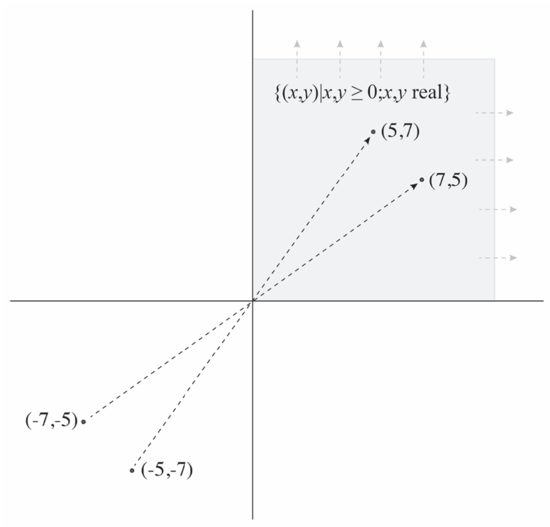

Figure 5: The restriction of the domain to $\Re_\circ^+$ with arguments selected from $\Re_\circ^+ \times \Re_\circ^+ = \left\{(x,y)\middle|x,y \geq 0; x,y \in \Re\right\}$.

As noted previously, the teacher's procedure has two options: one concerned with the addition of real numbers having the same sign and the other, with the addition of real numbers having different signs. The situation depicted in Figure 6 emphasises the restriction of the domain when the arguments for addition have the same sign. The restriction of the domain to $\Re_\circ^+ \times \Re_\circ^+ = \left\{(x,y)\middle|x,y \geq 0; x,y \in \Re\right\}$ is, however, mediated by an initial shift from $\Re \times \Re$ to $\left\{(x,y)\middle|x,y \geq 0; x,y \in \Re\right\} \cup \left\{(x,y)\middle|x,y \leq 0; x,y \in \Re\right\}$.

The shift in domain to $\left\{(x,y)\middle|x,y \geq 0; x,y \in \Re\right\} \cup \left\{(x,y)\middle|x,y \leq 0; x,y \in \Re\right\}$ is registered in the teacher's procedure as one of the possible outcomes of checking whether or not the signs of the arguments are the same, and this is used to produce a local region of compatibility for addition over $\left\{(x,y)\middle|x,y \geq 0; x,y \in \Re\right\} \cup \left\{(x,y)\middle|x,y \leq 0; x,y \in \Re\right\}$, with $f(x) = |x|$. Figure 6 shows a representation of the morphism made possible by the restriction of the domain of operation to $\left\{(x,y)\middle|x \geq y; x,y \in \Re_\circ^+\right\}$. To get from 12 to -12 the procedure instructs us to use "the common sign" as the sign of the final result of the computation. When the signs of the arguments for real number addition differ, however, the teacher's procedure requires students to use subtraction over the non-negative reals. This is so because once the elements of $\Re$ (e.g., say 5 and -7) have been subjected to the effects of the modulus function, we have to find an appropriate operation that will take 5 and 7 as arguments, but still keep us on a path that will eventually take us to the expected solution of -2. But now the collection $\Re^+_\circ \times \Re^+_\circ = \{(x,y) | x, y \in \Re^+_\circ\}$ cannot serve as domain of operation because the first argument for subtraction over $\Re^+_\circ$ must always be greater than or equal to the second argument to ensure the production of an output value (closure). The required domain is $\{(x,y) | x \geq y; x, y \in \Re^+_\circ\}$, a portion of which is shown as the shaded area in Figure 7.

$$\begin{array}{c} \left(\{(x,y)|x,y \geq 0; x,y \in \mathbb{R}\} \cup \{(x,y)|x,y \leq 0; x,y \in \mathbb{R}\},+\right) \\ \Big\downarrow \; f(x)=|x|, x \in \mathbb{R} \\ \left(\{(x,y)|x,y \in \mathbb{R}^+_\circ\},+\right) \end{array}$$

$$\begin{tikzcd} -7,-5 \arrow[r, "+"] \arrow[d] & -12 \arrow[d, "f"] \\ 7,5 \arrow[r, "+"'] & 12 \end{tikzcd}$$

Figure 6: *f* maps $\left(\{(x,y)|x,y \geq 0; x,y \in \Re\} \cup \{(x,y)|x,y \leq 0; x,y \in \Re\},+\right)$ **to** $\left(\{(x,y)|x \geq y; x,y \in \Re^+_\circ\},+\right)$**.**

However, not all elements of $\Re \times \Re$ are mapped to $\{(x,y) | x \geq y; x, y \in \Re^+_\circ\}$ by the modulus function. For example, if we apply the modulus function to each coordinate of the ordered pair (-5,7), preserving the order, we do not produce (7,5), which is what is required as the input to subtraction over $\Re^+_\circ$ by the teacher's procedure. It is here that the teacher's procedure requiring that the "smallest" number be subtracted from the "biggest" is of relevance. This criterion indicates a mapping from $\Re$ to $\Re^+_\circ$, followed by a mapping from $\Re^+_\circ$ to $\{(x,y) | x \geq y; x, y \in \Re^+_\circ\}$. This much can be discerned by noting that part of the procedure requires the computation $\text{MAX}\{|x|,|y|\} - \text{MIN}\{|x|,|y|\}$, viz., the "biggest" minus the "smallest" number in the teacher's terms, which means that elements of the required domain of operation are of the form $\left(\text{MAX}\{|x|,|y|\}, \text{MIN}\{|x|,|y|\}\right)$.

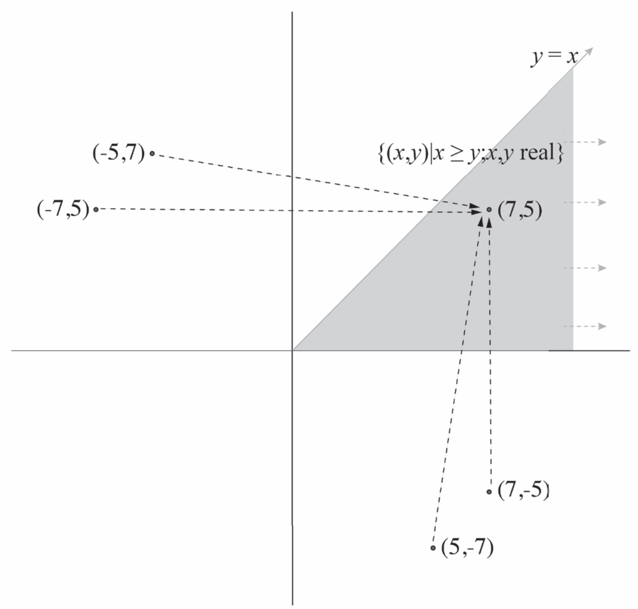

Figure 7: The restriction of the domain to selections from $\Re^{+}_{\circ}$ with arguments selected from $\left\{(x,y)\middle|x \geq y; x,y \in \Re^{+}_{\circ}\right\}$.

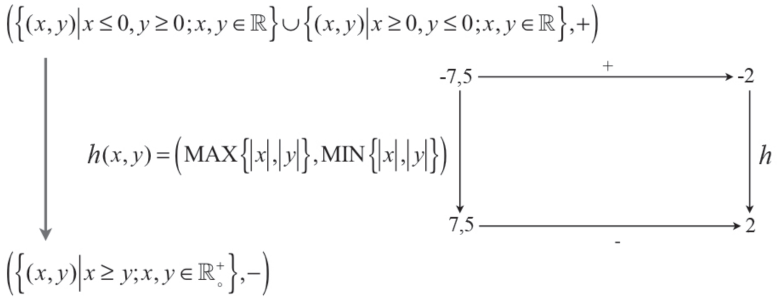

Figure 8: *h* maps $\left(\left\{(x,y)\middle|x \leq 0, y \geq 0; x,y \in \Re\right\} \cup \left\{(x,y)\middle|x \geq 0, y \leq 0; x,y \in \Re\right\},+\right)$ to $\left(\left\{(x,y)\middle|x \geq y; x,y \in \Re^{+}_{\circ}\right\},-\right)$.

The procedure first maps $\Re \times \Re$ to $\Re_\circ^+ \times \Re_\circ^+$, generating $\left(|x|,|y|\right) \in \Re_\circ^+ \times \Re_\circ^+$, and then maps $\Re_\circ^+ \times \Re_\circ^+$ to $\left\{(x,y) \middle| x \geq y; x,y \in \Re_\circ^+\right\}$, producing the required domain element for subtraction over $\Re_\circ^+$. Figure 8 shows a representation of the path generated by the teacher's procedure from $\left(\left\{(x,y) \middle| x \leq 0, y \geq 0; x,y \in \Re\right\} \cup \left\{(x,y) \middle| x \geq 0, y \leq 0; x,y \in \Re\right\}, +\right)$ to $\left(\{(x,y) \middle| x \geq y; x,y \in \Re_\circ^+\}, -\right)$. We define a function $h(x,y) = \left(\mathrm{MAX}\left\{|x|,|y|\right\}, \mathrm{MIN}\left\{|x|,|y|\right\}\right)$ which can be used to map $\left(\left\{(x,y) \middle| x \leq 0, y \geq 0; x,y \in \Re\right\} \cup \left\{(x,y) \middle| x \geq 0, y \leq 0; x,y \in \Re\right\}, +\right)$ to $\left(\{(x,y) \middle| x \geq y; x,y \in \Re_\circ^+\}, -\right)$. Having restricted the domain of operation by shrinking it from $\Re \times \Re$ to $\left\{(x,y) \middle| x \leq 0, y \geq 0; x,y \in \Re\right\} \cup \left\{(x,y) \middle| x \geq 0, y \leq 0; x,y \in \Re\right\}$, the teacher's method overcomes the non-compatibility of addition over $\mathbb{R}$ by using the function $h(x,y) = \left(\mathrm{MAX}\left\{|x|,|y|\right\}, \mathrm{MIN}\left\{|x|,|y|\right\}\right)$. The latter function is used to generate a suitable pair of arguments for the induced operation, which is easily recognised as subtraction over $\Re_\circ^+$, but where the choice of arguments is subjected to the restriction indicated by the collection $\left\{(x,y) \middle| x \geq y; x,y \in \Re_\circ^+\right\}$ to ensure closure of a sort. We need a little licence when thinking about h in relation to a single value rather than an ordered pair, as in the case of $h: -2 \rightarrow 2$. To get to the required solution of -2, the procedure instructs us to use the sign of the "biggest number" as the sign of the obtained value.

Let me summarise. The criteria entailed in the teacher's procedure for the addition of real numbers enable him and his students to perform computations over the non-negative reals by using a mapping that has the same effect on a real number as the modulus function, despite it being the case that addition and the modulus function are not compatible. The criteria generate restrictions of the domain of operation to appropriate subsets of $\Re \times \Re$, and in that way make it possible for the procedure to overcome the non-compatibility problems, so effecting the substitution of addition over $\Re$ by addition or subtraction over $\Re_\circ^+$, subject to restrictions on the selection of arguments from a suitable subset of $\Re_\circ^+ \times \Re_\circ^+$.

In its derivation of appropriate arguments in $\Re_\circ^+ \times \Re_\circ^+$ for addition, we saw that the teacher's procedure fashions an auxiliary domain of operation, along with suitable operations and operation-like manipulations, made up of character strings. This auxiliary domain of operation is employed to do the work that could be done by the modulus function. The passage between the auxiliary domain, *X*, and $\Re$ and $\Re_{\circ}^{+}$, is realised by implicit existential shifts from numbers to character strings and vice versa.

CONCLUDING REMARKS

Let me return briefly to some of the questions I posed in response to the extracts of teacher-student interaction on the issue of addition over the reals.

When the students assert that 5 must be subtracted from 7 to get the second term of the sequence $-7,-2,3,8,...$ it would appear that the students reason that, in needing to get -2 as a solution, they first need to get to 2, to which they can attach the minus sign. That's something of the way in which addition over the reals works for them. To subtract 5 from 7 is not an unreasonable response in the situation because a calculation like 7 – 5 would have been made along the way. The fact that most of the students are happy to accept the statement "$-7-5=-2$" as a record of the required computation suggests that the set of real numbers is not the domain of operation for them. It is as though they treat the situation as merely a new computational context for addition and subtraction over the non-negative reals, the latter probably being accorded the status of true numbers, as others have already noted (e.g., Gallardo, 2002; Sfard, 2007).

Much that is revealed by the analysis could be generated without considering the computational activity of the teacher and his students in the manner proposed here. However, the attempt to realise an observationally adequate description of the computational activity as regulated by the teacher's procedure, and in a manner oriented towards the Symbolic rather than the Imaginary, reveals a number of features of the pedagogic treatment of addition over the reals that might otherwise go unnoticed. The analysis indicates that operations and operation-like manipulations taking lexical elements as arguments and as values are employed. Whether or not teachers and their students realise that they perform such computations is beside the point. Such usage of lexical elements is not peculiar to the situation or the topic, as can be seen from, for example, Lima and Tall (2008), where different operations using lexical elements are shown to exist in their research context. Once it has been recognised, the use of such resources is found to be routine and quite extensive in the pedagogic situations of schooling. What this means is that rather than working only in a computational context like the field of reals, $(\Re,+,\times)$, where the arguments and values of operations are the same kind of thing, school mathematics as constituted in pedagogic situations very often has teachers and their students using auxiliary structures of the type described here to enable computations over the non-negative reals. The result is an internally inconsistent hybrid 'structure', where operations, arguments and values have to be restricted to various regions of the 'structure', and where movement across the 'structure' requires existential shifts to be made.

The analysis also makes it clear that we have to reconsider the manner in which we read the nature of content constituted in pedagogic situations. In the instance discussed here, addition over the reals is, arguably, not the content that is constituted, and so it would seem that the properties of the object $(\Re,+)$ have never been studied directly in the school in question. If we ask similar questions of the teaching of other topics, no doubt we'll find many instances of curriculum topics not being taught directly.

From my interactions with the teacher it was clear that he was perfectly adept at real number arithmetic of the type demanded by the school curriculum and that he would never make the kinds of errors that were common among his students. Yet, the syntax revealed in the procedure that he teaches his students for addition over the reals appears to generate criteria that are implicated in the procedure as constituted by his students, and his evaluations of his students' thinking are certainly attempts to stabilise those criteria. Are the criteria used by the teacher and his students the same? I don't believe that they are. There is also a suggestion in the statements of those students who were ignored by the teacher and most of the class that the students don't share the same criteria. However, since the students were not interviewed, I can't provide any further details on their thinking.

As suggested earlier, there are good reasons for questioning the existence of Bernsteinian "recognition and realisation rules" if such rules are taken to be common to participants in pedagogic situations, even when all participants appear to produce the same outcomes in some sense. It may well be the case that it is the work of pedagogic evaluation in marking out appropriate inputs (domains) and outputs (codomains), that comes closest to the idea of commonly shared criteria and "rules" of the Bernsteinian kind, but that is something that needs to be investigated rigorously.

I doubt that we can make much progress in mathematics education research by considering the constitution of mathematics in pedagogic situations only from the "outside". Rather, we do need to learn a great deal more about the "inside" of mathematical activity to get at what goes on in pedagogic situations, and a small step can be taken in that direction by striving to construct observationally adequate accounts of the mathematical activity of teachers and their students, starting with more precise descriptions of their computational activity. Constructing internalist accounts of thought, language and mathematical activity should in no way be construed as a disruption of attempts to study the impact and effect of the social on what comes to be constituted as mathematics in pedagogic situations. Internalist accounts can be used productively to situate the social in relation to the individual in a manner that changes our views on the growth of knowledge in an individual, but which still seeks to study the ways in which the social impacts on what comes to be constituted as knowledge in pedagogic situations.

NOTES

1. In this paper I draw together results from previously published work along with some new developments. There is, at times, substantial overlap with previously published papers, for which I apologise, but that has been necessitated by the need to present a reasonably comprehensive account of the ideas.
2. Words and sentences that are translated are indicated by underlining. The translations are indicated by being enclosed in pairs of braces.

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REFLECTIONS ON EVALUATION AND VALUES IN MATHEMATICS EDUCATION PRACTICES: A RESPONSE TO ZAIN DAVIS

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One of the important things that being involved with MES reminds us is that, when studying classrooms, we must look both within and beyond the walls of the classrooms in order to understand how whatever happens inside classrooms is related to the wider society. The social organisation of the pedagogic context involves not only relationships between teacher and students but also the social structures and practices of the school, local, national and global education systems, curricula and assessment. In reflecting on the paper offered to us by Zain Davis, it is notable that, while Davis has looked deeply into the detail of an interaction between teacher and students inside the classroom, the issues that this has raised for me force me to look beyond that interaction. I shall discuss the ways his paper have influenced my reflections: first on the practices of mathematics education research and then on the wider social structures of mathematics education practices.

REFLECTION 1: THE PRACTICES OF MATHEMATICS EDUCATION RESEARCH

Davis focuses on the *what* and the *how* of the construction of school mathematics knowledge. In doing so he seeks means of describing what happens in interactions in a mathematics classroom that "suspend one's content expectations". The notion of adequacy of description is an important reminder to us to avoid assumptions and to be careful about the ways in which we impose value judgements on the actions of teachers and students. In particular, I value the warning Davis provides against dismissing what happens in classrooms as not "proper" mathematics or as "just" manipulation. As he notes, "content substitutions" are a common feature of mathematics classrooms. That which we see as correct or real or proper or conceptual mathematics is approximated or substituted by some quasi-mathematics (itself a value-ridden term) that gives acceptable (in some sense) results within specific (possibly limited) contexts. It is, after all, common knowledge in the field of mathematics education research that students can get correct answers by 'incorrect', 'informal' or even 'non-mathematical' methods and that many teachers focus on procedural rather than conceptual knowledge or on partial narratives[1] such as "multiplication makes bigger" or on pedagogic metaphors such as "fractions are slices of pizza". By identifying these instances as not "proper" mathematics, we are imposing a particular form of external mathematical gaze upon the practices of mathematics classrooms, evaluating them according to alien criteria that are not the same criteria that apply within the practice. As an aside, I would liken this evaluative gaze to that often imposed on practices that the participants do not themselves identify as mathematical – for example in workplaces or so-called everyday activities – labelling them as mathematical practices and applying mathematical criteria (e.g. mathematical correctness and coherence) to analyse and evaluate them rather than considering the criteria internal to the practice (e.g. tradition, contextual appropriateness, legitimacy within the practice).

For mathematics educators there is a tension between the production of descriptions of what happens in classrooms or other educational contexts and the formation of evaluative judgements. We have past, present and future investments in participation in various practices of mathematics education – as teachers, teacher educators, curriculum designers and as students of mathematics ourselves. Each of these participations has involved the appropriation of evaluation criteria specific to the practice, reproducing what counts as legitimate mathematical and pedagogical knowledge and assuming pedagogic identities made available by the discourses of each practice. In many cases, we are or have been positioned not only as subject to such evaluations but also as authorities in control of the application of the criteria or even with the power to define what the criteria should be.

While research is a different practice that is not essentially pedagogic in nature, examining the discourse of mathematics education research reveals a pervasive, though admittedly not universal, evaluative component, perhaps especially when the researcher's gaze is upon the knowledge and the actions of teachers. This is apparent not only internally, in the ways in which researchers characterise teachers (as good, effective, sufficiently qualified or knowledgeable, reflective, etc.) and the quality of their mathematical knowledge and pedagogic communication (as deep, conceptual, procedural, etc.) but also institutionally in the common expectation by funders, reviewers and editors that research should have implications for practice and/or policy – that is, new expectations about what teachers *ought* to do. The moralising of the student implicated in the regulative discourse of the classroom is paralleled by the moralising of teachers in the discourse of mathematics education. So, the first reflection that Davis' paper has prompted for me pertains to my own practice as a mathematics educator and researcher. To what extent is it possible to suspend value judgements as I observe and describe mathematics classrooms and other mathematics education contexts? What research approaches and methodologies allow or do not allow the production of descriptions that attend to the operational specificities of the situation under study without ignoring, dismissing or disapproving those aspects that do not accord with our expectations?

REFLECTION 2: UNDERSTANDING MATHEMATICS CLASSROOM INTERACTIONS IN THEIR WIDER SOCIAL CONTEXT

The focus that Davis has offered us on the *what* and *how* of the construction of mathematical knowledge in classroom interaction provides a description that makes visible the internal operations and logic of mathematics classroom interactions. This enables us to adopt a viewpoint that steps aside from the evaluation imposed by hegemonic discourses of what mathematics 'really is'. On the other hand, it leaves open the question of *why* the construction of mathematical knowledge in this classroom takes this particular form.

Within the classroom the teacher may have the power to determine what is and what is not legitimate mathematical knowledge. But what is the basis upon which the teacher's choice of criteria is made? The classroom is not a closed system and the teacher is not a completely autonomous actor but is himself or herself subject to evaluation by others. This has always been the case to a greater or lesser extent: school managements, parents, school inspectors have judged the quality of teachers whether as part of formal evaluation systems or informally. It may be argued that the rise of neo-liberal discourses of managerialism and accountability in education has made evaluation of teachers a more explicit and ever present component of school life. Certainly this is the case in England with apparently ever increasing development of technologies of surveillance of teachers and schools. Such evaluative structures inevitably affect teachers' choices and actions, whether consciously or unconsciously, compliant or resistant, as they position themselves within school practices. Moreover, the texts produced in the classroom are not purely products of the immediate practice in the local context of a single classroom but are also shaped by the resources of other practices and social structures. The teacher's interaction with his students is located within a conjunction of specialised and everyday discourses of mathematics, school mathematics, pedagogy, curriculum and assessment, theories of learning, teaching and children as well as within the regulative structures of the education system.

We may speculate about the specific discourses that the teacher featuring in this example is drawing upon and the structures that regulate his choices (cf. Morgan, Tsatsaroni and Lerman, 2002). Perhaps these include:

- the academic mathematical discourse of his higher education experience
- school mathematics discourses from his own school experience of being taught operations with integers
- discourses of teaching and learning, whether specialised (originating in academic theory) or everyday (originating in the culture of the local community), that assign specific values to such ideas as: listening and responding to individual students; adapting knowledge to make is accessible to students; asserting teacher authority; achieving measurable outcomes; having a good "pace" for a lesson (Cowley, 2012); etc.
- "local" discourses about the capabilities and characteristics of the children in his class (Xu, 2011)
- official curriculum and assessment discourses

- school, local or national structures that regulate the selection and pacing of content and assessment procedures and criteria

This speculative list suggests a research agenda to investigate the context and origins of his practice.

I agree with Davis that we need to learn about the "inside" of mathematical activity in pedagogic situations. The insight his analysis offers into the transformation of mathematical knowledge from attention to mathematical objects and relations between them to attention to strings of characters and operations on these characters provides a concrete basis for characterising the possibilities for learning in this classroom. The method of analysis also provides a means of describing and distinguishing between the construction of mathematical knowledge and the possibilities for learning in different classrooms. Do the same transformations occur elsewhere? Are different kinds of transformations found where teachers, children, schools or communities have different social attributes?

Knowing *what* is going on is a fundamental aim of mathematics education research but understanding *why* is also critical, especially if we have any wish to effect changes. Understanding *why* demands attending to both "outside" and "inside" and the relations between them. Here I refer to Valero's (2009) conceptualisation of mathematics education as a network of social practices and her call for research to recognise its complexity by focussing study on the multiple sites within the network – the structures and interests of policy, economy and government, the practices and discourses of schools, mathematicians, curriculum developers and researchers as well as the practices in individual classrooms.

I asked earlier to what extent it is possible to suspend value judgements in order to produce an adequate observation and description, but I must also ask now: is it always desirable to suspend value judgements? Or are there value judgements that I might wish to continue to make as part of my practice as a mathematics educator and researcher and to impose on my description and analysis of data? One strand of the work of MES has always involved the integration of research in mathematics education with social action. This essentially involves holding and acting upon particular sets of values, whether these values are about the nature of mathematical knowledge constructed in classrooms or about the distribution of knowledge and other social goods among social groups. So once again, reading Davis' paper leads me to reflect on my own practice as a researcher, asking: how can I maintain and incorporate my values into my research practice while simultaneously suspending the assumptions they lead to in order to be able to produce adequate descriptions of mathematical activity in pedagogic situations?

NOTE

1. I use "narrative" here in the sense of Sfard's "endorsed narrative", that is, a statement or sequence of statements accepted as true within a mathematical discourse (Sfard, 2008).

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THE PRIVILEGING OF ENGLISH IN MATHEMATICS EDUCATION RESEARCH, JUST A NECESSARY EVIL?

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In mathematics education research, English has become the lingua franca in many situations. There are many advantages of having a lingua franca within such a diverse community. However in this paper, it is argued that the practices that are most affected by the need for a lingua franca, such as conference attendance and the writing of journal articles, also contribute to mathematics education research becoming monocultural, both in what is researched and how it is reported. Fictional dialogues are used to explore the construction of this monocultural nature of mathematics education research. In considering the collective praxis of researchers in this field, there is a need to identify the constraints on our possibilities for participating in those practices, along with the ways that those possibilities are affected by our participation. In this way, we not only begin a dialogue on these issues but have the possibility to locate other ways of participating.

PROVOCATIVE OR PARANOID?

In this paper, I explore how our ways of presenting mathematics education research are becoming increasingly constrained by regulations that we, as a community, have adopted, perhaps without enough thought, as a necessary component of working as academics. Are we colluding not just in our own oppression (Atkinson, 2004) but in that of others whose voices are reduced or removed when they are forced to use English? Some may feel that I am being paranoid, rather than provocative as I intend, in describing the outcome of these constraints to be the production of a monocultural mathematics education research community. In this paper, I hope to discuss how it is that we simultaneously mouth the need for diversity whilst excluding aspects of it through research dissemination processes. The paper has three parts. In the first part, I present the problem. In the second, I outline some factors that have contributed to the current situation and in the final section, I provide possibilities for changing the situation, something suggested by Harris (2005) that is still possible in the present, neo-liberal environment of higher education.

In choosing to describe our, the mathematics education research community's, research dissemination process as one that produces a monocultural community, I am inspired by the work of Jeanette Rhedding-Jones, who as an immigrant from Australia, works in early childhood education in Norway predominantly "for ethnic minorities and against racism" (2007, p. 38). For Rhedding-Jones (2007):

> It can thus be said that the articulations of monoculturalism may categorise ethnic minority children and adults as incompetent. This is in practice further qualified as 'developmental', 'social' and 'professional'. Hence language, domestic habits and public celebrations are not the only culturally constructed outcomes of racial and religious

> diversities, citizenships and the effects of media and migrations. For pre-school education and care this raises major problems regarding what it is that 'ought' to be happening. Is it the home and the parenting of the dominant culture or the minority cultures that should be being replicated? To what extent should or could they be blurred? Do dominant culture professionals hear, see and taste only their own cultural positioning? How are they to find other practices that are possible or desirable? Who will move out when minorities move in? (p. 40)

In reading Rhedding-Jones' work, the connections between these ideas on monoculturalism and the dissemination of mathematics education research may not be obvious. Rather than spell out immediately my reasons for considering this connection to be valid, I first want to present a series of imaginary vignettes based on situations in which I have been involved. I have chosen to present material in this way because I want to explore the situations so that "the dramatic 'frame' serves to *distance* the players from the subject in such a way as to ultimately *engage* them aesthetically and offer to them a simultaneous sense of recognition (things are as they seem) and the potential for change (things could be otherwise)" (Gallagher, 2005, p.83). Therefore, I beg an indulgence of the reader to consider Rhedding-Jones' (2007) description of monoculturalism while reading the vignettes.

Sweden

PhD Student: Can you look at this article and see if my English makes sense? I want to submit it to the journal *Mathematics Education for Today*?

Tamsin: It looks pretty good but why are you saying that you researched mathematics lessons in elementary schools? We don't have elementary schools in Sweden.

PhD Student: My supervisor thinks it is more acceptable if you use familiar terms for the reader.

Colleague: Can you look at my translation of this dialogue with a child who has Swedish as a second language? I need it to sound like a 10 year old child who has English as second language.

Tamsin: Okay, this is the best I can do but it will never have the nuances that you have in Swedish. Why don't you keep the original Swedish transcript in your paper?

Colleague: Can't, there isn't enough space. The journal only accepts articles of 20 pages. If I put in the original transcript and the translation, then the paper's too long.

Conference somewhere in the world

Presenter: We have used theories from the Arcadian researchers So-and-so and So-and-so. Their ideas have strongly influenced much of mathematics education research to do with this kind of technology in our country. In our paper we have extended these ideas in the following ways.

Audience member: But why have you not referenced This-person and That-person? Their articles are published in all the top journals. You need to make sure that you connect to the research literature.

Conference participant 1: How come those people from Arcadia never say anything in the discussions? Their English is pretty good.

Conference participant 2: I think they think we're too rude in the way we interact.

Conference participant 1: What do you mean?

Conference participant 2: You know, the way we talk over the top of each other when we get enthusiastic about something or want to disagree. If you aren't used to that kind of interaction you might find it a bit overwhelming, even if your English is very good.

In these vignettes, English is not just the lingua franca in which research is presented. The impact is wider, as Ernest (2009) suggested:

> This research literature, which incorporates the full range of academic publications including journals, texts, handbooks, monographs, and web sources is largely based in Northern and 'developed' countries, and is largely Anglophone at the high prestige end. Although journals, publishers and conference committees reach out to many countries for their editorial panels and members the locus of control remains firmly Eurocentric. This leads to the intensification of the ideological effect, as does the Eurocentricity of international research organizations and conferences. (p. 73)

The vignettes show how ideology operates at the local level where individuals chose to adopt specific practices, such as making research sound as though it comes from an English-speaking country. Terms such as "elementary school" only help those who cannot envision any other way that schooling could be organized. However, writing in English means that terms, such as primary or elementary schools, are not explained because they are considered to be self-evident. Sometimes, journal information for authors suggest that "given the international audience of SERJ [Statistics Education Research Journal], authors should make sure to provide sufficient details regarding terms, acronyms, concepts or issues which are country specific and whose understanding is essential to readers from other countries" (Statistics Education Research Journal, 2009). Although this journal also accepts submissions in French and Spanish, I suspect that terms such as primary or elementary would still not be included in their expectations about which terms need an explanation. For those who work in systems where schooling is organized differently, a translation is always required. This contributes to some ways of organizing schooling becoming "other" to the normal. Being confronted with the differences, those who do not have "normal" forms of schooling use these experiences to build their bicultural understandings about mathematics education research. They learn through this reflection what is required to get published in top journals, which includes leaving out details of their "abnormal" schooling practices and so presenting themselves as monocultural. As Atweh and Clarkson (2001) acknowledged, the format of conferences and journal articles does not support "a deep analysis of the context behind the research" (p. 85).

In the second and third vignettes, it can be seen that expectations about the ways that research should be presented affect what a presenter is "allowed" to do. If authors wish to include the original version of a transcript, then they must present their academic argument more succinctly to conform to space requirements. When a choice is made not to include the original transcripts, bilingual readers miss out on added information. When researchers are expected to (only) reference research published in English-language journals in order to be taken seriously, then there are some serious issues about what mathematics education research is valued and for what reasons (see Ernest, 2011). Jurak (2011) in his discussion of inequities between developed and developing countries illustrated the complexities that contribute to this situation, including having to write in English.

> It is likely that the quality of mathematics education is better in a developed country than in a developing country and eventually this quality differential will result in better teaching and learning of mathematics. Moreover, it is likely that the mathematics education community in the developing country does not have as much access or ownership of internet or knowledge of English as in the developed country. This by itself might generate an inequity between the two countries in terms of ownership of two essential tools for generating and sharing mathematics education knowledge, thus generating a chain reaction which results in an inequitable participation of the two countries in mathematics education at the international level. Even if a mathematics educator in the developing country succeeds in submitting a proposal to an international conference, it may not be accepted on the basis of inadequate 'quality' or questionable 'relevance' to the international community. If against all odds, a submission is accepted, its author will not likely have the financial resources to travel in order to participate in the conference. Obviously the interaction of these factors may eventually lead to the exclusion of the developing country from participating in mathematics education at the international level. (p. 131-132)

Although Jurak (2011) concentrated on what developing countries miss out on, I also consider that by excluding, in rational ways, the voices and opinions of non-English speaking, especially non-Western, countries, then the mathematics education community not only becomes poorer in its understanding of how different children learn mathematics but becomes inward looking in deciding what constitutes good mathematics education and good mathematics education research. Thus, the heterogeneity of English speakers also becomes hidden. Perceptions of how mathematics education should be presented do affect what content is considered to be valuable. It also ensures that some people are excluded from being present in discussions of mathematics education research.

The final vignette raises issues around academic discussions. Ways of being polite differ between versions of English as well as between languages (Kasper, 1997). Some forms of participation that are used in conferences will exclude some people who do not feel comfortable to interact in the necessary ways to ensure their voices are heard. Similarly, Atweh and Clarkson (2001) identified a lack of research about the impact of “cultural differences and norms in forms of establishing contacts and collaborations” (p.85) in mathematics education. If English-language speakers’, and in particular one dialect of English speakers’, ways of interacting dominate discussions, then other speakers will choose not to collaborate with them. In an article with Tony Trinick and Uenuku Fairhall, we problematised why teachers in Māori-immersion schools were excluding themselves from attending mathematics teacher conferences (Meaney, Trinick, & Fairhall, 2009). A reviewer comment on an earlier version of the paper was that nobody else was responsible for this exclusion, except the teachers. The fact that the organisers had done nothing but provide lip-service to Māori culture was not recognised as a contributing factor to the exclusion process. Ways that interactions are expected to occur will support research being done in only certain kinds of ways and this will lead to exclusion of some groups.

In this section, I have outlined some of the issues related to having English as the lingua franca in mathematics education research and suggest that one of the consequences of its dominance is that this research is becoming monocultural. Rather than embracing the diversity that is present in mathematics education, the use of English is encouraging us to do research that presents itself predominantly as of value and interest to native English speakers. Ernest (2009), in discussing how ideologies affect individuals’ mathematics education research practices, stated:

> It also leads to the ideological effect, whereby researchers in ‘developing’ countries are subject to and internalize the ideological and epistemological presuppositions and values of this dominant research culture. For to fail to do so is to be excluded from the high prestige channels for knowledge publication and dissemination. (p. 73)

Although native English speakers are not an homogenous group, the rapid adoption of similar policies across English-speaking countries, such as those to do with curriculum (Atweh & Clarkson, 2001), suggest that there are strong homogenising trends. Consequently, research from non-English speaking countries is either filtered out of the dissemination processes or made to take on the persona of being from an English-speaking country. In the next section I highlight some of the factors that have contributed to the strengthening of the dominance of English over the last two decades.

WHAT CONTRIBUTES TO MAKING ENGLISH SPEAKERS BLIND TO LANGUAGE ISSUES?

The valuing of English in mathematics education research has arisen because of the coming together of a number of different processes, in the same way that Jurak (2011) indicated that the valuing of English operates in conjunction with other factors. Bernstein's (2000) pedagogic device is one way of viewing how pedagogic communication, as a carrier of ideological messages, reproduces social inequality through the process of selecting what to teach in individual classrooms (Singh, 2002). As academics, the presentation of our research results through publications and conferences is a form of a pedagogical communication (see Beck (1999) and Beck and Young (2005) for other examples of how Bernstein's ideas have been used in regard to the university sector). Through his theory of the pedagogic device, Bernstein (1990, 2000) attempted to explain the "social grammar" which simultaneously reproduces and transforms knowledge within education systems. Bernstein (2000) suggested that "the device continuously regulates the ideal universe of potential meanings in such a way as to restrict or enhance their realisations" (2000, p. 27). It does this through a hierarchical set of rules:

1. *Distributive rules*: These rules distributed forms of knowledge to different social groups. In this way, distributed rules distributed different forms of consciousness to different groups. Distributive rules distributed access to the 'unthinkable', that is, the possibility of new knowledge, and access to the 'thinkable', that is, to official knowledge.
2. *Recontextualising rules*: These rules constructed the 'thinkable', official knowledge. They constructed pedagogic discourse: The 'what' and the 'how' of that discourse
3. *Evaluative rules*: These rules constructed pedagogic practice by providing the criteria to be transmitted and acquired. (p. 114)

In this paper, what is distributed, recontextualised and evaluated is knowledge about what is considered valuable in mathematics education research. Although the outcomes of research also change, this is a secondary effect of changing what is seen as valuable academic knowledge. The pedagogic device's primary purpose is to show how the reproduction of social inequities is achieved, through making invisible the decision-making process around curriculum selection, or in our case research selection (Bernstein, 1990, 2000). Therefore, the use of English as a lingua franca is both an outcome of the pedagogic device as well as an influence on how it operates. The dominance of English reinforces the selection of knowledge through each set of rules, thus resulting not just in the vignettes described earlier but in a tacit acceptance of the monocultural nature of mathematics education research.

Distributive Rules

Distributive rules provide different forms of knowledge to social groups, thus determining who has access to what knowledge, under what conditions (Bernstein, 2000). Although Apple (2001) also concentrated on schools, his ideas, like those of Bernstein, are relevant to the university sector. He saw education as an arena that has been heavily influenced, since the 1980s, by an alliance of political groups with separate agendas. The formation of this alliance has led to:

> The seemingly contradictory discourse of competition, markets, and choice on the one hand and accountability, performance objectives, standards, national testing, and national curriculum on the other has created such a din that it is hard to hear anything else. Even though these seem to embody different tendencies, they actually oddly reinforce each other and help cement conservative educational positions into our daily lives (Apple, 2001, p. 411)

Using the discourses identified by Apple (2001), societies, through their politicians and policy makers, have controlled what constitutes valid academic knowledge (Baert & Shipman, 2005). This has been a change from the European university tradition of much of the nineteenth and twentieth where universities and their members were considered to control the knowledge that they produced (Baert & Shipman, 2005). The dominance of the discourses of this political alliance, labelled neoliberal and neoconservatism (Beck, 1999; Apple, 2001), has arisen from wider societal issues such as the need for industries to have a more highly educated workforce to meet the change in production types (Beck & Young, 2005; Currie, 2005). These issues have led to a substantial increase in enrolment in higher education in English-speaking, as well as other, countries (Beck, 1999). The diversity in the needs of industry (market responsiveness), with the diversity in students who enrolled, enabled the knowledge that traditional universities had passed on to students to be branded as elitist (Beck, 1999; Baert & Shipman, 2005). By highlighting the issue of elitism, those who supported a neoliberal agenda have to some extent controlled the discussions about how university knowledge should be adjusted.

Although they have much less control, universities continue to have had some say in the development of the policies that they must abide by. Nevertheless, differences in types of universities that have arisen since the massification of higher education have contributed to them being unable to "speak with a single voice" (Vidovich, 2004, p.346). The perception that they have different needs has minimised their ability to present a unified opposition to government use of financial management to control what they, the universities, are allowed to do. This has contributed to governments being able to enforce their view that universities should become more market driven.

The adoption of these government policies, on a world-wide basis (Atkinson, 2004), has had an impact on the transnational mathematics education research community. Sriraman (2011), in Figure 1, highlighted the factors that he saw as affecting the research done in universities and used it to problematise how mathematics education research in Nordic countries was being forced to "borrow or mimic trends seen 'across the (Atlantic) pond'" (p. 76). However, only some factors in Figure 1 are controlled by the distributive rules. As a consequence of neoliberal/neoconservatism discourses, the distribution of knowledge has resulted in the corporatization of universities, with competition, marketisation and individual choice, and by controlling funding, through the imposition of the use of citation devices to achieve accountability and performance objectives. Institutional norms and dogmas are seen when recontextualising rules are in operation, whilst the impact on direction of local culture and scholarship is related to the operation of the evaluative rules.

Figure 1: Factors impacting Scholarship at Universities (Sriraman, 2012, p. 77)

The corporatization of universities is a result of neo-liberal reform agendas that highlight the importance of market forces "to ensure that only 'good' ones survive" (Apple, 2001, p. 412). In this way, universities are conceptualised as businesses, which then puts pressure on research to be considered relevant to this business' market. "Research in education is framed by the expectation that it will lead to improvements in both educational policy and practice and the view expressed by policy makers and practitioners is that it has not satisfactorily fulfilled these expectations" (Lingard & Blackmore, 1997, p. 5). The product at the centre of universities' business is knowledge, putting it firmly into what has been described as the knowledge economy (Ernest, 2009). Consequently, academics and the work they do has become part of an exchange system (Harris, 2005).

Although differences may appear in specific Western countries, almost none have been exempt from ideologies that consider knowledge to be a commodity and its production and exchange part of an economic (Ernest, 2009), rather than a cultural system. Comments made by mathematics education researchers in the Australasian region, suggested that the influence of US and UK policies on their own education systems were a form of colonisation (Atweh & Clarkson, 2002). Something similar could be said about the impact on developing countries. Thus, the knowledge that is distributed to universities is that they should see themselves as businesses and act accordingly.

Perceiving research as a commodity, within an education market, allows its quantity and the quality to be judged. In the last fifteen years, accountability processes have been put in place by some governments to try to ensure that the highest quality of knowledge is developed at the lowest cost (Adler, Ewing, & Taylor, 2009). Acceptance of the need for these accountability processes has affected what research is considered valuable and how this value is ascertained. Identifying who has produced high quality research provides the basis for determining the level of funding going to individual universities (Schneider, 2009). Universities that produce high quality research are given the most funding to continue doing this kind of research.

After concerns that some assessment practices simply contributed to an increase in quantity but not quality (Schneider, 2009), governments began to support the adoption of simple, numerical ways of assessing research quality, specifically journal impact factors or citation devices. Although originally proposed to help librarians chose appropriate journals (Bergstrom, 2007), counting how often an article is cited by other people has become a de facto way of determining quality (Schneider, 2009). Nevertheless, as Adler, et al. (2009) stated “governments, institutions, and even scientists themselves continue to draw unwarranted or even false conclusions from the misapplication of citation statistics” (p. 3).

Notwithstanding these concerns, measuring research quality in this way has become widespread. In Norway, university research funding is allocated based on institutions’ publication lists (Schneider, 2009). Prestigious publication outlets are given greater emphasis. As well, the type of publication has an impact on the points awarded. So an authored book published by a prestigious, academic publisher gains 8 publication points, whereas a chapter in an anthology published by a non-prestigious publisher would only receive 0.7 points. “The division of publication channels is made in order to give researchers incentives to focus their publication activity on a ‘selected number of prestigious channels’ within the research fields” (Schneider, 2009, p. 371). In the field of mathematics education research, only four journals, out of the twenty mathematics education publications noted in the Norwegian system, are considered to be at the most prestigious level, level 2. These are the *Journal for Research in Mathematics Education Research* (JRME)*, Educational Studies in Mathematics* (ESM)*, Journal of Mathematics Teacher Education* and *ZDM - The International Journal on Mathematics Education*. These journals, like all, bar one, of those at level 1, are published only in English. Of these four journals, only one has a non-English native speaker as its editor.

Not only have neo-liberal agendas controlled within-country funding, they also have contributed to a declining willingness by governments to fund improvements in education in developing countries. As Jacobson (1996) stated more than 15 years ago:

> We are experiencing a growing political conservatism in governments. There is less inclination to assist their less fortunate, and certainly not those in countries far away. The rich nations are becoming richer, and the poor poorer. The institutions set up to provide world co-operation, the United Nations, UNESCO, the World Bank, there are many, are being starved of funds and their activities curtailed. (p. 1252)

One example is that the translation of mathematics education research findings by UNESCO in the 1960s and 1970s is no longer readily financed, although there is some indication of changes in this area (see, for example Artigue, 2012). Not only is research primarily done in English-speaking countries – see Mesa (2004) for a breakdown of where JRME's articles come from – but it must be read in English, in expensive journals (Sriraman, 2012), or not at all.

My contention is that these conservative education agendas distributed by governments to universities as the knowledge that they should attend to become distilled into the practices, illustrated in the vignettes. The solidification of these policies in English-speaking countries has affected the mathematics education research community to a large degree because of the use of English as a lingua franca within this community. This is explored in more detail in the next sections.

Recontextualising rules

According to Bernstein (2000), recontextualising rules regulate 'what' should be taught and 'how' it should be taught and thus they form the pedagogical discourse. With regards to mathematics education research, recontextualising rules act as a sieve that influence how knowledge about what research is valuable, made available through the distributive rules, enters the pedagogical discourse of the university and the research field. As Harris (2005) stated "academic identity was related to subject discipline rather than to the institution itself" (p. 423) and consequently the mathematics education research community has been affected by these rules as well. Bernstein (2000) was particularly interested in the subject discipline and its control of knowledge. In my paper, the focus is on how the distributed knowledge from acceptance of neoliberal/neoconservatism agendas has affected the practices of universities and subject fields. The field of mathematics education research is transnational, making it simultaneously not affected by any one government's policies, whilst also being affected by all governments' policies. When government policies merge across the world (Baert & Shipman, 2005), then there are significant pressures on mathematics education research to become uniform in what it reports.

A result of corporatization of universities is that one university pits itself against another in order to gain the most government funding. Harris (2005) stated "it is increasingly important that academic activity contributes to the institution's overall strategy to maintain and improve its market position, which places more pressure on individuals to pursue and construct academic identities in line with corporate identity" (p. 426). This institutional dogma results in universities offering "rewards" to those staff who act in accordance with their mission statement. Sriraman (2011) noted that publication in a top-ranked journal can result in a university providing an academic with $US1000 research money. In parts of Scandinavia, the reward system is connected to the publications recognised by the Norwegian database, although the reward system may treat level 2 publications similarly to level 1.

An environment which supports competition between institutions can affect the sorts of co-operation that universities allow their staff members to engage in and this will have an impact on how disciplines develop.

> Only time will tell if the Nordic countries sustain their spirit of co-operation with one another, and the sharing of resources or whether they also succumb to the whims and vicissitudes of the competitive market economy. The goal of mathematics education is hopefully not to out-rank or polarize each other in arbitrary assessments, but to create a mathematically suave and literate society capable of solving its own problems – economic, social, migratory, political, or otherwise. (Sriraman, 2011, p. 77)

Competing for the available, limited funding can affect the kind of research that mathematics education researchers chose to do. "The 'purchasers' of research divide into two contrasting constituencies: the state, which traditionally sponsors 'pure' research for the perceived public good, and private businesses, more concerned with 'applied' research that can promise commercial payback" (Baert & Shipman, 2005, p.160). However, these roles are becoming blurred which has an impact on the type of research that mathematics educators, like other researchers, engage in. Thomas (2001) stated "as universities find it increasingly difficult to maintain their funding base for both teaching and research, gaining government contracts and tenders becomes a means for survival" (p. 106). These contracts and tenders have already identified the problem to be researched and often the mechanism for conducting the research. Therefore, "the increased dependence upon fee-for-service research has the potential to politically compromise the independence of research analysis and findings" (Lingard & Blackmore, 1997, p. 9). Thomas (2001) went on to describe how few open discussions are held about this control of funding because of a fear of creating divisions within the mathematics education research community because of different perceptions about the sort of research that researchers should be engaged in. However, as Atkinson (2004) stated, there is a risk that the control that the government exerts over funding research may mean that its "primary purpose is not to question or to critique but to serve policy" (p. 114).

Although Thomas was specifically referring to the situation in Australia, it is likely that similar situations occur in any country where universities have to operate in a neo-liberal environment of competition. Drawing from the work of Alan Bishop in the early 1990s, Atweh and Clarkson (2001) stated "although research in mathematics education is a relatively recent phenomena in many countries, research questions, methods, practices, and publications are becoming more standardized" (p.86). For example, when governments consider quantitative research as being more "scientific" than qualitative research, then research which is conducted for contracts and tenders is likely to have no option but to be quantitative. For example, the US National Mathematics Advisory Panel argued that mathematics education research should use large-scale, randomized control studies (English, 2010 cited by Ely, 2010). However, research of this kind can result in aspects of diversity being ignored because they are "hidden" within the statistics (Leder, 2012) or reified, in the case of socio-economic status being seen as a cause of poor mathematics achievement (Valero, Graven, Jurak, Martin, Meaney, & Penteado, 2012).

On the whole in the 1990s, educational research was recognised as being multidisciplinary in nature because of the types of questions that it was trying answer (Lingard & Blackmore, 1997). However, the standardization mentioned by Atweh and Clarkson (2001) has become more pronounced in recent years. For example, Heid (2010), as the editor of JRME, suggested that mathematics education should be clearly situated around issues to do with mathematics content. This has raised significant discussion in the US and elsewhere about what statements of this kind mean for mathematics education research (Martin, Gholson, & Leonard, 2010; Battista, 2010; Confrey, 2010). This discussion about what mathematics education research should be is connected to the need for it to be considered "scientific". There are two reasons why this connection is necessary. One is that in the US, the math wars have produced a very divisive discussion with mathematicians, heavily criticising the reform agenda of the National Council of Teachers of Mathematics (NCTM), who are the publishers of JRME (Klein, 2003). Their main concern was a perceived lack of mathematics in the mathematics education that NCTM's (1989, 2000) National Standards were promoting (Klein, 2003). If Martin et al.'s (2010) contention, that JRME predominately publishes articles on mathematics is correct, then their query about why Heid (2010) raised the point could be seen as a response to this ongoing debate. The other reason for raising an issue about the primary purpose of mathematics education research is to delineate it as a research field in its own right. Battista (2010) stated:

> I believe that it is important to maintain a distinct identity for the field of mathematics education research, a field that struggled for identity at its inception, and is struggling again to find a role in the political battles for control of the education system in this country. (p. 35)

The discussion about what constituted mathematics education as a field is not a recent phenomena (see Silver & Kilpatrick, 1994). However, in the current climate, the loss of identity as a research field becomes a significant problem if it affects how journals are rated and how the significance of its research is assessed.

Recontextualising rules take distributed knowledge about the need to put a value on mathematics education research and turns this into university and discipline activities, institutional dogmas and norms (see Sriraman, 2012), that give substance to how this valuation is achieved. The incentives, funding and determining of what it means to be part of a research community become Bernstein's (2000) pedagogic discourse in which individual mathematics education researchers operate. This pedagogic discourse has arisen from the knowledge distributed to universities and the mathematics education research community, predominantly from government, but it also affects the options available to individual researchers in how they decide upon their own investigations.

Evaluative rules

In the first section, I used a series of imaginary dialogues to suggest that the dissemination practices within mathematics education research are making it more monocultural. Understanding how evaluative rules act upon the recontextualised knowledge provide an explanation of how the practices illustrated in the dialogues have become a reality. Evaluative rules take recontextualised knowledge and transform it into the knowledge that individual researchers act upon when deciding what to do. Bernstein (2000) stated "evaluative rules act selectively on contents, the forms of transmission and the distribution to different groups of pupils in different contexts" (p. 115). With regards to mathematics education research, I suggest that it is at the level of research practices that the purpose of the pedagogic device as "a symbolic ruler of consciousness" (Bernstein, 2000, p. 36) is most evident.

The impact of marketisation of universities on academics' work has been documented for some time. Summarising some of this research, Currie (1998) stated "academics will experience the following changes: an intensification of work practices, a loss of autonomy, closer monitoring and appraisal, less participation in decision making, and a lack of personal development through work" (p. 18). Consequently, it can be said that these changes have an "impact on direction of local culture and scholarship" (Sriraman, 2012, p. 77)

Closer monitoring and appraisal of academics comes from having their work evaluated. The value given to citation devices by governments and transmitted down to the university affects an individual's decision about where to send his/her manuscripts. This is because choosing where to send manuscripts has an impact on his/her ability to gain grants or even tenure at his/her university (Bergstrom, 2007). These individual needs have overtaken considerations of whether specific conferences or journals would be the best places for the classrooms and teachers who were the participants in the research (Battista, 2010).

The emphasis on competition filtered down through the universities from neoliberal agendas and government policies also affects the co-operation between colleagues, even in the same institution. Baert and Shipman (2005) stated:

> The new pressure on academics to out-publish their colleagues, to be first to put their name to new discoveries, to compete for ever scarcer jobs and research funds, and to use their time originating new results rather than replicating and corroborating those of others, seems to erode collegial trust and the peer assessment it used to cultivate. (p. 169)

As well discussions about what constitutes, from a subject discipline perspective, mathematics education research, that will contribute to career advancement, affects the choices that individuals make. In responding to Martin et al.'s concerns about mathematics being the centre of mathematics education research, Confrey (2010) provided evidence of how mentoring about career choice has an impact on what research is undertaken:

> The field of mathematics education needs to be more diverse – who *does* the scholarship does matter – due to differences in experiences, priorities, interpretive frameworks, and identification. When I recently hosted a discussion of Martin et al.'s (2010) commentary, of the women who participated (African American and Caucasian, and all them professionals in educational research), more than half, including myself, had been at one time or another explicitly counseled *not* to study issues of race or gender as a scholarly enterprise for risk of being pigeonholed, and hence restricted in our subsequent professional opportunities. Therefore, I agree that the marginalization of scholarship referred to in the commentary is a widespread and unfortunate phenomenon reinforced by different forms of mentoring. (p. 26)

The pedagogic discourse that academics are enveloped in means that they often make unconscious decisions about what mathematics education research they should engage in. The discourse about ensuring that one solidifies one's market position through doing the "right" kind of research to be published in the "right" kind of journals makes it difficult to see how the requirement to use English contributes to mathematics education research becoming monocultural. There is a vicious circle where what gets cited and rewarded is most likely to appear in prestigious journals and the most appropriate way to get published is to do research in an area that is recognisable to English native speakers. For non-English speakers, as noted by Sriraman (2012), a replication of a study done in an English speaking context is unlikely to be publishable, even though the context is different and the study could be of benefit to the education system in their country. This is because the research would not be seen as adding anything to what English speakers want to know. On the other hand, the push for new results is tightly connected to building on what has been done before and reported in the prestigious journals.

WAYS FORWARD

In this paper, I provided some vignettes which were based on real events that troubled me in regard to the direction that the practices of mathematics education research were going. However, thinking through these issues has not been easy. I would certainly concur with Atkinson's (2004) suggestion that we are so "at-risk" of being controlled by the rhetoric which surrounds us, that trying to critique it means risking being seen as a heretic. The role of English in the vignettes seems to affect what is considered normal and in this way contributes to mathematics education becoming monocultural. Frowe (2001) stated that:

> Language enables participants to talk about the practice, to formulate regulative principles and engage in typically educational transactions, but it also provides an orientation towards the practice and helps constitute the nature of the practice. (p. 94)

The vignettes, provided as examples of the monocultural nature of mathematics education, did not occur because English-speaking researchers were simply thoughtless in how they acted. If this was the case then changing the situation would be relatively easy. Rather, the process that has lead to these kinds of dialogues is complex with many different, inter-related components. In the previous section, I used Bernstein's rules for the pedagogic device to show how these different components came together to bring about the tendencies for mathematics education research to become monocultural. In this section I want to discuss how the pedagogic device can be used for identifying possibilities for changing the situation.

There is hope that the forces channelling mathematics education research into becoming monocultural are not unstoppable. Although Bernstein (2000) suggested that it was only through distributive rules that the "unthinkable" can become thinkable, previous research has suggested that there are possibilities when each rule comes into operation (see for example, Meaney, Trinick, & Fairhall, 2010/2011).

The distribution rules that result in government policies determining much of what universities, and from them what academics choose to do as research, can be adjusted so that other knowledge becomes thinkable and thus actable upon. In the last fifteen years, Giddens' (1998) Third Way has been adopted by many Western countries (Humpage, 2006; Smyth, 2010), allowing "dialogic democracy" where dialogue has a primary role in decision making (Mouzelis, 2001). In Australia, the need for dialogue between all interested parties meant that the Australian Vice Chancellors' Committee changed its aim of trying to retain control of discussions about quality policy to one of working with State governments, which have legislation control over universities, in order to subvert the Federal government's desire to have its view accepted (Vidovich, 2004). Thus, the use of Third Way dialogues has the potential to allow other agendas to be contrasted with those of the neoliberal/neoconservatism agendas.

There are also possibilities for rethinking how distributed knowledge is turned into institutional dogmas and norms both by universities but also by the mathematics education discipline through the recontextualising rules. Thus, thinking about alternatives could lead to questioning of the distributive knowledge itself. Over a decade ago, Atweh and Clarkson (2001) noted that:

> Professional organizations planning international gatherings as well as editors of international journals should develop policies to encourage more equitable representations of views from developing countries. These may include multiple language presentations, differential fee structure and subsidies, and encouraging alternative research methodologies and styles of reports. It seems to us that as mathematics educators we are more concerned about standardization and uncontested acceptance of what constitutes good research at the expense of whose voices are represented. (p. 92)

In the time since this strongly worded suggestion was made, translation services have improved significantly, making these possibilities even easier to achieve. Nevertheless at the same time, the top-down forces from neoliberal agendas have further restricted possibilities for achieving these suggestions.

Of all the journals, JRME has had the most intense debates about how non-North American voices can be represented on its board (Silver, 2004) and in its publications

(Mesa, 2004). Mesa's (2004) editorial in the *Journal for Research in Mathematics Education*, not only problematised the issue of it only being published in English but promoted alternatives:

> Through JRME, we have accomplished much; we should be proud of it and thankful that NCTM has supported this enterprise. However, we are still far from the ideal situation with regard to making JRME an international journal. I believe that we, the mathematics education community, could play a bigger role in making this happen. What should guide our decision on this issue is that we have an opportunity to reach a larger and more diverse international community and, therefore, need to reach consensus on whether we are willing to provide the necessary financial resources to make that possible. If we make the journal for us, we should be willing to make sure that the journal be a forum for the worldwide community of mathematics education. (p. 4)

Although JRME is still publishing in English, it seems that its publication by an organisation rather than a publishing company means that such discussions are possible. The control of most mathematics education journals is within the hands of publishing companies whose main aim is to make profits (Sriraman, 2012). Time-deprived academics do not have the ability to produce high quality publications as had been done in the past. For example, *Mathematics Education Research Journal* is now published by Springer, partly to give it more prestige but also because it is very exhausting for a voluntary organisation to put a journal together regularly.

With regard to the recontextualising rules, Atweh and Clarkson's (2002) research, found that "in contrast with the official and university policies that promote marketization of educational delivery, mathematics educators often express more sincere humane and ethical reasons for being involved in international projects of development and research" (p.120). Researchers are able to see outside "the box" (Atkinson, 2004) to consider alternatives to only considering how to maximise their own careers. For example, mathematics education researchers make choices to attend conferences such as Commission Internationale pour L'Etude et L'Amelioration de L'Enseignement des Mathematiques [CIEAEM] which includes presentations in both French and English, with an expectation that presentations will be done in both languages (see http://ltee.org/cieaem64/scientificActivities.htm).

As well, academics are withdrawing their support for major publication houses (Samuelsson, 2012). At the beginning of 2012, a petition was started by mathematicians to boycott "a system in which commercial publishers are making money based on the work of mathematicians and subscription fees for libraries" (Fatima, 2012). It was aimed specifically at Elsevier publishing company. Since then Harvard University has publically supported its academics to make their research freely available online. Harvard's reasoning is that the fees that they pay for subscriptions are becoming exorbitant. It will be interesting to see how the move towards journals becoming open access affects the use of citations as a measure of research quality.

Changes to the monocultural tendencies of mathematics education research are possible through the different rules of the pedagogic device. However, it is more likely that these possibilities will become realities if there is discussion about the ways that our, the mathematics education research, community is controlled. To do this we need to continually question those practices that we engage in to ensure that we are not colluding with our own oppression as well as that of others.

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AN ENGLISH ONLY FOUNTAIN: A RESPONSE TO TAMSIN MEANEY'S CRITIQUE OF ENGLISH PRIVILEGE IN MATHEMATICS EDUCATION RESEARCH

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Tamsin Meaney (2013) writes in the introduction to her essay "The Privileging of English in Mathematics Education Research, Just a Necessary Evil?" that her purpose is to explore the ways that representation of mathematics education research (or knowledge) is increasingly constrained by the specific regulation of "English Only." She contends that we (i.e., members of the international mathematics education community) have adopted, perhaps without critical analysis, English Only as a necessary condition of working as members of a larger community who wish to cross national boarders. But is it really a necessary condition or "are we colluding not just in our own oppression… but in that of others whose voices are reduced or removed when they are forced to use English?" Meaney believes that for some her argument might seem to be provocative while to others it might seem to be paranoid. Nevertheless, what Meaney highlights could be called the *language diversity in knowledge production and dissemination paradox*: we simultaneously advocate for cultural diversity all the while we exclude language diversity, specifically, in regards to knowledge production and dissemination.

Table 1: ENGLISH ONLY Mathematics Education Research

Journals	Conferences
• Eurasia Journal of Mathematics Science and Technology Education • For the Learning of Mathematics • International Journal of Science and Mathematics Education • Journal for Research in Mathematics Education • Journal of Mathematical Behavior • Journal of Mathematics Teacher Education • Research in Mathematics Education • ZDM – The International Journal on Mathematics Education	• International Congress on Mathematical Education (ICME) • International Group for the Psychology of Mathematics Education (PME) • Mathematics Education and Society International Conference (MES) • National Council of Teachers of Mathematics Research Pre-session • North American Chapter of the International Group for the Psychology of Mathematics Education (PME-NA)

For instance, Table 1 provides a list of the "international?" journals[1] and conferences that require English Only submissions – so much for internationalism. If one juxtaposes her or his emotional responses (or lack thereof) to Table 1 with her or his emotional responses (hopefully) to Figure 1, she or he, I believe, is able to get Meaney's argument. That is to say, most (if not all) of us understand that the "WHITES ONLY" water fountain is an egregious injustice that delivers a resounding message of exclusion and marginalisation (such water fountains were commonly found in the Jim Crow South United States and Apartheid South Africa). And, in turn, most (all?) of us would strongly declare that such exclusionary and marginalising practices are unjust and would hopefully work toward eradicating such injustices. But why do we not react in like fashion to the unjust exclusion and marginalisation of the "ENGLISH ONLY" manuscript and proposal submission process? Why no emotional response to Table 1? Is it true, as Meaney suggests, that too many (most?) of us have accepted the oppression of English Only as a necessary component or evil of working as academics across national boarders?

Meaney's essay, I believe, is not intended so much to answer the question *Why English Only?* but more so to get us to ask the question and to begin to think of ways that we might work ourselves out of the language diversity paradox. She structures her argument by first establishing the exclusionary problem as a reality. Next, she provides some explanations of why English speaking mathematics educators, in particular, (too often?) have become "blind to language issues." And she concludes with some possible ways forward.

Figure 1: WHITES ONLY water fountain.

In this brief written reaction to Meaney's essay, my explicit purpose is to provoke an emotional response with the juxtaposition of the two visuals (Table 1 and Figure 1). However, while intentionally aiming for an emotional response, it is important to note that I am not suggesting that the injustices of Jim Crow and Apartheid were (are) one in the same nor that the injustices of English Only is somehow equivalent to the injustices of Jim Crow or Apartheid. But rather to note, borrowing from the Nobel Peace Prize Laureate and Civil Rights leader Dr. Martin Luther King, Jr. (1963/1998): "Injustice anywhere is a threat to justice everywhere" (p. 189).

In this context, the visual of the water fountain is apropos as it is in keeping with the often-used Western metaphor: *Drinking from the fountain of knowledge*. And it is in the limiting of knowledge that Meaney directs her focus as she refuses to simplify the reasons behind and consequences of English Only. Theoretically, she pulls from Bernstein and Apple to couch her argument in the larger discourse of neoliberalism.

Lipman (2011), in her recent book *The New Political Economy of Urban Education: Neoliberalism, Race, and the Right to the City*, describes neoliberalism as

> an ensemble of economic and social polices, forms of governance, and discourses and ideologies that promote individual self-interest, unrestricted flows of capital, deep reductions in the cost of labor, and sharp retrenchment of the public sphere. Neoliberals champion privatization of social goods and withdrawal of government from provision for social welfare on the premise that competitive markets are more effective and efficient. Neoliberalism is not just "out there" as a set of polices and explicit ideologies. It has developed as a new social imaginary, a common sense about how we think about society and our place in it. (p. 6)

Lipman's (2011) extended description of neoliberalism, I believe, frames Meaney's argument well. English Only has evidently become an uncritical common sense way of thinking about mathematics education knowledge production and dissemination. In many ways, policies and ideologies of neoliberalism have made ways out of the diversity language paradox of mathematics education appear to be somehow impossible. But are they, really?

Meaney notes that the mathematics education conference Commission Internationale pour l'Etude et l'Amélioration de l'Enseignement des Mathématiques (Commission for the Study and Improvement of Mathematics Teaching) (CIEAM; see http://www.cieaem.org/?q=node/12) includes presentations in both French and English. Similarly, one of the three non-English language journals included in the European Reference Index for the Humanities, *Revista Latinoamericana de Investigación en Matemática Educativa – Relime* (see http://www.clame.org.mx/relime/relimee.html), accepts and publishes manuscripts in Spanish, Portuguese, English, and French. These are just two examples of how it is indeed possible to find a way out of the language diversity paradox.

A PERSONAL CLOSING THOUGHT…

Elsewhere (Stinson, 2010), I wrote an editorial about my extraordinary experience at the Sixth International Mathematics Education and Society Conference (MES 6) held in Berlin, German during the spring of 2010. Below is an excerpt from that editorial:

> I must admit, however, that after the first agora (i.e., business meeting) of MES 6, I began to focus on the "structure" of MES 6 rather than its people. In so doing, I became somewhat disenchanted with the conference, given that I perceived some aspects of the structure of the agora to be too similar to the structures found in education conferences in the United States; structures that are designed (most often?) to maintain rather than transform the status quo. …
>
> Unfortunately, and in too many ways, I believe that even for members of ghettos it is difficult to think the unthought (cf. Foucault, 1969/1972) in our individual and

collective attempts to construct spaces that might be more ethical and just. In that, members of ghettos, like members of dominant groups, have been so discursively constituted within the multiplicities of unethical and unjust sociocultural and sociohistorical structures and discourses (cf. Foucault, 1969/1972) that we often – unintentionally, I suppose – duplicate the very structures and discourses that positioned us as members of ghettos in the first place. I include this brief, but important, critique of MES 6 to make clear that it was not without its flaws. (pp.34).

The specific disenchantment noted in the excerpt was in regards to what I perceived to be the silencing of a discussion about how language diversity might be embraced both at the conference and within the pages of the conference proceedings. I – a monolingual, English speaking mathematics educator – proposed the question. It was most disheartening when I perceived the very brief discussion (and somewhat negative reactions in general) to be more about why the status quo of English Only should be maintained rather than about how we might work ourselves out of the language diversity paradox. Here at MES 7, I am most hopeful that Meaney's critical, provocative, and timely essay will be the beginning of a thoughtful and fruitful discussion among members of what I believe to be one of the most thoughtful groups of mathematics educators in the world.

NOTES

1. The mathematics education journals listed are included in the European Reference Index for the Humanities (ERIH) and/or Social Sciences Citation Index (SSCI). It is important to note that the ERIH included three non-English mathematics education journals: *La matematica e la sua didattica*, *Nordisk matematikkdidaktikk / Nordic Studies in Mathematics Education*, and *Revista Latinoamericana de Investigación en Matemática Educativa – Relime*; the SSCI listed only English language journals. For more information about ERIH, see http://www.esf.org/research-areas/humanities/erih-european-reference-index-for-the-humanities.html; for more information about SSCI, see http://science.thomsonreuters.com/cgi-bin/jrnlst/jlresults.cgi?PC=SS&SC=HA.

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MATHEMATICS EDUCATION RESEARCH IN SOUTH AFRICA: A RESPONSE TO TAMSIN MEANEY

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As a researcher in South Africa I have realized that the state of mathematics research in South Africa has grown tremendously. In 2005 this was alluded to by Vithal, Adler and Keitel (2005) who indicated that the last decade had seen significant reform in the South African mathematics curriculum and the mathematics education research community has also grown markedly. These researchers indicated that research themes explored included: assessment; issues of language; aspects of radical pedagogy and progressive classroom practices; ethno mathematics; teacher education; and South African mathematics education research within both its local and international contexts. All this research in mathematics education has been mainly conducted in English, implying that in mathematics education research, English has become the lingua franca in many situations.

In South Africa mathematics education research has been mainly conducted in English especially in monolingual universities where the main language of teaching and learning is English. However, researchers whose first language is Afrikaans have been privileged to write mathematics education research in their home language whereas other English second language students are still doing research in English. Afrikaans students benefited because they study in dual language universities where the language of teaching and learning is both English and Afrikaans or where the language of learning and teaching is Afrikaans only. Even though Afrikaans speaking students do get an opportunity of doing their research studies in Afrikaans, when they have to present at mathematics conferences they have to present in English. This implies that practices such as conference presentations and the writing of journal articles also contribute to mathematics education research becoming monocultural.

WHAT IMPACT DOES THE MONOCULTURAL RESEARCH HAVE ON POLICY IMPLEMENTATION IN SOUTH AFRICAN SCHOOLS

As much as Vithal et al. (2005) argue that new agenda in mathematics education research needs to be debated in order to better understand the connections between curriculum research, reform, policy and practice, now in 2013 mathematics education research has not yet fully connected or addressed policy demands in South Africa with regards to the language of teaching and learning especially in the Foundation Phase. Foundation Phase (grades R to 3) and Intermediate Phase (grades 4 to 6) English second language teachers are equally affected by the use of the English language in mathematics education research. This implies that research reports that are written in English may still hamper the flow of pedagogical content knowledge in schools right from Foundation Phase to grade 12 learners because of the issue of multilingualism in schools.

According to the survey that comprised 5% of the sample (i.e. 70 schools randomly selected), regarding language of teaching and learning (LoLT) in Gauteng primary schools the outline was as follows:

Overview

- 61% of schools have adopted an African language LoLT in the Foundation Phase. Of these:
  - 41% have a single African language that has been adopted as LoLT
  - 20% are teaching in two or more African languages
  - In all of these schools officially the LoLT changes to English in the Intermediate Phase
- 28% are English medium schools. Of these:
  - 16% of schools have historically taught in English
  - 3% previously taught in an African language
  - 9% previously taught in Afrikaans
- 14 of the sample schools were historically Afrikaans medium schools. Of these 14 schools:
  - 6 are now teaching in English
  - 4 are Dual Medium (English and Afrikaans) schools. One of these is planning to change to full English LoLT in the near future. This means that
  - Half of the formerly Afrikaans schools have changed, or are in the process of changing, to English LoLT.

Needless to say, even though South African researchers address a diversity of interests and concerns by exploring and extending new directions in mathematics education research, particularly within the changing landscape of post-apartheid South Africa, English is still the dominant language in which research reports are being written. The question that arises then is "do the teachers from this diverse community benefit fruitfully from the mathematics education research in South Africa which is basically becoming monocultural?"

The implementation of Language in Education Policy in South Africa especially in the context of Gauteng demographics is complex because of a large number of English second language teachers and learners who do not share the same languages. The complexity is created by the fact that learners in the Foundation Phase have to be taught in their home language and those from Intermediate Phase have to be taught in English, which is not their main language. Some of these learners are not South

Africans and therefore they do not understand any of the eleven South African languages. This suggests that English second language mathematics teachers in the Foundation phase and Intermediate/Senior Phase (Intersen) may be facing a dilemma where Foundation phase teachers have to translate research findings into the learners' home language and Intersen teachers have to be able to interpret research finding to English second language learners who have to be taught in English. This raises the following question:

- Do English second language educators benefit from mathematics education research in South Africa?
- If so, to what extent?
- Would the English second language researchers agree to be taught mathematics education research in their home/main languages so that they may write their research reports in other languages other than English?

I am raising the issue of the third bullet because Setati (2008) argues that the choice of using English as the language of learning and teaching mathematics is the fact that it would create epistemological access for the learners otherwise maybe learners would be taught in their home or main languages until grade 12. The mathematics education researchers might be feeling the same way, who knows? What is more prevalent in the reasons for preference of English are: economic, political, and ideological factors (Setati, 2008). However the South African Education policy states that Foundation Phase learners need to be taught in their home languages because early education in mother tongue promotes language development and preliteracy skills (Pinnock, 2008). This implies that Setswana learners need to be taught in Setswana, Zulu learners need to be taught in Zulu, Sepedi learners need to be taught in Sepedi, etc. The questions that can then be raised are:

- Do these English second language teachers really benefit from the mathematics research?
- What is the purpose of conducting mathematics education research?
- What is the purpose of writing up the mathematics research journals?
- What is the purpose of presenting the mathematics education research findings?
- Who are the readers and the audience?
- Who understands better at a mathematics research conference? Is it the English second language teacher or the English first language teacher? Why?
- Amongst the two teachers, which one will find it easy to implement what he/she learned at the conferences?

I am basing my argument on the fact that some of the teachers who are teaching second language learners are often invited to the mathematics education conferences that I have attended in South Africa. These educators are English second language educators who do not even share the same educational background with some of the researchers. These teachers are encouraged to read journal articles written in English. They also need to implement the research findings from the reaserch reports in their mathematics classrooms, as journals are part of the membership benefits. Some of the findings and recommendations reported in these articles are on how they can improve the teaching of mathematics to second language learners in their classrooms. These presentations and journals articles are written in English, which is not the teachers' home or main language.

As a branch secretary for the Gauteng Association for Mathematics Education of South Africa (AMESA) committee I had an opportunity to invite teachers from our branch to attend the mathematics education conferences. These teachers showed much interest in attending "How I teach" sessions rather than plenary sessions that were based on research presentations.

Below is a vignette about one of the AMESA conferences that was held in Durban in 2010:

Dialogue with teachers attending the conference

Lindi: *why do you always sit outside during the plenary sessions*?

Teacher 1: *hayi (no), I always fall asleep there*

Lindi: *why*

Teacher 2: *phela yi lala class le session (this session is a sleeping session), they use big words, I try to understand but I just don't understand. thina si enjoya ama workshops (we enjoy workshops), not this*

Teacher 3: *re enjoya bo (we enjoy) 'how I teach' workshop because at least we know what they are talking about. This is not for us.*

Lindi: *why ama workshop (why workshops)*

Teacher 1: *because they talk about what we must teach and that's what we do everyday and how we should teach it*

Lindi: *but nalama plenary a relevant (but even the plenary sessions are still relevant, they relats to what is happening in class*

Teacher 2: *this one is for those that work at the universities, not us. Mina angibezwa ukuthi bathini (well I don't even understand what they are talking about).*

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THE MATHEMATICAL PRACTICES OF THOSE WITHOUT POWER

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Paulo Freire declared that: “The intellectual activity of those without power is always characterized as non-intellectual” (Freire & Macedo, 1987, p. 122). Inspired by Freire, I adapt his statement to argue that the mathematical practices of those without power are too often characterized as non-mathematical, hence my title.

I will talk about my work in progress with unschooled boat-builders in my native Bengal. Maritime trade and associated practices of boatbuilding have been around in India for thousands of years, as amply evidenced in archaeological markers. To set the scene, I first briefly discuss the devalorization of non-academic mathematical practices. I conclude with implications for alternative ways of conceptualizing mathematical knowledge, and by briefly addressing the open question of how this perspective might impact school mathematics. In my final remarks, I comment on forms of intellectual arrogance and their consequences.

THE DEVALORIZATION OF NON-ACADEMIC MATHEMATICAL PRACTICES

Part of the story of mathematical knowledge and power has to do with the Eurocentric (or racist, as Raju (2007) uncompromisingly characterizes it) narrative of the Western history of academic mathematics. Although that issue is rather beyond the scope of this talk, it is not unconnected. Raju, for example, suggests that there is a contrast between two streams of thought originating in Greece/Egypt and India/Arabia. While the former is anti-empirical, proof-oriented, and explicitly religious, the latter is pro-empirical, calculation-oriented, and has practical objectives. Raju’s thesis might be summarized in the following terms: “The mathematics of those whose work does not fit into (a constructed version of) the Greek tradition and its Western development since the Italian Renaissance is characterized as not real mathematics”.

Work within the tradition of Ethnomathematics has shown how very sophisticated mathematical practices exist in essentially all cultures (Bishop, 1988). The history of colonialism shows how such practices were often suppressed (Bishop, 1990). For example, Zaslavsky (1973, p. 131) commented that “it is incredible that African games were actually discouraged by the colonial education authorities in favor of ludo, snakes-and-ladders, and similar games of European origin” (not really incredible, actually, given the colonial mindset).

There are numerous examples of existing highly adaptive practices being overlaid by the practices of the colonizers. One case extensively analyzed by Raju (2007) relates to navigation, which at the time of Vasco da Gama's explorations around 1500 was much more advanced in India, so that Indian navigators guided him across the Indian Ocean after he had been cautiously making his way up the African coast. Another example has been analyzed by Gary Urton (2012) in relation to the Inka/Incas in Peru. When the Spanish conquest occurred, the highly developed methods of keeping records of economic activity, censuses, and so on, using khipu/quipus (knotted strings) were forcibly replaced by the Spanish system of accountancy.

As Gelsa Knijnik (Knijnik & Wanderer, 2012) has pointed out, mathematics is situated within what Wittgenstein termed "forms of life". Thus:

> ... the educational process developed by the Brazilian Landless Movement... throughout its history must be seen beyond schooling, since each Landless subject educates her/himself through her/his participation in the everyday life of their communities and also through the wide range of political activities developed by the Movement. In this enculturation process to which the Landless people are subjected, they learn how to use the language games that constitute their mathematics (p. 187).

Knijnik and Wanderer (2012) describe the current efforts to assimilate (without accommodation) the children of the Landless Movement into urban schools, under the guise of offering the same mathematical knowledge to all. This example illustrates a fundamental ideological faultline between those who see mathematics as universal, with mathematics education to be globally homogenized, and those who value cultural diversity (Greer, Mukhopadhyay, Nelson-Barber, & Powell, 2009; Mukhopadhyay & Roth, 2012).

A very moving account has been given by Munir Fasheh (Fasheh, 2012) of the mathematics of his mother in making dresses. This epiphany led him to radically re-evaluate his own mastery of academic mathematics:

> Why is my kind of mathematics considered valuable and worthy of being taught in schools and universities (almost all over the world) while my mother's kind of mathematics is totally ignored? Why is my kind of mathematics considered knowledge while hers is not?
>
> ... Gradually, I realized that the mathematics I studied and taught suppressed and won over my mother's mathematics through bullying; by devaluing, ignoring, and belittling her mathematics, and providing instead another mathematics that claimed to be neutral and universal – the only path into the future. It won not because it is superior or better but through being a tool serving interests of dominant political and economic powers, helping them in controlling people, suppressing their knowledges, and robbing them of what they have (p. 94).

TRADITIONAL BOATBUILDING ON THE BAY OF BENGAL

Currently I am engaged in an ethnographic investigation of vernacular engineering in the form of boat building in Frasergunj, a coastal village in West Bengal. Located about 130 kilometers south of Kolkata (Calcutta), it is a fishing village where the local fishermen commission the boat-builders to construct boats for deep-sea fishing. Unlike what one generally sees these days, these boats are made of wood and crafted totally by hand using very simple tools. Typically, a team of eight to ten men, varying in age and experience, works together for four to six months of the dry season to make boats some 60 feet in length that are used for trips of up to twelve days in the Bay of Bengal. My investigation grew out of a serendipitous encounter in a coastal village when I came across a team of men working together on a large boat, in what seemed like a choreographed series of movements, without any exchange of words. There were no drawings or blueprints visible; the tools in use were simple, some were homemade, and even looked too "primitive" to accomplish such an engineering feat. When asked about how they plan such an endeavor, the response of the head of the crew baffled and intrigued me, as he said: "Yes, we can follow plans if necessary, especially for government contracts, but that slows us down". By "plan", he apparently understood an engineering drawing or a blueprint. Starting from this incident, my interactions with these workers have led me to interrogate my own assumptions regarding thinking about the tasks, embedded mathematics, and tools in use, particularly in terms of use of language and representations. From the beginning, I have been intrigued by the question: "How do you know by looking at what you are building that the boat is correctly made, and will float well?"

The workers

The work

Employing ethnographic observations and occasional short interviews, for the last few years I have been investigating the complexities of cognition, in terms of collective concept formation, aptly termed *cognition in the wild* (Hutchins, 2000, 2005, 2012), in this non-Western context, in which the actors are functionally "illiterate", having had minimal formal schooling.

The craftsmen, *karigar* in Bengali, are carpenters who build large wooden structures using hand tools only. They come to Frasergunj each year from a neighboring village to work for six months of the dry season. Although not related in the formal sense of kinship, they hail from the same region in Bangladesh and by virtue of that have settled down in close proximity to one another. Thus, following the local tradition, they are “related” to one another; they are “uncles” and “cousins”, and an older member is even affectionately called “grandfather”. During the six months of their stay at Frasergunj, they build a temporary shelter close to the boat construction site. The shelter is a long rectangular dwelling with no windows but large openings at both ends that serve as doors. Similar to a dormitory, the men sleep in a row on the floor, and roll up their bedding, consisting of hand-stitched quilts from home and a mosquito net, in the morning as they go to work early. Small bags containing a few personal belongings sit next to each bed. The men work seven days a week, sunup to sundown, with extended breaks for breakfast and lunch. At night they either they go to the local market or sit around and work on making fishing nets which they use in their own village. They go home to their village from time to time for a couple of days. They talk about their home in the village where their family is left behind. The tools that they use regularly are kept in baskets and stored in one corner of the room.

(Some) Tools of the Trade

In recent years, Ranjan (I have their permission to use their real names), the head of the crew, has purchased a few power tools, such as an electric drill and an electric saw. One notices that in their collection of tools, they include their own handmade tools as well. Since they make their own tools they also keep up with the maintenance of the tools – instead of replacing, they consciously sharpen and repair them. Mihir proudly shows the tool – a groove planer – that he made under the supervision of his guru, his uncle in Bangladesh; he brought the tool along when he crossed the border. As stated earlier, the team consists of men of varying degrees of experience. There is always an apprentice, the youngest member, who is usually assigned to work with an experienced craftsman. The youngest member of the team, fourteen-year old Babu, is a school dropout. One of his daily tasks is to make sure that all tools are picked up from the site. This, surely, is one way to familiarize him with the tools.

Most of the men in the team have no formal schooling – either they have had no chance to attend school or have dropped out early because of poverty. Some of them can read but prefer not to, especially in the presence of an outsider. Samir, for example, aptly described their access to school as ‘we could not reach the door to the school’. They started working early in their lives, mostly to assist their families. They are highly articulate in their native tongue Bengali, and also speak Hindi.

In describing *cognition in the wild*, Hutchins (2000), characterizes it as “the distinction between the laboratory, where cognition is studied in captivity, and the everyday world, where human cognition adapts to its natural surroundings” (p. xiv). His perspective is particularly salient for this project for, although within the community of mathematics educators some of us are more passionate in unearthing mathematical thinking in cultural contexts, we often fail to recognize the essential role mathematics plays in activities of various workplaces. Every workplace exhibits complex interactions among its workers; for example, to make a product that has various parts, there is interdependence – in terms of design and construction – so that the parts could be assembled and integrated appropriately. Similarly the workers, who are at various levels of expertise, rely on one another to contribute effectively in completing the task at hand. This human interdependence is manifested as distributed cognition and is not just either social or cognitive in nature, but a complex interplay between the two.

In this study of vernacular boatbuilding, following Hutchins (2005, 2012), my focus shifts from individual expertise relating to concepts and skills to the ways in which these concepts and skills are distributed among the members of the group. Teaching and learning, as continuous processes, are based on informal apprenticeship. In this process, the men work together with clear understandings of their roles. They share their material tools, help one another, and offer friendly criticism. Being “uneducated” and poor, they have very little prospects for well-paying jobs. Generally speaking, they are loyal to and stay with the same crew. I have not heard them complain about the head of the crew, Ranjan, although they are vocal in voicing their opinions about their status within society. I have seen no evidence of recorded drawings, plans, formulae, or even bookkeeping. They report that everything is kept in the head and transmitted orally. During my observations on various problem-solving situations I have never seen even a diagram etched on the ground to explain the particular construction process. There is total reliance on oral and gestural communications.

As a part of this investigation (Mukhopadhyay, Liubov, Querol, & Engeström, in preparation) I shared a set of photographs from a boat-building site in Finland where the workmen are trying to revive the lost Finnish craft of wooden boat building. Although the Frasergunj boat builders responded enthusiastically to the photographs

– identifying the tools in use, the process of work, etc. – they had practically no comments on the sketch of a part of the boat that the head of the Finnish crew had done. When pressed for a response, Samir said, “You only need drawings when your supervisor is not around. *Dada* (meaning elder brother, referring to Ranjan) is always keeping a watch on the work that we do together. He intervenes when we need help or advice. If he had gone away – say, to the bazaar for a while – then we could use a drawing. He does better than what the drawing is about (that is, explaining).” His reluctance to discuss this particular photograph was baffling. In formal education, especially engineering and mathematics, people rely heavily on drawings as essential representations, but in this particular case of vernacular engineering, men with no formal education have no such need. Their understanding of the construction process is based on functionality as opposed to reliance on theories. They recognize their knowledge as “practical”, implying that it has developed from intense hands-on practice. They do not have any readily available vocabulary for the term “theory” as we understand the concept, and contrast their learning to formal training as “learning from books that were picked up at school.” Needless to say, formally trained engineers have very little interest in the folk engineering tradition. The boat-builders also have very little patience for the perceived arrogance of the “trained” engineers. Although these craftsmen are exceedingly polite, they do not fail to make comments such as that of Dilip: “The engineers do their drawings and then they leave. They do not know what the work actually is – they have missed out on working with tools. We often pick up the work from scratch and continue building.”

The rigor that a formally trained engineer might consider lacking in the work of the boat-builders contrasts with another form of rigorous validation; the boats are exceptionally functional and strong, and used over many years for extended oceanic travel. Despite the apparently “primitive” nature of their work, it is noteworthy that they adapt their traditional techniques to the changing technological needs of their society. Some of the technologies, such as wireless communication and the GPS, have hugely impacted the safety of the local fishermen. The boats have been motorized and are always fitted with wireless and GPS technologies. The workmen are also beginning to acquire and use power tools. One afternoon, Ranjan came back with a power drill that he had bought in a nearby town. There was a lot of obvious excitement when the others were taking the tool out of the box. “Save the warranty card,” said Ranjan, “the rest can go.” Clearly, the instructions, written in English, were of no use to them. The contrast in India (and elsewhere) between hi-tech and on-the-streets ingenuity is very noticeable. In the spirit of bricolage, people rely on their problem-solving abilities to repair and reclaim tools that would have been discarded in the technologically superior West. For a complex power tool, an “illiterate” worker does not need a manual.

IMPLICATIONS

Let me state at the outset that I do not see any direct link between my study of the boat-builders and school mathematics in the sense of a direct insertion of activities related to boat-building into a school curriculum. The connection to school mathematics of my work is more indirect. Work of the kind that I have described raises questions for the possibility of alternative conceptualizations of mathematical knowledge (Mukhopadhyay & Roth, 2012), and of learning. When we think about what these unschooled men, working with minimal tools and without external representations, can accomplish, it forces us to consider alternative models of cognition, and of learning/teaching. When a young member of the team and an experienced member work together inserting the staple-like pieces that secure successive planks together to form the hull, there is no separation between doing and learning. There is much to learn from the highly effective teamwork that achieves the object of making the boat, yet with little overt communication.

A major question that remains open is to what extent culturally relevant mathematics can be incorporated into school (and indeed university) curricula. There are cases of working to incorporate culturally embedded mathematical practices into school mathematics, such as the work of Lipka with the Yupik people in Alaska over more than twenty years (e.g. Lipka, Yanez, Andrew-Ihrke, & Adam 2009). Pinxten and Francois (2011) begin their paper with a long account of a journey undertaken by a Navajo boy in territory that outsiders would find impossible to navigate. In the course of this journey the boy relies on many forms of knowledge developed through experience. Later, they ask a very pointed question, namely for this boy, and children in general who are adapted to their physical and cultural environments, what is the added value of learning formal mathematics in school?

FINAL COMMENT

In relation to mathematics, as with all forms of intellectual activity, the assumption of superiority takes many forms.

"Pure" mathematics typically has higher status than "applied" (especially among pure mathematicians). At the very beginning of Galileo's "Dialogues concerning two new sciences", one of the philosophers, Sagreto, scoffs at the practical knowledge of Venetian artisans who told him that the supporting frameworks for larger vessels need to be relatively more robust than those for smaller vessels. Taking his cue from Aristotle, Sagreto argues that "mechanics has its foundations in geometry... [so] I do not see that the properties of circles, triangles, cylinders, cones and other solid figures will change with their size" (p. 2). Another philosopher, Salviati, representing Galileo's position, corrects him.

British colonial historians of the 19th century accused Indians of falsifying accounts of the development of Indian mathematics despite irrefutable documentary evidence (Almeida & Joseph, 2009; Raju, 2007; Sen, 2005). As pointed out by Almeida and

Joseph (2009), this rewriting of history has persisted until very recent times, and they suggest that:

> ... a possible reason for such puzzling standards in scholarship may have been the rising Eurocentrism that accompanied European colonisation. With this phenomenon, the assumption of white superiority became dominant over a wide range of activities, including the writing of the history of mathematics. The rise of nationalism in 19th-century Europe and the consequent search for the roots of European civilisation, led to an obsession with Greece and the myth of Greek culture as the cradle of all knowledge and values and Europe becoming heir to Greek learning and values (p. 174).

This thesis has been most fully developed by Raju (2007).

In terms of another dimension of assumed intellectual superiority, in their book "Loving and hating mathematics", Hersh and John-Steiner (2012) give many examples of the denial of the status of women within academic mathematics, a situation that is slowly improving.

As we contemplate the crises facing the planet today (D'Ambrosio, 2010), it is legitimate to ask how it is that the vaunted rationality of mathematics and science have played a role in bringing us to this pass, and what the ethical responsibilities of mathematicians and mathematics educators are in addressing the most pressing problem for humankind, survival with dignity.

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RESPONSE TO SWAPNA MUKHOPADHYAY: THE MATHEMATICAL PRACTICES OF THOSE WITHOUT POWER

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INTRODUCTION

Swapna Mukhopadhyay (2013), in her ethnographic study of the "vernacular engineering" of Bengalese craftsmen (*karigar*), states that her interest is in developing an understanding of how the *karigar* who are mainly "illiterate" and unschooled, manage to build boats that float without having any paper plans available to them.

The central claim made by Mukhopadhyay (2013) is that the "mathematical practices" of those that she refers to as "without power" are often described as non-mathematical and are consequently devalued. This claim is not a unique one since it resonates with claims made by others located in the Ethnomathematics tradition (see D'Ambrosio, 1985; Gerdes, 1985; Knijnik, 1993). Furthermore, Mukhopadhyay (2013) proposes that an understanding of practices such as boat building opens up possibilities for alternate conceptions of mathematical knowledge.

MATHEMATICS AS A KNOWLEDGE DOMAIN

Mukhopadhyay's (2013) paper is titled "The mathematical practices of those without power". I thus expected an explanation of what is meant by mathematical practices and in particular I expected a discussion of the practices of the Bengalese *karigar* to illustrate the nature of mathematics they engage in as they go about building boats. Nevertheless, it seems that for Mukhopadhyay, mathematics is embedded in the practices of the boat builders – an idea posited by others in the field of Ethnomathematics. I return to discuss the notion of embedded mathematics later. Mukhopadhyay's paper generates questions about what counts as a mathematical practice and how we recognise whether someone is doing mathematics.

Attempts to define what constitutes mathematical practices, inevitably leads to the question of what is mathematics, a question that has occupied philosophy of mathematics for centuries. Answers to such a question are complicated and fiercely contested. See Hersh (1997) for a discussion on the philosophy of mathematics. The complexity of answering the question is demonstrated by Courant and Robbins in their book, *What is Mathematics?,* who, instead of answering the question with a definitive statement about what mathematics is, resorted to showing solutions to a number of mathematical problems (Hersh, 1997). Inspired by Courant & Robbins' question and as an attack on Platonism, formalism and neo-Fregeneanism, Hersh (1997, p. xi) describes mathematics "as a human activity, a social phenomenon, part of human culture, historically evolved, and intelligible only in a social context." in his book, *What is Mathematics Really?*. Hersh's (ibid) definition of mathematics seems very general since it does not distinguish mathematics from any other human activity. Maclane (1986) argues that it is human activities that stimulate the very ideas that form the basis of mathematics, which he defines as the "formalisation of ideas needed to understand time, space and motion" (ibid, p. 414). Mathematics, he claims, is "an elaborate tightly connected network of formal systems, axiom systems, rules, and connections" (ibid, p. 417). While acknowledging that human activity acts as a stimulus for mathematics, Maclane does not equate human activity with mathematics.

I set out to interrogate what it means when we say that someone is doing mathematics. In order to do this, I momentarily take a step outside of mathematics to focus attention on cooking. In particular, I refer to Heston Blumenthal, a famous English chef and owner of the three-star Michelin[1] restaurant, *The Fat Duck*.

HESTON BLUMENTHAL IN SEARCH OF PERFECTION

The history of Heston Blumenthal's career as a chef is carefully documented in *The Fat Duck Cookbook* (Blumenthal, 2008). Blumenthal claims to be a self-taught chef who studied the cookery books of famous chefs in order to learn about the art of cooking. His fame as a chef derives from his scientific approach to cooking, having developed for example, the perfect potato chip using his triple cooked chip recipe (ibid, p. 51) or the perfectly cooked steak using the sous-vide method (ibid, p. 429-430). He developed an interest in perfecting Peking duck (a national Chinese dish), which is sought after for its thin, crispy skin and moist, succulent flesh. Producing perfectly, thin crispy duck skin while at the same time ensuring that the flesh is moist and succulent is by no means an easy task as Blumenthal soon discovered. In pursuit, of perfection, Blumenthal visited chefs in Beijing to source authentic Peking duck recipes. In addition, he set up a kitchen laboratory where his quests of perfecting particular cooking dishes were filmed as a BBC television cooking series. In the process, he consulted chemists and other scientists in order to understand the science underlying the process of producing crispy skin and perfecting other dishes.

Thus, Blumenthal appeals to chemistry to solve his cooking problems and to experiment with cooking dishes. His central problems such as "how do I make the perfect potato chip?", however, are not located in chemistry but in the field of cooking. Many Chinese chefs acquired their knowledge of cooking Peking duck through years of experience and perhaps through apprenticeship to experienced chefs, thus drawing on resources different to those used by Blumenthal. Most importantly however, is that Blumenthal does not consider himself to be a chemist. Although his work sparked a subfield of cooking, molecular gastronomy and he has published a number of research papers on different aspects of the science of cooking (see Blumenthal (2008)), he thinks about himself primarily as a chef who uses science as a tool to experiment with cooking and to pursue culinary problems. We can always describe cooking in terms of chemistry and physics. The question, however, is can we label anyone who engages in cooking a scientist?

BOAT BUILDING

Boat building is one of the oldest branches of engineering and is concerned with constructing the hulls of boats and, for sailboats, the masts and rigging. The first boats predate drawing and writing (Anderson & Anderson, 2003, p. 17). As Mukhopadhyay (2013) states “boatbuilding has been around in India for thousands of years, as amply evidenced by archaeological markers”. In her ethnographic study of the Bengalese boat builders, Mukhopadhyay asks the question: “How do you know by looking at what you are building that the boat is correctly made, and will float well?”

The idea of a boat or a vessel to transport people and things by water must have arisen from observing that wood floats. The problem faced by boat builders is not really about whether a boat will float but how wide or long the boat should be to hold a certain load. And if a hull is constructed, the problem is about making the hull waterproof. Once people invented boats that could hold the weight required, the problem evolved to how could we make boats that move faster, which lead to the development of sails and later motorised propellers (Anderson & Anderson, 2003).

The mathematics and physics underlying the floatation of boats is too extensive to deal with in this paper (see Biran (2003) for a detailed explanation of mathematics and physics involved). Archimedes formulated the natural laws underpinning buoyancy of objects. Firstly, he observed that “any floating object displaces its own weight of fluid” (Heath, 1897, p. 257). Secondly, he noted that “a body immersed in a fluid is subjected to an upwards force equal to the weight of the fluid displaced” (Biran, 2003, p. 23) The second observation referred to as Archimedes’ principle explains the natural law of buoyancy – that is, objects that are less dense than the fluids that they are partially or totally submerged in are subjected to an upward force by the fluid. Whether materials sink or float do not depend on their mass but on their density. Since density is defined as mass per unit volume, if you increase the volume of an object without increasing its mass, density decreases therefore increasing the buoyancy of the object. A boat will float if part of the boat remains above the water after it has displaced its mass. Thus the key to building a buoyant boat is to shape the boat so that its mass is displaced before the boat is submerged in water.

Although the Bengalese boat builders do not explicitly employ physics based on Archimedes’ principle in the construction of boats, they have to consider issues of density and buoyancy even if implicitly since they like everyone else are subject to the laws of nature. The Bengalese boat builders are quite capable of building seaworthy vessels using a wealth of knowledge, most probably accumulated over years and passed down from generation to generation. From Mukhopadhyay’s description, the elder boat builder, Ranjan Dada, has a clear idea of what the final product should look like and what processes, tools and material is required. He, therefore, does not need a paper plan to oversee the team of workers, who are entirely reliant on him for directing the different stages in constructing a boat.

On what basis can we claim that the Bengalese boat builders are engaged in doing mathematics? And, why do we want to refer to the Bengalese boat building as mathematics? I doubt that the boat builders characterise their work as mathematics. We can't deny that boat building involves arithmetic calculations, measurement and perhaps some geometry, but is this mathematics? In the same way that cooking entails measuring ingredients and perhaps calculating, can we claim that Heston Blumenthal is doing mathematics? A field of knowledge[2] is defined by the problems that it attempts to solve. In boat building, the problems involve issues related to the seaworthiness of boats, holding capacity and issues of waterproofing, to name a few. The problems that emerge in boat building are not problems that need to be pursued mathematically although it is always possible to use mathematics. However, we can't use boat building to solve mathematical problems.

A comparison with language serves as a useful counterpoint. The fact that we use language in many aspects of our lives – when we engage in conversations with others, read a book, watch a movie, send messages to others using a computer or a mobile phone and so on – does not mean that we are engaged in linguistics or semiotics. Similarly, it does not necessarily mean that when people engage in arithmetic and geometric calculations required for everyday activities or when craftsmen such as the Bengalese *Karigar* employ arithmetic and measurement in the building of boats, that we should label their practices as mathematics. This is not to discount the fact that it is always possible to employ mathematics to solve problems encountered in activities such as cooking or boat building. So, just such as we distinguish between linguistics and language use, so it seems that a distinction between mathematics and computations is required.

Why then is there an insistence on referring to the boat builder's practice as mathematics?

INTERROGATING THE NOTION OF EMBEDDED MATHEMATICS

Ethnomathematics, which emerged as a challenge to the dominance of Eurocentric conceptions of mathematics and the history of mathematics, claims that Eurocentric myths propagate the idea that mathematics was created by European males and as such devalue and ignore the contributions of colonized people to the body of mathematics knowledge (Powell & Frankenstein, 1997). In response to the Eurocentric myth, Ethnomathematics celebrates and uncovers mathematics that they claim is 'frozen' in the practices of different cultural groups (see for example, Gerdes, 1985; Knijnik, 1997, 2004; Mukhopadhyay, 2013).

Dowling (1998, p. 14) argues that Ethnomathematics "succeeds in celebrating non-European cultural practices only by describing them in European mathematical terms". Revealing the "embedded" mathematics content of practices that might be perceived as "primitive" serves as a mechanism to elevate the cultural practices of oppressed or formerly oppressed groups and liberate the practitioners. Similarly, Dowling's (ibid) critique can be levelled at Mukhopadhyay (2013), who claims that since the practices of the boat builders are not recognised as mathematics, their work is devalued. While labelling the practice of the Bengalese *Karigar* as mathematics appears to be an attempt to elevate their work, it could be argued that such re-labelling actually participates in devaluing the work and knowledge of the *Karigar*.

"Defrosting" mathematics that is said to be "embedded" in cultural practices is an attempt by Ethnomathematics to reverse the impact of colonialism on the cultural practices of colonized people. However, Dowling claims that such a position is a conservative one because it essentially achieves an insertion into the dominant ideology rather than subverting it. Although Dowling's (1998) critique reveals the potentially anti-emancipatory effect of Ethnomathematics, his analysis is not sufficiently radical.

Drawing on the work of Žižek (2000), we note that Ethnomathematics portrays the political and economic struggle against colonialism as a cultural struggle, a struggle for the rightful recognition of knowledge contribution, in this case mathematics. What we have is a case of a political struggle masquerading as a cultural struggle, which leaves intact and unquestioned the economic system, which structures capitalist societies. Ethnomathematics essentially represents a postmodern rewriting of the political struggle as a cultural struggle, which is an attempt at solving a partial hegemonic struggle without taking into account the structuring effect of the economic principle on all spheres of society (Žižek, 2000). This is not to say that that the marginalisation of the powerless should be reduced solely to an economic argument. As Žižek (ibid) argues, we do not need to choose between Marxist essentialist discourse and the postmodern relativist discourse. What we need is to understand the conditions that enable current hegemonic struggles. The following example illustrates the problem of focusing narrowly on cultural recognition.

The *San* people use *Hoodia*, a cactus-like plant growing in the Namib dessert, as an appetite suppressant. This knowledge was recognised by South Africa's Council for Scientific and Industrial Research, which appropriated and patented the knowledge. The patent was sold to *Unilever*, a multinational company, which produces and markets weight loss supplements using extracts from *Hoodia*. This is an instance of cultural recognition for the purpose of extracting surplus value.

UNDERSTANDING THE INTELLUCTAL-MANUAL DISTINCTION

Sohn-Rethel (1978) in his book, *Manual and Intellectual labour: a Critique of Epistemology*, locates the distribution of manual labour to working-class people and intellectual labour to the middle-class in the form that commodity-exchange takes within capitalist societies. From his analysis of Marx's notions of *commodity fetishism* and *real abstraction*, Sohn-Rethel (1978) argues that there is nothing essential about the distribution of manual and intellectual labour in capitalist societies. The distribution, he posits, is an effect of commodity exchange. He distinguishes between two modes of thought, the "practical" and the "theoretical", arguing that the two modes of thought result from an abstraction that is located in the act of the commodity exchange. He claims that it is the structure of commodity-exchange that although it is not thought itself, has the form of abstract thought. Sohn-Rethel argues, that in the act of exchange, commodities are detached from their specific use-values to become abstract units of exchange-value. As such, commodity-exchange provides the form of abstraction that is necessary for the elaboration of theoretical thought.

Thus, it is not the practice that individuals engage in that strips them of their power. The latter is an effect of an individual's position in the division of labour. Consequently, casting a mathematical gaze on the boat builder's practice does not alter the effects of capitalism in contemporary liberal democracies. As the case of the *San* illustrates, recognition of their knowledge does little to challenge the relations of power.

It is important to note, though, that the distribution of manual and intellectual labour along social class lines is a statistical phenomenon, and as such operates at the level of the group not at the level of the individual. It is quite possible for individuals to relate to knowledge in a way different to the general trend. Take for example, the poor Indian post-office clerk, Srinivasa Ramanujan, who without any formal education in mathematics and through self-study contributed significantly to field of mathematics, particularly in relation to number theory. Working in isolation, he independently produced results arrived at by other mathematicians but also produced unique results such as the *Ramanujan prime* and *Ramanujan theta function*. Mathematicians such as G. H. Hardy and Ken Ono later proved some of the results recorded by Ramanujan. His notebooks are currently still mined for new insights into mathematics (Andrews & Berndt, 2012). Ramanujan's story counters the idea that "those without power" are incapable of producing mathematics. The difference between the work of Ramanujan and that of the Bengalese boat builders is that his work is unmistakeably mathematics. The problems that he generated are problems that can only be solved using mathematics that exists or by developing new mathematical ideas.

CONCLUDING REMARKS

The plight of the disenfranchised remains a concern as we can see from the plethora of studies addressing the issue of equity in mathematics education and the continued under-performance of poor students in mathematics. But relabeling practices, such as boat building, as mathematics does very little to alter the structural effects of capitalist economies on the lives of marginalised people. The political agenda of Ethnomathematics is an admirable one. However, as discussed it fails to challenge the relations of power.

NOTES

1. Michelin stars are awarded by *Guide Michelin,* a series of annual guidebooks, to European restaurants as an index of the quality of the food served.

2. Here I use the term "field of knowledge" loosely. It is not restricted to disciplinary fields.

3. Commodity fetishism refers to social relations that are expressed in the value of a commodity, but where the value becomes detached from social relations and is treated as though it inheres in the commodity in the form of a particular monetary value.

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COMMENTS ON "THE MATHEMATICAL PRACTICES OF THOSE WITHOUT POWER"

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Mukhopadhyay's (2013) paper provoked my thinking. It confirmed ideas about the devaluing of the mathematical practices and cognitions of the poor and othered. The piece focuses on a group of fishermen in Bengal who build seaworthy boats that sail the deep oceans around India. I have focused my remarks around the idea of "those with out power", and the potential implications of Mukhopadhyay's work for mathematics classroom settings.

THE MATHEMATICS OF THE POOR

My grandmother had three daughters – my mother and my two aunts. When Eastertime came each year, she would need two dresses for each of her girls – one for the morning church service and the other for the Children's Program that afternoon. Those familiar with the African American Baptist church can appreciate the significance of this springtime event, including the pageantry and importance of looking one's best. Preparation began weeks, even months in advance. My grandmother would begin by cutting a brown paper bag open – making it flat. She would then draw the outline for the first dress – denoting the collar, the sleeves, yoke, and waistline in pencil. Once complete and cut out with the chosen piece of dress fabric, that same pattern became the backdrop for her next little girl's dress. A smaller version of the first, this new pattern honoured my mother's proportions (the middle child) with exacting precision. Finally, just as with the previous hand drawn pattern, my youngest aunt's dress was drawn from that same brown paper-bag pattern.

A pair of scissors, a well-used measuring tape, the bottom of a small can of Crisco shortening to draw round things, the side of a baking soda box for something straight. How did this young woman, who grew up sharecropping, design six exquisite dresses from two brown paper bags? When did she learn proportionality, principles of similarity, measurement, and design? When I asked my grandmother why she did not just purchase the patterns at the store, given all the work it took to make clothes for her three girls. She explained plainly how purchasing six separate patterns was not only far beyond her reach economically, but also how the store-bought products were completely inadequate for her task. "Your mom was always a little chubby and the patterns in the store did not account for that."

Mukhopadhyay shares the story of a group of fishermen working in a coastal village in Bengal. Like my grandmother used to, these boat makers design products of great value, ingenuity, precision, and quality. They use simple, sometimes handmade tools. They are resourceful. They throw away little, repair what gets broken, and take great care of what they have. They find written blueprints to be both unnecessary and limited in their usefulness. Can a written blueprint account for a knot or a particular unevenness in a piece of wood? Can a store-bought pattern account for a smaller, but chubbier child?

I find the precision of the fisherman (by way of their great attention to context) remarkable. As I was taught that the power of mathematics lay in its generalizability, in its ability to explain “any such” phenomena and to be context free; I found the fishermen’s responsiveness to context enlightening. How many times in working on a textbook math problem have we encountered solution methods for “special cases?” As in so many other hegemonic discourses, the narrative does not align with the reality. No one system of doing mathematics could account for the entire human experience. There are many ways of knowing and engaging. Mathematical complexity, elegance, and sophistication are not reserved only for the rulers. As Sleeter (1997) writes, “school mathematics is a very narrow subset of the range of mathematical thinking in which people have engaged” (p. 683).

Mukhopadhyay writes, “the practices of those without power are often characterized as non-mathematical”. She chronicles ways in which conquered peoples’ mathematical practices were deliberately devalued, and systematically replaced. The research she has cited including Raju’s (2007) discussion of the racist narrative of Western academic mathematics; Bishop’s 1990 explication of the role of Western cultural imperialism in convincing the rest of the world of their intellectual inferiority; Urton’s treatise of how the Incan quipo system of accounting was replaced by Spanish accounting practices, and the deliberate removal of African games during colonial occupation (Zaslavsky, 1973), ought to become required reading in teacher preparation and math methods courses around the globe. Overwhelmingly, those preparing to become teachers see poor, minority, landless, inner-city, Black, non-English speaking, (insert your local euphemism here), students as problems. Such students are in need of motivation, better parenting, more discipline, greater work ethic, more drills and fact building, self-efficacy, better values, more middle-classness, etc.

Mukhopadhyay writes, “I do not see any direct link between my study of boat-builders and school mathematics in direct insertion of activities related to boat-building into a school curriculum”. The links, she states are related to providing an, “alternative conceptualization of mathematical knowledge and learning”. Can such a new conceptualization be understated? How might schools look today if the cultural practices and ways of knowing of all communities were equally embraced? Because, as Mukhopadhyay shows through these six Bengali fishermen, powerless people do not need better values, they need better *valuing*.

Mukhopadhyay’s piece leaves the reader with much to consider about the nature of mathematics and implications for the formal classroom. I would like to add to those considerations, and push on Mukhopadhyay’s ideas related to “those without power”.

I offer that while the Bengali fishermen are currently “without power” they are poised to acquire some. I offer further commentary on those in schools today who I believe neither have power, nor are poised to obtain any. Mukhopadhyay’s piece, I suggest, might serve as strength-building for these particularly vulnerable students. These comments are offered in the spirit of collegiality and out of hopes of encouraging greater discussion and attention to the needs of students in schools. I begin with a brief story.

A STORY

I visited Africa (Ghana) for the first time in 2012. While I had longed to go since I was a child, the money, opportunity, and time to travel, had never previously presented itself to me. My experience in Ghana was one of great pain and transformation. Growing up as a conscious Black person in the US, one anticipates the joys and emotional complexities of going to Africa. The reality, however, is beyond what one could ever psychically portend. For the first time in my life, I was home. There was something empowering about knowing that I had a Family, a Culture, a People, and a place on the earth. I had come from somewhere.

Even though Western eyes see the people in Ghana as powerless, I found them to be incredibly powerful because they knew who they were. They were centered and assured, poor but not broken. They were builders, shoemakers, tailors, small business owners, braiders, and sellers. The fishermen in Mukhopadhyay’s text seem similarly powerful. They had technical skills in demand in their community (the fishermen spoke of having had Government contracts to build boats), they spoke several different languages, owned their own tools, possessed the know-how to repair them, operated in frugality, and had strong connections to a home community. This is in stark contrast to the Black youth I encounter in the US. These young people struggle to find any foothold in the academic world of schools and later find themselves at the lowest rung of the American economy. They are unskilled, do not possess their ancestor’s values of resourcefulness, and the one language that they speak is a stigmatized variety (Baugh, 1999). When you are landless, language-less, and have been taught that you are culture-less, then you are indeed powerless.

This powerlessness is not reserved to black students in the US. I have seen it amongst my Chicano students in California, have read about it in relation to Native children in Utah, Washington State, and North and South Dakota, and am certain that it exists around the globe. As communities become more de-centered through forces of globalization, it is likely that we will all encounter more of these students and communities in our educational settings. These may very well be the most vulnerable and volatile populations of students ever. Mukhopadhyay’s work speaks squarely to this dilemma. If these six fishermen of Frasergunj, with their great skill, and models of apprenticeship are struggling in the world’s economy, then what of the culturally de-centered masses that we encounter in schools everyday? While poor, these fishermen, my grandmother, and the Ghanaians seem better poised than many of the students I encounter today. By way of their cultural rootedness, these powerless people persist. This does not discount the incredible struggle for dignity of the materially dispossessed. Rather, it is to shed light on the task before us as mathematics educators. I am concerned that too much of our attention goes to the technical aspects of teaching mathematics – like standards and testing, when our central concern should be our students' centeredness in the world, their sense of who they are, and their very dignity (Bennett, 2003; Chavous, Bernat, Schmeelk-Cone, Caldwell, Kohn-Wood, & Zimmerman, 2003; Trubea, 1988). We need to take seriously the idea that those who do not know who they are, are desperately vulnerable both in this world and in the world to come. Taken as truth, the issues of culturally responsive pedagogy, and of whose mathematics get placed on the table is not peripheral, but central to our task as educators. I end with an excerpt from Malcolm X's speech, "Malcolm X on Afro-American History," which speaks to us still today:

> …when you are in tune with yourself, your very nature has harmony, has rhythm, has mathematics. You can build. You don't even need anybody to tech you how to build. You play music by ear. You dance by how you're feeling. You have it in you to do it. I know Black brickmasons from the south who have never been to school a day in their life. They throw more bricks together and you don't know how they learned how to do it, but they know how to do it. When you see one of those other people doing it, they've been to school- somebody had to teach them. But nobody teaches you always what you know how to do. It just comes to you. That's what makes you dangerous. When you come to yourself, a whole lot of other things will start coming to you, and the man knows it.

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SYMPOSIA

EXPLORING THE RELATIONSHIP BETWEEN IN-SERVICE MATHEMATICS TEACHER SUPPORT AND RETENTION

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The aim of this symposium is to stimulate active participation and debate on the relationship between teacher support and teacher retention. While large scale 'fix-it' Breen (1999) approaches to teacher support often work to alienate teachers from their profession and their professionalism, longer term teacher support in well-functioning communities of practice can work to strengthen teacher investment in the profession, enable teacher leadership and strengthen teacher professionalism. In this symposium we wish to draw on our experience as organising members of Discussion Group 11: Mathematics Teacher Retention at ICME-12, which illuminated similar and diverse experiences of issues across various contexts, to further debate, discuss and build insights to this topic at MES7.

INTRODUCTION AND RATIONALE

Teacher retention and particularly mathematics teacher retention seems to be a universal challenge even while the scale and nature of that challenge differs across various contexts. In the USA the modal number of years of experience of mathematics teachers is 1, over 20% of the teachers leave in the first year and over 55% leave in their first five years. While in South Africa such high turnover might not be the case an OECD (2008) report indicates that 55% of teachers indicate that they would leave the profession if they could.

During ICME12 we discovered that while there were many differences in our contexts and the challenges of working in those contexts our findings of what emerged from our longitudinal work with teachers indicated strong similarities. In particular notions of leadership, belonging, and shifting identities emerged across contexts. In this symposium each of our teams will present a brief stimulus presentation on our work in relation to mathematics teacher support with the aim that these will provide a stimulus for rich discussion. We also invite delegates to briefly share their experiences in relation to this topic. The discussion that follows will then focus on the following clusters of questions for discussion.

Discussion questions

1st Cluster: What purpose and value do communities of practice bring to mathematics teachers? Do communities of practice (cops) emerge as a by-product of professional development or are they purposefully created? What are the key enablers of cops that enable teacher support that promotes leadership, professionalism and teacher retention?

2nd Cluster: Is there a relationship between strengthened mathematical professional identities and teacher retention? If so what is the nature of this relationship? How does an absence of professional status and negative identities portrayed of teachers in some forms of 'professional development', research and/or the press affect teacher retention?

SHARING OUR EXPERIENCES

Some South African experiences

South Africa's radical post-apartheid curriculum change delineated new roles for teachers and teachers (and education) were charged with building a new democratic South Africa. Seventeen years later the dominant discourse is that our education system (and particularly mathematics education) is 'in crisis' (see Fleisch, 2008). Research into this crisis (and particularly the mathematics crisis) points towards teachers as one of several key factors responsible for our dismal performance on international (e.g., TIMMS), regional (Carnoy et al., 2011; SACMEQIII, 2010), and national departmental assessments (DoE, 2008) in mathematics.

Teacher morale is at an all-time low with a large percentage of teachers indicating that they would leave the profession if they could (OECD, 2008). Teacher attrition is far greater in subject areas such as mathematics and science, as these skills are highly sought after and thus these teachers are more able to get employment outside of the teaching profession.

Against this backdrop we have evidence across a range of teacher development projects in South Africa (e.g., Graven 2005; Graven 2012) that long term teacher support that positions teachers as partners and in which their experiences are taken as the basis from which engagement and learning takes place, enables teachers to re-invest in the profession with increased passion and confidence. The voices of teachers gathered across such projects point to the importance of the ethos of the in-service support; the importance of belonging to the community; the emergence of more confident forms of participation in multiple practices, the emergence of life long learner identities and the emergence of long term mathematics teaching trajectories (often with leadership roles). In the stimulus presentation we will share some teacher utterances in relation to each of these.

Some USA experiences

In the U.S., providing good teachers for all students goes beyond recruitment to teacher retention. Over the past two decades, the analyses of teacher employment patterns reveal that new recruits leave their school and teaching a short time after they enter resulting in the reference to teaching as a "revolving door". This "revolving door" is even more acute in urban and low-income districts (Smith & Ingersoll, 2003; Ingersoll & Perda, 2010; Ball, 2012; Pence, 2012).

Reasons for the lack of retention of new teachers and teacher in high-poverty schools are often described as "working conditions". High on the list of dimensions key to retaining teachers is that of support. Components of this support include professional and collegial support such as working collaboratively with colleagues; coherent, job-embedded, professional development; and increasing leadership opportunities (Johnson, 2006; Ingersoll & May, 2010, Pence 2012).

Building on these dimensions, a recent 5 year professional development project documented a decrease in yearly attrition from 20+% to 6.2%. Additional patterns included increased knowledge of content and content pedagogy, increased confidence, quality of teaching, and leadership, and development of communities of practice. Teacher reflections attributed these patterns to a multi-year professional development program that (1) went beyond mathematics content and pedagogy to focus on establishing teaching as a "noble" profession requiring work and preparation, growth that was complex, on-going, and supported the realization that there was a great deal to learn, and (2) built and supported a professional community of practice (Pence, 2012).

SESSION PLAN

First session

- Brief introduction to the aim of the symposium and the symposium organisers (5 minutes)
- Brief introductions of all and reason for participants interest (10 minutes)
- Stimulus presentations across the South African and Californian contexts (2 x 10 minutes + 10 minutes question and discussions following each)
- Discussion of key questions and other issues for further discussion in next session + introduction of 2nd session presenters (remainder of the first session)

Second session

- Stimulus inputs (pre-arranged and ad hoc) from audience participators followed by a few questions on each (up to 40 minutes)
- Discussion of key questions and discussion points that emerge from these stimulus inputs (40 minutes)
- Ideas for continued collaboration across contexts – where to from here? (10 minutes)

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THE SOCIAL FUNCTION OF MATHEMATICS EXAMINATION QUESTIONS

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In this symposium we will use a range of discursive approaches to explore the positions made available to students by examination questions in mathematics, and the ways in which students negotiate these positions. We will use examples from research to stimulate discussion around power relations, inequalities, expertise, the role of assessment and the possibilities of different theoretical approaches.

OVERVIEW

Assessment is important in positioning mathematics as a discipline and students in relation to mathematics and schools. It is a source of considerable power, anxiety and inequality (Black et al., 2009; Gipps & Murphy, 1994). This symposium held over a single session will take an unusual focus in looking not at the relationship between examination questions and mathematics, but in looking at their relationship with students' identities. Mathematics education research into assessment has looked at the relationship between assessment questions and mathematics (e.g., Van den Heuvel-Panhuizen, 2005), and the impact of assessment systems on student engagement and equity (e.g., Boaler & Wiliam, 2001). Yet wider education research evidences the profound impact of assessment on student identities (e.g., Reay & Wiliam, 1999). This symposium will thus be unusual within mathematics education, in taking up this focus on identity by exploring the positions made available to students within examination questions and their role in classroom practice.

The session will be split into two sections, the first focussing on how examination questions position students and the second on how students position themselves in relation to examination questions. Each section will begin with a short presentation of research data, followed by guided discussion.

HOW DO EXAMINATION QUESTIONS POSITION STUDENTS?

In this section Candia Morgan and Heather Mendick will use two contrasting discursive perspectives – social semiotic and Foucauldian respectively – to explore a selected question from an examination taken by students aged 16 in the UK.

From a social semiotic perspective, language not only construes our experience of the world but also construes our identities and relationships to each other and to our experiences (Halliday, 1978; Morgan, 2006). Powerful texts such as textbooks and examination papers provide specific positions for students, that is, ways in which students may interact with the text and act within the practice of school mathematics. Of course, it is possible for individuals to resist such positioning but the text provides a 'natural' way of reading (Hodge & Kress, 1993). The approach Candia will use is drawn from the *Evolution of the Discourse of School Mathematics* project (Tang, Morgan, & Sfard, 2012; Funded by the ESRC, grant reference: RES-062-23-2880) which views examination papers as a window onto the forms of mathematical activity expected of students. The analytic framework developed for this includes tools drawn from Systemic Functional Linguistics and Sfard's (2008) theory of commognition that provide insight into the positioning of the students in relation to mathematical activity.

From a Foucauldian perspective, language is not descriptive but constructive (Foucault, 1972; MacLure, 2003). Language is part of discourses, which are knowledges about objects, through which those objects (including ability, grades and even mathematics itself) come into being. Thus, analysing texts – be they examination questions, films or items of clothing – involves unpicking the discourses through which they make sense and that they, in turn, enact. Within this approach, identity is thought of as subjectivity to indicate it not as some fixed and stable essence but as always in-process, as one is positioned by and positions oneself within multiple, often contradictory, discourses (Davies, 1993). So although subjectivities cannot be read off from discourses in any straightforward way, discourses do set limits on what it is possible to say, think, feel and be and so are intimately connected with power. In her analysis, Heather will focus on the subject positions available to students who engage with the selected examination question.

Discussion questions

1. What do the two different theoretical perspectives bring out, and what other theoretical resources can we use to make sense of examination questions?
2. What difference does it make that this question is an 'examination question'?
3. Given the positions offered by this question, who can occupy them and in what ways? How does this feed into wider patterns of access in mathematics education?

HOW DO STUDENTS POSITION THEMSELVES IN RELATION TO EXAMINATION QUESTIONS?

In this second section Cathy Smith will shift the perspective to the ways that students engage with examination questions in constructing accounts of their mathematics practices, exploring some data from interviews with 16-18 year olds participating in advanced mathematics.

The analytic focus is what Foucault (1990) calls 'practices of the self': the discursive strategies that inscribe what it means to be a successful individual within a particular historical and social context. Practices of the self establish the norms and means by which people can explain themselves, govern themselves and attend to themselves *as* selves. Thus accounts of working on post-compulsory school mathematics can be read as constructing positions for students as adolescents, as learners and as 'choosers of mathematics'. It is then possible to examine the practices of becoming a mathematics student as practices that construct wider, social and political meanings (Smith, 2010).

In these accounts, examination questions function as a resource through which actors can position themselves in situated, shifting, subtle relations of power (Valero, 2005). The data excerpts that Cathy presents raise questions about how students make use of the authority that resides in examination questions to construct themselves in relation to mathematics and teachers. Working with examination questions typically comes towards the end of a programme, so that students have to negotiate their positions in terms of expertise. There is an expected graduation from apprenticeship in terms of mathematics practices and also in the practices of self-control and reflective self-knowledge that we attribute to experience (Rose, 1999). Students are scrutinised by examination questions but are positioned as able to prove themselves. Relationships between student, teacher and text are supplemented by the 'author'-ity of the examiners, and new positions emerge for students to express themselves as compliant, dependent and/or independent. One focus is on how students shift their accounts of identity and relationships in mathematics between talking about examination questions and talking about other classroom work, and what this might have to tell us about dominant discourses and possibilities for thinking otherwise.

Discussion questions

1. What power relations are played out through students and teachers working towards examination questions?
2. How does this affect the possibilities for students' relationships with mathematics?
3. How does the teacher's role as 'expert' impact on the possibility of students constructing themselves as autonomous and/or mathematicians?

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TEACHING MATHEMATICS FOR SOCIAL JUSTICE: CONVERSATIONS WITH EDUCATORS

Symposium Coordinators:

David W. Stinson — Georgia State University

Anita A. Wager — University of Wisconsin-Madison

Symposium Presenters – Tonya Gau Bartell, Brian R. Evans, Eric (Rico) Gutstein and Jacqueline Leonard

Symposium Discussants – Victoria Hand and Joi Spencer

Using Marilyn Frankenstein's germinal 1983 article "Critical Mathematics Education: An Application of Paulo Freire's Epistemology" and Ole Skovsmose's 1985 germinal article "Mathematics Education Versus Critical Education" as credible "start points", critical mathematics or more broadly, social justice mathematics, is marking three decades of empowering yet uncertain possibilities. Nonetheless, there are two recurring questions: What is it? and What does it "look like"? Drawing on the collective stories (and wisdom) of critical mathematics educators, this symposium aims to offer some open, non-definitive answers to these two questions.

AIMS OF SYMPOSIUM[1]

This symposium aims to engage MES7 delegates in a critical, interactive discussion on the recently released, edited volume *Teaching Mathematics for Social Justice: Conversations with Educators* (Wager & Stinson 2012), and on critical/social justice mathematics in general. The symposium participants include leading mathematics teacher educators and researchers, who have explored, developed, researched, and/or taught mathematics for social change. The symposium presenters will not only share personal narratives of how they came to do this important work but also offer theoretical, methodological, and pedagogical propositions in solidarity with others who might wish to explore the empowering uncertainties of teaching (and learning) mathematics for social justice.

The book in general was organized into three key sections intended to guide readers through the historical and theoretical development of critical/social justice mathematics, the teaching of teachers in how to teach mathematics for social justice, and the possibilities and challenges of teaching mathematics for social justice (TMfSJ) in classrooms. The symposium presenters are four contributing authors and the two co-editors who will provide, in turn, an overview of their respective chapter and the book in general. Two outside discussants will critique not only the work presented but also the open-ended challenges and promises of critical/social justice mathematics.

RELEVANCE OF SYMPOSIUM

The phrase *teaching for social justice* is increasingly visible within discourses surrounding education. Specifically, in teacher education, “social justice” is emphasized as part of teachers’ overall “diversity” or “multicultural” initial preparation or professional development (McDonald 2007). More generally, social justice is often found in the mission and vision statements of education organizations, in the overarching goals and objectives of education conferences and associations, in the titles of “special issues” of scholarly journals, and in the titles of an increasing number of books. After all, who in education would claim that they’re *not* for social justice?

The intent of the symposium (and of the book in general), however, is not to provide a definitive definition of social justice or, more specifically, critical/social justice mathematics but rather to provoke more questions and to stimulate new discussions about the many meanings of and possibilities for TMfSJ. In other words, echoing Bartell (2011), the symposium’s participants view teaching (mathematics) for social justice as a “sliding signifier,” which suggests that defining what teaching for social justice “actually means is struggled over, in the same way that concepts such as democracy are subject to different senses by different groups with sometimes radically different ideological and educational agendas” (Michael W. Apple, as quoted in Bartell 2011, p. 2).

Viewing TMfSJ as a sliding signifier springs from the symposium participants’ desire to ask MES7 delegates to enter into conversations as they travel on an individual and collective journey in discovering the possible meaning(s) of teaching for social justice in general and TMfSJ in particular. The metaphor *to travel* is borrowed from Marilyn Cochran-Smith’s (2004) book *Walking the Road: Race, Diversity, and Social Justice in Teacher Education* and Ole Skovsmose’s (2005) book *Travelling Through Education: Uncertainty, Mathematics, Responsibility*.

Cochran-Smith (2004) notes that her metaphor of traveling – or walking the road – “makes the case that doing teacher education for social justice is an ongoing, over-the-long-haul kind of process for prospective teachers as well as for teacher education practitioners, researchers, and policy analysts” (p. vxviii). Her metaphor of walking the road also represents her personal journey of over two decades in which she has focused seriously on issues of race, diversity, and social justice in teacher education practice, policy, and research at local, state, national, and international levels.

Skovsmose (2005), who positions social justice mathematics as just one approach to critical mathematics, continues to reconceptualize the open and uncertain possibilities of a critical mathematics education. In so doing, he not only speaks about traveling through different philosophical considerations but also physically traveling through different places around the world, experiencing different people, different cultures, different educational contexts—and different possibilities. Skovsmose claims that traveling through differences constitutes the turbulent development of critical mathematics, as aspirations and hopes are continuously recontextualized and reformulated, and uncertainties appear (Skovsmose 2009).

Similarly, although each symposium participant will provide her or his own unique, nuanced definition or description of critical/social justice mathematics, these descriptions have developed over time during her or his own journey and therefore are fluid and continue to change and adjust. Nonetheless, an overarching theme that is somewhat present in each description is a goal for teaching mathematics *about*, *with*, and *for* social justice (Wager 2008). Teaching mathematics *about* social justice refers to the context of lessons that explore critical (and oftentimes controversial) social issues using mathematics. Teaching mathematics *with* social justice refers to the pedagogical practices that encourage a co-created classroom and provides a classroom culture that encourages opportunities for equal participation and status. And teaching mathematics *for* social justice is the underlying belief that mathematics can and should be taught in a way that supports students in using mathematics to challenge the injustices of the status quo as they learn to read and *re*write their world (Gutstein 2006).

But in the end, neither Cochran-Smith (2004) or Skovsmose (2005) nor the symposium participants provide a simple, linear, or certain mapping of social justice for other travelers to journey. Indeed, Cochran-Smith notes that learning to teach for social justice, for teachers and teacher educators alike, "is a long road with 'unlearning' a rugged but unavoidable part of a journey during which people double back, turn around, start and stop, reach dead ends, and yet, sometimes, forge on" (p.xx). Likewise, Skovsmose claims that attempts to bring clarification or meanings to a concept such as critical (or social justice) mathematics often takes us in the opposite direction of any fixed meaning in which "clarification of 'something' brings us to consider 'everything'" (p. 216). We hope MES7 delegates will be inspired by the symposium participants' journeys and undertake their own journey of making meaning(s) of teaching (mathematics) for social justice, going through their own process of considering everything as they consider something—starting, stopping, and even sometimes doubling back. Undeniably, "TMfSJ is a journey, not a destination" (Stinson, Bidwell, & Powell 2012, p. 88).

PLAN OF SYMPOSIUM

The 90-minute symposium will be structured as follows:

1. Symposium coordinator (co-editor) will provide a brief introduction of the goals and objects of the symposium and a general overview of the motivation behind and the development of the book (10 minutes);
2. Symposium presenters (contributing authors) will provide brief overviews of their respective chapter (three chapters, 15 minutes each);
3. Symposium discussants will provide a critique of not only the work presented but also the open-ended challenges and promises of critical/social justice mathematics (15 minutes);

4. Symposium coordinator (co-editor) will facilitate a semi-structured question-and-answer session; possible questions include (20 minutes):
    a. Has critical/social justice mathematics moved to the "center" (as indicated by the support of the National Council of Teachers of Mathematics)? If so, at what cost?
    b. How might teachers begin to teach mathematics for social justice? How might teacher educators begin to teach teachers (pre- and in-service) how to teach mathematics for social justice?
    c. After 30 years, just where is the mathematics education community in regards to critical/social justice mathematics? What's next?

NOTES

1. The text from *Aims of Symposium* and *Relevance of Symposium* sections was extracted and revised from the introductory chapter of the book (Wager & Stinson, 2012).

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UNDERSTANDING THE PREVALENCE OF CONCRETE WORKING WITH NUMBER ACROSS TEACHING AND LEARNING IN FOUNDATION PHASE

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In this symposium, the aim is to bring together a group of delegates who have written about/are interested in issues relating to the ongoing prevalence of concrete strategies for working with number in Foundation Phase (Grades R-3) Numeracy classrooms in South Africa. The aim is to learn about what different perspectives on the 'problem' (over reliance on concrete ways of working with number) allow us to 'see' in terms of the phenomenon itself, and how and why it is produced. We also discuss what different lenses – sociological, sociocultural and discursive – might suggest in terms of implications for teacher education for improving access and success with early number learning through supporting the building of coherence and progression within instruction. Where interventions based on these different theoretical orientations have been attempted, the symposium will also allow space for presentation and discussion of these projects and their findings, taking in aspects of the curriculum contexts in which these interventions are located.

BACKGROUND AND RATIONALE

Schollar's (2008) data on learner work on number points to the existence of tally counting as a common strategy amongst learners well into the Intermediate Phase (grades 4-6). Baseline data within the Wits Maths Connect – Primary Project also notes that 'count all' based strategies – which rely on concrete counting, were seen across three quarters of the Grade 2 learner sample drawn from across the attainment range (Venkat, 2011). In their data set, even the highest attainers reverted to unit counting when the number range increased beyond the level of immediate recall. Broader analyses of teaching in this context have led to the identification of 'extreme localisation', a phenomenon within which disconnection within and between episodes in lessons produces an orientation to the ongoing production of answers by empirical verification, rather than through setting up some established facts that can be used in the derivation of further results (Venkat & Naidoo, 2012). Lack of coherence and disconnection in this work are understood through using tools drawn from systemic functional linguistics (Halliday & Hasan, 1985) and variation theory (Marton, Runesson, & Tsui, 2004; Watson & Mason, 2006)

The work of Hoadley (2007) and Ensor et al. (2009) has also pointed to the prevalence of concrete strategies in disadvantaged settings. Hoadley (2007) explains this prevalence in terms of more restricted orientations to text and meanings in these school settings. The work of Ensor et al. (2009) also points to teaching 'holding learners back' in concrete strategies through insufficient specialization of content and modes of representation. Allied with weak pacing of content, they note the ongoing presence of tasks focused on counting and linked with the ongoing presence of resources that allow and accept concrete counting as a means of solution of early number problems. This work from the Cape Town-based group draws on sociological theory to understand the prevalence of concrete strategies for working with number amongst teachers and learners – Bernsteinian lenses in Hoadley's work and Dowling's (1998) work based on the idea of 'specialisation strategies' as a requirement for leading into the esoteric domain of mathematics, in Ensor et al's study. Venkat & Askew (forthcoming) echo this finding through a specific focus on episodes discussing the ways in which teachers mediate number learning through the use of 'structured' artifacts in 'unstructured' ways. Building on Vygotskian ideas of artifact mediated action, and using Cole's (1996) distinction between artifacts and tools and Wertsch's (1995) emphasis on the relationship between artifacts and culture, they analyse teachers' use of resources within the teaching of early number. Recent data collected within our teacher education and classroom contexts also points to the ways in which concrete counting allied with column algorithms promotes a reduction of all number working to single digit number operation resulting in a bypassing of what literature would describe as good number sense, whilst allowing for the production of correct answers that allow the lesson to proceed.

In the body of work investigating the teaching of early number, theoretical positions drawn from sociological, sociocultural and discursive orientations have been drawn upon. Each of these orientations contains elements that reflect the broader concern with issues of equity, access and social justice that form the central interests of the MES community. At the broadest level, the impetus for the focus on early number teaching and learning remains a concern with the bimodal patterns of performance in mathematics in South Africa (Fleisch, 2008) where a minority of learners achieve highly, and the majority (which continues to be constituted as mainly poor and black learners) vastly underperform in relation to curriculum specifications. This a pattern appears to be lodged firmly in place by the end of Foundation Phase with poor national mean performance in numeracy established and seen in the Annual National Assessment results at the end of Grade 3 (Department for Basic Education, 2011a).

Interventions at the level of curriculum revision have abounded in this context of poor performance – the Foundations for Learning 'milestones' based curriculum was introduced in 2009, and the Curriculum and Assessment Policy Statement (CAPS) (Department for Basic Education, 2011b) was introduced in 2012 in the Foundation Phase. In the context of evidence of poor understandings of the 'level' associated with the Assessment Standards in the previous Revised National Curriculum Statement (DoE, 2002), the curriculum response has focused on increased specification of content and progression – as well as prescription of pacing and sequencing. This represents one national policy level response to addressing the problem of lack of progression from more concrete to more abstract conceptions of number.

In the session planned for this symposium, the aim is to focus on understanding the problem through the findings and theoretical frames that have been used to investigate the issue. We will also have some discussion around interventions linked to findings, including discussion of the CAPS model, and to engage with the ways in which support for instruction for early number learning that shifts to the more abstract number concepts and strategies required for engagement with Intermediate Phase mathematics, can be framed.

SYMPOSIUM MODEL – ONE NINETY MINUTE SESSION

Understanding the prevalence of concrete strategies for working with number across teaching and learning in FP

1. Venkat: Using Systemic Functional Linguistics and Variation theory (20 minutes)
2. Ensor, Hoadley, Galant – SPADE team: Using sociological theory (20 minutes)
3. Bowie: Using Vygotskyian theory to link teacher education with learner performance (20 minutes)
4. Discussion across theoretical frames, led by Professor Mike Askew as respondent (30 minutes)

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PROJECT PRESENTATIONS

POWERFUL MATHEMATICS: EMPOWERING LEARNERS

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I argue that in order to promote a just and democratic society, we need to privilege abstract and decontextualized mathematics rather than 'relevant' or everyday mathematics. I use Young's (2008) notion of 'powerful knowledge' to support my arguments.

INTRODUCTION

In this presentation I wish to enter into the debate about what sort of mathematics knowledge may be used to empower sectors of society which historically have little access to power, wealth or resources. I will argue that access to the canon of knowledge traditionally recognized by universities as 'mathematics', with all its abstractions and structures, is necessary for the promotion of democratic ideals in a society.

As I understand it, this position contrasts with some members of the critical mathematics education movement who espouse a view that "the main goal of a critical mathematical literacy is not to understand mathematical concepts better, although that is needed to achieve the goal. Rather it is to understand how to use mathematical ideas in struggles to make the world better" (Frankenstein, 2010).

At the outset I must state that like many of those in the critical maths education movement, I too hope that mathematics may provide a tool which promotes a critical viewing of society and which promotes democracy and equity. However my argument is that a version of mathematics which privileges an understanding of the social structure of society rather than an understanding of abstract mathematics (as codified by the academy of mathematicians), will merely reproduce the inequities and injustices in society. I will largely draw on a social realist view (Young, 2008) to frame my argument.

WHAT IS MATHEMATICS?

At this point, pertinent questions are: What do I mean by 'mathematics'? What is it and wherein lies its value? Historically there are two opposing views on these questions. On the one hand, there is Hogben's (1993) utilitarian view of mathematics:

> "Today, the lives and happiness of individuals depend more than most of us realize upon correct interpretation of public statistics kept by Government offices. Atomic power depends on calculations which may destroy us or may guarantee worldwide freedom from want…Without some understanding of mathematics we lack the language in which to talk intelligently about the forces that now fashion the future of our species" (cited in Sierpinska, p. 14, 1995).

In contrast to this, the famous British mathematician, Hardy (1992), of the mid twentieth century, exalts the decontextualized and abstract nature of mathematics: "Mathematics …must be justified as art if it can be justified at all" (cited in Sierpinska, p. 14, 1995). In essence, Hardy's view of mathematics privileges the internal connections within mathematical theory rather than any external connections to the real world.

Sfard (2008) gives a more elaborated view of the nature of mathematics. She argues that mathematics is a discourse characterised by several key features. It has its own words, for example, triangle, linear equation; its own visual mediators, for example, graphs, diagrams, symbols; specialised routines, including procedures such as long division and practices such as justifications and generalizations; and endorsed narratives, such as theorems and axioms, where the narratives are endorsed by expert mathematicians. School mathematics is a subset of this mathematics although its rules of engagement are made simpler and more transparent by the necessity of enculturating younger learners into its ways and means.

The question then is: what sort of mathematics should we be teaching in the classroom if we wish to promote a just and democratic society in which all citizens are able to critically engage? Currently there are at least four major positions. At the one extreme, there is a view that mathematics should be learned in those contexts in which it is expected to be used by its learners. This view is supported by many situated theorists, who question the notion of transfer. It is also often supported by many critical mathematics educators who recognise the worth and value of the everyday in the lives of the disempowered. A second vision, also subscribed by several critical mathematics educators, focuses on the "use of mathematics to understand relations of power, resource inequities, and disparate opportunities between different social groups and to understand explicit discrimination based on race, class, gender, language, and other differences" (Gutstein, 2003, p. 45). At the other extreme is the idea that mathematics should be taught as an abstract body of knowledge. It is an art (Hardy, 1992) and questions about its usefulness are not relevant to its value. A fourth position is the view that, in order to be ultimately useful in our complex world, the curriculum should promote a version of mathematics that is decontextualized, abstract and general. Such a version would ultimately increase the chances that the mathematics could usefully be applied to real-life situations.

POWERFUL KNOWLEDGE

Using Young's (2008) distinction between 'knowledge of the powerful' and 'powerful knowledge', I suggest that the fourth position (described above) is consistent with the notion of 'powerful knowledge'. Before arguing this point, I need to explain what is meant by these terms. 'Knowledge of the powerful' is the knowledge that the powerful use to keep themselves in power. Such knowledge may take the form of abstract and impersonal discourse and frequently undervalues everyday knowledge. Because of the anti-democratic nature of such knowledge and because of its role in reproducing the inequalities of society, many critical education theorists have argued for its rejection. In contrast, powerful knowledge

> refers to what the knowledge can do or what intellectual power it gives to those who have access to it. *Powerful knowledge* provides more reliable explanations and new ways of thinking about the world and acquiring it and can provide learners with a language for engaging in political, moral and other kinds of debates (Young, 2008, p. 14)

The question is: why is a version of mathematics that promotes abstractions and generalizations 'powerful knowledge'? Why do I regard a version of mathematics that focuses on the relevance of mathematics to the real world as 'less powerful', and thus, paradoxically, promoting the reproduction of societal inequalities?

Young (2008) argues that 'powerful knowledge' is context-independent. It is not tied to specific cases and this gives it its power. It can be generalised and ultimately can be applied to situations beyond the individual's experience. Furthermore, and in line with Young's (2008) arguments, this version of mathematics (which privileges abstractions, generalizations, proofs and conventionally acceptable representations) is powerful in its own terms. Understanding and having a knowledge of this body of mathematics enriches the thinking and intellectual life of the individual, much as say, the study of poetry may do.

THE NATURE OF MATHEMATICS IN THE REAL WORLD

Related to Young's notion of 'powerful knowledge' but from a different perspective, other educators (for example, Sierpinska, 1995) argue that much of what passes for 'mathematics' in everyday life is not mathematics. She argues that "mathematics has to do with representation and generalization; and not with solution of concrete or practical problems which can always be solved with ad hoc methods" (p. 4). Similarly, Christiansen (2007, p. 97) argues that "mathematics beyond simple arithmetic is not really central to performance in everyday situations, because what ever little mathematics is used, it is subordinated to the principles of the activity". In line with these arguments, I suggest that the mathematics that is required in real-life situations and that may contribute to the quality of life of a country's citizens, is often fuzzy and requires a profound understanding of what is often complex and complicated mathematics.

For example, in South Africa, we have recently had court cases and public debates in which the raising of revenue for the cost and upgrade of highways (necessary for a thriving economy) has been hotly debated: different revenue-producing methods (such as tolls on users of these roads or taxes on all drivers in South Africa) have been discussed in terms of efficiency, cost of raising revenue, fairness and so on. Economists, using sophisticated and complex mathematical models, have been employed by all sides to support the various arguments. The type of mathematics used in the economists' mathematical models is difficult – requiring a knowledge of abstract mathematical structures and operations. This is powerful mathematics in action; the results (not yet decided) of these debates, will impact enormously on the finances and daily life of individuals. Similarly, the financial crises in much of Europe and America may have been averted if more concerned citizens, and even politicians, had access to 'powerful mathematics'.

ACCESS TO POWERFUL MATHEMATICS

I am aware that I have not addressed the crucial issue of who gets access to 'powerful knowledge' and how. This is particularly problematic in South Africa where, because of apartheid education, the bulk of high school mathematics teachers themselves have not had access to powerful mathematics. This needs to be the subject of other research. My point here is that we, as mathematics educators, in South Africa and elsewhere, need to recognize the value of abstract, decontextualized mathematics. This is not to say that we do not try to make this mathematics accessible and relevant through applications to real-life phenomena. But these applications need to be in the service of mathematics rather than the other way round. It is only through this privileging of de-contextualized knowledge that citizens will ultimately be empowered to fight for a better and more just future.

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RESEARCH BASED TEACHER EDUCATION; DISCURSIVE POSITIONING OF TEACHER EDUCATORS IN NORWAY

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As researchers in teacher education institutions we are facing ideological and economic shifts involving restrictions on the possibilities of influencing themes and methodologies for research projects.

BACKGROUND

Over the last two decades there have been political discussions on the *quality* of teacher education in Norway, both for schools and for Early Childhood Education and Care (ECEC). This has resulted in a new specialised teacher education for primary and lower secondary school implemented from 2010, and a new revised teacher education for ECEC, starting in 2013. One common aspect is that a greater focus is placed on rhetoric's around a *research based education*. Our proposed project is investigating the term *research based education* and how this conception is positioning teacher educators as researchers. We have previously, through document analysis, examined ideological underpinnings for these new teacher education reforms (Braathe, 2012) and have found that travelling neo-liberal discourses been taken up, merged and transformed in relation to the Norwegian egalitarian school and kindergarten traditions.

THE GOVERNANCE TURN

Ozga (2008), and others, have pointed to the emerging new neo-liberal governance and defined it as a shift "from centralised and vertical hierarchical form of regulation to decentralised, horizontal, networked forms" (p. 266). The new governance produces a range of sophisticated instruments for the steering of education policy; standardisation, quality benchmarking, and data harmonisation. Governance shades into governmentality, particularly here, in our project, through attendance to the interdependence of governing and knowledge production of mathematics education. The new governance promotes the collection and use of comparative data on performing as a way of controlling and shaping behaviour of students and educators. These regulatory mechanisms "act as 'political technologies' which seek to bring persons, organisation and objectives into alignment" (p. 266).

Norway is strongly influenced by this turn in governance by international comparative studies and by the European harmonisation programs like for example the Bologna Accord. In this process education has been translated into learning. This transformation of the field of education is happening at the level of populations and institutions through the reshaping of the old institutions of schooling and post-compulsory education and their replacement with designs for lifelong learning (Braathe & Otterstad, in press). It is happening at the level of knowledge management and knowledge production and policy making, including the steering of research, and "it is happening through the new connections between governing and the creation of new political instruments that are devoted to the creation of data and to constant comparison of data about performance" (Ozga, 2008, p. 267).

Ozga (2008) also points to three key elements in this process that have direct bearing on the research quality relationship on which we are focusing. They are *data, comparison* and the role of *experts*. In this logic there can be no quality without comparison, and data become the resource for comparison. Ozga claims that "Experts develop the new political technologies through which comparison is made possible" (p. 267).

THE QUALITY ERA

Norway entered the Quality era in educational policy influenced by travelling neo-liberal discourses during the 1990's (Braathe 2012). One important influence and driving force was Norway's participation in the PISA international comparative studies. Following Ozga (2008), PISA is one of the most influential representatives of the political technologies. PISA results for Norway were mediocre both in Reading, Mathematics and Science. Since then *quality* of the educational system has appeared in all discourses from Kindergarten to University studies, mathematics education has received special interest. Research in mathematics education in Norway has during the last ten years directly addressed aspects of quality, producing data for comparison, either qualitative or quantitative (Hopfenbeck et al., 2012). Increased focus on mathematics education research is based on the identification of shortcomings in current practices. This has resulted in implementation of new approaches, which are frequently accompanied by a conceptualisation of 'progress' as moving towards an 'ideal' or improved state of affairs (Tzur et al., 2001). Recent policy in Norway aimed at educational improvement prioritises the development of *high quality* teaching as a means of addressing perceived underperformance in the school system.

In 2008 the Government put forward a Strategic Plan for Educational Research (Ministry of Education, 2008). In this, the Government is taking up the language of the governance turn, identified by Ozga (2008). The point of departure for the new strategy is that "[w]e know that the learning in school is too low, and that the students' abilities in reading, writing and mathematics is reduced, [and w]e know that student teachers do not get good enough knowledge on how they as teachers shall find, evaluate, and make use of results from educational research." (Ministry of Education 2008, p. 4, our translation). Additionally, these rhetorical questions are asked: "Are we using the resources well enough?… Do we get enough out of the economic investments?" (p. 4, our translation). As an answer the government put up some priorities as a strategy for resolving these political challenges:

The Government wants to:

- Strengthen and prioritise research where the traditions are weak in Norway, such as effect-research, longitudinal studies, and empiric research based on quantitative data.
- Follow up work with data on individual students.
- Strengthen the scientific competence on pedagogical measuring by establishing a unit for psychometric research.
- Consider if it, within the regulations for buying research assignments, is appropriate to prioritise some research environments to build specific research competencies. (Ministry of Education 2008, p. 14, our translation)

In addition, the universities and colleges engaged in research shall *disseminate the results* that lead to *innovation and value creation,* based on these. Politicians require that educational institutions shall provide updated research. In teacher education this means that the kindergarten and the school, as fields of knowledge, are challenged more than ever both in content, and in methodological and theoretical perspectives. In this official publication politicians require that research contributes to performance, innovation and value creation by signalling that teacher education institutions shall become accountable for this.

Such a shift is perceived to stand in contrast to individual researchers' 'research interests', possibly because education research can be said to have been driven out of researchers' *autonomy* and a tradition of democratic right to *free* research. This discursive shift creates some dilemmas, such as concerns about the research to be controlled from above and/or the research ethics guidelines for educational research to follow.

METHODOLOGY

Our project is based on documents like governmental White papers, strategic research-political documents both governmental and from the Norwegian Research Council, as well as evaluation reports on educational research in Nordic contexts, as text-based data for analysis. A general remark to the content of public publications is that they come with recommendations for investment in strategies *for qualitatively good research*, ie the strengthening of research communities at teacher training institutions. In addition, emphasis is placed on the findings that students during their studies do not get enough knowledge about teacher educators' research projects. We want therefore to investigate discursive formations of *qualitatively good research* articulated in these documents in relation to the new political technologies pointed to by Ozga (2008).

By analysing education policy texts on research-based teacher education from discursive perspectives, this means that we look at education as a language-based community. A language-based community consists, according to Foucault (2005) of the practices or discursive regularities contributing to individual research perspectives getting dominant position of power. Texts and text sections can serve as active participants, here related to national regulations for teacher training and knowledge and selected documents. Documents with its text, add premises for discursive production of research-based teacher education. We use Foucault's *discursive* thinking by looking critically for *production* of research power and research knowledge. Foucault argues that the statement can be read as discursive regularities. By examining specific methodological concepts regarding research in mathematics, we track some regularly discursive units/formations in the text material through the statements used. We are looking in the texts for surrounding linguistic designating nodal points, to look for patterns that help to create a certain discursive *order* around the selected concept, here mathematics education research projects. A discursive order of thinking implies a monitoring role and a position of power. This means that when the research-based teacher education is written into the documents, there is a diversity of *actors* and *networks* embedded in maths research that are keeping track of discourses on which research is in dominant positions. It is the different players we are interested in identifying in this project.

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EXTENDING THE COMMUNITY IN PROFESSIONAL LEARNING COMMUNITY

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In this paper, I draw on data from an ongoing teacher development programme that works with professional learning communities in and across schools in South Africa, in which teachers focus on learner errors as opportunities for learning, both for themselves and for learners. Analysing learner errors with an intention to understand the reasoning behind them leads to different attributions of blame and responsibility for learner errors. Understanding errors more deeply also suggests that a broader notion of community is required to support teachers to transform their teaching practices.

INTRODUCTION

Professional learning communities are increasingly seen as a sustainable and generative method of professional development in mathematics education (Clark & Borko, 2004; Jaworski, 2008). There are a number of definitions of professional learning communities, all of which emphasise two key aspects: professional and collective learning. Professional learning implies learning based on knowledge from practice together with knowledge from research. Collective learning provides support for teachers trying new ideas and shifting long-held practices, as well as providing for more comprehensive and coherent experiences for learners. School-based professional learning communities allow teachers to "coalesce around a shared vision of what counts for high-quality teaching and learning and begin to take collective responsibility for the students they teach" (Louis & Marks, 1998, p. 535).

In this paper, I draw on data from an ongoing teacher development programme that works with professional learning communities in and across schools in South Africa (Brodie & Shalem, 2011). In the project, teachers focus on learner errors as opportunities for learning, both for themselves and for learners. Learner errors are seen as reasoned and reasonable (Ball & Bass, 2003), and as entry points into both teachers' and learners' mathematical knowledge and practice. In the project teachers are continually positioned as both experts and learners. As experts, they contribute their knowledge of mathematics, their contexts and their learners, and as learners they deepen this knowledge, through interactions with other teachers, with project facilitators, and with research on learner errors. The facilitators take responsibility for supporting teachers to deepen their mathematical knowledge and for linking teachers' knowledge with new understandings from research (Jackson & Temperley, 2008). The facilitators participate in their own professional learning community (Nelson, Slavit, Perkins, & Hathorn, 2008), where they discuss the ideas emerging in the communities that they facilitate and how these ideas link with research. The facilitators and the research create a broader community for the teachers.

ERRORS: WHO IS TO BLAME?

We began the project intending to focus on two key ideas about errors in mathematics classrooms. First, errors are a normal part of the learning process and should be expected and embraced rather than denied and avoided (Smith, DiSessa, & Roschelle, 1993). Second, neither teachers nor learners are to blame for learner errors and at the same time both learners and teachers can learn from learner errors and engage with them in ways that transform learners' mathematical knowledge. The project activities and conceptual resources support teachers to see that errors are similar across different schools, and therefore cannot easily be attributed to learners or teachers. Research papers show that learners in "developed" countries make the same errors, so even "high" achievement is preceded by errors. The problematic in the South African context is why is it that errors persist for longer than in other countries, i.e. among our learners, errors seem to compound with time, rather than improve over time.

As we work with the teachers and analyze our data, it has become apparent that key shifts in thinking about blame and responsibility for errors take place as teachers talk about them. One of these shifts is in understanding how the experience and knowledge of other communities, beyond the local, support and can transform errors.

Our data has shown that teachers' first response is to blame learners for their errors. They argue that learners get confused easily, do not learn ideas properly the first time or do not practice enough. As teachers begin to interpret and explain learners' errors and begin to see errors as reasoned and reasonable, they may start blaming themselves, or other teachers, for learners' errors, finding the reasons for learners' errors in how they were taught. This move, from blaming learners to blaming teachers is a key first move in a process of development, which includes coming to understand how a broader notion of community can help teachers, facilitators and researchers to imagine new ways of teaching that engage and transform learner errors.

WHO IS THE COMMUNITY?

Two examples from our project illustrate how teachers begin to see beyond themselves and their contexts. In the first example, teachers were talking about errors that their learners had made in a recent test. They were shocked that all the Grade 9 learners in one teacher's, class had responded incorrectly to the question: multiply (a+b)(c+d) [1]. Melusi indicated that he clearly had not taught appropriately, while Buwane asked "what about the learners, did they practice enough?" Buwane said that usually one or two learners would get the answer correct and then "we can feel better about ourselves – at least we have evidence that we taught it well". For Buwane, the difference between a few learners being successful and no learners being successful was significant. He was challenged by the facilitator that in fact only a few learners being correct might not be a good indicator of teaching success, and by two other teachers who argued that they need to think about their teaching practices and whether they had engaged with the learners' errors enough in class before the test.

This discussion led to a focus on how they might shift their teaching practices. Buwane made two arguments: first that they could learn from the better-resourced schools in their district, where learners got better results (i.e., more correct answers), and second that it required many teachers to help to change teaching practices, the whole "community" of teachers. The first point was actually made in the negative – Buwane asked why the district subject advisor did not tell them about what the other schools were doing, suggesting that they needed to be helped from outside by others who are better than them. The second point reflected a sense that teachers could work together and help each other but that it needed to be a broader community of teachers, not only those in one school, because many teachers teach in similar ways and they would need substantial resources to try to shift [2]. The facilitator then pointed out how a section of the paper they had just read could help Melusi think about why his learners were making the errors.

In the second example, teachers had been grappling with why learners continue to put equal signs at the beginning of an equation, even when the teacher had explained carefully why they should not be there [3]. Considering about how they might explain the widespread and persistent errors across different schools and contexts, led the teachers to consider their own teachers as part of the community.

Lorraine: We tell them that because we grew up like that, we were told that

Joanne: We teach how we were taught

Nadine: Yes, we were all taught that

Andrea: I mean some of the stuff that I said, its pretty deeply ingrained in me as well.

Here the teachers extend their notion of community to a cross-generational community, which includes their own teachers. They acknowledge that they often teach as they were taught and later they argue that if they are going to be able to shift learning in significant ways, they will need to break the mould of their own learning and think more creatively about how to teach differently.

CONCLUSIONS

The work in our project tries to shift teachers from blaming learners for their errors to understanding why learners might make errors and why these errors can be seen as reasonable. Once teachers understand that errors are reasonable, they begin to blame themselves for learner errors. Our goal is to try to remove blame from both teachers and learners. Although a broader notion of community includes all, or many teachers, it helps to remove blame from particular teachers. One explanation for why particular errors are so persistent might be that teachers teach in similar ways, learning from their teachers and from each other, in ways that become ingrained, not only in individual teachers but in the social fabric of teaching. It is this social fabric that needs shifting, rather than individual teachers, and therefore needs a larger community.

NOTES

1. In the project we choose to work with the standard mathematics curriculum, arguing, together with Young (2008), that traditional subjects such as mathematics do represent powerful knowledge that all learners can learn and deserve access to. Moreover, traditional subjects can be taught in empowering ways, by developing a curriculum of engagement, rather than a curriculum of compliance (Young, 2010). See also Brodie, Slonimsky and Shalem (2010).

2. Professional learning communities within schools work in networked learning communities with other schools in the project, but here Buwane was talking about a much bigger "community".

3. The teacher knew from the project readings that this was a manifestation of an operational understanding of the equal sign. For example, the learner writes: $x - 2 = 5$ and on the next line: $= x = 7$.

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THE ROLE OF VOLUNTARY ONLINE TUTORING IN BUILDING STUDENTS' IDENTITIES AS COMMUNITY PARTICIPANTS

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New engineering students expect to start building their identities as engineers in their first year of study through the courses they take. Yet their first year curriculum contains mostly general subjects like mathematics and physics that are familiar from school and feel only loosely connected to the work they will do as engineers. Developing multiple self-identities as a mathematician, physicist, and engineer may ease the transition into university. This paper describes a voluntary online tutoring programme that enables university students to help high school students, primarily in mathematics. The concepts of mandatory and voluntary community service are discussed. The benefits of the programme are viewed through a 'community of practice' lens.

COMMUNITY SERVICE AND VOLUNTEERING

Community service is a graduation requirement at a growing number of higher education institutions (Gage & Thapa, 2012). The increasing requirement for community service reflects the hiring practices of employers: applicants are less likely to be shortlisted for jobs if they have no evidence of using their talents and leisure time to serve others (Jansen, 2012). Research corroborates this bias of employers, linking community service with qualities such as leadership ability, self-confidence, critical thinking and conflict resolution (Astin, Sax, & Avalos, 1999).

There is a difference between mandatory community service, such as a graduation requirement, and unforced volunteering although this distinction is not always made (Smith, 1999). Community service has been shown to be a successful means of personal development but it has been questioned whether enforced community service brings the same benefits as volunteering (McBride et al, 2006). Some education institutions have resisted enforcing 'volunteer' work on the grounds that it can negatively impact low-income students who need to work at paying jobs, that it disregards the benefits students gain from participating in university clubs and that it would create a need for more administration personnel (Allen, 2010). Institutions mandating community service must feel that a strong external incentive is necessary for undergraduate students to participate in a worthy experience that may challenge their world views.

VOLUNTEERING

Tutoring school-goers is a form of volunteering common in educational institutions. Often the target schools are poorly resourced and located in remote areas. In addition to their hours of engagement, potential volunteers might have to budget time and money for traveling to the venue. When volunteering takes place in areas with high crime rate, as is typical of low-income target areas, safety concerns may deter potential volunteers.

Given the demands of volunteering, what motivates students to volunteer? A review of literature by Handy et al. (2010) showed three dominant sources of motivation for volunteering that often coexist: value-based (or altruistic) motives include religious beliefs, supporting a personally important cause, helping others; utilitarian motives include gaining work experience, developing new skills, investigating potential careers, enhancing résumés, or making contacts; and social motives include growing a social network, volunteering because friends do, or responding to social pressures.

DR MATH

Dr Math is a mobile tutoring project offering free support to high school students. Questions are sent via instant messaging services (e.g., MXit, Google Talk) that students can access using mobile phones or internet-connected computers. Volunteer tutors reply as 'Dr Math' via an internet site. Tutors are encouraged to guide students towards answers rather than just giving answers. The tutor's code of conduct encourages the use of humor and sms language but not the sharing any personal information. All interactions are recorded. The service includes other high school subjects and help in at least four official South African languages. The project was developed by Laurie Butgereit of the Meraka Institute of the CSIR and has won a United Nations award ("UN award", 2011).

In this study, Dr Math tutoring was arranged in a computer laboratory during a class meeting time. This allowed peer networks to be established, as tutors (university students) asked for and gave their class-mates advice on how to reply to questions. They could access internet sites to look up definitions or examples that could help them reply to the questions posed.

Volunteering as a means to feeling more a part of the mathematics community

Lave and Wenger (1991) presented the view that people learn through the act of being part of a community of practice. Using Wegner's (2000) three-part description of what constitutes a community of practice, in this project the *domain* would be mathematics (and potentially other school subjects) up to first year university level, the set of *shared practices* would be conventions about writing mathematics, using 'sms-talk' abbreviations on mobile phones and the flow of written dialogue, and the *community* would include multiple subsets: the students in my class who hear me discuss the project (even if they choose not to partake), the school students who send in questions (Brew (2006) argues that students are part of the community), the staff at the Meraka Institute who monitor and research the Dr Math project, myself and other lecturers. In this project, the students volunteering as Dr Math tutors are simultaneously playing roles of teacher and learner as they search on internet sites for information before formulating a response.

There are numerous benefits of having students, particularly first year students, volunteer as Dr Math tutors. Students' sense of belonging to the university community would be strengthened through participation as a Dr Math tutor. They can pay forward the benefits they have received through their education, which can be personally rewarding and motivating. Developing an identity as a tutor can help to bring them from a peripheral position in the mathematics community (as a student) to a more central position. This could be especially helpful to students experiencing feelings of alienation and isolation at university, which has been shown to be evident in engineering students at the university where this study took place (Case, 2007). Dr Math tutoring may help raise the power status of normally low-powered social locations, e.g. youth, people with knowledge of township slang and English-second-language speakers. As such, it may be a tool to help transform the university environment to one more inclusive of a diverse population, which is a strategic goal of the university where this study took place.

The Dr Math project affords students a means of contributing to their home community by alerting students from their high school to the project and by volunteering as a tutor. It may also shift their perception of education. As they become contributors in the mathematics community and become more experienced at finding helpful resources on the internet or through peer networks, they may be less inclined to view learning as something that happens to them, directed by others, rather than something they have control over.

When Dr Math tutoring is done in a class setting, it can provide the lecturer with an opportunity to decrease the distance between student and lecturer (Habermas, 1984; Jansen, 2009). Bozalek (2011) suggests that the constructive criticism a lecturer gives to students will be more helpful and relevant if the lecturer has learnt about the students' cultural practices and resources. Perhaps the Dr Math project can provide opportunities for me to learn about my students' cultural practice of using sms-language and how they get, or avoid getting help, from the internet, peers or myself.

CONCLUSION

Most of the university students in this study were affected by a school system that promoted social inequality (Unterhalter, 2000) but they still managed to achieve school results that earned them a place at university. The Dr Math programme may change limited views of learning from school and help to develop social networks with peers. Forming an identity as a contributing participant may draw them closer to the centre of the mathematics community of practice. This may be especially valuable for first year students adapting to university. A multi-faceted self identity rather than one broad one as, say, an engineering student, may help them to succeed at university.

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STUDENTS' MATHEMATICAL IDENTITY CONSTRUCTION IN THEIR TRANSITION TO SECONDARY SCHOOL

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Research suggests that identity is a powerful way to consider individual experiences as fundamentally embedded in, and contributing to, broad social, cultural and political influences on learning mathematics. In this paper I propose a 'performance metaphor' (Butler, 1988) for learners' mathematical identity. I discuss how this definition of identity provides insights into the interconnectedness of the personal and the socio-political, using data from an ongoing longitudinal study of students' identity construction in their transition from primary to secondary school.

INTRODUCTION

In most countries students face a transition between their first school and a second school during their early teen years. This is an important time in students' mathematics education experience (Osborn, McNess, & Pollard, 2006). Research suggests that unsuccessful transitions are more likely for minority or underserved groups (Galton & Hargreaves, 2002). "School transfer acts like a prism, diffracting the social and academic trajectories of the children as they pass through it" (Noyes, 2006, p. 59), resulting in increasing disparities. Those students, who enter secondary school already disadvantaged in some way, seem to experience greater disadvantage through the process of transition.

It is useful to look at such transitions from a socio-cultural perspective and consider students' identity construction as implicated in the challenges encountered at this time (Hernandez-Martinez & Williams, 2011). My study uses a longitudinal design and the concept of mathematical identity to carefully examine transition to secondary school for a group of New Zealand students.

THEORETICAL BACKGROUND

Recent socio-cultural research within mathematics education has utilised identity as a framework to understand students' relationships with mathematics learning. Learning happens partly through a process of identity development (Esmonde, 2009).

This study draws on participative (e.g., Boaler & Greeno, 2000) and discursive (e.g., Sfard & Prusak, 2005) notions of identity, which have been useful in understanding the fundamentally social nature of identity and the importance of context in identity definition and expression. In this study I follow Chronaki (2011) and Hogan (2008) by extending Butler's (1988) use of a performative metaphor in the constitution of gender identity to mathematical identity. If we understand gender as one form of identity, then we can read her work with regards to any identity construction, including mathematical identity. Butler states that "...The acts by which gender is constituted bear similarities to performative acts within theatrical contexts" (Butler, 1988, p. 521). While honouring the participative and discursive nature of identity, the performative metaphor provides language and ideas for understanding identity construction as a social, cultural and political act.

METHODS

My research follows a diverse group of 22 students from two classes in two different schools as they move from their last year of primary education and into secondary school. In 2011 I conducted interviews with each student, observed them in class during mathematics lessons and interviewed their teachers. In 2012 I contacted the students again. They were in 9 schools, with 17 teachers. I interviewed and observed them twice, at the beginning of the school year and later in the year, and also interviewed their mathematics teachers. The interviews were semi-structured and interpretive (Kvale, 1996). I asked students about actual experiences in mathematics and encouraged expressions of identity (Lopez-Bonilla, 2011). In 2013 I will interview and observe these students for the last time, following their move into their second year of secondary school.

EMERGING THEMES

The notion of identity as performance (Butler, 1988) proved to be a very useful tool in understanding students' identity constructions. Performances express something about the performers, and take place on a stage. Any individual may give different performances on different stages, bringing the personal and the social together in a way that shows how they are intimately related – the stage framing and defining the possible performances that can take place.

During observations I looked at the 'stage' of the performances and asked questions such as: What performances are enabled or encouraged on this stage? In what ways does the teacher, as a co-performer, contribute to particular identity performances from their students? Are students allowed to take centre stage? Can students write their own scripts to perform on the classroom stage?

The physical environment of the classroom, the physical layout of the stage speaks a great deal to the type of performance required. In the two intermediate classes of my study all desks were placed in groups and all students moved about freely, utilising different workspaces. In contrast, upon the move to secondary school, most of the participants found themselves in mathematics classes that had individual desks arranged in pairs or rows (and occasionally single file) and facing the front of the room towards the whiteboard/smartboard and teacher. Of the 22 participants only two were in desk groups of four or six and two were in a U shape, again centered on whiteboard and teacher. These stage arrangements privilege certain performances – listening, passive receipt of information and assigning responsibility for mathematics learning to the teacher.

The stage is more than the classroom, however, as this metaphor draws in wider influences from the school, community and society at large. Parental expectations, curriculum requirements, school culture and resourcing are all expressed in the stage that students are offered for their performances.

The idea of the stage helps us to see the classroom in its wider social and political framework. In addition, the performative notion of identity helps us to recognise that an individual can perform different identities in different contexts, and that they may enact different identities at the same time. I asked my participants to reveal if they were able to enact a promising mathematical identity alongside the other performances they are compelled to give.

What sort of performance demonstrates a strong or promising mathematical identity? Some studies suggest that a performance of confidence within mathematics classes is often seen as indicative of competence in mathematics (Burton, 2004; Hardy, 2007). Yet for some students performing confidence is more problematic than for others; as demonstrated particularly by Pasifika students within the New Zealand context (Hunter & Anthony, 2011) and by girl learners in England (Mendick, 2005). Thus some groups of students may be marginalised by the conflation of confidence and competence in mathematical identity performances.

The constraints on a student's performance appear to be a product of the stage and the other required identity performances, such as 'girl' or 'Pasifika student' or 'low achiever'. The typical secondary classroom 'stage' I described earlier privileges certain kinds of performances over others. Demonstrating an identity of being 'strong at mathematics' requires a performance that may be at odds with what is privileged on this stage. For some students this performance may also be at odds with their other identity performances and therefore doubly hard to do.

CONCLUSION

The performance metaphor enables me to consider the ways the 'stage', upon which students perform, may work to constrain their mathematical identities. In my next analyses I am considering whether this stage works to privilege certain groups of students over others.

It also enables me to consider differing performances by an individual student and ask how and why performances change over time and context.

Finally it enables me to question how the different performances a student is compelled to enact concurrently may conflict with each other and perhaps ultimately cause the student to 'bow out' of mathematical performances in the future.

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LANGUAGE OF EVALUATION: IMPLICATIONS OF SELF AND TEACHER JUDGEMENT IN MIDDLE SCHOOL MATHEMATICS

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For more than a century, teachers have been expected to grade and academically sort students, using averaging and the 100-point scale, despite research clearly finding that the practice of assigning grades is pedagogically unsound (Butler, 1986; Deci, 1971) and the accuracy of such grading is questionable (Reeves, 2011; Marzano, 2008). Moreover, the frequency and importance of educational assessment has grown with the development of large scale testing and the more recent call for formative classroom assessment as a means to increase student achievement. Literature in the field of classroom assessment notes that, while well intentioned, the implementation of formative assessment has simply resulted in more of the same, more classroom-based testing and teacher evaluation, and is calling for a fundamental revision in practice whereby locus of authority is shifted to the students. Black and Wiliam (2010), authors of a germinal study on formative assessment, emphasise the need for a radical shift that "requires many teachers to fundamentally change how they relate to students, to become better listeners themselves, and to learn to promote, respect, encourage, and build on student contributions" (p. 47). While there is much written on the methodology, there is little research on the relational qualities of formative assessment. If formative assessment, as claimed, has the most significant impact on student achievement (Black & Wiliam, 1998), and "teachers enhance student learning more than any other aspect of schooling than can be controlled" (Marzano, 2006, p.1), the two need to be put together to better understand the interpersonal dynamics of classroom assessment and the impact on student learning. For this reason, my project presentation will focus on the language of evaluation in middle school mathematics' classrooms. As a method of investigation, I will be using discourse analysis as a general framework, relying in particular on Martin and White's (2005) model of evaluation and general theoretical framework of systemic functional linguistics (SFL) focusing on interpersonal meaning.

BACKGROUND

The educational community is engaged in a very passionate discussion over the practices of classroom assessment. Administrators, policy makers and parents are demanding performance measures and comparative data on educational standards and attainment targets while government policies, research, teacher professional development seminars and publishing houses are promoting the notion of ongoing assessment as a standard of effective classroom practice. Prior to the social and cultural changes of the 1960's, academic success was attributed to family background (Coleman, 1966) and the aim of classroom assessment was similar to a sieve, sifting and sorting students based on their perceived abilities (Bloom, 1971, p.7). In 1971, Bloom, Hasting, and Madauas ignited interest in classroom assessment with the publication of the *Handbook of Formative and Summative Evaluation of Student Learning*. In this work, Bloom et al. (1971) promoted the notion of teaching as a diagnostic enterprise and outlined a framework whereby teachers identify mastery goals, test students against those goals, diagnose difficulties and then prescribe remedial measures (pp. 117-141). This work planted the seed with regards to identifying classroom assessment as part of the learning process, shifting the focus from credentialing or certification to using classroom assessment strategically to foster learning.

In an effort to synthesise the glut of research on assessment practices in the decades since Blooms publication, in 1998 Paul Black and Dylan Wiliam published a meta-analysis finding that the use of formative assessment, defined as ongoing feedback on student performance, produced significant achievement gains particularly in low-achieving students. The effect size was between 0.4 and 0.7 (p. 141). An effect of 0.4 raises achievement of students by 35%. Since Black and Wiliam's research, an exhaustive list of publications, policies and professional development on the topic of formative assessment has encumbered the educational community and many argue that data and information gathering have, to a large degree, become the tail that wags the dog (Mansell & James, p. 13).

DESIGN

Despite the emphasis on assessment, I have not been able find research on the rich and complex interrelationships, and the assumptions, embedded in the interpersonal dynamics of classroom assessment even though it is accepted that the ability to co-participate provides the matrix for learning (Hanks, 1991). Language use offers a means to discover these dynamics by providing an avenue to engage with the structure, function and meaning of the processes while regarding the social aspects of the learning environment (Christie & Martin, 2007, p. 3). My research will involve textual analysis allowing for trends and meaning to emerge through the language used in a mathematics classroom. Vygotsky's (1986) recognition of the interplay among language, thought, and meaning making will support my use of Halliday's (1994) Systemic Functional Linguistics (SFL). In this tradition, I will also use Martin and White's (2005) appraisal linguistics to narrow the meaning structures found specifically in classroom assessment. Harré and van Langenhove (1999) theorization of positioning will inform my analysis of the human relationships within these practices and Wagner and Herbel-Eisenmann's (2009) elaboration of this theory in mathematics learning contexts will underpin my analysis of students' developing relationships within the discipline of mathematics.

A key aspect of the research will be the inclusion of the participants in the analysis. Fairclough's (1999) critical discourse analysis will provide a window on the relationship between the participants' language and learning through language awareness, for both teachers and students will be involved in the analysis of the language of evaluation and the role it plays in structuring knowledge. The reflective process will include an analysis of classroom discourse, interviews, focus groups and journals with SFL, and more specifically, appraisal linguistics, providing the framework for my analysis. International, national and provincial achievement data show a levelling off of subject matter growth in the middle years (OECD, 2009). For this reason, in addition to my personal experience as a middle school mathematics teacher, this study will focus on this age group.

In the presentation I will use sample transcripts to discuss language use and classroom assessment. In particular, I will look at micro-level evidence of classroom assessment, in relation to values and positioning as manifest in the linguistic exchanges between teachers and students. I will consider how those instances align with the findings in assessment literature, which I noted above. Narrowing the discussion to the (un)intended consequences of such exchanges will illuminate the cultural and political environments of the mathematics classroom. I will also ask about the value of stratifying the sample in such research to include representation from different urban, rural, gender and social economic demographics. Finally, I will question whether the issues of power relations in the classroom can be drawn out of the analysis on evaluative language and, if so, discuss the effects of this dynamic on teaching and learning mathematics.

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WHAT INFLUENCES THE STRUCTURE AND CONTENT OF MATHEMATICS TEACHER EDUCATION IN ICELAND?

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The development of mathematics teacher education is an ongoing project. We as teacher educators are interested in analyzing which political, social and cultural factors influence decisions about the structure and content of mathematics teacher education. In a period of change we have listed some questions we find interesting to study and discuss with other colleagues.

Our project is about looking into the mathematics teacher education for compulsory schools in Iceland and trying to understand what has influenced decisions about the structure and content of the its curriculum. We want to understand better the influencing factors with the intention to improve the program. Research has been dealing with what is important and what characterizes good mathematics teacher education (see Grevholm, 2006; Hammerness, Darling-Hammond, & Bransford 2005). Research results and theories give many ideas for planning mathematics teacher education but when it comes to putting the ideas into practice there are some political, social and cultural issues that need to be dealt with. In the project we want to learn more about those issues and to what extent they restrict us or help us to make the education coherent with the culture we live in. The underlying question is: Why is a small nation like Iceland making its own program, why do we not use programs from neighboring countries which we are closely culturally related to?

Teacher education for compulsory school teachers has been at university level (B.Ed. degree) since 1971. The program for educating compulsory school teachers has been under constant development during the last 10 years. The focus of the changes has been to increase opportunities for specialization and extend the study to a five year research based program (M.Ed. degree).

According to new legislation, from 2011 teachers need a master's degree in order to be certified as teachers. Compulsory school teachers are mainly educated at two universities; however only the University of Iceland (UI) offers specialization in mathematics. Student teachers specialize in their chosen subject along with general pedagogical studies. The tendency has been to increase specialization in school subjects. In the current curriculum, specialization in school subjects is 120 out of 300 ECTS compared to 50 out of 180 ECTS in the year 2000. University teachers in each subject area are responsible for the content and the structure of the specialization. In mathematics it is a result of discussions and negotiations between seven colleagues teaching mathematics and mathematics education. The members of the group have different backgrounds. Some are mathematicians and others have specialized in mathematics education and have a background in teaching. The authors of this paper came to the university from teaching in compulsory school and have taken part in teaching and developing mathematics teacher education in Iceland for over twenty years. We consider the development of mathematics teacher education as an ongoing project (Gunnarsdóttir & Pálsdóttir, 2011; Gunnarsdóttir & Pálsdóttir, 2010; Gunnarsdóttir, Kristinsdóttir, & Pálsdóttir, 2008). A new structure gives an opportunity to reflect on and analyze the current situation and to examine to which extent it is coherent to our ideas about good mathematics teacher education.

THE CONTENT AND THE STRUCTURE OF THE SPECIALIZATION IN MATHEMATICS

Teacher students at UI who specialize in mathematics can take 14 courses (5 or 10 ECTS), adding up to 120 ECTS. All the courses are specially designed for teacher students. In some the focus is on the teacher students learning of mathematics, some are with emphasis on mathematics education and in some the studying of mathematical content and mathematics education are combined. The table below gives an overview of the courses.

Year	Fall	Spring
2	Numbers, reasoning and number operations (10) Algebra and functions (10)	Mathematics teaching and learning in grades 7-10 (10, three weeks teaching practice) Geometry (10)
3 - 4	Number theory and algebra (10) Calculus (10) Diverse approaches in mathematics learning (10) Research in mathematics education (10)	Mathematics teaching (10, three weeks teaching practice) Discrete mathematics (5) In the realm of the real numbers (5) Linear algebra (5) Learning materials and use of software in mathematics education: GeoGebra (5) Subject-teaching (10, four weeks teaching practice)

Table 1: Overview of the specialization in mathematics

Throughout their studies the teacher students work with different mathematical content to strengthen their knowledge to draw upon in their teaching practice and future teaching. At the same time they become familiar with ideas about mathematics learning and teaching. Both fields of study, mathematics and mathematics education, get equal attention although the courses are mostly taught separately. During the five years master's program the students write both a bachelor (10 ECTS) and a master thesis (30 ECTS) which reflects the emphasis on research in teacher education. The theses are often linked to the students´ specialization.

Currently we are starting the fourth year of the master's program so the first teacher students taking the specialization will finish 2013. The plans are ready but are we confident that this is the way we want to go? Why does the program look like this? What are the influencing factors? What kind of questions should we ask ourselves in our attempt to search for answers to our questions?

INFLUENCING FACTORS

In order to get closer to the influencing factors on the mathematics teacher education we have put forward some preliminary questions which we are working on and would like to discuss at the conference. The questions regard **traditions and institutional culture** in teacher education, the **legislation** on teacher education and compulsory education, beliefs in **the society** about mathematics and learning, **research on mathematics teacher education**, **the school culture** in compulsory schools in Iceland, and the background and collaboration within **the collegial group.**

Traditions and institutional culture

- How is the teacher education at the University of Iceland organized?
- How is mathematics taught at university level?
- What are the traditions in mathematics teacher education at the university level?
- Who determines the structure of the program within the university?
- Why do we have separate courses for mathematics and mathematics education?
- What are the expectations of the teacher students and what is their background?

Legislation

- What is the framework we need to work within according to the law?
- What changes have been made due to new laws and regulations and why?
- Is there a consistency between law on teacher education and law on compulsory schools?
- How does the government's focus on inclusive education affect mathematics teacher education?

The society

- What ideas about mathematics, its importance and role appear in public debate?
- What ideas about mathematics teaching and learning appear in public debate?
- What sets off public debate on priorities in mathematics teaching and learning?
- How does teacher education respond to the public debate?

The research on mathematics education

- What conclusions can be drawn from research results about important aspects of mathematics teacher education?
- How has research on mathematics teaching and learning affected the program?
- How has the strengthening of mathematics education as research area, globally and locally, affected teacher education?

Compulsory school culture

- What effect does it have that teachers are educated as general teachers with a specialization?
- Why is teaching in grades 7–10 more subject orientated than in lower grades?
- What authority do teachers have in planning mathematics teaching and learning?
- How does it affect teachers as professionals to work in a small language community?

The collegial group

- What does the collegial group collaborate on? What authority does it have?
- What is the educational background of the individuals in the group?
- What is the groups´ relation to the practice field?
- What other roles do individuals in the collegial group have within the university?
- How does the group work together? Does the group have a common vision?
- What is the research focus of individuals in the group?

These are big questions but we find it important to reflect on many factors when trying to analyze and understand our program. We know that we cannot seek answers to all these questions and that we need to narrow our focus. Currently we are going through a phase of vital changes in our teacher education and therefore we find it useful to reflect on the current situation and evaluate our program. We believe it will be of great interest to discuss with other mathematics educators and researchers the factors influencing mathematics teacher education programs.

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CRITICAL DEMOCRATIC COMPETENCE AND CLASSROOM DISCUSSION – A PROJECT AT AN INITIAL STAGE

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The paper presents a recently initiated multidisciplinary project and its pilot study within mathematics education. The overall aim of the project is to study how classroom discussions require, develop and negotiate critical democratic competence in Norwegian lower secondary school and to gain new insight into explorative and argumentative discussions as an educative method for learning in several school subjects. The pilot study focuses on the characteristics of such discussions as well as teachers' motivations and experiences as facilitators for such discussions. In particular, we investigate students' development of mathematical argumentation while working with traffic safety, and where they apply statistics on death rates and safety fences.

BACKGROUND

Democracy and democratic values are highlighted as core issues in the national curriculum '*The Knowledge Promotion'* (2006), in Norway. Considerable attention is given to both the *understanding of democracy* and *democratic training*, building on research on democratic understanding amongst students in European secondary schools (Mikkelsen, Buk-Berge, Ellingsen, Fjeldstad, & Sund, 2001) and on critical evaluations of previous curricula (Haug, 2003; Hertzberg, 2003; Klette, 2003). A recent survey by The Norwegian Directorate for Education and Training (2012) stated that most subjects in the primary and secondary school include issues related to human rights and/or democracy, either expressed in the objectives of the subjects or stated in the competence aims. The principal aims of the education act state that education in Norway shall promote democracy (§1.2), and the core curriculum centers on *democratic ideals* as a fundamental and indisputable value, rendering democracy an essential and integral part of culture and society.

By means of mass media, teachers and students gain access to contemporary controversial issues, and students can observe, assess, and even engage in public debates. There is, however, a distinct difference between making knowledge relevant by using current issues as means of actualization and example, and bringing current news and issues into classrooms in order to problematize and question democratic values. By establishing this distinction, we wish to point to the fact that classroom discussion should not be considered as a closed field of 'hypothetical democracy', but should be seen as a sphere of social and public relevance, where students engage in actual, democratic exchange. Following Habermas' (1981) theory on communicative rationality, one might study classroom discussion in light of the distinction of communicative and instrumental/strategic actions.

Jerome and Algarra (2005) distinguished between deliberative debate, adversarial debate, and compromising. Deliberative debates are debates where the aim is to reach a consensus about the issue at hand, and are a common basis for political decisions. An aspect of democratic competence is the ability to criticize, evaluate and analyze applications of information in society (Blomhøj, 1992, 2003) which relates to PISA's understanding of literacy. Deliberative debate may deal with normative issues, and try to reach a common ethical basis for the political community (Held, 1996). An important step in analyzing the development of critical democratic competence in education will be to investigate the extent of such discussions and stimulate the use of such discussions, and to investigate how these overlap and interfere with other modes of student discourse, e.g., the use of social media.

The mathematics curriculum in Norway emphasizes that an active democracy is based on citizens' abilities to understand and critically evaluate quantitative information, analyses, and prognoses. Mathematical competence is necessary to understand and influence processes in society (Ministry of knowledge, 2006). Critical learning in mathematics requires mathemacy; that learners critically inquire mathematical concepts and structures, and how mathematics is understood and used in society. The quality of communication in the classroom influences the qualities of learning, and explorative communication can support the development of critical learning and critical democratic competence (Skovsmose, 2005; Alrø & Skovsmose, 2002; Johnsen-Høines & Alrø, 2012). In this project we want to gain insight into how discussions are knowledge-based and how the emerging learning and knowledge is influenced by the discussions.

PILOT STUDY

In the pilot study we investigate students' development of mathematical argumentation. The students have experienced an increasing traffic-related death rate in their neighborhood. Consequently, their teacher has chosen to work with road safety as a mathematical issue. With input from the local police, the pupils develop suggestions on how to improve traffic safety based on mathematical reasoning. They are working with statistics on death rates and safety fences while developing mathematical argumentation between themselves and with the police, orally as well as in writing. Hereby, the pupils get the opportunity to develop agency by using mathematics as a critical tool for investigating and analyzing problems related to their everyday life. This we denote as a student-driven project, as referred to by Greer, Verschaffel and Mukhopadhyay (2007).

This project builds further on the LCMP-project (Alrø & Johnsen-Høines, 2010; Hana, Hansen, Johnsen-Høines, Lilland & Rangnes, 2010; Rangnes, forthcoming) which showed that an assignment made possible by a socio-political context influences the intentionality, functionality and empowerment of students. The communication between students, teachers and representatives from workplaces had the potential for developing critical democratic competences.

RESEARCH QUESTION AND METHOD

The pilot study focuses on one of the project's research questions: *What characterizes argumentative/explorative classroom discussions when issues from public debates are included?* We will look in depth into communicative practices, learning situations and conditions for learning. We will study classroom dynamics and the actual events taking place when public debate is included in learning processes. We also ask: What are the characteristics of the situations where learning and critical democratic competence seem to be produced? The project will follow teachers and students to study in depth their motivations, knowledge and experiences. Video recordings will be used. Situations where discussions occur, will be analyzed and discussed, and both teachers and students will be partners in analyses of actual processes taking place. Speech act theory (Austin, 1962; Searle, 1969; Wunderlich, 1975) and analyses of different narrative and social patterns structuring classroom communication will be used as tools to investigate to what degree, and in what way, democracy is being established and negotiated in classroom discussions. The pilot study is currently developing forms of cooperation with the school teachers, where the school teachers take part in the research process.

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THE BELIEFS OF SECONDARY SCHOOL MATHEMATICS TEACHERS IN BILINGUAL RURAL CLASSROOMS IN SOUTH AFRICA

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Given the depth of literature about mathematics teachers' beliefs, and about mathematics' interaction with language in the classroom, what is it that mathematics teachers believe in and about their bilingual classrooms and practice? This paper seeks to explore this inductively with a small qualitative research study conducted in a rural South African secondary school, where isiXhosa is the home language of both the teachers and students, but the official language of teaching, learning and assessment is English.

INTRODUCTION

There is a great deal of literature that addresses the role of mathematics teachers beliefs in their classroom practice (Ernest, 1989; Thompson, 1992; Beswick, 2007; Nespor, 1987). An equally broad and diverse literature exists that describes the interaction and implications of mathematics and language, both for monolingual learners and classrooms (e.g., Pimm, 1987; Sfard et al., 1998) and in the multilingual classroom (Adler, 2001; Setati, 2008; Barwell, 2009). However, there is little examination of the intersection of these two bodies of literature: namely what mathematics teachers in multilingual classrooms believe about their work and how these beliefs affect and are affected by their practice.

A study investigating such a field would necessarily be socially positioned, given the socio-psychological nature of beliefs as well as that of language (Hymes, 1977). As indicated in Secada (1992), teachers do not operate in vacuums, and the social and political contexts of their work affect their beliefs and practices too. The next section briefly describes the current socio-political environment of the South African schooling system.

THE SOUTH AFRICAN CONTEXT

Mathematics attainment in South Africa is remarkably poor, especially in the light of the financial resources set aside for education in the national budget. Many theories have been espoused to explain the overall poor performance at all levels in the system across all subjects. However, the language-of-teaching-and-learning has become a particularly divided and contested issue.

Most South African students are learning in a language that is not their first and is not spoken at home or in their immediate communities. Although provision is made in the Language in Education Policy for School Governing Bodies to choose the language of teaching and learning for each school based on the desires of the parents and students, the reality is that many schools opt for English as it is seen as the only language which offers access to jobs, tertiary education and the formal economy (Setati, 2008). Yet most teachers operating in ex-DET schools (those that serve the poor majority in both rural and urban settings) are themselves not English first language speakers and, due to their own education under the Bantu Education Act of 1976, were denied access to acquiring powerful forms of English (Department of Basic Education, 2005).

The situation is further complicated by the stark rural/urban divide in South Africa: although students in the urban areas do not speak English at home, they have far more exposure to the language outside of the schooling environment, a resource their rural counterparts do not have. English is so remote in the rural areas of the country that such schools have been said to operate in an environment in which English is a foreign language (Setati & Adler, 2000; Adler et al., 2002). Yet these students will be officially taught in English, and will face their school-leaving assessment in English.

Teachers of mathematics in such rural schools are faced with the double challenge of teaching content *and* language to students who have very few resources outside of school that enable learning. Furthermore rural schools mathematics teachers have been poorly trained and have limited subject content knowledge.

Language issues in South African mathematics education

Mathematics offers up particular language challenges to English-language learners (Adler, 2001; Setati, 2008; Adler et al., 2002). Adler (2001) shows how dilemma-ridden a teacher's work can be when attempting to teach mathematics and language simultaneously in the classroom. That poor mathematics attainment closely correlates with language skills in South Africa is no secret (Simkins & Paterson, 2005). The fact that teachers need additional training not only on their subject content knowledge, but on negotiating the minefield of language learning is well documented (Adler, 2001). But what *perspectives* do the *teachers* who are faced with such challenges have on their work? Any form of research for improving practice must take such perspectives into account.

THIS STUDY

This particular study is intended to describe the beliefs of secondary mathematics teachers regarding their work as teachers, their subject – mathematics – and the role that they see language playing in their classrooms. The approach was an inductive one, focusing on the qualitative study of three mathematics teachers in a specific rural South African secondary school. The study took place over a period of 4.5 weeks in a small village in the rural Eastern Cape, where the predominant language of the students, teachers and community is isiXhosa. In this regard, this school was very typical of those described as "English as a Foreign-Language" learning environments as described by Setati and Adler (2000).

The school was a small one, with only one class of students per year cohort, and a total of 132 students in the entire school. The classes chosen for examination where in the GET band, that is Grades 8, 9 and 10, as this is the phase at which mathematics moves from the context-specific of primary learning to the more abstract, context-reduced forms of algebra and trigonometry. All three teachers in the school were represented within these grades.

Methodology

In keeping with the methodological techniques of qualitative belief studies outlined in Speer (2005) and Thompson (1992), the structure of the study was as follows: initial interviews were set up to profile the teachers' own educational histories, experiences of mathematics and linguistic repertoires, and to invite their reflections on what they believed about mathematics as a subject, as a focus of learning at school, and as an entity to be described with language. Beliefs about the nature of language and what constitutes effective schooling were also discussed.

After these initial interviews, classroom observations were arranged for each teacher and these were video-captured. Thereafter, a preliminary analysis was conducted on each video for evidence of beliefs or incidents of interest for discussion, and stimulated recall interviews were conducted with each teacher, which were also audio-recorded.

In addition to this primary data, a daily journal of my experiences as a researcher was kept which has proven to be a rich source of ethnographical and anecdotal data in describing the community at large and particularly my positioning within it as a white-middle class Anglophone woman in a homogenous rural, agrarian isiXhosa village.

THE WORK IN PROGRESS

At the time of submission of this project paper, the analysis on the data gathered as described above is in progress. Questions regarding the correct analytical tool are manifesting, particularly with regards to whether discourse analysis (e.g., Mercer 2005) should be used to find evidence of beliefs in transcripts of classroom talk.

In light of this, it is thought that the outcomes and process of this study would be of interest to the MES 7 community, not only methodologically, but also regarding the sociological nature of the questions being asked. It is the opinion of the author that the MES 7 community could offer much advice and productive input to this study.

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TEACHERS' COLLABORATIVE LEARNING AND STUDENTS' OPPORTUNITIES TO PARTICIPATE IN MATHEMATICAL REASONING

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The aim of this presentation is to briefly describe two ongoing studies, linked to each other. One study will be focusing on teachers' perception of their teaching before and after participating in school development projects. The other study focuses on teachers' moves in the classroom and the opportunities they provide for students to participate in mathematical reasoning. The two school development projects where empirical data is collected are the same for both studies. One project is a small project conducted by the authors of this paper and the other project is a national project, although we are just looking at a very small part of it. The national project is aiming to improve student achievement in Sweden through teacher development and will be available for all teachers in primary and secondary school.

INTRODUCTION

In Sweden there is an ambition to reform mathematics education. The government has given Skolverket (The National Agency of Education) responsibility for implementing a teaching development project for mathematics teachers. The initiative is aimed at all primary and secondary teachers teaching mathematics and will run from year 2012 to year 2016. The main purpose of these efforts is to increase student achievement in mathematics. Support material that is relevant to the development of teaching quality will be made available. The national initiative will be based on professional development by collaborative learning with professional support in the form of math facilitators. One of the reasons for the decision made by the government concerns teaching in Swedish schools, which to a large extent is organized by students working on their own with routine tasks in textbooks without sufficient guidance or feedback from the teacher. Such a design does not provide enough space for reasoning, argumentation, and opportunities to explore mathematical relationships and therefore, generally seen, a change in the Swedish teaching culture in mathematics teaching needs to take place. Students working too much on routine tasks does not match the new curriculum in Sweden from 2011, where competences such as reasoning are highlighted in a more prominent way than before.

Our interest regarding the upcoming initiative is to examine how teachers together can become aware of and develop their teaching. The focus will be on how teachers perceive their teaching and how they use various forms of feedback/follow up to enable students' participation in mathematical reasoning. It will culminate in two separate studies in which one will be the study of teachers' perception of their teaching of mathematics before and after the project. In the other study, the focus is on how the teachers, before and after participating in the project, use various types of feedback/follow up to provide opportunities for student participation in mathematical reasoning. We have previously carried out a small school development project together with four mathematics teachers, grade two, three, four and five. The project was conducted by us, but based on collaborative learning. In the project we have been interviewing each teacher, before and after the project. We have also video recorded some chosen lessons. We are at the moment analyzing the empirical data from this project. The coming national school development project will not be conducted by us, but we plan to collect the empirical data in the same way, and if possible using the same methods for analyzing, as in the first project. Collecting data from the national school development project will be done during the spring of 2013. In this project we will interview teachers and video record lessons from approximately five teachers. The teachers are the same in both research projects.

COLLABORATIVE LEARING AS TEACHER DEVELOPMENT

Some of the various school development models that have received considerable attention recently are the Learning Study, Lesson Study, Action Research and Design Research, all of which can be seen as collaborative learning and close to practice. The models are cyclic and the teachers themselves are involved in the design of the project. Characteristic of these development models is that the teachers work with colleagues discussing mathematics education issues concerning a specific mathematical content. The discussions are based on the teachers' own experiences from teaching, the students' displayed knowledge and on earlier research findings. A report made by Skolverket (The National Agency of Education) in 2011 shows thatteachers in the Learning Study considered attending the project as rewarding. The report shows that the use of Learning Study has influenced the development of student use of mathematical concepts, mathematical reasoning, selection and use of mathematical methods to perform calculations and solving of routine tasks, as well as oral and written communication of solutions for various tasks in mathematics. While taking part in a Learning Study the teachers have developed knowledge how to teach a specific competence with a specific content. This developed knowledge is seen as being a result of the discussions the teachers had where they were reflecting on the treatment of the content in the classroom. The reflection process seems to be very meaningful and successful. Black and William (2009) argue that to bring about lasting change in a school culture and teaching, teachers themselves need to be involved in the process and it should be done using small steps and over a long period of time. In the school development project, conducted by us, all teachers expressed that it has been rewarding to be able to have the discussions in which all the teachers took part. In the discussions the notion of reasoning was highlighted but also how the teachers could support it in their classroom through different actions. Not only the discussions but also the possibility to see and analyse their own teaching, through systematic reflection, was seen as very meaningful. In the final meeting the teachers stressed the importance of the mathematical tasks used during the lessons. The teachers were more critical to the textbook than before the project started and said that after the project they were thinking more about the goals of the lesson and the importance of selecting tasks according to the goals.

REASONING AND FOLLOW UPS

Mathematical reasoning is seen in many international frameworks as crucial for becoming mathematical proficient. How can teachers act in interaction with the students to promote their competence to justify ideas and conclusions, to create arguments and improve their conceptual understanding? A Swedish study (Bergqvist, Bergqvist, Boesen, Helenius, Lithner, Palm, & Palmberg, 2009) showed (with several exceptions and some variation) that students, in general, are provided with limited opportunities to develop their competence in reasoning. Generally, activities in the classroom focus mainly on routine tasks and mathematical procedures. The study was part of a quality review and observations from 64 lessons from schools in different areas in Sweden were conducted. There are many aspects to consider when creating opportunities for learners to develop their competence in reasoning mathematically. One aspect is the teacher-student interaction and the type of feedback teachers give to student contributions on a specific task or question. The feedback can be given in different forms and for different purposes. In their review of how student achievements are affected by different feedback, Hattie and Timperley (2007) present a model of feedback to enhance learning. One type of effective feedback is *feed forward* which answers the question *Where to next?* Björklund-Boistrup (2010) uses this model by Hattie and Timperley when she addresses assessment acts in the mathematics classroom. She finds feed forward as guiding and challenging in her study. In a study aiming to understand teachers' practices as they took up aspects of reform practice, the teachers' classroom talk was examined as teachers responded to, interacted with and took forward student contributions (Brodie, 2010). A set of codes for teachers moves were developed. The type of moves can differ from *affirm,* which indicates if the learner's contribution was right or wrong to *follow up,* which picks up students' contributions and engages with them in some way. We consider the different follow up moves as potential feed forward in different ways for the students. There are different follow up moves that the teacher can use. To develop mathematical knowledge the students need to be provided with opportunities to develop, for example, conceptual understanding, problem solving, argumentation and justification. The follow up moves can be seen as providing the students with these opportunities whilst taking the students' contributions forward in different ways in reasoning.

In our school development project we intend to use the follow up moves for analyzing the empirical data. From a socio-cultural perspective we see these moves as providing opportunities for students to engage in reasoning, either orally, in writing or by listening. The analysis of the data is not yet fully done but we can see indications showing that teachers use of follow ups increases.

DISCUSSION

To create a successful and sustainable teacher development, collaborative learning is seen as necessary (Stigler & Hiebert, 1999; Lewis, Perry, & Hurd, 2009). Borko (2004) argues that to create a successful Professional Learning Community (PLC) it requires structures of interaction that supports teachers to take risks and talk to each other about their teaching and how to develop it. An important factor pointed out is that to create good environments for PLCs that are sustainable, it requires facilitation that supports teachers in the above aspects (ibid.).To change a school (teaching) culture by a political decision will, if possible, take a long period of time. An evaluation of the national project to see if student achievement in mathematics has increased cannot be done until a few years from now. What we are interested in is to see what the teachers say about it and how their talk about teaching might have changed after taking part in the project and also if it has had any impact, even after a short period of time, in their teaching concerning students opportunity to engage in mathematical reasoning.

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ROLE MODELING AND LEARNER'S MATHEMATICAL IDENTITY IN A MENTORING PROGRAM IN TOWNSHIP SCHOOLS

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That the majority of South African learners are in underperforming schools, and these schools are typically in lower socio-economic contexts and largely in townships or rural areas, is common knowledge to educators and policymakers nation-wide (Human Sciences Research Council, 2011). Curriculum revision, while a strong feature of the post-apartheid education policy landscape, remains impotent in addressing the need to facilitate learning by fostering relationships between learning, teaching and school knowledge that enable learners to develop the 'can-do' spirit and identities associated with success. A recent pilot study (unpublished) suggested that such identities could be fostered through interactions between learners and university students/youths that come from similar socio-cultural, and economic backgrounds. Hence the setting up of a learners' mentoring program (LMP) as a structured social space – a second site of learning – where the salient feature is the 'mentoring' of school mathematics learners by university students currently studying in mathematics and science related degrees.

In this short paper, I briefly describe the elements and structuring of the project, and our orientation to mentoring in the project, through the notion of 'frientoring'. I present a brief view of voices of the learners and their mentors as these emerged in their weekly reflections and claim that the social space, from an experiential perspective, was indeed a location for fashioning 'can-do' mathematical identities in learners, simultaneously with emerging civic identities of mentors.

THE LMP

The LMP is a small part of a larger research and development project aimed at improving mathematics teaching and learning in ten secondary schools in one district in the Gauteng province. LMP recruited 40 mathematical and/or actuarial sciences, science majors and pre-service teachers (science and/or math) as mentors of selected secondary school mathematics learners in grades 9, 10 and 11, in three of the project schools. The program started with grade 9 learners in 2010 and continued with learners as they moved up in grades while a new cohort of grade 9 learners was added each year, in 2011 and 2012. Mentors and learners meet on Saturdays. In 2012, LMP conducted a 10-week (consecutive Saturdays) program in one of the three schools. Mentees are expected to bring homework or other problems from class that they wish to work on with support. The mentor's role then is to provide this, and not take the position of class teacher. These sessions were not another 'lesson' but a different space, and additional place where learning could be fostered.

Mentors for LMP were selected based on their letters of motivation indicating interests and social, cultural and academic characteristics considered suitable for interacting with and motivating learners to want to excel. Mentors were to work in close (vertical and horizontal) proximities (within interpersonal, social and public space (Hall, 1966, pp. 113-125)) with learners in one or two groups, and thus attended initial induction seminars to orient their activity and reflect on previous experience (Bandura, 1971).

Learners' who scored at levels 3 (40-49%), 4 (50-59%), and 5 (60-69%) marks were selected. Learners were organized into groups of up to five learners. They were encouraged to work together, within their groups, during the week, at school and/or at home to continue to practice mathematics and work on assignments. Discussions with learners and the use of Learner's Voluntary and Guided Action (LVGA) forms by mentors confirmed that most learners found ways to meet in class (when a teacher is late or absent) and sometimes in each other's homes to continue working together. Mentors assisted learners with school work and other assignments that learners were urged to bring with them as recorded in the LVGA.

MENTORING

I define mentoring as a location, in time and social space, where learning identities could be shaped by social interactions between a more knowledgeable 'other' (mentor) positioned as a role model and not a teacher, and a less knowledgeable mentee (Vygotsky, 1978). Mentoring occurs in the context of social interactions structured by participation in goal-oriented mathematical activities aided by purposeful, intentional or targeted dialogues. I therefore view the mentoring in LMP as a form of enculturation of a mentee into a certain community of practice (Wenger, 1998), constituted by a group of people with histories of success in school mathematics and the transition into university study.

In theorizing learners' mathematics identity formation and mentors' roles as symbols of academic possibilities and successes (role modeling), I engage the idea of *frientoring* (Brown, Davis, & McClendon, 1999) to explicate learners' and mentors' interactions as embodied in their weekly written reflections. In this paper, I use these reflections interpretively. Brown, Davis and McClendon (1999) coined the word *frientoring* to suggest the importance of friendship in mentoring. Unlike the traditional mentoring relationships which are often asymmetrical in nature where an older person mentors a younger protégé, *frientoring* is appropriate in the LMP as all mentors are university youths who are only a few years ahead of the learners they mentor and who could easily walk in the shoes of the learners (mentees). They, learners and mentors, engage in similar youthful practices, lingos and social networks in cyberspace. While mentees (learners) look up to mentors (university students) and can aspire to seeing themselves in mentors' positions someday, learners can also identify with their mentors as the hierarchical social boundaries are blurred (unlike a teacher, even of similar age as one of the mentors). It was therefore easier to temporarily and purposively collapse the power structure and invite learners to participate and contribute as equal partners and co-sharers of the space and artifacts of interactions and the intellectual stimulations generated as critical and creative thinkers and mathematical problem solvers. I argue that *frientoring* enables mentors and learners to communicate and relate in their mathematical learning trajectories (Wenger, 1998) as they identify with where they were, where they are currently and where they intend to be; these relate to operationalized notions of identity learning being the closing of the gap between actual identities and designated identities as posited by Sfard and Prusak (2005).

LEARNER AND MENTOR REFLECTIONS

I am in the process of analyzing the full data set collected in this intervention and associated research project. However, here I present excerpts of accumulating evidence of the identifying processes at work for both mentors and learners. With respect to the mentors themselves, frientoring generates empathy, a powerful affective response; as well as identification with civic worth.

> Looking back to the time when I was still in the same level as these learners that I are mentoring now, I recall being just an ordinary boy like every other learner. Most of us didn't have any motivation to do our school work. (Mokoane, 11/06/2011)

> ... Being a leader to some people makes you think differently on the things you want to do. I am saying this because mentoring changed my thinking because I saw myself as a leader to the learners and maybe a role model to some of them.... (Climant, grade 9 Mentor's written reflections 26/05/2012)

Here is an excerpt from a learner's written reflections.

> The program is helping me a lot about everything. Amu [Amukelani] is very good person; she is a sister to those [of us] who are wishing to have sisters; she is always there for us and took good care of us. I know that I have to learn from one another and she is teaching us to respect one another. I know that I do not have to play with my time too; I have to be a good and clever girl. I just want to thank you Amu for all things you are doing to us. (Gabavan, grade 9 learner's written reflections - 24/04/2012)

Such interaction also affords and motivates learners to want to find out what it takes to be successful like their mentors considering the fact that mentors are working to overcome similar potential circumstances that learners might be thinking could constrains their upward mobility. Interacting with mentors also provided context for learners to inquire about higher education and what it takes to achieve their future aspiration for higher education.

CONCLUSION

In conclusion, the mathematics identity formation of the learners (mentees) is rooted in their 'doing' (practice) of mathematics enabled by the environment (domain) of and the relationships in mentoring (community) engendered by the actions and activities of their mentors. The way mathematics learning is communicated and generated in an environment where it is safe to make mistakes and be humanely redirected to think critically through the steps leading to the correct answer, allows a learner to 'see' her own mis-step(s) and take corrective actions without being made to feel 'illegitimate' as a peripheral participant (Lave and Wenger, 1991); this provides the fulcrum for transformed practice and participation in mathematics learning, which changes attitudes-in-practice. In some ways, mentors provide social, cultural, emotional and educational supports that may not be available to learners at home, and perhaps, minimally available at their schools.

ACKNOWLEDGEMENTS

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RESEARCH PAPERS

ACCESS, EQUITY AND KNOWLEDGE IN USE IN SECONDARY SCHOOL MATHEMATICS IN SOUTH AFRICA

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What is an equity agenda in post-apartheid education? This descriptive paper engages this question through reflection on a research-led professional development project situated in the current South African context, and informed by previous related research. I draw from our experience and initial research to argue for the primacy of the epistemic for equity and in promoting democracy. I describe how and why current interventions with teachers, aimed towards improving the quality of teaching and learning of mathematics in our schools, requires a focus on their knowledge-in-use.

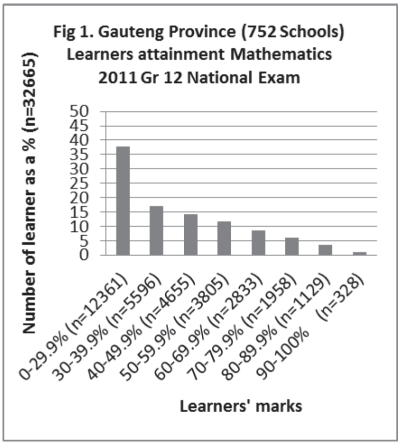
THE SOUTH AFRICAN EDUCATION CONTEXT

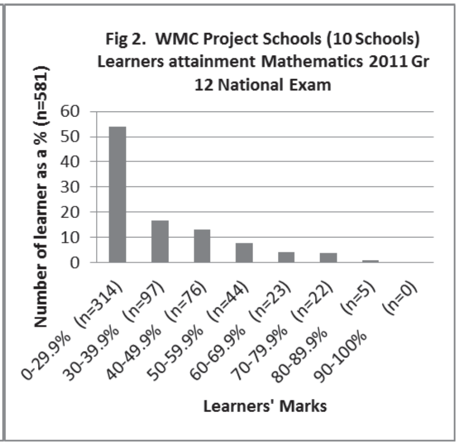
The South African post-apartheid education system is a telling case of how the rhetoric of 'transformation' or 'emancipation' meets an inequitable playing field, and the struggle for access to resources simultaneously fosters and impedes the democratic project. Our experience of change to democracy has revealed just how deep the inequalities still are. Experience has shown how, with the best will in the world and a lot of money (education gets the highest percentage of the GDP in the distribution of money from the state across all its activity), it takes time and an incremental approach to deal with the deep problems that beset education, and the management of these in contested social and political space.

2012 marks eighteen years of constitutional democracy. An enormous component of this has been the unravelling of the apartheid architecture. In education, for example, this has entailed the merging of numerous apartheid-created segregated and unequal (in terms of human and material resources) official departments of education into a unified national department with nine new provincial departments; and within each province there are restructured district offices cutting across previous apartheid boundaries, and servicing thousands of widely diverse schools. This structural undertaking has taken place in a highly politicised environment that encourages short term solutions and cadre deployment, which in turn have been a blockage to progress.

At the same time, there has been significant demographic shift into the cities from the rural areas in South Africa, but also into South Africa from neighbouring states to our north. While the move to cities is a worldwide phenomenon, in South Africa this has been a very rapid process. We have a changing class formation and particularly the emergence of a so-called black middle class. The middle class has expanded rapidly. What was a more exclusive racialised class is now a significantly multi-racial middle class. At the same time, however, inequality persists, indeed appears to be deepening. South Africa has one of the highest Gini coefficients in the world. And so provision of, and access to education in the new dispensation remains deeply inequitable, and this is reflected in patterns of learner participation and performance.

It is nevertheless important to note that educational access has improved. According to Taylor (2007), the high rate of learner enrolment in South Africa has ensured that, despite poverty and the impact of HIV/AIDS, access to education particularly at primary level is extensive. Taylor explains that overall school enrolment increased by 16.6% between 1991 and 2005, with most of this increase due to rapid expansion of secondary education which increased by 53.4% over this period overall. The Further Education (FET) band (Grades 10, 11 and 12) alone, grew by almost 70%. Success, however, is another story. It is the distribution of quality education that is highly inequitable.





The graph in Figure 1 above shows the results in the National Grade 12 Senior Certificate Examination in Mathematics in 2011 for the Gauteng Province, the second highest performing province in the country. Across 752 schools, over 50% of students who reach Grade 12 (and dropout rates before this are an issue too) scored below 40%. Of those that obtained a mark above 40%, only a very small portion exit with sufficiently good grades in mathematics to be able to enter university and study in the sciences. This performance curve is indicative of large numbers of learners in school, but simultaneously being failed by the system. And in South Africa, as in many other countries, mathematics together with proficiency in English, are social access subjects. Without mathematics and English, access to further study, and particularly to the professions, is restricted. Mathematics opens and closes doors, as does fluency in English. Figure 2 shows results for the schools participating in the Wits Maths Connect Secondary (WMCS) project that I describe and reflect on below. All ten schools are in the Gauteng Province. The performance pattern here mirrors the national picture. All of the learners represented in the bars in these graphs are in school. Yet, the majority failed mathematics in 2011, and the majority of those that passed, did so relatively poorly.

Morrow (2007) describes these qualitative inequities in terms of a distinction between 'institutional or formal access' on the one hand, and 'epistemic access' on the other in a similar way to which Bourdieu describes exclusion "from the inside" (Bourdieu, 1993). Elsewhere, this has been described as "education for all, learning for some" (Conference of Commonwealth Education Ministers, 2012, www.cedol.org). Referring to the Millenium Development Goals for education in the developing world, it is acknowledged that while many more are now in school, only some are actually learning. Learners enter the institution (the school), but their access to valued knowledge is restricted, and by inference, their opportunity to learn.

This very broad sketch of the South African education context is the context in which the Wits Maths Connect Secondary (WMCS) project is working.

THE WITS MATHS CONNECT SECONDARY PROJECT – WMCS

WMCS is a 5–year (2010 – 2014) research-led professional development project funded jointly by the state and the private sector, supported by the national research foundation, and located in the university. The project is aimed at improving the quality of teaching and learning of mathematics in ten [1] schools in one district in the Gauteng Province in South Africa, and strengthening the mathematics pipeline within the school and between the school and the university. In the light of the contextual discussion above, the project is concerned with understanding and improving (mathematics) epistemic access in these schools. Our shared goals with the schools is to change the performance curve, so that there are far less failures, and far more learners achieving higher results in mathematics in Grade 12. We're focusing with teachers on how they work mathematically in the classroom, and simultaneously researching this process.

The *'how' of WMCS PD work* is a function of wider schooling conditions and practices, and so I start here by locating the schools and teachers we work with. Drawing on Fleisch (2008), it is possible to describe the schooling system in South Africa as made up of three socio-economically related tiers. The bottom 20% of all schools are typically dysfunctional, but not simply from an educational perspective. These schools function in conditions of abject poverty, with learners suffering from hunger and poor health, problems the school itself is not able to address. Then there is the middle 60% of schools, that have been described as under-performing rather than dysfunctional schools. These schools are distributed across urban and rural areas, cities and townships. They are a function of a rapid shifting demography and migration to cities, and characterised by instability and unpredictability (Simkins, 2010) together with relatively high mobility, particularly of teaching staff. And then we have the top 20% of our schools – the high achieving tier of schools. These schools are in middle class, predominantly urban areas in the economic centres of the country. They are relatively stable, racially mixed and would compete with good schools elsewhere in the world.

The schools in the WMCS project are located in the middle tier – they are part of the 'under-performing' 60% of schools. Of the 10 schools, 5 schools are no-fee schools, in what were informal settlements, and now called townships. They are in typically poorer areas. The other five schools are in the suburbs of Johannesburg and are fee-paying. However, the level at which fees are set are relatively low, drawing learners from low to lower middle class conditions, and in most cases, learners travel into these schools from townships further afield.

In their analysis of the 'parallel economies of schooling in South Africa', Shalem & Hoadley (2009) provide further insight into the impact of economic inequality on teachers' work. While the majority of teachers teaching mathematics in the WMCS schools are qualified, given the historically unequal training of teachers under apartheid, they come with diverse training and education backgrounds, and thus the knowledge resources that support teaching in schools vary. Shalem & Hoadley describe resources in the school that support teaching as both material and cognitive, and go beyond what teachers bring to include the learners they teach. In poorer schools, teachers are working with learners who range in the extent to which they are physically healthy, cognitively prepared, and supported by a second site of learning (Bernstein, 2000). The data I present below amplifies that the learners in the WMCS schools are not cognitively prepared for the classes they are in. This compounds the difficulties of teachers' work. In addition to resources and learner conditions, Shalem & Hoadley identify specification of the curriculum, and the functionality of the school management as also mediating the quality of teachers work. These four quality conditions (resources, prepared learners, a specified curriculum, and functional school management) all impact significantly on teachers' work and so too their morale.

Following the tiers of schools described above, teachers in the top 20% of schools have access to all four indicators of quality, and thus work in optimum conditions. Teachers in the bottom 20% of schools have access to none. Teachers with access to some of these quality indicators are in the 'underperforming' middle band. Thus, the teachers we are working with in WMCS, being in this middle band, are not only in conditions of instability but they work with different levels of morale and support in terms of conditions of their work. As we have come to appreciate the conditions in the schools we are working with, so we are learning how the PD work we do can and must engage with this context.

The *'what' of the PD work, and inter-related research* has been impacted on by previous research, which in turn is influenced by research in the field of mathematics education in general and teacher education in particular. It is in this sense that the project is described as research-led. Early work on secondary teachers' knowledge of their practice in multilingual settings worked from an assumption that language (every learner has language to speak with) as a social/ cultural resource, and described the dilemmas of teaching this produced (Adler, 2001). This led to a larger project in teacher education, and a broader engagement with resources. In a context of limited material resources, and a lament by teachers that they 'lacked resources', we were interested in how teachers worked with resources – both new resources, and the resources they had in their classrooms. This led to a theorising of resources as a verb, and as we learned that the question was how the teacher worked with resources rather than what resources the teacher had or did not have. In other words, it is not what you have, but how you use what you have. A broader theorising of resources also emerged from this study, enabling us to identify and describe resources as not simply material. We saw teachers with more material resources, but poorer practices; and teachers working with very limited material resources, but using what they had in their mediation of mathematics (Adler, 2000).

Backgrounded in all this work, however, was knowledge as a resource and particularly mathematical knowledge for teaching – MKT (Ball et al., 2008), and so we shifted attention to teachers' knowledge in use in teaching, and the problem of how teachers might be supported and prepared for their mathematical work (Adler, 2012). We also studied how, in teacher education, mathematics and teaching (or pedagogy) as dual objects of attention co-exist and co-constitute each other, and how these shape what mathematics is privileged in teacher education practice (Adler & Davis, 2011; Parker & Adler, 2012). These latter works, in particular, have influenced the conceptualisation of the WMCS, and its focus on 'knowledge-in-use'.

WMCS – CONCEPTUAL VISION MEETS CONTEXTUAL GROUND

As a reminder, and now with its research led focus, WMCS professional development (PD) work aims to enhance mathematics teachers' professional knowledge, that is, their subject matter knowledge (SMK), pedagogic content knowledge (PCK), and curriculum knowledge (CK) in use in teaching (Shulman, 1986), or the MKT (Ball et al, 2008). The underlying assumption is that the capacity for informed mathematical judgement that underlies skilful teaching (and possibilities for increasing epistemic access) rests upon this professional knowledge base. WMCS aims at developing teachers' mathematical judgement, through deliberate teaching focused on key mathematical objects of learning, and researching this process together with mathematics teachers in selected secondary schools.

Initial in school observations

In 2010, together with graduate students, we spent the first six months visiting schools, attempting as far as possible, to 'live' inside classrooms with teachers, even in a limited way, and so establishing a sense of what the teachers were working with, their conditions, their learners, as well as their classroom practices. At the same time we piloted a diagnostic test on algebra that we administered in grades 8 and 10 in November 2010, and then in grade 9 and 11 in 2011, tracking a sample of learners.

We have reported on what from observation and discussion with teachers appeared as a pervasive culture that "there's no learning without teaching". Teaching was extensive with extra classes before and after school, and on Saturdays. Yet, learner performance remains poor. It appears that learners and teachers were unwittingly colluding in a culture where learning was equated with listening to the teacher, with learners saying. "When the teacher explains, its ok, but then later, on my own or in the test, I get confused". Exacerbating this was our increasing understanding of how due to the pressure of the National examination in Grade 12, the earlier grades in the school were neglected, and in many instances, teachers in Grades 8 and 9 classes were teaching mathematics 'out of subject expertise'. Learners reaching Grade 10, and the beginning of senior secondary mathematics came with significant backlogs of poor learning that placed teachers in situations where it was almost impossible to catch up.

Learners/learning and diagnostic assessment

Analysis of our diagnostic tests further illuminated the nature of the poor performance, and of imagination of on the ground practices.

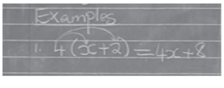
In 2011 (after piloting in 2010) we tested a sample of Grade 9 (n=1400+) and 11 (n=700+) learners in our schools, using the ICCAMs diagnostic test. Space limitations and the focus of the paper precludes more detailed discussion of the test which we have found useful in illuminating the ways in which learners at both levels in our schools interpret questions directed at algebra as generalised arithmetic, and use of letters. We are using our data to both track learners over time, and to inform our work with teachers. The ICCAMs test comes with a set of errors prevalent in the early learning of algebraic symbolic form, and thus a means for interpreting and categorising the errors made by our learners. It helps teachers see that common errors are not unique to our schools and learners, but are to be expected as learners begin formal algebra. Most illuminating for us, however, were not the anticipated errors, but rather the large array of seemingly arbitrary responses that did not fit the available codes. We introduced an additional code, together with sub-codes, for each question in an attempt to systematically record and make sense of the large array of errors. We were aware that in many cases, the majority of responses, particularly in Grade 9, but also at times in Grade 11, fell into this new set of codes. What we did not realise, and our current 2012 preparation for testing has revealed, is that notwithstanding the extra sub-codes, there remained a large portion of uncodable responses – in too many cases, a large percentage of learners produced responses that were too idiosyncratic and varied to warrant a particular coding. For example, consider the following item: Simplify (if possible): $3x - (y + x) = \ldots.$

The highlighted rows in Table 1 below provide the percentage of responses against the extended or additional codes we introduced. What stands out is the high percentage of responses that remained in the category 'other'. In addition to features, like the increase in errors of operation from Grade 9 to 11, and the misuse of the distributive law, the 63.5% of Grade 9, and 41% of Grade 11 that were coded 'other' in the second table was, for us, the most significant 'result'. While we can make some sense of the wrong answers coded (a) to (f) and a number of learners gave these wrong responses, the large number still in (h), other, reflects that on the one hand, the majority of learners in our schools, including a large number who are in Grade 11 mathematics, misrecognise or attach little recognisable meaning to algebraic symbolic form. The wide variation in responses suggests further that the messages about what counts as algebraic thinking and working in classroom practice is, largely, 'incoherent', and an indication that what learners have opportunity to participate in bears little resemblance to what counts as school algebra.

Table 1 – extended codes	**Grade 9 %**	**Grade 11 %**
Missing	8.4	7.1
$2x$-y	3.5	24
xy	2	0.8
$2x$	1.3	0.2
$2xy$	2.6	1.5
$3xyx$	2.8	0.2
$4x$-y	6.5	6
$3x^2y$	6.5	4.6
+/- $3x^2$-$3xy$; or =/- $3x^2$ +$3xy$	1	9.1
$3x^2$-y	2.1	5.3
Other	63.5	41

Teaching - classroom practices

Our more recent analysis of video recordings of teachers teaching algebra provides some insight into the practices that are implicated in producing such responses. I will illustrate through one lesson here, taught by a 'strong' teacher in the project: she is professionally qualified, has a degree in mathematics, and is an active participant in the project and all its activities. This was a Grade 9 algebra lesson on "The product of expressions". After going over homework involving applying the rules for 'exponents', the teacher (T) introduced the lesson for the day as "finding the product of expressions". She worked first on three sequenced examples of a monomial X binomial, inviting learners to contribute the 'steps' needed: $4(x + 2) = \ldots$; $4x(x + 2) = \ldots$ and $-4x(x + 2)$. The answer offered for the first example was $4x + 8$, and the teacher emphasises that "you multiply everything inside the bracket by the term outside", while illustrating this process with two curved lines as illustrated in the plate below. She then asked 'are we finished?', anticipating conjoining as we had discussed in examining learner test responses in the project.



There are answers of yes and no from the class, with some offering $12x$ as the final answer. A learner disagrees stating that '8 doesn't have a variable of x' which T revoices, and reminding learners that $4x$ and 8 are unlike terms, and she moves on to the next example: $4x(x + 2)$. She faces immediate difficulties as the first response from a learner is $5x + 10$. She draws attention to the meaning of $4x.x$ (as $4x^2$), referring back to the homework on exponent laws, and seeks another answer to the project. A different learner offers $6x$, and following disagreement from others in the class, moves on to obtain the correct project, $4x^2 + 8x$. A student then asks: "Mam, where does the $8x$ come from … the 2 doesn't have an x?"

There are a number of interesting issues in this short excerpt from the lesson. First is that through this lesson (and we see this in other lessons), the dominant 'explanation' for steps carried out, or legitimation for what counts as mathematical practice, is localised (i.e. valid for this example, rather than a class of examples) and carried by a visual image. The verbal description: "everything inside the bracket is multiplied by the term outside" now becomes a visually displayed strategy, and algebraic terms are recognised by how they look and what they 'have'. Reproduction by learners is thus likely to be by imitation, with learners managing to produce correct answers to a following 'problem' if it is presented immediately after in sufficiently similar form. This localisation and immediacy might well be a strategy teachers use as they come up against 'holes' in learners knowledge that have to be quickly 'plugged' so that the mathematics of the day can continue. Following our earlier work, we see that teachers' knowledge-in-use, and so what functions to ground or legitimate what counts in this class, is largely iconic (Davis, 2010). There is little 'mathematical' ground for why these products emerge as they do. Indeed, a trawl through the transcript indicates that references to products and their meaning is by assertions like 'brackets means multiply' and 'multiply everything inside by the term outside'. There are no additional representational forms (e.g. geometric, numerical) that might provide some meaning for these expressions and their products. Attention to operational sequences seems to lose sight of the object they are operating on (Artigue, 2009). The object of learning is out of focus – not made explicitly available to learn (Marton, Runesson, & Tsui, 2004).

REREADING ABSENCE

The challenge for the project, theoretically and politically, was how to (re)read these practices, and (re)design our professional development activity. What does it mean to disrupt what appears to be a deeply embedded social practice that learners and teachers co-constitute through their interactions in their classrooms? It has been productive to (re)describe the 'lack' we see in terms of participation in social practices. For many learners, their participation in class is participation in 'another' (non-mathematical) discourse and a particular social practice; and we need to understand this so as to be able to work with teachers on both its recognition and construction of different, more mathematically attuned, discourses and practices. For some learners of course there is non-participation, displayed by various forms of resistance.

In our work with teachers we focus on bringing the mathematics that they're working on into the foreground of their thinking. We work ourselves on the notion that pedagogy proceeds through professional judgment through the transmission of criteria, of what counts as legitimate in this practice (Bernstein, 2000). And judgement has epistemological as well as pedagogical entailments. In the wider teacher education discourse and education reform in South Africa, there is emphasis on learner centred practices which focuses on motivation and participation as being 'active' in class. We redescribe these with teachers as 'focusing on what the learners need to know and be able to do mathematically'. In Marton et al.'s (2004) terms, we thus bring both the direct and indirect object of learning into focus. We work with the teachers on key concepts and their teaching and learning, and their key features. Included in this, is careful and explicit attention to discursive demands. Teachers are invariably working in two or more languages – English as the language of instruction, learners' fluency in other languages – as well as between mathematical and everyday discourses. We emphasise the importance of learning to use mathematical words in legitimate ways and that this matters deeply for the learners' progress.

CONCLUSION - RETURNING TO ACCESS AND EQUITY

I began this paper with the question: what does it mean to have an equity agenda in education in post-apartheid South Africa? Through the WMCS this is interpreted as attention to epistemic access and teachers' knowledge in use. I have described our developing work with these goals. Critical work in mathematics education research asks important questions like: whose knowledge? Whose language? Whose problems? are privileged in mathematics learning in schools (Civil, 2012). Civil discusses the deep tensions in this work, and the tension I have foregrounded here is how confronting conditions and practices in the schools we are working in (and typical of the majority of schools in South Africa) we work so as not to produce a deficit discourse that unintentionally exacerbates the problems. In response to Civil, I have drawn on my earlier research as well as the recent work of Michael Young to distinguish 'powerful knowledge' from 'knowledge of the powerful' (Young, 2008). Epistemic access to (some – not all!) mathematics in school, is access to powerful knowledge. If we are to develop our fledgling democracy, our learners deserve no less. That is why WMCS focuses on 'objects of learning' and 'knowledge-in-use'.

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NOTES

1. There are actually 11 schools currently in the project, as one of the ten original schools was divided into two separate schools during 2010, a middle school and upper secondary school.

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FACILITATING MATHEMATICS LEARNING IN DIFFERENT CONTEXTS: THE VALUES PERSPECTIVE

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In this paper we discuss students' values in a teaching context where, pedagogically, the mathematical topics were connected to current societal issues. We follow the mathematics-learning story of a student named Henrik, an example of students' talk that demonstrates how student engagement changes with reference to different levels of learning contexts in and outside the mathematics classroom. Data were collected from a survey, interviews, spontaneous conversations, students' blogs and project logbooks. Changes in identity narratives appeared to be rooted in the relatively stable valuing of meaningfulness, fun, realism and technology. The extent to which the various contexts' valuing was aligned with Henrik's values facilitates our understanding of why and how he chose to engage (or not) with his mathematics learning. That is, sociocultural and personal valuing – and the extent to which these are aligned – promise to regulate and explain the role of contexts in promoting student engagement in, and hence learning of, mathematics in schools.

INTRODUCTION

Engagement in learning regulates the extent to which a learner interacts with the subject content, and is thus an important variable in mathematics learning. However, more recent evidence (see, for example, Andersson, 2011a) has suggested that engagement is not a trait, but rather a state of a mathematics learner that is affected by the contexts within which the learner finds him/herself. Contemporary research that identifies and labels particular learners as engaged (or not) so that 'something can be done about it' may not yet present the spectrum of experiences which (mathematics) learners go through as their learning contexts change.

Through the story of mathematics learning that developed for a student (in Sweden) named Henrik, this paper presents a window into the ways in which learners' engagement shifts with changing identity narratives in relation to the contexts in which the narratives were told. We will explore how these changing variables might be rooted in the cultural values, which are internalised within individuals' experienced learning contexts. Recognising what the various contexts value is important, we will argue, as it anchors change in engagement and identity narratives against a relatively stable variable, that is, values.

CONTEXT

In mathematics education research context tends to be restricted to the immediate context of a particular classroom or studied activity episode (Morgan, 2006). Efforts have been made to challenge this statement (Andersson, 2011a, 2011b). Contexts can be considered in a number of ways. Here, we propose a way of classifying these, namely, task contexts, situation contexts, socio-political school contexts, and societal contexts. First, task contexts are the referents to which particular tasks appeal in order to invite students to engage in mathematical activity. Task contexts are expressed in, for example, textbook exercises and through developed pedagogical projects (Wedege, 1999). Research reported by Stocker and Wagner (2007) who introduced tasks influenced by critical education exemplify research addressing the contexts in which exercises and tasks are presented and thus situated. Second, there are situation contexts, understood as the array of "current activities, the other participants, the tools available and other aspects of the immediate environment" (Morgan, 2006, p.221) in the classroom. A situation context thus also refers to the communicative understanding of contexts. Third, a wider socio-political school context refers to contexts outside classrooms that influence what occurs within the mathematics classrooms, operationalized through governmental policies on schools and the national curriculum, ideologies and school policies (Valero, 2004). This school context refers to layers of school organization that shape possibilities for engagement. These include, for example, school structures such as timetables and school leadership, as elaborated by Martin (2000) when addressing the complexity of reasons behind African-American youths' achievement or failure in mathematics education. Fourth, a societal context of societal discourses impact in mathematics classrooms. 'Specialness' when being 'good at mathematics' (Mendick et al., 2009) is an example of discourses within the societal context that impact on what occurs within classrooms.

These contexts exist within a socio-cultural setting, and as such they cannot be perceived as being free of the values which underlie cultures (Bishop, 2008). To the extent that contexts influence discourses in the mathematics learning process, it is useful for us to understand contexts also from the perspective of the cultural values that contribute to their occurrence. This is especially meaningful when we find ourselves analysing contexts that might be taking place across different cultures.

VALUES PORTRAYED THROUGH CONTEXTS

Values may be considered to be the window through which an individual views the world around him/her. They are the convictions that an individual has internalised as being important and worthwhile. The process of valuing, then, motivates how the learner utilises his/her cognitive skills and emotional dispositions to learning. That is, values might be regarded as a volitional construct. Often they contribute to the traits of the individual, who seeks to enact these values through the evaluations made, decisions selected, and actions taken. The various categories of values in mathematics education (see Bishop, 1996), then, represent "an individual's internalisation, 'cognitisation' and decontextualisation of affective constructs (such as beliefs and attitudes) in her socio-cultural context. Values related to mathematics education are inculcated through the nature of mathematics and through the individual's experience" (Seah, 2005, p. 43), thus becoming the personal convictions which an individual regards as being important (Seah & Kalogeropoulos, 2006) in the process of learning and teaching mathematics.

In our current study, we construct a learner's narrated identities grounded in various learning contexts. We have sought to identify the values that underlie these narrated identities which were portrayed through the contexts. In so doing, we propose a means of interpreting these identities and the ways these identities relate to (mathematics) learning.

BACKGROUND TO CURRENT RESEARCH

The data, which shed light on Henrik's mathematics learning experiences, constitute part of a one-year research study exploring upper secondary students' learning of mathematics within a social science program in Sweden. Students there commonly complete this program because it provides entry into university studies in the social sciences and language faculties. Also, students who do not enjoy mathematics and thus do not want to take the alternative natural science or technical study programs often see this social science program as a good option.

Annica, in collaboration with a mathematics teacher named Elin (pseudonym), introduced teaching sequences that enabled students' mathematics learning to be connected to societal topics inspired by different aspects from critical mathematics education (Skovsmose, 2005). How mathematical topics related to societal contexts regarding mathematics as a tool for identifying and analysing contemporary features in society was one important aspect. These aims matched the Swedish curriculum objectives, which asserted that mathematics education for social science students should "provide general civic competence and constitute an integral part of the chosen study orientation" (Ministry of Education, 2000). A second aspect concerned the epistemological point that an educational practice was considered to involve learning and becoming, rather than a simple transmission of knowledge (Skovsmose, 2005). A third aspect involved how power relations between the actors supported a classroom environment where students could become agentic in a positive way towards their learning and where students had access to and contributed to the discourse between participants (Andersson & Valero, in press).

Gathering information

In order to understand students' relationships with mathematics from their perspectives, ethnographic methods (Hammersley & Atkinson, 2007) were used for data collection. Annica established a trustful environment through engaging with the students in both formal and informal settings during all lessons in the course. In this way, she interacted closely with the students, and experienced the contexts and discourses. The research methods deployed included a survey, semi-structured interviews at the start and the end of the course, spontaneous conversations throughout the course, a student blog, and students' project logbooks. In the survey, students were asked about their prior experiences of mathematics learning and their personal goals in the current course, and hence these narratives referred to different context levels. The interviews also provided reflective data about the different context levels. The blog was a course activity and provided data mainly about task and situation contexts. Students' actions, hence their reflections of their agency (including resistance), were also evident in the blog. The logbooks provided data about the students' learning in relation to task and situation contexts. Annica's research-diary allowed the students' stories to be related to what went on in school and society at particular times.

Interpreting information

The data analysis process acknowledged Sfard and Prusak´s (2005) call to "equate identities with stories about persons" (p. 14) if the story is reified, endorsed and significant for the identity builder. Henrik (pseudonym) and the other student participants were the significant narrators of these identities and they drew on stories from their parents and their mathematics teacher (Andersson, 2011a). These stories were then located in relation to the different contexts in which they were told at those particular times. Talk about agency (including resistance) was also connected to the stories. In this way chronological storylines emerged where it became visible how contexts, agency, values and identity narratives were related.

In this paper we share the story told to us by one of the student participants, Henrik. In particular, four identity narratives in contexts from Henrik's course trajectory will exemplify changes in the students' narrations of themselves and how contexts impacted on the students' engagement through changes in their expressed narratives at particular times. We then filtered students' narratives further to reveal the culturally-based values that are internalised within Henrik's identity narratives.

HENRIK'S IDENTITY NARRATIVES

Valuing meaningfulness

> In general, I have always disliked mathematics; it has never felt meaningful for me. (…) The problem was not that I didn't understand, I was usually quick on that, the problem was rather that I became very tired of writing down the maths and focus on maths for a longer time. The most difficult was probably that I did not experience mathematics as meaningful (in lower secondary school), I could not relate this knowledge to something I would need further on. To sit down, and focus, calculating the same type of exercises again and again felt so meaningless. (Henrik, survey, 08-2009)

The story, as voiced here by Henrik, is definitely one we have heard before from many students, in many classrooms, across many countries. These traditional, repetitive, and predictable mathematics lessons has been called the school mathematics tradition by Cobb (1992), "the exercise paradigm" by Skovsmose (2001, p.123), simply traditional mathematics teaching by others, and criticised in the Swedish School Inspection quality report (2010). Clearly, these lesson modes did not appeal to Henrik in his school years of compulsory mathematics education. We can see in the quote above (and elsewhere in Henrik's articulation) how important it was for him as a learner for mathematics to be meaningful to him. That is, Henrik was valuing *meaningfulness*. In experiencing a misalignment of the valuing of *meaningfulness* between what he valued and what the task and socio-political (school) contexts valued, Henrik verified:

> Mathematics has been boring all the time. I think I ought to have had higher grades in mathematics, (…) but it became so boring, I tried to calculate but then I talked to friends instead. (…) I felt disappointed, like in ninth grade, the last year of lower secondary, I got G (pass), and can't I do better than this? (Henrik, interview, 09-2009)

In the quote above we see Henrik's valuing of *meaningfulness*. The school (socio-political) context did not appear to share this value, and the misalignment can explain Henrik's disengagement, shown through his choice to talk to friends in class.

Valuing fun

Henrik's valuing of *fun* is demonstrated in the quote below. During a project work period in which the students experienced possibilities for taking decisions on task and situation contexts, Henrik and Annica had the following blog conversation:

> Henrik: It was very comfy to choose and decide self [sic], we brainstormed together and came up with good ideas about what we could do. At the same time it felt very abstract that this could be possible, but it still was. Fun!! (Henrik, blog, 16-09-2009)
>
> Annica: What do you mean with the word 'abstract'? Can you please explain so I understand? (Annica, blog response, 16-09-2009)
>
> Henrik: Our goal was to become bakers (…). We searched for the world's best baker education and found one in France. I mean that this did not seem so trustworthy that it would be a real possibility… far from the position I am in today as an upper secondary student. But I really got the opportunity to live myself into the situation through this project, to step by step plan out what needed to be done to reach that goal. In that sense it became much more than only a mathematics task ☺ (Henrik, blog response, 17-09-2009, original smiley)

A change in the pedagogical tasks (to project work) has allowed for this task context to co-value *fun* with Henrik, which led him to reconnect with mathematics learning. Continuing the analysis of Henrik's blog comments during the project weeks, the words he uses are characterised by "today I checked", "we have not chosen yet…", "tomorrow we will" etc. indicating action and individual decision-taking for learning.

The first project was followed by two weeks of textbook work focusing on basic equations and algebraic simplification. According to the teacher, this book chapter was considered to be a repetition of prior lower secondary mathematics education and thus should not be too complicated for the students to carry out. Yet, Henrik made one blog comment during these two weeks:

> Today I have learnt new techniques on how to simplify equations, what this has meant for me personally I do not know. I have been concentrated [sic] so I am tired (Henrik, blog, 07-10-2009)

Here, his valuing of *meaningfulness* is evident, in that Henrik had found it important to focus and to persevere so that he could learn what he did not learn before. Interestingly, in finding meaning in this topic, Henrik has yet to understand what the skill would mean for him personally.

On another day, when Henrik was leaving the classroom after a lesson looking tired and low-spirited with hanging shoulders, Annica and Henrik had the following conversation, one that reinforced his valuing of *fun*:

> Annica: What's up?
>
> Henrik: Sometimes one comes here and knows one only has to do them, all the boring exercises, to learn
>
> Annica: What do you mean?
>
> Henrik: Well, somebody has to make the decisions for one, because I don't know what I have to do to learn this stuff. (Researcher's field notes)

Valuing technology and realism

A statistical project commenced as a cross-subject collaboration with the environmental science subject at the end of a semester. The topic was focused on the ecological footprints people left on earth, with possibilities for the students to plan and take decisions on both content and task contexts. During the initial phase of the project the group Henrik participated in experienced some collaboration problems. Henrik and another boy in the group wanted to continue a prior bakery project, but now in relation to ecological issues and sustainable development. But the third boy did not like this idea, so it took the group some time to compromise on the focus of their project. In the end they decided on a food topic, hence to work with consumption of 'Kravmärkt' (organic) food. After the topic was settled the group worked well and were focused (Researcher, field notes; Henrik's log book).

What amazed us all – teachers, researchers but also Henrik's father – was the groups' decision to work on their project using the web-based forum Google Docs, thereby demonstrating the learners' valuing of *technology*. As Henrik's father commented:

> I am astonished, through Google, in real-time, they sit in different locations and rectifies their writings, so their project is very much alive and in progress. I believe they have developed (personally) and that they learn a lot through this way of working (Henrik's father, interview, 11-2009)

The students in all groups accounted for creativity in a way that they reified Lange's (2010) statement that creativity and choice-making are prime aspects of agency. Analysing Henrik's log book during the project work, the typical reflective statements during this period were formulated in line with: "we have been thinking about how to manipulate the diagrams to our advantage", "we could not find a way to formulate a question that was answerable on…" (Henrik, logbook). Here we have a situation in which both the classroom and personal contexts value *technology*, an alignment which facilitated learning and student agency.

Valuing technology, realism and fun.

In the following excerpt Henrik reflects on his project work in relation to his learning of mathematics. It appears that his engagement with learning through the project work came about not just through the alignment of the valuing of *technology*, but also through the valuing of *realism*:

> I have learnt much more this way than if I only had done book calculations. I believe I have received more useful knowledge, because this way of working is more real with a stronger connection to the 'real' reality (Henrik, logbook, conclusions)
>
> I learnt new mathematics in the last project we had, the investigation, how to make ones interests to have impact on others through mathematics and still learn, that has been fun. It was harder, and required more work but was more fun (Henrik, interview, 12-2009)

The last phase covers quite a long period of time, where the students worked both with 'traditionally planned mathematics', education with textbook work – and with a geometry project. Annica received three e-mails from Henrik during this time, and here we present two excerpts from these e-mails as they show a change in his way of talking about himself:

> I am very sorry that have not responded to your e-mail until now. The reason is that there has been some mathematics school work that has not felt meaningful and thus I lost some of my study motivation (…) Before the Easter holiday the topic in maths was functions. That was 'usual' mathematics however with some practical laboratory tasks that made those lessons more interesting. We then got examined with a usual (written) math test.
>
> At this time I have got back some of my study motivation because we now do some interesting and meaningful stuff, in maths it is geometry. We could choose, for the geometry sequence, to get examined through a "usual" math test or, if one wanted, to suggest another way. I decided to do a report about Tetra Pak's [1] legendary milk package.
>
> (I will) calculate volume, angles, area etc. that I then will use to create a smaller miniature milk package model. After that I will reflect on usefulness, how much material is required and a conclusion about how it can get better and improve. If I have time I will create a personal variety with a miniature milk package containing 1 dl. That feels meaningful! ☺ (Henrik, e-mail, 24-04-2010, original smiley).

Again we can notice a shift in Henrik's way of narrating his identities as a mathematics learner.

The contrast between Henrik's ways of talking about himself during the different sequences is obvious: "has not felt meaningful", "I lost some of my study motivation" and "we then got examined with" – indicating passivity, meaninglessness and lack of power during the functions sequence – compared to "we do some interesting and meaningful", "I decided to do a rapport", "I then will use", "how it can get better and improve" and "if I have time I will" – all indicating action, meaningfulness and ownership. The project work has indeed been very significant in Henrik's learning experience, not just because the task context exemplified the alignment with his valuing of *technology* and *realism*, but also because it allowed for the socio-political (i.e. classroom) context to co-value *meaningfulness* and *fun*.

CONCLUDING REMARKS

Thus, examining Henrik's stories through the values perspective has allowed us to further interpret socio-culturally Henrik's learning experience across the different learning contexts and across the different pedagogical tasks within the different contexts, which underlied the activities within the classroom. As we see above, Henrik values *meaningfulness* and *fun*. However, as the learning contexts changed, and as the pedagogical tasks within each of the contexts changed as well, there were times when these values were not aligned with what the tasks valued, leading Henrik to feel unmotivated and excluded. In this sense, the act of valuing relates to learner agency. Values are volitional variables that regulate actions such as motivation and engagement (Seah & Wong, 2012). There were also other tasks (such as the Tetra Pak project) in which *meaningfulness* and *fun* were embedded. However, in such an instance, the alignment of these two values between Henrik and the respective contexts gave us a learner who was able to talk about himself and his learning in a positive and proud manner. Here, the data suggest that the situation contexts are capable of supporting the valuing of both *meaningfulness* and *fun*, although they do not appear to embody the values by virtue of their own characteristics. Rather, the pedagogical tasks need to be designed in such a way as to portray these values, which were also shared by Henrik.

Of course, depending on what Henrik's peers value personally, the portrayal of *meaningfulness* and *fun* in particular pedagogical tasks within the situation context may be in conflict with other learners. Similarly, while Henrik's own values were aligned with the valuing of *realism* and *technology*, the narratives of his peers in the same classroom may yield stories of disengagement and lack of motivation.

Learners engage to different degrees in different contexts (Andersson, 2011a) and when given different pedagogical tasks (e.g. Sullivan, 2010). While these may imply that classroom teachers plan their professional repertoire in ways which expose their students to different tasks in different contexts (e.g. Sullivan, 2010), there may be a more purposeful approach to lesson planning that allows for the customisation of pedagogies to students' learning preferences, thereby optimising learning outcomes. As we saw in Henrik's case, the different contexts and/or tasks within the situation contexts which enabled him to develop interest, confidence and performance might be more usefully understood in terms of the common values they espoused, values which were aligned with Henrik’s personally-held values, and which included *meaningfulness*, *fun*, *realism* and *technology*. Thus, a teacher’s understanding of what most of his/her students value – and/or what particular students value – and the subsequent structuring of the various contexts within which teaching takes place, would go a long way towards empowering teachers to facilitate student interest, confidence and performance in school mathematics.

NOTES

1. Tetra Pak, founded by R. Rausing, is a legendary industry in this part of Sweden, famous for invented the milk package Tetra Pak. So Henrik connected his geometry project to society and a geographically important invention for the region.

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THE COMPLEXITY OF INTERWEAVING MATHEMATICAL AND SOCIOPOLITICAL CONTENT

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This paper presents a nuanced understanding of the complexities that emerge in a classroom where students and teacher co-created a classroom to read the mathematical word (learn mathematics) and read the world with mathematics (understand social reality using mathematics) using generative themes (key social contradictions) from students' lives. Using a theoretical lens synthesized from Vygotskian and Freirean perspectives, this paper follows the trajectory of classroom interactions to understand how teacher and students interwove mathematical and sociopolitical dimensions in this classroom, how the teacher scaffolded these two dimensions, and the tensions therein.

INTRODUCTION

In this paper, we discuss the ways in which students and teacher navigated mathematical and sociopolitical content in one of the units of study from a year-long class [1] (September 2008 – June 2009) titled "Quantitative literacy through investigating urban social reality" (or, informally, the "math for social justice" [M4SJ] class). This class was offered to students in their senior year at the Greater Lawndale/Little Village School for Social Justice (henceforth referred to as Sojo). Sojo is one of four small schools in the Little Village Lawndale High School (LVLHS), a neighborhood Chicago Public School (CPS), born out of a community struggle, serving the communities of North Lawndale and Little Village that are home to low-income and working-class families of color. The student population at LVLHS is entirely Black and Latino/a, and all students in this class were from Sojo's first graduating senior class.

This year-long class was guided and inspired by Paulo Freire's work to read [understand] the world and write [change] the world. Students and the teacher (Rico Gutstein, a white male, university professor, henceforth referred to as Rico or teacher interchangeably) co-created a classroom where, together, they investigated contexts from students' lives (generative themes) mathematically and sociopolitically. These contexts (elections, displacement, HIV-AIDS in their communities, criminalization of youth/people of color, and sexism), were suggested by students or proposed by the teacher and accepted by students (Balasubramanian, 2012; Gutstein, 2012).

Two of the several purposes for this class were a) supporting the learning of rich mathematical ideas through an investigation of sociopolitical contexts from students' lives and the broader world, and b) supporting the growth of students' sociopolitical analyses and sense of social agency through a mathematical investigation of these contexts. Students chose this class while in their junior year from the three options available to them: M4SJ (only offered in 2008-2009), year four of IMP (Sojo's regular math curriculum), or pre-calculus. Twenty-one students selected to enrol in this class, six were African American, 15 were Latino/a. Fifteen students were females (of whom four were African American), and six were males (of whom two were African American).

Mathematics teaching efforts based on critical pedagogy suggest that sociopolitical realities and students' lived experiences can indeed be powerful and meaningful contexts for students (and adults) to learn mathematics and use mathematics as a tool to investigate social conditions. See for example, Frankenstein (1983), Gutstein (2006, 2012), Turner (2003), Varley Gutiérrez (2009). However, unlike the literature on interactions in reform-oriented classrooms (e.g., Walshaw & Antony, 2008), research on "up-close" interactions between class participants in critical math classrooms is limited. Gutstein (2006), Turner (2003), and Varley Gutiérrez (2009) are some exceptions. In this paper, we trace and analyze the trajectory of classroom interactions from one of the units of study in this class. This analysis adds to our collective understanding of how the mathematical and sociopolitical dimensions are in a dance in such classrooms, and how the interactions between teacher and students in such a classroom can facilitate the interweaving of the two dimensions.

We used a framework synthesized from Vygotskian and Freirean perspectives (Balasubramanian, 2012) and considered this classroom as a space for joint activity. Reading the world (understanding the world) and reading the mathematical word (understanding the mathematics) were two of the many purposes of this class (Freire & Macedo, 1987; Gutstein, 2006). This joint activity was mediated by the content (generative themes and mathematical ideas), talk (interaction patterns and norms), and was facilitated by the teacher. The objects of investigation were generative themes from students' lives and social reality, and the mediating artifacts were the curricula, texts, video, and the interactions. A dialogic stance permeated this activity and was central to this class. Teacher and students together developed normative ways of interacting with each other and created a dialogic space that allowed for intellectual openness, critique, creativity, questioning, collaboration, and sharing power and authority in order to achieve the goals of reading the world and reading the mathematical word.

Using a qualitative approach drawing from methods in both ethnography and discourse analysis, we investigated the dialectical relationship between the sociopolitical and mathematical dimensions as seen in teacher-student interactions. We collected [2] and used several types of data including researcher field notes, teacher journals, student surveys, student work, audio and video data of classroom interactions, the entire curriculum, class assignments, homework assignments, journal assignments, and PowerPoint made by students for two community presentations.

TRACING THE TRAJECTORY OF HIV-AIDS UNIT

The HIV-AIDS unit was the third unit that students studied in the academic year and it lasted 7 weeks. The purpose of this unit was to support students in understanding the HIV-AIDS epidemic in their communities and Chicago, through mathematical and sociopolitical analysis. Rico (the teacher) and two graduate students from the curriculum development team contacted health professionals and researched medical journals to get more information related to the spread, prevention, and treatment of HIV-AIDS and to develop a curricular outline for this unit.

Mathematically, students worked on the following in the unit: creating a *discrete dynamical system* [3] (DDS) with one and two variables; finding equilibrium values (algebraically and graphically); creating a DDS for disease spread (SI models [4]); simulating disease spread using DDS on calculators; interpreting graphs, statistics, pie charts and other visual representation of data; analyzing proportionality and disproportionality; and predicting using linear and cubic regression. Sociopolitically, a key focus of this unit was to support students in examining the role of social forces, in addition to individual behavior, as a factor that influences the rates of HIV-AIDS spread, infection, and recovery. Towards the end of this unit, students considered how to share their work in the presentations with their communities, and they completed a final, take-home exam.

The initial plan for this unit was for students to develop mathematical models for the spread of HIV-AIDS in students' communities and

> …then think about tweaking them based on more (for example) gender equity so to reduce "survival sex," or better/more accessible HIV testing or full free access to condoms everywhere. The idea would be to make a mathematical argument that we'd have less deaths if we did one or more of these things. (Teacher Journal, 3/19/09)

Students spent the first two weeks extending the mathematical ideas of developing a DDS, which students had studied in the previous unit and used to model mortgages. Although students had some experience with the DDS, several had difficulties combining terms as well as understanding recursive functions. Problems also emerged as students tried to make sense of the algebraic process of finding the equilibrium value for dynamical systems with one or two variables. In one instance, Rico journaled,

> It was a pretty dismal affair, with students tuning out and me teaching at them … students' intuitions and conceptual understandings of math are not that developed and their knowledge of integers is weak, and their knowledge of how to manipulate algebra is weak, etc. (Teacher Journal, 3/16/09)

Towards the end of the second week, he recognized that developing a model for disease spread in this context was not trivial. This approach was contrived and impractical in this situation because data for students' communities was difficult to find, creating a model for disease spread in this instance was beyond the scope of this class, and most importantly, the mathematical-sociopolitical connections through this approach were not very obvious (Balasubramanian, 2012; Gutstein, 2012).

> At this point I don't know if students see the mathematics as being connected to social reality. Yes, (the spread of) HIV-AIDS can be modeled this way, so what? How does understanding this model help me make an argument to change the discourse on AIDS? Can I use the model to make an argument for increased social responsibility in how we are dealing with AIDS? Can I use this model to make an argument to show that our current approach to dealing with this epidemic is futile and so on? We need to tie the mathematics as being helpful in understanding and changing the social reality. (Researcher Field Notes, 3/19/09)

Moreover, apart from a brief discussion on what HIV-AIDS is and how it spreads, there was little initial clarity on how the mathematical ideas connected to the sociopolitical context nor much discussion on the sociopolitical context of HIV-AIDS itself. In order to engage students in a sociopolitical analysis, Rico asked students to read the second chapter (henceforth referred to as the myths chapter) from the book *Global AIDS: Myths & Facts* (Irwin, Millen & Fallows, 2003). This chapter presented scenarios of people affected with HIV-AIDS with details of their social, economic, cultural, and other factors that constrained the individual choices they could make (e.g., for safe sex practices or jobs) and in turn made them more susceptible to HIV-AIDS infection.

Students read and presented sections of this chapter over a period of three to four days. During this time, the complexity of understanding the social factors discussed in this book chapter and the difficulty of going beyond the discourse of individual responsibility came to fore. In one instance, Greg [5] and Jenny talked about Rakhi, a woman in India, infected with HIV-AIDS because of unprotected sex with her husband. An interesting conversation ensued, mainly between students. Julie asked, "How come they [Rakhi and her husband] cannot afford to buy condoms?" Jenny pointed out that one, "her [Rakhi's] husband didn't want to use the condoms," two, "she [Rakhi] didn't have no money to go buy condoms," and three, that "of course she [Rakhi] can't just force it and put it on him." Rico then asked Jenny to say more about why she (Rakhi) could not force her husband. Jenny responded,

> Jenny: He doesn't want to use condoms just like other boys our age don't want to use condoms. He didn't want to use condoms and she got AIDS. She couldn't force him, that was her husband.
>
> Greg: Probably she want some too.
>
> Jenny: No, it ain't even that. She, he forced her to have sex with him.
>
> Greg: So you saying he raped her?
>
> Carlton: They was married.
>
> Jenny: Don't say that he raped her but they married so it's not rape.
>
> (Video, 3/23/09)

Several students in the class simultaneously disagreed and said, “Yes, it is” indicating that they considered it rape. Jenny, however, persisted and responded, “No it’s not. She’s obligated to have sex with him. In other cultures, that’s how it is. She doesn’t have a choice. She’s a woman.” Rico revoiced Jenny’s contribution and steered the conversation towards a discussion about choice and power. Although Jenny seemed to indicate that Rakhi was limited in her power to choose (“can’t just force him”, “could not force him”), later in the conversation, she raised the concern that the authors of this chapter “seem to be saying that poverty is the issue without really saying that individual choice is also important to consider and that there are some people who are promiscuous and that behavior cannot be excused” and “sometimes women have no choice, but that women should be able to refuse if their partner does not want to use condoms, knowing well that they are at risk and that this book does not address that” (Researcher Field Notes, 3/23/09).

As Rico noted in his journal that day, “She [Jenny] is conflicted, it appears, because she believes both that the point the book is making is valid, but feels that it is probably too strong” (Teacher Journal, 3/23/09). Other students felt conflicted and frustrated too in trying to differentiate the nuances related to race, economics, gender, and cultural stereotypes that this chapter raised to indicate that social forces influenced individual behaviour. This juxtaposing of the two contrasting discourses (individual behaviour versus social forces) was a central part of reading the world in this unit. Since one of the sociopolitical goals for this unit was for students to recognize the ways in which social factors limit individual choices, Rico brought this piece of text into the classroom for reading, made this viewpoint (on social factors) explicit, and opened up a space to dialogue about it.

The teacher’s role in facilitating the ensuing conversations was crucial for allowing the tension between individual choices and social forces to emerge in these discussions. He used this text and dialogue to explicitly direct students’ attention to social factors by bringing the relatively abstract idea of social factors constraining individual choices in direct contrast with the everyday notion of individual behavior into the classroom and facilitated conversations around it. Doing so required him to think from the perspective of social factors discourse. As Vygotsky (1987) posited, it was this difference (between teacher and students thinking) that created the possibility for these conversations to unfold in joint activity around this piece of text (Balasubramanian, 2012).

After students finished presenting the key ideas from the sections of the myths chapter assigned to their groups, Rico asked them to think about the relation between racism, poverty, and high rates of HIV-AIDS infection. By doing so, he tried to bring to fore the mathematical-sociopolitical connection. Ellen and Gema responded.

Ellen: Isn’t that kind of, it’s like, if you live in poverty then, well, AIDS and poverty connect, because, like, when you’re poor, you don’t have as much resources and stuff like that. And you are more closed out and, like, you know white people, well I am not saying they are rich or whatever, but, like, they have more resources and more places to go, and more information to know and stuff.

Gema: Well, then it also fits in, like the stereotypes, like, many people that live in minority communities are mostly are living in poverty, and they tend to watch television. And sometimes they [TV] tell that they [TV viewers] should act as the way people, as they are being portrayed in the media. And that tends to lead into, like this, they start thinking that that's the way they should live, leading them to do stuff that they wouldn't do otherwise.

(Video, 3/24/09)

Here, Ellen and Gema began to consider the possibility of poverty influencing HIV-AIDS infection rates in communities. The conversations on the myths chapter also brought to fore a clearer and more pragmatic mathematical-sociopolitical connection and the kind of mathematics required for reading the world in this unit. Subsequently, Rico decided to focus on the mathematics of disproportionality for a few weeks. This decision was not an easy one to make since he also wanted students to learn the "rigorous" mathematics of the DDS. In hindsight, we suggest that this decision was consistent with the larger mathematical and socio-political goals of this course for the following reasons. First, although the mathematics of creating DDS for modeling HIV-AIDS transmission was more challenging and rigorous, it did not, in this situation, support a better sociopolitical understanding of HIV-AIDS infection and death rates. Second, Rico chose to foreground the disproportionality in HIV-AIDS infection and transmission rates as he wanted students to find ways to explain these data that went beyond the myth of bad and dangerous individual behavior. Finally, contrary to what one expected, the mathematical idea of disproportionality was not easy for many students due to their profound mis-education in the racialized U.S. public education system (Martin, 2006).

In this instance, shifting to the mathematics of disproportionality was not "dumbing down" the curriculum for students. We argue that, instead, it was an appropriate decision. Mathematically, it proceeded from students' level of understanding and challenged them, in sync with reform mathematics pedagogy. Sociopolitically, the mathematics of disproportionality was required for reading the world. In other words, this decision to shift mathematical content supported both reading the world and reading the mathematical word, and was indeed necessary to create coherence between the goals and the content in this joint activity. Following this decision, Rico brought in data on HIV-AIDS diagnoses in Illinois and Chicago for 2006, and students spent a significant portion of time investigating these data and working on the idea of disproportionality.

While doing so, Roxanne wondered why the infection rate for Latinos (16% of new HIV-AIDS cases in 2006 in Chicago) was less than that for whites (25% of new HIV-AIDS cases in 2006 in Chicago) if poverty is a factor influencing HIV-AIDS infection rates, since students knew that Latinos were overall poorer than whites. Ann further argued that if Latinas/os were disproportionately under-represented (16% of new HIV-AIDS cases in 2006 in Chicago when constituting 26% of the population) and African Americans disproportionally overrepresented (56% of new HIV-AIDS cases in 2006 in Chicago when constituting 36.8% of the population, (2009)) then the disproportionality could not be based on income. Instead Ann argued it was "about promiscuous behavior and not poverty." Here, Ellen, Gema, Ann, and Roxanne were interweaving the mathematical and sociopolitical dimensions as they tried to make sense of the data (mathematical) and connect them to their sociological analysis (sociopolitical).

Towards the end of this unit, Rico shared some data on HIV-AIDS infection rates from North Lawndale, and students worked to create a dynamical system for disease spread in North Lawndale. Rico asked students to consider how they could talk about the disproportionality in the upcoming presentations in their communities.

Ann:	How are we supposed to, how are we supposed to explain, when we don't know.
Rico:	Know what?
Ann:	Answering why it's disproportionate.
Rico:	Okay, so why did we do all this work with these, why did we spend a week discussing this [referring to the myths chapter]? What was the, all that, you know, do you have any sense what, how looks your explanation for that [pointing to the pictorial representation of the disproportionality, a pie chart of the infection rates and the population distribution in Chicago]?
Ann:	But an assumption is an assumption
Rico:	What do you mean an assumption is an assumption?
Ann:	Like, it's, I mean, I don't think there's a certain fact that we can say this has to be the reason for it to exist. That's basically an opinion.
Rico:	Basically what?
Ann:	Opinion.
Rico:	So what is your opinion? And does your opinion matter?
Ann:	May not, but
Carlton:	We only convince other people [inaudible] facts.
Rico:	Okay, so what facts do we have?
Ann:	Numbers, statistics, but you are, that is going to the math, that was we explaining what is happening, but you are asking to ex, tell you why.

(Audio, 4/21/09)

Rico wanted students to develop their own sociopolitical analysis for the disparity visible in the data to ensure that they did not leave class with the bad and dangerous behavior myth (addressed by the myths chapter). However, Ann was emphatic that students could not explain the disparity. Another student, Vanessa, concurred: "But I don't think you can explain it." Ann's comment "that's basically an opinion," and Carlton's view that "we only convince other people" with facts such as numbers and statistics point to the tension and complexity that students perceived while trying to interweave the mathematical data and the sociopolitical analysis.

The conversation towards the end of the class ended abruptly with the ring of the bell and Rico wound up the class saying, "Obviously there is more conversation to be had here," implying that work was in progress to make sense of this connection for students. Whether all students had some sense of the mathematical-sociopolitical connection is unclear from the available data from this class. Providing a simple explanation for why Latinos are disproportionately underrepresented while Blacks are overrepresented is neither possible nor was it the aim of this unit. Moreover, it is not possible to expect that students or the teacher can resolve this and other complex issues that emerged in this unit within a few days. Allowing space for uncertainties, ambiguities, and open questions is an integral part of a dialogic problem-posing approach (Gutstein, 2012).

DISCUSSION

Several points emerge as we trace the trajectory of this unit. First, the mathematical and sociopolitical dimensions were interwoven in multiple ways at different times/levels (Balasubramanian, 2012). One form emerged when either student or teacher brought together the mathematical and sociopolitical dimensions in a single utterance (for example, Rico's questions or student statements connecting the two dimensions). Another form of interweaving is in the teacher's pedagogical decisions to foreground, background, or interrelate the dimensions, mindful of both mathematical and sociopolitical goals for this class (e.g., shifting to the mathematics of disproportionality). Finally, there is the interweaving at the entire unit (and year) level, when multiple forms and instances of interconnecting occur to read the world and read the mathematical word. The teacher's decision to shift to the mathematics of disproportionality, the choice of the myths chapter, the discussion that ensued in class, the teacher's role in pushing students to think beyond individual factors and yet allowing them the space to work through the tensions that arose for them, and students' participation, all contributed to the interweaving of the mathematical and sociopolitical dimensions during this unit.

Second, the teacher facilitated the movement between the two dimensions through his pedagogical acts and decisions in the classroom. Students in turn participated in and contributed – by asking questions, responding to teacher and peers, bringing in their awareness of the world and mathematics, and by beginning to connect the mathematical and sociopolitical (as we saw Ann, Roxanne, Greg, Jenny, Ellen, Gema and others do). We suggest that this is consistent with Sfard's (1998) perspective of the participatory metaphor of learning (in this scenario, learning to read the world and read the mathematical word).

Third, part of the teacher's role was to decide when to shift and leave one dimension to go to the other, and return later. Rico facilitated the interweaving of the two dimensions based on what he considered the potential mathematical-sociopolitical connection in this unit, and simultaneously continued to refine and be open to changing it. Although Rico started out with a different mathematical-sociopolitical connection, it changed during the course of the unit, and he made pedagogical decisions to bring coherence between the long terms goals, content, and classroom interactions. This suggests that the mathematical-sociopolitical connection of each unit is (and must be) a central consideration in the teacher's pedagogical decisions, at various temporal levels, to shift between the two dimensions and the content/context to focus on. The teacher needs to have a sense of the (potential and possible) connection at the unit level and/or at least be open to them emerging, as happened in the HIV-AIDS unit.

Finally, the dialogic space that students and teacher co-created to engage in complex conversations and struggle to make sense of the world and the data was a crucial factor in facilitating the interweaving of the two dimensions (Balasubramanian, 2012). As one student said in survey they completed after the first semester, "I like the collectiveness between us students. I like that we had built enough trust and honesty to where we can say anything and not be afraid of harsh criticism."

In conclusion, interweaving mathematical and sociopolitical dimensions in a classroom not only requires a curriculum that supports mathematical and sociopolitical content, but it also demands a closer attention to the interactions between teachers and students.

NOTES

1. The class was listed as a college bridge class. That is, students co-enrolled at their school and University of Illinois at Chicago (UIC). However, they did not receive college mathematics credit for this course because it was listed as an elective at the College of Education.

2. The first author was a participant observer in this class for the entire academic year.

3. Discrete dynamical systems can be used to model and analyze many real-world problems such as population growth, compound interest and annuities, radioactive decay, pollution control, and medication dosages.

4. SI models are simple models that use two variables (representing the susceptible and infected populations) to simulate the transmission of a disease. Later on, students developed a model using three variables - the susceptible population, population infected with HIV population, and population with AIDS.

5. All students names used in this paper are pseudonyms.

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DESIGNING WRITTEN TASKS IN THE PEDAGOGIC RECONTEXTUALISING FIELD: PROPOSING A THEORETICAL MODEL

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Task design is an emerging theme in mathematics education. Based on the assumption that tasks have a role to play in pedagogic practices, the topic is taken as a problematic issue. This paper is an attempt to make a theoretical contribution on designing written tasks at the level of what Basil Bernstein calls the pedagogic recontextualising field. Through Bernsteinian lenses, I propose a theoretical model based on three key concepts for analysing task design in the mentioned context: frame of reference, reverse recontextualisation and task markers. Examples and Bernsteinian concepts will be brought together to build the argument.

INTRODUCTION

The focus on task design is increasingly present in the mathematics education agenda. An example of this is the ICMI Study 22 which has the purpose of producing a state-of-the-art of the topic (ICMI, 2012). The word "task" has wide use in literature. Tasks may be conceptualised as statements in curriculum materials, set up by teachers, or enacted by teachers and students (Stein, et al., 2000). I will use the term "task" in a broad sense for now. I will follow the document of the ICMI Study 22 (ICMI, 2012), in which task is taken as "anything that a teacher uses to demonstrate mathematics, to pursue interactively with students, or to ask students to do something" (p.10). In this sense, the term "task" is wider than others used in literature such as curriculum resources (Adler, 2012) or curriculum materials (Remillard, 2005). Tasks and designing tasks give rise to important contributions in teacher education, which is well documented in a recent book edited by Zaslavsky and Sullivan (2011).

A number of studies have suggested that teachers act selectively in appropriating tasks to their classrooms (Remillard, 2005; Silver & Herbst, 2008; Choppin, 2011). Besides, tasks provide different types of mathematical opportunities for student learning (Stein et al., 2000; Silver & Herbst, 2008). That is to say they have a role to play in the support of teaching and learning mathematics. One of the questions that requires clarification is: What is the role of designing tasks in mathematics education? Or, in order to be more specific, let me replace the question with this one: How are tasks designed in mathematics education?

The question demands a theoretical formulation. Different responses are possible, depending on the theoretical framework. Lerman (2010) points out that parallel understandings are possible in mathematics education research. Every theory structures (and even builds) phenomena and makes corresponding questions. Since symbolic control is not clearly addressed in the document of the ICMI Study 22

(ICMI, 2012), I am interested here in developing a theoretical model that draws on Bernstein's (1990, 1996) theory. In fact, the research question that I ask is a very ambitious question and I am aware this paper is a first step. In particular, the model will locate task design in what Bernstein (1990, 1996) calls the *pedagogic recontextualising field* (PRF), which includes curriculum materials, authors, teacher education programmes, and so on. I discuss the concept of PRF in more detail later.

Bernstein's (1990, 1996) theory has attracted the attention of mathematics education researchers (among others, Lerman & Zevenbergen, 2004; Lerman, 2010; Jablonka & Gellert, 2011; Morgan, 2012). Also, in the series of Mathematics Education and Society conferences, the Bernsteinian framework has been used as an analytic tool for critical issues (for example, Jablonka & Gellert, 2010; Kanes, Morgan, & Tsatsaroni, 2010). Briefly, the theory is concerned with how power and control are translated in principles for pedagogic communication (Bernstein, 1990, 1996). In this paper I bring together Bernsteinian concepts and some examples to develop a theoretical model for designing tasks that is based on three notions: frame of reference, reverse recontextualisation, and task markers. Next I present each notion.

FRAME OF REFERENCE

Let me start with an example. In 2012, my colleague Andreia Oliveira [1] and I took part in an in-service teacher education programme called, in Brazilian Portuguese, "Ensino Médio em Ação" (Secondary Teaching in Action) known as EM-AÇÃO (translated to English: in-action). The aim was to support teachers of the Brazilian state of Bahia to implement changes in their mathematics teaching. The programme was organised and supported by Anisio Teixeira Institute, a state teacher education centre. We as teacher educators concentrated on discussing the learning milieus such as those theorised in Alro and Skovsmose (2002). At a certain point, we asked the participating teachers to design a written task in the form of a mathematical investigation for secondary school level. Our proposal derived from the argument that designing tasks is a powerful way to support teacher learning mathematically and pedagogically (Watson & Mason, 2007; Zaslavsky & Sullivan, 2011).

Some researchers conceptualise tasks as a mediation tool that provide affordances and limitations for human actions (Watson & Mason, 2007; Sullivan, Jorgensen, & Mousley, 2011). In this sense, every task communicates something to someone, which leads me to see it as a text in a Bernsteinian perspective (Bernstein, 1990, 1996). Texts may assume different forms such as oral, written, gestural, and so on. Then teachers at the EM-AÇÃO programme were required to produce a written task, which is considered a type of text.

If we take the definition of task presented in the first paragraph of this paper, creating a task itself is a task, which in turn is taking place in a sort of context that is socially organised to regulate the circulation of texts to classrooms. As mentioned before, that is an example of what Bernstein (1990, 2000) calls *pedagogic recontextualising field*.

Its function is to delocate the texts produced in the scientific field (in this case, mathematics) and to relocate them to be moved, for instance, to classrooms (Bernstein, 1990, 1996).

The task design proposed to the teachers is part of the pedagogic practice carried out in an in-service teacher education programme. Following Bernstein (1990, 1996), pedagogic practices are characterised in terms of the relationship between transmitter and acquirer [2] in an organisational context. Every pedagogic practice operates by principles (Bernstein, 1990, 1996). In EM-AÇÃO, mathematical investigations, problem solving, and mathematical modelling were the learning milieus approached through lesson simulations, analysis of classroom episodes, and reading and discussing classroom tales and articles. The programme was designed to socialise teachers into what Alro and Skovsmose (2002) call landscapes of investigations. At the point when the task design was proposed, the teachers seemed aware of the legitimate texts expected in that pedagogic practice. The relationship between teacher educators and schoolteachers operated a control on what was legitimate (what was accepted) to communicate and how to do this. So teachers seemed aware that the programme emphasized a perspective of investigations/explorations for school mathematics. They can be said to recognise the rules about what to say. Bernstein (1990, 1996) would call these the *rules of recognition*.

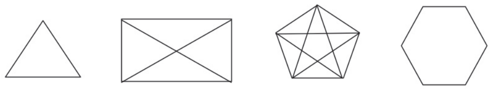
In order to create a task of mathematical investigation for secondary students, the schoolteachers in that context were expected to operate according to another kind of rules, those called *rules of realisation* by Bernstein (1990, 1996). In other words, they were expected to design a task by addressing the principles that operated the EM-AÇÃO programme, since the symbolic control is expressed in communicative actions among the agents at a given context.

A number of times, I approached a pair of experienced teachers, Emilia Souza and Ivanildo Porto [3], who were discussing how to design the task. Their questions grasped the principles that oriented the communication in the programme: Is it related to pure mathematics, or isn't it? Let's make open questions. Let's bring students to make explorations. Besides, as a teacher educator, I legitimated the texts produced by them. Sometimes, when they mentioned features not related to mathematical investigations, such as closed-ended questions, I put questions for debate about the qualities of mathematical investigations.

The discursive control is present in the interactions among the agents, which shapes the task design. In the end, the pair of teachers presented the task shown in Figure 1.

Task

Consider the polygons shown below.



1. Draw the diagonals of the fourth polygon.

2. What is the fifth polygon? And, the sixth?

3. How many diagonals does the fourth polygon have?

4. Raise a conjecture about the relationship between the number of sides and the number of diagonals.

Figure1: Written task designed by schoolteachers Marcos and Maria (translated from Brazilian Portuguese]

The written task presented in Figure 1 matches the principles of pedagogic practice of the EM-AÇÃO programme. There is consonance between the task itself and the setting where it was produced. When one searches to understand how a text is produced, then one is advised to look for its production conditions (Morgan, 2012). The main argument I derived from the example is the task mirrored the pedagogic context where it was designed.

Let us consider another pedagogic context reported in literature. For example, the reinvention principle is often addressed in the programmes and materials based on realistic mathematics education (Gravemeijer, 2004). Analogously, task design here mirrors the contexts based on such perspective. This argument might be extended for publishing houses and all contexts of the pedagogic recontextualising field.

The example used was about written tasks, but I think it is possible to suggest that any task design takes place in a *frame of reference* that places conditions on what is valid or not for the task itself. Those conditions refer to the principles that orientate what is a legitimate task or not. The frame of reference provides limitations for what is possible. Metaphorically it is like a painter who cannot go beyond the frame of the screen. The frame of reference operates a communicative limit for the way of addressing principles by task designers (be they teachers or publishers).

However I put forward for consideration that the frame of reference is not the unique driving force for task designers. In my work as a teacher educator, I have developed some insights into the conditions imposed by the tradition of school mathematics, which I discuss in detail in the next section.

REVERSE RECONTEXTUALISATION

The interaction with Emilia and Ivanildo while they were designing their task provided me with some insights. The teachers often asked questions about what Brazilian students would be able to do at the secondary level and how the task would get into the school curriculum. They also were worried about students' investigation skills, as students were not used to investigations. Below I recall a short conversation for illustration:

Ivanildo:	They are not familiar to this kind of tasks. Certainly they will ask us to make an example.
Emilia:	That is why the questions one, two and three are important. They will function as scaffolding. Perhaps we could organise the calculations in a table at blackboard so making easy to see the relationship.
Ivanildo:	I think we cannot go too far, otherwise students will not be able to approach the task.

Conversations like this made me aware that the teachers were visibly considering the pedagogic relationship that existed in their classrooms. As they were trying to fit the task to the pedagogic principles addressed in the EM-AÇÃO programme, they were, at the same time, trying to balance these principles with the principles that regulate the pedagogic practices in their schools.

The same has been reported in Lewis et al. (2011). The authors show teachers evoking the features of their classrooms while they were designing tasks in a programme based on the modality of lesson study. Ainley, Pratt and Hansen (2006) have called it the *planning paradox* in order to name the imagination about the trajectory of students' actions.

According to Bernstein (1990, 1996), there is insulation between the pedagogic recontextualising field and classrooms (which is called the *field of reproduction*, in a Bernsteinian terminology). It suggests a particular nature and role for both fields in the process of symbolic distribution. Basil Bernstein presents the notion of pedagogic recontextualisation to conceptualise the move of texts from the pedagogic recontextualising field to the field of reproduction. Jablonka and Gellert (2010) extend this Bernsteinian concept and introduce the idea of dual recontextualisation to characterize the discourse of school mathematics in terms of texts moved from both professional mathematics and everyday practices.

On the other hand, the example from EM-AÇÃO mentioned in this section suggests a movement in the reverse direction. It seems that agents are addressing principles of the practices developed at the level of the field of reproduction, while they are designing tasks at the level of the pedagogic recontextualising field. In this case, the agents seem to be operating a *reverse recontextualization*.

In order for the idea of reverse recontextualisation to be accepted, it must be connected with the notion of frame of reference. In the EM-AÇÃO example, the teachers' actions in designing tasks may be explained in terms of dealing with both frame of reference and reverse recontextualisation. Instead of viewing these as acting in combination, I shall suggest that they are in conflict, as the insulation between them is grounded in different logics. This leads me to see task designers in the pedagogic recontextualising field as agents operating according to two different sets of principles. Written tasks are a product of this, as represent in Figure 2.

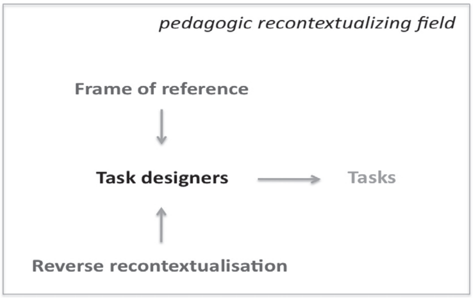


Figure 2: Diagram representing the conflict between frame of reference and reserve recontextualisation.

As a consequence, we would expect tasks to show evidence of the conflict between the frame of reference and the reverse recontextualisation. I shall name this evidence, *task markers*. In the next section I identify some possible markers in tasks.

TASK MARKERS

If markers are the necessary way to draw conclusions about the production conditions of tasks, then the next step is characterise them. In order to do that, I draw on the task designed by Emilia and Ivanildo who attended the EM-AÇÃO programme (Figure 1).

The task (Figure 1) has reference in pure mathematics as the statement talks about polygons. Alro and Skovsmose (2002) propose three contexts of reference for mathematics tasks: mathematics, semi-reality (fictional situations), and reality. Let us think about many possibilities between two extremes represented by pure mathematics-based and reality-based situations. Note that semi-reality situations are located between both extremes. Analogously, task statements are different combinations between reality-elicited and mathematics-elicited references [4]. I use the same term used by Alro and Skovsmose (2002) to name this marker, *context of reference*.

Written tasks provide some semiotic signals that communicate the level of rigour, which is related to the context of reference. In Figure 1, for instance, question 4 asks students to raise a conjecture, which is more formal rather than other possible ways of asking. So let us consider many possibilities for the *use of language*, a range of possibilities from a strong to a week rigour as it appears as strong or weak control. I take this as a marker.

Figure 1 also shows a certain structure composed of a starting statement, in this case, the representation of some polygons and their diagonals followed by some questions. The first questions require attention to a numerical mathematical relationship. The last question draws attention to a generalising relationship. Some would likely identify this kind of task structure as opened-end. However, the structure of tasks may vary largely, which leads me to consider a continuity between closed-ended and the opened-ended tasks. I call this marker *structure*.

The pair of teachers made selective decisions about what to represent in tasks and thus also about the curriculum content and skills students are expected to work on. In the example, the teachers looked interested in approaching a property of polygons. Tasks may be classified as either *high* or *low level*, as they require memorization or investigations and explorations, respectively (Stein et al., 2000). I will keep this terminology, but I shall use this simply to denote the complexity of mathematical relationships students are demanded to deal with. It suggests a sort of marker, which I will name here *distribution*. It refers to decisions about which parts of curriculum knowledge are selected, connected and approached in tasks.

Last, I shall say that tasks are not fully explicit about the quality of the relationship between teachers and students, because tasks are not taken as determinants of pedagogic practices, but as conditionings. However, the task in Figure 1 communicates expectations of a dialogical pattern of interaction, since it presents open questions. In contrast, a closed-ended task may not explicitly encourage so much dialogical interaction. Bernstein (1990, 1996) refers to *insulation* between the subjects engaged in a pedagogic practice. A strong insulation means a strong control by the transmitter, whereas a weak insulation means that the transmitter has less control of the communication. Then let us consider the *pedagogic relationship* as a marker, as tasks suggest the quality of the insulation between teachers and students.

Five task markers have been proposed so far, which may be used as tools to analyse the qualities of tasks. The term "quality" is not synonymous with measures, instead it is viewed as attributes. The diagram in Figure 3 summarises these task markers. Note that line segments are used to represent the qualities of the markers because I want to emphasize their variation. Metaphorically let us think about this as a segment of real numbers, so the quality of any marker may assume any position.

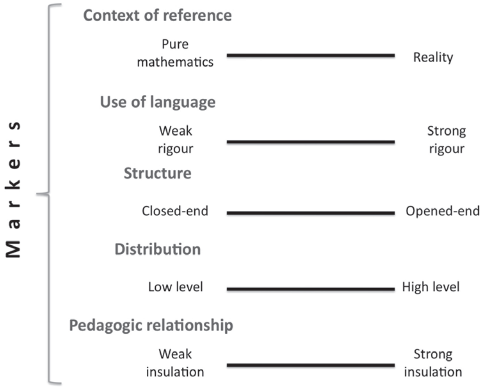


Figure 3: Task markers and their variation qualities

Not surprisingly the markers raised here are not exhaustive. Other markers can be identified and added to the diagram in the Figure 3. Furth empirical studies will be useful for this purpose.

FINAL REMARKS

Throughout this paper, the term "task" was used as a sort of text, which is a result of discursive control. In particular, I referred to designing written tasks in the pedagogic recontextualising field, a filed which includes the production of curriculum materials and teacher educations programmes.

The theoretical model proposed allows us to see the design of written tasks in terms of a conflict between what is described as frame of reference and reverse recontextualisation. Such conflict is not a problem to be eliminated, when viewed in terms of Ainley, Pratt and Hansen's (2006) notion of the planning paradox. From a Bernsteinian point of view, conflicts are part of pedagogic contexts, since pedagogic practices are insulated from others. Further work should concentrate on examining how task designers deal with these conflicts.

The diagram in Figure 4 brings together, in a schematic way, all concepts proposed in this paper, wherein the task markers are portrayed as results of these conflicts. The theoretical model has two potential uses: an analytical tool for research, and a pedagogic tool for teacher education. Future use of the model will provide opportunities for refining the theoretical model.

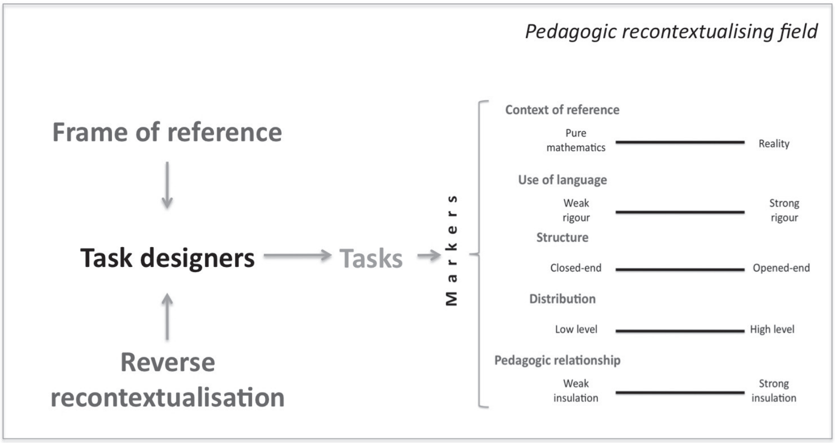


Figure 4: A theoretical model to describe task design at the PRF

NOTES

1 Andreia Oliveira authorised me to quote her real name in the paper.

2 The terms *acquirer* and *transmitter* refer to those who take part in a pedagogic relationship (Bernstein, 1990, 1996). They are not restricted to the tradition of school mathematics.

3 Emilia Souza and Ivanildo Porto also authorised me to quote their real names in the paper.

4 The term "reality" is problematic and deserves a substantial discussion, but herein I am only using Alro and Skovsmose's (2002) terminology.

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STRONG IS THE SILENCE: CHALLENGING SYSTEMS OF PRIVILEGE AND OPPRESSION IN MATHEMATICS TEACHER EDUCATION

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Mathematics teacher educators (MTEs) are often silent about the systems of privilege and oppression (e.g., racism, classism, sexism, heterosexism, ableism) within which we operate. In particular, while MTEs have begun to talk about these issues in relation to the preparation of mathematics teachers (MTs) and mathematics teaching, we rarely talk about them with respect to our own preparation and the preparation of future MTEs. As a result, our research agendas, frameworks, approaches, and strategies for taking action toward equitable systems within the programs in which we work (i.e., preparing future MTEs and future MTs) are underspecified and underconceptualized. Our hypothesis is that concentrated attention to thoughtful discussion and action related to identifying, understanding, and confronting systems of privilege and oppression can improve our work as MTEs and, ultimately, will impact MTs' and students' learning experiences in mathematics classrooms, especially students who have been historically underserved in schools.

We believe that we need to break this silence and provide venues in which to plan and take thoughtful action in relationship to systems of privilege and oppression, develop strategies for working on these systems amongst ourselves and with our graduate and undergraduate students, and enable us to invite others into such conversations. In this discussion document, we provide a rationale for the need to break this silence. We recognize, up front, that the points that we will make are grounded primarily in the context of the United States. We hope, however, that we can not only spur cross-national discussion about these issues but also learn more about our own perspectives, assumptions, and biases from engaging in a discussion with participants from other countries and contexts.

RATIONALE FOR DISCUSSION

Although we recognize that our rationale for the need to break the silence is primarily based in our context, we raise three important points:

1) Although schools in the U.S. are rapidly becoming more diverse in terms of race, class, and language – all potential sources of privilege and oppression – MTs and MTEs remain fairly homogeneous along these demographic lines (Hollins & Guzman, 2005).

2) Although the literature on preparing teachers in the U.S. to work in diverse classrooms, schools, and communities has recently been growing, there is a paucity of work on preparing MTEs to facilitate this kind of work. This

includes not only preparing graduate students to be new MTEs but also examining the work currently conducted by practicing MTEs themselves (see McLeman, Vomvoridi-Ivanovic & Chval, 2012, for initial work examining the practice of MTEs).

3) Although the growing literature on equity in mathematics education has been framed in various ways to address issues of *oppression* and (sometimes) emancipation, we think that anti-oppression activism also requires confronting the *privilege* granted by institutions and society through addressing *interlocking systems of privilege and oppression* in order for our mathematics education community to thoughtfully avoid replicating imperialism (i.e., enabling the powerful to act and speak on behalf of the oppressed).

We say more about each of these points here.

Increasingly diverse schools and relatively homogeneous teaching populations

A reality in mathematics education is that while the teaching population in public schools and in universities in the U.S. has remained fairly homogeneous in terms of race, class, and language facility (i.e., White, middle class, and English monolingual), K-12 student populations are growing more and more diverse in these ways. For example, nationally, 43% of students enrolled in public schools are students of color (Fry, 2007), whereas nearly 90% of teachers in the U.S. are White (National Center for Education Information, 2005). Scholars have argued that these differences have serious implications for teaching and learning (Ladson-Billings, 1994; Gay, 2010; Larson & Ovando, 2001).

Consider the issue of racial difference, for example. It is typical for White teachers to claim to be "color-blind" and treat all students the same (Bell, 2002). This color-blindness, however, masks the inequities created by class, race and power (Johnson, 2002). Without explicit attention to racial identity development in all MTs and MTEs, it is likely that White teachers will unintentionally negatively impact the performance of students of color and undermine multicultural practices and policies (Lawrence & Bunche, 1996). Research also suggests the following patterns of White teachers confronting race and equity issues: White elementary teachers are often ignorant about racial inequality; if confronted with inequity, feel blamed for injustices and act defensively toward presentations on issues of social inequality and White privilege; tend to approach issues of inequality from a personal perspective rather than as societal, systemic, and institutional manifestations; and want to be told what to do in a multicultural classroom, how to teach "others" rather than to explore the impact of their attitudes on multicultural teaching effectiveness (Cooney & Akintude, 1999). As Taylor and Kitchen (2008) stated, "it is well-documented that teachers hold lower expectations for students of color and those from poor families than they do for White middle class students (Ferguson, 1998; Grant, 1989; Knapp & Woolvertson, 1995; Zeichner, 1996)" (p. 112). Scholars have argued, in fact, that these lower expectations are not unique to White teachers. As Bell (2002) pointed out:

> Though teachers of color are less likely than their White counterparts to deny the existence of racism or to cling to dominant ideology about color blindness and dramatic social progress (Bell, 2003; Thompson, 1998), they may benefit from the ways that racism is internalized by members of subordinated groups, and issues of collusion and horizontal oppression among different groups of color (Hardiman & Jackson, 1997).

Given the current underachievement in mathematics of many students of color and students who live in poverty, we need to stop being silent and address these issues explicitly in the mathematics education community, particularly among MTEs.

A primary focus on mathematics teachers, not mathematics teacher educators

Recent literature that considers how this fairly homogeneous teaching population works with students who are racially, economically, and linguistically different from them highlights the increasing attention to *teachers* and *teaching in K-12 public schools*. Yet, in order to create systems of equitable work, it is imperative that these issues be explored and considered in relationship to *MTEs*.

In the Conference Board of Mathematical Sciences report on *U.S Doctorates in Mathematics Education*, the following five "needs" were identified for the preparation of PhD students in mathematics education:

1) To learn about diversity/equity in all of their coursework and to develop national leaders in this area;
2) To learn "core knowledge" and have common experiences related to diversity/equity issues across institutions within doctoral programs in mathematics education;
3) To have professional experiences in a diversity of settings;
4) To develop an appreciation of diversity/equity issues even if diversity/equity is not central in the research they undertake; and
5) To develop an appreciation of theoretical frameworks related to diversity/equity and have knowledge of the research that has been undertaken that relates to diversity/equity in mathematics education (Taylor & Kitchen, 2008, pp. 112-114).

Each of these needs is important and requires careful consideration in order to prepare MTEs to understand how to move beyond a "missionary or cannibal" approach (e.g., Martin, 2007) and to understand what thoughtful collaboration to dismantle systems of privilege and oppression may look like. In fact, even when MTEs have begun to unpack some of these ideas, there is always more work to be done. In the first meeting of the authors of this paper to discuss the issues put forth here, for example, we were able to quickly generate tensions that occur for us in our work as MTEs (some of which have also been reported in the literature):

> As a White teacher educator, I often find that White prospective teachers tend to just agree with me. How do I get them to more deeply engage with these issues? (See also Gillespie, Ashbaugh, & DeFoire, 2002.)
>
> What can I do when my students resist my talk about race because they think I have an 'agenda'? (See also Aguirre, 2009.)
>
> An issue I have run into is that MTs want to jump in to "solve the problem." What can I do to get them to sit with these issues and tackle them thoughtfully? (i.e., They want to be in charge of the solution rather than working carefully in partnership on solutions, which relates to our earlier points about avoiding a missionary approach and how potentially fast, careless, and well intentioned contributions can lead to perpetuating imperialism.)
>
> There is such a lack of comfortableness with talking about issues of privilege. I'm not sure how to tackle that sometimes. For example, in one class a prospective teacher said something about a child and her parents not caring and other prospective teachers in the class pushed back, sometimes in good ways but at other times in potentially damaging ways. How do I get those good ways to happen more often?
>
> As a White MTE, I'm unsure how to handle it when discussing these issues in settings where there are students from many different racial backgrounds. For example, what do I do when a student of color voices some of the meta-narratives that indicate that outcomes are all about hard work and do not relate to things like race?
>
> How do I unpack my own privilege and what it does in the ways we interact and engage with things like readings for the course?

As can be seen in these examples, even when university faculty have been engaging in work related to focusing on issues of privilege and oppression, we definitely do not have answers for the many dilemmas that we confront in this work. We are left with many questions, for example: How might we better structure these conversations? What knowledge of systems of privilege and oppression is reasonable for new teachers and MTEs to take with them into settings where there are multiple narratives about these systems? What are some reasonable action strategies for actually addressing these broader systems that prospective teachers and MTEs can take with them when they leave?

Understanding oppression and privilege and interlocking systems

Equity research has become a growing line of research in the past two decades in mathematics education. In particular, the early and prevalent line of this equity work focuses on the "achievement gap" and access issues. There have been debates, however, about whether this is an overly limited way to consider issues of equity. For example, Gutierrez (2007) offered a framework for equity that includes the achievement and access issues (which she calls the "dominant axis") but pushes mathematics educators to consider issues of identity and power (which she calls the "critical axis" of equity work). In education research more generally, Ladson-Billings (2006) suggests that the achievement gap be re-named the "education debt." By choosing to rename the issue, she argues, the focus can shift from being only about individual student's achievement on narrow standardized tests to also considering historical and systemic issues in the institution of schooling. As policy researchers have argued, how problems are framed shapes responses made by policy makers and mathematics educators (Choppin, Wagner, & Herbel-Eisenmann, 2011). If, for example, we also focused on the "education debt" rather than just the "achievement gap," the manner in which changes are made would need to be different. For instance, we might examine and change policies and programs that support students and partner with communities to change schooling, rather than doing things like add test preparation to our curriculum. Thus, the ways in which these issues are framed matter to the realities of students and families.

Some mathematics educators and teacher educators have recently focused on issues of identity and power, often adopting frames like teaching mathematics for social justice (see the 2009 special issues in the *Journal of Mathematics Teacher Education*) or that of critical mathematics education. In these perspectives, the goal of education relates to emancipation and dismantling systems of oppression at the interpersonal, institutional, and cultural levels. For example, Gutstein (2006) draws on the work of Paulo Freire to teach students how to read and write the world with mathematics. That is, when he has students use mathematics to analyze social, political, and economic situations that relate to issues of oppression, he teaches them to "read" the world with mathematics; when he has students generate and engage in action related to these issues of oppression, he is teaching them to "write" the world with mathematics. In this literature systems of oppression are explicitly named and unpacked. Yet, in order to dismantle systems of oppression, we believe that the interlocking system of privilege must also be interrogated. Using a mathematical analogy, we see privilege and oppression as complementary sets that must both be considered together in order to understand the system.

If we look beyond mathematics education literature, however, systems of privilege are examined. For example, there is a growing literature that uses Whiteness theory to understand how prospective teachers work in diverse schools (e.g., Cochran-Smith, 1995, 2000; McIntyre, 1997; Paley, 1979). A couple of exceptions to this work in mathematics education have also used Whiteness theory to explore aspects of their own identity in mathematics teacher education work (e.g., Gregson, 2001; Gutstein, 2003).

Understanding and acknowledging privilege is not enough. When MTEs and MTs have not critically examined their own place in the systems of privilege and oppression, they frequently bring a deficit model and exhibit behaviours that are patronizing because they view this work through a lens of charity rather than justice.

In our identification of the problem, we stated that MTEs are often silent about systems of privilege and oppression. Yet, it is imperative that we:

- Deepen awareness of how oppression, privilege and power are at work in all relationships and organizations;
- Invite people with privilege to recognize and unlearn the habits and practices that protect their privilege;
- Nurture collaborative action and authentic relationships across differences of race, age, gender, dis/abilities, class, and sexual identity;
- Equip organizations (in this case, academic programs) to recognize, and then take action to decrease the disparity between their current practices and their inclusive ideals; and
- Encourage MTEs to explore and deepen their resources for social change and to connect our resources and the resources of MTs and students.

CONCLUDING THOUGHTS

It is time to consider MTEs knowledge and practice – their preparation and their research agendas, frameworks, approaches, and strategies for action toward equity – in relation to the interlocking systems of privilege and oppression within which they (we) operate. One way to address the goals set forth in this paper may be to engage MTEs in both thoughtful reflection and action related to identifying, understanding, and confronting systems of privilege and oppression. The experience of people working together around issues of race and class can be profound and transformative and can result in deep and spreading changes in scholarship, teaching and programmatic work that creates widening effects (Apol, 2011; Apol & Herbel-Eisenmann, 2012).

The work of breaking the silence starts now. Although the points we have made are grounded primarily in the context of the United States, cross-national discussion with participants from other countries and contexts about these issues can further support this work and support us in learning more about our own perspectives, assumptions, and biases. To that end, we hope to engage in discussion around the following questions related to this paper:

- In what ways, if any, do these rationales apply more generally to contexts beyond the U.S.?
- What additional rationales are important to consider for this work?
- How does the situating of this work in the U.S. context mask the assumptions we may be making in the framing of this work?

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POWER RELATIONS IN MATHEMATICS EDUCATION: RESEARCHING ASSESSMENT DISCOURSES IN DAY-TO-DAY COMMUNICATION IN MATHEMATICS CLASSROOMS

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In mathematics classrooms as well as in research in mathematics education it is possible to identify various power relations. Here we draw attention to power relations between researcher and teacher during classroom research and also power relations in implicit and explicit assessment acts in communications between teacher and student in the mathematics classroom. We describe a basis for a planned action research project within a critical mathematics education approach. We are drawing on a model by Skovsmose and Borba, and adding a Foucaultian concept of discourse. We include tentative analytical tools as well as methodological considerations.

A basis for this paper is a recently started research project where we investigate some aspects of the situation in Swedish mathematics classrooms regarding equity (Björklund Boistrup & Norén, 2012). These aspects, such as ethnic backgrounds and socio-economic circumstances, are becoming more problematic than earlier (National Agency of Education, 2012). This problem area is not isolated to Sweden and we know from other research that teachers' expectations and demands, as well as local circumstances, segregation, poverty and social problems limit opportunities for students' achievement (Arora, 2005). The planned research project aims to connect this problem area to a specific aspect of classroom communication, namely classroom assessment (here taken in a broad sense). We know from several earlier studies that assessment taking place in classroom communication is affecting students' achievements (Black & Wiliam, 1998; Hattie; 2009), which is why we have chosen to specifically research this.

The project will consist of quantitative as well as qualitative studies. This paper is connected to one of the qualitative studies and to a research question where we ask how teachers and researchers collaboratively can develop classroom assessment practices in the mathematics classroom. This question is also relevant for another research project starting in September 2012 where one of the authors (Björklund Boistrup) is engaged in action research studies with teachers in two Swedish municipalities with a focus on assessment (taken in a broad sense) aspects in mathematics classroom communication. The latter studies constitute pilot studies for the first mentioned project.

CRITICAL CLASSROOM RESEARCH

We position this paper within a critical approach. As Skovsmose (2012) does, we find it important to explore various sites for teaching and learning mathematics, and to go beyond the "prototypic mathematics classroom" (p. 344) research. A central theme when researching within a critical approach in mathematics education is inequities between different actors in the mathematics classroom (Vithal, 2004). These inequities may concern different groups of students (National Agency of Education, 2012; Norén & Björklund Boistrup, 2013) as well as power relations between researchers, teachers and/or students (Skovsmose & Borba, 2004). Additionally, and equally significantly, is the way that the mathematics classroom is part of (and affected by) institutional and discursive aspects in a broader context. Valero (e.g. 2004) argues for a research process that takes into account the social arenas in which the classroom is immersed. In elaborating on the presence of institutions, it can be argued that communications in mathematics classrooms are situated in contexts characterised by dominant (mathematics) education discourses, the use of artefacts developed over time, framings in terms of specific resources for learning, division of time, structures within and between schools, classification of students into schools and learning groups, established routines, classroom structure and authoritative rules (Selander, 2008; Björklund Boistrup & Selander, 2009).

Regarding relations between student, teachers, and/or researchers, Skovsmose and Borba (2004) highlight how research processes in critical action research include all these actors. They present a model that illustrates what such research may address (Figure 1). The authors argue that critical classroom research is about change. That is, not only, as a researcher, to capture and describe notions in the mathematics classroom, but also to go beyond this and "bring about some input to the empirical material from a situation which has *not* taken place" (Skovsmose & Borba, 2004, p.210, italics in original).

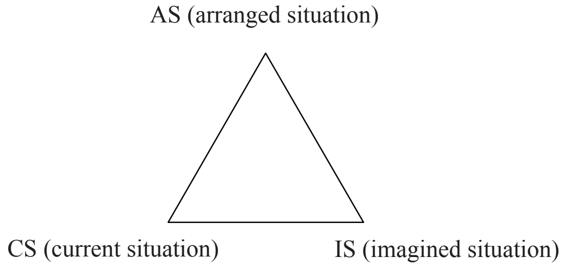

Figure 1. Skovsmose & Borba (2004), model of critical mathematics education, illustrating what research may address

In the model, CS refers to the *current situation* in the mathematics classroom before any substantial changes are introduced. IS corresponds to a vision about possible alternatives, an *imagined situation*, where the learning environment for the students might be different. The third corner of the model illustrates the *arranged situation*. This situation is different from the current situation but also from the imagined situation. One could say that the arranged situation is "a practical alternative which emerges from a negotiation involving the researchers and teachers, and possibly also students, parents, and administrators" (Skovsmose & Borba, 2004, p. 214).

We find the model by Skovsmose and Borba (2004) to be a powerful tool when researchers, teachers, and/or students conduct action research in mathematics classrooms. However, what is only partly incorporated in the model is the notion of the classroom as part of and affected by a broader institutional context. In order to include this notion more strongly we use a Foucaultian concept of discourse. Discourses are then recognised as practices structured through power relations that enact different identities and activities (Foucault, 1993). With a dynamic view on discourse, drawing on Foucault (1993), neither researchers and teachers nor students are to be seen as imprisoned in a discourse. Each actor may be part of a long-term change of the discourse and "leave" it and instead take active agency in another discourse (e.g., Norén, 2011). Discourse, according to Foucault, is often understood as encompassing entire disciplines, but can also be conceptualised as smaller discourses related to specific interests in a discipline. The latter view of discourse is adopted here (see Walkerdine, 1988; Björklund Boistrup, 2010a, 2010b; Norén, 2010).

ASSESSMENT ASPECTS IN MATHEMATICS CLASSROOM COMMUNICATION

In this paper we understand assessment in a broad sense to include, not only traditional tests and project work, but also aspects in day-to-day teacher student interactions (Morgan, 2000; Watson, 2000). One example here is where teachers aim to find out students' mathematics knowing towards providing "scaffolding" to their learning. Adopting a critical approach incorporates an acknowledgement of different, multiple positions that teachers and students (can) adopt vis-à-vis assessment in the mathematics classroom. This includes an interest in whose and what kind of knowing is represented in assessment in mathematics and also how this is connected to the broader social context (Morgan, 2000). Mellin-Olsen (1993), similarly, considers a specific power relation when he asks where the student is as a subject in the assessment of mathematics (see also Cotton, 2004). He attests that the student is often treated as an object, as 'the one who is assessed'. Another example is Foucault (2003), who writes about the role of assessment in education. He argues that, in assessment, surveillance is combined with normalisation. Through the assessment, there is both qualification and classification taking place, as well as the exercise of power and education of a specific knowing.

For a student, a teacher's assessment can be shown through feedback. One could say that without first making some kind of assessment of what a student displays, it would be very hard for the teacher to provide any feedback at all (Björklund Boistrup, 2010 a, 2010b). In earlier studies by the authors we construed discourses related to assessment in the mathematics classroom. In Norén (2010, 2011), the interest is in students with minority backgrounds in mathematics education. Norén construed discourses considered to be products of selective traditions: the public, traditional mathematics education, and language discourses in mathematics classrooms. She argues that power relations in the broader society are repeated in these discourse practices. Her findings also show that the students in the classrooms are not passive recipients but agents of their learning and empowerment. In a situation when the students are taking a National test, which the teacher administers, a discourse that normalises Swedish is enabled. In the beginning the teacher introduces the discourse “Swedish only” despite that the “normal” discourse in this classroom is bilingual and both Swedish and Arabic were used. Despite that this particular test was a group test, where communication is necessary, bilingual communication is not supported. Through actions by the students, the discourse is, after a while, changed, when the teacher explains one Swedish word in Arabic.

In Björklund Boistrup (2010a, 2010b) four assessment discourses in mathematics with a specific interest in feedback are construed. The first one, “Do it quick and do it right” has connections to a traditional mathematics classroom practice. The focus of the feedback in this discourse is on whether an answer is right or wrong, or on the number of accomplished items. The second discourse, “Anything goes”, is quite opposite to this traditional discourse, and is one where students’ performances that can be regarded as mathematically inappropriate are left unchallenged. Here teachers’ approval of students’ work is common. In the third discourse, “Openness to mathematics”, there are several instances of feedback both from teacher to student and vice versa. Often the focus is on processes towards an answer of an item. Different communicational resources (for example speech, drawings, manipulatives) are acknowledged and at times the teacher promotes or restricts the use of resources depending upon the meaning-making demonstrated by the student(s). Finally, the fourth discourse, “Reasoning takes time”, goes a step further, with a slower pace and an emphasis on mathematics processes such as reasoning/arguing, inquiring/problem-solving and defining/describing. Silences are common and the possibility (for teacher and student) to be silent seems to serve the mathematics focus.

These discourses are not stages in a taxonomy towards “better” assessment in mathematics classrooms. Instead they are analytical constructs construed from analyses and they constitute tentative tools for describing assessment practices in mathematics classrooms. For the first two discourses the lack of focus on mathematics processes produces low affordances for students’ learning of mathematics, despite the seeming openness of the second discourse. In the third and fourth discourse, there are affordances for students’ learning of mathematics with special attention given to basic skills in discourse three and attention to processes like reasoning and problem solving in discourse four. The power relations between teacher and students are significantly different in these four discourses. In the first discourse the main agent is the teacher, and the affordances for students’ active agency are not high. In the second discourse, the teacher, takes on the role as the one who evaluates students’ performances, in this case, in terms of “good”. The student is then positioned as the one who is being assessed. In discourse three and four, the teacher more often provides descriptive rather than evaluative feedback and also more often invites students to give feedback concerning the teaching. Here the power relations between teacher and student are more equal.

RESEARCHING COMMUNICATION IN MATHEMATICS CLASSROOMS

In the following sections, we describe how we coordinate (Prediger et al., 2008) the model by Skovsmose and Borba (2004, see Figure 1) with a Foucaltian concept of discourse. We also use earlier research described here as analytical starting point. We describe a plan for a critical research project in a mathematics classroom where power relations in classroom assessment in a broad sense are investigated.

Pedagogical imagination

The process of *pedagogical imagination* (Skovsmose & Borba, 2004) is, in the model in Figure 1, positioned between CS (current situation) and IS (imagined situation). Here the researchers and teachers conceptually explore educational alternatives to the current situation. In the projects described in this paper the focus of the pedagogical imagination is a changed assessment practice in the mathematics classroom where the affordances for students' active agency and learning of mathematics are qualitatively different. One source for this imagination is the findings in research described in the previous section. However, it is possible to imagine also other assessment discourses in the mathematics classroom. One example could be an assessment discourse with a focus also on a critical awareness of the role of mathematics in society and people's life. Here the notion of mathematics is not conceptualised as something inevitably good, but as something that can imbue different consequences for people depending on how it is used (Skovsmose, 2005). Another source for this process is the teachers' knowledge about the work as a mathematics teacher in school today as well as other knowledge. This knowledge is essential in a critical classroom research project. The imagination and decision making in this process are linked to co-operation between teachers and researchers. More importantly, this "co-operation includes negotiation and deliberation. Deliberation is based on the idea that nobody has access to unquestionable knowledge" (Skovsmose & Borba, 2004, p. 217).

Practical organisation

The process of *practical organisation* (Skovsmose & Borba, 2004) is positioned between CS (current situation) and AS (arranged situation). Whereas there are no limits during the process of pedagogical imagination, the research process encounters reality during the practical organisation of the project. This process has the current situation as point of departure. In co-operation between teacher and researcher and also other agents such as administrators, a 'pragmatic' solution will be the arranged situation. This situation is not the same as the imagined situation but it is the one that was possible to accomplish in negotiations. In the projects in this paper, these negotiations also address constraints and possibilities of the institution of school. This may include frames such as group sizes or number of teachers in a student group. It may also concern decisions on a municipal level concerning certain assessment materials that the teacher has to use. We find the constraints and possibilities of the institution of school to be significant enough to be the focus of a process on its own and we will come back to this after the description of the explorative reasoning.

Explorative reasoning

The process of *explorative reasoning* (Skovsmose & Borba, 2004) has its position between AS (arranged situation) and IS (imagined situation). Explorative reasoning provides a means to draw conclusions not only in relation to the arranged situation but also in relation to the imagined situation. Teachers and researchers have learnt about assessment in mathematics classrooms through analysis of the arranged situation. When also including the imagined situation in the analysis it will be possible to look through such data:

> In particular, it is relevant to make conclusions about the imagined situation based on what we have observed with respect to the arranged situation. In this way this later situation turns into a window through which we might better grasp and qualify the imagined situation (Skovsmose & Borba, 2004, p. 219).

Also this process is a process of negotiation between teachers and researchers (and possibly also students). This way of collaboratively conducting research with teachers is a way to qualify the research. The agents which the research concerns are part of the research process. This is an essential aspect of participatory research in a critical approach. In Björklund Boistrup (2010a, 2010b) the analysis and findings were discussed with the teachers but the teachers were not fully included in the research process. In our current projects we change the participants' roles fundamentally and by doing this the power relations between teachers and researchers.

Scrutinising the institutional context

We adopt a Foucaultian concept of discourse as a next step, which is a process closely related to the previous explorative reasoning. We call this process *scrutinising the institutional context.* Here teachers and researchers jointly will analyse the institutional context and how it affects classroom communication and assessment in mathematics. While the situation in the classroom is in focus in the process of explorative reasoning, the institutional context is in focus in the process described in this paragraph. One power relation where institutional rules affect classroom work is that teachers are expected to follow steering documents in the day-to-day classroom work. However, we argue that other forces affect assessment practices in mathematics classrooms as well. One force is the power executed through dominant discourses. The discourse "Do it quick and do it right" corresponds to a high degree to a traditional discourse of assessment in mathematics. In trying to critically investigate mathematics classroom work, and to go beyond "prototypic mathematics classroom" research, it is essential to bring in the power executed by dominant and normalising discourses and in collaboration between teacher and researcher go beyond these discourses and explore new possible assessment practices in mathematics classrooms.

The assessment discourses described earlier in this paper are a starting point for the process of scrutinising the institutional context and during the process we expect other discourses to be construed. The indirect impact of the institution can be conceptualised in terms of what kinds of discourses are affecting teacher-student communications in mathematics. It is possible to find differences between the current situation and the arranged situation. One finding may be that the "presence" of a traditional assessment discourse, "Do it quick and do it right", will have decreased. When comparing the arranged situation with the imagined situation, it will be possible to further investigate the institutional context. Here the direct impact of the institution will be in focus. This direct impact can be related to institutional traces such as decisions made on other "levels" than the classrooms, for example the municipality making decisions that directly affect classroom work in mathematics. Also here the previously mentioned construed discourses will provide initial analytical tools. If, as an example, there is assessment material in mathematics that all teachers have to use with their students, this material will have a direct effect on the assessment practice in the mathematics classroom. In turn, the assessment acts in mathematics that the material is affording may have a substantial effect on the possible arranged situation.

FINDINGS FROM A PILOT STUDY

During August 2012 – January 2013 a pilot study in two Swedish municipalities was performed (Björklund Boistrup & Samuelsson, work in progress, a and b). We then followed the methodology outlined in this paper. The participants were four teachers and two researchers in each of two action research projects. In both part-studies, implicit assessment acts in the mathematics classroom were investigated and here we describe one of these studies.

In one of the studies, the notion of silences in teacher-student communications during students' independent work was in focus. As described earlier, silences were typical for the assessment discourse Reasoning takes time, and here they were specifically addressed. During the process of *pedagogical imagination*, teachers and researchers, formulated together, relying on earlier research (e.g., Björklund Boistrup, 2010b), an *imagined situation* with more silences in teacher-student communications than in the *current situation*. We posed questions about how this change would be beneficial (or not) for teachers' feedback and for students' agency and learning of mathematics. During the process of *practical organisation* we engaged in the teachers' experiences so far of being more silent in communications with students. On our way to the *arranged situation* we problematized the notion of silences as single phenomena and we brought in other notions that were connected to the presence of silences. One notion was that we developed questions where silences served the purpose of giving the teacher time to formulate feedback and the student time to reflect over mathematical processes such as problem-solving and reasoning. The findings formulated during the process of *explorative reasoning* indicate that when the number of silences increases in combination with other notions, such as the questions asked by the teacher, the affordances for students' agency and learning of mathematics increase during the communications. In the end of the project, we engaged in the fourth process, *scrutinising the institutional context*. The teachers gave account of a dominant traditional discourse as something that may impede teachers from taking on a more silent and listening role in the mathematics classroom, with a change of power relations as a consequence. The teachers mentioned positive factors on a local level which facilitated a changed assessment practice in the mathematics classroom, where the action research project was mentioned as one part.

CONCLUDING REMARKS

As experienced in the pilot study, the model by Skovsmose and Borba (2004) provides a structure for a methodology where the power relations between teacher and researcher are coherent with a critical approach and, hence, both the researchers' and teachers' perspectives are part of the research process. Furthermore, bringing in a Foucaltian concept of discourse provides analytical tools for addressing the institutional context. As we see it, a student, teacher, and/or researcher always take active agency in discourses. The discourse can affect the individual in terms of who has the authority to act, what to communicate (assessment) on, and how communication is (can be) constituted. In this paper it concerns both power relation between teacher and researcher during research and power relations between teacher and student in communication in mathematics classrooms. It could be said here that power is executed through assessment and other acts. The individual, on the other hand, has the possibility to take active agency in another discourse instead, or be part of a long-term change in the discourse. The power relations between teacher and student are clearly not equal, and teachers have specific responsibilities in the assessment practice. In a dynamic view of assessment discourses there are opportunities for teachers and, to some extent, students in the mathematics classroom to take active agency in the teaching and learning through participation in potential alternative assessment discourses. This is not something straightforward since there also are power relations between classroom practices and institutions. The methodology described in this paper allows these power relations to be addressed and acted on.

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