PROCEEDINGS OF THE SEVENTH INTERNATIONAL MATHEMATICS EDUCATION AND SOCIETY CONFERENCE

Volume 2

Margot Berger, Karin Brodie, Vera Frith and Kate le Roux (Editors)

Cape Town, South Africa
2nd to 7th April 2013

MES 7

Proceedings of the Seventh International Mathematics Education and Society Conference
Edited by Margot Berger, Karin Brodie, Vera Frith and Kate le Roux
First published in April 2013
Published by MES 7
Printed by Hansa Print (Pty) Ltd, Cape Town

ISSN 2077-9933

MES 7 Conference Logo
The MES 7 logo design is based on the engraved markings found on the Blombos ochre plaque, thought by many to be the world's first symbolic object. This artefact was found by Christopher Henshilwood in 1991 in a small cave near Still Bay on the southern Cape coast of South Africa, and has been reliably dated to about 75 000 years ago. Although there is some debate about the meaning of these markings, many believe that they are evidence for symbolic thought processes as the systematic pattern suggests that the markings represent information rather than decoration.

Acknowledgements
The conference organisers would like to acknowledge the financial support of the following institutions: Rhodes University, University of Cape Town, University of KwaZulu-Natal, University of the Witwatersrand, National Research Foundation of South Africa. We also acknowledge the role of the South African Mathematics Foundation and the Centre for Higher Education Development at the University of Cape Town in underwriting the costs of the conference.

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THE INTERPLAY OF THE SOCIAL, PEDAGOGICAL AND MATHEMATICAL IN A MATHEMATICS TEXTBOOK

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This paper presents an analysis of the geometry chapters of a grade 10 South African school textbook designed to comply with the first post-apartheid curriculum. The paper looks at the effect of three strong forces (social redress, mathematics and learner-centred pedagogy) prioritised in the curriculum on the mathematics constituted by the textbook and argues that the interplay of these forces has significant effects on how each of them come to be realised in the textbook.

INTRODUCTION

The textbook is ubiquitous in mathematics classrooms in South Africa and is seen to be supporting teaching. However the textbook is a social product and as such is shaped by the interacting forces in the educational arena. In this paper I report on an aspect of a larger study in which I investigated the interplay and effect of these forces on what came to be reflected as school geometry in the curriculum documents and textbooks in grade 10 in South Africa during the immediate post-apartheid era.

THE CONTEXT

The first national democratic elections in South Africa in 1994 heralded a period of large-scale social and political change. ANC policy from this period promoted the importance of values like equality, democracy and redress within education. (Cross, Mungadi et al., 2002). Given the role that education had played in supporting apartheid there was a clear imperative to overhaul the education system.

The process of curriculum reconstruction that faced the new South African government was not an easy one. It needed to both counter the effects of apartheid, promote values of social justice, give voice to the previously marginalized and be a vehicle for promoting economic growth allowing South Africa to compete in a global market. Although outcomes based education (OBE) was projected as the underlying philosophy of the post-apartheid curriculum in South Africa, it has been argued that OBE has not been well-understood or, in fact, implemented in South Africa and been conflated with notions from the dominant discourse of education reform (Jansen, 1998). Harley and Wedekind (2004) argue that this was exacerbated by inadequate training about OBE and thus "complex issues of pedagogy with major implications for teachers' personal and professional identity were reduced to simplistic dichotomies" (p. 200). This, they argue, saw the old undesirable "teacher-centred" education being replaced with "learner-centred" education and the old prescription of content replaced with outcomes. The new curriculum thus was seen as one that prioritized learner-centred education with an active learner and teacher as facilitator, group work and the importance of process skills and application of knowledge above content for its own sake. (Jansen, 1998; Harley & Wedekind, 2004; Chisholm & Leyendecker, 2008; Engelbrecht & Harding 2008). Although much of this resonates with approaches to education internationally (Chisholm, 2005), in South Africa it represented a rupture with the past and took on a strong moral and political undertone as they were deemed part of the project of reversing the negative effects of apartheid and building a new democracy.

Thus within the process of creating the mathematics curriculum there were three powerful forces vying for attention: the need to redress the inequalities of apartheid which had the strength of a moral imperative, the presence of mathematics, a strong vertical discourse, and an education theory that put an active learner at the centre. The version of the mathematics curriculum called the National Curriculum Statement for Mathematics (NCSM) that was produced and implemented in South Africa during that time is currently being superseded by a newer version of the curriculum that eschews OBE. However the particular time of rapid and radical change with a strong commitment to social equity and nation-building in which the NCSM was created makes it a particularly interesting curriculum to investigate.

RECONTEXTUALISATION

Bernstein uses the notion of the pedagogic device to describe "the relay or ensemble of rules or procedures via which knowledge (intellectual, practical, expressive, official or local knowledge) is converted into pedagogic communication" (Singh, 2002, p. 573). Of particular importance to my study is the aspect of the pedagogic device that Bernstein terms recontextualisation. According to Maton and Muller (2007) the field of recontextualisation is where selections are made from knowledge produced in the field of production (typically universities) and transformed into educational knowledge. They describe curriculum policy and textbooks as typical sites in the field of recontextualisation. In most studies recontextualisation is discussed in relation to the academic discipline the school subject relates to. For example, Morgan, Tsatsaroni and Lerman (2002, p. 448) state that school mathematics is "a pedagogic discourse formed through the recontextualising of the specialised discourse of mathematics". Bernstein extends this to say "the recontextualising principle not only recontextualises the what of pedagogic discourse, what discourse is to become subject and content of pedagogic practice. It also recontextualises the how; that is the theory of instruction." (Bernstein, 1996, p. 49). Ferreira, Neves and Morais (Ferreira, Morais et al., 2008) and Neves and Morais (Neves & Morais, 2001) drawing on Bernstein's work describe the general regulative discourse as representing the dominant principles of society and argue that within the field of recontextualisation this discourse is transformed to produce pedagogic discourse. Thus Bernstein, and those drawing on his work, suggest that in the process of creation of school mathematics we will see the recontextualisation of mathematics, as well as of the theory of instruction and the general regulative discourse. These three strands relate closely to the three forces (social imperative for redress, the discipline of mathematics, and a learner-centred, constructivist theory of education) at play in the curriculum reconstruction process in South Africa discussed earlier.

In this paper I exemplify the interplay between these forces and show how their recontextualisation co-constituted the geometry chapters of the best-selling grade 10 mathematics textbook in South Africa, Classroom Mathematics (Laridon, Barnes et al., 2004).

METHODOLOGY

In order to analyse the geometry chapters in the textbook I drew on the work of Valverde, Bianchi et al. (2002) in their analysis of textbooks as part of the TIMSS study. Like them I divided the geometry chapters up into blocks, where blocks are either narrative elements (which tell stories, state facts and principles through narration) or task-based elements (which prescribe tasks for the learners to engage either as classroom activities or as exercise sets). Within the category of narrative blocks I make a distinction between instructional narratives which are narrative blocks central to the core instructional trajectory of the chapter and extra-information narratives that provide information which is not part of the core development of the chapter. These extra-information narratives are often little historical notes or a general discussion of applications of the mathematics provided in blocks with a heading "Did you know?"

In this paper I report on two aspects of the analysis that was done:

In the first case I discuss how the imperative for social redress and critical citizenship has been portrayed in the literature and within the NCSM and use these to develop indicators for the analysis of the textbook. The block-by-block analysis of the geometry chapters of the textbook in relation to these indicators is presented and discussed.

In the second case I show how the focus on learner-centred teaching on learners being involved in the practice of mathematics resulted in a large proportion of textbook devoted to learner work. I examine in particular those blocks suggested as class-based activities (rather than practice exercises) and look at some of the tensions that emerge in this style of textbook writing.

RECONTEXTUALISING THE SOCIAL DISCOURSE AND MATHEMATICS

With the first democratic elections in South Africa, the strong discourse centred around redressing the imbalances of apartheid and building a democratic culture that values human rights (Chisholm, 2005). This discourse within the NCSM also has roots in the thinking of people and organisations who worked in opposition to apartheid mathematics education. In particular organisations like the National Education Coordinating Committee Mathematics Commission promoted the notion of People's Mathematics. People's Mathematics was a response to the context in South Africa at the time of apartheid, but had much in common with the varieties of Critical Mathematics Education (Julie, 2004). Skovmose (1985) defines critical education as having three key elements: an assumption of critical competence on the part of teacher and student, a critical distance to content, and a critical engagement with social problems. Aspects of mathematics reform that emerge as having their roots in critical education include a concern with equity (Gustein, Middleton et al., 2005) and ensuring previously disempowered students have access to mathematics (Gutstein, 2003) and ensuring all students are able to find themselves and their cultural heritage in mathematics (Herzig, 2005). These notions link to the notion of critical competence put forward by Skovmose. Skovmose's "critical distance to content" is reflected in a questioning of the nature of mathematics as irrefutable, ready-made truth. The work of people like Frankenstein (1993), Gutstein (2003) and Brantlinger (2011) in developing mathematics courses based around key problems in the social and political issues that influence the students' lives or reflect inequalities in terms of race, gender and class in society reflect the concern with a "critical engagement with social problems".

A reading of this literature together with an analysis of the way it played out in the NCSM indicated the following three core strands of critical mathematics education present in the NCSM

1. Mathematics is important and thus all learners must have access to mathematics. Although this includes systemic considerations it also includes countering the Eurocentric bias in the history of mathematics, using contexts that are familiar to learners, avoiding negative stereotyping and ensuring all learners can find themselves and their cultures in the text.
2. Mathematics can be used to question and challenge social and economic injustices
3. Mathematics is a human creation and hence mathematics and the way it is used is open to question and critique.

The analysis of the 134 blocks relating to geometry in the textbook revealed the following picture:

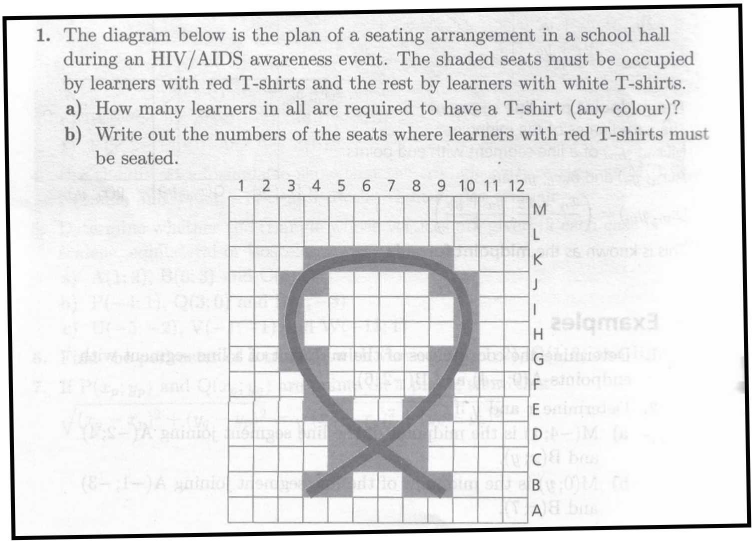
	Aspect 1: Inclusivity	Aspect 2: challenging injustice	Aspect 3: Maths is a human creation
Number of blocks	5	1	1

Table 1: The number of block in the geometry chapters related to each aspect

Of these seven instances, five made reference to or made use of South African cultural product and cultural practices. These blocks were thus included as showing aspect 1 as they could be seen as attempting to counter a Eurocentric bias and allow learners to find themselves and their culture in the textbook. However only one of these five instances required that learners actually do some mathematical work with the cultural product. In the rest the cultural products are merely used as illustrations that accompany the work. The cultural products are thus not integral to the mathematical development in the textbook.

Similarly the only discussion of mathematics as a human creation that is open to question occurs in an extra-information narrative and thus is also not central to the mathematical work of the chapter.

Only one block showed indicators relating to aspect 2. In this block the use of the HIV/AIDS context suggested as an attempt to reflect the curriculum concern with using relevant social contexts.



1. The diagram below is the plan of a seating arrangement in a school hall during an HIV/AIDS awareness event. The shaded seats must be occupied by learners with red T-shirts and the rest by learners with white T-shirts.
   a) How many learners in all are required to have a T-shirt (any colour)?
   b) Write out the numbers of the seats where learners with red T-shirts must be seated.

Figure 1: Textbook extract related to HIV/AIDS (Laridon et al., 2005, p. 84)

This question appeared in an exercise in a chapter on linear functions and analytical geometry and in a section entitled “The midpoint of a line segment”. The grid system used is different to the Cartesian plane that is used for the analytic geometry in the rest of the chapter. The question does not refer to line segments or midpoints. The question requires learners to count the number of squares in the grid and name the shaded grid squares. The mathematical demands of the question are thus low and not central to the mathematical development in the chapter. In addition although the HIV/AIDS context is used it could be omitted entirely without changing the demands of the question. The model (the grid with shaded squares) has been provided for the learners.

In this analysis we see that despite a strong imperative towards social redress this has only been incorporated into a small number of blocks in the geometry chapters and, for the most part, is not integrated into the main body of mathematical work. In the block where it is brought into the main section of the work, there is both a trivialization of the mathematical content and no substantive engagement with the social issue.

RECONTEXTUALISING PEDAGOGY AND MATHEMATICS

One of the core differences in the version of the textbook written for the NCSM and previous editions of the textbook was the difference between the kind of work that learners were asked to engage in. In comparing the edition written for the NCSM to a previous edition, the most obvious difference was the task-based blocks in the old edition consisted almost entirely of exercise sets whereas the new edition included classroom-based activities and exercises in which new ideas were explored. To get a sense of the different kind of work that learners were asked to engage in in the old and new editions of the textbook I enumerated the imperatives in the task-based blocks in the chapter on quadrilaterals that existed in both the old and new editions of the book. Table 2 below shows the five most frequently occurring imperatives in the task-based blocks in the old version of the textbook and shows what proportion of the imperatives in the task-based blocks in the new version of the textbook they constituted. Table 3 below shows the six most frequently occurring imperatives in the task-based blocks in the new version of the textbook and shows what proportion of the imperatives in the task-based blocks in the old version of the textbook they constituted.

	prove	calculate	determine	construct	name
new edition	17%	3%	0%	0%	1%
old edition	52%	17%	9%	6%	3%

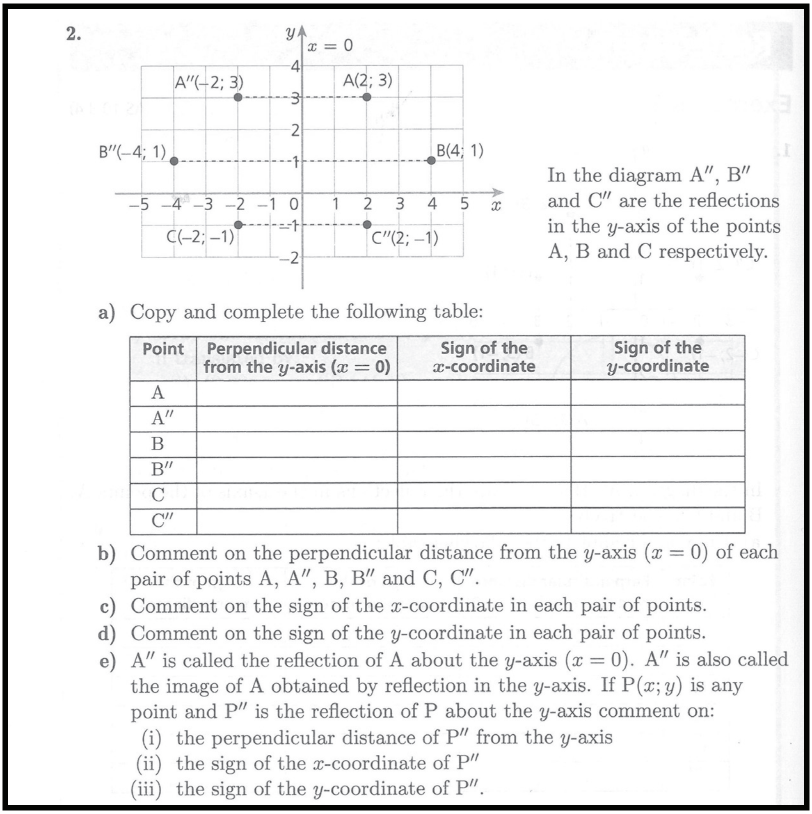
Table 2: The prevalence (as a percentage of all imperatives in each edition) of the five most frequently occurring imperatives in the task-based blocks of the old textbook

	prove	write	draw	work in groups or pairs	use	discuss
new edition	17%	15%	8%	6%	7%	5%
old edition	52%	0%	1%	0%	0%	1%

Table 3: The prevalence (as a percentage of all imperatives in each edition) of the six most frequently occurring imperatives in the task-based blocks of the new textbook

Clearly in the old version of the textbook, learners were predominantly engaged in providing proofs and doing calculations. This forms a much smaller proportion of learners' work in the textbook written for the NCSM. In the new edition we see a large number of words like 'draw' suggesting a far greater focus on discovery by the learners. The presence of words like 'discuss' and 'work in groups or pairs' suggests a focus on learners' thinking. These types of ideas align, at least on a surface level, with a more learner-centred approach coupled with discovery learning, which were influential theories of learning at the time of creation of the NCSM. However a closer look at the activities themselves and the manner in which they fit into the mathematical development of the chapters indicates a significant tension between the mathematical development and pedagogical intentions and thus a recontextualisation of both. I illustrate this by looking at the example of how the notion of reflection in transformation geometry was introduced and developed.

The chapter on transformation geometry starts with definitions of the transformations (translations and reflections) that will be worked with in the chapter. In this opening instructional narrative reflections are introduced as being defined by the line of reflection and a special note is made of the fact that this line bisects the distance between a point and its reflection at right angles. Thereafter there are a number of blocks in which learners work with translations. When learners return to working with reflections later in the chapter they begin with the following activity:



2.

In the diagram A″, B″ and C″ are the reflections in the y-axis of the points A, B and C respectively.

a) Copy and complete the following table:

Point	Perpendicular distance from the y-axis ($x = 0$)	Sign of the x-coordinate	Sign of the y-coordinate
A			
A″			
B			
B″			
C			
C″			

b) Comment on the perpendicular distance from the y-axis ($x = 0$) of each pair of points A, A″, B, B″ and C, C″.

c) Comment on the sign of the x-coordinate in each pair of points.

d) Comment on the sign of the y-coordinate in each pair of points.

e) A″ is called the reflection of A about the y-axis ($x = 0$). A″ is also called the image of A obtained by reflection in the y-axis. If $P(x; y)$ is any point and P″ is the reflection of P about the y-axis comment on:

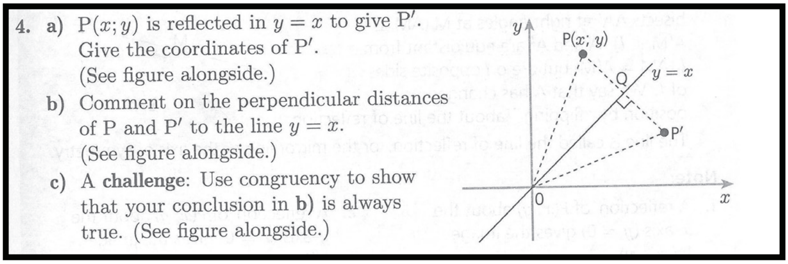
(i) the perpendicular distance of P″ from the y-axis

(ii) the sign of the x-coordinate of P″

(iii) the sign of the y-coordinate of P″.

Figure 2: Textbook extract from activity (Laridon et al., 2005, p. 199)

In this exploration learners are "discovering" that if the *y*-axis is the line of reflection then it is the perpendicular bisector of a point and its image. This fact however was part of the original definition of the reflection and thus produces a circularity in the mathematical development. This circularity is further perpetuated in the exercise that follows in that learners are asked to prove that PQ = P'Q which is a fact that is part of the definition of a reflection.



4. a) $P(x; y)$ is reflected in $y = x$ to give P'. Give the coordinates of P'. (See figure alongside.)

b) Comment on the perpendicular distances of P and P' to the line $y = x$. (See figure alongside.)

c) A **challenge**: Use congruency to show that your conclusion in **b)** is always true. (See figure alongside.)

Figure 3: Textbook extract proof task (Laridon et al., 2005, p. 201)

We see here two distortions that occur. The first is a distortion in the mathematical coherence. Learners are being asked to discover, and then prove, something that has been given as part of the definition. This arises from attempting to involve the learner as an active constructor of mathematics. However doing this requires careful consideration of the mathematical difficulties it can produce. An informal understanding of reflection (e.g., it is where the image of the dot will end up if you fold the paper in half along the reflection line) can clearly precede a formal definition of reflection. Exploring the informal idea leads to observation that one needs to define the line of reflection as the perpendicular bisector. This touches at the heart of a paradoxical situation in mathematics: although the process of defining often starts with an awareness of the existence of an object which is then defined, the object only actually exists mathematically once it is defined. This difficulty, and the dilemmas it causes for teaching and learning, has been highlighted by researchers (Kuzniak & Rauscher, 2005; Nachlieli & Tabach, 2012; Fischbein 1993).

The second distortion is in the way learner-centred teaching and discovery learning comes to be realized in the textbook. Although, as we see in the above example, activities require learners to discover certain properties, the exploration is tightly prescribed. Learners are pointed clearly to what needs to be discovered. This is necessitated by the fact that the activity fits into an unfolding mathematical story in the textbook. The learners cannot be left to truly investigate and explore on their own as the facts discovered in the activity are required for use in the next exercise.

CONCLUSION

The purpose of the analysis was to look at the effect of three strong forces (social redress, mathematics and learner-centred pedagogy) prioritised in the NCSM on the mathematics constituted by the textbook. What the analysis has shown is that each of these forces is not only powerful, but also complex to realise in anything other than a watered-down form. From the analysis we see how in the process of recontextualisation of each of these forces into the textbook, the interplay between them has a distorting effect. The need to cover mathematical content meant that issues of redress were largely side-lined. Where social issues were brought into the mathematical work both they and the mathematical work were trivialised. The desire to promote active learning and discovery by learners had the effect of weakening the mathematical coherence. The need for the explorations by the learners to lead to conclusions that fit a predetermined mathematical story leads to highly prescriptive "explorations." The textbook analysis demonstrates some of the difficulty experienced in trying to reflect a curriculum shaped by three powerful imperatives: the need to redress the inequalities of apartheid, the need to present coherent mathematics, and a desire to promote learner-centred teaching and discovery learning.

ACKNOWLEDGEMENTS

This paper forms part of the QUANTUM research project on Mathematics for Teaching, directed by Jill Adler, at the University of the Witwatersrand. This material is based upon work supported by the National Research Foundation under Grant number FA2006031800003. Any opinion, findings and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Research Foundation.

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MATHEMATICS TEACHER IDENTITY IN A PROFESSIONAL LEARNING COMMUNITY

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Teacher identity has always been regarded as significant in research about teacher professional development, but has received little attention in research about professional learning communities. In this paper I discuss the relationship between teacher identity and professional learning communities through a study of two teachers. I show that participation in the professional learning community influenced shifts in both teachers' identities, and in turn the teachers' evolving identities explained their participation in the professional learning community. The results highlight the importance of paying attention to teacher identity in analyzing teacher learning and participation in a professional learning community.

INTRODUCTION

Teacher identity has long been seen as a key construct in research about teacher professional development and practice. Studies that have researched teacher identity have focused mainly on pre-service teachers. The focus has been on developing knowledge about how pre-service teachers' identities evolve within professional development programmes (e.g. Drake, Spillane, & Hufferd-Ackles, 2001; Samuel & Stephens, 2000; Smith, 2006). The significance of teacher identity is based on the influence of identity on "teachers' sense of purpose, self-efficacy, motivation, commitment, job satisfaction and effectiveness" (Day, Kington, Stobart, & Sammons, 2006, p. 601). Thus teacher identity is believed to have an influence on the decisions that teachers make in relation to their professional practice and the affective attitudes which they develop about teaching as a profession.

Although professional learning communities have received a lot of attention in research about teacher professional development, little attention has been paid to the role of teacher identity in teacher learning and participation in such communities. Research has highlighted the importance of features that support the success of professional learning communities. Such features include: having a challenging focus; the creation of productive relationships through trust; collaboration for joint benefit; and engagement in rigorous enquiry (Katz, Earl, & Jaafar, 2009). In addition to these features professional learning communities emphasize group coherence, collaboration and collective learning for their implementation (Stoll & Louis, 2007). Group coherence and collaboration suggest the need for a group identity if professional learning communities are to succeed in supporting collective learning. A group identity means having a strong sense of belonging to the group and a collective responsibility for the learning efforts of the group. These ideas place identity at the core of developing successful professional learning communities.

CONCEPTUAL FRAMEWORK

The central concepts in this paper are teacher identity and professional learning community. A professional learning community is "a group of teachers sharing and critically interrogating their practice in an on-going, reflective, collaborative, inclusive, learning-oriented, growth-promoting way" (Stoll & Louis, 2007, p. 2). The goals of the collaboration include: challenging practice, focused professional learning, deepening understanding, proposing and testing solutions, and improving the quality of classroom instruction (Katz et al., 2009). In the study discussed in this paper, five mathematics teachers and the researcher, constituted as a professional learning community, collaborated in analyzing learners' errors in order to determine the learners' learning needs, which in turn informed the teachers' learning needs (Brodie, 2011).

Wenger's (1998) view of identity as modes of belonging was used to guide the analysis of the teachers' identities in the study. Wenger (1998) describes three modes of belonging which he argues are worth considering in order to make sense of identity formation in communities of practice. The three modes of belonging are: engagement, imagination and alignment.

Engagement is the "active process of involvement in mutual processes of negotiation of meaning" (Wenger, 1998, p. 173). Wenger (1998) further argues that engagement entails the formation of a community of practice and is a source of identity formation. In this study engagement refers to participation with others in the activities of the professional learning community. Through such active participation the teachers developed identities of belonging in the professional learning community.

Imagination "refers to the process of expanding our self by transcending our time and space and creating new images of the world and ourselves" (Wenger, 1998, p. 176). Wenger likens imagination to "looking at an apple seed and seeing a tree" (p. 176). Imagination is thus a creative process through which we create images of ourselves and others beyond the immediate time and context; hence it is a source of identity formation. In the study, analyzing the teachers' identities meant paying attention to their imaginations of themselves and others beyond the immediate activities of the professional learning community.

Alignment refers to "coordinating our energy and activities in order to fit within broader structures and contribute to broader enterprises" (Wenger, 1998, p. 174). Through alignment broader enterprises are formed and participants are connected by the "coordination of their energies, actions and practices" (Wenger, 1998, p. 179). In the study alignment refers to how the teachers identified with the achievements of the professional learning community and how they saw these achievements as forming part of their professional practice.

Why teacher identity in a professional learning community?

A fundamental aspect of Wenger's view of identity is the contiguity of learning and identity. Learning occurs within the practices of communities in social and cultural contexts (Wenger, 1998) and is regarded as becoming a better participant in practice (Brodie, 2005). Thus learning is viewed as a process of becoming a member of a community of practice, and is a way of "being in the social world, not a way of coming to know about it" (Hanks, 1991, p. 24).

Successful professional learning communities emphasize collective teacher learning (Katz & Earl, 2010). Such learning depends on collective inquiry that challenges taken-for-granted assumptions about the teachers' professional practice (Katz, et al., 2009). Participation is fundamental for teacher learning in professional learning communities. Through participation teachers construct identities of community membership, which may or may not support their participation in the community. Thus teacher identity influences the agency of individuals to participate or not to participate in the practices of the professional learning community, which implies that identity influences teacher participation, and hence teacher learning, in a professional learning community.

Participation and learning in a professional learning community entails shifts in knowledge, teaching practices and identity. Understanding how shifts in identity occur requires attention to the teachers' evolving identities as members of a community. In the study described in this paper mathematics teacher identity was considered an important social construct in order to understand how the teachers' evolving sense of membership in the professional learning community influenced their participation in the community and alignment with the practices of the community.

THE STUDY

The study was conducted in one township high school in Johannesburg over a period of twelve months. The professional learning community engaged in a number of professional development activities guided by the concept of data-informed practice. The main goals of the research were to understand how teacher learning through collective inquiry supported shifts in the teachers' knowledge, practices and identities. The professional learning community analyzed the errors made by learners on a test set by the teachers. The process involved analyzing the validity and reasoning in the errors; identifying learners' learning needs in the form of critical mathematics concepts; deepening the teachers' professional knowledge about the identified critical concepts; and using that knowledge to collectively plan innovative lessons, teach the lessons and reflect on the lessons as a community.

In this paper I explore shifts in two teachers' identities through their participation in the professional learning community. The paper focuses specifically on the teachers' identities of belonging or membership in the professional learning community. The data consisted of interviews, which were conducted before and after the professional learning activities. The data also include two focus-group interviews, which were conducted during the activities of the professional learning community. The purpose of the interviews was partly to understand the teachers' developing identities through participation in the activities of the professional learning community.

The two teachers whose results are presented in this paper were Janeth [1] and Tandeka. They were selected for their different identities before and during the professional learning activities, which influenced their different participation in community with others. The two serve as different examples of the significance of teacher identity in a professional learning community.

Janeth held a Diploma in Education [2] with specialization in teaching at secondary school level, and an Advanced Certificate in Education (ACE) [3]. Her major teaching subjects were Mathematics and Physical Science. She had eleven years teaching experience in a number of secondary schools. She had been in the current school for three years. During the year of this study she was teaching Grade 11 Mathematical Literacy. Tandeka held a Senior Primary Teaching Diploma [4]. Her major teaching subjects were Mathematics and Physical Science. She had five years teaching experience, three of them in the current school. During the year of the study she was teaching Mathematics and Natural Science to Grade 7 classes.

The teachers' identities before the professional learning community activities

Before the project began Janeth described her engagement with other mathematics teachers as a way of sharing knowledge through consulting a colleague teaching the same subject or grade level with her.

> I am teaching maths lit with someone now we are doing teamwork, if we have a problem with…go there, mister J [5]… I have a problem with this please help me, can you please help me, I like to work with these educators

She however viewed some ways of engagement as exploitative rather than collaborative. She described her collaboration with Jeffrey the previous year as being more to Jeffrey's advantage than hers.

> I don't know but most like mister S [6]…, I was teaching with mister S… last year grade eight and nine, most of the time I was the one who was doing all the assessment, if he is struggling with this, mam can you please help me

Janeth's remarks about her engagement with other mathematics teachers before the professional learning activities reflect a sense of community, which was limited to those teaching the same grade level or subject with her. She imagined such engagement as a way of learning from each other. She described some of the engagement as exploitative, something that might constrain collaboration with the teachers she viewed as constituting her community.

Before the project began Tandeka described her engagement with the other teachers as based on consultation with individual colleagues when she had challenges. She saw herself as learning from the other teachers whom she regarded as more knowledgeable than her.

> Ja, for, for our department especially, if I find that no, in this chapter I do but not so much, I manage to take Mister M [7]…, 'Excuse me please help me, how can you', and then we go to my class to explain to the learners. You find that they understand him more than what I do … Yes, or else I go to Mister M…, 'Mister M… please teach me', and then we sit down. 'Here we do this and this, this, this' and then I find a way … If I find somewhere there is a trick I don't have a way to go forward, I, I ask

She remarked that none of the mathematics teachers had ever consulted her for help. From her description of her engagement with other mathematics teachers, Tandeka saw herself as a learner who consulted the others for purposes of being taught, and was in turn not consulted by the others. Her sense of community was limited to individual colleagues whom she consulted for assistance, and in that community she located herself as a learner.

A common feature in the two teachers' sense of community before their participation in the professional learning community was the limited engagement with the mathematics teachers in the school. None of them made reference to engagement with all the mathematics teachers in the school as their community. They described their engagement as limited to certain individuals whom they consulted when the need arose. A distinguishing feature in their ways of engagement with others was how they imagined themselves in the contexts of engagement. Janeth saw the engagement as both collaborative and exploitative. She imagined herself as learning through consultation. She also imagined herself as being exploited in some of the ways in which one teacher engaged with her. Tandeka's description of her engagement with others alludes to an imagination of herself as learning from others, rather than collaborating with them. Such imagination may be based on a self-perception of being less knowledgeable than the other teachers; thus locating herself as a learner in her engagement with other mathematics teachers. Tandeka was the only mathematics teacher whose professional qualifications were for teaching in a primary school, and she had the least teaching experience among all the mathematics teachers.

The teachers' identities after their participation in the professional learning community

After their participation in the professional learning community both teachers' descriptions of their engagement with others showed some shifts in their identities. Janeth made comments that alluded to her imagination of herself as a member of the professional learning community, and the need to discuss with the community when teaching difficult mathematics topics. She saw herself as learning in the community. Her reference to the professional learning community as 'my group' alludes to an identity of belonging in community with other mathematics teachers.

> Ja, its clearer, for us to do that group discussion, we the teachers ne, aah I am telling you, aayi M…, we are going somewhere. Because sometimes you will be having, eh maybe okay I am going to prepare the Probability, ne. Firstly at least I must sit with my group and then do the discussion, maybe I am going to give them the learners wrong information

Janeth had a vision of the community as continuing in the following year and ultimately leading to better learner results.

> At least even next year I see that we are going somewhere from this year up until, I am telling you we are going to get the results. We must not stop doing that, even when you are gone, uhm, we must continue because other learners are coming from primary so they need our assistance, ja and we must continue to provide it

She however still perceived the community members differently and regarded herself as more disposed to work more closely with fellow mathematics teachers teaching Further Education and Training (FET) with her, including one teacher who did not participate in the professional learning community.

> I like to work with other teachers. Like myself and Mister M… we are teaching maths literacy, we are working together, teamwork is there M… but the problem is, like Mister, like T [8]… and M… they won't go to FET, they didn't train for secondary, ne, ja, so that's the problem

Janeth's comments reflect some shift in her sense of belonging to a community. She identified herself as a member of the professional learning community based on collaborative discussions that supported learning and the possibility of achieving better results. Such an identity contributed to her positive vision of the community as continuing in future. Thus her identity supported her imagination of the sustainability of the professional learning community. An interesting feature of Janeth's evolving identity of belonging was her reference to working closely with a teacher who was not a member of the professional learning community, and her perception of some of the teachers in the professional learning community as not qualified enough to teach FET. Imagining herself as more knowledgeable than some of the teachers in the professional learning community militates against engagement that supports sharing knowledge and learning from each other which are key to effective teacher learning in a community. Thus Janeth's developing identity both supported and constrained her engagement with others, which explained the way she participated in the professional learning community.

In the professional learning community, Janeth did not participate a lot. At times she seemed to lose track of the conversations and appeared to be disengaged. In most cases she needed to be asked for her opinion or contribution before she opened up. When she did contribute, her contributions were well thought-out. Janeth's perceptions of having another community to support her could have been a source of her reluctance to actively participate in the professional learning community, leading to disengagement in the professional learning community.

After her participation in the professional learning community Tandeka indicated some shifts in her identity. She still saw herself as a learner but felt it was better to consult the professional learning community rather than individual teachers. She saw sharing ideas in the professional learning community as generating more information.

> Sometimes you feel that the information you get from that teacher, that colleague is not enough rather than when you are in a team. Someone can give one another and you find that you have more ideas rather than asking one person … Ja because you may find that there is a problem, you go to another teacher you find that no even from that teacher its difficult, but when we are here sometimes we get *inaudible* from you [9], so it helps

Tandeka expressed her wish for the professional learning community to continue, under the facilitation of someone else.

> Uhm to me I can say if we can continue we learn a lot … Ja, if we can do this we can have more information … If, even if you can leave and then someone manages to do this each week… more of this

Evident in Tandeka's comments is a shift in her ways of engagement with other mathematics teachers. She viewed engaging with others in the professional learning community as more productive than engaging with individual colleagues. She saw herself as a member of the community, which would learn a lot if they continued the collaboration. She saw the continuation of the community as beneficial in terms of collective learning. This shift was likely to support her sustained participation in the professional learning community as a way of collective learning. Tandeka however maintained an identity of learning from the others as opposed to learning with others. Such an identity could explain the way she participated in the community. In the professional learning community Tandeka was reserved and said very little in the community, although she was attentive and sometimes made useful remarks which helped push the conversations further. She listened more than she contributed. When asked directly about her opinion or thinking about the issue under discussion she made useful contributions, indicating engagement with the conversations of the professional learning community. She was committed to the community and did all the tasks in preparation for sessions.

CONCLUSION

The results above highlight the significance of teacher identity in a professional learning community. Using Wenger's (1998) idea of engagement as a source of identity, the results show that prior to the professional learning community both teachers' ways of engagement with others did not support the development of identities of membership of the entire community of mathematics teachers in the school. For both teachers their ways of engagement with other mathematics teachers were sources of identities that supported a limited view of teacher community. Both teachers engaged with other teachers for purposes of seeking assistance, which indicates an awareness of the significance of learning from others. However, Janeth's imagination of being exploited might have been a source of her disengagement from participation in community with others. Tandeka's imagination of herself as learning from others would support teacher learning in community. Imagining herself as a learner, or as less knowledgeable, limited her participation and sharing of information with others in the professional learning community.

After their participation in the professional learning community, both teachers had developed identities of belonging to the professional learning community. Both made comments that showed that they identified themselves as members of the professional learning community whose focus was collective learning. They both imagined the community as sustainable based on learning together in community. For both teachers their identities before and during the professional learning activities influenced their participation in the community. Janeth's imagination of being more qualified than some other teachers and her imagination of belonging to another community of FET teachers could explain her somewhat disengaged participation in the professional learning community. For Tandeka the persistence of her learner identity in a way constrained her contribution in the professional learning community although she was engaged all the time.

The results highlight how a focus on teacher identity as dynamic and shifting in response to participation in a professional learning community helped to explain how the teachers engaged with others before and during the activities of the professional learning community. This illustrates the relationship between identity and participation in a community of practice (Wenger, 1998). The results allude to the need to pay attention to teacher identity in the initiation and implementation of teacher professional learning communities. Such knowledge can help the understanding of how teacher identity as a source of group cohesion and group identity supports teachers' participation and learning in community with others.

NOTES

1. All the teachers' names are pseudonyms.
2. This was a 3-year teaching qualification awarded to non-graduate trainee teachers by the former Colleges of Education in South Africa.
3. The ACE is offered by universities to in-service teachers who want to upgrade their qualifications and extend their knowledge and teaching in their area of specialization.
4. This was a three-year teaching qualification awarded by some former Colleges of Education in South Africa to non-graduate trainee teachers specializing in teaching at Senior Primary School Level.
5. Jeffrey was one of the teachers in the professional learning community.
6. S… was Jeffrey's surname.
7. M… referred to Mandla, one of the teachers in the professional learning community.
8. T referred to Tandeka.

9. In apparent reference to me as the facilitator.

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WHEN THE WORLD IS NOT THE PROBLEM: REAL-WORLD CONTEXTS IN ANALOGIES[1]

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This paper considers real-world contexts in the teaching of mathematics. An in-depth qualitative analysis of mathematics lessons in two urban high schools explores how real-world contexts serve as sources of analogies for introducing mathematical concepts and procedures. This study identifies and describes the effective discursive practice of elaboration which supports and develops analogies. Elaboration and analogies offer additional means for teachers to "center" (Tate, 2005) instruction on urban students' lived experiences.

INTRODUCTION

Our focus on teachers' use of real-world contexts emerges from a research and professional development project on culturally relevant mathematics pedagogy (CureMap). CureMap consists of three dimensions: teaching mathematics for understanding; centering instruction on students, their experiences, and their communities; and creating opportunities for students to think critically about and with mathematics (Rubel & Chu, 2012). Real-world contexts (RWCs), defined as objects, situations, or practices that exist independent of the mathematics lesson, span the CureMap framework in multiple ways. For instance, teaching mathematics for understanding includes making explicit connections between school mathematics and out of school phenomena. Similarly, centering instruction on students and their experiences invites the inclusion of RWCs that are local and/or relevant to students. Developing students' ability to be critical with mathematics suggests using mathematics as a lens through which to view and analyze RWCs.

The correspondence between problem solving in school and mathematical thinking about a RWC is not always direct. For instance, individuals can be successful in solving mathematical problems outside of school, in well-defined, situated contexts, but then have difficulty with similar, in-school tasks (e.g., Noss, Hoyles, & Pozzi, 2002; Saxe, 1988). Further, when solving problems based on RWCs in school, students frequently do not take realistic considerations into account (e.g., Carpenter, Lindquist, Matthews, & Silver, 1983). For instance, when asked how many two-foot boards can be cut from two five-foot boards, a common response is five, with the reasoning that ten divided by two is five (Verschaffel, De Corte, & Lasure, 1994).

A growing body of research has also demonstrated that students sometimes take into account realistic considerations *more* than intended by teachers, or by curriculum or assessment designers. The realistic features of the task and students' out of school knowledge about the RWC end up distracting or misleading students from the intended mathematical objectives. This tendency is more frequent among students from non-dominant groups, whether it be according to gender (Boaler, 1994), language (Zevenbergen, 2000), race (Tate, 2005), or social class (Lubienski, 2000).

Despite these various challenges associated with RWCs and student learning, RWCs are also potentially significant means for building mathematical understanding (e.g., Baranes, Perry, & Stigler, 1989; Koedinger & Nathan; 2004, Moses & Cobb, 2001; National Council of Teachers of Mathematics, 2000; Van den Heuvel-Panhuizen, 2005). In this vein, CureMap advocates including RWC's, provided that these contexts are familiar and relevant to students.

ANALOGIES AND ELABORATION

The flow of understanding between mathematics and RWCs lies along a spectrum. At one end of this spectrum, students can develop understanding *about* RWCs *with* mathematics. Fully realized examples include mathematical modeling (e.g., Lesh & Doerr, 2003). In these instances, a cyclical process iteratively refines multiple transitions between salient aspects of RWCs and mathematical representations. At the other end of the spectrum, students can develop understanding *about* mathematics *with* RWCs. In these cases, teachers create analogies between RWC's and mathematical procedures, concepts, or rules. This paper focuses on this latter category of instances where the RWC is not the primary object of study but rather is the basis for developing understanding of mathematical concepts or procedures.

An analogy consists of a relational mapping that relates a source, which can be a RWC, to a mathematical target (Gentner, 1983). This view of analogy subsumes metaphors, with the distinction being in the degree to which similarities are made explicit. That is, the mapping can relate not only source and target objects but can also connect attributes or properties, relationships, and operations. For example, a common analogy relates a mass scale as a source to the target of algebraic open sentences, thought of as equations. The property of a scale being balanced can be mapped to the property of equality within an open sentence. The operations of adding or removing the same mass from the sides of a scale can be mapped to performing identical arithmetical operations to the two sides of an open sentence.

An enduring tension in the development of analogies within the classroom is the extent to which students and the teacher share the work of developing the analogy and making inferences about the target. Teachers typically exert a great deal of control over analogies, as they introduce the source, identify the target, and provide the mapping between the source and the target (Richland, Holyoak, & Stigler, 2004). Developing an analogy within a mathematics lesson is supported by the practice of elaboration, a teacher's whole-class discursive moves which highlight salient aspects of the RWC as well as the relational mapping to mathematics. Elaboration is likely necessary to articulate a sufficient understanding of the RWC to allow for a relational mapping from the RWC to the target. This role of elaboration of RWCs in analogies is parallel to findings about lessons in which a mathematical task is set in a RWC. Lessons in which teachers set up the mathematical task by leading discussions focused on key and salient contextual features of the RWC subsequently had summative whole-class discussions which had higher levels of academic rigor (Jackson et al., 2011).

A key form of elaboration of RWCs is to connect the RWC to students' individual experiences, through a process known as "contexting" (Evans, 1999). Contexting is organized by guiding questions like: "Does this (RWC) remind you of anything that you currently do?" and "Does this (RWC) remind you of any earlier experiences?" These types of questions serve to first make personal, and possibly affective, connections with a real-world context before trying to apply interpretations specific to a school task. Such approaches have also been described as allowing students to "resonate in" a RWC problem, by finding and discussing correspondences and connections with their personal real-life experiences (Chapman, 2006). These resonances are of further potential value as students assess the reasonableness of the mathematical solution by referring to their experiences with the RWC (Depaepe, De Corte, & Verschaffel, 2010).

The research was conducted in two high schools in low-income, urban neighborhoods in a large city in the United States. One school is a majority Latino school, and the other school is a majority African-American school. In both cases, students have histories of achievement on middle school exams which have been correlated with disengagement with the regime of schooling (e.g., Davis & Martin, 2008). Our analysis includes specific lenses for what counts as local or relevant to students, as well as the broader opportunities for student participation provided by both the substance and process of elaboration.

RESEARCH QUESTIONS

This paper investigates two connected research questions:

- To what extent and how do teachers and students use RWCs in analogies?
- How do teachers and students elaborate upon RWCs as sources of analogies?

The answers to the first research question will focus on the frequency and mathematical topics with which analogies using RWCs are used. The answers to the second question will focus on the process through which teachers and students connect sense-making about RWCs to mathematical concepts and procedures.

DATA SOURCES AND ANALYSIS

Six teachers at two high schools were observed over two school years in a total of 106 observations. Each teacher was observed either seventeen or eighteen times in eight rounds spread across the school year. A detailed, narrative description was written for each lesson.

Lessons that included analogies between RWCs and mathematics were identified. In each case, the source and target of each analogy was identified. Then, for each of these analogies, the extent to which the RWC was elaborated was analyzed. This concept of elaboration considers whether and how a RWC is explained, interpreted, paraphrased, or expanded upon by students or the teacher in the lesson.

RESULTS

Fifty out of the 106 lessons (47%) contained RWCs, and thirteen lessons (12%) included analogies with RWCs. Table 1 displays the sources and targets of the analogies in the observed lessons. Compared to other uses of RWCs in lessons, analogies were infrequent. Most (37) of the lessons involving RWCs cast story problems in terms of the RWC. For instance, multiple lessons on the topic of quadratic functions included story problems involving projectile motion. The greater prevalence of RWCs in story problems was consistent with the less frequent use of RWCs in geometry lessons.

Teacher	Source	Target
A	Negative space images	Area of composite figures
A	Various "middles"	Constructing the centroid
A	Image of balanced breakdancer	Centroid and midsegment of triangles
A	Casino or "house" advantage	Expected value of a game
B	Translating words in languages	Translating literal expressions
C	Weight of feathers and coal	Operations on equations
C	Combining purchases of burgers and fries	Systems of linear equations
D	Crowding people into the room	Density of rational numbers
D	Legal arguments	Proofs
E	Parts of a pie	Parts of a circle
F	Climbing mountains	Operations on signed numbers
F	Societal expectations around dating and gender	Functions and non-functions

Table 1: Analogies using real-world contexts as sources

In three of these 13 lessons, the analogies created by the teacher did not map well to the target mathematical idea. For instance, in a lesson about the distribution of rational numbers on the number-line (9/21/09), Mr. D [2] asked students to think about fitting more and more people into the room. He said crowdedness was a way of thinking about the density of the number line. This RWC does not map well to the mathematical target because rational numbers, unlike people, do not occupy space. This example illustrates the essential role of teacher content knowledge in developing analogies that promote student understanding. In four other instances, analogies were mentioned at the beginning of a lesson as an initial motivation or explanation but were not referred to again in the lesson.

Sustained use of a central analogy for refining mathematical statements

One of the observed lessons, on determining whether relations are functions, referred to a central analogy throughout the lesson. This analogy was based upon students' knowledge about contemporary attitudes toward dating practices for boys and girls. Ms. F asked students to think of the domain as girls, and the range as boys. With pairings as relationships, Ms. F then solicited students' views about boys who have more than one girlfriend, in contrast with girls who have more than one boyfriend. Ms. F noted how there was a bias against girls who have more than one boyfriend, but not boys who have more than one girlfriend. Ms. F then had the class use this analogy as a way of remembering the test definition for a function. This "rule" of society was rephrased in many ways. For example, one student said, "A girl can't have more than one boyfriend."

Beyond serving as a motivating definition, this analogy was referred to repeatedly by students to support their reasoning about whether a particular mathematical relation was a function. After students, working in groups, analyzed a collection of either ordered pairs, mapping diagrams, tables, or graphs, they presented to the whole class a rule for determining whether or not the given relations were functions. These rules were phrased in terms of their assigned representation. Of particular interest is how Ms. F facilitated the whole-class discussion as the group given ordered pairs shared their work with the class. This discussion was marked by multiple transitions back and forth between the RWC of boys and girls dating and the more abstract setting of domain, range, and values of x and y. This discussion is presented in Table 2. Exchanges in the mathematical context are left-justified in plain text, while those phrased in the context of boys and girls dating are right-justified in bold text.

This lively discussion engaged many students and explored multiple revisions of the rule for ordered pairs. Although Ms. F initially referred to the RWC as she circulated with the small groups in asking questions or probing student thinking, in the remainder of the discussion she made statements largely in terms of x's, y's and the domain. Each of the four revisions made by students, noted in Table 2, are phrased in mathematical terms. The RWC of the analogy allowed other students a legitimate way to both demonstrate their individual understanding and contribute to the collective development of a representation-specific definition.

Similarly, for the representation of mapping diagrams, students' statements became more precise. At first, they spoke of counter-examples as "if it has two different arrows from the x-side to the y-side". Later, a student phrased the rule as "x cannot have more than one arrow point to different y's… or guys." In this last statement, the hybrid use of symbolic and RWC language points to flexibility in students' mathematical definitions of key test conditions.

Section	Whole-class Discussion
Original version and examples	A student in the group says, "by first looking at the *x*-value, if the *x*-value has more than one *y*, it is not a function." **As the group presents, students give examples and explanations such as: "because x has multiple boyfriends, and you can't have that... a girl can't have multiple boyfriends..." and "Both the boy and the girl have multiple spouses".** **Ms. F asks a student which girl, and he answers "Girl 3".**
Revision #1	Ms. F then asks the group who can "restate" the rule. A group member says, "If *y* is repeated and *x* is different".
Counter-examples to "new" rule	Two students then provide counterexamples. Student A says that *y* could be a constant and *x* can't, but that *y* could also be different numbers. **Student B says, "If *y* the boy has multiple girls…"** This statement is rephrased as "*x* has different numbers, *y* the same..."
Revision #2	Ms. F returns to the group member's statement, "If *y* is repeated and *x* is different". He now says "repeating values in the *x*". Ms. F draws the class's attention to an example on the board where the value of 2 in the domain is paired with multiple values in the range. **Student C explains why this specific example is not a function "*x* is the girl, she keeps going every man house."** Ms. F says, "right, you get it."
Revision #3	Student D says, "*x* cannot be repeating more than once, but *y* can."
Ms. F's counter-example	Ms. F then says that they will need to "adjust" this statement. Ms. F writes on the board the ordered pair (2,4) three times. **Student E exclaims, "She with the same guy!"**
Revision #4	Student F says "If *x* is repeating more than once with a different *y*". Ms. F restates the rule as "*x* cannot be repeating with different *y*-values".

Table 2: Whole-class discussion by setting of statements.

Elaboration: Facilitating entry

Seven of thirteen lessons with analogies included elaboration, varying according to breadth and depth. Breadth gauges how many students participate and provide different alternative or personal responses to a question about RWCs. Depth measures the level of detail within a sustained contribution by a single student or the teacher. In three cases, this elaboration was developed in Ms. A's classroom to substantially support student understanding. In one of Ms. A's lessons, however, although students elaborated on the fairness of casinos as the lesson began, this discussion was not connected to the teacher-directed exposition of expected value.

As an example of a lesson with broad elaboration, in a lesson about the mathematical topic of geometric area of composite figures, Ms. A showed students three images involving positive and negative space. Students described what they saw:

> For the faces/vase optical illusion, students explain both how they see faces in profile as well as a "candle holder" or a "pimp cup". A student picks up a trophy sitting on the teacher's desk and shows it to those around him. As Ms. A shows another image of a dog eating a cat eating a rat, students clamor to contribute. Others express their disbelief or make references to "therapy", or Rorsharch images. Students then talk to each other and help each other with a negative space image in which "EVIL" is written inside "GOOD". Ms. A summarizes these examples, telling the class about negative space as a concept and technique in art, mentioning the students' art teacher. (12/15/09).

In this lesson, the cumulative effect of these three examples was to provide a range of instances of negative space in popular images, which then supported the variety of composite figures which groups of students then analyzed in terms of geometric area.

A second example of a RWC elaborated broadly and more deeply was in a lesson about a triangle's centroid. Ms. A asked students to write down everything that they knew about "middle." She gave prompts such as "middle child", "middle of the week", and "middle of the road". Ms. A then asked students to share their ideas with the class. Students enthusiastically raised their hands, sharing different opinions, perspectives, and experiences. Through their responses, students took the perspective of being in the middle and having the same amount of something (e.g., days of the week or brothers) on either side. This diverse assortment of RWCs directly supported the notion of centroid which students then physically constructed and investigated with triangles cut out of cardstock.

The lesson that immediately followed, also in Ms. A's class, includes an instance of deep elaboration. Ms. A projected a photographic image of a classmate break-dancing and balanced upside-down on one hand, with the following prompt: "How does this dancer relate to what we were talking about yesterday?" In the ensuing whole class discussion, a student said:

> on the three sides… equal balance, just sitting on the tip of my pen. His body got equal balance, the top part is equal balanced to the bottom part.. not too heavy on the hand and so… on the triangle we did the slice and all that, equal weights, sitting on the tip of our pen. (2/12/10)

Ms. A further elaborated, drawing upon what other students had said and engaging another student's question:

> Ms. A asks what those slices were and then names two students Edwin and Katheryn, crediting them as having described it: "cut it at an angle, cut it in half". She says that Katheryn had said it was the midpoint, and the name of the line itself is the median. Martin asks what the dot is, and Ms. A says the hand is the centroid and the arms and legs

> are the medians. Ms. A calls them cross-sections of balance, gesturing across her own hips and talking about the balance and weight coming down. (2/12/10)

Here, as in the previous lesson, analogies connected RWCs to mathematical ideas. The remainder of the lesson was focused on non-contextualized problems involving medians and midsegments of triangles. The deep elaboration in the introduction of this concept allowed students to directly connect the previous lesson to the new topic.

DISCUSSION

These three analogies in Ms. A's class are notable for how they promoted conceptual understanding of geometry, a subject infrequently connected to RWCs among observed lessons. In addition, the analogies offer examples along a broad range of options for prompts that elicit different modes of student participation. Images invite student to *describe* and *associate.* Multiple prompts, verbal or visual, allow students to *compare* and *contrast*. Students *connect* and *map* as in the elaborated analogies between a breakdancer and a triangle. These invitations are an alternate approach to the "relevant" in culturally relevant. Rather than being a feature of a specified problem context relative to a group of students, relevance *emerges* from how students' experiences are invited, explored, and connected to noncontextualized mathematics. Similarly, the function analogy based on dating broadened access to more students *qualifying*, *clarifying*, and *rephrasing* a key mathematical definition. None of these analogies, except perhaps for breakdancing on the subway, was intrinsically urban. Rather, each analogy invited students to enter with their lived experiences, some of which reflected the urban context.

Beyond specific analogies, analogical reasoning is an important form of higher order thinking. Because it focuses on mapping *structures* to one another, analogy is useful in multi-representational reasoning, with correlating components of different representations. Analogical reasoning also supports mathematical habits of mind, such as algebraic abstraction (English & Sharry, 1996). Although this paper has focused on RWC analogies, analogies can also be purely mathematical. Experience and confidence in analogies grounded in familiar or local RWCs can support students' non-contextualized analogical reasoning.

Regarding CureMap's third dimension, analogical reasoning can support students becoming more critical *about* mathematics. If students frequently explore and develop analogies, they can analyze disanalogies to criticize limitations of mathematical explanations and algorithms. For example, how does the common analogy from balanced mass scales to solving equations extend to inequalities? In the case of inequalities, multiplying by negative numbers requires students to revisit, revise, rephrase, and refine the analogy with reference to the RWC. These doubts can lead either to criticizing the commonplace analogy, or to extending the analogy to negative weights, using structures such as balloons or counterweights on pulleys.

In terms of teacher professional development, this paper has identified and proposes three effective strategies for teachers to implement to deepen urban students' conceptual understanding of mathematics. First, teachers could benefit from an explicit focus on the structure of analogies, with a focus on the mapping of structures. Second, professional development should emphasize providing students with a variety of options and modalities for elaboration, both in terms of social participation as well as conceptual content. Finally, teachers ought to be encouraged to engage in lesson planning that anticipates students connecting their knowledge of RWCs to mathematical procedures in rich and nuanced ways. Each of these three shifts in professional development can foster the participation and understanding of students in urban settings, even if they do not involve story problems set in RWCs.

NOTES

1. This material is based on work supported by the National Science Foundation under grant no. 0742614. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Science Foundation.

2. All proper names in this paper are pseudonyms.

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QUANTITATIVE LITERACY AND EPISTEMOLOGICAL ACCESS AT UNIVERSITY: REFLECTIONS ON USING THE THRESHOLD CONCEPT FRAMEWORK FOR RESEARCH

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Many students in South Africa are poorly prepared to meet the quantitative literacy (numeracy) requirements of academic disciplines. Courses for such students must provide epistemological access to disciplinary practices, including access to the quantitative concepts which are regarded as threshold concepts for numerate practice across disciplines. Using the threshold concepts framework for enhancing our reflective practice has made explicit some of our implicit assumptions and expectations and made us focus less on covering course content and more on students' thinking and communication. It has allowed us to design a manageable project that has produced timeous changes in the way we teach and given greater insight into the ways curriculum for quantitative literacy could be structured.

INTRODUCTION

It is commonly accepted that all citizens should be numerate (quantitatively literate) in order to participate effectively in an increasingly quantitative world [1]. Most numeracy practitioners would agree that a disposition and ability to think critically about quantitative information is an essential aspect of numerate practice (Johnson & Yasukawa, 2007; Jablonka, 2003; Best, 2007; Steen, 2001). For example Johnson (2007, p. 54) defines numeracy as "a critical awareness that builds bridges between mathematics and the real world".

Many students in South Africa are poorly prepared to meet the quantitative literacy requirements in university curricula. These curricula do not just require understanding and application of mathematical and statistical techniques, but also crucially demand that a student displays a critical stance and is able to use the language associated with quantitative concepts appropriately (Frith et al., 2010). The fact that many students are unable to do these things is largely as a result of educational disadvantage resulting from the structure and history of education in our country. Inequities in the outcomes of schooling lead to inequity of access to higher education, but also if they are not addressed explicitly by the university curriculum, to inequities in outcomes, which are both economically and socially unacceptable.

Morrow (2009, p. 77) originally (and subsequently many other educational researchers in South Africa, see for example Bozalek, Garraway, & McKenna, 2012; Boughey, 2005) used the term 'epistemological access' to refer to access to the academic practices and knowledge of the academic disciplines, as opposed to merely physical access to the institution. Paying attention to the numeracy demands of the higher education curriculum is an important element of a strategy to provide this epistemological access and hence to address inequity of outcomes in higher education. 'Epistemological access' is understood to mean access to the key concepts and procedures of an academic practice and to the ways of acting and communicating authentically in that practice. It is generally understood that most of our students will not gain this kind of access without teaching that pays attention to making the implicit academic practices and underlying knowledge systems more explicit.

> " ...both the academic staff and the students need to become explicitly aware of their discipline's 'epistemological core', of the kind of knowledge valued by the discipline, of what kinds of knowledge are excluded from it and of which linguistic constructions are best used to represent those values." (Clarence, 2010, p. 37)

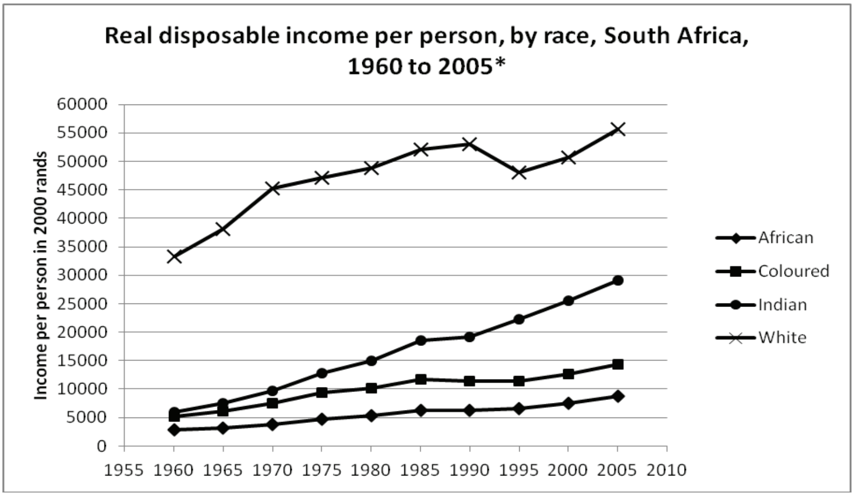
For this reason we view quantitative literacy as a component of a broader conception of academic literacy. Thus in our courses we try to focus on making meaning from quantitative information in the contexts of the disciplines as well as on the use of appropriate language for expressing quantitative ideas.

In an earlier paper (Frith et al., 2010) we discussed the tensions that arise for both students and lecturers as a result of learning mathematical and statistical content through engagement with disciplinary contexts and we raised questions about what kind of research methodology would be most appropriate for us to investigate teaching and learning in these courses. In this paper we will discuss the usefulness and appropriateness for our purposes of one approach that we have subsequently explored, namely the use of the threshold concept framework (Meyer & Land, 2003). Using this approach in studying learning in quantitative literacy is somewhat innovative, as traditionally the idea of threshold concepts has been applied to much more clearly-defined academic disciplines like economics, biology and physics. The threshold concept framework has apparently also not been used often in Mathematics education research (Pettersson, 2010)

CONTEXT OF OUR RESEARCH

In the previous paper (Frith et al., 2010) we introduced our research project which investigates the effectiveness of the curriculum of our quantitative literacy courses at the University of Cape Town. One of these courses is for law students (called 'Law that Counts') and is intended to assist students to develop appropriate quantitative literacy for their discipline (Frith, 2012) and in this way to promote 'epistemological access'. The curriculum uses contexts that have a social justice focus which we judge to be relevant to Law. In line with our conception of quantitative literacy (Frith et al., 2010), one of our stated outcomes in these courses is that students should be able to reason critically about quantitative information in the disciplinary context, as well as in the broader social setting [2].

Initially one aim of our research project was to determine to what extent we were effectively promoting or achieving this outcome. Once we began to analyse our context-based learning materials, our classroom interactions and the students' responses to assessment questions it became apparent that the students in these courses were hardly ever thinking critically in the way that we would like. Our reflection on why this is the case led to the realisation that too many of the students were still struggling at the level of understanding the mathematical concepts (and techniques) to allow for us to focus effectively on developing their critical awareness. If we think of numeracy as “an aggregate of skills, knowledge, beliefs, dispositions, habits of mind, communication capabilities, and problem solving skills that people need in order to engage effectively in quantitative situations arising in life and work” (Steen, 2001, p. 7), then our students in fact still need to acquire the skills and knowledge, before we can really help them develop appropriate habits of mind, communication capabilities and problem-solving skills. For example, most students could not begin to interpret or critique the data in Figure 1 (which we will refer to again later) because they are not able to make mathematically meaningful comparisons between values shown in the chart (by answering the question shown below the chart)[3]. This realisation has redirected our focus in teaching and in research on trying to understand how best to promote students’ learning of the necessary mathematical and statistical concepts and procedures.



** Personal disposable income per person is calculated using total Gross Domestic Product (GDP) divided by the total population. All incomes expressed using 2000 as the base year.*

Question: Consider the real disposable income per person for Indians and Whites. Without doing any calculations, say which race group experienced the greater percentage increase in real income per person from 1960 to 2005. Explain your answer.

Figure 1: Example of a question to assess students’ ability to make comparisons of relative changes.

THRESHOLD CONCEPTS

The threshold concepts framework (Meyer & Land, 2003) provides a means to describe the learning of concepts which are the foundations of disciplines or practices in higher education. Threshold concepts can be seen as gateways to thinking and practising and communicating authentically in a particular discourse and as such can provide a way of focussing on aspects of the development of a student's academic literacy. These concepts are likely to be troublesome for the student and ultimately transformative, leading to a shift in perception and use of language (Meyer & Land, 2003). This change in perspective is unlikely to be forgotten and is thus irreversible. The time taken for the process of internalising a threshold concept (and thus effecting a transition from one way of thinking to another) will vary depending on how troublesome the concept is for a learner. During this transition a learner experiences a "state of liminality … a suspended state in which understanding approximates to a kind of mimicry or lack of authenticity" (Meyer & Land, 2003, p. 13).

We propose that there are mathematical and statistical concepts that act as threshold concepts for numeracy across disciplines. While much of the work on threshold concepts reported in the literature has focused on concepts within disciplines such as economics, accounting, philosophy and computer science, some authors have highlighted concepts that seem to fall outside of specific academic disciplines (belonging in the realm of academic literacy) and yet underlie important concepts in these disciplines. For example, Ross et al. (2010) suggest that the threshold concepts in biology are not the difficult biological concepts like cellular metabolic processes, but rather the "ability to work with concepts and processes; ... randomness and probability, proportional reasoning, spatial and temporal scales, thinking at a microscopic level" (p. 165). Quinnell and Thompson (2010) examined student learning in life sciences and medicine and found that obstacles often occurred when students attempt to practise academic numeracy within these disciplines.

We suggest that achieving epistemological access (to a discipline or practice) necessarily involves mastering the threshold concepts of that discipline, including those that may be intrinsic to academic numeracy practised in that discipline. Thus the threshold concepts framework provides an approach to assessing aspects of the success or otherwise of curriculum interventions to promote epistemological access.

THE VALUE OF THE THRESHOLD CONCEPT APPROACH FOR REFLECTING ON TEACHING AND LEARNING

A number of authors have discussed the usefulness of university teachers researching threshold concepts for promoting their reflective practice (Irvine & Carmichael, 2009) and for developing "actionable theory" (Meyer, 2010, p. 198). Threshold concepts "provide a powerful heuristic to interrogate the cause of troublesome content knowledge ... and to aid in the development of teaching interventions ..." (Ross et al., 2012, p. 165).

Our research approach based on the threshold concept framework enables us to design a manageable project consisting of an in-depth investigation of the teaching and learning of one important and pervasive concept at a time. By focussing on one concept we can analyse data collected relatively quickly, and then rapidly put into practice changes in teaching that the results of the analysis suggest to us. A new cycle of data collection and analysis can then be carried out.

To illustrate the advantages of this research approach, we briefly discuss one example of a study that has already led in a relatively short space of time to some effective changes in teaching. In this study we examined how students in the 'Law that Counts' course learn the concept that we term 'proportional comparison'. We use this term to describe the concept of comparing quantities in relative and in absolute terms (and the associated language and reasoning). This is a concept we believe students must master in order to be able to think critically about data. Students must understand that the significance of changes or differences can be either obscured or exaggerated by describing them in either absolute or relative terms only. For example the seriousness of the consequences for an individual of a superficially impressive claim that, for instance, a person's behaviour increases their risk of accident by 80% will be very dependent on whether the risk was initially quite large or initially very small (that is, whether the reported change comes off a small or a large base).

We suggest that this kind of proportional reasoning is a threshold concept for academic numeracy and necessary for critical reasoning about data in society. A student who has not crossed this threshold will be unable to assess the validity of many arguments based on quantitative evidence and would thus not be regarded as numerate. It is of course not the only threshold concept, but rather one in a "web of threshold concepts" (Ross et al., 2010) which make up the "troublesome knowledge" (Meyer & Land, 2003) in our courses.

We used a framework developed through an iterative process of coding and re-coding to analyse students' answers to selected questions in two different assessments in 2011. The question from the second assessment has already been shown in Figure 1. These questions referred to changes represented graphically and students were asked to explain (without calculation) which of two data categories experienced the greater percentage change over time. These questions expected students to make several observations and then bring them together in the correct logical relationship in order to make the appropriate comparison. Students were expected to observe from the graphs (a) the magnitude of the absolute changes, (b) that the absolute changes for both categories were approximately the same, but (c) that for one of the categories the percentage change was calculated from a much smaller initial value (came off a smaller base). They were then expected to conclude from these observations that the resultant percentage change would be greater for this data category.

<table>
<tr><td>Code</td><td>Description</td><td>Stage in threshold concept framework</td></tr>
<tr><td>(a)</td><td>Compare the sizes of absolute changes</td><td rowspan="6">Pre-liminal:
Answers that were coded (a), (b) or (c) only
Liminal:
Combinations of codes (a) to (e), but excluding (g)
Code (f) can be present in answers regarded as Pre-liminal or Liminal</td></tr>
<tr><td>(b)</td><td>Recognise that absolute changes are roughly equal</td></tr>
<tr><td>(c)</td><td>Compare positions of initial values</td></tr>
<tr><td>(d)</td><td>Recognise that percentage change is a fraction whose base is important</td></tr>
<tr><td>(e)</td><td>Misconception that larger denominator means fraction is larger</td></tr>
<tr><td>(f)</td><td>Distracted by the context in which this kind of reasoning was previously experienced</td></tr>
<tr><td>(g)</td><td>Correct conclusion, comparing fractions, reasoning with components (a), (b), (c) and (d)</td><td>At the threshold</td></tr>
<tr><td>(h)</td><td>No comprehensible explanation provided</td><td>Not classifiable</td></tr>
</table>

Table 1: Framework for the classification of students answers to 'proportional comparison' questions.

An outline of the framework for analysing the student responses is shown in Table 1. The categories in the framework emerged from examining the students' responses (not from any pre-conceived ideas we had about the questions) and it was during this process that we realised that these categories corresponded to the elements of the reasoning required for a full expression of the understanding of the concept. So one benefit of coding students' answers was making explicit the implicit expectations we had when posing questions of this kind. The research helped us to recognise that such questions represent a significant degree of complexity for students.

	2011		2012	
	First assessment	Second assessment	First assessment	Second assessment
	(n=36)	(n=32)	(n=53)	(n=44)
Pre-liminal	42%	66%	34%	46%
Liminal	36%	25%	45%	25%
At the threshold	11%	3%	9%	23%
Unclassifiable	11%	6%	11%	7%

Table 2: Summary of results of analysis of the same assessment questions in 2011 and 2012.

The results of the analysis of assessment questions using this framework in 2011 are shown in the left-hand side of Table 2. The following are examples of responses to the question in the second assessment (see Figure 1) that would be regarded as 'Pre-liminal' and 'Liminal' respectively: "Whites because the final and initial amounts are greater than the Indian amounts" and "Indians because they started off a smaller base than the whites". In contrast the following response would be considered to be at the threshold: "Indians: although the percentage point increase is almost the same as for the whites, Indians begin at a significantly lower base value, which means the percentage increase is higher". The analysis revealed that many students' answers revealed an awareness of only one or two elements of the correct reasoning and that very few of the students could be classified as having mastered the threshold concept of 'proportional comparison' in the first assessment. Given that in the final assessment only one student gave a complete answer to this question, it could be argued that in fact only one student crossed over this threshold in our course in spite of our conscious attempts to teach this concept.

The investigation in 2011 prompted us to introduce changes in our teaching of this concept in 2012, based on ideas suggested by the analysis and by the literature. An approach often suggested is to encourage "metareflection" (Orsini-Jones, 2010, p.286) and to improve students' awareness of the process of moving through the liminal space in various ways. For example, Kabo and Baillie (2009) suggest that learning activities can be designed around requiring students to classify other students' utterances on a "spectrum of liminality" and thus make them more aware of the critical aspects of the threshold concept.

In 2012 we maintained a sustained emphasis on the 'proportional comparison' concept and the appropriate use of language to express it, introducing many different representations including graphical, diagrammatic and verbal in various different contexts. We also introduced some specific activities, in particular an activity intended to develop students' metacognitive awareness, as suggested by Kabo and Baillie (2009). In this activity (based on the first assessment question), we first discussed the idea of the threshold concept, and of 'proportional comparison' as an example of such a concept. We then discussed the elements of a correct answer showing the students the framework (Table 1). Students then critiqued examples of other students' answers (from 2011) and attempted to classify them according to the framework. The object of this exercise was to make students more explicitly conscious of the elements involved in proportional reasoning and to improve their metacognitive awareness of how they learn in general. Student interview data indicated that for some students this exercise made a powerful impression.

The same questions that were analysed in 2011 were included in assessments and analysed in 2012. Table 2 also contains a summary of the results of this analysis. Comparison of the results from 2011 and 2012 reveals that although there were modest gains in students' understanding of this concept as a result of what one of our students referred to as our "being on a mission" to teach this concept better, it remains very troublesome for the majority. From the research in both 2011 and 2012 we gained new insight into the extent of the troublesomeness of this concept and concluded that it is unrealistic to expect all students to cross this threshold in the time available in a one-semester course. This has implications for the manner in which quantitative literacy provision in the curriculum should be structured. It supports our argument that quantitative literacy should ideally be integrated into the teaching of the disciplines and cannot be adequately addressed only in a short introductory add-on course (Frith et al., 2010; Frith, 2012). A similar conclusion was reached by Orsini-Jones (2010) in writing about sentence structure as a threshold concept in linguistics: " ... the threshold concept identified is complex and cannot be crossed in one academic year by many students. Further explorations of long-term curricular interventions are needed ..."

The example of a research project briefly described above reveals how one key concept, 'proportional comparison', which we initially assumed was quite simple (and did not originally make very explicit) was shown to be in fact a typical threshold concept in terms of its troublesome nature and the time it takes students to assimilate. This understanding (and the threshold concept literature) has provided pointers to approaches we might employ to more effectively promote the development of this concept.

CONCLUSION

Our research approach based on the threshold concept framework enables us to design a manageable project consisting of an in-depth investigation of the teaching and learning of one important and pervasive concept at a time. By focussing on one concept we can analyse data collected relatively quickly, and then rapidly put into practice changes in teaching suggested by the analysis.

Paying attention to the quantitative literacy demands of the higher education curriculum (academic numeracy) is an important element of a strategy to provide epistemological access and hence to address inequity of outcomes in higher education. We find the theory of threshold concepts to be a useful framework for researching students' learning of key knowledge and practices in academic numeracy and facilitating our own reflective practice. Researching in this way has made us focus less on covering topics in the curriculum and more on the students' experience. In addition the construction of a framework for analysing students' written answers has made us more aware of the implicit demands of the kind of reasoning we expect of our students. This awareness is a necessary condition for effectively promoting epistemological access.

NOTES

1. We use the terms 'numeracy' and 'quantitative literacy' (QL) interchangeably in this paper.

2. The course we provide for law students is almost identical to the one for social sciences students. Although in this paper we focus on research we have done in the former course, our insights apply equally to learning in the latter.

3. Race classification in South Africa is used as a social (rather than a biological) construct in the measurement of success (or otherwise) of transformation in society.

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TEACHING THROUGH ETHNOMATHEMATICS: POSSIBILITIES AND DILEMMAS

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Ethnomathematics is at a stage in its development where it needs further theorization combined with critique. As a relatively young field, the concept itself remains vigorously contested with no clear consensus about its epistemological status. It is largely marginalized by, and in turn critical of, mainstream mathematics education. Very careful thought needs to be given to how Ethnomathematics might, in practice, enhance mathematics education. Such considerations inevitably lead into a minefield of exploding concepts, full of dilemmas, tensions, contradictions, open questions, and deeply political. Through the lens of ethnomathematics we see hegemonic struggles between opposing worldviews.

EMERGENCE OF ETHNOMATHEMATICS

The ~~intellectual~~ *mathematical* activity of those without power is always characterized as ~~non-intellectural~~ *non-mathematical* (adapted as indicated from Freire & Macedo, 1987, p.122)

Growth

Ubi D'Ambrosio brought Ethnomathematics into prominence, as a term and a perspective on mathematics as a human activity, at the 1984 ICME meeting in Australia (D'Ambrosio, 1984). Notable precursors include Claudia Zaslavsky's book on African mathematics (Zaslavsky, 1973), the work of Paulus Gerdes in Mozambique (e.g., Gerdes, 1981), Pinxten's deep exploration of Navajo epistemology of space (Pinxten, van Dooren, & Soberon, 1987) and the work of Ascher and Ascher (1981) on Incan quipus.

More broadly, significant shifts in several domains have contributed to a supportive ethos for the emergence and development of Ethnomathematics.

- In anthropology, the inherently racist view of a scale of human development from "primitive" to "civilized" has been deconstructed.
- In psychology, through developments such as cultural psychology (Cole, 1998) and the recognition of the situated nature of most cognition, it became clear (to some, at least) that experiments conducted on a small and biased sample of humanity in artificial situations could not support universal theories of human behaviour.
- In the philosophy of mathematics, Platonism became increasingly hard to maintain (e.g., Hersh, 1997; Restivo, 1993) as it became recognized that mathematics is a human, and hence cultural, activity.
- In the history of mathematics, the critique of the Eurocentric (or, as Raju (2007) uncompromisingly puts it, racist) narrative developed.

- And within mathematics education itself, a critical mass has formed (Greer & Skovsmose, 2012).

Ethnomathematics is now established as an area for research and pedagogical development, with organizations, conferences, publications.

Politics

D'Ambrosio (1985, p. 47) reminded us that "colonialism grew together in a symbiotic relationship with modern science, in particular with mathematics, and technology", and Bishop (1990) characterized Western mathematics as "the secret weapon of cultural imperialism". Taking Freire's declaration that the two great themes of the contemporary world are oppression and resistance, we can point to the growth of resistance in the context of countries gaining nominal independence whilst facing economic and other new forms of colonialism, in particular the colonization of the mind. Thus, following the gaining of nominal independence by many countries, colonialism is carried on by other means. Vithal and Skovsmose (1997, pp. 131-134) refer to "modernization theory" by which they mean the claim that industrialization is a progressive force that can spread prosperity, not least through mathematics education, from the developed countries, along with democracy. In this context, ethnomathematics "can be interpreted as a reaction to the cultural imperialism which is built into modernization theory" (p. 132); Gerdes (1985) described it as a vital part of emancipatory education.

Resistance within mathematics education has not been confined to academia. The example that immediately comes to mind is "People's Mathematics" led by Cyril Julie in South Africa (e.g., Vithal, 2003, pp. 27-35). Another example is use of mathematics with the Landless Movement in Brazil, extensively studied by Knijnik (e.g., Knijnik & Wanderer, 2012).

Another form of ongoing imperialism is linguistic, in particular in relation to English. The field of mathematics education is weakened by insufficient incorporation of work in languages other than English. Despite the many languages spoken by MES participants, these proceedings are in English, including this paper by a monolingual Irishman who cannot speak his native language.

Towards theorization

Ethnomathematics, as an area of research and as a body of theoretical ideas, has matured to the point where clearer lines of theorization are possible and desirable, in parallel with critique (Pinxten & Francois, 2011; Vithal & Skovsmose, 1997). There are many complexities and dilemmas within this minefield of exploding concepts, beginning with the word itself.

Vithal and Skovsmose (1997) identify four main strands:

- combatting the Eurocentric narrative of the history of academic mathematics
- mathematical practices of other cultures that have not been absorbed into academic mathematic

- the mathematics of occupational and other cultural groups
- possible applications of ethnomathematics in enhancing mathematics education. Vithal and Skovsmose (1997, p. 135) commented that "there are relatively few detailed descriptions of the actual implementation and outcomes of the use of such an approach within the formal school system". Some examples are cited below.

POSSIBILITIES FOR TEACHING

Countering the Eurocentric narrative

It seems uncontroversial to state that countering the Eurocentric narrative of the development of academic mathematics should be an aim of teaching at all levels, including college. At the most general level, as expressed by Dias (2002, p. 205):

The understanding of what is "human" and what should be regarded as "development" has taken... a specific turn since the coming up of the "Invention of Man" toward the end of the sixteenth century, and of the subsequent hegemonic construction of the alien, subaltern (non-European) other. The consequence has been the steady marginalization, separation, and subordination of difference and diversity of the world-wide existing human and cultural experience and its multifaceted expressions. This domination structure has as its correlates the privileging and selective imposition of reduced cognitive structures, of one-sided interpretation patterns, or restricted scientific and technical solutions and of monolinguistic habits.

This characterization is clearly reflected in the construction of a Eurocentric narrative for the history of academic mathematics (quite apart from the negation of many cultures' mathematical practices as non-mathematical). However, through the work of scholars such as Joseph (1991) and Raju (2007) (and see the collection edited by Powell and Frankenstein, 1997), a counternarrative is being constructed. In particular, Raju (2007) argues in detail for the existence of two main streams within academic mathematics. One, emerging in Greece and Egypt, was anti-empirical, proof-oriented, and explicitly religious; the other, in India and Arabia, was, by contrast, pro-empirical, calculation-oriented, and aimed towards practical objectives.

Raju also argues that mathematics, far from the timeless and acultural endeavour portrayed in the Platonist view, is culturally dependent. In particular, European mathematics is based on two-valued logic, which is not the only possible choice and is not viable for all situations.

From this perspective, mathematics teaching at all levels should, as a minimum, make clear the contributions on non-Western cultures to the development of academic mathematics. Students should know that there is considerable evidence of development of calculus in India prior to Newton and Leibnitz, as analysed in depth by Raju (2007).

Adapting cultural knowledge for the classroom

Beyond academic mathematics, there are non-academic practices (or practices that have been excluded from academic status) in all cultures, as Bishop (1988) pointed out. As a first step, presenting examples of cultural practices in which significantly complex mathematical activity can be discerned (e.g., Zaslavsky, 1996) has the function of disrupting the deeply entrenched view of mathematics as solely a European achievement. In seeking to move beyond that beginning, a dilemma faced by those studying the mathematics embedded in practices of cultures other than their own is the general dilemma of the anthropologist (and also the historian), namely that of interpreting from the point of view of a cultural and epistemological outsider. Further, this dilemma implies the danger of reformulating those practices for educational purposes in ways that prise them from their cultural contexts. For example, Native American activities may be presented as competitive games of chance, which does not represent what they mean in the communities in which they originate.

Scholars/researchers such as Pinxten, Gerdes, and Lipka have devoted many years of hard work to immerse themselves in the cultures they have studied, enabling them to provide detailed analyses of alternative epistemologies. They have all, also, developed curriculum materials in close collaboration with the knowledge holders within the respective cultures (e.g., Lipka, Yanez, Andrew-Ihrke, & Adam, 2009).

A further aspect of bringing the ethnomathematical perspective to bear on mathematics education is how teachers should be prepared for culturally responsive mathematics education (Greer, Mukhopadhyay, Nelson-Barber, & Powell, 2009). A rare account of work directly with teachers has been given by Ferreira (in press).

DILEMMAS

The end of innocence

Attempts to move mathematics teaching beyond the immaculate transmission of a fixed body of abstract knowledge inevitably complicate the job of teaching and give rise to complexities, tensions, dilemmas (Vithal, 2003). In the case of ethnomathematics, these ramifications are heavily political. In a strong critique paying particular attention to the context of South Africa during Apartheid, Skovsmose and Vithal (1997) analysed the way in which ethnomathematics was exploited and distorted in defence of Apartheid-era educational injustice.

The biggest dilemma

In a very telling exchange, Atweh and Clarkson (2001, p. 87) describe interactions at a conference when the president of the African Mathematical Union (Kuku, 1995) "warned against the overemphasis on culturally oriented curricula for developing countries that act against their ability to progress and compete in an increasingly globalized world". Such an attitude is, of course, consistent with almost universal rhetoric to the effect that mathematics (and science) education are essential for economic competitiveness (a rhetoric that itself is certainly open to critique).

As Barton (2008, pp. 167-8) pointed out, "an understanding of [near-universal conventional] mathematics and a world-language such as English… [represent] access to communication, further educational opportunities, employment, and development". That brute fact leads to the dilemma of what/how to teach mathematics to indigenous groups who "learn mathematics in a distinct cultural-linguistic context – how can they study an international subject while maintaining the integrity of a minority world view?" (p. 142).

As indicated by the quotation just given, there are both parallels in relation to language and intersections between language and mathematics education and in the underlying ideologies (Greer & Mukhopadhyay, in press), with a major fault line between those who promote global homogenization and those who value cultural diversity (Skutnabb-Kangas, 2000). (There are also, of course, many differences – it is significant, for example, that we are at ease speaking of either "language" or "languages", whereas "mathematics" remains stubbornly singular). Setati's work in South Africa (e.g., Setati & Planas, 2012) and parallel work in many parts of the world illustrate how students, parents, and teachers are aware of the cultural capital of speaking English and other dominant languages.

In general, those who advocate resistance to hegemony, whether of academic mathematics, or of English as a dominant language, acknowledge the arguments for keeping open avenues, whether individual or collective, to the economic benefits that may depend on mastering those discourses. Thus, in relation to language, bilingualism/ biliteracy in education is advocated, for example, by the Project for the Study of Alternative Education in South Africa (www.praesa.org.za) established by Neville Alexander in 1992. What would be the equivalent in mathematics education? Children could learn both universal academic mathematics and mathematics relating to their own cultures. But, in my view, more is needed for a well-balanced curricular diet.

Seeking balance; recognizing diversity

Aspirations to put an Ethnomathematical slant on classroom mathematics may be seen in the context of broader attempts to move beyond typical mainstream mathematics education that is dominated by academic mathematics. As well as teaching that is related to the cultural backgrounds of students, and as pointed out in Vithal & Skovsmose (1997), preparation through mathematics education for agentive citizenship needs to address "mathematics in action" (Skovsmose, 2005) and the ways in which mathematics formats reality.

Arguably the best example of how this may be done is the work of Gutstein (2012) in Chicago. Following the Freirean notion of generative themes (Freire, 1970/1998), Gutstein works with Chicano and African-American students to use mathematics in the analysis of issues important in their lives, such as sexism, population movements, AIDS. Although Gutstein does not characterize his work in terms of ethnomathematics, it is arguable that the work he is doing relates to the ethnomathematics of the sociopolitical milieux of his students.

Gutstein (2006) advocates a balanced and integrated approach combining three types of mathematical knowledge: Classical, Community, and Critical. An inclusive stance is taken by Pinxten and Francois (2011) in which they argue for ethnomathematics as the total of mathematical practices, of which academic mathematics is one particular sort. Two general arguments are made for the retention of "Classical Mathematics", at least as an option for every student, namely the educational/economic opportunities that it affords, and, secondly, the Freirean argument of the necessity to speak the dominant discourse.

The search for an exemplary curriculum is illusionary, given that diversity in mathematics education does not merely relate to the students, but also to (a) sites in which mathematics is taught/ learned, (b) forms of mathematics in action, and (c) educational possibilities (Skovsmose, 2012). Consider three types of classroom in terms of the students: (a) one that serves a homogeneous class within an indigenous community; (b) a classroom containing students from multiple cultures; (c) a classroom serving predominantly white children of rich parents. Putting an ethnomathemical slant on the mathematics education for each of these, and many other possibilities, asks for different foci, beyond the common aim of presenting mathematics as a multicultural, human activity and discussing how mathematics relates to society. In particular, I would argue that, even in a situation of cultural homogeity within a classroom, material should not be restricted to the ethnomathematics of that particular culture.

Pinxten and Francois (2011) propose "multimathemacy" as a pedagogical principle so that "the diversity of all mathematical practices coming from the students' background can now enter the learning context" (p. 271). Following Skovsmose, they also emphasize the student's "foreground" by which is meant the way in which they see their educational and life opportunities. This leads to an interesting question as to what a formal education in mathematics adds to the quality of life of a student for whom it has no relevance.

The all-but-universal rhetoric about mathematics (and science) education being essential for economic competitiveness is combined with a focus solely on narrowly defined academic mathematics that is increasingly homogenized (and international comparisons exacerbate the nationalistic and competitive motivations).

CONTRASTING WORLDVIEWS

Besides being the intellectual father of Ethnomathematics, D'Ambrosio has passionately called for mathematicians and mathematics educators to acknowledge their ethical responsibilities (e.g., D'Ambrosio, 2010). Underlining both endeavours is respect for the Other. Ubi's perspective points to fundamental differences in worldviews. Most extremely, but by no means exclusively, in the United States, mathematics (and science) education are linked not merely with economic competitiveness but also with military strength, as opposed to Ubi's characterization of the most urgent problem for humankind being survival with dignity. Through the ethnomathematical lens, we also develop respect for the intellectual activity of those without power and without credentials, such as the unschooled boatbuilders in Bengal who, with minimal tools, no formal training, and no plans, construct fishing boats that pass the high stakes tests of days in the open ocean.

Above all, ethnomathematics straddles the ideological faultline between those who would homogenize humanity (predominantly in the service of capitalistic hegemony) and those who value cultural diversity.

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MATHEMATICS TEACHERS' REPRESENTATIONS OF AUTHORITY[1]

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Though issues of authority abound in education and schooling, mathematics teachers' perspectives on authority have not been investigated sufficiently. We describe a tool that we developed to initiate dialogue with teachers about authority – using a diagram to represent authority in their classrooms. Using mathematics teacher's authority diagrams, we investigated what sources of authority they represented and how the teachers related these sources to each other. The diversity in their representations showed us that research on authority in classrooms has merely scratched the surface.

INTRODUCTION

Issues of authority abound in education and schooling. For example, at the broad system level, authority appears in the monitoring of students' performance. At the classroom level, authority occurs in the teacher-student relationship. In mathematics education, authority is especially important because of the discipline's characteristic interest in truth and proof. A quantitative investigation of a large body of transcripts from secondary mathematics classes corroborated the prominence of authority in mathematics classroom discourse, but showed that authority structures were commonly contingent on social positioning, rather than reasoning (Herbel-Eisenmann, Wagner, & Cortes, 2010). Authority was placed unquestioned in the teacher and in accepted mathematical procedures.

In most research on teaching and teacher education that addresses authority, it is not a central object of inquiry. The few theorizations of authority do not provide insight into how mathematics teachers think about it. Amit and Fried (2005) provided the most substantive work on authority in mathematics education contexts, with theorization that is substantiated with some interviews with students but not with teachers. Knowing how mathematics teachers think about authority is imperative to understanding issues of authority and agency in mathematics classrooms, but we have not found research that focuses on teachers' conceptualization of authority.

In this paper we describe a tool that we have developed to initiate dialogue with mathematics teachers about authority in their classrooms – using a diagram to represent how they think authority works in their classrooms. Our analysis of the diagrams teachers created and discussed in our work together helps answer our question: *How do mathematics teachers think about authority in their classrooms?*

AUTHORITY IN CLASSROOMS

Authority is one of many resources teachers employ for control and has been defined in an educational context as "a social relationship in which some people are granted the legitimacy to lead and others agree to follow" (Pace & Hemmings, 2007, p. 6). This relationship is highly negotiable. Students rely on a web of authority relations with friends and family members as well as with the teacher (Amit & Fried, 2005). Although Pace and Hemmings and Amit and Fried consider authority to being negotiated between people, tools and artefacts can also be considered authorities, especially in the context of schooling. For example, textbooks play a prominent role in what and how content is taught, especially in mathematics and science classrooms. Additionally, other tools (like graphing calculators) have been shown to be resources students rely on as they consider the correctness of their work. Nevertheless, the role of these resources in the classroom is mediated by teachers (e.g., Herbel-Eisenmann's (2009) analysis of the use of mathematics textbooks in classrooms).

We see the idea of positioning as being important to work on authority because it recognizes that relationships necessarily involve issues of control, authority, and power. These issues appear at many levels, including interactions within a classroom (in one-on-one interaction, small groups and whole-class interaction) and between people in the class and stakeholders outside of it. Harré and van Langenhove (1999) described positioning as the ways in which people use action and speech to arrange social structures. In their theorization of positioning, they showed how clues in word choice or associated actions evoke images of known storylines and positions within that story. For example, a teacher may see herself as a coach while the student sees her as a drill sergeant. Such multiplicity of possible storylines demonstrates that various authority relationships may be envisioned in any particular situation.

Distinction between the actual positioning or authority structures and what people say about these power relations is significant. Using interviews with mathematics students, Amit and Fried (2005) claimed to look at the way authority actually is in classrooms, but one might argue that they were only looking at what students said about authority. Authority is only a conceptualization, however, so like with positioning theory, there is no empirical authority relationship. There are only people's perceptions or attributions of authority. Thus any account of authority is contestable.

METHOD

The primary data in this article are diagrams generated by teachers to describe the way they see authority at work in their classrooms. These data were supplemented by transcripts of our recorded dialogue with teachers, as they described their diagrams and asked questions of each other. The contexts for these dialogues varied, though they are set in research studies oriented primarily around professional development for participant teachers. They were all set in different cities in Eastern Canada. Because of the professional development focus in these contexts, we focused on prompting reflection and discussion among participants, which proved useful for our research question too. In particular we add focus to our research question by asking:

1) What sources of authority do teachers represent in diagrams depicting classroom authority? 2) How do they conceptualize the relationship among these sources?

The first context in which we had teachers draw diagrams was at the outset of our study engaging secondary mathematics teachers in conversation about authority structures in their classrooms. We interviewed each teacher and asked him/her to describe his/her view of authority in his/her classroom. Preceding the instructions for drawing the diagrams, we asked the following questions: 1) What or whom do your students see as authorities in your classroom? 2) How do your students know something is right in mathematics? 3) How do your students know what to do in mathematics? 4) How do you, as a teacher, know what is right and what to do in mathematics? After listening to the teachers' answers to these questions, we drew for each teacher a thick dot on a blank paper or blackboard and said, "This dot is you." We then invited them to use symbols, lines, words, or whatever they needed to show how authority works in their classroom to complete the diagram. As the teachers were drawing, they talked about what they were drawing, and we asked them questions like "What does this arrow mean?" or "Why did you use a dotted line to connect those parts instead of something else?" (We reported on the connections between their diagrams and their unique contexts in Herbel-Eisenmann and Wagner (2009)). The diversity in their diagrams and descriptions prompted us to use this line of questioning in our other interactions with teachers and pre-service teachers, both in and outside of research contexts.

The second research context was a two-day professional development for grades 6-9 mathematics teachers. The session focused on discourse in mathematics classrooms and was led by six mathematics education researchers. The context was different from our first context because the teachers were not alone when making their diagrams. They did not talk about their answers to the four questions about authority, but instead were given time to make notes and reflect on those questions. When drawing their diagrams, they could probably see other's diagrams in their periphery but they did not talk during that time. When the diagrams were complete, they in turn described their diagrams to the group. For each description, the group was invited to ask questions and make comments. Questions of clarification were encouraged – "Why did you … in your diagram?" The third research context was another two-day session in a different city, modelled on the two-day session described above.

Our analysis began with our identifying and categorizing depicted 'sources' of authority in the diagrams. Whether or not teachers talked about these 'sources' as showing authority, we call them 'sources' because they are potential sources. Even a passive person might be called a source of authority because one's acquiescence to another person in a relationship is part of what gives that other person authority. Items we do not take as sources of authority include the arrows and lines, which we took as representing connections between the sources of authority in the diagrams (though we recognize that this exclusion could be problematic).

Our interpretation of what the symbols were supposed to represent and how teachers thought about these objects is informed by the teachers' accompanying discussion about their diagrams. Connecting this commentary with the diagrams also highlighted the extensive choices teachers made when drawing, all of which relate to the teachers' conceptions of the sources of authority. Even our choice to represent a person as a dot (and teachers' choice to follow suit, using dots to represent people) is a significant choice that relates to a complex array of alternative possibilities in relation to the teacher's experiences, though the dot itself appears very simple.

We have been asked why we asked teachers to represent themselves with a dot. We could have left it open as to how or where they represent themselves in their diagrams. In our view, using a non-dimensional dot to represent the self helps focus the diagrams on the relationships and interaction more than on personal identity. If a teacher was to consider how she would represent herself, we would expect the focus of her attention in the exercise to be significantly different. With a non-dimensional dot, her representations of herself would more likely be in terms of her relationships with others, rather than on identity markers (e.g. gender, clothing).

After looking at the objects in the diagrams we focused our attention on the symbols used to indicate relationship among the objects. For example, many teachers used arrows to connect objects. The placement and direction of such arrows suggests a teacher's sense of which objects connect and of how they connect.

It appeared to us that most teachers drew their sources of authority first and then connected them, but we believe that some of the teachers began their drawing with symbols suggesting relationships and added in sources afterwards. Nevertheless, our interaction with the teachers facilitated our understanding of what in their diagrams were of primary importance to them and what their symbols meant to them.

TEACHERS' VIEWS ON AUTHORITY: SOURCES OF AUTHORITY

We begin our findings with a focus on differences in who and what teachers included in their diagrams, which we refer to as sources of authority. Table 1 lists the items shown in the 34 diagrams with their frequencies (how many different diagrams they appear in). We see more diversity of sources of authority in these diagrams than what we have seen reported in the literature. For example, we have seen self, students, families, and peers as sources of authority in the literature but have not seen professional teaching organizations cited as authority sources in classrooms. Additionally, the category we label as "processes/actions" encompass a range of sources that we have rarely seen in work on authority. Because there is such an extensive list in each column, we highlight a few ideas about each of the columns rather than discuss each item teachers included.

Following our instructions to the teachers, every diagram included the self, the dot in the centre. Our choice to specify that the teacher be represented as a dot likely influenced the teachers' choices of how to represent others in their diagrams – for example, many teachers used dots to represent students and other people as well.

Some teachers used symbols other than dots to represent individuals. Perhaps this choice suggests that teachers wanted to say something about different identities of students. (They could use dots for each person if they wanted to focus on the interactions.) Some of these teachers talked about wanting to indicate that students made choices in their classrooms. In these cases, the different depictions of students might say less about individual student identities and more about the teacher's understanding of the choices a student could make on any particular day.

people	processes/actions	classroom objects	disciplinary artefacts
self (34)	questioning (4)	textbooks (10)	math curriculum (10)
students (22)	communicating (3)	black/whiteboard (7)	prior skills (4)
family (7)	discussing (3)	calculator (4)	ideas (3)
other teachers (4)	directing (2)	computer (3)	prior problems (3)
administration (3)	giving feedback (2)	desks (3)	expected answers (2)
department of education (2)	answering (1)	manipulatives (3)	classroom rules (1)
professional learning community (2)	confirming (1)	books (2)	daily routine (1)
church (1)	comparing (1)	materials (2)	methods (1)
groups (1)	discovering (1)	posters (2)	prior experiences (2)
NCTM (1)	disrupting (1)	resources (2)	questions (1)
school board (1)	estimating (1)	ruler (2)	roles (1)
sport teams (1)	focusing (1)	handouts (1)	tests/exams (1)
tutor (1)	guiding (1)	paper (1)	"There is no math god" (1)
vague others (1)	instructing (1)	technology (1)	topic (1)
	investigating (1)		
	justifying (1)		
	memorizing (1)		
	planning (1)		
	positioning (1)		
	practicing (1)		
	prompting (1)		
	questioning (1)		
	raising hand (1)		
	understanding (1)		

Table 1: items represented in 34 authority diagrams and their frequencies

Even among the diagrams that use dots for people, there are distinctions between kinds of dots, as we showed in our early reporting on authority diagrams (Herbel-Eisenmann & Wagner, 2009). Dawn and Jill[2], whose diagrams are given in that reporting, used open and closed dots to distinguish between kinds of people. Dawn also used an x to represent the student as distinct from teachers who were dots. Mark, whose diagram is also discussed in our previous reporting, used sizes of dots to represent the relative weight of authority ascribed to individuals. Not all students were depicted as equal, demonstrating an awareness of complex differences in relationships – there are not only teacher-student relationships but there are many kinds of student-student authority relationships.

Other teachers distinguished between students in other ways. For example, Dallas (Figure 1) used rectangles to represent his students (perhaps these are desks with metonymic connections to the students), and showed some as having questions and others as not engaging. Another teacher used a dot to represent a group of students instead of a single student. We wonder what this view of students as collectives instead of as individuals means for his teaching.

Thirteen teachers focused on actions or processes (column 2 in Table 1) by using symbols and words (which are also symbols) to explicate what happens in the interactions among the sources of authority. These thirteen teachers came up with 24 different processes to describe the nature of the interaction. This diversity indicates that teachers' views on authority differ significantly and that one could expect other teachers making diagrams to include more processes yet. Sometimes teachers sitting close together when drawing seemed to borrow ideas from each other, but the diversity of responses is evidence that the teachers had significantly different points of view and the desire to express these views.

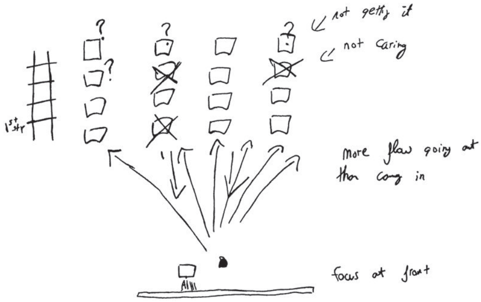

Figure 1: Dallas' authority diagram

Manipulatives, which are concrete objects used to help model mathematical ideas (column 3) appeared in a number of diagrams as sources of authority and garnered some discussion when teachers presented their diagrams to each other. When asked to say more about manipulatives as an authority in her diagram, Rochelle said,

> We're doing algebra now so they need the tiles on the desk with them. If they're not sure if they got it right they can go to their tiles and they can use them to play around, and make sure it [the answer] works. And if your tiles don't give you the same answer, well, that is giving you feedback—to see, to see if they have the right answer.

Manipulatives are inanimate objects, but Rochelle attributed authority to them, much like looking up answers in the back of a textbook or checking one's work on a calculator. Students can "go to the manipulatives" to test their ideas. However, the manipulatives are controlled by the student and are used as a tool. It is interesting to us that Rochelle and two other teachers (including Louise, see Figure 2 in the next section) represent manipulatives as separate from the self though they are extensions of the self. This split is reminiscent of the split self described by Rotman (2008). He described how people doing mathematics embody different roles within themselves; the 'thinker' instructs the 'scribbler' what actions to do to inform further thinking. A student's work with manipulatives or with a calculator is an example of the scribbling (or rote action) performed to explore one's thoughts, which seem to operate separately in a conversation within oneself. Similarly, the 'ideas', depicted by three teachers, may reflect the 'thinker', the other side of the self, described by Rotman.

Of the disciplinary artefacts (column 4) teachers referenced, not surprisingly, the mathematics curriculum was the most prevalent source mentioned. The suggestion that "prior" skills, problems, and expectations might be sources of authority was interesting because these items suggest attention to what has previously been described in the literature as "common knowledge" (Edwards & Mercer, 1987) in classrooms. Other disciplinary artefacts appeared as part of the set of the social norms that guided the classroom work such as rules, routines, and roles.

TEACHERS' VIEWS ON AUTHORITY: POSITIONING OF SOURCES

In addition to the various depictions of sources of authority in the diagrams, we found variation in how the sources were arranged. For example, seven of the 34 teachers arranged their diagrams to depict the physical arrangements of their classrooms. For instance, Dallas' diagram (Figure 1) shows the arrangement of desks with himself and the blackboard in front. Louise (Figure 2), one of the three teachers who depicted manipulatives as sources of authority, showed how the desks were arranged in groups in her classroom and showed one wall with some postings on it and a computer centre up against it. She positioned herself in the centre of the class. The arrows, however, are relatively figurative. She talked about herself at the centre moving outwards: "the teacher is in the centre and circulates to as many students as possible." She also said that students made choices about how and where to work on their problems thus there are different kinds of arrangements of her students, but these differences depict structures of their interaction (or social positioning) more than physical positioning.

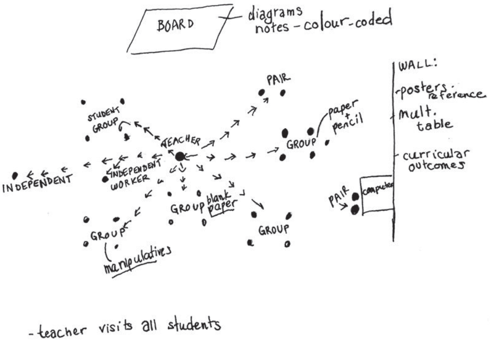

Figure 2: Louise's authority diagram

By contrast, Mark did not use arrows in his diagram. He did, however, use lines, which he said indicated his movement throughout the class. He talked about balance without mentioning direction (no beginning and no end): "I'm really all over the place." Mark and some other teachers had their dot at the front of the room in their diagrams, and talked about their position at the front of the room. Though they did not say that this was significant in terms of how authority works in mathematics, we suggest that this physical positioning, who stands in front of the others, has a powerful impact on social positioning.

Most significantly, 27 of the 34 teachers did *not* organize their diagrams to reflect the arrangement of their classrooms. They saw their diagrams more metaphorically. In most diagrams, the teachers used lines, arrows and other symbols to help represent the metaphoric relationships among the objects in their diagrams.

We believe that the metaphors used to connect the sources of authority are as important as the sources themselves. Of the 34 authority diagrams, 24 had arrows connecting people to other sources of authority but there were differences among the arrows. For example, Dawn said her arrows represented someone looking to an authority, and Jill used arrows to indicate the flow of authority in communications, thus Dawn's arrows and Jill's appear to be in opposite directions. Also, Louise (Figure 2) used broken arrows, which she did not explain in discussion. These differences suggest different ways of thinking about authority.

Other metaphors appeared as well. Dallas (Figure 1) depicted a ladder and described a relationship that had him making decisions about what students should do. With this description, Dallas garnered sympathy from at least one other teacher who helped him finish his sentences:

> Wagner: What's the railway track?
>
> Dallas: Oh that's actually a ladder. I have some kids that don't get math. They just shut down when I ask them a question, but if I can say, "Okay, but you know this because we talked about it the other day. You know this, you know that." So then we go up another step and they do good. But it's me trying to break it up evenly. So that's the strategy.
>
> Woman: That's how they know what to do. You lead them through it.
>
> Dallas: Yeah, with each individual one it's—
>
> Woman: It's exhausting.
>
> Dallas: Yeah, that's the word.

Jean's diagram (Figure 3) employed another metaphor. The mirror and window may not look very central to the diagram, but they were the first aspects of the diagram about which she talked. Before describing the images that represented what she called 'influences' on students, she said, "I thought it was important that kids have a window to see forward and also a mirror to see a reflection of themselves, and that it was important to reflect." She did not say what artefacts students might use as prompts for reflection. Perhaps they used their manipulatives, as described by other teachers, or perhaps they relied on memories. Nevertheless, her comments remind us of Skovsmose's (2005) suggestion that both background and foreground are important for working with students. Jean's metaphor also reminds us of Gutiérrez (2011), who also used a mirror/window metaphor to say that students should be able to use mathematics to look out at the world but also to see and recognize themselves.

Yet another metaphor was described by Joanne (Figure 4). She saw authority as something that can be passed from one person to another but in the short time given to think about this she was still seeking imagery that recognized the fluidity of relationships and authority.

> If I'm in the centre, I drew a spiral, meaning that I'm everywhere in the classroom. I looked at it as communication. And authority is kind of passed on to the students through

> giving them, I guess, the understanding or the ability to problem solve and to work through and to help each other. So it leaves me anywhere along here you can find students. And there's arrows going every which way so it's been passed between the students and between myself. And I talked about the ripple effect. Even though the authority might be given to me to teach these students then ultimately I'm passing it on to them to conduct their own learning. […] I drew those [radial lines] kind of like a radar […]. It's kind of pulsing out, [like a] spider web, interconnected. I was going to have the web and the radar and the ripple effect all happening at once. The important thing is I'm not anywhere per se and the students aren't in any exact position. It's all transient.

Oyler (1996) challenged possession metaphors for authority. Such metaphors suggest authority is like a finite resource and that one person's increased authority implies someone else's loss. Joanne's description demonstrates for us her struggle to conceptualize authority differently from the dominant possession metaphor.

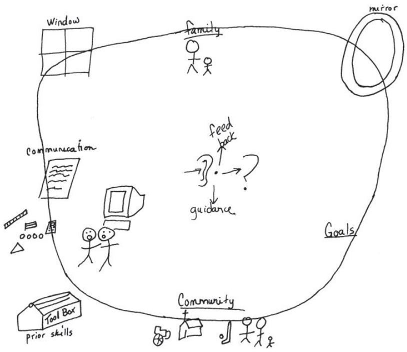

Figure 3: Jean's authority diagram

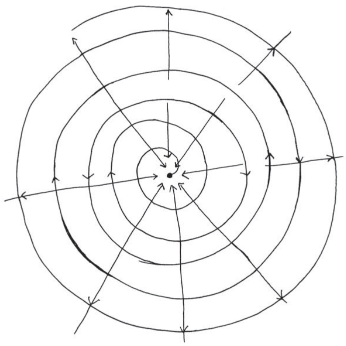

Figure 4: Joanne's authority diagram

IMPLICATIONS

The diverse representations of authority generated in the diagrams illuminate various perspectives teachers work from when thinking about the authority in their classrooms. This reminds us that any account of the way authority works in a particular situation is contestable. There can be diverse accounts and there can be no authoritative account of the authority working in a particular context.

It is also clear from the diversity in the diagrams that scholarship has not yet exhausted the useful ways of conceptualizing authority. Some of the images being used by the teachers to represent authority were unlike representations we have read in the literature. Thus there is potential to investigate these and other representations and images, and what they mean to teachers. We note that the teachers in our research work in the same geographic region, and thus we wonder whether there would be even greater diversity represented by teachers coming from more diverse cultures.

NOTES

1. This research was supported by the National Science Foundation (Grant No. 0347906) and the Social Sciences and Humanities Research Council ("Positioning and Authority in Mathematics Classrooms"). Opinions, findings, conclusions and recommendations do not necessarily reflect the views of the granting bodies.
2. All names of teachers are pseudonyms.

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MATHS IS ACTUALLY EVERYONE'S PROBLEM: SOCIOCULTURAL FACTORS EFFECTING THE (RE)LEARNING OF MATHEMATICS IN A FOUNDATIONAL MODULE

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This paper describes three cycles of reflective practice in a foundational mathematics module required for future teachers who had not passed Grade 12 Mathematics. Sociocultural factors that might have contributed to their lack of success at school mathematics and potentially impede progress in this module are identified and discussed. Evidence of the influence of sociocultural factors on the learning in this module is then extracted from the extensive data collected over three years. Language issues were found to impede mathematical learning for second language English speakers but the other factors occurred across all students supporting the student statement that in this module cohorts "mathematics is actually everyone's problem

INTRODUCTION AND BACKGROUND

The legacy of apartheid and post-apartheid schooling in poor townships and rural areas is very visible in the poor mathematical skills of many students entering tertiary education. A national and institutional requirement for future teachers is "sufficient numeracy" which is taken to mean a Grade 12 pass in Mathematics. Without this, they are required to pass a foundational mathematics module, Mathematical Literacy for Educators (MLE), in order to qualify as teachers. My work as the curriculum developer and teacher-researcher of this foundational module forms the background to this paper. I expected a group of students with one thing in common - Maths had not been a great success for them. The diversity of the group in terms of gender and race was not unexpected, but I was surprised at the diversity in age. As the lecturer it was both my professional duty and my personal mission to make the mathematics accessible to them, and to convince them that as adults they would be able to cope with the basic mathematics required in this module. As the module unfolded, and over the years it became clear that this was also an issue of redress for the very inadequate schooling most had received, and that with strong advocacy, funding could be found to provide the necessary academic and moral support. In order to enable learning, I needed to draw on my experience, the literature and later the mathematics autobiographies of the students themselves to identify factors that could make a difference to the way in which the mathematics was learnt. I will briefly outline the interventions and then discuss the role of language, past schooling experience, gender and culture in the learning in this module.

FACTORS AFFECTING LEARNING MATHEMATICS

Mathematics learning is clearly linked to many cognitive factors but in this paper I am focussing on the personal, social, cultural and schooling factors that influence mathematics learning in terms of actual mathematics performance and in shaping the affective response to learning mathematics.

Affective factors

Affect comprises beliefs, attitudes, and emotions; these three areas are ranked in order of increasing intensity and decreasing stability. Evans (2000) reports "a substantial measure of agreement about the affective variables that might be expected to influence thinking and performance in mathematics in older students and adults" (p. 44). These are mathematics anxiety (the emotional response to mathematics), confidence, with the associated ideas of self-efficacy and locus of control (the attitudinal aspect of affect) and three variables, perceived usefulness of mathematics, perceived difficulty of mathematics and finding mathematics interesting and/or enjoyable, which together comprise beliefs about the nature of mathematics. Affective factors that influence further study of mathematics are thought to have their roots in schools, and more particularly in interactions with teachers. Ellsworth and Buss (2000) found that a major influence on attitudes of students were the comments (supportive or demeaning) made by teachers to students. This view is supported by Benn (1997) who remarks that many students, often twenty years or more later, recall the negative comments made by their teachers, and by Ashcraft (2002) who suggests that risk factors for Mathematics Anxiety include exposure to teachers who were impatient with errors and held learners accountable for their lack of understanding and subjected them to public displays of their incompetence. This view is supported by Michael Goldenberg who, in response to the online version of Sparks (2011), wrote "Math anxiety is not something people are born with: they catch it from others. However, there are carriers who are not themselves suffering from the disease. Contemptuousness from mathematics teachers can readily drive someone into math anxiety, I strongly suspect". Mathematics anxiety has consequences for mathematics performance. Tobias (1993, p. 100) claims that "negative attitudes… can powerfully inhibit intellect and curiosity and can keep us from learning what is well within our power to understand". The presence of maths anxiety is thought to interfere with the working memory – as noted by Sparks (2011), mathematically anxious people use up the brain power needed to solve the mathematics problem on worrying. Stress can result when the culture of the teaching situation differs from the students' culture. Stiff (1990) contrasts the culture of traditional mathematics classrooms in the USA that includes working independently and responding in an orderly and structured manner in classroom situations with the culture of African American students that includes working in support groups and valuing the personal relationships that can be nurtured using a "conversational style" discourse and perhaps leaving one's seat to answer a question. The tension that is sure to result from these different cultures sends the message to the African American students: "You are not the type of mathematics student we want" (Stiff, 1990, p. 156).

Language factors

After surveying the literature, Posel and Zeller (2011) conclude that most studies "suggest that language proficiency and literacy skills, particularly in English are low among South Africans" (p. 117). It is not surprising then that many African learners enter tertiary institutions without adequate English proficiency (Weideman & Van Rensburg, cited in Posel & Zeller, 2011). Language difficulties in mathematics learning include adapting to instruction exclusively in English, reading academic texts and course notes, and dealing with mathematical language.

After extensive analysis of the South African dataset arising from the Third International Mathematics and Science Study - Repeat (TIMSS_R), Howie (2002) claimed that "Pupils who spoke either English or Afrikaans at home achieved higher scores than those who did not" (p. 258). Given that the language of learning and teaching for the pupils tested would have been, according to South African government policy, either English or Afrikaans, this finding can be understood as indicating that those whose schooling was in their home language did better on the mathematics tests than those pupils whose schooling was in their second language. Setati (2005) concedes that the statistics indicate this, but cautions that the explanation is more complex than the fact that learners are taught and assessed in their second language. "The major disadvantage is not that they are not fluent in the language of learning and teaching, but it is the in the fact that learning mathematics in their main language would create opportunities for them to engage in a range of mathematical discourses" (Setati, 2005, p. 103). It is common practice in South African schools for teachers to code switch in mathematics lessons, interspersing the English instruction with explanations in the vernacular.

General reading skills which facilitate correct interpretation of print based materials such as lecture notes, assignments and test instructions, are an important language competence. Bohlman and Pretorius (2002), reporting on their research among students in a Mathematics Access module at the University of South Africa, claim a robust relationship between reading ability and academic performance in mathematics. Their results suggest that "while high reading scores do not guarantee mathematical success, a low reading score does limit mathematical achievement. In other words, poor reading ability… seems to function as a barrier to effective mathematical performance" (Bohlman & Pretorius, 2002, p. 201).

Language issues in learning mathematics are compounded when the difficulty in distinguishing between mathematical language and ordinary language is overlaid with the difficulty in interpreting the nuances of the language of instruction. Rangecroft (2002) speaks of the complexity of dealing with words which have different meanings in Ordinary English (OE) and Mathematical English (ME). For example, the word *range* in OE could refer to a series of mountains or a collection of the new season's fashion, in ME the range is a set of values that the *y*-values in a function may assume or the difference between highest and lowest scores in a dataset. "In an

English-medium classroom of multilingual learners who do not speak English as a first language, the confusion between OE and ME is complicated by the fact that both languages (OE and ME) are new to the learners" (Setati, 2005, p. 81).

Gender and age factors

The TIMSS 2003 study reported that, in line with the international results, the difference in the South African national average mathematics scale scores for girls and boys was not statistically significant. In most countries, including South Africa, there were equitable participation rates in mathematics classes between girls and boys (Reddy, 2006). It seems then, at school level at least, gender is becoming a decreasingly significant factor in the achievement and participation of girls in mathematics. Affective factors, however, are still thought to be important. "Boys and girls differ in how they explain to themselves (and to the researchers who interview them) both their current and past successes and failures in mathematics" (Tobias, 1993, p. 80). Girls typically attribute their success to effort, luck or a helpful environment, and their failure to a lack of ability. In contrast, boys attribute their success to their ability, and their failure to insufficient effort. This matters, since while on the one hand, such attributions erode the confidence of girls who become fearful that their effort will be insufficient to master more advanced mathematics, on the other hand, those who attribute success to ability have "every reason to expect success in the future, because ability will remain relatively constant" (Meyer & Koehler, 1990, p. 66). A more recent large scale study in Germany found that even when controlling for academic achievement, girls reported significantly more anxiety, hopeless and shame (Frenzel, Pekrun, & Goetz, 2007).

There is extensive literature on adults learning mathematics (see for example Benn, 1997; Duffin & Simpson, 2000; Evans, 2000) often in the context of "second chance" courses and this is very pertinent to the MLE module where first, the content of the work (very basic mathematics) is mismatched to the age and university level of the student, and second, many of the students were older than the school leavers one might expect in a first year module and hence are more likely to have family responsibilities. Duffin and Simpson (2000) suggest that the aim should be to move adult learners to see maths as a goal (a state the learner wants to be in, and through their actions tries to approach) rather than an anti-goal (a state the learner wishes to avoid, and through their actions, tries to move away from). Finally, they warn that: "The fact of that failure is often, of itself, sufficient to have made mathematical situations anti-goals for the learners involved and this brings with it emotional indicators which can prevent an otherwise intelligent adult from attempting any form of mathematical task" (p. 97).

MODULE PEDAGOGY AND DATA COLLECTION

The three cohorts of students in the MLE module (N=254) formed a very diverse group aged from 17 to 45, representing all race groups in South Africa, both genders and coming from schools ranging from exclusive private boarding schools to the poorest rural schools. What they had in common was an unsuccessful mathematics background. The gender distribution within the three cohorts was more or less constant with about 60% of the students in each cycle of the MLE module being female. The racial composition of the student cohort in the MLE module, although representing all race groups, was largely African (75%), with 10% White students, 9% Asian students and 6% Coloured students. Due to the mature age of many students, the racial classification is significant since all, or at least most of, their schooling was in pre-democracy days when some groups suffered significant disadvantage. Somewhat less than half (45%) of the students indicated that their schools were poorly resourced, 30% considered their school adequately resourced, 18% attended either ex-Model C schools or Private schools, and 8% were unspecified. About 25% of the students were over 25 years old in their first year of study.

The detail of the interventions and data collection for the first three cohorts can be found in Hobden (2007) and what follows here is a brief summary to provide the context for the discussion which follows. The first year of the module could be described as exploratory with fairly routine teaching with ad hoc efforts at support and encouragement. The second cycle was more innovative in that I integrated a course in teaching methods for senior students with practical work in providing tutorial support for the MLE students. In practice, this led to four subgroups within the large cohort each with a senior student leader. The third year saw the introduction of self-help resources in the form of a CD containing the notes, past question papers, some downloaded freeware basic arithmetic practice exercises and links to useful internet sites. Key lectures were videotaped and the CDs of the lectures were made available for students to borrow, and multiple copies of basic mathematics books, suitable for adults were purchased and made available in the library. The printed notes provided to students were annotated with links to all these resources.

Each year, the students filled in biographical questionnaires, wrote their mathematics autobiographies and completed pre-and post-module questionnaires. Thirty eight students accepted the invitation to be interviewed about their experiences in the module. Additional data was collected from the tutors involved with the module over the three years.

FINDINGS

In this section I will present claims based on the evidence of the differentiating role played by personal factors such as age, home language, schooling experience, culture and gender in the engagement with the MLE foundational module. van Groenestijn notes that such factors are intertwined and sometimes difficult to separate, contending that "everybody carries a backpack filled with a mix of real-life experiences and school knowledge and skills, built on a variety of language, mathematical, cultural, social and emotional aspects… These aspects should not be seen as loose elements but as an entity. They can be distinguished but not separated from each other" (cited in Kaye, 2002, Mieke van Groenestijn section, para. 3).

Bad mathematical experiences at school were carried into tertiary studies

My experience in teaching the MLE module indicated that many students entered with extremely limited mathematical proficiency and a crippling lack of a confidence and disposition to learn mathematics. Some students seem to have started out apprehensively and in the full expectation of failure, following the trend noted by Ingleton and O'Reagan (2002) for the successes and failures of primary school, with their effect on self-identity and self-esteem to be reinforced even into the years of tertiary learning. It does not seem possible for them to "just get over it". This phenomena is well known and has been described by Tobias (1993) in terms of a "dropped stitch". Maths anxious people recall incidents and people who negatively affected their learning during their schooling, but cannot explain why, in later years, they have not returned to pick up where they left off. The mathematics autobiographies written by the students at the beginning of the module revealed many bad school experiences that left students with negative attitudes to mathematics (Hobden & Mitchell, 2011). For example, a student wrote [1]: *My math teacher could actually tell us that he doesn't care... I tried to cope with the situation until I couldn't get more than 20% correct. After that I said to hell with maths.* The apprehension felt at starting the module was expressed by a student who said *when they said I must do Maths Literacy, eish! I was very worried, I was like 'Oh this thing I've tried to go away with it it's still following me now*'. Race and gender were not significant differentiating factors in this regard – as the title of this paper suggests – maths was everybody's problem.

English language competence impeded progress in learning mathematics for those who spoke English as a second language

While reading ability was not tested in this study, there was ample evidence of misinterpretation of words in the questionnaires used as instruments. For example, a small group of African students, in response to the question "What would you identify as <u>gaps</u> in your mathematical knowledge and skills?" wrote *You can also identify <u>gap</u> as a space between words or something…* and the following comment, presumably a misinterpretation of the item related to the <u>pacing</u> of the lectures: *I think the lectures are <u>spaced</u> apart correctly except for Friday I do not think that it is the right thing to learn maths in the afternoons.* It was noticeable that some of the African students were very slow readers, sometimes seeming to be reading word by word which is a big disadvantage in timed tests. An African student explained the mathematical struggles of her peers saying *they feel difficulties because the way they are taught in school… they are taught English in Zulu, other subjects in Zulu you know.* This was supported by another student explaining why he was finding the module work difficult: *As a person from a rural area, Maths was not difficult to me* (at school) *because it was taught in first language and then sometimes explained into*

Zulu language. The tutors noted that students were recreating the code switching environment familiar to them from the bilingual classrooms of their school days by forming informal study groups where the work could be discussed in their home language. Most of the lectures were delivered by myself, a native English speaker with an accent and turn of phrase that could take isiZulu speakers a while to grow accustomed to. For example, when asked in an interview if he could understand the language in the lectures, a student replied in the affirmative, and then paused and continued *but not as from the word go. I started understanding at the end of the semester.* Although the quantitative parts of the module evaluation indicated mean agreement that the language of the lectures were understood, the open responses hinted at language difficulties. Examples included; [I had] *no experiences in mathematics but I can manage if I can put more pressure in my English improvement* and from another student*: English is not our language… I couldn't understand some of the terms and some of the handouts written in English... I used to try and write it in my language and understand it here and there and see what is being asked or what the section is talking about.* One student spoke of another aspect of language difficulty, that is the Mathematical English that is interspersed in mathematics classrooms with the Ordinary English in everyday use. *It* (the language of the MLE module) *was a big problem for me. Sometimes I could take a dictionary and look for a word, for the meaning of the word. Sometimes I couldn't even understand even when I looked up the meaning and I have to ask someone to explain it to me.* She went on to cite as examples *possibilities, probabilities and all those words. They were so difficult for me to understand.* The mean scores for agreement with the post-module questionnaire statement "I was able to cope with the language, pace and level of difficulty in the lectures" were disaggregated by race. The African students' mean score was calculated to be 3.6 (slight disagreement) compared to the mean of 4.2 (moderately strong agreement) for the white students. This statistically significant result points to the self-reported problems African students had with the pace, level of difficulty and language of the lectures. The language factor clearly differentiated between race groups and made the task of learning in the MLE module especially difficult for African students.

Gender differences were slight and predicted by the literature

Commonalities between the male and female students predominated, but there were some quantitative results showing gender differences, as well as anecdotal observations. The only statistical difference found in the pre-module questionnaire was in the attribution of mathematics ability. The literature indicates that boys tend to ascribe their success to ability whereas girls tend to ascribe their success to effort or outside help. This gendered pattern of attribution was also found in this study where the male students ascribed more influence to the Self factor (a composite of items pertaining to how clever they were born, ability to think logically and the parent's mathematics ability) than did the female students. When the second cohort were asked if they studied alone or with others, 60% of the male students reported studying alone, compared to just 48% of the female students. This seems to conform to the preference noted by Boaler, Brown and Rhodes (2003) for girls to prefer a more social and collaborative style of learning. From personal observation, it was apparent that the female students were more constrained by family responsibilities especially childcare and housework. Two female students, wrote in a refection that they did not have much time to practise due to *house work and baby work* and having *an 8 month old baby to look after*, and a mature student explained how she had to see to four children and sort out the house when she returned from the university. The family responsibility issue was found across all race groups. This is not to say that male students are not constrained by their families at all – one male student explained how he was given charge of the family tuckshop as soon as he got home and was often called to do errands when he was trying to study. Nevertheless, female students fit the profile of adult learners more closely than their male counterparts of the same age who typically stayed in the residences apart from their families.

Cultural differences occurred in interpersonal relationships and study methods

There was no specific mention of racial tensions in any of the student writing or interviews, but some were surprised to see white students in the class and expressed a wish to interact more with other race groups. This is well illustrated by the following extract from an interview: *I thought that there is no white person who has a Mathematics problem. So when we were in Maths literacy I saw there were lots of white people and I thought that if they had the same problem then why not me, and so we came to realise that Maths is actually everyone's problem. I think you should split the people randomly. Get to know each other. Not black together.* There were several instances in the interviews where students alluded to cultural norms to explain some behaviours. For example, the student counselling service offered to students who needed help adjusting to university life was rejected in favour of peer study groups; the student saying *I think one thing that can help them is the workshop not the counselling thing because most of us don't believe in counselling as blacks, you know we don't believe in going there.* A student brought up the issue of cultural differences in the interview when she spoke of the reticence felt by young Zulu people in the questioning of older people: *Ja now… I need to ask some questions when you teaching but ah I'm scared because you know… in our culture it's not easy to give the person if she or he's older than you many questions now I'm scared to ask you 'I don't understand that part'.* When the third cohort was asked how they had prepared for a data handling test, it was clear that only the African students were drawn to study in peer groups, the rest indicating that they studied alone. This could be a reflection of a culture that prefers a more social style of learning as suggested by Stiff (1990), but it should be noted that the majority of African students were in the residence and so the social methods of study were easier for them than for students who travel home and must perforce work alone. However students of all race groups responded very positively to the tutorials and expressed great appreciation and regard for the tutors

CONCLUDING REMARKS

I concur with Benn (1997) that all too often "equity can be seen as pretending not to notice, as removing consideration of gender, class or race on the grounds of equal opportunities. However, if we remove these definitions from our discourse, then we cannot challenge prejudice when it occurs" (p. 137). In this paper I have tried to look for differences to inform my pedagogic practices. Commonalities, mainly the problems with mathematics, outweighed differences.

As the three cohorts of the module unfolded, I tried to be responsive to problems and issues that arose. I organised smaller groups with tutors so that reticent students could ask questions and get more individual help. In response to those who indicated that the pace of lectures was too fast and they battled with the language – I videoed the lectures so they could watch again to check their understanding. Many students had missed out on basics at school, so I secured funding to make a CD of basic arithmetic programmes, and bought a collection of suitable books to consult. As it turned out, the books and CDs were largely ignored and students developed a dependence on the tutors and their more able peers. Noting this, for the fourth cycle of the module, I organised and paid peer tutors for the next year.

Who was in control of their learning? Looking back I am challenged by the thought that through my good intentions to smooth the path before the students, I was directing learning more than encouraging self-regulated learning. Is it presumptuous to decide what teaching and learning strategies are best for other people? How does one personally overcome the notion that one knows best? These are issue that I wish to reflect on and take forward.

NOTES

1. Italicised words are verbatim quotes extracted from student writing or interviews.

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STUDENTS' MATHEMATICAL IDENTITIES IN RELATIONSHIP TO AN INTENSIVE MATHEMATICS SUMMER PROGRAM

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Summer programs can provide support for students of color, who have historically been marginalized by the system of schooling, to be successful in mathematics. Nonetheless, we know little about students' experiences in these opportunities. This paper is based on an ongoing empirical study, which employs a case study approach, to better understand African American students' developing mathematical identity in a STEM focused summer program. The author also investigates aspects of students' mathematics socialization as it unfolds in the summer program. Preliminary results suggest that students showed confidence in their ability to perform well in mathematics, subscribed to the myth of meritocracy, and that peer interaction in the summer program positively affected their mathematical identity.

INTRODUCTION

The in-school experiences of students under the age of 18 have been the primary focus of research in mathematics education in the United States; though the evidence related to these experiences suggests the education system can no longer rely solely on K-12 schools to provide *all* of the necessary tools for students to succeed. Children spend only 20% of their time in school, so it is important that researchers also examine the influence of other environments other than school on their learning (Miller, 2003). Out-of-school experiences can contribute substantively to the learning that occurs in the classroom. In fact, there are out-of-school programs that can help alleviate the stress on teachers during the school year. Although most of the few empirical studies focus on academic achievement, there can be other aspects of out-of-school programs that need attention, such as the dispositions of students in out-of-school contexts.

Research has shown there are benefits of summer programs for students (e.g. Cooper, Nye, Charlton, Lindsey, & Greathouse, 1996), however, the mathematics education field knows little about students' experiences in these out-of-school experiences. Summer programs offer a platform to examine the benefits that these additional opportunities could have on a students' educational experience within traditional classrooms. There are many ways to explore the benefits of summer programs on students' "opportunities to learn", defined here as taking into account "not only students' access to mathematical content and discourse practices but also their access to (positional) identities as knowers and doers of mathematics" (Cobb & Gresalfi, 2006, p. 53). In *Adding It Up,* the National Research Council (2001) also broadened conceptions of student learning arguing that "students' disposition toward mathematics is a major factor in determining their educational success" (2001, p.131). This disposition, which is a necessary component of student achievement, in combination with many other factors, can be a part of a student's mathematical identity. Mathematics education researchers have a growing interest in student identity because the identities that students construct have an impact on their learning (Nasir & Cobb, 2007). With the exception of a few scholars (Berry, 2008; Nasir, 2002; Taylor, 2009), few studies examine students' out-of-school experiences *and* identity. Much of the mathematics education research on identity has focused on the construct of mathematical identity of middle and high school students as it relates to in-school learning. Also, little empirical research exists that describes the impact of these experiences on the disciplinary identities of the participants. This paper offers narratives from students going beyond more typical program descriptions, and focuses on which specific activities in these programs seem to work, for whom, and why.

This on-going practicum study focuses on how four students of color think a month long summer program helps shape or maintain their mathematical identities and other aspects of their in-school experiences. In this short paper, I focus on mathematical identity, which relates to affective constructs, such as students' beliefs, confidence, and attitudes (Fennema & Sherman, 1976). These affective traits are important because they relate to students' persistence or willingness to study mathematics (Boaler & Greeno, 2000; Hembree, 1990; Sherman & Fennema, 1977). This study also investigates aspects of students' *mathematics socialization*. According to Martin (2000), mathematics socialization is the collection of experiences that "facilitate, legitimize, or inhibit meaningful participation in mathematics" (p. 206); this can occur in various contexts, such as peer interaction, school, family, and in this study, in a summer program.

SIGNIFICANCE OF THIS WORK

Students in this summer camp come from different schools and many of these students of color are a minority in their own schools. In the summer program, however, they are the majority. Fordham and Ogbu (1986) propose the framework of *fictive kinship* to help understand how a sense of collective identity enters into the process of schooling and affects academic achievement. Such a summer camp may create a sense of community that can develop a cultural and academic cohort among the students. Although the students' peers have a role in shaping the mathematical identities of these students, the structure and components of the program may also play a part.

Teachers in the summer camp have less to balance than teachers in a school setting; the extra focus on students can contribute to a sense of importance, self-worth, or improved teacher-student relationships. This program is structured so that class sizes are typically smaller than the public school these students attend. Historically in the United States, minority students often suffer from tracking in schools. African Americans especially are unfairly placed in the lower mathematics classes in their schools. Many scholars argue that tracking exacerbates racial and class segregation and disadvantages those students most in need of exposure to rigorous curriculum and high standards (e.g., Archbald & Kelher, 2008). Students in this summer program are not tracked. For example, all rising ninth grade students will be enrolled in Algebra, and some are placed in Geometry or Algebra 2 based on the results of the pre-test given at the beginning of the program.

Because there has been a historic collective struggle within the African American community for educational equity and access as a means to upward mobility (Carter, 2008), we need to address African American students' goal of attaining equity and social mobility. Many African American students graduate with a high school diploma without having taken classes, specifically mathematics classes, which prepare students' to position themselves to do well academically in college. As mentioned earlier, mathematics is not only a part of students' day-to-day functioning, but it provides students access to diverse career choices. The type of summer program that I examine allows African American students to position themselves in a career trajectory of working in the fields of science, technology, engineering, and mathematics (STEM).

RELEVANT LITERATURE

Out-of-school experiences

As Kirkland and Hull (2011) suggest, "such [out-of-school] experiences can complement, extend and diverge from understanding ways of participating around curricula that are used in a traditional school day" (p. 171). Students' experiences in out-of-school contexts can shed light on how students perform in school and in what ways we can enhance or supplement their performance in school. Although examining students' academic performance has been popular, it is a limited way to gauge students' experiences or opportunities to learn. In the past 15 years there has been an abundance of research on out-of-school settings. However, the vast majority of this research has been in the form of evaluation studies, which focus on evaluating the viability of such programs (Kirkland & Hull, 2011). One issue with such work is that the people in charge of the programs sometimes do these evaluation studies, raising conflict of interest issues.

Mathematics education researchers examine identity as one of those aspects that could help researchers, teachers, and students understand students' experience as it relates to mathematics. Research on mathematical identity and identity in mathematics education has been conducted mostly in relationship to the context of *in-school* mathematics experiences (e.g., Martin, 2000; Nasir, 2002; Spencer, 2009). To date, little empirical research in out-of-school experiences, specifically in summer programs, related to mathematical identities has been done. I anticipate that a focus on summer programs can offer additional insights into students' opportunities to learn, beyond what researchers have already learned from studying students' out-of-school experiences.

Identity in Mathematics Education Research

In recent years, investigations focused on identity have become common in mathematics education research (Boaler, 1999; Cobb & Hodge, 2002; Martin 2000; Sfard & Prusak, 2005). Studying identity allows researchers to broaden the view of students' mathematical learning to include examining the ways students think about themselves in relation to the mathematics. Mathematics education researchers have conceptualized identity differently as *narrative* (e.g., Sfard & Prusak, 2005), *positioning* (e.g., Wagner & Herbel-Eisenmann, 2009), and *beliefs* (e.g., Martin, 2000). Conceptualizations of identity offer an understanding of the connection between individuals that seem to be productive for understanding how learners relate to and experience school mathematics (e.g., Sfard & Prusak, 2005; Stentoft & Valero, 2009). For example, Grootenboer and Zevenbergen (2008) defined mathematics identity as a person's "beliefs, attitudes, emotions, and dispositions" (p. 243) about mathematics and a person's mathematical capabilities to do mathematics. Cobb, Gresalfi, and Hodge (2009), in fact, contend that researchers should study "the extent to which students have developed a commitment to, and have come to see value in, mathematics as it is realized in the classroom" (p. 41).

Mathematics education researchers examine identity as one of those aspects that could help researchers, teachers, and students understand students' experiences as it relates to mathematics. Research on mathematics identity and identity in mathematics education has been conducted primarily in relationship to the context of *in-school* mathematics experiences (e.g., Martin, 2000; Nasir, 2002; Spencer, 2009).I now turn my attention to the more particular concept that I focus on in this study – mathematical identity – to describe relevant literature and the particular framework I use in this paper.

Mathematical Identities Framework

Research on students' beliefs has contributed to the development of the construct of mathematical identity. Grootenboer and Zevenbergen (2008), for example, defined mathematical identity as a person's "beliefs, attitudes, emotions, and dispositions" (p. 243) about mathematics and a person's mathematical capabilities to do mathematics. According to Martin (2000), *mathematical identity* refers to the beliefs that individuals develop about their ability to perform or participate in a mathematical context and use mathematics to change their lives. Mathematical identity is a useful idea because it "includes the broader context of the learning environment and all the dimensions of learners' selves that they bring to the classroom" instead of only focusing on students' achievement in mathematics (Grootenboer & Zevenbergen, 2008, p. 243).

Martin (2000) conceptualized and studied the mathematical identities of African American students, focusing on sociocultural factors and how they relate to in-school mathematics experiences. He found that *sociohistorical factors* (historical discrimination regarding policies and practices among African Americans), *community factors* (parent and guardian beliefs about the importance of mathematics), *school factors* (influence of teachers and institutional beliefs) and *individual agency* (students' personal beliefs about their abilities to do mathematics, motivation to learn, and the importance of learning mathematics) were all important to one's developing mathematical identity.

For this paper, I focus specifically on Martin's (2000) concept of *individual agency*, which includes students beliefs about (a) "their ability to perform in mathematical contexts," (b) "the importance of mathematical knowledge," (c) "constraints and opportunities in mathematical contexts," and (d) "the resulting motivations and strategies used to obtain mathematics knowledge" (p. 19). I pay attention to students' individual agency, in particular, because I will only be able to spend time with students during their summer camp and will not be able to explore the broader aspects of mathematical identity in such a short time period and within this more bounded space. If students consistently suggest that these other aspects are salient to their mathematical identity, however, I will not ignore those aspects of their experience.

The first aspect of students' beliefs addresses students' self-esteem with respect to mathematics. People readily articulate whether they are or are not "good at math". Students' beliefs about their mathematical abilities are important because confidence can lead to persistence and effort to be successful. This notion relates to students' confidence to do mathematics. Confidence in one's ability to do mathematics can be important when students struggle with the subject. The second part of Martin's (2000) framework for individual agency is students' beliefs about the importance of mathematics as it relates to their future. Students may imagine a future in which mathematics helps them meet an academic or personal goal (Ekert, 2001). The last aspect of Martin's framework is beliefs about motivations and strategies used to obtain mathematics knowledge. This aspect implies students' form an opinion about what it means to 'obtain' mathematical knowledge and develop the motivations and strategies to acquire various types of knowledge.

Although using Martin's definition of mathematical identity provides a thorough framework to define mathematics identity, his framework was not clearly operationalized in terms of helping another analyst identify these aspects in interviews or other data. Therefore, I will augment his work with Sfard and Prusak's (2005) work to help me to articulate aspects of someone's mathematical identity. By focusing on students' narratives about their mathematical identity, I will be able to capture students' perspectives in relationship to factors that shape their mathematical identity and aspects that aid in contributing or hindering their success in mathematics.

RESEARCH METHODS

This paper will feature qualitative case studies (in progress) of four rising 10th grade African American students (two males and two females) in the United States. Case studies concentrate on a single phenomenon – here, mathematics identities in a summer program – providing an in-depth examination of that phenomenon (Stake, 2004; Merriam, 2002; Yin, 1984). Each single case also builds a unique context about students' experiences attending this similar summer programs. Each case study will examine each student's realities and complexities related to the development of their mathematical identity in this summer program, as well as students' descriptions of its impact on their schooling experience.

This summer program at a university in the Midwest United States offers Latino/a and African American students an intensive 4-week summer program. This program operates on four principles: convivial (a philosophy that focuses on "making people feel welcome"), routine of success (create an environment that communicates the students are important and part of a community), accountability (holding students to academic and behavioural standards set by Science Bound), and share responsibility (the success of the program is directly related to their collective success as a team). This summer program is an opportunity for these students to see their possibilities, beyond their current academic and socio-economic situations. So, when they return to a regular classroom routine they have strategies and dispositions to enhance their academic and social abilities.

This program essentially covers three-quarters of the year's worth of Algebra in four weeks. Students are exposed to topics they will see in the upcoming school year. The books are similar to the textbooks used by their school districts, so they are learning the same material they will see in the following school year. Research also indicates that students in the program benefit by developing more positive attitudes toward science and improve their academic achievement. In the past seven years, the program has seen 98% of the students attend college, 80% attend a four-year university, and 63% of university-attending students major in a STEM field (Hargrave, 2011).

DATA COLLECTION AND ANALYSIS

The data sources for this study are currently being collected and include weekly audio-journals and interviews (Barone, 2011). The students in this study will be able to share their narratives through their weekly audio journals and weekly follow-up interviews in regards to their audio journal. These data sources will allow students to present themselves in a way that is unique and distinctive as they are having these experiences, even though they may be connecting their current experience to ones from the past. The primary data source for this study will be information that the student provides in semi-structured interviews. I will then review responses to questions asked in relation to the four components of mathematical identity: (a) "their ability to perform in mathematical contexts," (b) "the importance of mathematical knowledge," (c) "constraints and opportunities in mathematical contexts," and (d) "the resulting motivations and strategies used to obtain mathematics knowledge" (Martin, 2000, p. 19). Any responses that reflect similar themes will be grouped together. I will look for dominant themes within and across the interview transcripts. In analyzing these statements, I will pay close attention to repetition, tone and affect as well as how they are telling their stories.

After the program concludes, two months into the school year, I will conduct another 60-minute semi-structured interview. During this interview, I will read each student the finding and ask for comments on the text (e.g., What do you agree or disagree with? Is there anything you would like to clarify? Is there anything you would like to elaborate on?). I will also ask them some follow up questions, based on information they previously mentioned. These questions will not be the same for each student because the case findings will be slightly different. This final interview will allow me to probe student's perspectives on the ways in which they thought their summer experiences might influence their current schooling experience.

PRELIMINARY FINDINGS AND DISCUSSION

On-going analysis of the data sources suggests that the program is helping these students, in multiple ways, with their individual agency. First, it is providing them with information and strategies that can give them the confidence to perform well in their mathematics class. Next, even though the participants recognize the importance of mathematical knowledge, the program helps reiterate this knowledge on a daily basis. Last, the participants continuously refer to the careers in the STEM fields that the program helps them learn more about. It is through these opportunities, that this program helps maintain a positive mathematical identity as it relates to the participants' individual agency.

In the initial interview, the four participants' were very motivated to do well, not only in mathematics, but also in high school, so they can have a successful career. They acknowledged mathematics as a gatekeeper for their career. Although this program may not have been the main reason these participants had a positive mathematical identity, the program continues to play a role in developing and maintaining that identity. For example, one of the participants noted that he was homeless for four years and seeing his father get through that adversity showed him that he could get through everything. In a later interview, the participant said that when he wants to give up in mathematics during the school year, he thinks of how much he learned through the summer and realized he can get through his mathematics class. All of the participants noted that the program director and teachers set high expectations for them through their rules ("on time is late") and using phrases like "without a challenge, there is no change". When students wanted to give up on the mathematics, I often heard that phrase from the teachers and their peers in class. Participants noted these high expectations and support from their peers were lacking in their in-school experiences.

As noted above, these participants entered the initial interview with a positive mathematical identity by being confident in their performance in mathematics, stating that mathematics knowledge was important to possess, and using their future career and a full scholarship from the program as motivation to do well in mathematics. The participants spoke very passionately about being successful and being an example for their families. One participant stated, "I *have* (emphasis added) to pave the way for my younger brother and show him that people in our family can be successful". This participant wants to be an engineer and knows mathematics will play an important role in achieving that goal. When asked how he knows that, he noted that the summer program has a career exploration assignment where they have to explore various careers in the STEM field. Through this assignment, he saw his mathematical identity as a designated identity, he related his narrative to the future, and being successful was significant to him due to him caring about how his brother sees him.

Finally, participants appreciated the determination and hard work of their peers in the program, saying their peers in the program were smart and the participants wanted to make sure they studied hard to remain competitive with their peers. This was not always the case in their respective schools. The participants noted that they still work hard during the school year, but they work even harder during the summer program. All of the participants believed that the harder they work the more successful they will become, otherwise known as the myth of meritocracy, which McIntosh (1990) defines as the myth that democratic choice is equally available to all.

CONCLUSION

The work presented in this short paper can help to better understand the role an intensive summer experience might have in developing students' mathematical identity, especially students who are typically under-represented in STEM careers. This study can help to inform the design of such programs and policies, and funding related to this kind of work. This case study examining students' mathematical identity in a summer program will contribute to a research-based understanding of how out of school experiences can contribute to students' developing mathematical literacy. Martin (2000) noted that it is important for researcher to develop effective strategies for helping communities highlight the importance of mathematics beyond the school context. The Learn and Earn summer program provides an opportunity to engage students in discussions beyond what occurs in school. It is my goal that the strategies used by the program that develop a positive mathematical identity can be adopted by other programs and/or integrated within school contexts. Intensive summer programs are not only useful, but also necessary for improving the mathematical identity of African American students. It is hoped that once the mathematics education field reads about the positive impact this program has on

African American students, then the structures and programmatic design can be adapted and adopted at other institutions that could run programs similar to this one.

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MAINTAINING THE FOCUS ON MATHEMATICS AND THE SOCIAL IN MATHEMATICS EDUCATION RESEARCH

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A concern expressed in mathematics education research is that research methodologies focusing narrowly on mathematics tend to deflect attention away from the social, political and cultural issues pertaining to mathematics education (Gutiérrez, 2010). However, literature attending to socio-political concerns has been inclined to neglect mathematics (Valero & Matos, 2000). In this paper, I explore possible reasons for the tendency to de-emphasise mathematics in socio-political mathematics education research by focusing particularly on research that deploys methodological resources that rely on the distinction between mathematics and the 'everyday'. I illustrate that the tendency for research focusing on socio-political concerns to either ignore or misread mathematics is a consequence of the analytic categories constructed by the methodological resources deployed.

INTRODUCTION

Valero and Matos (2000) express a concern that research attending to socio-political issues has been inclined to neglect mathematics. They claim that the "dilemma of mathematical specificity" or the tendency for mathematics to "vanish or to be questioned" arises when adopting socio-political approaches to mathematics education research (Valero & Matos, 2000, p. 398). Although they argue that the tendency to ignore mathematics results from shifting attention to research objects located outside the "didactic triad" of the teacher, student and mathematics content, they seem to suggest that reasons for the emergence of the "dilemma of mathematical specificity' in mathematics education research requires further investigation.

While socio-political mathematics education research is accused of ignoring mathematics, research focusing too closely on mathematics, as is the case with some research located in the field of psychology of mathematics education, is charged with ignoring social, political and cultural issues (Gutiérrez, 2010). Thus it appears that there is a necessary tension between socio-political concerns on the one hand and mathematics on the other. In other words, that an emphasis on the socio-political necessarily implies a de-emphasis on mathematics and a focus on mathematics, a de-emphasis on the social. This raises the question: does a formal necessity to de-emphasise mathematics when focusing on the social exist?

This paper attempts to explain the de-emphasis or backgrounding of mathematics in some research focusing on socio-political concerns. I set out to demonstrate that the problem identified by Valero and Matos (2000) may be related to the insufficiency of the descriptions of mathematics generated by the methodological resources deployed in some mathematics education research concerned with the social. Such research recruits methodological resources from a range of fields such as psychology, sociology, linguistics, and anthropology. Below I discuss two examples of research, concerned with social class differences in mathematics achievement, which deploy methodological resources that rely on the mathematics versus ‘everyday’ distinction. Firstly I discuss the work of Barry Cooper and his colleagues who locate their work within the sociology of mathematics education. Then I move to focus on the work of Veel (2000) who deploys methodologies recruited from Systemic Functional Linguistics (SFL).

OVER-DETERMINATION BY THE SOCIAL

Barry Cooper and his colleagues conducted a large-scale study in the United Kingdom, investigating the relationship between students’ performances on mathematics test items and their social class membership. The results of the study are published in a number of papers produced by Cooper and his colleagues (see Cooper & Dunne, 2000). Their study found that working-class students performed considerably poorer than their middle-class counterparts on ‘realistic’ test items. A ‘realistic’ item is defined by Cooper and Dunne (2000, p. 84) as an item which “contains either persons or non-mathematical objects from ‘everyday’ settings”.

A test item that involves finding the price of a box of popcorn given two bits of information: (1) a coke and a box of popcorn cost 90p and (2) two cokes and a box of popcorn costs £1.45 is classified as a ‘realistic’ item where according to Cooper and Dunne the boundary between mathematical knowledge and everyday knowledge is weak (Cooper & Dunne, 2000, p. 84-85). Cooper’s categorisation of references to ‘everyday’ objects as non-mathematical is problematic. Cooper’s distinction between mathematical and non-mathematical objects is non-sensical since all the objects referred to in a mathematics problem are mathematical because they connected by mathematical operations and therefore constitute particular mathematical relationships.

Cooper (1998) reports on interviews with two students, Diane (a middle class student) and Mike (a working-class student) as they worked on ‘realistic’ test items. One of the items concerned a sorting task with the following instruction: “The children in Year 6 cleared the rubbish from the sports field. Choose a way of sorting the rubbish and sort it. One has been done for you.” The items (a newspaper, can, mustard jar, fountain pen, milk carton, and soda can) were to be sorted into two groups indicated by circles. In the question, the soda can was linked to one circle or group. Diane’s response to the question was as follows:

> I sorted the things this way for two reasons. 1. The things I put in circle A were all 3d, and the one I put in circle B was 2d. 2. The things I put in circle A were all containers of some sort. The one I put in circle B (the newspaper), was not. (Cooper, 1998, p. 519)

Mike’s response to the question was as follows:

> The children in year 6 … [He continues reading silently. He writes the words meatle [sic] and glass in the left-hand circle, and the words paper and card in the right-hand circle. He then draws lines linking the mustard pot, pen, and soda can to the LH circle, and the newspaper and milk carton to the RH circle, while muttering :] Metal. And glass. Metal and glass. (Cooper, 1998, p. 519)

Cooper (1998, p. 20) concluded that the middle-class student (Diane) drew on esoteric knowledge to answer the question and the working-class student (Mike) recruited his ‘everyday’ knowledge of the materials the items were made of to answer the question. Cooper’s (1998) explanation for the difference in responses of middle-class and working-class students to ‘realistic’ test items is based on purported differences in their orientation to meaning – a proposition derived from Bernstein’s (2000) and Holland’s (1981) recontextualisation of Luria’s (1976) analysis of the relation between the mode of production and cognitive habits understood as semantic orientation. Orientation to meaning, where meaning is identified as context-independent or context-dependent by Bernstein and Holland, is recognised as the “selection and organization of meaning, of what is seen as relevant and taken as the focus of attention in any situation” (Holland, 1981, p. 1). Bernstein (2000) proposes that a student’s orientation to meaning is a function of his/her social class membership. According to him, working-class students are predisposed to a context-dependent orientation to meaning, referred to as a *restricted code* whereas middle-class students possess both context-dependent and context-independent orientations to meaning, referred to as an *elaborated code*. Middle-class students, according to Bernstein are, consequently, more likely to acquire the specialised knowledge presented in schooling, which privileges the elaborated code.

Bernstein’s proposition regarding orientation to meaning as a function of social class relies on Bernstein’s (2000) concept of classification that refers to:

> relations between categories, these relations being given by their degree of insulation from each other. Thus strong insulation created categories, clearly bounded, with a space for the development of a specialised identity, whereas the weaker the insulation, the less specialised the category. (Bernstein, 2000, p. 99)

In Cooper (1998) classification refers to the ostensible proclivity of working-class students to misrecognise the classificatory principle privileged in schooling. Thus, he claims that working-class students tend to use knowledge of everyday contexts in school mathematics contexts.

According to Cooper, Diane used two principles to sort the items. The first, ‘dimensionality’ (Cooper, 1998, p. 520), used to group all the items except the newspaper as three-dimensional objects and the newspaper as a two-dimensional object, is recognised by Cooper as a context-independent or esoteric property. The second, sorting the items into containers or non-containers, represents for Cooper a context-dependent principle based on ‘everyday’ experience (Cooper, 1998, p. 520). Diane, according to Cooper, displays both context-independent meanings and context-dependent meanings and she privileges context-independent meanings in school contexts. Mike, on the other hand, sorted the articles into two groups, those that were made from glass and metal and those that were made from paper and card. Cooper claimed that Mike used his ‘everyday’ knowledge to sort the items corresponding to the purported restricted orientation to meaning of working-class students as proposed by Bernstein. Cooper does, however, acknowledge that Mike’s sorting principle refers to a general property of the materials that the items are made of.

However, Cooper fails to recognise that Diane’s use of the dimensions of objects as a sorting principle is comparable to common-sensical description of objects. For example, objects such as CDs or mirrors are commonly referred to as two-dimensional because it is often the surface of the object that is referred to. So although Cooper describes dimensionality as context-independent, Diane’s usage of the dimension as a sorting principle appears to be more like ‘everyday’ usage of dimension rather than a strictly mathematical notion of dimension. Mike’s categories, on the other hand, could be considered as context-independent since he is using the properties of the objects as sorting criteria.

Furthermore, the question itself is open-ended since it asks students to respond in anyway they like. So Mike’s response should be considered as appropriate and apparently it was accepted as correct by markers of the test. Cooper’s failure to recognise Diane’s sorting criteria as common-sensical and his consideration of Mike’s sorting principle as an ‘everyday’ criterion seems to be over-determined by his deployment Bernstein’s proposition regarding an individual’s orientation to meaning as a function of social class membership. Thus, Cooper’s analysis appears to be based on an *a priori* notion of the cognitive attributes of students with respect to social class and is, therefore, essentialist.

I agree with Cooper and his colleagues that the working-class students in comparison to middle class students fail to solve the test items satisfactorily. However, their deployment of Bernstein’s proposition on orientation to meaning produces a misreading of the empirical and generates inadequate descriptions of mathematics constituted by the students in their study. The orientation to meaning of the students may still be an issue but not in the way in which the construct is described in the work of Cooper and his colleagues. Consequently, the methodological resources employed by Cooper and his colleagues suffers from descriptive inadequacy with respect to mathematics and raises questions about the explanatory adequacy of their deployment of Bernstein’s proposition regarding the social class basis of an individual’s orientation to meaning. It seems that Cooper’s research constitutes the student’s social class membership as the central research object rather than their mathematical productions.

WHAT ABOUT THE MATHEMATICS?

A number of studies in mathematics education are concerned with the role of language in teaching and learning of mathematics, particularly in relation to social class differences in mathematics achievement. Here, I consider mathematics education research that employs Systemic Functional Linguistics (SFL) as methodological resources. The central methodological resources, employed in mathematics education research using SFL, derive from the work of Michael Halliday, Ruqaya Hasan and J.R. Martin. Bernstein's (1971) proposition on semantic orientation is often used in conjunction with SFL as a resource for explaining social class differences in performance in mathematics. In particular, Bernstein's initial notion of sociolinguistic code in terms of the linguistic form of utterances is explicitly and sometimes implicitly used as a proposition in SFL studies (see Christie (2006) for a series of SFL studies using Bernstein's theory).

Veel (2006) discusses the potential of a number of SFL methodological resources to describe mathematics teaching and learning. In one example, he discusses the interactions of a group of students engaged in solving the following mathematical problem: "a five-metre length of fencing timber costs $8.00 and fence posts cost $5.00 each. If 9 metres of timber are needed to fence a triangular paddock and a fence post is needed for each metre of fence, explain whether the fence timber will cost more and why?" (Veel, 2006, p. 200). Part of the students' discussion of the problem is shown below:

M_1:	Is it worth trying to link this up? (Mmmmm) Three there, three there, three there.
F:	Yeah. Two more.
M_1:	A five-metre length of fencing timber costs $8.
F:	Mmmmm. One of these cost $8 and the other one. One post cost $5.
M_1:	No. You've gotta work out the triangle, how many posts you need.
F:	Yeah … wouldn't you wanna find … 9 posts.
M_1:	Yeah, you need how many posts.
F:	Wouldn't you want to find the perimeter, 'cause you're putting on a fence and –
M_1:	Yeah.
M_2:	All right. Ugh. First the um per perimeter of the ah fence is 9 lengths. Right.
F:	Yeah*
M_1:	Yeah*
M_2:	Is given. And …

M_1: Each length's …

M_2: But they don't give you (Mmm) the length of the triangle. It says 9 lengths. What is lengths?

M_1: Each length is 5 metres.

M_2: Each length is 5 metres. Okay, fair enough.

M_1: So the perimeter is 45.

M_2: Uh. Okay, so if there's 9 lengths, right …

F: Yeah.

(Extract from Veel, 2006, p. 200, * in original)

Veel (2006) uses Martin's concept of *speech roles*, that is, the extent to which the teacher and students occupy primary and secondary knower positions and Halliday's concept of *lexical density* (the average number of 'content words' per clause) to analyse the students' discussion of the problem. He concludes that the student talk has a low lexical density and therefore resembles 'everyday talk' rather than 'mathematical talk' (Veel, 2006, p. 203). However, the recognition rule for 'content words' is implicit in his analysis, that is, what is 'content' is not defined. For example, in the students' discussion of the paddock problem, it appears that words such as 'fence', 'timber' and 'post' are not counted as 'content words'. However, these words are part of the problem and should count because, as discussed below, the words are signifiers for objects central to the solution of the mathematical problem. The objects such as 'fence', 'timber' and 'post' are related through particular mathematical operations and therefore constitute particular mathematical relationships and therefore cannot be considered as 'everyday' talk. Thus, Veel's deployment of lexical density as an analytic resource and Cooper's use of Bernstein's notion of classification both rely on the false distinction between mathematics and the 'everyday'.

Furthermore, Veel does not provide any commentary on the poorly structured problem and the mathematical quality of students' responses. The solution to the problem can simply be read off the problem because the fence timber cost is $8 and the fence post is $5, making the fence timber costs more than the fence post. However, it seems that the question actually entails calculating the cost of timber required for fencing the paddock and the cost of the fence posts required to fence the paddock. The solution requires recognising the perimeter of the paddock as 9 metres that enables one to calculate that 2 five-metre lengths of fencing timber and 9 fencing posts are required. The calculations of the cost of the timber and the posts are then straightforward. The cost of the posts is 9 x $5 = $45 and the cost of the fencing timber is 2 x $8 = $16. So, cost of the fencing posts is the greater cost involved. The students, however, calculate that they need 45 posts that cost $225 and 45m of fencing timber costing $360.

The students recognise the objects ‘perimeter’, ‘number of fence posts’, ‘fencing-timber cost’ and ‘fencing post cost’ required for solving the problem but confuse ‘length’ with ‘metre’ in reading the perimeter as 9 lengths rather than as 9 metres. In addition, they establish an incorrect relationship between ‘length’ and ‘metre’, that is, that length equals five metres. In their cost calculations, the students use 45 metres as the perimeter of the paddock rather than 9 metres, producing an incorrect solution to the problem. Veel (2006, p. 201), however, focuses on the linguistic aspects such as lexical density and assigns students to the primary and secondary knower positions without taking into account the mathematics produced. In fact, student M_1, accorded primary knower status by Veel in the discussion, is the one who introduces the relationship between ‘length’ and ‘metre’ (“each length is 5 metres”) that results in the incorrect solution produced by the students. In other words, the primary knower position for Veel is not related to knowledge. The primary knower position is bestowed on the dominant speaker irrespective of knowledge produced by the speaker.

Thus, Veel’s analysis of mathematics classroom discourse focuses primarily on linguistic and social relations in the classroom and tends to ignore the quality of mathematics produced by the students. What is meant by content, a crucial aspect of recognising lexical density, is not made explicit in his analysis. Furthermore, Veel’s employment of primary and secondary knower does not take into account the mathematical content produced by the knowers in the data. As such the methodological resources in Veel’s work tend to focus chiefly on linguistic aspects and social relations with the effect of detracting from the mathematics generated in the pedagogic context.

In both studies considered above, I attempted to show how the methodological resources deployed either skews the reading of mathematics from the data as is the case with Cooper or bypasses the mathematics as is evident with Veel.

CONCLUDING REMARKS

Concerns regarding the lack of attention to mathematics in socio-political mathematics education research (Valero & Matos, 2000) and the tendency to ignore socio-political concerns in research focusing too closely on mathematics (Gutiérrez, 2010) has set up an opposition between mathematics and the social. Such an opposition is false and implies that there is a formal necessity to de-emphasise mathematics when focusing on the social. However, there appears to be no formal necessity to ignore or background mathematics in research concerned with social as demonstrated for example in the work of Dowling (1998) who examines the distribution of different forms of mathematics to different groups of students on the basis of their social class membership. Dowling (1998) illustrates that it is possible to focus on the social and maintain a focus on mathematics. In other words, that there is no formal necessity for a de-emphasis on mathematics in research concerned socio-political issues.

The aforementioned discussion of the research conducted by Cooper and Dunne (2000) and Veel (2006) illustrates that contrary to a formal necessity to de-emphasis mathematics, there appears to be a tendency to either ignore or misread mathematics. The proclivity to background mathematics in some socio-political mathematics education research arises as a consequence of the analytic categories established by methodological resources deployed.

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SCHOOL MATHEMATICS AS A "GAME": BEING EXPLICIT AND CONSISTENT

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Bourdieu's games analogy helps to explain the differential outcomes in education. Being able to engage with the game and to play the game of school is synonymous with success. Drawing on data from successful schools that achieved success in numeracy for Indigenous students, the paper draws on the games analogy to frame the responses made by teachers and principals when they talked about their practice. Most notably, the foci of their strategies was on ensuring that the rules of the game are made explicit so that learners can engage with, and successfully play, the game of school mathematics.

NUMERACY IN REMOTE INDIGENOUS COMMUNITIES: BACKGROUND

Within the Australian context, the challenge of education provision and educational success for Indigenous Australians has gained considerable political and educational momentum in recent times. Outcomes from the National Assessment Plan for Literacy and Numeracy (NAPLAN) (Australian Curriculum Assessment and Reporting Authority, 2010b) along with results from international testing has highlighted the poor performance on numeracy (and literacy) for Indigenous Australians. McGaw (2004) in his analysis of Australian performance on international tests indicated that nationally the country scored well but there were serious concerns in terms of equity. More recently, Leder (2012) highlighted the consistent gap in achievement on NAPLAN between Indigenous and non-Australian students with the general population across each of the year levels for NAPLAN testing indicating little improvement since McGaw's concerns were raised.

The context where this study was conducted is quite different from other states within Australia. It covers a large area (approximately 1.35 million square kilometres) but with a relatively small population. While nationally, Indigenous Australians make up approximately 3% of the population, more than 35% of the this region's population is Indigenous (Maher, 2011). While there are approximately thirty Indigenous languages, tuition in most communities is undertaken in English and most often by an early career teacher. Further, there is a high turnover of staff in remote communities creating challenges for sustainability of programs (Maher, 2011). The reasons for the high turnover are many and complex but typically include the isolation of living remote along with issues of language and culture (Lyons, Cooksey, Panizzon, Parnell, & Pegg, 2006). Attracting, training and retaining teaching and administrative staff in remote areas is a significant challenge for education systems (Giles, 2010). The sheer geographic isolation of remote communities creates challenges for staffing, particularly in training and the mentoring of early career teachers and principals (Giles, 2010). The implications of high turnover for teachers, programs and learners are significant. Many schools do not have sustained programs due to the turnover and changing interests of staff. Further, the communities are unable to create relationships with staff since they do not remain long enough in community.

Schooling is an enterprise that has particular goals. In terms of this paper, one of the key goals of schooling is to enable students' access to dominant forms of knowledge, most notably in mathematics (and literacy). But access to this dominant knowledge is often elusive and exclusive for some groups of students. Bourdieu's project enables theorisation of this access through his notion of "game". This is not to trivialise the process but rather to highlight that being successful in a game is knowing how to play the game and to accumulate the 'trump cards' in that game. For some, playing the game of school is easier than for others due to familiarity with the game. This paper explores how educationalists in some of the most challenging schools in Australia attempt to make the game more accessible to their students.

APPROACH

This paper draws on research that documented the success of eighteen schools identified as being successful in the 2010 NAPLAN testing. This was defined as being better than "similar schools" [1], or for having exceeded the normal gains. The aim of the project was to document the practices of these schools that may have contributed to their successes in national testing. The project developed case reports for each school. This paper reports on the common themes across the schools with regard to the importance of consistency and explicitness when planning mathematics experiences in remote Indigenous communities.

Method

Eighteen schools across the region had produced outcomes that exceeded expectations or which had improved significantly in the past reporting period. Departmental personnel identified schools that were performing above similar schools (as identified by the National Testing/Reporting process), utilising data for individual schools available on the *My School* website (Australian Curriculum Assessment and Reporting Authority, 2010a). Systemic-wide data, available only to educational systems, was also used but which is inaccessible to the general public.

The eighteen participating schools were spread were in urban, regional and remote settings. All except one school had large proportions of Indigenous students attending. For the purposes of this paper, the predominantly non-Indigenous school has not been included. The remote schools, (n=14) were serving communities with mostly, if exclusively, Indigenous students. The socio-economic demographics of all the schools were low, and many of the remote sites were very low, according to indices accessible from the *My School* website. Only publicly available data were accessed to develop profiles of each school. The research team requested that no further information be provided by education authorities about individual schools, particularly around curriculum and pedagogy, so as not to influence the data collection.

Ten schools were visited and staff from the remaining eight very/remote schools were interviewed by phone. The very remote schools were difficult to access due to location, cost involved in travelling to remote sites, and the time it would take to cover these vast and isolated communities. Many of these schools were on dirt tracks hundreds of kilometres from main roads. However, three very remote schools were included in the local visits to ensure that factors specific to such sites were not overlooked. Local visits were generally conducted over 1-2 days with two research staff attending each site. Interviews with key staff at these schools were tape recorded and later transcribed. Interviews conducted by telephone were notated and expanded notes produced at the end of each interview. Local site visits also included classroom observations.

Interviews were semi-structured and focused on developing a sense of the context of the school; practices that were implemented that were seen to be contributing to the school's success; outcomes that were observed/recorded by staff; and issues around curriculum development, implementation, assessment and leadership. Finally, interviewees were encouraged to raise any other relevant points related to the work of the school. Because the schools were so diverse, the questions needed to be broad to enable responses contiguous with their local setting.

While the project focused on both literacy and numeracy, the primary concern of this paper is the numeracy component of the research. However, it is noted that for many of the students, English is a second or other language. In many of the remote communities, English is a foreign language as the communities are strong in their home language and culture.

MAKING THE GAME EXPLICIT AND CONSISTENT

In coming to understand what made these schools successful, the staff in the school identified a range of practices that they believed had been instrumental in increasing student performance in the national testing program. Most were also clear that learning was much more than that which was represented in the tests. The school principals and teachers expressed a need to have clear expectations of the students and families. By making it clear and explicit as to the rules of the game, then students and families knew what the game was and how to play the game. This approach is not dissimilar to that advocated by Delpitt (2007) in her work in disadvantaged schools in the United States. She advocated that the liberal enterprise of schooling masked the agenda of literacy learning for students – most often Afro-American and/or Hispanic learners. By hiding the goal of activities, the students were not able to engage effectively with the activity. She concluded that strategies more akin to direct teaching may be of greater value for these students since they could identify the real purpose of an activity and hence engage more substantially with the intended goals of the teaching episode. Her thesis of being open and explicit resonates with the experiences cited in this paper. Although there was not an explicit recognition of Bourdieu's game analogy in the teacher's responses in this study, the practices being adopted across the schools indicated that the game of schooling was being made explicit to students and families, and in so doing, creating opportunities to engage with the game.

Consistency in Structures

At a fundamental level, the ways in which the schools operated were governed by timetabling. The timetable provided the rules for the game – students and communities new how the game of schooling operated. There is a high degree of flexibility in remote communities around timetabling and schools were able to build their school day around the particular needs of those communities. In some sites this meant the school, or game, commenced later in the morning as the community was one where sleeping late was commonplace. In other communities, the school started early in the day when it was cooler and then finished earlier to avoid the heat of the day. In other sites, the week was broken up so as to cater for community needs – Monday to Thursday were longer days with Friday being a short day to enable the school to operate a movie night on Friday evenings. Whatever timetable was constructed it was constrained by legislative requirements of operating a nominal number of hours each week and how those hours were built were shaped by community life. What was evident across the schools regardless of the timetable structure was the need for the structure to be made explicit to the teachers, students and communities.

Typically schools, or the game, operated a three-part day and were structured around a relatively predictable schedule. This general structure seemed to operate in most of the remote settings where there was a high degree of flexibility around the general daily routines. The typical structure was articulated as being something like:

> Principal: The school is run as a school that you would find in an urban setting. It runs to a timetable that is set and this operates everyday according to that timetable. There is [a] 2-hour literacy block, a 2-hour numeracy block and a 1.5 hour integrated studies block. (Remote, telephone interview)

All schools ensured that the community knew the general school timetable in terms of start time and the overall structure of the day. But this also meant that in some communities, principals needed to be proactive in enabling students to gain access to the timetable, that is, they needed to make the rules of the game transparent to the players in order for them to engage with the game. This was done in many ways, including the display of the school program in the local community stores or health clinics so that the community and support people in the community were able to support students and families to play the game of attending school.

But life in remote communities is governed by their own internal procedures. Many homes do not have clocks and the day-to-day life operates by mechanisms outside those of the school structure. Students know that school starts by various cues –

these may be the teachers or support people coming around the community with "troopies" [troopcarriers] to collect students; or by noticing the teachers driving to school. These were subtle cues to engage with the game. Cues, rather than clocks, were used to identify when to go to school. Other schools were more proactive in their approach, thus being more overt with the rules of the game:

> Principal: Families in the community don't have clocks in their homes so they take cues from others about times, including when to go to school. So that the community would know when it was time to get up and get ready for school, and also when to start school, we installed a public address system. This acts as a clock for the community. When they hear the music, then they know it is time to get ready for school. They can then hear when it is time to leave home, and when school starts. Without this, we found that they would be coming at all times and this disrupted learning. Now they know what the times are and school runs a lot more effectively. Families also know it is home time when they hear that music, or lunch time for that matter. (Very remote, interview)

The daily timetable shaped the day, but a number of teachers also indicated that they operated their lessons in a similar manner where they made the structure of the lessons transparent to the children. Both Delpitt (2005) in her work in literacy and the Accelerated Literacy (2007) program specifically designed for Aboriginal learners and operating in many Indigenous communities, take an approach that makes teaching of literacy structured and explicit. The format of the lessons was highly structured and followed a very set process. Teachers tried to operate their mathematics lessons in ways that was familiar to students and in so doing were trying to make the rules of the game somewhat more explicit to the students:

> Teacher: I make my [maths] lesson as much the same as I can each day without them being boring. The kids need to know what is going to happen in a lesson so that they are not guessing what I am going to do. When I do that [do not make it clear], that is when I lose them. If they know what is going to happen each day, then they are happy to be involved. (Remote, personal interview)

Similarly, another teacher explained his structure for his teaching of mathematics:

> Teacher: For me, the most important thing is consistency in the lesson. The students need to know what we are going to do. So we start off with some warm up exercises – to get their maths brains working. They like this as it is usually fun. It helps them know that we are doing maths now. We then have a chat about what we are going to do next. I try to put the work into some practical situation for them so that they know why they might need to know this stuff. Rosie (Aboriginal teaching assistant) sits with me and helps to explain some of the things to the children in language [the students' home language]. (Very remote, telephone interview)

The teacher then goes on to explain further how he structured the lesson, but what is clear in his interview is the need for a structure to the lesson so that the students know what and how the lesson will develop in order that they could play the game.

> Teacher: I find that when I deviate from the way I usually have the maths lesson is when I have the most problems. The kids can't follow me. It's like "hey, we don't do that now" and they focus on that rather than what I am trying to teach. I have found the lessons work best when I keep to the same structure each day. (Very remote, telephone interview)

While it may be easy to criticise the approach as being too structured and hence little scope to deviate, some teachers believed that the structure was critical to success. Students needed to know the rules of engagement if they were to play the game with some success. One teacher explained the success of being explicit, consistent and to some degree, repetitive with maths as enabling the students to see something clear about their learning.

> Teacher: They like it 'cause their lives are so unsettled and all over the place. So it's something they know is going to happen every day. I mean a razzle dazzle teacher will come and say "that's too repetitive" but I think that's what makes it work [being consistent]. (Very remote, personal interview)

Having structure to the lesson enabled the students to know what was going to happen, that is, they could recognise the rules for playing. In these cases, the structure was enabling the students to gain access to the game of mathematics (and schooling) because the teachers were transparent about the process through which they would be teaching. Students did not have to second-guess the teaching process.

Making the processes transparent to the students also meant that teachers were expected to model the desired behaviours to the students. As one principal said:

> Principal: Having expectations made explicit to the students (and communities) was important but equally important was the expectation of the teachers to model these expectations to the students (Very remote, telephone interview)

Another principal similarly argued that teachers have to model the expectations to the students. This principal had indicated in the earlier part of the interview that it was very easy in remote communities to become complacent and slip in the protocols of the school structure. A key role of the principal was to ensure that the standards that were expected in mainstream schooling were not diminished by teaching remotely.

> Teaching Principal: The teachers here also know that if we say that classes start back at 1.00pm, then they must be at the classroom before that time, ready for the students. The children have to see the teachers ready for them at the times we nominate or else it does not work. Teachers have to model appropriate behaviours. (Very remote, telephone interview)

The principals and teachers recognised the importance of having structures to timetables and lessons that were made explicit to the various parties involved in the education process in order that the rules for playing were transparent and could be engaged with successfully.

Consistency in Programs

While consistency in the structures of the school and lessons were important, so too was consistency in the mathematics programs. With the high turnover of staff, there is often instability in the approaches being used at schools. In these contexts, the game of school mathematics varied considerably with the change of staff. To use Bourdieu's metaphor, the constant change is akin to playing chess, then Twister, and then poker. The games changed constantly so it was difficult to understand what was happening and what was expected, let alone engage successfully in the game when there was constant change. It is common for teachers to remain for short periods of time in remote settings. This is for a range of reasons, least of which is the lifestyle and the compatibility of the teachers within those conditions. But high turnover of staff often resulted in new approaches being implemented with the new staff coming on board. Many of the new staff were recent graduates from various states across Australia, but also international graduates who were seeking adventure in these remote locations. Across all schools, staff made mention of the multiple programs that had been used by the previous staff. It was apparent from the experiences that teachers and principals came to sites where the predecessors had undertaken a range of different programs but there had been little sense of any sustainable programs over time.

Many of the schools had reported building new maths programs and assessment regimes as these were not evident prior to their arrival at the schools. It seemed that in most schools, the task of the incoming staff was to create a new program that would become the standard program for the school.

> Head of Curriculum: When I arrived each class teacher was planning their own themes so there was no coordination across the school at all and they were also trying to fit in every curriculum area every week, so the curriculum was very boxy, an hour of this and an hour of that. The Nelson maths books had been purchased. There hadn't been PD [professional development] or anything but the books had been purchased. (Regional primary school, personal interview)

From here this school developed a whole-school approach so that there would be consistency across the school:

> Head of Curriculum: We established a curriculum committee. We started actually working on beliefs about teaching and learning [of mathematics] so that we're all coming from a, the same belief system. [we then developed an] Integrated plan, started to write policies now for English and maths assessment and reporting so it was actually documented what Red Dust[2] School was on about in those areas. Whole school assessment at that time was non-existent when I arrived, in a formal way so we started with just with some simple checklists based on [the curriculum frameworks used in that region]. (Regional primary school, personal interview)

The school has since gone on to develop a range of school-based assessments for mathematics so that all students are profiled and interventionist pedagogy is then used to align with the students' levels of achievement. Unlike literacy, the school could not find an assessment program that met the needs of the school so worked on one that was suitable. This process has taken approximately four years but is now in operation and working across the school, with all teachers using the same system. This was reported across many of the sites so that the role of the incoming staff – either administration or classroom teachers – had been to rebuild programs. These successful schools were creating games with robust (and transparent) rules with which students and families could engage. Most of the schools were building a shared curriculum across the school and incorporating assessment tools to match the mathematics curriculum:

> Teaching Principal: I don't think it matters too much what program you use but rather the consistency in your program is more important than the substance of the program. ... The kids just need a common and consistent approach. That seems to work more than the program itself. (Remote, telephone interview)

The comments across all sites supported the need for consistency and explicitness in the approach and organisation of the timetable across the day, but also within numeracy blocks. There was consistency among the views of principals and teachers that students needed to know how the program would be organised so that they could engage with the events and learning.

> Teacher: One thing that does work is if the students know what to expect in lessons. It is when it is the unknown that they get confused and will disengage and muck up. It is important that the school has an approach that is common so that the students and communities get a sense of what we are trying to achieve. Then they know what is going on and how to work with us. We now have a common curriculum and way of teaching that all the teachers use. So the students may go into another class but they know what to expect. If the mother or father or aunty comes into any class, they see pretty much the same things so they know what is going on. (Very remote, telephone interview)

Overall, the importance of the school working as a team to have a common approach seemed to dominate the views expressed in this study. Schools went to various lengths to build common approaches to the organisation of their mathematics learning and curriculum. Professional development for teachers in remote settings is difficult as the distances to travel for professional learning are vast and often take two days of travel each way. Having teachers come into the communities to relieve teachers while they undertake professional learning is also challenging and in many cases the school would do after-school professional learning to ensure that there were common threads being developed among the staff.

> Principal: The big challenge in remote sites is how to ensure that the teachers know the same things and will teach the same way. We have teachers here who come from all over the place so there are no common or shared understandings. I have had to do a lot of after-school workshops to try to get them all on the same page. I have brought in a few experts to help us with that (Remote, personal interview).

For all the schools, there was a priority to have a common and shared vision of the mathematics curriculum and teaching, as well as assessment. Enacting a shared vision was challenging due to the conditions of the remote locations.

COMMUNITY ENGAGEMENT AND CHANGE

Some of the participants also recognised that the explicitness and consistency was important for the wider community so that they too could understand the routines of the school. A number of the schools held regular meetings with community to explain the practices and organisation of the school along with the expectations of the staff and programs. Most important was the need to create opportunities for the community to engage with the information being provided. One principal had worked across many communities before taking up her current position. In her previous role she worked closely with community members who often bemoaned the changes that new staff would bring to the school.

> Principal: One of the things I always heard was that communities were tired of the change – every time a new principal came in, there was another change. They did not get a chance to know the old things before the new changes were implemented. It was good for me as I realise that it may be better to stick with the old things for the benefit of the community and the kids. (Very remote, telephone interview)

She also argued that it gave little time to embed something new, and that too much change was so commonplace in remote sites that it was better to persevere with a program or innovation for a bit longer in the hope that the change itself was not the issue.

> Teaching Principal: The important thing about the routines is not to change them. Hang in there with them and get them embedded. The more change, the less likely they are to succeed. (Remote, telephone interview)

It would appear that change for change's sake may be detrimental to learning – for the students and communities.

CONCLUSION

From the research undertaken, it would appear that one of the keys to success in remote education was to provide consistent approaches to mathematics that were made explicit to teachers, students and community. In this way, Bourdieu's analogy of the game is most appropriate. Participants were suggesting that it was important for students to know "how to play the game". This was being achieved by making the rules of the game – school mathematics – explicit to all parties. The rules were being embedded in consistent approaches to school mathematics. By taking such approaches, the rules of the game were both explicit and consistent, enabling students to engage fairly with the game being played.

NOTES

1. Similar schools is a term used to note schools that are similar in make up to the target school. This includes variables such as size, location and student background. This better allows schools to compare their progress against similar schools rather than the national scores.
2. Pseudonyms are used in this paper to protect the identity of schools and participants in accordance with University ethics requirements.

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SOCIO-ECONOMIC STATUS AND RURALITY AS SCHOLASTIC MORTALITY: EXPLORING FAILURE AT MATHEMATICS

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Within the Australian context, some students are more at risk of failing at mathematics than others. Australia performs poorly in terms of equity outcomes in mathematics. This paper explores two significant social contexts – low socio-economic status and rurality – two characteristics where students have traditionally performed poorly in mathematics. The first of two key foci of the paper is the extent to which social backgrounds constrain mathematics outcomes. The second focus draws on a range of literatures to explore the variables that may contribute to this systemic failure. We frame the project using Pierre Bourdieu's theoretical ideas. Together, these two foci provide us with a rationale for more detailed work to be undertaken to explore the extent and impact of variables in these two contexts. This work forms the basis for a three-year project in which we explore pedagogies associated with these outcomes.

THE CONTEXT OF THE RESEARCH

Australians may perform moderately well on international tests such as TIMMS but it has also been acknowledged that while, at a national level the results are positive, there is considerable variation across the country as a whole where some groups of learners in some locations are performing quite poorly. The implications for equity are profound as the results suggest that there are some specific characteristics identifying students who perform considerably below expected standards. Most notably in the Australian context one of those characteristics is that of ethnicity, where remote Aboriginal students perform significantly below the national benchmarks (MCEETYA, 2009). Yet it is also recognised that other factors such as social-economic background and geographical location also impact on educational outcomes; the compounding of these factors is also an important consideration given the tendency for many Indigenous communities to live in remote geographic locations. In the Australian context "remote" and "rural" are not particularly synonymous and both have their own specific socio-economic structures. However, while remote Indigenous Australians are the most likely to perform well below national benchmarks, they constitute a very small portion of the overall population. Indeed, the specific impact of such a small representation was a key consideration in a national project aimed at enhancing mathematical outcomes for Indigenous Australians. This project specifically targeted urban and regional Indigenous students (Australian Association of Mathematics Teachers, 2008) in recognition that the issues around remote education were quite different from those of urban and regional settings.

Aside from these issues of remote Indigenous education, other factors – most notably low socio-economic background and rurality – are key variables that impact on considerable numbers of Australian students (including of course Indigenous students). In order to provide a suitable framing for understanding how educational practices are themselves implicated in the success or failure of students, we draw upon the sociological ideas of Bourdieu which help us understand how embedded social practices may enhance or hinder the learning of students from low socio-economic status backgrounds, or students who live in rural settings. Many of these students are in schools often described by employing authorities as "difficult-to-staff schools" either due to the demographics of the communities or the physical location of the schools (Berry, 2005). This is not an issue unique to Australia as the United Kingdom has similar issues with its so called "failing schools" which are usually in inner city, low socio-economic status (SES) communities, or communities with high racial diversity. In many states of Australia, teachers are employed by the state and to ensure that these difficult-to-staff schools have teachers, teaching staff are provided with incentives to teach (generally for short periods of time) in these schools. Many new graduates, for example, take the opportunity to gain extra credits that can be accumulated and exchanged for teaching positions in sites that are preferred by many teachers. The need to ensure quality teachers in these contexts has been widely recognised in Australia as well as internationally (Prince, 2002).

In spite of its theoretical difficulties we have chosen to retain a commitment to the term 'social class' to refer to the relative educational and economic (dis)advantage experienced by members of the wider Australian community. However we accept there is considerable tension over a clear definition of social class, particularly in a nation such as Australia which popularly considers itself an egalitarian society. We adopt a position on class as a construct created so as to explain a particular phenomenon rather than representing real categories or objects. It then becomes possible to theorise sets of people who occupy particular positions within the social strata. Consequently,

> Classes [are] sets of agents who occupy similar positions and who, in being placed in similar conditions and subjected to similar conditionings, have every likelihood of having similar dispositions and interests and therefore of producing similar practices and adopting similar stances. The "class on paper" has the theoretical existence which is that of theories… It is not really a class, an actual class, in the sense of a group, a group that mobilizes for struggle; at most it is a probable class, inasmuch as it is a set of agents which will present fewer hindrances to efforts of mobilization than other sets of agents (Bourdieu, 1985, p. 198).

We suggest that this theorisation could be similarly applied to people living in rural and remote areas, and also applied to how rurality is defined. Bourdieu goes on to expand his categories, arguing:

> This “class on paper” has the theoretical existence which belongs to theories: as the product of explanatory classification… it allows one to *explain* and predict the properties of things classified – including their propensity to constitute groups. (Bourdieu, 1991, p.232)

Using Bourdieu’s constructs it becomes possible to understand how class operates as the embodiment of culture into what is referred to as a class (or rural) habitus. This habitus is a “system of durable, transposable dispositions which functions as the generative basis of structured, objectively unified practices” (Bourdieu, 1979, p. viii). Using this approach to understanding social groupings, we can think about groups of people (and in particular learners of mathematics) who share similar dispositions, similar attributes, and thereby a similar habitus. This similarity within the collective, and hence difference from others, is what makes the construct of class such a powerful one, albeit a difficult one for which to create a tight definition. The classed habitus provides a lens for seeing and interpreting the world and for interacting with the social world. The capacity to be successful in school mathematics, for example, can then be seen as a process of aligning the home habitus, whether based on social class or geographical location, with the school institutional habitus – the cultural norms and dispositions represented through the school mathematics curriculum (Reay, David, & Ball, 2005). Accessing school mathematics thus becomes a task of ‘cracking the code’ that is represented through the classroom practices of mathematics education (Zevenbergen, 2000).

This paper is the beginning of a larger project, the main aim of which is to understand the already well-established phenomenon of social inclusion/exclusion through mathematics education. For this paper, we follow Bourdieu’s advocacy for the use of statistical confirmation of social class and other social categories to clarify some of the underlying structures of educational success and failure. To this end, the remainder of this paper draws on Australian data from the national testing of 2009 to illustrate the resonances between social and geographical background and achievement in school mathematics.

SOCIAL BACKGROUND AND NUMERACY

To ascertain the relationship between factors of social background, rurality and performance in mathematics, we drew on the national testing data from the Australian Curriculum Assessment and Reporting Agency (ACARA). These data are available from a national site – My School (Australian Curriculum Assessment and Reporting Authority, 2010). On a school-by-school basis, data are provided about the number of students, the percentage of indigenous students, the level of relative social dis/advantage (as indicated by the Index of Community Socio-Educational Community Advantage - ICSEA score) and the mean score for the school in the years of performance. The ICSEA data are constructed by the Australian Curriculum Assessment and Reporting Authority based on information about parental occupation, school education, non-school education and language background that has been obtained from school records in conjunction with data obtained from the Australian Bureau of Statistics (ABS) census data. Current testing is undertaken in Years 3, 5, 7 and 9.

We are aware of the controversy over the quality of such assessment data and debates about such testing and the data they yield. On the one hand the Australian Curriculum, Assessment and Reporting Authority (ACARA), the administrators of the tests and analysis, claim:

> The reliability of NAPLAN tests is high and that they can be used with confidence and are fit for purpose. The rigorous processes that are carried out during the development of NAPLAN each year ensure that the results are reliable and comparable between years. (ACARA, 2010)

On the other hand the Australian Primary Principals' Association (APPA) has its reservations:

> Currently details describing the reliability and validity of the NAPLAN tests are kept from the public. This means that it is not possible to estimate the confidence that can be attributed to differences in test scores. In regard to school performance reporting it is conceivable that differences between high and low performers may be due to measurement error. (APPA, 2009, p. 4)

We use this data because it is the only extensive database available to us – while we support the APPA claim that the use of such data should not serve to distort or destabilise the educational system. Moreover, we have used these data to establish relationships and differences across schools rather than making judgements about the performance of individual schools or about specific assessment items within tests.

At present, Australia does not have a national curriculum and consequently there are distinct differences in the nature and organisation of schooling across the country. In order to establish a representational data source, we needed to consider the country's diverse geographic representation (urban and rural) and the fact that there are differences in schooling across states. As a result, we opted to only use the data from Years 5 and 9 so as to reduce possible differences cause by different commencing ages (which would have most impact on Year 3 data) and for the transition to secondary school (which would have the most impact on Year 7 data as some states commence secondary studies in Year 7 while others start in Year 8).

One site is a major city with a population of over 1 million people and where there is considerable socio-economic diversity, the second site is within a rural area (in another state) that has a mixture of regional centres and small farming/agriculture regions. Schools vary in size from very small (30) to large (over 800). We are confident that these regions represent the distribution of schools across Australia. A total of 676 schools were part of the sample (see Table 2 below).

For this paper and for the sake of early simplicity, we draw only on the data for the ICSEA scores and the school numeracy scores. More complex multivariate analysis will be undertaken at a later stage to explore the interactions of other factors such as school size and size of the indigenous populations. The purpose of this paper is only to explore the intersection of social background and numeracy.

SCHOOL-BASED SES AND NUMERACY DATA

For data on social class, the protocol adopted by ACARA is that the standardised national average ICSEA score is 1000 with each standard deviation being 100. In terms of the NAPLAN scores, the national average in 2009 for Grade 5 was 487 and Grade 9 was 589. The scores are modelled so that there is a progressive increase as the students progress through bands indicating a nominal growth pattern over time.

For our cohort, there were statistically significant relationships between the ICSEA variable and NAPLAN numeracy performance for Grade 5 pupils [$r = 0.53$, $p \leq .01$] and Grade 9 pupils [$r = 0.59$, $p \leq .01$]. These data suggest a *moderately strong* yet significant relationship between social advantage (ICSEA) and the school's numeracy performance (NAPLAN) across the two geographic locations.

However interesting patterns emerge when we separate out the relationship between ICSEA and NAPLAN variables by geographic location (urban and rural) and Grade (Grades 5 and 9). Whilst for both Grade 5 and Grade 9 all correlations are statistically significant ($p \leq .01$), there is a much stronger relationship between urban schools than those in the rural cohorts (Table 1**,** Note: *$p \leq .01$).).

Year level	Urban	Rural
Year 5	0.716* (n = 228)	0.289* (n = 150)
Year 9	0.739* (n = 52)	0.430* (n = 73)

Table 1: Correlations between SES and Numeracy Scores

In order to provide some more detailed understanding of the underlying patterns in these data, we undertook ANOVAs on mean scores (for both ICSEA and NAPLAN results) at Grades 5 and 9 levels using geographical location (urban and rural) as the dependent variable. Given the national standardisation procedures conducted on these data (and the large sample sizes), we were satisfied that the underlying conditions for ANOVA have been met for these data, including the sample distribution and homogeneity of variances. Means and standard deviations are reported in Table 2.

Grade	Geographic Region	N	ICSEA Mean *(S.D)*	NAPLAN Mean *(S.D)*
Grade 5	Urban	281	1016.9	476.2
			83.1	*32.0*
	Rural	268	989.1	485.1
			53.8	*30.4*
Grade 9	Urban	52	997.5	572.3
			71.4	*32.3*
	Rural	75	993.8	579.9
			49.0	*27.6*

Table 2: Means (and Standard Deviations) for ICSEA and NAPLAN by Region

ANOVAs revealed statistically significant differences on ICSEA scores by geographic location across the Grade 5 cohorts ($F_{1,\ 548}$= 21.58, $p \leq .01$; effect size Cohen's d = 1.84). The mean ICSEA scores are higher for urban locations than for rural. Furthermore, for Grade 5 pupils, there was a statistically significant difference in NAPLAN scores ($F_{1,\ 377}$ = 7.25, $p \leq .01$; effect size Cohen's d = .75) with mean scores for urban students higher than that of students in rural locations.

Interestingly however, in contrast, there is no statistically significant difference between the ICSEA scores of students in urban and rural areas across the Grade 9 cohort ($F_{1,\ 126}$ = 0.117, $p > .05$). In terms of differences in the performance of Grade 9 students on NAPLAN scores, the ANOVA revealed no statistically significant differences in the means of the cohorts by location ($F_{1,\ 124}$ = 2.01, $p > .05$).

These data further strengthen our hypothesis that numeracy performance, as measured through high-stakes national testing, is strongly influenced by social measures though the relationship is complex, intersecting with geographical location in ways we have yet to understand. We are, however, able to draw on a number of research outcomes to hypothesise why these may be the case, and these will be the basis for a much larger research project to better understand the observations reported in this paper. It is known that many rural students (of high SES background) are likely to leave rural areas to attend boarding colleges when they reach high school, so this may impact on the data for rural areas. Further, we also know that it is difficult to staff rural or hard-to-staff schools with teachers who have strong mathematical content knowledge. This may impact on the numeracy experiences of learners as international research suggests that strong content knowledge of teachers positively influences learning outcomes (Davis & Simmt, 2006). Many of these schools are also staffed by early career teachers who are not strong in pedagogical content knowledge.

The impact of this practice is worthy of investigation so that we can understand the impact on learning for our targeted (low SES and rural) students.

CONCLUSIONS

These analyses indicate that there remains a strong correlation between social background and geographical location and numeracy outcomes. This is not a new phenomenon despite nearly 40 years of educational research and intervention into alleviating this discrepancy and despite successive national governments claiming to close the education disparity. We now seek to move forward from these hypotheses by undertaking further investigations that explore the reasons for these outcomes. It is not our intention to subscribe to deficit models and focus on characteristics of individual students but rather to examine the social, practical and policy contexts within which these results occur.

The reasons for such correlations are complex and need to be considered carefully (Cogan, Schmidt et al., 2001). There are multiple reasons for the observations in the data provided in this paper. First it has been shown elsewhere that the quality of the teachers *vis a vis* their mathematical backgrounds is an important factor in pupil learning outcomes. For example, it has been shown that the mathematical background of the teacher has a significant positive relationship with learning gains, particularly in the early years of schooling (Hill, Rowan et al., 2005, p. 371). Also, the teachers' mathematical background has been particularly salient in low SES schools where there is a much larger effect in low SES schools than in high SES schools (Nye, Konstantopoulos et al., 2004). However, mathematical content knowledge must be considered in concert with pedagogic content knowledge which has similarly been found to affect learning outcomes (Baumert, Kunter et al., 2010). Other issues may also be implicated in producing the outcomes noted in this paper and are worthy of further exploration.

BOURDIEU AND SCHOLASTIC MORTALITY

To understand this phenomenon, we draw on Bourdieu's work, particularly that related to education, albeit it, higher education. However, his constructs apply to the school context and we will locate our work within this framework. It has been well recognised that his work on habitus, field and capital help to understand the ways in which practices (such as mathematics teaching) enable some students greater or lesser access to important knowledge and hence power. Using this framework, we are able to see the habitus which Bourdieu describes as "the product of history [that] produces individual and collective practices" (1977, p. 82) such that it can be seen that the familial circumstances create particular ways of seeing and acting in a social world that are influential but not deterministic in how that student acts in the world of school. He argues that the habitus is integral to the generation of social practices as well such that "system of durable, transposable dispositions which functions as the generative basis of structured, objectively unified practices' (1979, p. vii). The student in the mathematics classroom may come to school with a particular habitus that may predispose him/her to act in the classroom in particular ways and these ways of interaction and displays of particular knowledge may, in turn, be rewarded or not by the teacher. In this way, the habitus thus can become a form of culture that is differentially acknowledged and rewarded by the teacher and system. For Bourdieu, this habitus as a representation of culture now becomes a form of capital that can be exchanged for other goods. These goods may be in the form of grades, certificates and so on. But this may not be the case for all students and as such Bourdieu's framework allows for a theorisation of why some students are more at risk of failing school mathematics, not due to some innate or inherent natural ability but due to the structuring practices within the school system. These practices acknowledge and reward particular aspects of the culture and hence create a form of symbolic violence for those students whose culture does not resonate with the school practices. Within this framework, the failure of some students becomes a form of scholastic mortality – death due to the lack of resonance between the practices within the field of school mathematics and the culture of the student. While there may be critique that this is a deterministic position, by recognising that failure may be due to culture rather than something more innate, educators can become aware of the cultural disparities to create opportunities for learning – or restructuring the familial habitus – so that students can learn the code of school mathematics.

Using Bourdieu's theoretical framework, we are able to theorise that the differential outcomes noted in this paper may be representations of the inconsistent alliance of classed and geographic habitus (the internalised culture) of students with the practices of school mathematics. That urban, high status students are more likely to be successful in school mathematics has been shown to be statistically highly significant. Similarly, students with low status or rural habitus are less likely to perform well in the practices of school mathematics. Bourdieu's work enables us to understand this as being related to the ways in which the social heritage of learners, that is their social background, aligns more or less with that of the school (Jorgensen & Sullivan, 2010). Where there is not a strong alignment with the social heritage as represented through the habitus, there is a greater chance of scholastic mortality. This theoretical position has been endorsed by the statistical analysis undertaken in this paper.

FUTURE DIRECTIONS

The work presented in this paper is the starting point for a three-year project [1] in which we are exploring the nexus between social/educational practices in urban (wealthy and poor) and rural communities in Australia. More detailed work to be undertaken with regard to these data will allow us to theorise, in greater detail, the ways in which practices are implicated in the outcomes explored herein. It is our intent to explore the cultural practices in these contexts in order to better understand the complexity around the structuring practices adopted and enacted by schools in these settings. The initial data presented here confirms the marked differences in performance of students in these areas. The project will identify schools that conform to the anticipated trajectories of school performance *vis a vis* social and geographical location – high SES/urban and high performance, low SES/rural and low performance as these confirm the general educational trends. Understanding the confirmatory practices will enable us to develop understandings of the general observable trends in education. We are also seeking to explore those contexts where the trends are reversed – high SES but low performance; and low SES and high performance. These sites may help in understanding the practices that are enacted that enable the constitution of habitus and practices to enable access to school mathematics.

Our initial statistical analysis of the regions under investigation have shown that the hypothesis underpinning this project is robust. Using a range of factors drawn from the research literature that are thought to be powerful in mathematics – such as years of experience, experience in working with the nominated cohorts, teacher knowledge (pedagogic content knowledge and mathematics content knowledge), leadership and reform in schools **–** we are now in the process of surveying teachers working in these. From this we will identify eight schools (four in urban setting and four in a rural setting) that align with the confirmatory and non-confirmatory model. These schools will form case studies from which we intend to develop detailed analyses of classroom practices that may (or may not) be contributing to scholastic success or mortality. Bourdieu's work will be most useful in framing our observations and data in order to build a coherent and comprehensive study of school mathematics and how it contributes, or not, to success for particular cohorts of students.

NOTES

1. This project is funded by the Australian Research Council under its Discovery Grants scheme.

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IDENTITY WORK IN TRANSITIONING BETWEEN CONTEXTS

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The paper explores – mainly at a theoretical level - the relationship between identity work involved in children's transitioning between school mathematics contexts and other important contexts such as their homes. The exploration is then used to draw up a methodological framework for identifying contexts between which children transition and the identity work involved.

INTRODUCTION

In this paper I explore the relationship, in mathematics education, between the notion of transitioning between contexts and the notion of identity work. More precisely, I look at identity work as a feature of transitioning between contexts and sketch a methodological framework for identifying contexts and identity work. While actual analysis of data will await a later paper, related research have been done on the same set of data (Lange, 2007, 2008a, 2008b; 2009; Lange & Meaney, 2010; Lange & Meaney, 2011). This body of research considers identity work that children are forced to engage in when they transition between school mathematical contexts and other important contexts, such as their home, in which they live their lives. Another background to the paper is that much research using the notion of identity in my reading leaves this notion poorly defined or not defined at all, especially in regards to what counts as instances of identity in data and what not. Furthermore, when multiplicity and fluidity of identity is posited, as is usually the case (Bishop, 2012), little consideration is given to the psychological workings of such constructions. The resulting conceptualisation of identity does not correspond to my experience of my identity. As I hesitate to apply it to myself, I am uneasy about it being applied to others.

TRANSITIONING BETWEEN CONTEXTS

Meaney and Lange (2012 forthcoming) saw "contexts as enactments of systems of knowledge within social practices" and adjustment to new contexts as always involving learning. School mathematics is a social practice whose elements according to Fairclough (2003) are (a) action and interaction; (b) social relations; (c) persons (with beliefs, attitudes, histories, etc.); (d) the material world; and (e) discourse.

> Classroom teaching articulates together particular ways of using language (on part of both teachers and learners) with the social relation of the classroom, the structuring use of the classroom as a physical space, and so forth. … Social events are casually shaped by (networks of) social practices –social practices define particular ways of acting, and although actual events may more or less diverge from these definitions and expectations (because they cut across different social practices, and because of the causal powers of social agents), they are still partly shaped by them. (Fairclough, 2003, p. 25)

Thus, while the social practice of school mathematics frames the social events of mathematics classrooms, e.g. children becoming conversant with school mathematical forms of reflection (Radford, 2008), social practices from contexts between which the children transition, may cut across and co-shape social events.

Meaney and Lange (2012 forthcoming) emphasised transition not only as a transitioning from whatever context the child comes from ("background") *into* the context of school mathematics education but also from the latter into out-of-school contexts in which the child is participating, not least the home context. Emphasising the two-way nature of transitioning troubles assumptions of self-evident, natural "goodness" of school mathematics education by pointing to the fact that the children have to transition "back" to the contexts in which they live their out-of-school lives. If the contexts, between which children transition, are based on different or even conflicting systems of knowledge and social practices, then the reflection required in adjusting to the contexts may be detrimental to children's capacity to enact the knowledge systems in one or both contexts. This could be the case with the system of school mathematics knowledge and a system of cultural knowledge at home. Thus, accentuating transitioning between contexts as a two-way process has a particular edge with regards to children from non-dominant social groups, such as colonised Indigenous populations, non-Western immigrants in Western societies, working class or rural populations in (post-)industrial societies; in short, non-middle-class groups in societies dominated by (often white) middle-class. In these circumstances, transitioning into a school mathematics context may involve loosing connections with the home context, in which case the learner's horizons of possibilities for their future will be narrowed (Meaney & Lange, 2012 forthcoming). An Indigenous group "may well recognize that schooling provides the skills necessary to survive in a technological world, but it will also blame the school for alienating students from their home culture, whether deliberately or unintentionally" (Cantoni, 1991, p. 34).

IDENTITY

Adapting to a context – assimilating and accommodating to invoke Piagetian notions – requires effort, "work", not only in adapting to a different physical architecture, but, in terms of identity, more importantly in adapting to a different social (positioning, hierarchy, power, agency, language) and body-mind (psychological) "architecture".

The etymology of *identity* probably goes back to Latin *identidem* meaning repeatedly, a contraction of *idem et idem*, same and same (Mariam-Webster Dictionary (online)). Hence, meaningful notions of identity have to reflect what is repeated, what is the same, of a person, a body-mind, across contexts.

While being very critical of definitions of identity that rely on expressions about "who one is", Anna Sfard and Anna Prusak (2005), in arguing for their strictly discursive definition of identity, recognised the psychological need for speaking about constancy, sameness, in the processes of change in lived lives.

> Why this overpowering proclivity for "is-sentences"? Paradoxically, the reason may be exactly the same as the one that formerly evoked our concern: We cannot do without the is-sentences because of their reifying quality. Our relations with the world and with other people change continually, sensitive to our every action. Metaphorically speaking, identifying is an attempt to overcome the fluidity of change by collapsing a video clip into a snapshot. The use of is-sentences, which do the job of "freezing the picture" and turning properties of actions into properties of actors, is grounded in the experience-engendered expectation – indeed, hope – that despite the process of change, much of what we see now will repeat itself in a similar situation tomorrow. Based of this assumption, identity talk makes us able to cope with new situations in terms of our past experience and gives us tools to plan for the future. (p. 16)

Jessica Bishop (2012) gave a comprehensive overview of notions of identity in research literature. On this basis, she defined identity in terms of social position and "who one is" in a given community:

> I define *identity* as a dynamic view of self, negotiated in a specific social context and informed by past history, events, personal narratives, routines, and ways of participating. An identity is who one is in a given community and, as such, both individually and collectively defined. (p. 38; italics in original)

In a similar manner, she took *mathematics identity* to mean

> "the ideas, often tacit, one has about who he or she is with respect to the subject of mathematics and its corresponding activities. ...This includes a person's ways of talking, acting, and being and the ways in which others position one with respect to mathematics. ...Identity is situated; learned; stable and predictable, yet malleable; and is both individual and collective" (p. 39).

Substituting community with context, identity then is a question of "who one is" in a given context. Transitioning between contexts involves different forms of "who one is". However, it is not clear in Bishop's otherwise thorough analysis of her data how students in psychological terms manage enactments of different identities.

James Paul Gee (2000) defined identity in terms of performances in society not unlike Bishop (2012) (which is not surprising given that Bishop among others has Gee as a source of inspiration) but further made a distinction between such multiple identities and core identity:

> The "kind of person" one is recognised as "being", at a given time and place, can change from moment to moment in the interaction, can change from context to context, and, of course, can be ambiguous and unstable.
>
> Being recognized as a certain "kind of person", in a given context, is what I mean here by "identity". In this sense of the term, all people have multiple identities connected not to their "internal states" but to their performances in society. This is not to deny that each of us has what we might call a "core identity" that holds more uniformly, for ourselves and others, across contexts. (p. 99)

Ways of being in the world, being a certain "kind of person", happens in social groups and at least in some instances is a matter of choice.

> Since reading and thinking are social achievements connected to social groups, we can all read and think in different ways when we read and think as members (or as if we were members) of different groups. … Any specific way of reading and thinking is, in fact, a way of being in the world, a way of being a certain "kind of person," a way of taking on a certain sort of identity. In that sense each of us has multiple identities. Even a priest can read the Bible "as a priest," "as a literary critic," "as a historian," even "as a male" or "as an African American" (priest, literary critic, historian, or ethnic group member), even if he chooses to privilege one way of reading–one identity–over another. (Gee, 2003, p. 3)

Unlike many other writers on identity, Gee is clear about the relationship between different identities. It is not only that different identities are enacted at different times and locales (teacher in the morning at school, consumer in the afternoon in the supermarket, parent in the evening at home, and lover at night in a bedroom) but also that in situations where more identities could be enacted you may privilege one rather than others.

CORE IDENTITY

In a study of a group of college women's multiple dimensions of identity, Jones and McEwen (2000) found that the participants distinguished emphatically between their multiple "outside identities" and their core "inner identity" which they kept to themselves.

> The core was frequently described by participants as their "inner identity" or "inside self" as contrasted with what they referred to as their "outside" identity or the "facts" of their identity. Outside identities were easily named by others and interpreted by the participants as less meaningful than the complexities of their inside identities which they guarded and kept close to themselves and made less susceptible to outside influence. The words these women used to describe their core included intelligent, kind, a good friend, compassionate, independent. They resisted using terms that conveyed external definition and identity categories to describe their core sense of self. To these young women, labels lacked complexity, accuracy, and personal relevancy. They believed that labels rarely touched the core of an individual's sense of self. For them, individual identity was experienced and lived at far greater depth than such categories suggested or permitted. (p.408-409)

Studies like this suggest that a viable notion of identity as a research tool in (mathematics) education needs to account for people's expressed sense of core identity in the flux of "outside" multiple identities.

Contrary to other writers, Gee (2003) recognised the psychological "materiality" of having multiple identities by pointing out that a person's multiple identities are kept together by the body and a core identity:

> This [having multiple identities] does not mean we all have multiple personality disorder. We each have a core identity that relates to all our other identities (as a woman, feminist, wife, ethnic of a certain sort, biologist, Catholic, etc.). We have this core identity thanks to being in one and the same body over time and thanks to being able to tell ourselves a reasonably (but only reasonably) coherent life story in which we are the "hero" (or, at least, the central character). But as we take on new identities or transform old ones, this core identity changes and transforms as well. (p. 4)

The inherent temporal dimension of core identity, Gee (2000) addressed by seeing core identity as combination of a trajectory in Discourse space and its narrativisation. Discourse with a capital d is defined as "socially accepted associations among ways of using language, of thinking, valuing, and interacting, in the 'right' places and at the 'right' times with the 'right' objects ...that can be used to identify oneself as a member of a socially meaningful group or 'social network'" (Gee, 1999, p. 26).

> Discourses can give us one way to define what I earlier called a person's "core identity". Each person has had a unique trajectory through "Discourse space". That is, he or she has, through time, in a certain order, had specific experiences within specific Discourses (i.e., been recognized, at a time and place, one way and not another), some recurring and others not. This trajectory and the person's own narrativization (Mishler, 2000) of it are what constitute his or her (never fully formed or always potentially changing) "core identity". The Discourses are social and historical, but the person's trajectory and narrativization are individual (though an individuality that is fully socially formed and informed). (Gee, 2000, p. 111)

Context and Discourse have much in common. Children's transitioning between contexts constitutes their trajectory in Discourse space – or part thereof. Therefore, children, who transition between the context (Discourse) of school mathematics and other contexts (Discourses), need to tell coherent life stories that integrate the narrative counterparts of their lived experiences (Sfard & Prusak, 2005) in these contexts into a sufficiently coherent whole and with sufficient temporal continuity.

IDENTITY WORK

Depending upon the compatibility of the systems of knowledge and social practices constituting the context of school mathematics and other important contexts for children such as home, the formation of a coherent life story may require more or less effort on part of the children. Transitioning between contexts involves identity work because the adaptation (assimilation, accommodation) to different contexts involves dealing with questions of "who am I" and "who do I want to be", or in Gee's terms, as what "kind of person" am I recognised or do I want to be recognised. With reference to Sfard and Prusak's (2005) narrative definition of identity, the question is what narratives are reifying, endorsable and significant in each context, which narratives are the same and which are different across contexts. It takes effort on the part of the identity builder to form "snapshots" that are reifying (transform stream of events into "things"); to select or decide stories for endorsement; and to make decisions about which stories count as significant. Further it requires work to form designated identities, that is, hopes, dreams, desires, and intentions for identities to be actual in the future. In the case of contradicting narratives, especially when relevant to the same context, it takes energy to resolve the conflict sufficiently to allow for maintaining the cohesion of the life story. This process of identifying is what I call identity work.

Bishop (2012) studied enactment of identity in a mathematics classroom and saw every communicational move as affording an opportunity for participants to negotiate their identity. Her explicit emphasis was on "the work accomplished through discourse during human interaction and communication" (p. 44). From my definition, enactment of identity would count as identity work.

Using the idea of one body "housing" a core identity allows for a slightly different description of identity work involved in transitioning between contexts. Moving the body from one context to another implies calling to the fore another identity according to the new context. This, however, does not do away with other identities "residing" in the body, which makes the physical transition from one context to another. The context of school mathematics may bring forward, and certainly is expected to do so, a child's identity as a school student in a mathematics classroom while some of the child's other identities, such as football player and daddy's darling, fade into the background of his/her awareness. Other identities, such as belonging to an ethnic minority, may not so easily fade away, but rather become contextualised in another way. The point here is that none of the identities have disappeared. They are part of the "trajectory in Discourse space", and, at the end of the day, all of them have to lend themselves to the narrativisation of a "reasonably coherent life story". Telling this life story is identity work and the effort required depends on the contexts to be transitioned. The more aligned the contexts are in their systems of knowledge and social practices, the less effort is required to form a coherent narrative. The more different the contexts are, the more unaligned are the identities, "the kinds of persons", that goes with them, and that, because of the one body "living" them, all need to be brought into some sort of narrative cohesion. While adapting to different contexts always involves learning (Meaney & Lange, 2012 forthcoming), it may be that incompatible differences between contexts make such demands on the transitioning child's reflective capacity that little is left to engage in the Discourse of the context.

IDENTITY WORK AND LEARNING

Some mathematics education researchers discuss mathematic learning in terms of identity. In an article outlining possible fields for collaboration with literacy researchers, Paul Cobb (2004) identified students developing "a sense of affiliation with mathematical activity" and their "emerging identities" as such promising fields.

> Students' development of a sense of affiliation with mathematical activity and thus the cultivation of what might be termed their *mathematical interest* is essential to the success

> of a design experiment. /... A central issue for mathematics educators concerns the process by which students' emerging identities in the mathematics classroom might, over time, involve changes in their more enduring sense of who they are and who they want to become. (p. 336, italics in original)

An "enduring sense of who they are who they want to become" is another expression for "core identity" and Cobb pointed to this concept of Gee's (2000, 2003) as being "of critical importance" in the collaborative endeavours he envisioned.

From studying learning involved in video games, Gee (2005) distilled *learning principles* that unequivocally connected *deep learning*, which can be taken to be the kind of learning that Cobb wanted to happen in design experiments and saw as requiring "a sense of affiliation", with development of identity, that is, ways of knowing and acting characteristic of practitioners in an academic area.

> Principle: Deep learning requires an extended commitment and such a commitment is powerfully recruited when people take on a new identity they value and in which they become heavily invested – whether this be a child 'being a scientist doing science' in a classroom or an adult taking on a new role at work. (Gee, 2005, p. 7)

> School is often built around the 'content fetish', the idea that an academic area like biology or social science is constituted by some definitive list of facts or body of information that can be tested in a standardized way. But academic areas are not first and foremost bodies of facts, they are, rather, first and foremost, the activities and ways of knowing through which such facts are generated, defended, and modified. Such activities and ways of knowing are carried out by people who adopt certain sorts of identities, that is, adopt certain ways with words, actions, and interactions, as well as certain values, attitudes, and beliefs. /.../

> Ironically, when learners adopt and practice such an identity and engage in the forms of talk and action connected to it, facts come free – they are learned as part and parcel of being a certain sort of person needing to do certain sorts of things for one's own purposes and goals (Shaffer, 2004). Out of the context of identity and activity, facts are hard to learn and last in the learner's mind a very short time indeed. (Gee, 2005, p. 8)

If deep mathematics learning, relational understanding in Skemp's (1976) term, is what mathematics education should aim at, then Gee's analysis seems to suggest that policy makers and curriculum authors have to let go of primarily conceiving of school mathematics in terms of content. Instead they should focus on developing contexts for school mathematics in which "fact comes free" with the identities of such the context. This is truly a daunting task given present day policy trends. However, the research reported in Meaney and Lange (2012 forthcoming) gives some indication of what these contexts might look like. Such contexts could ease the identity work load and hence possibly facilitating children's transitioning between contexts.

METHODOLOGICAL CONSIDERATIONS

Gee (2000) described four intertwined perspectives on identity. Each is characterised by a process, a power, and a source of the power through which the process works. They are summarised in Table 1.

Label	Perspective	Process	Power	Source of power	(Extreme) Forms
N-Identities	1. Nature-identity: a state	developed from	forces	in nature	
I-Identities	2. Institution-identity: a position	authorized by	authorities	within institutions	Calling Imposition
D-Identities	3. Discourse-identity: an individual trait	recognized in	the discourse/ dialogue	of/with "rational" individuals	Ascription Achievement
A-identities	4. Affinity-identity: experiences	shared in	the practice	of "affinity groups"	Institutionally sanctioned Not institutionally sanctioned

Table 1: Four ways to view identity. Reproduced and eleborated from Gee (2000)

A preliminary re-analysis of the data analyses in earlier work (Lange, 2007, 2008a, 2008b, 2009; Lange & Meaney, 2010; Lange & Meaney, 2011) suggests that the contexts between which 10-11 year old children typically transitioned included home (family life), school teaching, school leisure (non-teaching times and spaces such as breaks and school day-care), leisure (out-of-school). Their futures were also contexts for identity work. Theoretically, this is supported by Cobb's (2004) taking "who they want to become" as identity, Sfard and Prusak's (2005) distinction between actual and designated (hoped-for) identities and Ole Skovsmose's (2005) pointing to students' foreground as strongly involved in forming students' learning intentions.

Gee's perspectives on identities can be combined in a matrix with these contexts. In this matrix identities can be placed in relation to the context in which they are "active". Table 2 sketches some possible context-identity pairs.

Context / Identity	Home / family	School (teaching)	Leisure at school (breaks a.o.)	Leisure out of school	Future
N-identities	Child, sibling				
I-identities		Student Bad in maths	Younger/older ("smaller"/"larger")		Education Job
D-identities		Good classmate	Friend, playmate,	Friend, playmate	
A-identities				Counter strike-player Playstation player	

Table 2. Identities versus contexts with some examples

This matrix is meant to serve as a theoretical-methodological lens for identifying and analysing in detail the identity work involved in children's transitioning between school mathematics contexts and other important contexts as well as inspire the mathematics education community to also take a serious interest in mathematics learners' transition *out of*, and not only *into*, school mathematics contexts.

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"I JUST MAKE SURE THAT I GO FOR IT": A MATHEMATICS STUDENT'S TRANSITION TO AND THROUGH UNIVERSITY

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This paper uses a post-structuralist perspective of identity from Fairclough and Norton to investigate the transition from school mathematics to advanced mathematics at university. Four longitudinal interviews with a successful science student were analysed using discourse analysis. I argue that the student invests or does not invest in various social identities, thus succeeding despite the structural constraints of his background. The analysis illuminates the considerable individual material and mental work required for this investment and suggests that the constraints remain a presence throughout the transition.

INTRODUCTION

South African education has undergone extensive structural, policy and curriculum reform in recent years, with the aim of increasing formal and epistemological access. Yet access to education in the sciences at both school and university remain a function of the related constructs of race, socio-economic class, geographical location and language (e.g., Reddy, 2006; Scott, Yeld, & Hendry, 2007). In 2009 South African universities accepted the first cohort of school leavers who had completed their schooling on a new curriculum. Thabo (pseudonym), a Black [1] student in this cohort, talks about living in a "township" and learning school mathematics in "our language" (iSepedi). At school he is "the best learner in school who passed maths", and is advised by a teacher to study science at university. However, the university positions Thabo as educationally disadvantaged and places him in a four year degree programme in which he completes first-year foundation courses over two years before tackling mainstream courses. Thabo fails his first university mathematics test, yet he makes "sure that I go for it" and wins awards for his performance on the foundation courses. He then fails his first advanced mathematics test in a mainstream course which is "too theoretical, it is all about proofs". Yet he passes this course (with 50%), thus completing the required mathematics courses for his science degree.

A student's transition from school to university mathematics can be viewed from various perspectives, for example, in terms of the "fit" between curricula (e.g., Engelbrecht, Harding and Phiri, 2010) or in terms of the student's personal motivation (e.g., Gibney, Moore, Murphy, & O'Sullivan, 2011). Such studies foreground differences between mathematical practices, yet they may assume unproblematic transfer across practices that "fit" or may locate difficulties in the individual student. Other perspectives look further to the background of a student (e.g., Frempong, Ma, & Mensah, 2012), describing this as "Black" and/or "second-language". Such studies point to the structural constraints on transition, but as noted by McGee and Martin (2011), categories such as race may become static and used to explain performance.

A growing body of research in mathematics education views the school/university transition as the interplay between student agency and the wider social structure (e.g., Lerman, 2012; Smith, 2010). Valero (2009) argues that the concept of identity plays "a pivotal role" (p. 218) in understanding this interplay. In this paper I use Fairclough's (2003) concept of *identity*, supplemented with the notions of *investment* and *imagined community* (Norton, 2010), to investigate the interview texts of Thabo, a successful mathematics student. In particular, I ask what social identities he identifies as set up for him in the practices in which he acts and how he describes himself as investing or not investing in these identities.

THEORETICAL FRAMEWORK

Fairclough (2003) uses three levels of the social to account for the relationship between agency and structure; the concrete *social event* (e.g., what happens in the school mathematics classroom), the abstract *structure* (e.g., race and the English language), and *social practice* which mediates the relationship between event and structure. A social practice such as school mathematics controls "the selection of certain structural possibilities" (p. 23), thus defining how a student should act in class. Yet a social event is only partly shaped by social practice, this due in part to the agency or "causal powers" (p. 22) of participants such as students.

Fairclough's (2003) concept of *identity* has two parts. The first accounts for structure; *social identity* is defined by one's circumstances and early socialization (e.g., Thabo speaks iSepedi at home) and one's later socialization into particular subject positions (e.g., Thabo is the top mathematics student at school). The second part recognises individual agency; *personal identity* is the personal investment made in the subject positions on offer. An individual's identity is a product of the dialectical relationship between social and personal identity. Norton's (2010) notion of *investment* recognises the "socially and historically structured relationship" (p.353) of a student to learning. She argues that a student *invests* in learning, knowing that this brings with it symbolic and material resources. Investment involves aspiring to an *imagined community* that a student aspires to join and which offers particular subject positions in the future.

METHODOLOGY

The four interview texts in this paper were produced in a wider longitudinal study of the 2009 cohort of students at a South African university [2]. Thabo was interviewed by a trained interviewer in the first half of each academic year. The first three interviews were structured, focusing on Thabo's educational experiences. His final interview was semi-structured, drawing on the object Thabo identified as representing meaning in his life. The interviews were transcribed to represent the verbal interaction. The study has ethical clearance from the university at which it is located. Thabo's texts were selected for this paper since he participated fully in the study and he completed the required advanced mathematics course for his science degree.

McLeod (2003) notes that "identities do not simply reveal themselves in interviews" (p. 203). I regard an interview as an instance of "recontextualized social practice" (van Leeuwen, 2008, p. 3), practice being recontextualized in two respects. Firstly, the initial practice that is talked about in an interview (e.g., school mathematics) is "inserted" (p. 14) in the practice of research interviews, with the interaction influenced by the research goals. The second recontextualization involves time; Gordon and Lahelma (2003) argue that the present is "constantly reflected in relation to the past, as well as plans for the future" (p. 252).

The analysis draws on tools from Fairclough's (2003) method of critical discourse analysis. In the interview Thabo uses language to give meaning to the recontextualized practices, both material (e.g., university) and symbolic (e.g., language), in which he acts. He does this by *representing* these practices, *identifying* himself and others in these practices, and *acting* by enacting relationships with others and with texts. Using Fairclough (2003), in interaction with the transcripts, I have identified certain textual features that play a role in these three meanings. For example, naming signals a particular way of representing the world, transitivity identifies a student as active or a thinker, and pronouns set up a relationship to others.

THABO'S TRANSITION

This section is structured according to the practices in which Thabo acts; for each practice I describe the social identities identified by Thabo and his investment in these identities. Since many analyses can be constructed from longitudinal interviews (McLeod, 2003), I link the description closely to the textual strategies as evidence to show how this particular analysis "is possible" (p. 209).

Being a top mathematics student at a disadvantaged school

Thabo's representation of his schooling positions him as disadvantaged in terms of his race, language, socio-economic status and educational background. His use of negation and choice of adjectives sets up a contrast between home/school and university. He represents his school as a Black school by describing what it is not; "I am not used to Whites and Coloureds, I never went to a Coloured school or a White school". Thabo's use of negation represents the language for learning mathematics at his school and at the university as different; "my school, we don't talk English,... maths we do it in our language". He describes his school in terms of absences, for example, "lack of materials" and "shortage of electricity". In the following text Thabo's use of negation, pronouns and emphasis to describe people at university ("people here", "you") positions him ("I") as lacking socio-economically:

> People here have money, no, I'm just an ordinary person, I'm not even rich too, I wish to have those laptops but I don't think I fit in,... Back home no-one will tell you about money… the money that you eat for lunch here, I can make use of it for a <u>week</u> there, so it's a lot, could spend R100 for a <u>whole</u> week, here you could R100 for a <u>day</u>, it's nothing,….

Despite these constraints, Thabo represents his home and school with fondness, suggested by the possessive pronouns "my" and "our" to talk about language-use at school and his description of "ordinary" people "back home". He resists his positioning as disadvantaged by investing in the social identity of top student in his school, an identity set up for him by teachers and other students. He invests in this positioning by acting like a teacher for other students:

> ... I would behave like a teacher and then after school we had some sort of a study group, we studied together,..., it's not like I didn't know these things because I was just helping those who didn't know. If I wanted to go home there would be a lot of stress, some people would come to me crying, "Hey, I don't understand".

Thabo identifies himself as part of a study group ("we") in which participants have specific roles; he ("I") helps others ("some people", "those who didn't know"), the negation and pronouns identifying him as someone who does know. He reinforces the difference by recruiting the text of his peers in which they describe their own lack of understanding. Thabo's individual agency in investing in the top student identity is suggested by his repetition of and emphasis on the material process of studying in the following description of his daily routine at school:

> I study whole day, I would study during the class lessons when the teacher teaches other people, I would study. I would study after school, go home, eat, study... my self-esteem was high that I was in Grade 12, I managed to go this far so nothing can stop me.

Thabo identifies himself as joining an imagined community of students at university ("tertiaries"), a social identity set up by his school science teacher who recommends he study university science. Thabo suggests that such an identity is rare amongst his peers ("people", "they"):

> Where I come from, it's not a tertiary environment that people would dream of going to tertiary one day, it's just that they only want Grade 12... they don't love tertiaries.

Thus, as a school student, investing in the social identity of someone who will go to university not only involves the active process of studying very hard, but also the mental processes of thinking and dreaming about this imagined future.

Being a university mathematics student

Thabo's identity as a mathematics student changes relative to that of his peers. As the top student at school he is the object of interest; "people wanted to be like me, they wanted to know how I do things". Arriving at university, Thabo continues to invest in this school identity by tutoring school students. However he fails his first university mathematics test and now other successful students ("they") are in view:

> Some of the students, they passed, I used to fail and then I just wanted to be like them, but I approached them and then try to figure out how they do it.

Resisting the social identity of failing student, Thabo actively approaches these students to "figure out" what they do. After two years of studying foundation mathematics he is winning awards for his performance and, once again, becomes the object of interest to his peers who "want to put themselves close to me". However, in his third year of study he is failing advanced mathematics, but he resists this identity by representing different degrees of failing:

> I studied for those tests but still I couldn't do, not that I failed failed, you know, I managed to get close to the passing mark, but not really pass.

This resistance is accompanied by Thabo's changing description of academic success, from being "the best student" in school mathematics, to achieving 65% in foundation mathematics, to someone with his "background" being one of only two students ("it is this White guy and it is me") taking a "tough" combination of subjects. He also questions the value of university assessments in defining success:

> … you can't say you don't know something if you are not passing… you have those people who can tell you answers if you ask them but they can't pass tests, you know there is a contradiction there.

Thabo is also "angry" about having tutors assess his performance, a critique related to the academic knowledge of tutors who are "only one year above you" and favouritism based on race and personal relationships.

Being a student who loves mathematics

Thabo points to his own passion for his imagined university community by identifying other school students' lack of "love" for this community. He also consistently talks of a relationship of "love" for mathematics, no matter his representation of the practice and his positioning in that practice. He represents both school and foundation mathematics as being about "numbers", but suggests that the calculations in the latter practice are "more advanced" and require problem solving skills. He links his love of the subject to his identity as a good student; "I'm good in numbers and I love maths". Advanced mathematics is represented as different to school and foundation mathematics:

> ... I haven't figured it out yet because I'm used to maths as you know, you know you deal with numbers and stuff, but there is this other module, they do some weird stuff, ja, they call it linear algebra, it is more, it's too theoretical, it is all about proofs and stuff, you don't even see digits there, that's the thing, so it's one of my modules that I don't even see light but I am planning to,... I am sure this term I'm going to be comfortable with that module.

Thabo uses pronouns to distinguish between what he ("I") and the interviewer ("you know") recognise as mathematics, and linear algebra which is done by others ("they"). Advanced mathematics is represented as opaque, yet he expresses confidence in his ability to invest in this new social identity. This investment involves his personal agency to "figure it out". Despite this challenge, Thabo still expresses a passion for the subject, a passion that lies, firstly, in his past positioning as a good student with natural ability; "... if your mind is in, if it is based on mathematics... you always find ways to solve problems". Secondly, he invests in the new identity because he sees mathematics figuring completely (repetition of "all") in his degree programme which is "all about doing maths all the time".

Being a university student who studies mathematics in English

Thabo does not allow his initial difficulties with foundation mathematics to challenge his identity as a good mathematics student. Rather, he explains his difficulties with reference to his new social identity as a student who studies mathematics in English. He emphasises (using repetition and negation) that he is not comfortable with this new identity; "No, absolutely, no, I'm not". In tentatively identifying his lecturer as a "White guy", Thabo also suggests his difficulties are related to his race (not "White"). Thabo identifies himself as a volunteer public speaker at school and he actively chooses a variety of African languages to talk to other university students; "I speak any language that I want to speak". In contrast, his use of English in academic spaces at university is passive (it "happened to him") and he no longer "talks a lot":

> … it's uncomfortable, but I talk. I don't know what happened to me, I used to be someone who talks a lot, especially in front of people, I did public speaking in high school, I did debate, I don't know what happened to me when I came here.

Yet Thabo invests, through material and mental action, in this new social identity:

> I just make sure when I sit somewhere and people are talking, I just grasp the accent, the way they say the words and then I put them in my mind.

Thabo's feeling of discomfort learning in English continues during his academic career. However, he also ascribes his later difficulties to the action of taking "too many courses". It seems he was investing in the social identity of good student, set up by his university student advisor; "he told me, no man you can manage".

Not being a foundation student/being a foundation student

Thabo's placement in a university foundation programme positions him as disadvantaged, a social identity in which he alternately invests and does not invest. Initially "happy" with this placement, he soon resists this identity as it challenges his love of studying. His "normal", "very fast" pace is contrasted to the "slow" pace of his foundation courses:

> … in high school I studied very, very, very fast, I used to be a fast learner, but here it's slowing my pace, ja, I don't normally do this.

However, in his second year of study Thabo interacts with mainstream students in some courses and he repositions himself in relation to these students His new social identity as someone who doesn't "know everything" opens the space for him to make use of the "unlimited help" offered by the foundation programme:

> Okay, in the foundation programme they do like, I don't know how to put it, what can I say, they feel sorry for you, like they help you, you have unlimited help,… they know, they understand you, they take it that you don't know everything and here [the

> mainstream] they, okay, you managed to get into mainstream and then you know everything, so they just touch you up and they just leave you.

Thabo's difficulty knowing "how to put" his new identity points to the personal struggle that this investment involves. Yet, in his third and fourth years he draws on his identity as a Black foundation student to account for his academic difficulties. He describes the courses with the adjectives "difficult" and "very tough" and he recruits the texts of university staff to confirm his representation; "they tell me this is a tough combination". Yet he also uses his identity as disadvantaged to celebrate his achievement in making the transition, identifying himself along with one "White guy" as the only students in the "whole of the university" taking this combination.

Being alone at university

While Thabo may value the "unlimited help" offered to him as a foundation student, making use of this resource involves a repositioning, from student who teaches others to a student who needs help. Thabo attributes his difficulty investing in this new subject position to his identity as someone "from a disadvantaged area" who does not know anything. Rather than approaching a lecturer with questions straight after a lecture, he uses the textbook and the internet to "prepare what I'm going to ask". His use of the pronoun "you" (rather than the first person "I") suggests that this is the way people like him should act; "if you don't know anything, you can't ask".

Thabo's positioning as different to his school peers was on account of his superior academic knowledge. He also identifies himself as different to others at university, but this time his superiority is replaced with a sense of being completely alone. In contrast to his school identity, he uses negation to position himself as inferior in relation to his lecturer's academic knowledge, use of language and race:

> … it's a problem, most of the lecturers are White, so when they set the papers, they understand, they know how to write questions and you come there with your less knowledge of it and then you can't answer it well.

Although he studies with peers at times, he usually studies alone. His investment in his position as a successful student involves both the material action of repetitive practice, but also telling himself that he can invest in this social identity "next time":

> I… tell myself that next time I'm going to do better and study more and make sure I practice and do everything all over again, all over again.

Responding to a question about what has contributed to his successful transition, Thabo's use of negation and repetition reinforces the sense that he works alone:

> Nothing, to be honest, nothing,… This question that you have just asked me made me realise that I have actually been doing everything alone, you know, nothing, my sister's family, they never contact me about stuff like that, never – nothing.

Being a successful university student involves not investing in other social identities, positioning that illuminates the sense that Thabo is alone at university. He does not socialize "because … I am here to study" and does not pursue his "love" of acting. Thabo's meaningful object, presented at his fourth interview is a picture of people "performing drama", something Thabo represents as "exactly what I used to do a while back". However, being a science student is the natural thing to do, and he assumes that the interviewer shares this view ("you know") about "how people are":

> You know how people are, they give you this potential that you didn't see, like, you good with maths, why can't you just do this. And you know you follow them because you you are good with that.

In addition, Thabo identifies the symbolic and material power associated with being a scientist, a choice that provides "a faster way up the ladder" than becoming an actor. Gaining this power is part of investing in the social identity set up for him by his family who "see me as a successful scientist" and his identity as a Black person ("we"), "… we are Black people, we have families we have to take care of and we need to go back home". Thabo's investment in this social identity involves taking two specializations; one (which he "is not … passionate about") and another so that he can "just do research". However, investing in this identity also involves not investing in certain home identities; he studies during the vacation at home as any other identity is "so not me", and he clears his mind completely of thoughts of home when at university ("everything that happens at home just stays there").

DISCUSSION AND CONCLUSIONS

Thabo completes the mathematics courses for his science degree, despite the structural constraints he identifies and which are mentioned in the literature as affecting performance in mathematics in South Africa. His investment, in response to his changing social identities, takes on different forms, a result that both confirms and supplements other post-structuralist identity research on the transition. Thabo invests in social identities that remain stable in his transition, for example, the student who loves mathematics (e.g., Bartholomew, Darragh, Ell, & Saunders, 2011) and who imagines himself in a community that does mathematics (e.g., Black et al., 2010). However, continued investment in a social identity may require redefinition of that identity. To avoid the role of assessment in his positioning as good student (e.g., Boylen & Povey, 2009), Thabo removes assessment from his representation of success. He also invests in new social identities (the student who learns mathematics in English) and resists other identities (those that prevent him from studying). Lastly, Thabo alternates between investing and not investing in the social identity of foundation student.

Yet this analysis suggests that neither the structural constraints nor the personal struggle involved in the transition can be under-estimated. Firstly, the constraints remain present in the power relations between Thabo, his peers and his lecturers. Thabo actually harnesses his disadvantaged identity for "positive agency" (McGee & Martin, 2011, p. 1349), using it to explain his difficulties and celebrate newly defined success. While McGee and Martin (2011) intentionally focus on the construct of race, this study in South Africa points to the inter-connectedness of race, language, socio-economic status and education in defining disadvantage. Secondly, I use the word "work" to emphasise the considerable individual agency required in the investments (e.g., Smith, 2010). For Thabo, this agency involves material action like studying hard and mental action like thinking about his future. His mental action is also relational, in that it involves reflecting on others (e.g., McGee & Martin, 2011). There is a strong sense in this analysis that Thabo is working alone, and that the changing social identities impact on his sense of himself as a person (e.g., Black, Mendick, Rodd, & Solomon, 2009).

The story of Thabo's transition develops our understanding of how the successful mathematics student works to overcome the constraints of his background. Yet his investment work is relational, and thus points to the role of institutions in positioning students. As educators we should be asking questions such as, "How do we support a student to develop a mathematical identity in a way that does not require resistance to home and other university identities?", and "What support enables a student to negotiate the power of language, race and knowledge to make use of the resources on offer, rather than doing the investment work alone?"

NOTES

1. I use the term *Black* (for *Black African*), *Coloured*, and *White* for race groups since this terminology is used in reporting educational participation and performance in South Africa.

2. The study is financially supported by the Andrew W. Mellon Foundation. The work of the two interviewers Bonani Dube and Judy Sacks is gratefully acknowledged.

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THE PURSUIT OF PROGRESS

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In this chapter I use poststructural analysis to examine the production of progress within key discursive spaces. Specifically I use policy and case studies with student-teachers to argue that the pursuit of (linear and uniform) progress drives mathematical learning in the classroom. However, I contend that the notion that progress is linear and equitable is a myth; it is instead a fictional production of overt rationality and neoliberalism. As such prescriptive versions of the classroom and of mathematics thrive and the classroom becomes inequitable.

INTRODUCTION

"No single idea has been more important than, perhaps as important as, the idea of progress in Western civilisation for nearly three thousand years" (Nisbet, 1980, p. 4). For New Labour, the UK government from 1997-2010, the pursuit of progress was evident from the start of their term in office. They set the task "to challenge schools to raise standards continuously and to apply pressure where they do not." (Department for Education Employment (DFEE), 1997, p. 3). As such, (and as I argue in this article) progress has come to dominate contemporary classrooms and schools. It is difficult to argue against common sense taken for granted notions such as progress, especially ones that appear so rational. In this article, I do not do this, instead, and following Foucault, I examine what happens when something is produced as universally good. Moreover I ask what happens when progress has a position of prominence within schools and classrooms and as a consequence, if progress is beneficial to education and to mathematics. To do this, I look at progress in relation to certain discursive spaces. Primarily I examine how progress is produced in educational research, in educational policy and by a small group of six student-teachers of mathematics, who were part of a wider longitudinal study. Specifically I compare each discourse searching for moments of cohesion and tension.

In order to analyse these discursive spaces, I primarily draw upon poststructuralism and the work of Foucault both openly and tacitly. Specifically I use his interpretation of discourses, such that they are constructive rather than descriptive. In addition, discourses are permitted truths and thus "authorise what can and cannot be said" (Britzman, 2000, p. 36) and who is entitled to speak (MacLure, 2003). Foucault's work is also valuable as he critiqued the supremacy of reason, rationality and progress (Foucault, 1970).

> Humanity does not gradually progress from combat to combat until it arrives at universal reciprocity, where the rule of law finally replaces warfare; humanity installs each of its violences in a system of rules and thus proceeds from domination to domination. (Foucault, 1979, p. 151)

Hence using Foucault allows me to question both assumed notions of good and the assumed linear nature of progress. Furthermore it allows me to show how power/knowledge and governance work to produce subjects.

Specifically in this article I argue that the pursuit of (linear and uniform) progress drives mathematical learning in the classroom. However, I contend that the notion that progress is linear and hence equitable is a myth; instead it is a fictional production of overt rationality, which itself is a product of neoliberalism.

LITERATURE

Nisbet argues that "humans since the beginning of time have been interested in the idea of progress and in ways of showing how achievement is possible" (Nisbet, 1980). Thus it is unsurprising that progress is seen as a key aspect of modernity; it is a "a fundamental Enlightenment precept, the thesis that humanity is making steady, if uneven and ambivalent, progress towards greater freedom, equality, prosperity, rationality, or peace" (Brown, 2001, p. 6). Hence, progress is persuasive as it is a strong symbol of a modern, prosperous nation and is thus a goal of most Western societies. "We have coined no political substitute for progressive understandings of where we have come from and where we are going" (Brown, 2001, p. 3). Thus the overall pull of progress and its emancipatory narrative is difficult to resist, especially in educational policy (Mendick, 2011). The United Kingdom (and England), is one such nation that views itself as a key builder of both a modern society and of the wider modern world and one way they can achieve this is through education.

In the current regime of truth "people think of schooling as the major institution by which to improve society" (Popkewitz, 1988, p. 78). Hence education always already evokes a relationship to progress. This relationship functions in different ways. In the first instance education is constructed as having responsibility for economic growth and progress. This maxim has international validity, such that "powerful supranational organizations, such as the OECD and the World Bank, view education primarily as a tool for improving economic performance" (Gilead, 2012, p. 113). Specifically, within New Labour's neoliberalism,

> equity and enterprise, technological change and economic progress are tied together within the efforts, talents and qualities of individual people and the national collective – the 'us' and the 'we'. (Ball, 2008, p. 17).

Hence economic progress is constructed around the promise of benefitting both the individual and the national, which is a key assertion of neoliberalism. Moreover, a belief in self improvement is required, one which is based upon the notion of the production of the progressive subject. "Modern man... is not the man who goes off to discover himself, his secrets and the hidden truth; he is the man who tries to invent himself" (Foucault, 1994, p. 50). And within modern times, producing him/herself requires advocating progress, in particular of the self. Thus education has an emancipatory narrative where progress and redemption can be sought by the individual and society. Hence the notion of progress can be found in the human agency that Foucault critiques.

However the value of progress is not just produced through government and national economic agendas. Progress can also be found implicitly and explicitly through educational research discourse; again it is evoked as a common sense good and rarely questioned. Both Dale and Mendick (who similarly questions progress in mathematics education) point out that the nature of educational research is concerned with improvement and this is different to other types of research.

> My own and others' readings of mathematics/education policy, practice and research indicates the persistence here of the anchoring narrative of progress, as speaker after speaker "straightforwardly invokes the premise of progress... As Dale points out, this is very different from researchers in sociology of religion or the family where the research agenda operates independently of their personal views on the social roles of religion or family. In contrast, in the sociology of education, education is treated less an object of study than as a resource. (Mendick, 2011, p. 50)

Thus the majority of educationalists seek to improve education, rather than study it for its own sake. Hence progress is always already present, seems to be amplified for mathematics education. Through its foundations in science, reason and rationality, mathematics education always already evokes a promise of progress. As Mendick states,

> The compulsory status of mathematics in the school curriculum attests to this and to its centrality to the 'progressive' project of compulsory education (Jivaji, 2011), while quantitative methods provide the apparatus for measuring progress. (Mendick, 2011, p.51)

Furthermore mathematics is one of the key measures by which a school's progress is evaluated. Specifically the school is judged through the discourses that position mathematics as the basis of child development and of the normal human being (Walkerdine, 1997), which is reflected in the high status of its grades. Hence mathematics education is caught up in the production of, and producing, progress.

Thus progress in the mathematics classroom is constructed as measurable, which suits a neoliberal government which demands data as evidence of its success. From this, it can appear unproblematic to visualise a similar rational, straightforward view of progress. Next I examine how progress is constructed specifically through policy.

POLICY AND POLICY DOCUMENTS

To analyse the policy documents I went through a process of thematic coding. Once I had decided upon dominant themes, I performed the content analysis again but through a closer reading of the text. I "selected the policy documents partly systematically and partly eclectically" (Llewellyn & Mendick, 2011, p. 50); they are all documents from the New Labour regime. Specifically I argue that within New Labour's policy documentation, a very specific, target driven, measurable version of progress is constructed as the definition of success. Thus learning is tied to levels and pupils become commodities. For this to be achievable learning and progress are constructed as linear, and equitable. However I contend that this is a myth and there are different rules for different groups of pupils.

New Labour's neoliberal version of progress

New Labour's version of education, is based upon the production of a complete transformation; one such that "the 'depth, breadth and pace of change' and 'level of government activity' in education is 'unprecedented' (Coffield, 2006, p. 2)" (Ball, 2008, p. 2). From the outset they positioned education as a key symbol of government and progress as an indicator of educational (and their) success. As such progress needs to be both visible and measurable. This accountability is part of a wider shift in education towards a neoliberal managerial discourse (Ball, 1994, 2008). Language such as targets, performance management and appraisal entered the world of teacher discourse, as education became increasingly influenced by marketisation and the business world (Ball, 2008). As such, education is constructed as a system where pupils, teachers and schools are accountable; where parents (and wider society) became the consumers and pupils are measurable products fit for conversion (Llewellyn & Mendick, 2011). Hence for validity there is an emphasis on data and measurable progress. This is evident from New Labour's first white paper; for example, they state that "school performance tables will be more useful, showing the rate of progress pupils have made as well as their absolute levels of achievement"(DfEE, 1997, p. 6) (emphasising progress over the end result). Specifically they will "focus more on the progress made between different stages" (DfEE, 1997, p. 26). This assumes that progress and learning is comparable between schools and between pupils, thus it must be measurable and uniform. In addition, the use of "rate of progress" suggests speed, and that progress can (and should) be constant. Hence, like machines, pupils are capable of progressing at the same rate continuously throughout school. This linearity is given validity in mathematics through the objective driven structure of the mathematics curriculum for England which is largely based around psychological theories of development that mostly construct learning mathematics as a progressive, linear process. It is also enforced through the National Numeracy Strategy (NNS), where objectives from the curriculum are broken down. It is further legitimised through the Making Good Progress series (part of the NNS), which lists rules for moving between each level.

This both produces and is a production of success in education, which is fabricated upon these constructed levels and targets (Llewellyn & Mendick, 2011). It is authenticated by surveillance in schools, and by outside agencies and structures (e.g. league tables). The government's position is that "the rigorous use of target-setting has led to high standards and consistent year-on-year increases in the proportion of pupils who reach or exceed national expectations" (Department for Education and Skills (DFES), 2001, p. 10), which the statistics may say. However an alternative argument is that pupils are better at achieving targets as teachers are better at test preparation (Brown, Askew, Millet, & Rhodes, 2003). This demonstrates how power relations are both realised and rationalised. And how and overtly mechanistic construct of progress become the conception of children's learning, which becomes levels, and in a Foucauldian sense, this becomes the 'truth'.

Progress as inequity

All of the extracts already referenced are indicative of language of the wider policy documents. For example "pupils needs to be tracked on a regular basis and obstacles to progress identified and addressed" (Department for Children Schools and Families (DCSF), 2009, p. 27) in addition teachers are asked to "track progress and to tell pupils how they can do better" (DCFS, 2007a, p.64); suggesting simplicity and responsibility (on both the pupil and the teacher). I suggest that as progress becomes specific, simplified and measurable so do perspectives on learning and the pupil. Consequently pupils are devoid of diversity and instead are positioned as rational automata programmed to move through predestined levels (Llewellyn & Mendick, 2011). However policy implies meeting pupils' individual needs and ensuring progress are supportive of each other.

> Every pupil will go to a school where they are taught in a way that meets their needs, where their progress is regularly checked and where additional needs are spotted early and quickly addressed. (DCFS, 2009, p. 7)

Simple unequivocal suppositions such as this are a trait of government policy documents (Curtis, 2006) and again evoke authority through rationalisation. The policy statement above also indicates for progress to be real, pupils need to be under constant observation (and aware of it). One way the government propose to achieve equity is via personalised learning, which is demonstrated in the already mentioned 'Making Good Progress' series of NNS documents. For instance within these documents it is stated that "teachers use their detailed knowledge of each pupil's progress to provide more accurate support, more differentiated teaching and more personal provision" (DCFS, 2007a, p. 66). The suggestion is with appropriate support, everyone can be fixed to progress 'normally'. The documents do contain one acknowledgement that this type or progress is not straightforward. Indeed the existence of the Making Good Progress implies it is not only vital but difficult, in spite of the straightforward language that is found within.

> However, even with equal access and despite everyone's best efforts, children do not progress at the same rates. Many children who do well at Key Stage 1 are unable to maintain their progress during Key Stage 2 and slow down or stall completely. The conversion rate in 2006 to Level 5 at the end of Key Stage 2 for pupils who finished Key Stage 1 with a Level 3 in English was 78 per cent; the corresponding conversion rate for mathematics was 74 per cent. (DCFS, 2007b, p. 2)

In the above extract, economistic language is particularly evident; pupils are constructed as machines with conversion rates where the desired outcome is to reach the prescribed targets. The fabricated aim of the document is for children to progress at the rate of the designated norm, and consequently anyone that does not is constructed as deviant. Hence, a concern is that this may fix some 'unnaturally' to the norm which is an impossible ideal and one that many pupils and teachers may struggle with. Therefore I argue that personalised learning, if devised through specified targets, can be the vehicle of normalisation.

Progress and speed

New Labour also place substantial importance on the rate of progress, specifically that it must be quick. For instance they position low attaining pupils as "pupils who lose momentum"(DCFS, 2007b) or "slow moving" (DCFS, 2008) pupils. This works within neoliberalism as it acknowledges an entrepreneurial attitude. Hence speed is given an elevated position and quick learners become effective learners. This excludes other versions of learning, for example ones where pupils rely on caution or time. This draws on familiar stories of mathematics as fast paced and leads to slower learners being constructed as non-mathematicians (Walkerdine, 1998). However, there is one of group of pupils who are allowed and encouraged to be different to the norm - those who are labelled 'gifted and talented' (though this is more often realised as speed). In terms of progress, the government state that "it will be easier for young people to accelerate through the system - early achievement at Key Stage 3 or AS levels will be recognised in the achievement and attainment tables"(DfES, 2005, p.57). Other policy documents contain praise for schools that do this, thus validating their position.

As with the majority of government documents, the discourse is one of neoliberal rationality; it is decidedly unambiguous and unquestioning. This consistent and unified approach affords authority and suggests the statements are almost factual, self evident and commonsensical.

STUDENT-TEACHERS

The student-teachers were students on a three year undergraduate teaching training degree from 2006-2009. Their interviews were carried out at key points in each year. I analysed the interviews similarly to the policy documents described earlier. Specifically, I argue that the student-teachers are aware of the need for progress. It is produced as the dominant measure of learning in the classroom. However, they find it difficult to reconcile these rational expectations with fuzzy 'real' practice. Most of the tension is constructed around pupils and teachers that do not fit the normalised expectations of the rational machines. This can lead to frustration and blame, which can position teachers as deviant.

The pressure of progress

All of the student-teachers enacted the discourse of the need for progress within their classrooms. For example, Louise is aware that she must push the pupils on; for instance she states "if I say I'm working with a target group I need to work with that group 'cause I need to push them on"; which could exclude non-targeted pupils.

Whilst Louise seems to be negotiating progress, the other student-teachers produce the push for progress as problematic. For instance, the section below (which is indicative of the wider discussion and taken from a group discussion) demonstrates that tension is evident between the pressure to obtain targets and the perceptions of pupils' learning and needs in the classroom.

Anna: What do you think of having targets?

Nicola: I think there's too many of them. Because you are changing, like sometimes you're changing topic like do one week on it and then you're on to the next thing and by the end of the second week they've forgotten what you did the first week because you haven't got the time to go back over.

Sophie: I think you do need some targets set but there's so much to do in the year, some kids you can go and do it one week, they'll be perfect at it, go and do a couple of weeks of other work and then you come back and they'll have completely forgotten it. So you can't win... because you were trying to meet all the targets and all the strands, you had to skimp on certain weeks and certain bits and pieces.

Above, the push to meet targets and hence the push for progress governs the classroom. In particular it affects the mathematics curriculum (such that topics need to be covered), superseding other conceptions of learning.

The right kind of mathematician

This group interview extract also demonstrates how the notion of progress within mathematics lessons is responsible for producing an acceptable version of the mathematician (also shown in the policy documents). Specifically, both Sophie and Nicola highlight that you are not allowed to forget things in mathematics - the pursuit of progress forbids it. However Kate demonstrates how, in practice, haste can restrict progress.

Kate: So they were, a lot of the more able children were like, we've done this but they were actually making mistakes because they were just wizzing through it so we were getting them to slow down

Louise also constructs ideas of the right and wrong type of learning, she states "I think I remember what I was like when I was 6 or 7 and I just wanted to play but there's not the time, not when we need to learn not when we already have such short sessions". In this discursive space, the regime of truth with regards to learning is focused around piecemeal activities, short term targets, and activities that show measurable rates of progress; which is similar to policy.

Tension between emotion and the rational

Returning to Nicola and Sophie's discussion, blame is present within the discourse and is attached to the pursuit of progress in the system. However blame is also attached to the pupils for not being able to keep up with the system, the pupils are positioned as deviant, who "still didn't get it" (again this models the construction of pupils within policy). Specifically there is a conflation of levels and learning, and of targets and achievement; moreover there is a clear expectation that pupils should meet these targets, almost as if they were products of a 'disciplining machine', which is similar to policy's production of pupils as rational automata. Similar sentiments are shown throughout Kate's interviews.

Jane also performs frustration. She is aware that she needs to keep all pupils moving forward but she also feels the need to consolidate the work and make mathematics comfortable. She positions the pupils who "still didn't get it" as abnormal. Hence for Kate, Nicola, Sophie and Jane there is tension between the rational expectations of policy and their fuzzy classroom experiences. However Jane is different to the other student-teachers, such that she is starting to take some of the responsibility onto herself. She attempts to contain their lack of progress within her own production of herself.

Jane: but he still can't work it out in the way that we were working it out and that worries me because I don't know where to go with him now... He just still doesn't understand

Anna: Do you feel as a teacher that they should all understand [1] or

Jane: Not that they should all understand. I never think it's anything to do with them. I always immediately think that I should have done something different with those children and I just worry that I don't know what to do

Jane knows she should be ensuring that pupils make progress as simply as determined by the rational and linear policy documents. However in practice the pupils are not robots, so something must be wrong with them or wrong with Jane. Hence she positions herself as deviant, as an abnormal and unsuccessful teacher. Moreover she states that she's "got to keep moving on", demonstrating how easily pace merges with progress, which is a conflation that is mirrored in the discourse of policy. This leads to a particular version of mathematics being available.

Conflation of pace and progress

The majority of the student-teacher interview data highlights the consequences that occur when progress and pace are the dominant drivers of the mathematics lesson and in particular how this relates to different ability groups. For example Kate states the "red table picked it up quicker because they're more high ability. But greens, they didn't even reach reception level". This demonstrates the desire for a linear and continuous version of progress (and thus success) and in addition (and as discussed) how rigid expectations encourage culpability to be placed upon the pupils. All of the student-teachers produced the completion of the work as the goal of mathematics classrooms.

Overall it is clear that the student-teachers are aware that they have to convert their pupils into levels and what each table (or group) should be working at. This encourages the teacher to construct the pupils as levels, in a similar fashion to the policy documents. This can have consequences for the classroom, it becomes one driven by pace and targets and by prescription and rationality. This can create tension, especially between the rational expectations and the fuzzy experiences of actual practice and actual pupils.

IN CONCLUSION

It is clear from the discussions above that the movement towards a business model of education has had an effect on the discourse of the classroom. As such the student-teachers are caught up in a constructed production of rationality that does not model their practice; furthermore it does not allow any space for deviance, for example through emotion or through different ways of learning.

Progress is such a dominant driver of education that it supersedes all other conceptions of learning in the classroom. As such certain versions of the classroom and of mathematics (particularly ones that conflate progress and fast pace) thrive whilst others, and pupils, that do not conform are deemed abnormal. Moreover the notion that progress is linear, continuous and equitable is a myth; this is a fictional production of overt rationality. Whilst personalisation may work for some, that it encourages progress for all is a similar facade, instead personalisation can be the vehicle of normalisation.

In spite of the chimera of the gifted and talented pupil, it is the normal child that is wanted by education systems and by society. It is the normal child that can help alleviate society's ills and perform as a machine; particularly whilst education remains a political tool.

NOTE

1. Being able to understand is conflated with being able to do and hence progress see Anna Llewellyn (2012)

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SUCCESS, FAILURE AND DROPOUT AT UNIVERSITY ENTRANCE: AN ANALYSIS OF THEIR SOCIOCULTURAL CONSTRUCTION

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This paper reports on research that aims to analyse the learning processes involved in the entrance to the university, focussing on a degree programme in which mathematics has a strong presence in the curriculum. This degree programme is also characterised by having high dropout and failure rates. In this paper, failure and dropout are understood as social, cultural and historical constructions instead of understanding them as features of the students. Using a qualitative research methodology, the data were collected during six months of ethnographic field work focussed on a first year logic course for computer science majors. Three aspects that, day by day, build the sense of success, failure and dropout are analysed: the labels used to categorise the students, the teacher-student relations and the explanations around the dropout in the first year.

INTRODUCTION

This paper reports on research that aims to analyse the situated learning process involved in the entrance to the university. The attention is focused on a career characterised by giving special importance to mathematics in the curriculum and having high dropout and failure rates.

Failure and dropout problems at university have been observed and studied in different articles by different researchers (Capelari, 2009; Ezcurra, 2011; Lucero & Viamonte Leme, 2010). According to Ezcurra (2011), the apparent democratization of Argentinean universities, observed through cold statistics of high coverage rates, became opaque with a reality that is difficult to ignore: failure and dropout.

In turn, the fact that mathematics is frequently associated with failure in different educational levels cannot be ignored (Giménez et al., 2007a; Nunes Caraher et al., 1993; Knijnik, 1996). Giménez et al. (2007b) stressed that in a society strongly oriented towards professional occupation, where credentials are necessary for entering the labour market, the fact that failure in mathematics truncates someone's professional career is clearly a form of exclusion. Therefore, failure in mathematics leads to a situation in which the access to better opportunities for professional development becomes problematic. In this situation, dropout is the more obvious manifestation of a series of small failures in the school experience: lack of sense for mathematical contents, impossibility of doing homework without extra help, not having someone to ask for help and support, etc.

In this paper learning is considered from a situated perspective, shedding new light on failure, success and dropout at the university entrance. From this theoretical approach, failure, success and dropout are understood as sociocultural constructions which are daily produced in school practices.

SITUATING LEARNING, SUCCESS AND FAILURE

According to Lave (1996a), the research on socially situated activity assumes that people and the social world of the activity cannot be analysed separately; rather, there are mutually constituent relations between people, the activities that people carry out, and the situations in which these activities are carried out. This assumption implies that activities, as well as situations, become flexible and changing entities. In order to construct a theory that could embrace these mutual relations, the author suggests encompassing people, activities and situations into the category called *social practice*.

The researchers that support this theory conceive that participation in everyday life is a continuous process of understanding in practice. In this way, learning is understood as an aspect of everyday practice; learning is an integral part of the activity in and with the world. Consequently, learning is considered as a social, historical, and culturally situated process involving participation in social practices and identity construction. Instead of reducing learning to a brain activity, this theory focuses on the whole person that learns, considering her as a person acting in the social world (Lave & Wenger, 1991). That learning occurs is not the problem. What is complexly problematic is what is learned (Lave, 1996a).

In mathematics education much research has been drawn on situated learning theory (Winbourne, 2008; Winbourne & Watson, 1998; Boylan, 2005; Pinto dos Santos & Matos, 2008; etc.). But the construction of success and failure in mathematics using situated activity theory is still a little explored issue in mathematics education research.

In view of the fact that situated activity has an heterogeneous, collective, and multifocal character – because different individuals that contribute to the activity know different things, have different interests, and are located in different social places – conflict is an unavoidable aspect of human experience. An analysis of the changing participation in these conflictive practices should be centred on answering the following questions: What are the disagreements on what is relevant? When, and to what extent, is something worth knowing and doing? What to make of ambiguous circumstances? What is convenient for whom? What to do next when one does not know what to expect? Who cares most about what? (Lave, 1996a). These questions are of great importance in the analysis of the entrance to a university degree programme marked by dropout and failure. Throughout this article, the different answers that first year students and teachers gave to those questions will be presented.

If learning is always present during participation in practice, the category "learning failure" becomes blurred when it is considered in isolation. Wenger (1998) points out that “even failing to learn what is expected in a given situation usually involves learning something else instead” (p. 8). Regarding failure, the situated learning theory suggests analysing how different institutions create “learners”, “learning”, and “things to learn” as products of the socially situated activity (Lave, 1996a). In this article we will focus on the social construction of different kinds of “learners”: “*those who are gifted”*, “*those who struggle”*, “*those who have previous difficulties”*. From this perspective, success and failure are viewed, not as attributes of individuals, but as sociocultural constructions. They are positions and social processes which are usual and active. Success and failure are also socially organised identities (Lave, 1996a).

Mc Dermott and Varenne (1995) developed a theoretical perspective that is compatible with that of Lave. According to these authors, the analysis of the students who fail at school has been contextualized in two different ways. Firstly, a strand is focused on what is wrong “inside the students”, in their cognitive, linguistic, and social development. Failure is a problem of the student and the research aims at identifying, defining, and repairing the problem. These theories, deeply concerned with individual differences, abilities, and disabilities, “in the last instance blame marginalised people for being marginal” (Lave, 1996b, p. 149).Secondly, the other strand is centred on what people around these students do in order to make their life so seemingly unproductive. The attention shifts from “inside the students” to the world they live in. This second contextualization invites self-criticism, turning failure into a problem in which many people are involved: teachers, students, classmates, parents, curriculum designers and educational researchers. In this approach, the interest turns to the

> “arrangements among persons, ideas, opportunities, constraints, and interpretations (...) that allow or even require that certain facts be searched for, discovered, measured, and made consequential as label relevant” (Mc Dermott et al., 2006, p. 13).

Based on the theoretical perspectives presented by Lave, McDermott and their colleagues, this article analyses the sociocultural practices which lead to pre-established labels such as success or failure, rather than individual features, being used to justify failure.

THE RESEARCH

The research was carried out in a computer science degree at a public Argentinean university. The curriculum of this degree programme is organised in semester courses. The first year includes courses on Algebra, Discrete Mathematics, and Calculus. There is also a logic course called Introduction to Algorithms, the syllabus of which includes mathematical notions such as propositional calculus, quantifiers, recursion and induction proofs. The students spend six to eight hours per day attending courses. In this setting, the desertion level is high [1] and a significant number of students cannot pass the exams.

The main aim of the research was to analyse and describe the situated learning processes in the entrance to the degree programme. In particular, the research seeks to describe the social, cultural and institutional construction of success and failure in the first year.

Consistent with the research aim and the theoretical perspective, a qualitative research methodology was chosen. Six months of ethnographic field work and interviews with the research participants were carried out.

During the field work, focussed on the course called Introduction to Algorithms, a regular contact with four first-year students and three teachers was established. Their everyday experiences and difficulties were observed and recorded. The students – Florencia, Gabriel, Judith and Francisco [2] – arrived at the university with different previous experiences. Two of them had recently finished high school and the other two had started other university degrees without finishing them. None of the students was working at that time. They performed differently in the courses. Florencia dropped out after not passing any of the exams. Judith, Francisco and Gabriel passed Introduction to Algorithms with different grades. Francisco and Gabriel did not pass Discrete Mathematics while Judith managed to pass all the courses of the first semester. The teachers – Pablo, Juan and Lorena – also had different academic trajectories inside and outside Argentina. Juan, Pablo and Lorena started teaching at the university in 1997, 2002 and 2005 respectively.

Following the description of an ethnographic research developed by Rockwell (2009), the data collection sought to document the non-documented of the university entrance, that is, the familiar, hidden and unconscious facts and routines. At the same time, the data collection aimed at understanding the local knowledge of the first-year students and teachers. The ways the participants understood their experience were analysed through the theoretical perspective.

The analysis aimed at giving a privileged place to the voices of all the classroom participants, focussing on their experiences, their difficulties and the sense the students attributed to their first year at the university. During the analysis, a dialogue between these different voices arose.

The analysis of the data, considering success and failure as sociocultural constructions, gives rise to different intertwined aspects. In the rest of the article three of these aspects that contribute to understanding failure and dropout problems from a different perspective will be described: labels to categorise the students, the teacher-student relations and the different ways of understanding dropout.

Although this research was conducted in a specific university degree programme, the analysis encourages reflection on first year trajectories at university and the recognition of similarities with other school settings characterized by failure and dropout.

A DIALOGUE BETWEEN STUDENTS AND TEACHERS ABOUT SUCCESS AND FAILURE

While talking with teachers and students about issues related to success and failure in mathematics and dropout at the university, a dialectical interplay was established between perceiving the other and constructing the other.

Labels to categorize the students

According to McDermott and Varenne (1995), success and failure are not two separate issues but two sides of the same coin. This way, when the three teachers involved on the research spoke about their students, they defined two interrelated groups: the group of "*those who are gifted*" and the group of "*those who struggle*", "*those who have previous difficulties*".

The first group represented, according the teachers, 10% to 15% of the first year student population. During an interview, the teachers described this group as follows:

Pablo: They already come [to the university] with a gift – I don't know where it comes from – for formal work and for them it's obvious, as it should be for anyone who is trained for manipulating formal symbols.

Lorena: They are guys who deal better with the abstraction. I don't know why it is.

When teachers spoke about the "*gifted*" students they could easily identify. But it became difficult for them to explain why these students had the competences to deal with abstraction or formal symbols. One of the characteristics that teachers assigned to these students was the existence of a background knowledge that ensured them a good performance in the first year. In this case, it seems that the teachers' role was blurred, being largely relegated just to recover previously acquired knowledge. An emerging problem from this description is: if the gifted students already have a background knowledge that guarantees a good performance in the first year, then the real learning opportunities offered to them seem to be limited.

The second group was the one of "*those who struggle*". According to Juan, the teacher's role should be centred on such students, helping them to find a good pace that allows them to "make progress, pass the courses, and feel that they are learning". The same teacher highlighted that these students had a rather blurred idea of the career: "they come with little idea of where they come". As a consequence, Juan thought it was difficult to motivate them to study curriculum topics that were not naturally interesting for these students.

The students involved in the research also clearly perceived these two groups. In their daily conversations the references to these labels always appeared. Considering the students' performance, their own perceptions and classmates' perceptions of them co-constructed the labels. The following dialogue, recorded during the field work, shows the interplay between different perceptions about Francisco' performance:

[During lunch break four students were talking about their experiences at high school]

Francisco: I was dumb! Well, I'm still being a bit dumb!

Gabriel: He always pretends not to know anything but then he does very well at the exams!

The students also recognised those *"who really get it", "who were doing really very well"*, *"who were geeks"*. The members of this group embodied the meaning of being a successful student in the first year. Among the four students involved in the research, three of them passed Introduction to Algorithms with different grades. Nevertheless, none of them considered himself or herself as a member of this group. They worked hard to pass the exams, so, identifying themselves as *"those who were gifted"* would have meant not to recognise that their performance was the result of their hard work. For example, Judith, one of these students, said: "I was not a gifted student, I studied more". Others perceived themselves closer to the group of "those who struggle" expressing their difficulties following the pace of the courses with sentences like "this career overwhelm me". Francisco chose to see himself in an intermediate position: "I wasn't going as well as other students but I wasn't going as bad as others. I was in the middle, as an average student. A normal student". For the students the spectrum of positions between success and failure was broader since, while keeping in mind the two extremes recognised by the teachers, they were able to locate themselves in intermediate positions.

This analysis shows that the first year practices involved the situated construction of different kind of "learners" and their associated identities. The labels to differentiate these "learners" were based on three ideas. Firstly, success in mathematics. Secondly, the possession of outstanding abilities and, thirdly, the effort a student should make in order to pass the exams. Thus, success and failure were labels established in the classroom even before the students entered the university. The degree programme was a setting trained in searching and locating differential performances. In this way, none of the first year participants could ignore or be unaware of them. Besides, the perceptions of successful students structured the perceptions of students with difficulties and vice versa.

The teacher-student relations

Another aspect associated with success and failure in mathematics was the relations that teachers could establish with the groups of students described in the previous section. With the *"gifted"* students teachers built harmonious relationships, based on the possibility of empathising with them:

Juan: When I was a student everything was easy for me, I had a very good performance, so I identify myself with them and I understand what they are thinking.

Lorena: If you throw a glance at the classroom, they are the only ones you see [...] you don't realise why the students doing well have a good performance, because that is what is normal, that is what you understand [She refers to the way the students solve the problems].

These sentences highlight the historical construction of success and failure. Teachers' trajectories as students and their years of membership in communities related to mathematics and computer science played a fundamental role in the kind of relations they could establish with their students.

Although teachers claimed that they wanted to help the *"students who struggle"*, the relations that teachers could construct with these students were problematic:

Lorena: These people are a bit hard to treat. Because you look at them and think: [speaking quietly] you aren't understanding anything! Zero! Telling you the truth, it is a bit hard because I don't know how to cope with that.

Frustration and lack of resources were two aspects that pervade the relationships that teachers constructed with this group of students. Another aspect was the teachers' difficulty putting themselves in the place of the weak students. According to the teachers, such students were *"those who came with previous difficulties"*, with a poor mathematical knowledge background and problematic learning experiences. They did not recognise any connection between their students' problems and their teaching practices. The teachers also showed difficulties perceiving why some topics could be really problematic for their students. They continuously gave opinions about some topics in the classroom and their teaching approaches using expressions like: "[this topic] isn't difficult but the students aren't familiar with it", "I did a work for primary school, I mean, I did all the calculations [he refers to the resolution step by step of problems on the blackboard]". One of the students involved in the research described his teacher's difficulties in this respect:

Gabriel: If you asked a question, they [the teachers] told you the same they told before [...] You're one step behind, you're trying to understand things from another point of view and you can't make the connection: What's he [the teacher] talking about? What does this have to do with the things I already know? I couldn't make that connection. I wonder how is it possible that a teacher can't interpret the student in front of him.

In their daily practices, teachers constantly showed their points of view about what topics were relevant, what activities and mathematical notions were worthy of being known and which practices were important enough to spend more time on. All these points of view were strongly associated with the teachers' ideas about what topics were difficult or not. Teachers did not seem to be very flexible about that. The students frequently did not share those points of view, staying "one step behind" their teachers.

Gabriel's experience emphasises the communication problems between students and teachers. It seems that during the lectures the teachers were talking to *"those who were gifted"*. This analysis provides evidence that the sociocultural construction of success and failure is present in everyday practices, influencing the interactions between students and teachers and generating different learning settings.

Ways of understanding dropout

A last aspect related to success and failure was the sense attributed to dropout. Lorena's view on this issue stressed the gap between the students' expectations about the degree programme and the approach to programming in the curriculum:

> Lorena: [When the students enter the university] they see that this [the career] is something else. I think that if they do well they will continue and if they do poorly they will give up. The students' expectations are very vague, so we finally impose to them what we think it [the career] should be.

In fact, the conception of programming underlying the course syllabus – a formal activity deeply related to mathematical logic – contrasted with the students' initial ideas about programming: "I didn't think that programming will be so related to logic and mathematics". In this situation, the students had difficulties relating mathematical notions, such as formal proof, with their previous ideas about programming: "when we studied the axioms [of propositional calculus] I wondered: what is it for? I don't know how it would be helpful for me". Considering these discrepancies about the relevant mathematical practices involved in programming, the students delayed the sense-making of such practices during the first year.

In addition to these contrasts, students mentioned different intertwined factors to explain dropout:

> Francisco: [dropout] happens because you feel bad because you do poorly at the exams, I think it's normal. Many people think that they don't understand anything, that they have to study a lot of topics, that the teachers go too fast and that the workload in the courses is heavy. These are the factors: study, time, usually... it [the degree programme] isn't what you thought.

The causes of dropout were related to both personal and collective aspects. Considering the difficulties and demands recognized by the students, it seems that the first year was not for a "common" student.

The students' test marks were an extremely dominant indicator when they thought about dropping out the career. The way Florencia explained her decision highlighted this fact: "I dropped out [the degree programme] because I failed the exams", "After the exams, I realised that I hadn't learnt anything, so, why should I continue wasting time?"

Explanations around success, failure and dropout constructed by students and teachers decontextualized the problem. For all of them the student himself was solely responsible: "**I** didn't learn anything", "**I** failed". This way, failure was a personal experience that stressed the fact that the student did not have certain skills. It was very difficult for the students to consider that other people could be involved in their experience of failure. Only Gabriel was more critical about the learning opportunities the degree programme offered to him:

Gabriel: What about the time I dedicated to the degree programme? Where does my effort go? That's what I don't quite understand. What am I doing wrong for not getting what I want?

The situation repeats itself year after year: about 50% of the students fail the exams and many of them drop the degree programme. This way, failure is one of the most common and socially organised trajectories during the degree programme. The approach developed by McDermott and Varenne (1995) allows us to highlight that the students' problems responding to the demands of the degree programme are not independent of the arrangements and access opportunities offered by the institution.

FINAL REMARKS

In this article, social practices linked to mathematical success and failure and to dropout in a university setting have been studied. The labels used to categorise the students were analysed as social, cultural and historically situated constructions. The analysis carried out reveals the ties between the labels used to categorise the students and the research participants' personal histories. It also shows how these labels permeated daily practices and influenced the students' identity construction. Day by day, a wide range of practices contributed to construct the sense of success and failure in mathematics. Some of them, such as tests, were specific and concrete. Others, a dialogue between classmates, a teacher's casual comment, were ephemeral. To develop an ethnographic field work was a key issue in order to discover and describe all these practices. The analysis of these practices is relevant because it shows that they are at the heart of the production and reproduction of the sociocultural order in which we live. At the same time, it can contribute to promote processes of transformation and change.

NOTES

1. The degree programme has an average dropout rate of 50% for the past ten years.

2. Pseudonymous were used to refer to teachers and students in order to preserve their identities.

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A FRAMEWORK TO UNDERSTAND THE CONTRIBUTION OF QUANTITATIVE LITERACY TO THE SOCIAL JUSTICE AGENDA: A PILOT STUDY

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This paper reports on a pilot study that aims to develop a framework through which we interrogate the contribution of the course materials in a Quantitative Literacy (QL) course towards the promotion and appreciation of a socially just society. Through a naturalistic inquiry of the course materials, a framework that uses, as domains, the Bloom's taxonomy to analyse the cognitive levels of the tasks in the materials against QL competency and knowledge areas within the context of social justice is developed. Preliminary findings from this pilot study show that there is merit in the use of the framework to analyse the contribution of QL course materials towards the social justice agenda.

INTRODUCTION

The primary aim of the Quantitative Literacy (QL) courses for humanities and law at the University of Cape Town is to equip students for the quantitative demands of their studies in higher education. Given our context of teaching in higher education in a post-apartheid South Africa with all its challenges, our work is driven by a concern for social justice and a belief that students should have a critical/questioning stance in civic matters. This concern for social justice is embedded in the process of sensitizing students to the extensive social problems in our country (Frith et al., 2010). The teaching of QL has an embedded social justice agenda in that it prepares students to participate in societal change and make them aware of the environment (Jablonka, 2003) while giving them social capital (Yasukawa et. al., 2008).

The courses are primarily context driven with the QL embedded in contexts such as Children's Rights in South Africa, prison overcrowding and xenophobia (Frith et al., 2010). Given that QL is taught through contexts with inherent difficulties such as issues of language and those that arise in the contexts themselves, the scheme of sensitizing students to explicit social problems in our society seemingly has a hidden agenda of social justice that is never made explicit to students, nor is it articulated anywhere. Furthermore, the course(s) are considered to be courses in mathematics by the students as they are coded as other courses in the Mathematics Department.

Is there a hidden agenda of social justice in the QL courses for humanities and law? This paper forms a part of a larger study which aims to: (i) understand whether the current structure and presentation of the courses constitute a hidden agenda and, (ii) if so, to explore the nature of this agenda. This paper, therefore reports on the development of tools necessary to carry out this analysis.

We support the view of a quantitatively literate person as one who exhibits the competencies in disciplinary contexts (Frith & Prince, 2009). This would include the behaviour of someone with a critical stance (Skovsmose, 2000). If a social justice agenda does exist, then QL competencies with a critical stance should be observed in the context of social justice (Wiest et al., 2007) or at least the courses should (unashamedly) seek to contribute to this ideal. This article reports on a pilot to develop a framework for understanding the contribution of course materials, including assessments, to such an agenda.

THEORETICAL LANDSCAPE

This study utilises the cognitive apprenticeship theory as an instructional model within the broader social constructivist paradigm. The aim of the cognitive apprenticeship theory is to "address the problem of inert knowledge and to make the thinking process of a learning activity visible to both the students and the teacher" (Ghefaili, 2003, p.1). In our view the cognitive apprenticeship theory has roots in and is strongly influenced by: socio-cultural theory of learning, Vygotsky's Zone of Proximal Development (ZPD), situated cognition and traditional apprenticeship. These four philosophical notions are by no means the only ones that have inspired the development of the cognitive apprenticeship theory (Ghefaili, 2003). Critical to cognitive apprenticeship theory is the notion of the "situated cognition" or "situated learning". Orgill (2007) posits that knowledge cannot exist as a separate entity in the mind of an individual, but is generated as an individual interacts with his or her immediate environment. Essentially what QL does is to allow the use of mathematical tools such as: data analysis, graphing, and modelling to explore authentic real life contexts that provoke a sense of social justice thus empowering students to seek change (Lesser & Blake, 2006). Thus QL becomes a tool to scrutinize the social environments, promote social justice awareness and serves as a vehicle to further an agenda for social transformation towards a more just society (Gonzalez, 2009).

Knowledge construction under situated learning requires an environment where students, working in communities of practice, engage with each other and the materials of instruction. Within these communities of practice, students share and deepen understandings, and create knowledge from collective learning opportunities in the course (Brown et al., 1989; Ghetaili, 2003; Macklin, 2007; Orgill, 2007; Wenger, 1998). We, therefore, argue that in so far as the cognitive apprenticeship theory is concerned, the challenge is to situate learning and teaching activities in contexts that make sense to students. Brown, Collins and Duguid (1989) argue that conventional approaches to education and training no longer provide students with authentic activities, and therefore are not fully productive for any meaningful learning to take place. These conventional approaches in the form of formal schooling were intended to represent a shift away from the traditional apprenticeship learning model which was largely focussed on skills development. In traditional apprenticeship learning tasks are usually observable and learning is wholly located in the workplace. Although both instructional models focus on skills development, the cognitive apprenticeship theory, as an alternative to conventional approaches to education and training, aims to "produce graduates with equal thinking and performance capabilities" (Bockarie, 2002, p. 48). In cognitive apprenticeship theory, then, the aim is to give the student the opportunity to generalise the skill, know when a skill can be applied and that the skill can be transferred to solve unfamiliar problems in many different settings or contexts (Collins, 2006; Collins et al., 1991; Ertmer, 1995; Hendricks, 2001). By way of contrast, the underpinnings of the cognitive apprenticeship theory also differs from the school curriculum approach in the sense that the latter places huge emphasis on the problems that appear in textbooks and class presentations, thus depriving students from reflections and exploration of new ideas. In the QL courses there are no prescribed textbooks, hence the instructional materials are excerpts from media and research reports and census data among others.

The sociocultural theory of learning has great implications for the cognitive apprenticeship theory. A feature of the sociocultural theory of learning pertinent to human development is that higher order functions in the QL course develop through social interactions. Vygotsky (1979) acknowledges that in order to understand the human development of an individual we need to study both that individual and the external social world associated with him or her. Kublin et al. (1998, p. 287) argues that "Vygotsky (1986) described learning as being embedded within social events and occurring as a child interacts with people, objects, and events in the environment". Human development and learning factors such as communication and linguistic styles, and academic backgrounds are born out of social and cultural interactions (Packer & Goicoechea, 2000). We are aware that the sociocultural theory of learning is more robust than this brief preview discussed here, however, the elements of the theory highlighted will help to show the connection between cognitive apprenticeship theory and the sociocultural theory of learning.

We now examine the association between Vygotsky's Zone of Proximal Development (ZPD) and the cognitive apprenticeship model. The ZPD represents the gap in cognitive development between the actual development level which is determined by independent problem solving and the potential development as determined by problem solving under the guidance or in collaboration with someone who is more experienced – potentially a teacher, parent or even a capable peer. Within the ZPD, through social and cultural interactions, students receive instructional support from experienced peers and teachers in a particular QL context. Throughout these social and cultural interactions, social tools, such as language and other sign systems play key roles in the cognitive development and learning (Bockarie, 2002; Dennen, 2006; Ghefaili, 2003; Tudge, 1990; Zuengler, 2006). After internalising the skill or information the student will be in a position to independently carry out a similar problem solving situation. The social and cultural interactions within the ZPD are critical to the cognitive development and culture of an individual in that they allow them to participate in the QL course learning activities that would have been inaccessible to them through their own attempts. We posit that the cognitive apprenticeship learning takes place in the ZPD and that the ZPD is an important factor to consider when scaffolding QL learning activities (Bean & Steven, 2002; Rogoff, 1990; Rogoff & Wertsch, 1984).

In sum, the cognitive apprenticeship theory has two important implications for cognitive learning. First, students internalize the skills they have learned to enable them to carry out tasks or solve problems independently. Second, students are able to generalise concepts they have learned so that they can apply their acquired skills to identical contexts and use their current knowledge base as a starting point for further learning (Bockarie, 2002).

FRAMEWORK FOR DEVELOPING A QUANTITATIVELY LITERATE PERSON

In this section we outline how the framework for understanding the contributions of course materials in teaching of QL through social justice was developed. The framework is divided into four domains, namely: content categories, expected QL competencies, Bloom's taxonomy and social justice agenda. With regard to content categories, we deal with the nature of QL content and the related specific learning objectives for each content area. A content review on Unit 1: Children's Rights was carried out. Once all the content areas were listed, re-classification of the content areas into the following five categories: number sense and representation, proportions, statistical data representations, compound growth and everyday personal finance took place. Table 1 below shows the categories and the specific learning objectives associated with each.

Table 1: Content categories in QL & specific learning objectives

Content	Specific learning objectives
Number (or quantity) sense and number (quantity) representations	Orders of magnitudes; size factors of numbers; scientific notation; absolute numbers; significant figures and decimal places; accuracy.
Proportions	Percentage changes and percentage points; Relative numbers (e.g. per 1000); ratios; fractions; proportions.
Statistical data representations	Tables, charts and graphs; measures of central tendency; measures of spread; scatter plots and lines of best fit; box-and-whisker plots; probability; co-efficient of variation; trends lines; samples and sampling
Compound growth	Compound growth; growth factors; growth rates; growth trends
Everyday personal finance	Inflation rates; interest rate and compound interests; actuals and real earnings; time value of money; annuities.

As alluded to earlier in this paper, the mathematics content in the QL course is situated in contexts. Each context is accompanied by tasks for students. These tasks demand particular mathematical and critical thinking competencies from the students. Table 2 elucidates the core descriptors in each competency strand. The approach employed to identify the five content categories was replicated to determine the seven competency categories from the tasks. Therefore, as part of our data analysis, we also seek to establish and quantify the distribution of competencies within the content categories.

Part of the framework of this study was the use of the revised Bloom's Taxonomy to analyse the cognitive levels of context tasks from the course materials. This is the first time that the tasks from the course materials have been subjected to an analysis using the Bloom's Taxonomy. Ideally, one would prefer a situation where tasks incorporate higher levels of the Bloom's taxonomy since they tend to foster higher order thinking. The revised Bloom's Taxonomy is a two dimensional framework which consists of the knowledge dimension and cognitive process dimension. This study considers the knowledge dimension to be represented by the competency categories such as: factual, procedural, conceptual or metacognitive knowledge. The six cognitive process dimensions are represented by illustrative verbs: remembering, understanding, applying, analysing, evaluating, and creating (for more details on Bloom's taxonomy visit: www.nmlink.com/~dondark/hrd/bloom.html).

The final component of the framework considers the distributions of the social justice (SJ) agenda within the QL contexts and related tasks. We sought to establish if the SJ agenda is explicit in the QL context and/or related tasks or not explicit in either of them. From this analysis, we hope to be able to tell which of the three categories under the SJ agenda are predominant in the course materials. The framework which evolved from the analysis of our course material is summarised in Table 3. After developing the framework it was validated using Unit 1 and Test 1of the course materials.

Table 2: Expected QL competencies

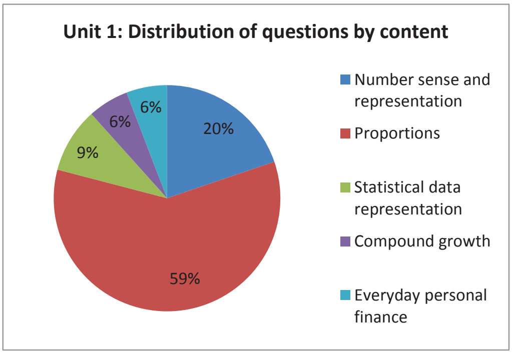
Competencies	Core descriptors of the competency
Comparing numbers	Conversions of numbers from one form to another
Reading from tables, charts, figures and text	Meaning making of numbers in texts, charts, tables, graphs and figures; identify trends in data; comparing data in charts, tables, graphs, figures and texts
Procedural competencies	Routine calculations; relationships between quantities; substitution and manipulation of formulae
Social justice beyond the classroom	Finding solutions for Social Justice agenda beyond the classroom activity; applying the social changes to advocate for real changes; being part of the solution to a SJ agenda
Writing skills	Communicating information effectively; clarifying thinking; synthesizing information thus increasing comprehension; explaining understandings of concepts and ideas; applying acquired knowledge to new unfamiliar situations
Understanding and applying data representation methods	Familiarisation of with data representation methods (tables, charts, scatter plots, box-and- whisker plots etc.); analysing data presented in various data representation methods; interpretation of data presented in various data representations methods
Critical thinking skills	Involves asking questions about the content of the contexts; examining evidence; analysing assumptions and biases; tolerating ambiguity; alternative interpretations

Table 3: Summary of the framework

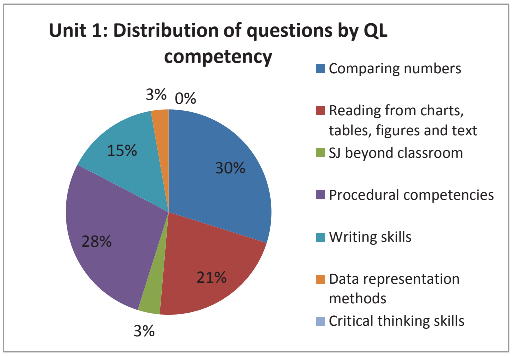
Course content	Competencies	Bloom's taxonomy	Social Justice agenda
• Number sense and representation • Proportions • Statistical data representation • Compound growth • Everyday personal finance	• Comparing numbers • Reading from charts, tables, figures and texts • SJ beyond classroom • Procedural competencies • Writing skills • Data representation methods • Critical thinking skills	• Remembering • Understanding • Applying • Analysing • Evaluating • Creating	• SJ not explicit in both context and question • SJ explicit in one of them • SJ explicit in both context and question

VALIDATION OF THE FRAMEWORK

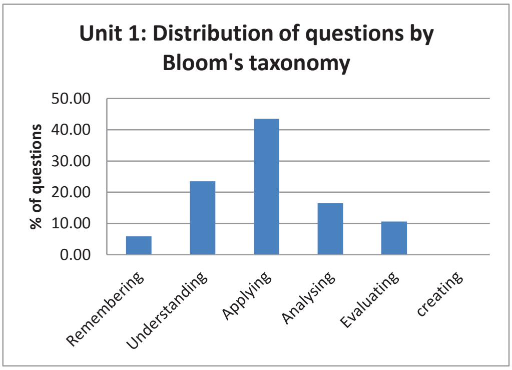
Unit 1, as well as the Test 1 that occurs immediately after the completion of the unit of the QL course materials were analysed for the development of the framework. The document analysis yielded distributions of the questions in Unit 1 and the Test 1 by QL competency, mathematical content, as well as Bloom's taxonomy. Both the unit 1 and test 1 use a social justice context explicitly so there was nothing to analyse on the social justice domain.



Findings showed that while a large percentage (59%) of questions dealt with the mathematical content area of *proportions*, a significant proportion (20%) of questions were based on *number sense and representation*. This can be expected as it was by design to place an emphasis on those content areas. However, as depicted in the QL competency distribution chart, there were no questions testing critical thinking skills in QL competency, despite there being an even spread of questions in QL competencies on *comparing numbers*, *reading from charts/texts/ tables* and *procedural competencies*.



Almost half the number of questions (44%) was pitched at the application level on Bloom's taxonomy and none at the uppermost level of creating. This, however, is the first unit and pitching it at this level could be expected as well as the fact that development of students occurs on the different domains.



An analysis of expected frequencies of questions in Unit 1 of the various domains against each other revealed a statistical significant difference only in the distribution of questions in *comparing numbers* competency within the *number sense and representation* content area with the level of remembering having a higher than expected frequency (residual *p*-value of 0.00297).

A basic analysis of expected frequencies of questions on the various domains between Unit 1 and the Test 1 revealed no statistical difference in the QL content domain whereas a highly significant difference on the QL competencies domain and a significant difference on the Blooms category domain were detected. Lower than expected number of questions on *comparing numbers* (*p*-value 0.00613), higher than expected number of questions on *reading from charts* (*p*-value 0.00101) and higher than expected number of questions on the *applying* level (*p*-value 0.00506) were observed.

CONCLUSION

This study has developed and validated a framework to understand the contribution of QL to the social justice agenda. Preliminary findings from this pilot study show that there is merit in the use of the framework to analyse the contribution of QL course materials towards the social justice agenda. Using the framework will allow QL facilitators to ascertain whether the QL course materials provide the opportunity to sensitise students to the extensive social issues in their communities. In addition, the framework allows QL facilitators to assess the articulation between QL course assessments and the course learning outcomes. Further research will focus on applying the framework to broader QL course materials in order to interrogate the various domains.

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THE TEACHING OF MATHEMATICS IN RURAL SCHOOLS IN BRAZIL: WHAT TEACHERS SAY[1]

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The majority of research in mathematics education in Brazil focuses on aspects related to urban schools. Although there has been a discussion for decades about the need for a curriculum in rural schools that considers the importance of the local culture and the concept of context-based knowledge, there is still poor teaching of mathematics in these schools. This paper discusses research data that addresses the teaching of mathematics in rural schools located in Pernambuco, Brazil. We focus on the discourse of teachers who participated in the empirical study by analyzing their views on rural education and on their performance in mathematics teaching. Generally, the teachers were unaware of the rural schools' specificities. The results lead us to reflect on the possibilities of teaching mathematics to empower rural communities.

INTRODUCTION

In Brazil, rural spaces incorporate areas such as the forest, livestock farms, agricultural areas, fisheries, mines, quarries and extraction plants. Social movements and organizations, as well the governmental education legislation, use the term "field education" to refer to the teaching and learning processes which are developed in these contexts. According to Operational Guidelines for Basic Education in Field Schools (Brasil, 2002), the term "field" emphasises those spaces as a field of possibilities. In fact, there is a theoretical and ideological debate about the terms "rural education" and "field education" which is beyond the scope of this research paper. For lack of an English term that differentiates and expresses the education of people living in areas of the field, this article will use the term rural education.

According to the 2000 Census (Brasil, 2003), 18.8% of the population of Brazil lives in rural areas and 32.7% of this rural population aged over 15 are illiterate, while in urban areas this ratio tends to be lower, at about 10%.

Historically, the organization of schooling for the rural population in Brazil did not consider the particularities of these contexts. Educational policies were not concerned with schooling that considered the rural reality. In this sense, the curricular content, teaching methodologies and pedagogical proposals of the urban schools were transported to the rural ones. This led to schooling that ignored the potential for better development of rural citizens and did not guarantee their right to this development (Monteiro, Leitão, & Asseker, 2009).

According to Souza (2006), the origin and organization of Brazilian schools is connected with the social and political bases of the unequal distribution of land and with slavery. It was only during the 1880s that the government started to develop formal education for rural areas. However, Souza states that the first Brazilian rural education system began in the 1920s. This initiative was called “patronato”, with the aim of domesticating rural workers. This initiative was intended to improve agricultural production and to prevent overpopulation of cities. The “patronato” is characterized by the transport of knowledge and practice from urban schools to rural schools, without any consideration of the diversity, the specific experiences and the knowledge associated with rural life.

From the 1980s, several social organizations and movements demanded more attention to the education developed for rural inhabitants and a curriculum that considered problems and possibilities of rural education. This curriculum should give value to the different knowledge and needs of education in rural areas (Arroyo, Caldart, & Molina, 2004). However, Brazil is a country with continental dimensions and many different contexts of rural areas. In some regions of Brazil, the “patronato” continues to exist.

The contemporary school needs to seek an intensive dialogue between the various actors involved in the learning process. The teacher needs to realize that no group is homogeneous and that there are specificities related to each student involved in the educational process. For example, he/she needs to respect students’ skills and values. However, when it comes to rural education, the specificities most often are overlooked.

We consider that rural schools have a particular role in society, because they are situated in rural communities where people have unique culturally constituted experiences, and where the means of production are linked to local reality, and yet at the same time are part of global reality.

In this paper we reflect on the teaching of mathematics in rural schools by making a link with socio-historical aspects of rural education. We analyse data from a research study developed with teachers of rural schools in Pernambuco, and discuss what these teachers say about the socio-cultural reality of rural contexts and uses of resources in teaching mathematics.

THE TEACHING OF MATHEMATICS IN RURAL SCHOOLS

Most research on mathematics education in Brazil focuses on schooling in urban areas. Therefore, this paper presents a discussion that is fairly rare among Brazilian academic studies.

Generally speaking, in Brazil, there is a common perception that rural schools are institutions without resources to provide good teaching, and therefore do not provide the elements to guarantee students’ learning processes. In fact, statistics (Brasil, 2005) indicate that in Brazil many state primary schools (including those located in rural areas) do not have basic infrastructural equipment (approximately 22 000 schools do not have toilets and 27 000 do not have electricity). However, this aspect cannot be the only factor which influences the students’ performance, because in the last few years the government has implemented important projects which are changing the infrastructure of many state schools, but the level of achievement remains very low. Therefore it seems that the effective teaching of mathematics is a complex issue, and the increase in quantity of resources available is not a factor that will solve this issue. .

Knijnik (2004) argues that mathematical knowledge has been linked to the economic power of dominant social classes. The contents of mathematics and practices of teaching and learning mathematics in school would be linked to the values and interests of the dominant class. Under these conditions, schools would be presenting only a limited mathematical knowledge, denying the importance of knowledge and practices related to specific contexts, such as those developed in rural areas. Knijnik (2004) emphasizes that mathematics curricula should make knowledge accessible to students of all classes and social contexts. Thus, diverse populations, regardless of their social position, should have access to the relationship between the practices of everyday life and schooling.

Knijnik has investigated these practices in an environment of social movements, such as the Landless' Movement and rural communities (Knijnik, 1996, 1998, 2004). She highlights that students should be led to think and rethink mathematical situations from their realities, especially those professional activities which include local mathematical practices.

Garnica and Martins (2006) argue that stereotypes about rural schools are linked to historical aspects of mathematics teaching. These authors analyzed interviews of rural school inspectors, teachers and students in 1950 and 1970. In the interviews respondents were asked to report their experiences with the teaching and learning of mathematics. The analyses of the responses suggested that they reduced the teaching of mathematics to the instruction of methods of calculating (addition, subtraction, multiplication and division), and memorizing multiplication tables.

Adler (2001) emphasises that to understand social practices which are developed in schools, it is necessary to analyse the access to certain resources. Mathematics educators need to develop a wider conceptualization of resources in the teaching of mathematics which embraces material objects as well as activities and processes which constitute and emerge from diverse educational practices related to the teaching of mathematics.

Adler (2000) argues that general descriptions of resources in teacher education (pre-service and in-service) seem to be ineffective. Instead, teacher education programmes should work with teachers in order to help them to learn how to conceptualise and approach different resources (human, material, process, etc.). Teachers should understand resources as mediators which can amplify the possibilities of their teaching.

Melo, Leitão and Alves (2007) discuss some aspects of teacher education in the reality of rural education in Pernambuco. They report that among rural school teachers who participated their study, 25% had completed only a middle-level teaching qualification (corresponding to professionalizing course at high school level). Therefore those teachers did not graduate with an education certificate from faculty or university. These authors reported that participants said that the lack of resources and teacher education affecting rural schools contributes to students' failure in mathematics.

Asseker and Monteiro (2008) argue that it is important to change teachers' perspectives on teaching mathematics and to make them value the local culture as providing fundamental resources for the teaching and learning of mathematics.

METHODOLOGY

In this paper we discuss data from semi-structured interviews conducted with 12 teachers from 4 rural schools situated in the Pernambuco state of Brazil. These schools belong to a network consisting of 90 rural schools. Interviews were conducted from May 2007 until June 2007 and each took about 1 hour and 30 minutes to complete.

The main criteria used to include the 4 schools and the 12 teachers in the data collection were: 1) the teacher's availability and 2) the school's accessibility. The rural schools where the interviews were carried out have only two classrooms and each teacher works with a class group comprised of students in up to four different school years and hence with a large range of ages.

The local education authorities have developed an educational program for this type of class group. It is called *Escola Ativa* (Active School) and was suggested by the Ministry of Education of the Federal Government. The aim of this program is to improve the quality of teaching in primary schools in the poorest rural regions, and to reduce the gap between children's age and the school year they are attending. According to Piza and Sena (2001), the *Escola Ativa* combines different experiential strategies with the aim of stimulating the learning, collaboration and the participation of the students.

The aim of the analysis of the interviews was to investigate the participants' discourse, including their opinions about rural education and their own practice as mathematics teacher. Each teacher was interviewed individually. All interviews were recorded on a digital audio player and then transcribed into protocols. Significant passages were selected, based on a content analysis approach.

RESULTS

All participants were female, with an age ranging from 22 to 53 years (mean age: 35 years). Eleven of them have a higher education certificate; 9 in Pedagogy, 1 in Portuguese Language, and 1 in Accountancy. They have a professional experience ranging from 4 to 25 years (mean: 13 years of experience). One participant has a middle teaching level qualification (*Magistério)* which is equivalent to high school level. In this section, we focus on 10 interview questions related to teachers' opinions about rural education and the teaching of mathematics.

Generally, the teachers seem to have a conception of rural education associated with the students' realities. However, they conceptualize rural realities as restricted and inferior compared to the students who live in urban areas. Within the scope of this paper we cannot report all relevant extracts we transcribed from the 12 interviews. The following interview extracts, from 4 different teachers, exemplify the teachers' conceptualisation of rural education.

Interviewer: What is rural education to you?

Zilda: (...) Because you work... You can work with the children the concrete. In the rural area there is the concrete to work with them, you work with animals, plants, water... all these things in a rural area, in a rural school. In town you do not work in concrete... it is very difficult. So, to me rural is all this, because of that, the environment, because of the difference of environment.

Interviewer: Do you consider this school as a rural school?

Zilda: Well, although the school is located in rural area, my students… I do not consider them as students from rural area.

Interviewer: No?

Zilda: No, not because they have, they have access, so they have knowledge… town boy has, today they have. The same knowledge that a town boy has… not before, when I first came here, they did not have, twenty years ago it was very difficult, isn't it? There was not electricity here, today he has knowledge… the boys around here they already participate in the city things, today they have more live in the city, the local boys go to the mall, go... So they have much access to the city; the only difference is because they live far from the city centre.

(Zilda is 53 years old, graduated in Pedagogy, 25 years of teaching experience)

Zilda's point of view about rural school is heavily linked with the idea of agriculture. In this sense, her opinion deviates from what is discussed by official contemporary documents and by social movements.

In the interviews, the participants told us that the local government education secretary provides in-service education for mathematics teaching. However the participants considered that this support does not adequately prepare them to teach this school subject.

Interviewer: How about the teaching of mathematics, have you received orientation?

Nadia: I have received a different methodology to innovate, isn't it? This is because day-by-day children need to increasingly have to follow the changes, isn't it? Playing games, developing everyday situations, that's what we have been working since the beginning of the *active school* which work on it, there must be change, cannot be that traditional.

Interviewer: Specifically in mathematics?

Nadia: Especially in mathematics. It is what they most beat up, right? Lots of games, lots of logical situations that we have to work out, to escape from the sameness, isn't it? More elaborations of problem situations which do not have to be those traditional algorithms, after they catch those systems, we can develop only problem situations, isn't it? With the four basic operations… and so on with a fraction… and so on.

Interviewer: Do you think the guidelines that the secretary gives are enough?

Nadia: No. My experience is also taken into account. I seek a little bit of my experience because it counts. Especially because, not everything that is discussed in the meetings is the reality that we find in the classroom, right? So we have to adjust, sometimes is the issue that I told you, we prepare all different, isn't it? Those instructions we received... However, sometimes we escape from those instructions a little bit to reach the student, the student's knowledge, isn't it? I fit myself into his situation, into his need, right? I think this is education, isn't it? We cannot say that it has to be to that side and the child does not understand, that does not work, right? I have to see what is the best way to bring knowledge to them, that's how I do in those 20 years, isn't it?

(Nadia is 48 years old, graduated in Pedagogy, 20 years of teaching experience)

As we can see in the extract from Nadia's interview, she considers the educational secretary's guidelines as not linked to rural classrooms. In order to deal with this limitation, she recognizes the need to adjust the instructions that she receives. Therefore, when she does the adjustment she clearly is mobilising a kind of resource to teach mathematics.

Another example of the use of resources in teaching mathematics in the context of rural schools is provided in Julia's interview. In this extract from her interview, we could infer that she generally emphasises the idea of resource as only including material objects. However, when we specifically asked her what "resource" means, she seemed to consider other aspects.

Interviewer: What is resource for you?

Julia: It is your creativity. You can take the best resource, the best material and then place it in a classroom, but if you do not have creativity it does not make any difference… I achieve many things in my classroom playing with them. I put them at the playground and start playing, playing… Sometimes

> I want to teach one thing, but then many other things emerge and then I embrace these things and teach even things that were not planned.

(Julia is 38 years old, middle teaching level qualification, 15 years of teaching experience)

Julia makes explicit her unplanned strategies to use resources in her teaching of mathematics. The extract above suggests that she also considers the nonmaterial dimension of resources. For example, she emphasises creativity as a resource and mentioned that the process involved in the use of a resource is more important than the resource itself.

The analyses of the interviews showed that 7 teachers positively evaluated their performances when they teach mathematics, as the extract below exemplifies.

Interviewer: Yes, its performance as a teacher of mathematics. How would you describe your way of acting as a mathematics teacher?

Claudia: Look, Mathematics for everybody, that's how we usually speak, is a “big deal” [literal translation: monster with seven heads]. I do not like math, you know? But whenever I go to math class we have to search, you have to study and you also work with the resource that you have on hand, but I'm not in love with mathematics don’t you see? [Laughs]

Interviewer: Hum-hum.

Claudia: No... Well, I study mathematics more when I'm teaching, or studying for a competition, so now I have no FEAR, FEAR [said with emphasis]. The math is not what you say, it is not a big deal, a bogeyman, and sometimes the math becomes a thing simple for us, only we do not know how to use it not so? Because so maths always is in the life of the people. When you wake up, you already have a timetable to get up, get ready for ... then it is math, right?

(Claudia is 41 years old, graduated in Accountancy, 10 years of teaching experience)

It seems that Claudia is conscious about her role as a resource to teach mathematics. When Claudia stresses the need to search for and study new and different ways to teach, she recognises that this attitude can overcome restricted ideas about mathematics.

Another aspect observed in the speech of teachers is the lack of knowledge of the specific reality of the countryside which is different from that of urban areas. The teachers tended to regard the countryside as they would an urban area, and thus reproduce in multi-grade classrooms in rural areas the knowledge, methodologies, mathematical content and goals that are specific to the reality of cities. This perspective can be seen in the words of teacher Julia when she assigns to students a lack of interest in learning.

Interviewer: Do you think there is a difference between teaching in rural and the urban area?

Julia: It has a lot of difference. I think teaching in the city is easier, students learn faster, parents encourage the children and students have access to TV, newspapers, DVD's, magazines, shopping lists that her mother makes... , here there are none of these things. Look. Even homework, they do not do... They do not want to learn because they think they know what they need to do what parents do, work in the field.

(Julia is 38 years old, middle teaching level qualification, 15 years of teaching experience)

This teacher speaks of the lack of motivation of students to engage with scholarly knowledge and it can be inferred that this is because the curriculum does not reflect the mathematical content of the reality of the field.

All teachers reported that they did not identify a difference between teaching in the countryside and the city. This is presumably because the initial training received by the teachers did not offer enough support specific to teaching in rural areas.

It is essential to address issues related to the conceptualization of resources as an extension of the teacher in the practices associated with teaching mathematics at school, especially given the rural reality, so that students can construct meaningful mathematical knowledge.

FINAL REMARKS

The characteristics of the current Brazilian rural schools are a result of socio-historical processes. They include an unequal social stratification that attributes less value to the rural population. Only in recent years has rural education begun to have a particular approach which tries to build an identity that respects rural realities.

Our data analysis indicates that the pedagogical organisation of rural schools does not yet consider the particularities of rural education. Generally speaking, the teaching of mathematics has a peripheral status within their pedagogical planning because less time is spent in class on this subject than on other school subjects. On the other hand, the teachers' conceptualisation and use of resources to teach mathematics is diverse. They emphasised material resources, but also saw their attitudes to overcome resistances and limitations in this school subject as a resource.

However, the discussion about a wider conceptualization of resources in the teaching of mathematics (Adler, 2000) involves the consideration of different types of resources, such as material, process, human, socio-cultural, etc. Therefore, in order to approach the concept and use of resources it is necessary to analyse the social practices which teachers develop in classrooms but it is also important to understand the macro-cultural context.

Our study suggests that the conceptualisation and the use of resources should be a crucial topic in teacher education (pre-service and in-service). The teacher education program needs to provide situations in which teachers can learn about the elements and processes related to the types and uses of resources in the teaching of mathematics, considering the socio-cultural context in which the schooling processes are developed.

NOTES

1. This paper is based on discussion of research projects which had financial support from CNPq – Conselho Nacional de Desenvolvimento Científico e Tecnológico, and FACEPE – Fundação de Amparo à Ciência e Tecnologia do Estado de Pernambuco.

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TEXTBOOKS OF MATHEMATICS AS LITERARY MICROCOSM: WHY AND HOW CAN THESE TEXTS BE ANALYSED

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The aim of this paper is to show how analysis of mathematical textbooks can contribute to the effort of uncovering the roots of common biases and prejudices in the society, for example the stereotypical gender roles. The authors build on the fact that the world of word problem assignments and expository narrative of school textbooks of mathematics is a fictional world resembling the microcosm of any work of art, which allows their analysis in the framework of literary theory. The presenting author of this paper is also an author of one set of mathematical textbooks used in the Czech Republic. Thus the paper includes first-hand experience and intentions when conceiving the word problems and narratives in her textbooks.

INTRODUCTION

The need to create a solid methodology or framework for textbook analysis is well known. As the international textbook market is flooded with hundreds of different textbooks designed for the same age group, researchers and educators have been working on projects trying to somehow compare them. Most common are international comparative studies of the textbook content.

However, the aspects that can be analysed are numerous, depending on the objective of the whole project. The analyst may focus on the content, language or didactical aspects. It is possible to study the difficulty and comprehensibility of the text, its correspondence to the language children and pupils may know from children's books, validity and reliability of the textbook, its readability, age appropriateness, the ratio of compulsory and extensional subject matter, the ratio of textual and non-textual components etc. It is equally important to study the outcomes and effects of the work with the textbook, or teachers', pupils' and parents' attitudes (Průcha, 1998). While it seems relatively easy to make quantitative analyses, e.g. the ratio of verbal and non-verbal elements, content of curricular units, or difficulty of the language used (there are even formulas helping to evaluate some of the variables), an analysis of the semantic content of the story the textbooks tell, is much less easy.

Recent developments in the world, globalization, integration processes in Europe, multiculturalism, racial tension and other thorny issues has resulted in the interest in the content of textbooks, not in the sense of curricular topics, but of the discourse used by their authors, of the stories they tell, the authors' opinions and views that influence future generations, and their beliefs. That is why the debates over the content of school textbooks are sites of considerable educational and political conflict. According to Crawford (2003, p. 1), "evidence from national education systems across the globe strongly suggests that the manufacture of textbook content is the result of competition between powerful groups who see it as being central in the creation of collective national memory designed to meet specific cultural, economic, ideological and social imperatives".

Comparative analyses of content of textbooks seem to be of special interest to historians and also supranational organizations that are well aware of the formative role of the stories and ideologies, and that seek to promote international understanding by improving the picture of international history and different nations. The League of Nations in the 1920s, UNESCO and the Council of Europe oversaw or oversee bilateral or multilateral textbook projects whose aim is improvement of history textbooks and stories they tell (Nicholls, 2003). The leading position in this area was taken by the Georg Eckert Institute (GEI) for International Textbook Research in Braunschweig in Germany, which is an accredited and internationally connected reference centre for textbook research. Its central competence lies in the application oriented research of collective patterns of interpretation, concepts of identity, and representations as conveyed through national education and, as such, also institutionally secured. The GEI contributes to the deconstruction of prejudices and concepts of the enemy and develops recommendations for the objectification and advancement of instructional media (http://www.gei.de/en/the-institute.html). The span of its attention is limited to history, geography and social studies textbooks, thus ignoring textbooks of other subjects, e.g. mathematics and foreign languages. It is certainly not only history, geography or social studies textbooks and stereotypical images of nations that are worth attention. Gender roles and stereotypes of men and women presented in textbooks are of equal interest and have been studied, for example, by Apple (1986) and Sleeter and Grant (1991).

The authors of this paper are convinced that mathematical textbooks are especially important for uncovering the roots of some of the most widespread prejudices, as mathematicians try to show that mathematics is applicable and must be applied in the solution of everyday problems and situations. This effort can be best observed in word problems, which are designed with the intention of simulating the solution of problems from real life. Consequently, the process of word problem posing closely resembles the process of conceiving the microcosm of a novel. The world and characters are merely fictional and only pretend to "represent" the real world and reality. The same can also be observed in expository narrative in textbooks of mathematics which prefer a narrative rather than encyclopaedic form of presenting new subject matter. This narrative winds through the whole textbook and is usually built on a child coming across mathematics in everyday life. If texts in textbooks are taken for what they are, that is, texts created by an author with some intention, for some readers, they may be analysed using the frameworks of literary criticism. As literary theory is only a soft theory (Iser, 2009), there are of course a number of different frameworks, none of which is correct or incorrect. However, only some can be employed successfully in textbook analysis, as the objective of texts in textbooks is not, for example, to have aesthetic value.

TEXTBOOK ANALYSIS RESEARCH

Literature focusing on the issue shows that most attention has been paid to history textbooks. They are specific in nature as they tell stories of history of different nations and are prone to heavy influence by the governing ideologies. Modern national identity is based on the nation's self-perception, which means it is crucially important what story of the national history children get. In her diploma thesis (Moraová, 2001), the first author of this paper studied how long it takes for new discoveries and interpretations of one key historical event of national history to enter school textbooks. Her analysis leads to the conclusion that the authors of textbooks of history tend to ignore historiography and narrate stories compatible with the official view of the establishment. Her detailed analysis also showed that authors of school textbooks tend to copy from preceding school textbooks rather than draw information from scientific journals. They are either ignorant of the latest developments, conform to the demands of the society, or both.

Textbooks of mathematics are different. They either offer a selection of scientific facts and discoveries in encyclopaedic form, or they present word problems and expository narratives with a world of their own. Word problems and expository narrative texts, unlike history, do not claim to be presenting scientific facts. They are meant to simulate real world and show mathematical problems people can come across in everyday life. Undoubtedly the priority of the author is still the mathematical content, the data the pupil is supposed to handle, but this mathematics is embedded in a world that, although fictional in nature, pretends to be a "representation" of the real world. Are the authors aware of this fact and do they pay any attention to this literary aspect of their production?

For the needs of this paper and of the analysis of the world presented in textbooks, we will focus on Pingel's (1999) criteria types [1] of texts/modes of presentation and content analysis as well as textbook sector components, especially adoption procedures and structures of publishing houses.

WHY SHOULD WE STUDY THE MICROCOSM OF WORD PROBLEMS AND EXPOSITORY NARRATIVE IN SCHOOL TEXTBOOKS?

A typical analysis of mathematical textbooks deals with the analysis of curricular content and perspective of presentation (according the Pingel's scheme). Content analysis for a mathematician or mathematics educator means analysis of what mathematics is included in the textbook and how it is treated. In their comparative analysis of mathematics textbooks for elementary school, McCrory, Siedel and Stylianides (in review) study three major curricular topics – fractions, multiplication, reasoning and proof. They look for the similarities and differences in their presentation, in the level of detail, depth and breadth of approaches, presentation of material, and functionality of the book. They pay no attention to the fictional world of these textbooks although they do point out that textbooks may be placed on a scale from purely encyclopaedic to narrative. What stories the narrative textbooks tell, however, is not studied.

Analysis of curricular content, its presentation etc. is the typical, but not the exclusive approach to mathematics textbook analysis. For example, Novotná and Moraová (2005) focus on the language and cultural obstacles pupils learning mathematics in a foreign language may face. The authors build on the fact that the cultural background of textbooks published in different countries varies and may result in the pupils' failure to understand the assignments. This type of content analysis is closer to the framework proposed in this paper, as it is sensitive to the fact that textbooks mirror real life and values of the society in which it originates.

The fact is that textbooks are cultural artefacts and in their production and their use inside classrooms a range of issues to do with ideology, politics and values are confronted, which in themselves function at a variety of different levels of power, status and influence. Textbooks present narratives and stories offering a core of cultural knowledge which future generations are expected to both assimilate and support. School textbooks are crucial organs in the process of constructing legitimated ideologies and beliefs and are a reflection of the history, knowledge and values considered important by powerful groups in society. They tend to enforce and reinforce cultural homogeneity through the promotion of shared attitudes (Crawford, 2003).

Textbooks belong to educational discourse. Discourse has, in theory, always been associated with power, regulation and ideology. Discourses are seen to affect our views on all things; it is not possible to avoid discourse. As according to Foucault (2002), discourse is a practice that systematically creates the objects it speaks about, that is, discourse constructs knowledge, it governs, through the production of categories of knowledge and assemblages of texts, what it is possible to talk about and what is not (the taken for granted rules of inclusion/exclusion). If discourse is constitutive in the sense that it constitutes its objects, it constructs social reality. Every author including the author of school textbooks always makes a selection from a number of possible statements. Every statement is articulated as a difference of other possible statements. Thus he/she creates a specific picture of what is normal in the world and excludes all the other possible worlds. In the context of school textbooks one must therefore ask why the author chose the given microcosm and excluded all the others.

Textbooks are also associated with habitus, the set of socially learned dispositions, skills and ways of acting that are often taken for granted, and which are acquired through the activities and experiences of everyday life and with symbolic violence, that is, the tacit almost unconscious modes of cultural/social domination occurring within the everyday social habits maintained over conscious subjects as defined by the French sociologist Pierre Bourdieu. He claims that the school system (of which textbooks are an inseparable part) is the typical example of constituents of symbolic violence (Kraus, 2008).

Teun Adrianus van Dijk (1998), the Dutch scholar in the fields of text linguistics, discourse analysis and critical discourse analysis, also points out that some discourses legitimate the opinions, beliefs and ideologies of the governing classes while other discourses do the opposite, that is, are subversive, produced by those who oppose the governing classes.

This of course opens a lot of questions about who becomes the author of a textbook of mathematics, which organs and institutions are involved in the process of their publishing, what is the cultural background of the community of the author and the readers (teachers and pupils). How is the author's productive work regulated? Who are the powerful groups and organs and what kind of pressure do they impose on the author? Is the author aware of the fact that he/she is creating a new world in the textbook? Is his/her intention to acculturate the pupils, to hand over the beliefs and attitudes he/she finds natural, or does he/she do this unconsciously? Do teachers pay any attention to the cultural value borne by the texts in the textbook? Are there any textbooks in which the created world is subversive? If so, how is it accepted by the public?

HOW SHOULD WE STUDY THE MICROCOSM OF WORD PROBLEMS

If the texts in textbooks are to be analysed, this can best be done in the context of literary theory and linguistics, as they offer the tools and frameworks for this analysis. As Iser (2009) says, literary theory is crucial in all social sciences as it provides possible tools for interpretation of texts, which had for a very long time been regarded as natural but which is by no way a straightforward process. This role can now be given to literary theory as its focus has gradually been moving away from the aesthetic values of any work of art to its meaning, from the author to the reader, and to the act of reading and understanding and is thus applicable to any discourse, any text or set of texts.

Mimesis in textbooks of mathematics

Mimesis, that is, the relation of the world or the referent to literary text, has always been one of the key concepts of any literary theory from the time of ancient Greece. It was a concept discussed both by Plato and Aristotle, who in his *Poetics* regarded mimesis as imitation of nature. Correspondence of literature to the physical world was understood as a model for beauty and truth. The concept of mimesis as representation of reality, of literary text as a window to the world, reached its peak in the 19^{th} century when the bourgeoisie used literature as an instrument for dissemination of its ideology. Also Marxist theories, for example Georg Lukács (1971), have for a long time taken literature as a mirror of life.

However, the developments in art and in social sciences in the 20th century put an abrupt stop to this conception. Building on Saussure's structuralism, modern literary theory claims that reference is a mere illusion and literature never speaks of anything but literature itself, it is self-referential. (Compagnon is not so radical in his opinions, he points out that Saussure never said that language did not refer to the world, he spoke of the sign as arbitrary, not the statement (Compagnon, 2009).) Also, the attention of literary theory has shifted from the author and his/her intention to the reader and the act of reading, for the meaning of any work of literature is constituted both by the author in the act of writing and the reader in the act of reading. In gestalt theory perception is no longer perceived as passive imprinting of stimuli on the brain, but as projection of one's mind on the surrounding world, that is, perception is performative, it creates the meaning (Iser, 2009). If meaning depends on the act of reading, it is different for every reader and the world of literature cannot be understood as having one specific referent in the real world. Also it implies that the world outside is not a fact but a social construction created in communication and in reading (Kraus, 2008).

This means that literature is autonomous, according to modern theory there is nothing in the world that literature refers to, this world is a construction, probable illusion, delusion of truth, imitation, fiction, virtual reality, the author's and reader's invention (Compagnon, 2009). Literature does not represent the world, it creates it. Exactly the same applies to texts in textbooks despite the fact that there are some major differences between literature and textbooks. For one thing, the aim of the author of a textbook is not to produce extraordinary aesthetic value. The author of a textbook of mathematics is more interested in the subject matter, and the narrative he/she writes is a narrative guided by the needs of mathematics exposition. Connected to this is the fact that the public does not tend to perceive textbooks as works of art and focus on their scientific content, which makes them more susceptible to referential illusion. On the other hand, the life of textbooks is much shorter than the life of literature, and consequently the author and the pupils are usually members of the same interpreting community (Kraus, 2008) and thus the author's intention and the pupils' reading and reconstruction of meaning are closer than when reading a work of art written in a different era.

There are many different definitions of *mimesis.* For example Baudrillard (1998) introduces the concept of simulacrum which he defines as a virtual copy of a non-existent original, which is more real than reality, the so called hyperreality. In other words, the readers are not only unable to distinguish between reality and simulation of reality, but they prefer the simulation and trust it. For Adorno, a work of literature evokes something non-existent; Ingraden speaks of illusion of reality, anthropologists of art not as a copy of reality but as a symbol. In reader-response theory, reference of literary text is not negated, but the documentary value of a work of art is still disputed as it transplants social and cultural fragments extracted from their original social and cultural field in consequence of which literature affects these systems (in

Compagnon, 2009). For Barthes (1997) *mimesis* is repressive and is connected to ideology. Barthes claims that every regime has its manipulative manuscript which generates its time, space, inhabitants and myths. It is a constructed world, not reality. This definitely holds true about the world of word problems.

This implies that the world of word problems and expository narrative must not be analysed with the intention of discovering the referent, as a documentary description of the world it stems from, because that can never really be reconstructed. It must be analysed with respect to its performative nature, to what it incurs, what it does, what it changes, what effect it has on cultural values of the pupils, with respect to what ideology or discourses it supports.

Performativity

The concept of performativity of language comes from Austin (1962) and Searle (1969), who show that language has the power of doing things (e.g. acts as promising, ordering, greeting, warning, inviting and congratulating). In other words by saying something, we do something. Austin himself excludes literature from speech acts, as utterances in art are not made in appropriate circumstances. But, in his Speech Acts in Literature, Hillis Miller claims that it is the work of literature as a whole which is performative (Hillis Miller, 2001, in Iser, 2009), that is, that it does something, brings about some change.

Performativity of literature and literary texts is one of the concepts that most literary theories agree on, although its explanation is based on different grounds. The problem with *mimesis* is that readers of fictional texts are susceptible to referential illusion and believe that the text refers to real world (Compagnon, 2009). According to the Thomas theorem, if men define situations as real, they are real in their consequences (Thomas, 1928, in Kraus, 2008, p.19). This also means that interpretation of a situation shown in a text causes action. In other words, fictional texts are performative. Also Northrop Frye speaks of *mimesis* in the sense of causing effect outside fiction, that is, in the world (Compagnon, 2009). Marxist literary theorists perceive a work of art not only as a mirror of reality, but also as the model that the society should reach (Iser, 2009), that is, they also perceive it as performative.

This of course means that expository narrative texts and word problems in textbooks of mathematics are performative and have impact on the beliefs and attitudes of at least one future generation. Are their authors aware of the responsibility they take when writing these texts?

Illustration: Perspective of an author of a school textbook of mathematics

The following is an explanation of the presenting author of the circumstances of production of a set of mathematics textbooks for lower secondary schools:

> The set of textbooks Matematika s Betkou was written in response to call of parents who needed to persuade their children that they do not need to be afraid of mathematics. Our decision was to present new subject matter through expository narrative with a family who encounter mathematics in their everyday life. This was a pioneer approach in textbooks of mathematics, known in our country only from foreign language textbooks. We hoped the expository narrative would draw the pupils into the centre of action. Originally the family was given the name Pokorný and the children common names Katka and Jirka. But piloting of the textbook at schools showed that children with the same first name or surname were likely to be in every class. This led to long discussions on similarity of the textbook characters' experience and experience of the pupils. To avoid this, the family was renamed – they were given the name Ajnštajn (phonetic transcription of the name Albert Einstein in Czech) and the children (son and daughter) were given uncommon names Betka (in reference to the Greek letter *beta*) and Kryšpín (a non-existent name, though it may be used in some families to call a boy called Kristián). The main character is the girl Betka, who is as old as the pupils using the textbook. This was our intention, we wanted to show girls, whose attitude to mathematics tends to be regarded as more off-hand, that Betka and therefore all girls can solve mathematical problems and non-standard situations successfully and with invention.
>
> The reviewer appointed by the Ministry of Education thought the use of expository narrative was unnecessary. However, we managed to persuade her that the story narrated was an important motivational component. The textbook was officially certified by the Ministry of Education for use at lower secondary school level. The publishing house did not enter the process of writing this textbook; there were no interventions from their side.

What do the paragraphs above say about the textbook in the perspective of the discussed framework? The authors of this set of textbooks decided to produce expository narrative rather than present new concepts in encyclopaedic form, with the intention of motivating the children, of making them involved. The pupils' reaction to the original names shows that pupils are victims of referential illusion and take the expository story as if it were real life. If they saw the family as fictional, that is, the narrative as self-referential and they would not try to compare themselves to it.

It is also interesting that the world created is that of a four member family, which is believed to be the optimum number of children by society and is most supported by state institutions in the country of origin. It is also interesting because all three authors of the textbook have two children and thus may unconsciously perceive this number of children as the most probable illustration of reality. From a gender perspective, it is interesting that the girl is younger than the boy, who is in a position of authority and one who knows more. Again, two of the three authors have an older son and a younger daughter, which may be the unconscious motivation of this conception.

Obviously, the authors were unaware of the performative aspect of their text.

CONCLUSION

The paper introduces a new perspective on textbook analysis. Unlike most research in the area of mathematics textbooks, it does not focus on the subject matter, mathematical concepts and their presentation. Rather, it is interested in the world which is created by their authors in their effort to simulate real life situations. Using frameworks of literary theory and social sciences, the authors point out that textbooks are part of educational discourse and therefore must be studied and analysed in their connection to power, the establishment and its effort to control its citizens, to ideology, and to habitus. With respect to power and ideology, one must also study the adoption procedures, the mechanisms of giving a textbook official certification (in cases where it is needed) and also the mechanisms for how schools select the sets of textbooks they use and who in the schools is responsible for this selection.

One must also be aware of the performative nature of texts in textbooks as even though textbooks cannot refer to real world and reality, if the pupils believe the textbooks do present reality, these textbooks enforce or reinforce certain attitudes, beliefs and opinions of these pupils. If we look for sources of prejudice and stereotypical perceptions, and for discursive practices, regulation and power, we must carefully deconstruct the stories that textbooks tell and also the stories they do not tell. Textbooks contribute to the process of formation of social reality and help create socially constructed categories. For example, with respect to gender we must ask what images of people we come across in textbooks and, if we agree with Butler (1990) that gender is performative and socially constructed, we must analyse what gender roles the textbook texts support or spread. But we can also study the issue of minorities, of other nations, age groups, etc. The outlined framework is universal and the analysis may contribute to discussion of any of the current social phenomena and issues.

School textbooks of mathematics do not tell any complex stories of history of nations, their society, culture and values. Yet they are artefacts produced within some social background and are part of some discourse (in the case of state certification of textbooks, the discourse is legitimated by the state). They create some microcosm that influences the beliefs of their users. This influence is even stronger as it is much more covert than in history textbooks and the general public is usually unaware of it, tending to focus on the mathematical, rather than the literary aspects.

NOTES

1. A universal framework for textbook analysis is provided in the UNESCO *Guidebook on Textbook Research and Textbook Revision* (1999, p.48). This document presents the following framework for textbook analysis: Analysis of **textbook sector components** (educational system, guidelines/curriculum, adoption procedures, structures of publishing houses), **formal criteria** (bibliographic references, target group – school level, type of school, dissemination), **types of texts/mode of presentation** (author's intentions, descriptive author's text – narrative, illustrations/photos/maps, tables/statistics, sources, exercises), **analysis of content** (factual

accuracy/completeness/errors, up-to-date portrayal, topic selection/emphasis (balance)/representativeness, extent of differentiation, proportion of facts and views/interpretation), **perspective of presentation** (comparative/contrastive approach, problem-oriented, rationality/evocation of emotions).

ACKNOWLEDGEMENT

The research was partly supported by the project GACR P407/12/1939.

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NUMERACY, MATHEMATICS AND ABORIGINAL[1] LEARNERS: DEVELOPING RESPONSIVE MATHEMATICS PEDAGOGY

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Aboriginal students' mathematics outcomes in Australia's schools continue to remain substantially behind those of their non-Aboriginal counterparts[2]. Developing responsive mathematics pedagogy that will improve these outcomes is the goal of the national project Make It Count. This theoretical paper derives from the work of the project's Clusters and presents a model for Responsive Mathematics Pedagogy (RMP) based on the different pedagogies being developed by the Clusters. These pedagogies are responses to the cultural, social and academic needs of Aboriginal learners. Three case studies, drawn from the project, illustrate how RMP can cater for the differing and varied needs of Aboriginal students.

INTRODUCTION

At the heart of quality teaching of students in mathematics are the professional judgments about teaching and learning – judgments based on teachers' knowledge, experience and evidence in relation to pedagogy, their students, and mathematics. If teachers are to provide quality experiences in mathematics they will need deep content knowledge and deep pedagogical knowledge in mathematics (such as those outlined in the Standards for Excellence in Teaching Mathematics in Australian Schools (AAMT, 2006)). However, for many Aboriginal students, this is not enough and mathematics remains a gatekeeper, or a gate that is kept locked, to future prospects. According to Perso (2003) a teacher's approach to teaching Aboriginal students mathematics needs to respond explicitly to Aboriginal people and their culture, Aboriginal children's mathematics understandings, and explicit mathematics teaching. Perso's research provided a foundation for the work of Make It Count and the subsequent development of the model presented in this paper. Make It Count is a project of the Australian Association of Mathematics Teachers (AAMT) Inc., funded by the Australian Government as part of the Closing the Gap initiative.

The work of Make It Count includes consideration of what constitutes a highly effective teacher of mathematics in the context of Australia's Aboriginal learners. We know that teachers need to have high levels of mathematical pedagogical content knowledge but, beyond this, are able to teach in ways that are "relevant and responsive to social realities and the cultural and racial identities" (Martin, 2007, p.18) and academic identities of their Aboriginal students. Make It Count brings together two "fringe dwellers" in the educational landscape – Aboriginal education and mathematics education (the major focus of funding has been literacy rather than numeracy) – and through their intersection, is developing cultural competency in its teachers and their mathematical pedagogical content knowledge in an endeavour to improve learning outcomes of Aboriginal students (Figure 1).

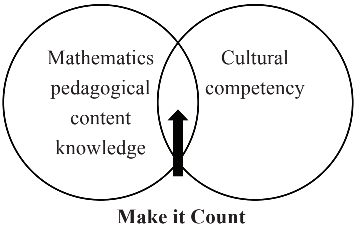

Figure 1. Make It Count focus: Developing responsive mathematics pedagogy

THE MAKE IT COUNT: NUMERACY, MATHEMATICS AND INDIGENOUS LEARNERS PROJECT

> Much of the present teaching of mathematics, particularly in the primary years, has Aboriginal students doing mathematics that is not related to their world and their everyday experiences. As a result, by the time many Aboriginal students have reached the latter years of primary school they have been alienated from mathematics. (Matthews, Howard, & Perry, 2003, pp. 12-13)

Make It Count is an Australian-wide project striving to improve the learning outcomes of Aboriginal and Torres Strait Islander students in mathematics through whole-school, sustainable, evidence-based practice. It is part of the Australian Government's 'Closing the gap – expansion of intensive literacy and numeracy programs initiative'. The project has established eight Clusters of schools across Australia to find something new, or adapt something old, which will make a difference through the development of RMP. It needs to be made clear that RMP does not mean mathematics worksheets with boomerangs around the border or "counting the number of beats from a clapstick"; Aboriginal content that 'is tokenistic, separated from the core content and treated as an interesting or fun activity. These approaches only marginalise Aboriginal learners further from mainstream education' (Yunkaporta, 2012). Rather, RMP means valuing what Aboriginal learners bring to the classroom and responding to it in ways that develop deep mathematical knowledge.

The eight Clusters are engaging with RMP in different ways, with a different focus, but generally their approach can be described in Figure 1. Cluster teachers are increasing their mathematical pedagogical content knowledge – they need to know the mathematics well and the different ways it can be taught – and they are developing cultural competency so they are responsive to the learning needs of Aboriginal students. In other words, teachers need to know the mathematics, know how to teach the mathematics well, and know how to teach Aboriginal students.

Clusters are developing cultural competency in educators with the view that this might improve pedagogy. McAllister and Irvine (2000) describe cultural competency as:

> …one who has achieved an advanced level in the process of becoming intercultural and whose cognitive, affective, and behavioural characteristics are not limited but are open to growth beyond the psychological parameters of only one culture. (p. 4)

Clusters' approaches to this, and ways for teaching mathematics, includes learning that: is investigative and problem solving; is explicit, scaffolded and sequenced; provides one to one tutoring that 'front-ends' Aboriginal learners in the mathematics in preparation for the classroom learning; contextualising, mathematising and transferring learning; builds resilience in Aboriginal students; and uses Aboriginal pedagogies and processes. Whatever their approach, the Clusters are designing learning that entails being intentional (being very clear and articulate about what mathematics Aboriginal students are to learn), being responsive (the pedagogies and practices teachers draw on to teach the mathematics), and being effective (how the teacher will know if their students get it)[3].

It is clear that there are different ways to teach mathematics suggesting that a repertoire of practices based on deep understandings of pedagogy is necessary. Teachers need to be able to understand how mathematics and pedagogy (and the practices within pedagogy) connect to Aboriginal students and their cultural and social realities. One Cluster has a focus that is strongly focused on academic inclusion through the use of scaffolded, sequenced learning; another has a focus on cultural inclusion through the infusion of Aboriginal pedagogies and processes; a third has a focus on social inclusion through the use of contexts that are authentic and meaningful to Aboriginal students. In the next section we describe the work of these three Clusters and use it to propose a new model that describes the elements of responsive mathematics pedagogy.

To develop RMP teachers and students need to engage in reciprocal learning - a mediation between home and school. This two-way learning is integral in the development of repertoires for participating in practices (Gutierrez & Rogoff, 2003) that are culturally, socially and academically responsive. Through the development of responsive mathematics pedagogy teachers are able to 'develop dexterity in determining which approach from their repertoire is appropriate under which circumstances' (Rogoff, 2003).

RESPONSIVE MATHEMATICS PEDAGOGY

Make It Count is a work in progress and the following illustrates the project's progress so far in developing mathematics pedagogy that is responsive to, and inclusive of, the cultural, social and academic needs of Aboriginal learners. It reflects the pedagogical frameworks devised by various education systems around Australia but, at the same time, raises questions about the polarity in mathematical practices around the country.

The project is developing a series of messages from the Clusters about responsive mathematics pedagogy. They are based on what has worked for schools involved in the project and are supported by evidence that they have worked in improving outcomes for students. Some of these are illustrated in the following three case studies. These consist of an outline of the relevant Cluster's focus, data collected and what the data have told those involved. These are followed by relevant Cluster findings drawn from all eight of the Clusters.

Cluster A: Academically responsive mathematics pedagogy

Cluster A is applying principles of Gray's Accelerated Literacy (AL) pedagogy to the teaching of mathematics that is scaffolded and explicit in Reception to Year 7 classes. AL has been shown to be highly effective in enhancing literacy for Aboriginal students and this is why it is being adapted for the teaching of mathematics. Gray, (2007) describes this approach to teaching and learning as an organised teaching sequence that provides careful development of an explicit and supportive classroom interaction process, a teaching/learning context that is systematic and inclusive (Gray 2007, p. 4). He outlines four central concepts behind what is now called the National Accelerated Literacy Program (NALP) in Australia. They are:

- The notion of discourse as a primary goal for teaching
- The application of the Zone of Proximal Development (Vygotsky, 1978) as a strategy for achieving access to high-level performance within academic/literate discourses
- The staging of a teaching sequence structured around the two concepts above
- The integration of scaffolding (Wood, Bruner & Ross, 1976) as the framework for teaching /learning processes within the Zone of Proximal Development and the NALP teaching sequence in particular (Gray, 2007, pp. 4-5).

When the Cluster first became involved with the Make It Count project it became apparent to the teachers that they lacked adequate mathematical content knowledge. AL is underpinned by detailed knowledge of literacy and its development – they were short of the mathematics equivalent of this knowledge base. To address this they embraced the Big Ideas in Number program developed by the Department of Education and Children's Services in South Australia in consultation with its developer, Professor Di Siemon from the RMIT University in Victoria, Australia. When designing learning, they were able to be explicit about the learning goal and intention, which is articulated clearly in the first step of the learning sequence the Cluster has developed. Basically, the learning sequence begins with low order orientation which includes welcoming students into mathematical discourse. It moves to a clear articulation of the learning goal for the whole lesson followed by high order orientation which includes drawing on knowledge from other lessons. Finally, at the end of the sequence, handover is reached through joint conceptualisation and meaning making.

Ruby is a middle primary teacher in an urban school in Cluster A. She is very experienced in using the AL pedagogy and explicitly teaching the cultural orientation necessary to engage in school learning. Ruby discusses the change in Andrew, one of her Aboriginal students:

> At the beginning of the year and throughout most of term one Andrew presented as a quiet, under achiever who was lacking in confidence particularly in the area of mathematics… Towards the middle of term 1, I changed my pedagogy in this subject area and brought it into line with how I taught Accelerated Literacy…Andrew in particular, began to shine in the lessons. He gradually became more confident to offer answers. He now asks questions when he doesn't understand. He shows that he enjoys mathematics and is eager to share his knowledge with others. He goes home and shares what he knows with his parents who have been delighted with this transformation. They have written notes in his diary.

The diary entries by Andrew's mother provided Ruby with rich data that reinforced the pedagogy. Ruby continues with an analysis of Andrew's diary:

> This was not set as homework. He just went home and did it himself every night for the week. (I always say to the children that they can practise Maths at home. Until this, hardly any children ever did extra Maths at home.)
>
> Andrew's mum commented on his enthusiasm in his diary. "He conquered his frustration of the Maths", wrote Mum.

Cluster findings relating to academic responsiveness include:

1. Practice explicit teaching with a defined and planned learning goal for each lesson.

2. Remember that strong mathematics skills build learners' confidence and enthusiasm for mathematics, and this encourages risk taking which is an integral part of mathematics learning.

Cluster B: Culturally responsive mathematics pedagogy – Centring Aboriginal values and processes for cultural inclusion

Orange Public School (OPS) began working with the 8ways pedagogy of learning (Figure 2) as part of their participation in the Make It Count project.

> This Aboriginal pedagogy framework is expressed as eight interconnected pedagogies involving narrative-driven learning, visualised learning processes, hands-on/reflective techniques, use of symbols/metaphors, land-based learning, indirect/synergistic logic, modelled/scaffolded genre mastery, and connectedness to community. But these can change in different settings. (Yunkaporta, 2012)

Figure 2: 8 Aboriginal Ways of Learning - Tell a story. Make a plan. Think and do. Draw it. Take it outside. Try a new way. Watch first, then do. Share it with others.

The school localised the framework to become: Yarn up. Learning map. Model work. Hands on. Different ways. Symbols and images. Assessment item. Community links. A Yarn up could be about times when mathematics has been used to solve real problems in everyday life. Teachers highlight the importance of yarning as a way of creating and passing on knowledge in Aboriginal culture. Symbols and images could be about using visuals and creating symbols to help students understand and remember content. Teachers at OPS promote this as an Aboriginal form of communication (Yunkaporta, 2012).

The project findings to date have highlighted the importance of explicitly modelling assessment items and tasks, including the provision of key ideas, indicators and criteria for students at the beginning of the learning sequences. All mathematics units are now designed according to the school-community's Aboriginal learning cycle.

Community connectedness has proved to be the key factor for success in Aboriginal student engagement, and is the most clearly demonstrable outcome of the program to date. There are 15 to 20 Aboriginal community members regularly attending meetings, as opposed to negligible community involvement prior to the project. Additionally, pride in Aboriginal identity has measurably increased, with more students being willing and proud to self-identify and assert their Indigeneity. There has been a 15% increase in the number of students officially identifying as Aboriginal since the beginning of the project. This indicates a significant shift in school values and attitudes towards Aboriginal culture and community.

This shift all began with the establishment of a community advisory group for Aboriginal pedagogy, with a particular mathematics focus. The advisory group continues to have direct input into unit planning and curriculum design. They also put out a quarterly newsletter. In addition, they have established an ongoing project whereby community members are leading the planning and construction of an Aboriginal space in the school, including an ethno-garden and a roundhouse. The garden symbolically represents the Aboriginal learning cycle used in teaching and learning across the school, incorporating sacred sites, songlines, dreaming stories and local histories.

Cluster findings relating to cultural responsiveness include:

1) Know that for Aboriginal students the sense of community and belonging is very important to learning

2) Design mathematical learning experiences that have family and community significance.

Cluster C: Socially Responsive Mathematics Pedagogy

From an initial focus on learning through context, the Alberton Cluster of schools sought to link mathematics and context through the deliberate acts of mathematisation and contextualisation (Thornton, Statton, & Mountzouris, 2012). The model illustrated in Figure 3 evolved after almost two years of discussion and reflection on teaching practices among the teachers at the Cluster schools. Teachers were challenged to think about how they might make the teaching and learning of literacy and numeracy more meaningful by embedding it in contexts such as art, design, technology, sport or enterprise. They then spent some time exploring and evaluating this approach in practice.

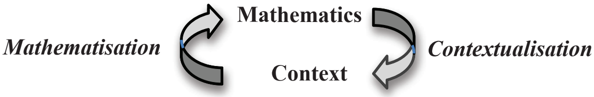

Figure 3: The Alberton Cluster model of mathematisation and contextualisation (Thornton, Statton, & Mountzouris, 2012).

The teachers in the team felt that the approach to teaching and learning in the Make It Count project was not only making their teaching more connected and purposeful, but that it was also engaging disinclined learners, particularly Aboriginal students, and encouraging them to maintain positive attitudes to mathematics beyond primary school. The teachers described this as increased mathematical resilience (Johnstone-Wilder & Lee, 2010), and identified five key aspects: having a growth mindset shown through behaviours such as learning from mistakes; meta-cognition shown through a willingness to reflect on answers and problem solving processes; adaptability shown through a willingness to try new strategies or start again; inter-personal aspects of learning such as seeing asking questions as clever rather than an admission of lack of knowledge; and a sense of purpose shown by a student's desire to seek meaning in his or her learning.

Debra is a year 4 student at Alberton school. She is a smart and precocious student who, like all young learners, had a range of behaviours and attitudes that were productive and some others that were clearly counterproductive to learning mathematics well. Prior to the Make It Count, she seemed disconnected from learning, reacting negatively to the shock of a new set of circumstances, such as a new space or a relief teacher, often challenging staff or failing to follow organisational structures. She would respond to learning tasks by stating “I’m not doing this!” or “I need help” and would sit in her seat distracting the students around her.

Through the explicit focus on connecting mathematics and context through the processes of mathematisation and contextualisation, Debra has made great shifts in her approach to mathematics. In a lesson on measurement she was asked if, like Aunty Alicia a prominent Aboriginal leader in the community who was studying horticulture, she had used a tape measure. She replied that she had and became very excited about the prospect including Aboriginal plants in the native garden project the class was undertaking. In an ensuing lesson on place value, arising from an investigation of hundreds, tens and units from the tape measure, Debra worked with a partner, asked for assistance and did not become withdrawn or angry when she made mistakes. Debra’s teacher attributed this to the greater mathematical resilience developed as a direct result of the connection between mathematics and context.

Cluster findings relating to social responsiveness include:

- Know mathematics that is beyond the classroom so you can teach mathematics through rich contexts.
- Help learners develop and maintain positive attitudes to mathematics. Develop positive dispositions, resilience, and skills of effective mathematical learning and thinking that equip learners to solve problems then transfer and apply their understandings, whatever the context.

SUMMARY

The diversity of approaches highlights the three critical dimensions of responsive mathematics pedagogy: academic inclusion, cultural inclusion and social inclusion. No one of these is enough in its own right. The three elements interact in a dynamic and generative way. Cluster A has focused strongly on academic inclusion through explicit teaching and scaffolding, and has noticed greater engagement and sense of pride among students in their mathematics learning. The challenge now is to create stronger cultural links and to look at how meaningful contexts can become embedded throughout their mathematics. Cluster B has focused strongly on cultural inclusion through the 8 ways of learning and noticed that students feel a greater sense of pride and identity as Aboriginal people. The challenge is to infuse this pedagogy throughout mathematics teaching in a way that leads to deeper understanding of mathematical ideas. Cluster C has focused strongly on social inclusion through meaningful contexts and has begun to look at how to draw out the significant mathematics from these contexts. The challenge is to infuse pedagogical approaches that draw on Aboriginal ways of knowing.

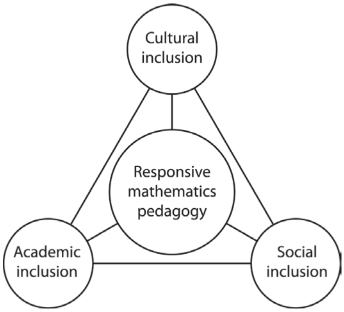

Figure 4: Responsive mathematics pedagogy

There are also findings that are potentially conflicting. For example Cluster A has adopted an approach that is driven strongly by the mathematics curriculum, emphasising the developmental progression that is necessary for further learning within the Big Ideas in Number framework. On the other hand Cluster C has adopted a dynamic and emergent approach to curriculum, teaching content as it arises from the contexts and projects being undertaken by the students. Perhaps there is no one right answer, but it is critical that the research community investigate further questions concerning curriculum content and pedagogy that promotes academic, cultural and social inclusion for Indigenous students.

It is becoming clear in Make It Count that teachers who successfully engage in responsive mathematics pedagogy possess (a) deep content knowledge, (b) strong pedagogical content knowledge, and (c) strong culturally, socially and academically relevant pedagogies. They have a deep commitment to Indigenous students and these students are empowered as a result of their mathematical experiences (Ernest, 2002) and are able to make second-by-second to lesson-by-lesson to term-by-term judgements about practices and pedagogies that engage and inspire Aboriginal students in mathematics and numeracy. Clusters are realising that perhaps they need to be looking at each other for ways to achieve this. The model we have described summarises these components of responsive mathematics pedagogy, and provides a theoretical framework that can be used to inform further research and practice that enhances the mathematical learning and experiences of Aboriginal students.

NOTES

1. Indigenous' is the terminology used by the Australian Government for Aboriginal and Torres Strait Islander people. In this paper the term 'Aboriginal' is used for Australian Aboriginal people.
2. Thomson, De Bortoli, Nicholas, Hillman, & Buckley (2011).
3. Adapted from the Department of Education & Children's Services, South Australia Teaching for Effective Learning Review Tools.

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WHAT IS A GROUP? THEORETICAL CONSIDERATIONS WHEN RESEARCHING AFFORDANCES FOR MULTILINGUAL STUDENTS' MATHEMATICAL LEARNING

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In this paper we outline theoretical and political considerations when researching affordances for multilingual students' mathematics learning during classroom communication. On one hand we address the difficulties when categorising students into groups in research and how this can be counteractive since it can reinforce stereotypes. On the other hand we address the significance of doing research also concerning groups of students since this can provide understandings that go beyond deficient models regarding students' language backgrounds. We discuss a basis for an analytical framework for a newly started project focusing on a specific aspect of communication between teacher and student, namely assessment (here taken in a broad sense) in mathematics.

In everyday life, as well as in research, people are categorized in different groups for different reasons. Common and vernacular group categories are for example boys and girls, men and women, young and old people. In educational terms groups can be monolingual students, bilingual students, first and second language speakers, immigrant students, students with special needs, gifted students, high achievers, low achievers, etc. To categorize people into groups is a way to describe them and by the categorisation, "stories" are told about the people in a group category.

Categorization in educational terms also is involved with how a nation's educational system distributes opportunities to learn mathematics, as well as other subjects in school, among various groups of students. In official school/educational documents, policies, and reports, students are categorized into groups (for example by the Swedish Agency of Education, in TIMSS and in PISA data). Reports on various groups of students' achievements in mathematics tell stories about students with different language backgrounds (immigrant students) and socioeconomically disadvantaged students that characterise them as low achievers (National Agency of Education/Skolverket, 2012).

In this paper we will outline some theoretical and political considerations to enable us to focus on different groups of students, in this paper, multilingual students, in relation to communications during mathematical classroom work. We are concerned about focusing on particular groups of students, since this can be counterproductive and reinforce stereotypes. On the other hand it can also provide understandings that go beyond deficit models regarding students' language backgrounds (Khisty, 1995; Norén, 2010).

SUPPORTING EQUITY IN MATHEMATICS CLASSROOMS?

When researching and discussing how to support equity in mathematics classrooms there are several aspects that need to be addressed. In this section, as background, we describe some of these.

The Swedish school system and the mathematics classroom

The Swedish school system today does not achieve the objective that the school should compensate for differences between students' backgrounds. The differences between different groups of students' academic performances are increasing. A recently published report from the National Agency of Education (2012) is partly based on analyses of national tests and international comparisons, such as the PISA study. The report shows that the range in student performance in reading has increased between 2000 and 2009, and the same development can be seen in mathematics and English. While the proportion of pupils who reach the highest possible rating increases, so does the proportion of students who do not qualified for access to secondary education.

In our new research project, "Supporting equity in mathematics classrooms: The role of assessment in day-to-day communication", we focus on various groups of students, and their day-to-day classroom communication. The project consists of quantitative as well as qualitative studies. We address some features of the situation in various Swedish mathematics classrooms where equity aspects, such as students' ethnic and language backgrounds as well as socio-economic circumstances and parents' educational levels, is becoming more problematic than earlier (see also Björklund Boistrup & Norén, 2012, Björklund Boistrup & Norén, 2013). It is a general problem area in Sweden and elsewhere, that teachers' expectations and demands, as well as local circumstances, segregation, poverty and social problems limit opportunities for students' achievement (Arora, 2005).

What does it mean to be viewed as "normal"

Atweh and Bland (2002) and Atweh and Keitel (2007) problematized equity as a concept for social justice. The literature in mathematics education refers to issues related to gender, race, multicultural and multilingual aspects, as well as socioeconomic factors. We think it is vital to also problematize grouping and categorisation, as these phenomena are the implicit basis for discussions on social justice and equity. We will come back to define the concept of groups. At ICME 12 (2012) the Survey Team 5's proceedings on socioeconomic influences on mathematics education was reported. In general, countries in different parts of the world produce similar results about "normal" school achievement and different groups of students are compared to those "normal" expectations on students (Valero, Graven, Jurdak, Martin, Meaney, & Penteado, 2012).

In so-called prototype mathematics classroom we can find "normal" groups of students. In this paper we look beyond a prototype mathematics classroom, where mainstream studies in mathematics education usually have been conducted. In that kind of research, conceptions of learning do not take complexities and conflicts into account. In research conducted in a prototype mathematics classroom students are well equipped and willing to learn (Skovsmose, 2012). On the other hand:

> Mathematics education can operate in very many different ways, depending on the site. One can consider how a particular mathematics classroom may function for immigrants in Denmark, for Indian students in Brazil, from students in a favela environment. There is no context-independent interpretation of how mathematics education might function (Skovsmose, 2012 p. 344).

Classroom communication

When considering how teachers interact with different groups of students, there is a need for considering types of classroom practices, such as communication and assessment acts (in a broad sense), that will support all groups of students in the mathematics classroom. A support for this is the discussion by Gorgorió and Planas (2005) of teachers' construction of each students' possibilities on the basis of certain social representations established by micro-contexts (p. 1180). They discussed the role of social representations in teachers' expectations towards different students. They found teachers often refer to social constructions and not to students as individuals. A public discourse around immigrant students, contextualized in Barcelona, was a source of problems rather than a resource for learning. The finding is the same in Civil's (2012) overview on mathematics teaching and learning of immigrant students in southern European countries. César and Favilli (2005) report that mathematics teachers in Italy, Portugal and Spain, seem to have perceptions of immigrant students based on the students' countries of origin.

The view on assessment in this project is that in every situation in mathematics classrooms, there are acts taking place that can be analysed in terms of classroom assessment. Assessment acts consist of not only traditional tests and project work but also, and most importantly, aspects in day-to-day teacher student interactions, for example where teachers aim to find out students' mathematics knowing towards providing scaffolding to their learning (Morgan, 2000; Watson, 2000). Classroom assessment is regarded as the lens through which we view institutionally situated teacher-student communication in the mathematics classroom. This lens is essential in order to capture acts that provide more or less affordances for students' active agency and learning in mathematics classrooms.

These aspects are not commonly addressed in mathematics education research and our project will provide insights with essential implications for various groups of students and their engagement (agency) and learning in mathematics classrooms.

Multilingual students in the Swedish context

Although immigrant students [1] in Swedish schools do not belong to a homogenous group, they are defined as a group, who often have gaps in their knowledge, due to poor skills in the Swedish language. Having poor language skills (in Swedish) is a common explanatory factor for immigrant students' often low achievements in mathematics, and other school subjects. Deficit discourses call for remediation in the students themselves to become more "Swedish". Deficit discourses regularly cause teachers to view immigrant students as disadvantaged, and to have low expectations on their performance in the mathematics classroom. The mathematics classroom is constructed not only as a place to learn mathematics, but also as a place to become more "Swedish". The interplay between the formulation of the attributes and the strategies of remediation result in the formation of strong deficit discourses on multilingual learners with immigrant background in Sweden (Norén, 2010).

There has been research that explains how social processes influence immigrant students' experiences at school, and students' and teachers' positioning in classroom discourse (Martin-Jones & Saxena, 1996; Heller, 1999). Haglund (2002) implies that multilingual students have to cope with neutralization of difference and reproduction of Swedish as the legitimate language at the Swedish school. The multilingual students' experiences may be demonstrated in negative attitudes towards multilingualism and cultural diversity. Alrø, Skovsmose & Valero (2007) found similar results in Denmark, where the students in their study represented the "sameness" approach about integration as the public discourse. The students didn't pay much attention to diversity and it was not a resource for teaching and learning.

Teachers categorize multilingual students in terms of their every-day school practices (Gruber, 2007). One of Gruber's interesting observations is that immigrant students who do well in their schoolwork, are categorized as immigrants by teachers to a lesser extent than those who do not prosper. These students are categorized as "almost Swedish". The main focus of Gruber's study was on construction of difference, with an emphasis on how ethnicity is turned into a basic category for the social organisation of the school. Her interest was also on how construction of ethnicity is bound up with social complexity and how they interact with other categorisations, especially gender and class. Ethnicity is understood as socially constructed, and as generating processes whereby people are divided into groups on the basis of language, religion, culture or geographical origin. According to Gruber ethnicity also embraces ideas about gender that "generates a number of pupil categories that are assigned different positions in every day school life" (p. 223). We are interested in these complex intersections, and assume that they are present in the day-to-day communication, affected by discourse; in the mathematics classrooms we will study.

BASIS FOR AN ANALYTICAL FRAMEWORK

We view the mathematics classroom from an analysis of communication in the mathematics classroom, and will primarily build on two notions, discourse (Foucault, 1993), and agency as defined in Björklund Boistrup (2010), Norén (2010) and Andersson & Norén (2011), but we also need to consider and engage with other concepts such as "groups". In this section we describe possible analytical tools for the new project. On the one hand these tools will serve the purpose of accepting the difficulties when categorising students into groups in research. On the other hand they will serve the purpose of doing research concerning groups of students since this can provide understandings that go beyond deficit models regarding students' language backgrounds in mathematics education.

What is a group?

People are differentiated according to social groups such as men and women, age groups, religious groups, racial and ethnic groups, etc. Groups are fundamentally entwined with the identities of the people described as belonging to them, with particular consequences for how people understand one another and themselves. "A social group is a collective of persons differentiated from at least one other group by cultural forms, practices, or way of life" (Young, 1990, p. 53), and it exists only in relation to at least one other group. A group can be seen as an expression of social relations. Group differentiation is probably both an inevitable and a desirable aspect of social life.

In accordance with Young we, in this paper, view social groups as produced by social processes (p. 58), and group differentiation as "multiple cross-cutting, fluid, and shifting" (p. 59). We see a group as not having substantive essence and there is "no common nature that members of a group share" (p. 58-59). When conceptualising social groups in a relational and fluid approach, it is possible to view groups as not permanent.

Continuing to build on Young we argue that problems of stereotyping groups exist because people, from an individualistic point of view, wrongly believe that "group identification makes a difference to the capacities, temperament, or virtues of group members" (p. 57). One could say that in all kinds of categorizing and sorting some focuses are categorized as valuable and others marginalized (Bowker & Star, 1999).

For groups that are categorized as disadvantaged, distributive models of social justice are often suggested in educational compensatory programs (Atweh, 2009). One example of such a group classified as disadvantaged in Sweden is multilingual students/immigrant students. Distributive models seldom question the curriculum itself or its assessment practices. The reasons for inequality between groups are seldom taken into account and so these models might even create inequality. Models of social justice that do not take the role of social structures and relations into account when determining individual or group achievement, will fail.

A third model of recognition and redistribution may incorporate the two models mentioned (Atweh & Keitel, 2007). Atweh and Keitel emphasise that research on marginalised groups may give "voice to the voiceless". We argue that research questions and methodologies that respect the students we research (as individuals but also as groups), are of great importance for the benefit of new knowledge about students' engagement in mathematics learning.

Categorisation, groups and discourse

Skovsmose and Borba (2004) describe the complexity of, for example, quantitatively categorizing students into groups in critical research. Simultaneously they argue that there may be essential findings that can be lost if this complexity would exclude research on quantitative research concerning different groups. What needs to be taken into account is that research based on specific categories, such as gender or race comes with limitations (Stentoft & Valero, 2009). Focuses on groups may overlook identities that are made relevant in classroom discourse and determine inclusion or exclusion of various groups (or individuals) in the mathematics classroom. Categorisation of groups as "normal", high achievers, low achievers, immigrants, Swedish, etc. can lead to the stereotyping of certain (groups of) students as engaged in learning, and some (groups of) students as not engaging in learning.

With a combination of a critical approach (Skovsmose, 1994, 2005; Skovsmose & Borba, 2004) and Foucault's (1993) concept of discourse we view the mathematics classroom and the social practices going on there as part of and affected by broad institutional and societal contexts. These theoretical approaches also help us to bring in the understanding that within groups there are always individuals. Discourses are then recognised as social practices structured through power relations that enact different identities and activities (Foucault, 1993). Consequently also categorization of groups is an effect of discourse. With this dynamic view of discourse, individual students are not to be seen as imprisoned in a group (affected by discourse). By taking active agency, individuals, as well as groups, can be part of long-term changes in discourse, or "leave" a discourse and instead engage in other discourses enacted in a mathematics classroom (Norén, 2010). Agency is not just individual, it is exercised within social practices (Andersson & Norén, 2011), and thus we understand agency as an effect of discourse.

Discourse, according to Foucault, is often understood as encompassing entire disciplines, but can also be conceptualised as smaller discourses related to specific interests in a discipline. The latter view on discourse is adopted in our research (see Walkerdine, 1988; Björklund Boistrup, 2010; Norén, 2010).

Classroom discourse, agency and affordances

Foucault (2003) writes about the role of assessment in education. He argues that, in assessment, surveillance is combined with normalisation. Through the assessment, there is both qualification and classification taking place, as well as the exercise of power and education of a specific knowing. From research in mathematics classrooms Björklund Boistrup (2010) construed four assessment discourses in mathematics. The focus was mainly on feedback. In one of the discourses, "Reasoning takes time", the emphasis was more on mathematics processes such as reasoning/arguing, inquiring/problem solving and defining/describing, than in the other three discourses. At times teachers promoted or restricted the use of communicative resources dependent upon the meaning-making demonstrated by the student(s), and different semiotic resources were acknowledged. In discourses where students and teacher are on more equal terms relative to power relations, affordances for students' active agency were high. This was in discourses where teachers gave descriptive feedback, and where students were invited to give feedback on the teaching.

In our on-going research we will address the intersections of assessment discourses, discourses normalizing "Swedish-ness", or promoting bilingual mathematics learning, as well as opportunities for students' active agency and affordances for learning mathematics.

CONCLUDING REMARKS

The basis for the analytical framework described in this paper will be developed further during the project. We argue for a critical exploration of day-to-day classroom communication, and intersections of dominant assessment and normalising discourses in multilingual mathematics classrooms. We argue for the need for research to focus on different groups of students, in this paper, multilingual students. In our research the need is not to speak for or "give them voice", but to get new knowledge on school mathematics practices in non-prototype classrooms, for the benefit of learning mathematics for different (groups of) students. We will use categories already existing in public and official documents, and will not create any categories of our own.

However we find that it is necessary to see each group of students as a floating group of individuals or agents, socially constructed, and as an effect of discourses. The theoretical framework described here enables us to study non-prototype mathematics classrooms, in a manner that takes complexities and conflicts into account.

NOTES

1. In day-to-day discourse, based on the Swedish National Agency on Education categorizations of students born abroad and students who have one parent born abroad, used in statistical reports on students' achievement.

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EXPLORING IDENTITY POSITIONING AS AN ANALYTICAL TOOL

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The aim of this paper is to explore how a socio-political analysis can contribute to a deeper understanding of critical aspects for prospective primary mathematics teachers' identity construction during teacher training. The question we ask is: How may power relations in university settings affect prospective mathematics teachers' identity positioning? We elaborate on the elusive concepts of identity, positioning and power relations, seen as dynamic and changeable, as these represent three interconnected parts of research analysis in an ongoing larger project. In this paper we clarify the theoretical stance, ground the concepts historically and strive to connect them to research analysis. In this way we show that power relations and subject positioning in social settings are critical aspects and need to be taken into account if we aim at understanding prospective teachers' identities.

INTRODUCTION

Research focusing on the situation of becoming teachers at Swedish universities focuses mainly on specific mathematical topics. These studies indicate that prospective teachers have difficulties in using correct mathematical vocabulary in lower secondary school (Nilsson, 2005); in understandings of functions through threshold concepts (Pettersson, 2012); and in understanding students' perceptions regarding teaching and learning of functions (Hansson, 2006). Very little (not to say no) attention has been given to prospective mathematics teachers' own experiences of, or challenges of, being part of teacher education.

In this paper we show how power, identity and positioning can be helpful concepts for analysing and hence promote understanding of the dynamics within discourses in a teacher education programme. The overall research question we ask concerns relationships between power relations in university settings and prospective mathematics teachers' discursive subject positioning. We ask this question because we want to explore possibilities, challenges and constraints that teacher students experience, what they talk about and how they act with regard to these experiences. These issues imply a move beyond socio-cultural theories with a focus on culture and participation structures and, in Gutiérrez' (2010) words, also "privilege the voices of subordinated groups and forefront the politics and power dynamics that arise from sites of interaction" (p. 3). By drawing on empirical work undertaken in a larger study, we illustrate how identity, positioning and power are simultaneously working and thus become possible tools for understanding the complexity of prospective teachers' situation.

Implications of taking a socio-political approach

Mathematics education research has undergone what Lerman (2000) calls *the social turn*, which is defined as "the emergence into the mathematics education research community of theories that see meaning, thinking and reasoning as products of social activity" (p. 23). This turn has made us rethink learning as a social activity and the situated theories of Lave and Wenger have become powerful for understanding learning as becoming in practice (Wenger, 1998); for instance as prospective mathematics teachers become practicing teachers (e.g., Jaworski, 2006; García, Sánchez, & Escudero, 2006). From the socio-cultural theoretical perspective it is possible to understand how social practices set the rules for how we act and what we do in a practice. In this research we adhere to Andersson's (2011) understanding that "(mathematics) classrooms are spaces of socially organised practices that, in different ways, shape how individuals are expected to, allowed to and/or required to act" (p. 215).

However, the social and cultural theoretical approaches do not necessarily address political issues in mathematics education (Gutiérrez, 2010; Valero, 2004). If we want to address issues of justice and equity in mathematics education it is important to both pose critical questions regarding mathematics education, like *why* and *for whom,* and also to let the research itself undergo critical scrutiny to ask why we construct research the way we do (Pais, Stentoft, & Valero, 2010). The socio-political turn in mathematics education research offers an additional layer that highlights issues of power at play in these interactions, thereby helping us better reflect on and contribute to the complexity in our society (Gutiérrez, 2010).

A socio-political approach supports us to rethink learning as a political and social activity (Valero, 2004; Gutiérrez, 2010), which provides an additional dimension to learning through participation in communities of practice (Wenger, 1998). The approach stresses the importance of *transparency* to make the familiar seem strange and to make taken-for-granted roles more explicit; of *subjectivity* to see the individual as constantly in the making, defying categories; and *agency/voice* to understand individuals negotiating and sometimes showing resistance to the discourse (Gutiérrez, 2010). Thus, if we want to understand prospective teachers' concerns during teacher training, we need to move behind the scenes in the well-known contexts and focus on factors that affect both actions and reactions in social settings.

IDENTITIES, POSITIONING AND POWER

In this section we place identity, positioning and power within the socio-political theoretical tradition to illustrate how we see the three elusive concepts working together in the present study.

Identity

There is agreement that identity is a key term in contemporary social analyses (Brown & McNamara, 2011) and that identity is multi-facetted and dynamic in its character. However, the range of interpretations regarding how identity can be understood varies ontologically; for Wenger (1998) our identities are formed through participation in communities of practice, whereas Gee (2000) suggests that peoples' multiple identities imply "being recognized as a 'kind of person', in a given context" connected to their performances in society (p. 99). From a socio-political stance, identity is something you do and not something you are (Gutiérrez, 2010). We are inevitably drawn into power relations in different contexts, which in turn suggest that identity might be "consented to through constant social negotiation" (Walshaw, 2004, p. 65).

Stentoft and Valero (2010) argue that a poststructural perspective allows them to "think of identity in terms of fragile identification processes embedded in discourse and, therefore, tightly related to peoples' actions and participation in on-going discursive practices" (p. 62). This on-going changing relationship requires a more nuanced understanding of the relationship between the individual and that to which one identifies, and therefore *identity* often is replaced by *subjectivity* (Walshaw, 2004, p. 67). We do not replace identity by subjectivity in this study. We use identity as a dynamic concept and understand identities as "fractured and fragmented […], complex and multiple" (p. 80), and follow Brown and McNamara (2011) suggesting *identifications* as ways of making sense of what individuals experience: "[T]here are no identities as such. There are just identifications with particular ways of making sense of the world that shape that person's sense of his self and his actions" (p. 27).

Positioning

Positioning is a conversational phenomenon through which the actors are positioned by themselves or by others (cf. Davies & Harré, 1999; Holland, Lachiotte, Skinner, & Cain, 1998; Setati, 2008). In contemporary socio-political research positioning is central, and used as indicator of how power relations determine discourses and how individuals take up different positioning as a consequence of these discourses (Davies & Harré, 1999). Positioning is dynamic, which implies that one single person will enact different storylines in parallel, depending on whether the situation is well known or rare, if the person is experienced or a newcomer. Posed in other words: what a person tells or does to position him- or herself, will differ and change, depending on the situation (Davies & Harré, 1999) or context (Andersson, 2011). This could happen almost simultaneously. Positioning could also be strategic, meaning that people will tell different stories about themselves depending on how they want to be presented (van Langenhove & Harré, 1999).

> [T]he catalogue of kinds of positions that exist here and now will not necessarily be found at other places and times. In so far as the content of a position is defined in terms of rights, duties and obligations of speaking with respect to the social forces of what can be said, and these 'moral' properties are locally and momentarily specified, positions will be unstable in content as well. (van Langenhove & Harré , 1999, p. 29)

Holland et al. (1998) distinguish between figurative identities and positional identities, and write that “figurative identities are about signs that evoke storylines or plots among generic characters; positional identities are about acts that constitute relations of hierarchy, distance, or perhaps affiliation” (p. 128). This indicates that figurative identities have not so much to do with political concerns and power relations, and that the socio-political connection is much stronger when we are acting out our positional identities. This can be compared with the strong political writing of Davies and Harré (1999) about how our experiences and lived histories impact our subject positioning:

> ‘Positioning’ and ‘subject position’[…] permit us to think of ourselves as choosing subjects, locating ourselves in conversations according to those narrative forms with which we are familiar and bringing to those narratives our own subjective lived histories through which we have learnt metaphors, characters and plot. (p. 41)

The social forces and ‘moral’ properties that permit different subject positions are, as we can see, related to political issues and power relations. We are in this study thus using positioning as verbs (Wagner & Herbel-Eisenmann, 2009), as something one *does* to choose subject and to identify oneself in relation to how power relations determine the actual discourse. This study is focusing mainly on individuals’ positioning through *actions* (van Langenhove & Harré, 1999; Holland, et.al., 1998). Actions are here defined as what individuals say and do to position themselves in, or in relation to, a situation, and found in stories from interviews, utterances in classroom interactions as well as in physical movements, such as raising one’s hand, standing in front of the class etc. Individuals’ actions are hence interpreted as positionings in relation to different situations, such as group work or problem solving in whole class; other individuals, for example, teachers, classmates, the researcher etc.; and in relation to mathematical content. Therefore we do not separate figurative and positional identities, and we also strive for using *positioning* and *subject positioning*, to avoid too many words that have almost the same meaning.

Power

What does power mean within mathematics education? How can power be understood and brought into mathematics education research as a useful tool? These questions have been discussed by Valero (2004), who outlines three ways of interpreting power and its consequences for research in mathematics education. First, she strongly rejects the assumption that power is within the mathematics subject, that mathematics is a powerful knowledge and that mathematics education empowers people. “Saying that mathematics is powerful means that mathematics itself can exert power [and hence that] mathematics is given a life of its own that it does not have” (p. 13). Second, power can be seen as a capacity of people, or groups of people, to maintain social structures of inclusion and exclusion. This conception of power, rooted in the Marxist and Critical traditions, have been challenged in ethnomathematics and equity research and highlights the “necessity of questioning both mathematics and mathematics education practices" and "of incorporating c*ritique* as an essential element of a socio-political approach" (p. 15).

The third way of defining power – and the one for which Valero argues – is related to postmodern and poststructuralist understandings and seen as "situational, relational and in constant transformation" (p. 15). This means that we cannot see power as either stable or intrinsic to social class, for instance, or that power is within mathematics education or in mathematics itself. Valero expresses this different way of defining power "as a relational capacity of social actors to position themselves in different situations and through the use of various resources of power" (p. 15). Power could then be used as an analytic tool in different contexts without limitations regarding race, gender, etc. This third way of defining power resonates with the aim of this study. There are no highly conflictive situations and extreme inequalities that will be in focus. Instead, it is the mundane and compulsory activities within the mathematics teacher programme that are central and that have the potential to reveal how power relations can affect prospective mathematics teachers' acts of positioning and identification. We sum up this section by clarifying that our starting point regarding identity, positioning and power is that these concepts are understood as dynamic, interrelated and situated. These unstable relations are not only theoretical constructions, but also powerful methodological and analytical tools in this study.

METHODOLOGY

A socio-political approach in research comprises not only relevant theories and methodologies, but also visibility of the researcher and consciousness of how consistency between these is maintained. Following Andersson (2011), this study is socio-political with a "little p", by being aware of political issues, power relations and having a fair relation with the participants: "A researcher acknowledging a 'little p' emphasises an awareness of political issues, is sensitive to power and relationships, and cares for research participants through a researcher 'attitude'" (p. 30). The socio-political "little p" approach supports the way data was collected. By taking a socio-political stance in the study we strive to give the student teachers a voice. A voice, which could be both a reference to singular stories and occasions, as well as to more common concerns, shared by more than single prospective teachers.

Methods

Ethnographic data, such as interviews, course documents and a rich research journal, was collected over a two-year period in situations typical to the teacher education programme. Through Kicki's participation in daily routines and developing ongoing relationships with the students, we learnt about the social setting, its discourses and contexts (Emerson, Fretz, & Shaw, 1995). This approach made us sensitive to individual reactions and unforeseen incidents, but also conscious of careful writing "that leads to empathetic understanding of the social world of others" (p. 72).

Interviews were open, to allow spaces for discussing what participants find important. It was important for us to be aware of how the interview situation could affect which stories were told. Van Langenhove and Harré (1999) emphasize that, within conversations, it is important to grasp the dynamic character of positioning. The conversants are jointly creating both the story-line and the illocutionary force of speech-acts. This implies that a speaker, who takes up a position by opening a conversation, can change position during the communication. Kicki as interviewer played an important role by posing questions, but that did not necessarily mean that the further conversation structure was determined. The interviewee could choose whether or not to adopt the position that the interviewer initiated, as seen in the transcript below. This approach is in line both with the aim of the study and with the socio-political theoretical tradition.

Foregrounded in this paper is one particular interview with a prospective mathematics teacher, Theresa (pseudonym), who expresses concerns about institutional constraints, language and mathematics, and a transcript from a mathematics class where Theresa is an active participant. The data was thus generated from two different contexts: the interview context with Theresa and Kicki involved, and a mathematics lecture.

The data processing was done in three steps. First, a summary of the situation was written to grasp central issues and actual concerns for Theresa. Second, her different positioning(s) in relation to each concern was noted, together with relations of power that affected this positioning. At the same time we observed how she identified herself due to each specific concern. As a third step, a reflective note was added to each situation. This note, inspired by Andersson (2011), was written from the basis of the ethnographic work and in a fruitful way allowed the talk to be connected to the specific contexts in which they were told. In the following section we present results from the analysis of an interview transcript and of transcribed field notes, both conducted in September 2010.

TRANSCRIPTS AS AN EXAMPLE

In the following example we illustrate how identity, positioning and power are simultaneously working and how the dynamics between Theresa's positioning, the present power relations and her identification can be interpreted:

Kicki: How... this course you take... How do you feel about it?

Theresa: Well… quite good. I don't know… yes.

Kicki: You are often involved and respond to questions and… when it comes to mathematics and stuff. Are you confident?

Theresa: Yes, yes, yes! In KOMVUX [1] I have had many… what can I say, eh… good grades all the time. I have not… But, what I have learnt here is a lot more about language. That is my problem. And… It is about life experience too… what one needs. Since I haven't had any child in the Swedish… eh, school, I have no experience of that. And I haven't been working either. Everything affects me from the beginning and I have had a bad mentor who pressed me very much. But maths…

Kicki asks Theresa to tell more about the specific course and invites her to do this by saying that she has recognized that Theresa is active during the lectures and that she sometimes poses questions to the teacher. Theresa positions herself in different ways with regard to this: first, in relation to the mathematics she has learned in Sweden, second in relation to her language difficulties and lack of cultural experiences and third, in relation to her supervising teacher, who she does not like. She does not take up the position of herself being an active student. Theresa expresses empowerment regarding mathematics, but disempowerment with regard to the demands she experiences regarding language and to be knowledgeable about the Swedish school system. She identifies herself as one who is positive towards and knowledgeable about mathematics, but at the same time frustrated, because of her language concerns and of being novice in the Swedish culture.

Theresa interrupts Kicki's attempt to pose the next question and continues to talk about mathematics and about her difficulties in understanding the content of the compendium [2]:

Theresa: I've tried two teachers and I do not understand anything, still, what is written in the compendium. So she [the teacher] said to me, what she explains is what is important. Not what is written in the compendium.

Kicki: And you think you can follow along as she explains?

Theresa: Yes, I understand what she says, but I don't understand why and how to use it later. I don't know. But…

Kicki: [Poses a leading question.] Could you ask questions about that, when... because you sometimes do? Like 'how… isn't it supposed to be like this?' or…

Theresa: Yes, but I am a little unsure of the language, and because of that I… I have many, many questions [giggles]. And since she is a bit… I don't know… she is a bit unsure. Or… this is the first time she teaches this course. I don't know how to pose questions. Because… when it comes to mathematics I can manage independently. I do not need the teacher so much. I'll be fine if I read a lot myself.

In this discussion, Theresa positions herself in relation to mathematics, in relation to written and oral language, and in relation to the teacher. Kicki poses a leading question regarding Theresa's activity in class and this time she takes up the position of herself being an active student. Theresa expresses disempowerment with regard to institutional constraints, such as written mathematical text in the compendium and posing questions to her teacher. Still she is empowered by her mathematical knowledge and says that she will be able to pass the course without the teacher's help. Theresa identifies herself as knowledgeable in mathematics and independent of teacher support. At the same time she identifies herself as having language difficulties and therefore dependent on teacher's explanations.

What follows below is a short extract from one of the first lectures in the mathematics course (field notes, September, 2012).

> The teacher talks about primes and prime factorization and asks: 'What is a prime?'
>
> Classmate: A number one can divide by one and [the number] itself.
>
> The teacher asks if the class can give an example.
>
> Class: (in chorus) One.
>
> Teacher: No, that is more like a convention. [She writes 2,3,5,7,11,13,17,19,23, ... on the black-board]. Another number that can be written as a product of as small numbers as possible [writes 324=2*162=2*3*9*9=2*3*3*3*3*3] Prime factorization is unique! There is exactly one way to prime factorize each number.
>
> Theresa, who usually sits silently, raises her hand and asks:
>
> Theresa: May I ask you a question? I understand the first line [2*162], but where does the three come from [2*3*9*9]?
>
> The teacher re-writes 2*3*54=2*3*27=2*3*2*9*3=2*3*2*3*3*3

Theresa does not get any response to her question. The teacher just corrects her mistake and returns to the agenda without considering whether she got it right or not – which she did not [2*3*54≠2*3*27].

In this extract Theresa is positioning herself in relation to the mathematics content by raising her hand and asking for clarifications. She is empowered through her mathematical strengths, but does not question the new writing. No one in the class comments which indicates that there might be power relations present that prevent questioning of the correctness of the teacher's writings. Theresa identifies herself as knowing the mathematical content and willing to share her knowledge with others by asking for clarification. However she is not questioning the teacher.

CONCLUSIONS

The results illustrate how we can interpret subject positioning, power relations and identities that are working simultaneously in educational settings and also in a research situation/context. What the analysis also illustrates is the importance of reflexivity in every step of the research process (Emerson, Fretz, & Shaw, 1995). Through conscious reflexivity we can see that power relations in these contexts have been related to individual opportunities and constraints, as well as institutional and intrapersonal relations. By, for instance, posing the leading question in the second extract we may have got a different answer from Theresa, than what had been the case if she had continued what she was talking about without being interrupted. In this case Theresa returned to her language concerns and the teacher-student relation, instead of the mathematical applicability she started talking about. Hence we need to be aware of, that power relations and the acts of positioning also involve the researcher, who affects the discourse. This, in turn, will affect our understanding of the phenomena. The analysis indicates that power relations and subject positioning in different contexts affect the student teacher's identities. We can also see that students can enact different identities almost simultaneously, due to the fact that the identities are dependent on relations of power as well as on one's positioning in the actual context.

What we have shown in this paper is a way of using identity, power, and positioning in a complementary way to understanding the process of becoming a teacher; affected by power-relations and by the individual's positioning of self in social settings. This approach verifies that mathematics teacher education is not a neutral activity, emphasised by Gutiérrez (2010), and we propose "alternative – and complementary – forms of interpreting, explaining and understanding mathematics education practices", as suggested by Valero (2004, p. 16). We argue that we should not only focus the on the fragility of identities (Stentoft & Valero, 2010), or merely focus on power relations as affecting identities (Walshaw, 2004). We need to take into account the three interrelated factors, namely subject positioning, power relations and identifications as mutually interacting in social settings. This approach provides a complementary way of understanding becoming teachers' situations and concerns during teacher training, an understanding that can provide new dimensions on learning to teach mathematics.

NOTES

1. Municipal adult education
2. The compendium contains advanced mathematical texts and tasks at university level.

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IDENTITY DEVELOPMENT THROUGH MATHEMATICS DISCOURSE: TEACHER LEARNING ABOUT POSITIONING

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Rich discourse in secondary mathematics classrooms can support students in learning about mathematics and developing of their identities as mathematical learners and doers, or positioning. We describe a teacher education experience that supported teachers in analysing discourse practices and linking those practices to positioning. We trace teacher learning along a hypothetical learning trajectory, identifying activity features and learning goals that support knowledge of positioning.

The discourse that teachers and students use defines both the intellectual and social structures of the mathematics classroom. Purposeful classroom discourse (Herbel-Eisenmann & Cirillo, 2009) involves attending to the influence of the classroom talk on both the mathematical trajectory of the classroom and on students' identities as mathematical learners. Increased accountability to the rigor of mathematics taught in the secondary classroom has brought closer attention to the mathematics taught in the United States (NGA & CCSSO, 2010). Less attention has been paid to supporting teachers in accessing and shaping students' identities as knowers and doers of mathematics. In this article, we describe a professional development experience that provided teachers with opportunities to learn about purposeful classroom discourse. We hypothesize a learning trajectory across several activities designed to address issues of positioning – that is, the "ways in which [teachers] use action and speech to arrange social structures" in the mathematics classroom (Wagner & Herbel-Eisenmann, 2009, p. 2). In this report, we describe the ways in which teachers engaged in that set of activities and describe how that engagement informed a revised learning trajectory.

POSITIONING: SUPPORTING IDENTITY DEVELOPMENT IN STUDENTS

The ways in which teachers position students in the mathematics classroom has a direct influence on their developing identities, dispositions as learners, and beliefs about learning and doing mathematics. When students' opportunities to know and do mathematics are limited to reproducing procedures and memorizing facts, they often lack confidence and creativity in their mathematical ability and emerge from secondary mathematics disillusioned with the discipline (Boaler, 1998). Engaging students in making sense of and reasoning through problems is a promising path towards richer conceptions of knowing and doing mathematics. Enacting this sort of mathematics requires teachers to reconceptualise their roles as the mathematical authority (Webel, 2010). Discourse influences this work, as the words that teachers use and the contexts in which they use them frames the opportunities students have to act as mathematical creators and authorities. By supporting teachers in analysing the nature of their classroom discourse, we anticipate that they will become better able to attend to their students' identities as mathematical learners.

MATHEMATICS DISCOURSE IN SECONDARY CLASSROOMS: A CASE-BASED PROFESSIONAL DEVELOPMENT CURRICULUM

For the past decade, we have worked with mathematics teachers to support the development of their classroom discourse practices. Rich discourse supports students' engagement in argumentation and conceptual explanations in ways that improve learning (e.g., Chapin, O'Connor, & Anderson, 2009). Teachers need to establish classroom norms with meaningful discourse at the center (e.g., Staples, 2007). Mathematics teachers' discourse patterns, however, remain rather traditional (Stigler & Hiebert, 1999), even when teachers attempt to change their practice in this way (Herbel-Eisenmann, Lubienski, & Id-Deen, 2006)}. In response to this evidence, we developed and piloted professional development (PD) materials for supporting secondary mathematics teachers in enacting purposeful discourse in their classrooms. These practice-based materials are built around narrative or video cases that problematize aspects of classroom discourse and support teachers in examining their own discourse practices. We focus on discourse at the secondary level because mathematical language and meanings become increasingly complex beginning in middle school and most discourse-related work in mathematics education has focused on elementary school classrooms.

This work builds on that of Chapin, O'Connor, and Anderson (2009), who focused on mathematics discourse in elementary classrooms. Our materials modify their "talk moves" to make: (a) the context of secondary school mathematics classrooms central; and (b) students' opportunities to learn mathematics vital to decisions about their use. Using six Teacher Discourse Moves as a centerpiece, our materials support secondary mathematics teachers in becoming purposeful about developing productive and powerful classroom discourse. By productive, we mean how discourse supports students' access to mathematics content and discourse practices. By powerful, we refer to how discourse supports students' developing identities as mathematical knowers and doers. As mathematics teachers learn about, contemplate, and plan for discourse, they become more purposeful about how they enact their classroom discourse practices (Herbel-Eisenmann & Cirillo, 2009). In this analysis, we trace teachers' opportunities to learn about powerful discourse and ways they can access and support students' identities as mathematical knowers and doers. The design of the materials incorporated a hypothetical learning trajectory (HLT) related to positioning. We use the HLT to analyze teacher talk along that trajectory to portray the ways in which teachers came to understand the role of discourse in positioning students. Two questions guided our analysis: In what ways do teachers talk about the concept of positioning in secondary mathematics? To what extent do teachers integrate ideas of positioning with mathematics teaching and learning?

METHOD

This analysis draws from a pilot of the PD materials by the development team at a large public university in the Midwest United States. Nine middle and high school teachers from three local districts, ranging from suburban to rural fringe, were recruited to participate. Participants varied in teaching experience from 0-17 years. Eleven PD sessions were held across the academic year, each lasting three hours. Participants were exposed to three types of interrelated learning activities: solving rich mathematics tasks, analysing narrative and video cases of teachers using those tasks in classroom practice (including the use of video, student work, and transcripts), and collecting data and asking questions about their own classroom discourse.

The materials use a case-based approach to teacher learning (Sykes & Bird, 1992). This approach sidesteps limitations of a theory-to-practice approach, which leaves the important work of translating general theories into specific pedagogical practices to teachers. Rather, the analysis of carefully-selected cases brings into relief general principles related to teaching and learning. Case-based research in mathematics education has underscored the importance of considering multiple perspectives on the mathematics. New insights are afforded when teachers first engage with a mathematical task as learners, analyse a narrative or video case of a teacher using the task with students, and reflect on implications for practice (Smith & Friel, 2008).

In considering how teachers might learn about the role of positioning in mathematics class, we theorized that a similar sequence of activities would support learning about positioning. As such, we built an activity sequence that first engaged teachers in considering a student's perspective on positioning, then the analysis of several cases of teaching focused on the teacher's role in positioning students, then reflection on their own practices in positioning students. This HLT is illustrated in Figure 1 below.

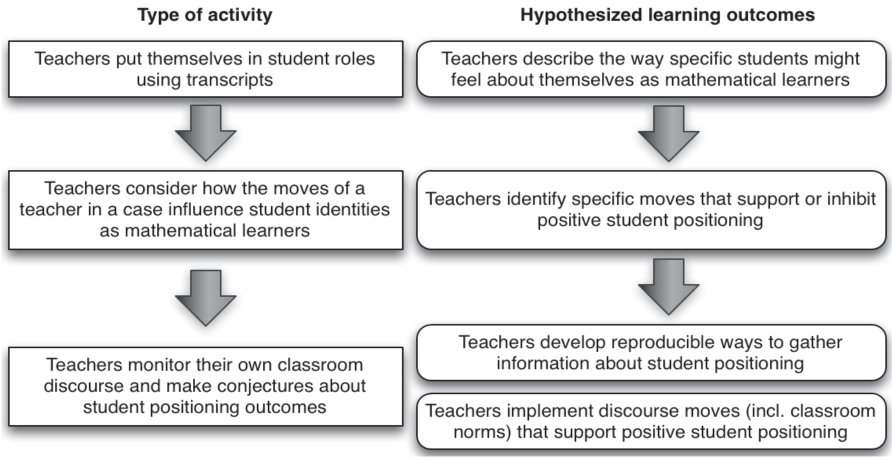

Figure 1. Initial Hypothetical Teacher Learning Trajectory for Student Positioning

Data Collection and Analysis

The enactment of the pilot in general, and the positioning activities in particular, were conceptualized as a design experiment. The research team drafted the initial set of materials with learning goals in mind related to purposeful discourse. PD sessions were video recorded with field notes taken, and the team met to debrief each session and make changes to future plans to best support teachers in meeting the learning goals. Particular attention as paid to the goals related to positioning, as this thread of the materials was relatively new. Additionally, we sought to enact the activities in the HLT to furthering our goals for positioning while respecting the conceptions of positioning that teachers possessed at the start of the PD. Data from the recorded PD sessions, field notes, and exit interviews were used in this analysis.

The HLT served as the data analysis frame. We identified activities corresponding to the types in the left column of the HLT. We examined field notes to identify other activities related to positioning of types not represented in the initial HLT. For these activities, we examined research team's planning notes and discussions to identify the rationale for changes. To describe teachers' opportunities to learn, we analysed the discourse in each of the positioning activities. We identified key themes in the discourse that describe the ways in which teachers were making sense of positioning and related these themes back to the goals in the HLT. Our results consist of the themes identified in each activity, and a revised HLT based on our analysis.

RESULTS

In this section, we describe each positioning activity and map the activity to aspects of the HLT. We then analyse the discourse in which teachers engaged, constituting the opportunities to learn. Finally, we describe the research team's reflections on each activity and the ways in which the enactment influenced the revised HLT.

Sessions 2 & 3: Creating an imagined classroom transcript

The first positioning activity asked teachers to create an imagined transcript of students discussing an area and perimeter task that the teachers had solved earlier in the session. The goals for teachers were to consider how students may come to understand the key mathematical ideas, and how students might position themselves as mathematical learners. Groups of three started their transcripts during session 2. As they began working, they focused on the teaching issues – such as document camera use and the task's location in the curriculum – rather than on student thinking.

Teachers read and commented on one another's transcripts prior to session 3, which began with a discussion of what they noticed and wondered about those transcripts. Two of the three groups enacted the task and transcribed rather than imaging the transcript. As teachers were just at the start of opening up their discourse, this resulted in strongly teacher-led discourse. When asked what they noticed, Bobby identified this immediately, saying that they were really "pushing students along."

While the conversation focused primarily on evaluating the teacher's role in each of the transcripts, there were some moments in which positioning was considered:

Kelly: I noticed in our scenario with the kids that as soon as the final group got up to present the general example, the class kind of shut down, and Xander had to say, "I can tell you have questions, ask these guys they're the experts…" Kids were muttering, "They cheated, how could they come up with that?"

At the same time, Kathy voiced what we suspected teachers were thinking, that the activity felt contrived and inauthentic: "I'm glad you guys did this in person, because [in a prior PD] we were asked to talk like students and it never turned out the way it is in real life." Teachers valued the discussion but it did not meet the goal of teachers describing how students experience positioning. An adjustment was necessary.

Session 4: Read-aloud of the Hidden Triangles Exploration case, Part 1

The case discussed in Session 4 featured Mr North leading a group of students in an exploration of triangle congruence. The case was intended to illuminate and problematize different discourse patterns. The first part of the case consisted of transcript of Mr North's initial review of concepts and launch of the task, using rapid cycles of Initiate-Respond-Evaluate (IRE) discourse patterns (Stigler & Hiebert, 1999). After the failure of the imagined transcript activity, the research team hypothesized that enacting student roles through a read-aloud of the case might help teachers consider how students experience positioning. This discussion following the read-aloud was designed for teachers to notice ways in which participation was limited by the closed nature of the IRE pattern.

The initial discussion of the case focused primarily on the ways in which IRE might be effective in moving through a great deal of mathematics quickly. Teachers noted that students gave short answers without explanations. Some defended the value of this discourse pattern for getting lots of students to talk and to stay engaged. Teachers also wondered whether students were volunteering or being called on randomly, which prompted teachers to consider the role of the non-speaking students as the teacher pressed for a definition of congruent, as exemplified by Deidre's statement:

Deidre: And you know the thing that could be happening there, if I know what congruent means and I'm ready to do the exploration, I'm going to get on my cell phone and not pay any attention.

This read-aloud began to achieve the first-level goals of the HLT: teachers began to consider how students may feel as mathematical learners. There was also some movement towards the second level, considering the teacher's role, as Kyle noted at the end of the 15-minute discussion: "I can understand the rush, we've gotta get through this, but I just feel… there was a lot of stuff that [Mr North] could've done differently." The next discussion of the case was designed to link the notion how students might have felt to Mr North's moves as a teacher in a later part of the lesson.

Session 5: Read-aloud of the Hidden Triangles Exploration case, Part 2

In Session 5, teachers read another excerpt from Mr North's class, during a mid-lesson discussion where students share solutions and featured denser student discourse. The goals were for teachers to identify teacher discourse moves and to consider how those moves influenced both the mathematics and student positioning.

The 10-minute discussion started with teachers speculating about how students might have felt. With some prompting from the facilitator, participants began using their experiences from the read-aloud to make stronger claims about student positioning. The discussion began with Xander noticing that Mr North's use of revoicing helped clarify Kiley's thinking because "she was getting fuzzy at the end and losing focus." This contribution moved teachers to a discussion about how the revoicing positioned students' ideas as important. Deidre and Maggie added to these ideas, wondering how students felt when their ideas weren't taken up as compared to Kiley's positioning as a mathematical expert. Teachers were then asked to reflect on how they felt when they voiced the lines of their assigned student, prompting multiple perspectives on how students may have felt. The facilitator recapped the discussion:

Carol: Maybe moving the math forward positioned José in that moment as his ideas or the solution you came up with as not being important in that moment. So sometimes they don't work together, sometimes they do.

In this discussion, we saw evidence of teachers achieving the second-level goal of the HLT: identifying specific moves that support or inhibit positive student positioning. Teachers made connections between the moves made by Mr North and how those moves may have positioned his students. However, the amount of criticism levied at some of Mr North's decisions made the team reticent to press teachers to immediately analyse their own practice with students around positioning. We instead created an activity for teachers to consider relationships between norms and positioning more broadly. This would inform the research team about teachers' current practices and ease the transition to more detailed discussions of practice.

Session 7: Setting goals for productive and powerful discourse

Given the criticism of Mr North by the group of teachers, we felt it necessary to learn more about what the teachers already do in their own practice before engaging in planning activities around their own practice. We developed an activity that served this purpose and began the shift in focus from another teacher's practice to their own, asking teachers to consider the ways in which a mathematical and social goal of their choosing might inform planning for rich classroom discourse. The conversation about social goals almost immediately focused on the teachers' own norms and expectations and their influence on positioning for their students. Deidre, the group's most experienced teacher, opened up the discussion about setting norms by saying, "I'll say that I'm bad at this. I am *bad* at this. My norm and expectation in my class feels like anything goes, as long as it doesn't make me mad."

Other teachers offered examples from their practice about classroom norms related to social goals and speculated on the impact of norms on positioning. For example, several teachers noted the importance of modelling for students the value of offering an incorrect solution. They noted how establishing that norm might encourage more student contributions, even if they might be wrong, enriching the mathematical discourse. After about 12 minutes, the facilitator shifted conversation to the ways in which specific discourse moves influence positioning. Participants offered suggestions of moves that might move forward their social goals. This conversation made connections between teacher moves and positioning, reinforcing the second level HLT goals, and used the idea of norms to situate teachers in their own practice.

Session 7: Examining expectations for communication and participation

Following this activity, the team wanted to move from broader norms to specific evidence that one might collect from one's own classroom, the third-level goals in the HLT. Teachers revisited a classroom transcript in which Ms Krusi conducts a whole-class discussion of an algebra task. With Ms Krusi's classroom norms in mind, teachers identified ways those norms influenced student positioning. Kyle began by noting that Ms Krusi did not seem to evaluate student responses even when they were incorrect. Deidre noted that Ms Krusi seemed to know which students she was going to call on in advance. These initial contributions focused on the intentionality of the teacher's actions, further evidence of having made gains on our second-level goals. Teachers shifted to talking about the broader impact of such moves, noting how the teacher's actions reflected her norms and expectations for participation and influenced positioning. Kelly shared what her group noticed about ways in which the teacher's norms and actions encouraged broad participation:

> Kelly: We found when we were looking at the norms, that technique [of calling on many students in an intentional way] must've helped everybody participate… It wasn't one student dominating that section of the discussion. And within that same section also it was okay for the kids to ask questions.

She then provided examples in which students felt comfortable asking questions when they were confused and linked this to Ms Krusi's norms. Kathy noted that these classroom norms take time to establish and that the teacher must be intentional in establishing and enforcing norms for powerful discourse: "I think [teachers] make this a habit. It's not something that they bring out every couple of weeks. It's something that has to be done pretty consistently."

Participants concluded with a few more examples linking expectations in Ms Krusi's classroom around the topics of authority (e.g., Ms Krusi has the authority to interrupt) to ways to assess the possible influence on positioning with students (students continued to contribute). This discussion showed evidence that teachers were able to identify practices for setting and reinforcing norms that influenced positioning, and were beginning to identify ways in which they could collect data about positioning from students. Following this session, teachers engaged in a series of Connecting to Practice activities related to the third-level HLT goals, in which they implemented the Teacher Discourse Moves and identified ways in which their moves influenced student positioning. While teachers reported on these moves and student behaviours, these discussions felt mathematically thin. The team decided to explicitly address this tension between mathematical and social goals in the final session.

Session 11: Navigating tensions between productive and powerful goals

Carol's comment in Session 5 had framed positioning, and the notion of powerful discourse, as a construct that can sometimes work in concert with the mathematical goals of a lesson, and other times can be in conflict. After looking at positioning in their own classroom, teachers' reporting out of the relationships between discourse and positioning were frequently amathemematical. In our final session, we elected to have an open discussion about the use of productive discourse in the classroom, and the tensions between mathematical (productive) and social (powerful) goals.

Teachers discussed in small groups the nature of this tension and ways to be planful about navigating those tensions. They initially focused on what they would want to do to set norms next year, and how those expectations would support the mathematical work in which they wanted students to engage. Bridget reported that she wanted to better negotiate social goals and ways of working at the start of the year. Deidre echoed this: "I just assume they know how to participate in different modes of teaching," going on to note that students get confused as to their roles as learners (talk, listen, use technology) on any given day. After teachers recalled stories of students that thought they were good at procedural mathematics and hit a conceptual wall later, Kathy summarized the conversation:

> Kathy: They have an expectation about what math class is supposed to be, and it's probably (limited) – [we should] start with that at the beginning of the year, and I recognize that now, and say that we're going to approach this differently, and you may not (take on the same roles) you're used to.

This discussion shows teachers integrating the setting of social goals and positioning students in different roles in the classroom with the opportunity for students to engage in richer, more meaningful mathematical experiences. The conversation that was initially focused on the tensions between mathematical and social goals evolved into a discussion of the connections between the two, which was our intended goal for what teachers might learn about positioning through engagement in the materials.

Looking back on the six activities related to positioning, teachers met many of the hypothesized learning outcomes in the original HLT, supported by two key additions to the HLT. First, considering how general norms and expectations can influence positioning in Session 7 supported teachers in being able to examine their own practice in depth. Second, the final discussion on the tensions between mathematical and social goals helped teachers make concrete plans for addressing positioning in their classrooms. The revised HLT in Figure 2 reflects these shifts.

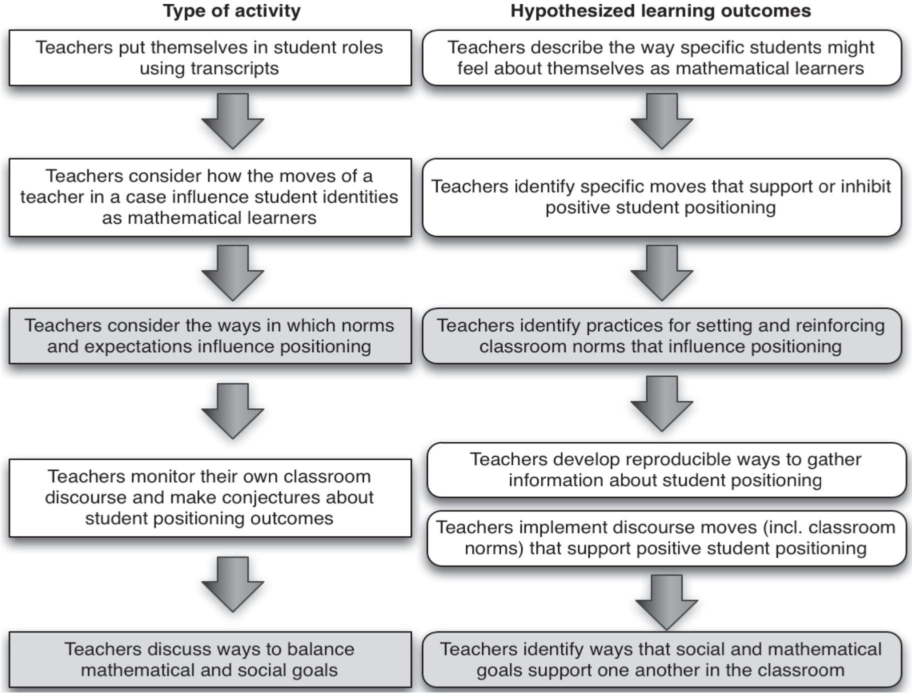

Figure 2. Revised Hypothetical Teacher Learning Trajectory after Implementation

DISCUSSION

In this study, we sought to investigate the ways teachers made sense of positioning and how they integrated positioning with the teaching and learning of mathematics in secondary classrooms. The initial HLT envisioned a sequence of activities in which teachers first considered positioning from a student perspective, then identified moves in another teacher's practice and their implications for positioning, followed by the investigation of positioning and implementation of discourse moves that support positioning in their own classrooms.

Looking back on the enactment of the PD activity sequence, teachers were able to identify aspects of student positioning and link particular discourse moves to positioning implications. The revised HLT in Figure 2 indicates some key areas in which teachers might need additional support in considering issues of positioning. First, considering positioning from a student's perspective was challenging for teachers in our first activity. While we found the imagined transcript activity useful for other goals related to learning about powerful discourse, it did not serve to help teachers think about student positioning. Having teachers take on student roles through a read-aloud enactment of a classroom seemed to be an important step in achieving the first-level goals. Second, while linking teacher moves to student positioning was a natural transition for teachers, there may be a tendency to be overly critical of a teacher's practice in this regard. In transitioning a group of teachers from discussing an outside teacher's practice to analysing their own, a broader discussion about classroom norms and practices seemed to ease the criticism and allow a safe space to consider their own teaching. Finally, teachers may consider positioning as more closely linked to student behaviour and classroom management than the mathematical learning of students. We found the closing discussion of the tension between mathematical and social goals to be effective in linking the two concepts.

Implications

By providing insight into the research team's iterative reflection process – repeatedly making design decisions based on information about what participants seemed to know and understand about student positioning – this article illuminates the ways in which professional development efforts can be responsive to teachers' needs, particularly for a concept like positioning that can be challenging for teachers to discuss openly and honestly. That is, by using the HLT as a frame, we were able to create experiences that encouraged forward movement on the trajectory by making informed design decisions. These results served to confirm the hypothetical learning trajectory, as well as suggesting some additional detail that served to ease transitions from looking at positioning from the student perspective, from the teacher perspective, and ultimately in one's own practice.

Limitations and Future Research

Our results are tempered by the fact that our pilot teachers volunteered for the PD and as such, were already motivated to consider their classroom discourse practices. Enacting a similar set of activities with a larger and more diverse group of teachers would provide more information about the nature of the HLT and how teachers learn about student positioning. Our ability to investigate changes to teachers' classroom practice was limited to self-report data from the group; following teachers into their classrooms to examine the ways in which their discourse positions students is an important next step in this line of research.

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WHEN KNOWLEDGE LOSES RELEVANCE - A MATHEMATICS CLASSROOM IN A CONTEXT OF SOCIAL AND INSTITUTIONAL SEGREGATION

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In this paper, I will investigate a mathematics classroom that brings together students who all share unfavorable social backgrounds and a lack of success in primary school. Using the Bernsteinian concepts of the pedagogic device and the pedagogic code, I will show how the process of transmission discharged the pedagogised knowledge of any kind of power and value that it could carry. With this analysis, I want to contribute to a deepened understanding of how institutionally and socially segregated educational contexts may amplify the 'lack of aspiration' for official knowledge by students from underprivileged backgrounds.

AN INTRODUCTORY VIGNETTE

"You surely all know basic operations from primary school. Actually, you all are able to do the addition quite well, also beyond the tens, but what I noticed then is that you very unfortunately forgot how subtraction, division and multiplication worked again." Therefore, the teacher announces a repetition of the basic operations for the next weeks. *"I will explain subtraction to you once again"*. However, the teacher's announcement *"I explain"* in turn is substituted firstly by *"I'll do an exercise for practice"* and finally by the demand *"so is one of you able to compute* 333 minus 18 *at the front, for the class?*" After two students fail at the blackboard to either carry out the computation or to provide its verbalisation the teacher goes through the computation herself: *"Well the first thing is a plusnumber okay?"* She writes a plus in front of the number. *"This is a plusnumber from which I shall deduct eighteen, I take the latter number",* she points at each digit. *"From eight to three doesn't work right? The three is in fact smaller than the eight. I borrow a ten from the row in front",* she writes a small one. Pointing at one digit after the other, she goes on: *"From eight to thirteen are five, okay? I write down the one here because I have borrowed it from the row before. One plus one is two. I add up to three that's one. Don't need to borrow a ten and from zero to three is three, okay? Solution gets underlined twice! ... Any questions?"*

INTRODUCTION

The students in this classroom are in the age of twelve to fourteen. They are in the seventh grade. The lesson this vignette represents is the very first mathematics lesson for which the teacher and the students come together in secondary school. None of the students has a German ethnic background. Most of them have unemployed parents, and most of the parents are receivers of social aid money. The school is a part of the lower-stream of the German secondary school system, which stratifies students to three different types of schools according to their achievement in primary school. Thus, right before the summer holidays, these students have had an official testimony that they belong to the lower third of achievers. In the German school system, the tradition of ability-streaming is based on a common sense of different types of abilities, that in turn - without any scientific legitimation - apparently justifies the stratification of streams (Rösner, 2007): There are “practically” able learners and “theoretically” able learners and the creation of homogenuous learning groups is supposed to make learning more effective for *all*.

My starting point for the analysis is the teacher’s utterance that “*this desire to enhance, this... this thirst for knowledge or so, sadly that’s not there*” which she addressed in an extensive interview, when reasoning about why her students hardly show any indications of a wish to achieve. As the title indicates, my argument will develop in a way that identifies a lack of relevance of the knowledge transmitted as one of the crucial keys to answer the question behind the teacher’s observation. My use of the term *relevance* is not to be understood in terms of *use values* or in terms of *relevance for the students' experiential environments*. Here, relevance means the carrying of principles of power that operate inside *and* outside the institution of school. By this I want to add to the often-used distinction of *use value* and *exchange value* a third type, namely an *epistemic value*. This line of thought is based in the Bernsteinian sociology of education and inspired by Maton’s (2010) distinction of social and epistemic relations of knowledge. While the reading of the vignette may give rise to a variety of different explanatory attempts (e.g. psycho-analysis or didactics have been suggested to me), in this paper I will restrict myself to the concepts of pedagogic device and pedagogic code as conceptualized by Bernstein (2000). The focus here is on the relation between knowledge and power. Among theories of social reproduction, Bernstein is said to have provided the most sophisticated model of how knowledge and power get interrelated in institutions of knowledge reproduction, namely the *pedagogic device*.

LINKING KNOWLEDGE AND POWER - THE PEDAGOGIC DEVICE

Bernstein (2000) defines the pedagogic device as consisting of three rules, the distributive rule, the recontextualising rule and the evaluative rule, that stand in a hierarchical relation to each other. On the level of the *distributive rule*, categories of meanings are created, brought into relation and distributed to categories of social groups. The field of production of knowledge is canonically engaged in this rule. Drawing on Durkheim’s division of labour and the derivative emergence of specialization, Bernstein maintains that on the level of the distributive rule there is a stratification of *mundane* and *esoteric* meanings. In this stratification, esoteric meanings inevitably take the dominant role, as it is those meanings that transcend the spatial and temporal materiality and that bear the potentials to think yet unthinkable solutions that draw on (contextually) external frames of reference. Social groups that are given access to esoteric meanings therefore tend to dominate social groups whose access is restricted to mundane meanings. However, orders of meanings (e.g. knowledges, discourses) are not transferable from their locale of production to their locale of reproduction (e.g. schools) directly. In order to enable transmission and acquisition, there needs to be a selection, a relating and a hierarchising of a new pedagogised set of meanings. Along with this process, called the *recontextualising rule*, the distribution of different meanings to different social groups is reconsidered and so the field of recontextualisation is another social arena for the ordering of meanings. This second arena is not structurally independent from the distributive rule. However, several scholars have shown how recontextualisers on different levels (e.g. ME-researchers, ME-didacticians, text-book-authors/publishers or teachers) tend to reproduce the dominance of the esoteric over the mundane. For example, Dowling (1998) has shown how British ability-oriented textbooks construct a close connection between low-ability and the mundane and between high-ability and the esoteric. Cooper & Dunne (2000) have shown how standardized assessments in the UK demand an orientation to esoteric meaning even though recontextualising these meanings in mundane contexts. Hoadley (2007), in a South African context, demonstrates in detail how classroom interactions in white middle-class primary schools prepare students to penetrate an esoteric order of meanings, while non-white working-class students are prepared to act within mundane contexts. The list of sociological research that indicates the reproduction of the dominance of the esoteric over the mundane by the recontextualisers of knowledge is long, and not restricted to mathematics. On the third, lowest level, these orders of meanings must be translated into consciousness of individuals and social groups in pedagogic interaction. This level is called the *evaluative rule*, as according to Bernstein, consciousness is constructed through evaluation. This is not to be misunderstood as a behaviourist perspective, but rooted by Bernstein in Mead's symbolic interactionism. While it is on this level that consciousness is created through pedagogic interaction, the evaluative rule is not independent of the two other rules in interweaving knowledge and power. It is settled on the lowest of the three hierarchical levels, thus it is not the social arena for an effective *distribution* of power relations, but the arena where these power relations can become *visible* or *invisible*, the arena where access to the established power relations may be opened or denied. Whether or not such opening may happen depends on a) the *coding orientation* of students, namely whether they have already acquired access to the dominant hierarchy of meanings and b) the *pedagogic code* that translates power and control relations into pedagogic practice and thereby may reveal or conceal the dominant hierarchy of meanings.

PEDAGOGIC CODES

In Bernstein's terminology, codes are "specific semiotic grammars that regulate the acquisition, reproduction, and legitimation of fundamental rules of exclusion, inclusion, and appropriation by which and through which subjects are selectively created, positioned, and oppositioned" (Bernstein, 1990, p. 47). These grammars are determined by particular distributions of *power,* and modes of *control* to access, maintain and disrupt these distributions of power. A pedagogic code is then a semiotic grammar, which regulates power and control in a pedagogic setting.

Classification as the carrier of power relations

For Bernstein, *power* is rooted in the social division of labour and the derivative need for shared symbolic meanings. These are rooted in trans-contextual systems of meanings to enable communication among members of different contextual groups:

> Power relations, in this perspective, create boundaries, legitimise boundaries, reproduce boundaries, between different categories of groups, gender, class, race, different categories of discourses, different categories of agents. Thus power always operates to produce dislocations, to produce punctuations in social space. (Bernstein, 2000, p. 5).

Then it is the concept of *classification* that translates power relations into relations between categories. Classification thus for Bernstein does not work through positivist definitions of categories, but exactly through "the dislocation in the potential flow of discourse which is crucial to the specialisation of any category." (p. 6). Thus, "[i]n the case of strong classification, we have strong insulation between categories. [...] In the case of weak classification, we have less specialised discourses, less specialised identities, less specialised voices." (p. 6). In the context of the pedagogic device this means that it is particularly strong classification that realises an accumulation of power by carrying the potential of an alternative order.

Framing as the carrier of control relations

Classifications construct the stratifications, distributions and locations of the social space. However, the categories constructed need to be maintained in interaction: in order to exist, distributions of power need to be controlled. Thus while classification constructs *what* counts as legitimate communication, framing regulates *how* legitimate communication is to be realised, and how it is to be acquired.

> "Framing is about *who* controls *what*. [...] Framing refers to the nature of control over:
>
> - the selection of the communication;
> - its sequencing (what comes first, what comes second);
> - its pacing (the rate of expected acquisition);
> - the criteria; and
> - the control over the social base which makes transmission possible." (Bernstein, 2000, p.13)

Taken together, classification (as providing a given distribution of power) and framing (as the principle of control over this distribution) translate into communicative principles, namely a pedagogic code. Providing the means of power and control within a particular category, the pedagogic code produces a particular orientation to meaning, a particular symbolic consciousness within that category.

Fine-tuning a model of classification for analysis of mathematics classroom discourse

While the five dimensions of framing proposed by Bernstein appear completely sufficient for our analysis, we (Straehler-Pohl & Gellert, 2012) saw the need to elaborate the concept of classification for our empirical analysis. Our model was inspired by Dowling's (1998) model of classification but tried to coherently implement three dimensions, namely what discourse and knowledge are about (content), the role of language in constructing discourse and knowledge (communicational means), and the ‘conceptual level’ of discourse and knowledge (praxeological organisation).

Classification of content: Concerning the classification of content, for the time being we will refer to Dowling (1998). He stresses the notion of connotative links between categories. An element of one category may or may not have such connotative links to another category. As an example, a question about students’ shopping experiences within a mathematics class (thus an element of the category school mathematics) may also be reasonable in the context (or category) of actual shopping. Thus the availability of connotative links is high, the classification of content is accordingly low.

Classification of communicational means: Inspired by the fruitful connection of SFL and Bernstein in the last decades, we aimed at grounding the classification of communication in a social semiotic perspective. Here we refer to the distinction between contextualised and decontextualised language, as proposed by Hasan (2001). Hasan distinguishes between *actual* and *virtual* contexts that words can refer to. What defines a context as actual is that it is at least *potentially* sensible. Thus, no matter whether this context is dislocated in time and space, the contextualised language may well suffice to communicate the intended meanings. A virtual context in contrast is neither sensible in the here-and-now nor anywhere-at-anytime, but *constituted* by the verbal act itself. Decontextualised language thus brings a context into being that does not yet exist. The more the communicational means tend to refer to virtual contexts, the more are the meanings of utterances determined by particular registers and genres. Strong classification of communicational means is thus characterised by a use of strongly decontextualised language. In a weakly classified communication, contextualised language seems sufficient for an unproblematic communication: the addressee is free to construct meanings from the context s/he is experiencing, has experienced or is likely to experience in an already ‘thinkable’ future.

Classification of praxeological organisation: To conceptualise the classification of ‘mathematical quality’, we integrated the concept of praxeology from the Anthropological Theory of Didactics (ATD, Chevallard, 1999) with classification (Straehler-Pohl & Gellert, forthcoming). Within ATD, mathematics is considered a human activity that is concerned with the study of certain *types of problems*. Types of problems, together with the *techniques* to solve these problems form a practical block (the know-how) of human activity. Thus, doing mathematics consists in studying with the aim of solving problems of a given type. However, human practices rarely exist without a discursive environment that aims at describing, explaining and justifying the techniques employed to solve the given problems. Consequently there is a second, a knowledge block (the know-why). This block provides the mathematical discourse necessary to interpret and justify the practical block. As the practical block, it is structured on two levels: first, the *technology* (the 'logos' of the 'techné') that provides a justification for a technique, and second, on a deeper level the *theory*, that constitutes the justification of technology in a broader discursive environment. Together, these two blocks with four elements form a *praxeology,* indicating that practice (praxis) and discourse about practice (logos) are inseparably linked.

In mathematics classrooms, solving problems of a certain type is frequently treated as a discourse *of its own*, where a solution is legitimised by a documentation of a chosen technique and the correctness of the solution itself. In such a praxeology, the technological frame of reference remains tacit and open: there is no specialisation or insulation within the 'logos', it remains an indefinite sphere of possible legitimations. Thus, we describe an activity that leaves the 'logos' unspecific and arbitrary as weakly classified. An activity that demands a legitimising of a technique in technology and thus a location of the frame of reference can in turn be described as strongly classified.

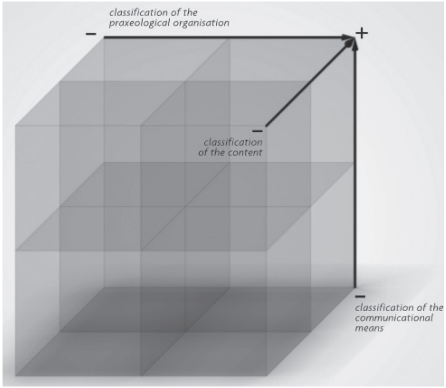


Figure 1: A three-dimensional model of classification (Straehler-Pohl & Gellert, forthcoming)

Finally, we come to a three-dimensional model that allows an investigation of mathematics classroom discourse and knowledge, as illustrated in Figure 1.

THE PEDAGOGIC CODE IN THE INTRODUCTORY VIGNETTE

Now that I have laid out the analytical frame of reference for my analysis, I would like to invite the reader to jump back to the beginning of the paper for a second reading of the introductory vignette and just then go on with the analysis of the pedagogic code in this episode. I have chosen exactly this episode as a key-incident (Erickson, 1986), as it represents the very common organisation of classroom interaction throughout the 630 minutes that I have videotaped during the first three weeks of mathematics lessons in secondary school. Thus, I claim the pedagogic code in this episode as the dominant code of mathematics transmission in this classroom.

Classification of content

The content involved was written subtraction. The numbers were given by the teachers as an example, exclusively designed for computation; they were not derived from some apparent real-world example. It is quite unambiguous, that the discourse is about a content that is (very) strongly classified: $+C_C$.

Classification of communicational means

The teacher sought an experiential frame of reference right from the beginning: she referred to experiences that students have made in their primary school career. So even though the theme "basic operations" is of non-experiential nature, she located it within an actually lived-through reality. When the teacher formulated the students' deficits concerning three of the four operations, she reported on "*forgetting*" instead of not *understanding* and on "*how they work again*" instead of *what they mean*. However, when she declared to "*explain*" subtraction, we could assume a shift towards a more decontextualised talk. Nevertheless, again virtual discourse is replaced by action straight away: instead of an explanation or a discussion, a student was asked to *proceed* the computation at the blackboard and to *accompany* his actions by telling, what he was *doing*. The role of verbal communication in this case is strictly ancillary. As such, it does not construe any meaning beyond the writing on the blackboard. However, the teacher seemed to consider the articulation of the procedure as the core of the learning-opportunity and thus proceeded herself. When introducing the (non-existent) term "*plusnumber*", its meaning was not constructed through verbal action, but remained implicit. It was only associated *visually* with a plus-sign, the teacher physically put in front of the number. The term "plusnumber" seemed an unhandy attempt to avoid the supposedly discouraging technicality of the term positive number. Thus, we can assert that the introduction of such a term a) reveals the expectation of an unreadiness for decontextualised technical language and b) pretends a contextualised character of the concept: a plusnumber is a number that has a plus *in front* of it. The classification of communicational means is weak: C_{CM}-.

Classification of praxeological organisation

The very first sentence already indicates an activity with a weakly classified praxeological organisation: Telling an adolescent, that s/he will "*surely*" know what s/he is supposed to do subsequently "*from primary school*" has a connotation of opening up a world of common sense. A world that is so easy to handle that it actually does not need any particular frame of reference. That the teacher assumed that her students "*forgot, how subtraction worked again*" is metaphorically indicative of the praxeological organisation of the activity: the task of written subtraction calls for remembering a technique, irrespective of understanding or explicating any technological legitimation. This interpretative assumption is acknowledged in the further episode, especially in the end, when the teacher demonstrated the procedure. Only a very small number of the many procedural steps went along with a reference to a legitimation; however, all of these moments were realised implicitly. Recognizing potentially technological moments presupposes an already existent insight in the place value system and the constancy of sums. For example for the subtraction of ones, she says: "*From eight to three doesn't work right? The three is in fact smaller than the eight. I borrow a ten from the row in front*". The possibility of "*borrowing*" tens from the "*row in front*" is not reflected as a universal possibility made available by the structure of the place value system but as a simple procedure. The classification of praxeological means is weak: C_{PO}-.

Framing analysis

The teacher exercised the control over the selection overtly and unambiguously: the choice of a "*primary school*" topic was announced by the teacher, and certainly not a choice made by the adolescent students ($+F_{sel}$). As the selection, the sequencing is fixed. First there was some sort of learning by following a demonstration and verbalisation, then learning by repetition of similar tasks on the worksheet. While the two failing students challenged that sequence by refusing to verbalise, the teacher insisted on the sequence and finally carried out the verbalisation herself ($+F_{seq}$). It is quite unambiguous what is considered as a legitimate text, namely the correct reproduction of the algorithm for written subtraction, and the teacher overtly has the control over the criteria, thus according to the evaluation criteria, framing is also strong ($+F_{e.c.}$). In contrast, the control over pacing and social order is to be classified as ambiguous. At times the pace was fast and regulated, at others it was left to the students ($\pm F_{pac}$). The teacher partly controlled turns and partly accepted autonomous turns ($\pm F_{soc}$).

Discussion of the pedagogic code in the introductory vignette

It is quite remarkable that it is particularly and solely the content that is strongly classified. This is particularly remarkable, as the existence of the lower stream is legitimised by an apparent optimising of learning for supposedly practically able learners. Finally, the only thing that characterises, or specifies, discourse and knowledge is that they are precisely not concerned with the practical, extra-mathematical world. If we remind ourselves of Bernstein's notion of classification as the carrier of power relations, we can say concerning the structure of knowledge that the only power it carries is the power of the institution of school. Thus in terms of power, it supposedly has an *exchange value* that is redeemable within the institution, but no *epistemic value*. In summary, the dominant pedagogic code realises a transmission-oriented pedagogy, in which knowledge is a carrier of exchange values.

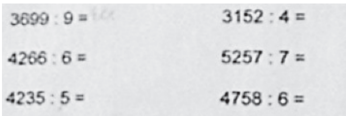
While this vignette was representative of the majority of discourse in that classroom, I would now like to introduce a second vignette, in order to show what happens when the strong classification of content is questioned. It represents one of the very few instances when references to extra-mathematical contexts were made.

VIGNETTE 2: SHARING CANDIES

"*What does it mean, 'divided'? ... I have a bag of candies with a hundred candies inside and there are ten students in the class. If I want to divide them now, what does it mean?*" Yassir answers: "*Everyone gets ten candies.*" The teacher now wants to know how Yassir got his answer. "*Say, I have four candies... I give him two, and I still have two.*" Nodding, the teacher acknowledges his answer: "*So you distribute the number of candies to the number of students.*" After going through a few examples, where reasonable amounts of candies shall be shared among reasonable amounts of students, the students are left to a work-sheet, with problems similar to 3699:9= . Almost all students struggle with the work sheet. "*What did we say just a minute ago? There I just have a different number.*" She writes 3699:9 at the blackboard and together with Dragan, she demonstrates the written algorithm. Frustrated with Dragan's problems proceeding autonomously, she addresses the class "*You know what that is? That is primary school, third grade*". The teacher then starts from scratch and erases the written algorithm and asks "*3699 divided by nine. What does that mean? I have 3699 candies, among how many students shall I distribute them?... And what do I want to know? What do I want to calculate?*"

Discussion of vignette 2

In the beginning of this episode, the strong insulation of mathematical and everyday meaning is broken. The teacher claimed that division and sharing candies do not only *share* a common meaning, but that the meaning of division is *derivable* straight from the context of sharing candies. Division and sharing candies seem exchangeable for one another. Thus, at this point the classification of content is weakened ($-C_C$). However, that the teacher came back to this metaphor for 3699:9 revealed the absurdity of this claim. Any recontextualisation of meaning from the mundane context of sharing candies to the esoteric context of written division would not improve but rather hinder the ability to use the written algorithm. Finally, the weakening of the classification of content did not last for long, but - considering the demands of the worksheet (see Figure 2) - was discarded as fast as it was introduced ($+C_C$).



3699 : 9 =	3152 : 4 =
4266 : 6 =	5257 : 7 =
4235 : 5 =	4758 : 6 =

Figure 2: Extract from the work-sheet

Further, by apparently legitimising the technique of written division by a reference to candies, the teacher obscured that she was not about to introduce a *technology* within a mathematical frame of reference ($-C_{PO}$). Ideologically, sharing candies was introduced as the technology that justifies the technique of division. As the caricature of sharing 3699 among 9 students was not explicated as a caricature, the evaluation criteria become extremely implicit ($-F_{e.c.}$).

DISCUSSION

Finally, while the dominant pedagogic code exemplified in the first vignette produces knowledge that carries no power beyond that of the institution, the second vignette reveals that even the promise of institutional power is a myth: when the teacher introduced the *mundane* context of sharing candies as the technological frame of reference for the *esoteric* context of the written algorithm for division, she cut it off all its power relations. We can assert that the students do have a feeling that in terms of power, sharing candies is an *irrelevant activity in their life* and that any institutional activity that can be derived from this activity must be *irrelevant* within the institution of schooling – it has lost its exchange value. Finally, it is not knowledge itself – namely the knowledge of basic operations – that *is* irrelevant, but exactly its recontextualisation that *makes* it irrelevant. In her frustration the teacher did not even try to maintain the illusion of institutional power of the knowledge transmitted: "*You know what that is? That is primary school, third grade".* So when the teacher observed that "*this thirst for knowledge, sadly that's not there*"*,* the reasons might well be that the knowledge transmitted hardly is a carrier of power. Already discharged from its epistemic value, the only power mathematical knowledge *could* promise here is the power of the institution. But even that is a myth: the mathematics to be transmitted is a marginalised "primary school" mathematical knowledge, it is as trivial as sharing candies. It is irrelevant.

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THE POLITICAL, THE PERSONAL, IMPLICATIONS FOR MATHEMATICS EDUCATION: THE CASE OF CHICAGO AND TUCSON

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When educational policy at the national, state, or local level determine school action for "low-performing" schools; which programs or schools remain funded; or "who" and "what" is allowed to be taught in schools, important questions are raised about the means by which these decisions are made. After decades of U.S. school policy and reform attempting to provide equitable opportunities for students, the case of Chicago, Illinois and Tucson, Arizona demonstrate shifts to reaching political goals through legislation negatively impacting communities, teachers, and students. The evidence supporting such goals becomes questionable claims, techniques, and framing. This paper sets out to explore the issues around these two very different political and demographic areas, providing implications for mathematics education.

INTRODUCTION

As educational systems across the United States continue to be the target of concern for countless policy initiatives, it becomes difficult to distinguish between approaches of research and reform that have students' best interest in mind. This incessant idea of a deteriorating public education and instruction is not a new idea, taking height after the Sputnik launches (1957) and later the report A Nation at Risk: The Imperative for Educational Reform (1983) (Zhao, 2009). Political leaders, headlines, news articles, and television news continue to help paint a grim picture of particular public neighborhood schools, communities, teachers, and students (Berliner & Biddle, 1995). Two specific cases that have reached new heights, gaining national attention, and strongly impacting schools, communities, students, teachers, and the like are that of "school action" on public schools in Chicago, Illinois (Chicago Public Schools) and the banning of ethnic studies (and in effect Mexican-American studies) in Tucson, Arizona (Tucson Unified School District).

Two states within the United States, with very different historical, political (Chicago, Illinois historically Democrat/liberal; Tucson, Arizona historically Republican/conservative), and demographic environments, both facing political/educational controversy. In this paper, we set out to understand the factors involved in the "school actions" affecting these two educational systems. We begin by providing our framework for understanding these situations and then provide a description of the historical/political context of these two areas, focusing particularly on educational policies and initiatives. We further look at the evidence, claims, and framing available used as the primary source for executing these school actions, and provide implications for mathematics education.

THEORETICAL FRAMEWORK

In understanding the issues developing around public education in Chicago and Tucson, we draw from a postmodern feminist perspective and the conception of Framing. A postmodern feminist epistemology accepts that knowledge is always provisional, open-ended and relational (Wallin, 2001). Therefore, ways of knowing and master narratives are located in historical and cultural contexts (Benhabib, Butler, Cornell, & Fraser, 1994). In this sense, we understand that there cannot necessarily be direct connections between the two situations as they are contextually specific. However, we will attempt to investigate the direction and impact of educational policy on the situations involving public education.

An important factor in these two situations is the framing involved. An issue frame is a "central organizing idea or story line that provides meaning to an unfolding strip of events, weaving a connection among them. The frame suggests what the controversy is about, the essence of the issue" (Gamson & Modigliani, 1987, p. 143). Through this cognitive process some considerations are given a central role, while other considerations are either placed in the background or pushed aside altogether. In these cases we look at the explicit frames, which involve outright statements regarding the way an issue "must" be understood (Winter, 2008). Studies of issue framing demonstrate how frames affect people's opinions by making certain considerations seem more important than others, in turn, affecting the way people judge the issue (Hurwitz & Peffley, 2005). In essence, framing demonstrates a process that creates a tighter link between the consideration and the policy attitude, while also bringing to the fore the importance of the consideration as a decisional measure (Winter, 2008). We will use this idea of framing to understand the way in which these issues are being framed to continue pushing forth a political end.

BACKGROUND

The United States has a long history of educational policy and reform, although some key points in history have significantly impacted national and state educational policies and practices. One important shift in education was a new conservative era that focused on standardized measures and competition following the election of Ronal Reagan in 1980 (Reagan, 1984). Then in 1983, the report A Nation at Risk (supported by Ronald Reagan) called for sweeping reforms in public education and teacher training (Gándara & Contreras, 2009). However, a Nation at Risk was a report, not legal mandate, so did not have as much direct impact as what would follow in 2001. In 2001, the controversial No Child Left Behind Act (NCLB) was approved by Congress and signed into law by Republican President George W. Bush ("Education Policy Timeline", 2012). The law, which reauthorized the Elementary and Secondary Education Act (ESEA) of 1965 and replaced the Bilingual Education Act of 1968, mandated high-stakes testing, held schools accountable for student achievement, and penalized schools for not meeting adequate yearly progress (AYP) ("Education Policy", 2012).

In 2008, Democrat Barack Obama was elected the 44th President of the United States. With his election came hope for substantial changes in the No Child Left Behind Act. However, with two ongoing wars and a focus on the nation's economic problems, the reauthorization of No Child Left Behind Act would not come as anticipated. This fact became evident as Arne Duncan, former CEO of Chicago Public Schools and Secretary of Education under the Obama administration, incorporated turnaround/charter schools as a preferred strategy in the federal "Race to the Top" competition ("Race to the Top", 2012). Duncan also established a School Improvement Grants Program, which in part provided grants to support establishing turnaround/charter schools across the country ("Race to the Top", 2012).

Turnaround/charter strategies have been and still are usually focused on improving test scores, in order to avoid being labeled as "a failing school" as set out by No Child Left Behind measures ("Education Policy", 2012). The focus on standardized measures has fueled numerous national, state, and local initiatives that use these measures as the determining factor of whether schools are doing well, teachers are teaching well, and students are learning. Finally in 2011, President Barack Obama announced that the United States Department of Education was inviting each state's educational agency to request flexibility regarding some requirements of the No Child Left Behind Act ("ESEA Flexibility", 2012). This is an ongoing process.

CHICAGO IN CONTEXT

Particularly for Chicago, in 1980, during the election of Ronald Reagan, the Illinois State Legislature called for the replacement of all School Board members ("History and Context", 2012). In 1987, teachers went on strike for a record of 19 days. This was during the time when the Secretary of Education at the time, William Bennett, declared Chicago's Public Schools as the "worst in the nation" ("History and Context", 2012). In 1988, with the lobbying for school reform, the Chicago School Reform Act (PA85-1418), established control of Chicago Public Schools to Local School Councils (LSCs) and principals (Brown, Gutstein, & Lipman, 2009). Individual schools were granted substantial responsibilities such as hiring principals, monitoring budgets, and developing school improvement plans (Brown, Gutstein, & Lipman, 2009). Then in 1989, the School Board was replaced, again ("History and Context", 2012). In 1994, Chicago Public Schools Superintendent at the time, Argie Johnson, set out to identify low-performing schools for varying levels of remediation. Schools with the lowest averages of students who met or exceeded state standards were placed on "probation" ("History and Context", 2012).

In 1995, the Chicago School Amendatory Act (PA89-15) was passed by the Illinois State Legislature to institutionalize mayoral control over the district with a district chief executive officer (CEO) being appointed by the mayor (Brown, Gutstein, & Lipman, 2009). Mayor Richard J. Daley and his CEO focused on accountability, high-stakes testing, and ending social promotion ("History and Context", 2012). Chicago's 1995 school reform based on high stakes testing and accountability provided a model for the 2001 No Child Left Behind (NCLB) federal education legislation (Lipman & Haines, 2007). In 2001, No Child Left Behind (NCLB) drove Chicago Public Schools accountability standards for schools and teachers (Lipman & Haines, 2007). NCLB aimed to tie student achievement to district accountability and federal funding.

In 2004, Mayor Daley announced a Chicago Public Schools two-part reform initiative known as Renaissance 2010 (Ren2010). This initiative called for closing 60 to 70 public schools and opening 100 new schools, where two thirds of the schools would be run by private organizations and staffed by teachers and staff who would not be part of the Chicago Public Schools Unions (Lipman & Haines, 2007; Brown, Gutstein, & Lipman, 2009). Schools recommended for school action continue to face reassignment boundary change (attendance areas boundary change involving reassignment of currently enrolled students); phase-out (gradual cessation of enrollment in certain grades each school year until a school closes or is consolidated with another school); co-location; school closure (closing of a school and assigning all students enrolled at that school to one or more designated receiving schools); or consolidation (consolidation of two or more schools by closing one or more schools and reassigning the students to another school) (“Guidelines”, 2011).

ARIZONA IN CONTEXT

Arizona’s long history of animosity towards undocumented immigrants has infiltrated school policy decisions. In 2000, voters endorsed a requirement for English immersion in schools, banning bilingual education (Proposition 203) (Schmidt, 2000). It passed 63 percent to 37 percent. Furthermore, in 2002, under the No Child Left Behind Act, English language learners were required to meet three annual measurable achievement objectives (AMAOs): (1) make annual progress, (2) attain English proficiency, and (3) meet Annual Yearly Progress (AYP) requirements set by their states and measured by state standardized tests (Ofelia, Kleifgen, & Falchi, 2008). In 2004, Arizona voters approved Proposition 200, which denied public benefits to undocumented individuals to the country (“Timeline: Immigration”, 2010). It passed 56 percent to 44 percent. In 2006, voters supported various ballot measures related to undocumented immigrants including: requiring undocumented Arizona residents to pay out-of-state college tuition, denying bail to undocumented immigrants charged with a crime, and making English the state’s official language (“Timeline: Immigration”, 2010). Each of these ballot measures passed with a 70 percent or higher of the vote.

In 2009, Republican Jan Brewer ascended to the Governor's Office when Governor Janet Napolitano resigned from her position to join the Obama administration as Homeland Security Secretary. With the resignation of Napolitano came a shift to a very conservative approach by Brewer. In 2010, Arizona Senator Russell Pearce introduced Senate Bill 1070 (“Timeline: Immigration”, 2010). SB 1070 passed the Senate with a 17-13 vote; an amended version of SB 1070 passed the House with a

35-21 vote; and the Senate gave the final approval to amend SB 1070 with a 17-11 vote ("Timeline: Immigration", 2010). That same year, the bill made it to Governor Jan Brewer, who signed SB 1070 into law ("Capitol Media Services", 2012). This Bill included provisions on registration documents for undocumented immigrants; state penalties relating to immigration law enforcement including trespassing, harboring and transporting undocumented immigrants; and law enforcement officer's responsibility to determine an individual's immigration status during a "lawful stop, detention or arrest", or during a "lawful contact" (Senate Bill 1070, 2010). Three separate lawsuits challenging this law's constitutionality were filed in federal court ("Capitol Media Services", 2012). Governor Brewer signed HB 2162, which included provisions intended to address the racial profiling concerns. Upon these changes, Arizona's new immigration law was scheduled to go into effect.

Arizona continued to gain national attention when in 2011, Brewer approved House Bill 2281, which bans schools from teaching classes that are designed for students of a particular ethnic group, promote resentment, or advocate ethnic solidarity over treating pupils as individuals (House Bill 2281, 2010). The bill also bans classes that promote the overthrow of the U.S. government. Under the law, the state can withhold 10 percent of funding for any school district that refuses to change its courses. This resulted in the ban of ethnic studies in Arizona and in effect Mexican-American Studies in the Tucson Unified School District. Teachers and students from the program spent the entire year challenging in federal court the constitutionality of HB2281 and the state's ruling ("The Opposition", 2012). But before any final court decision was made, on January 10, 2012, the Tucson Unified School District (TUSD) school board voted to immediately cease Mexican-American Studies classes so as not to lose state funding ("The Opposition", 2012).

THE PROBLEM WITH EVIDENCE AND FRAMING

Chicago. The situations in both Chicago and Tucson demonstrate significant actions taken negatively affecting public education. While supporters (particularly from the mayoral office, CEO, school board) of "school actions" in Chicago utilize mainly quantitative measures (i.e., test scores) as evidence for schools being recommended for remediation, supporters of Arizona's HB2281 (governor, legislature, school board) ignore quantitative evidence as a factor. This Framing by political leaders works to shape the way these issues are understood. This becomes an issue not only when this limited data is used as a basis for action, but also when counter evidence remains left out of the conversation. Particularly for Chicago, Renaissance 2010 was (and is) marketed as an opportunity to bring in new partners with creative approaches to education (Brown, Gutstein, & Lipman, 2009). This became the Framing initially for the larger populace to support this initiative. The criteria by which schools have been (since the initiation of Renaissance 2010) and continue to be exempt from being considered for a school action are:

(1) an elementary school has an Illinois Standards Achievement Test (ISAT) composite meets or exceeds score at or above its geographic network average in the previous school year;

(2) a high school with a 5-year cohort graduation rate at or above its geographic network average in the previous school year; or

(3) any school that scored at or above the 25th percentile on the trend and growth component of the Performance Policy in the previous school year. ("Guidelines", 2011, p. 1)

However, even given this criteria and given the reasons for certain schools chosen for school action (i.e., community meetings highlighting solely ISAT scores to support claims), schools chosen for the various actions are not necessarily the worst performing of all Chicago's Public Schools. This emphasis on tests scores has even occurred as "Guidelines" (2011) states that in making a decision on which school actions to propose to the Chicago Board of Education ("Board"), the CEO would also consider other information including, but not limited to: "student safety data, school culture and climate, enrollment estimates, the quality of the school facility, family and community feedback, or whether the school has recently undergone any school actions, changes in academic focus or actions taken pursuant to 105 ILCS 5/34-8.3" (p. 2).

So the quantitative data used as evidence only becomes test scores, where other quantitative data (including school improvements and counter-evidence) and qualitative data are completely excluded. This was particularly important in a study claiming that on average, Chicago elementary/middle schools that underwent reform made significant improvements over time (Luppescu, Allensworth, Moore, de la Torre, Murphy, & Jagesic, 2011). However, four years after these interventions, the gap in test scores between reformed elementary/middle schools and the average of schools that did not undergo reform decreased by almost half in reading and by almost two-thirds in mathematics. Furthermore, on average, Chicago high schools that underwent reform efforts did not perform differently than similar schools in terms of absences in grades nine through 12 or in terms of the percent of students on-track to graduate by the end of ninth grade (Luppescu, Allensworth, Moore, de la Torre, Murphy, & Jagesic, 2011).

The Turnaround Schools (one school action), in particular, have been the subject of extensive news coverage and publicity by their advocates, who include Mayor Richard M. Daley and Mayor Rahm Emanuel and both their CEO's and School Boards. However, in contrast to the support by these political leaders and the claims of there effectiveness:

the 33 high-poverty, high- achieving neighborhood elementary schools with fully-empowered Local School Councils (LSCs) that are achieving above the citywide average for all Chicago elementary schools, and far above any Elementary

> Turnaround School, four of which were briefly profiled in Section 1, are (with a few exceptions) virtually unknown to the public. (“Chicago Democratically-Led”, 2012, p. 5)

This study highlighted the reality of quantitative data that had not been given to the public, aiding in the Framing of the issue by political leaders. This data did not support the Framing of the issue, and therefore, remained irrelevant to meeting the political ends set forth. And unfortunately, although these reports provided evidence contrary to the evidence, claims, and Framing around these school actions, the school actions continued to be executed.

Tucson. In Tucson, the passage of HB 2281 set out to ban programs that: (1) promote the overthrow of the U.S. government, (2) promote resentment toward a race or class of people, (3) are designed primarily for pupils of a particular ethnic group, and (4) advocate ethnic solidarity instead of treating pupils as individuals (House Bill 2281, 2010). Ethnic studies (and in effect the Tucson Unified School District’s Mexican American studies (MAS) program) were claimed to have violated this state law. The Framing of this issue has been hearsay, where there has been no documented evidence or claims by initiators (governor, legislature, school board) to suggest a violation of this law and no quantitative data either to demonstrate how numbers are impacted by such programs. Furthermore, there has been not evidence linked to student learning that support this claim.

An audit mandated by John Huppenthal (state schools superintendent) showed that the Mexican-American Studies (MAS) program had helped raise student achievement, and students who participated in the program were more likely to attend college (“The Opposition”, 2012). In fact, students in the Mexican-American Studies (MAS) program outperformed their peers on Arizona’s state standardized tests in reading (by 45 percentage points), writing (by 59 percentage points), and math (by 33 percentage points) (Ginwright & Cammarota 2011). Furthermore, students who participate in the program enroll in post-secondary institutions at a rate of 67 percent, which is well above the national average. Also, the pedagogies used in Tucson’s Mexican-American Studies (MAS) classes promote, encourage and support students to be actively involved in their communities (Cammarota & Romero 2009). This strategy has in turn been shown to correlate with increased classroom engagement. And again, although this evidence was available to the governor, legislature, and school board, although not readily provided to the public, these actions continued.

DISCUSSION

In Chicago, the claims of particular neighborhood schools in need of “being closed, fixed, or changed” and in Tucson, the claims of why certain programs within schools are seen as “a threat to teaching hatred” lack supportive “evidence”. Even without this supportive evidence, school actions continue, with no real democratic process to determine the best actions. Similarly to the Manufactured Crisis (Berliner & Biddle, 1995), we can view these situations as appearing within a specific historical context "led by identifiable critics whose political goals can be furthered by scapegoating [schools, communities, and] educators" (p. 4). This is particularly evident through the questionable claims and techniques—data available for claims, suppression of contradictory evidence, and distortion of research findings. Since 1997, CPS has initiated five distinct reforms that aim to dramatically improve low-performing schools in a short time. These initiatives are Reconstitution (seven high schools), School Closure and Restart (six elementary schools and two high schools), placement into the School Turnaround Specialist Program (STSP) model (four elementary schools), placement into the Academy for Urban School Leadership (AUSL) model (ten elementary schools and two high schools), and placement into the CPS Office of School Improvement (OSI) model (two elementary schools and three high schools) (Brown, Gutstein, & Lipman, 2009). Since 2000, Arizona has been in full force for anti-immigrant initiatives strongly impacting schools. From making English the only language of instruction in schools (Proposition 203) to banning Ethnic studies (and in effect Mexican-American studies), Arizona officials continue to use hearsay as the main evidence for such claims. This leaves the questions of how and for whom can mathematics education (regarding data presented and available) be used to either support or suppress inequitable initiatives for schools, communities, and students.

IMPLICATIONS FOR MATHEMATICS EDUCATION

The overuse or underuse of quantitative measures to push forth an agenda are evident in the cases of Chicago, Illinois and Tucson, Arizona. Along with such ideas as students having democratic access to mathematical ideas (Malloy, 2002), schools, communities, parents, teachers, and students should have access to ways in which mathematics is used as a tool for supporting or suppressing an inequitable situation. In research, more investigations need to be made to understand the ways quantitative data in relation with qualitative data is used or not used in inequitable ways, such as in the cases of Chicago and Tucson. In practice, democratic education as Malloy (2002) discusses can be used in all spaces. With these tools such as mathematical skills, knowledge, and understanding, individuals can become educated citizens who use their political rights to shape their government and their personal futures (Malloy, 2002). As efforts have continued to combat these inequities, further access to this mathematics can help to see the power of mathematics to address ills in our society. This becomes especially important in a "democratic" society that no longer demonstrates democratic processes for actions affecting schools, communities, students, parents, and teachers.

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DISBURSING AUTHORITY AMONG MATHEMATICS STUDENTS

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This longitudinal case study of a high school mathematics teacher paying attention to the way authority works in his classroom follows him from one school to another. His students' resistance to his wish for them to exercise their own authority was frustrating. He eventually had an explicit discussion about authority with them, which seemed to catalyse change. We analyse the classroom discourse in the various settings using categories that describe authority relationships in mathematics classrooms.

Mathematics is often characterized as having an interest in certainty. Thus authority is central to the discipline. Mathematics comprises truth claims, which are supposed to be authoritative. Authority is far from simple in mathematics classrooms. For the past few years, we have worked with teachers to consider ways of developing their repertoires for handling authority issues. This paper presents a case study of one teacher's experience working with authority, both to understand how he considers authority and how he negotiates it in his practice. His situation had special challenges relating to authority because he changed schools part way through the research. In a new school he had to develop students' confidence in his authority.

AUTHORITY IN MATHEMATICS CLASSROOMS

Authority is one of the many resources teachers employ for control and has been defined in an educational context as "a social relationship in which some people are granted the legitimacy to lead and others agree to follow" (Pace & Hemmings, 2007, p. 6). This relationship is highly negotiable and students rely on a web of authority relations including friends and family members as well as the teacher (Amit & Fried, 2005). Educational research related to teacher authority often makes distinctions between different types of authority (e.g., Amit & Fried, 2005; Pace & Hemmings, 2007). Most relevant here are the distinctions made between being an authority because of one's content knowledge and being an authority because of one's position (e.g., Skemp, 1979). Pace (2003) showed that these become blended as participants interact in classrooms.

We demonstrated this blending in a recent computer-aided corpus analysis of pervasive language patterns in mathematics classrooms; classroom discourse encodes the structuring of authority in many ways (Herbel-Eisenmann & Wagner, 2010). The most common pervasive discourse patterns explicitly called on the teacher's *personal authority* (e.g., 'I want you to…') and suggested the expectation that students rely on the authority of their teacher. Another prevalent authority structure suggested that the discipline had to be followed, which we called *demands of the discourse as authority*. Language patterns that include combinations like 'we need to' and 'we have to' explicitly identify obligations suggesting that anyone must follow certain rules. These rules, which come from outside the personal relationships, may be attributed to the discipline of mathematics (or perhaps the discipline of school mathematics). A related authority structure suggested a discourse that obscured the presence of authority but in which actions were predictable, which we called *more subtle discursive authority*. With this category there is no explicit reference obligation, but rather a sense of predetermination. Discourse that include patterns like ‘we are going to’ and ‘it is going to’ suggest that there are no decisions to be made. The results are inevitable. Because participants in the discourse do not have authority, the authority rests outside somehow. Other less common patterns suggested *personal latitude*, which recognized that classroom participants could make decisions, and thus had authority.

This distinction between personal authority and disciplinary authority has also been explored through the lens of positioning theory (Wagner & Herbel-Eisenmann, 2009). When people grant authority to the discipline (which is *transcendent* or outside the experience of people participating in the discourse) through their practices, it is different from authority being granted to people with agency in the classroom (who are *immanent*). As Schoenfeld (1992) pointed out, however, the development of internal authority is rare in students, who have “little idea, much less confident, that they can serve as arbiters of mathematical correctness, either individually or collectively” (p. 62).

Even if we agree that students should develop their own sense of mathematical authority, it is problematic to say that teachers should cede their authority. Teachers are reluctant to entertain the idea of giving up authority, partly because of the implications for the teacher’s necessary social authority, but also because they know that their mathematical authority is necessary for teaching. Chazan and Ball (1999) confront this tension through descriptions of two teaching situations, in which they were reluctant to express their authority but realized the necessity of it. Yet, little has been done to date to try to better understand authority and positioning issues in a “reform” mathematics classroom. As Chazan and Ball stated, being told “not to tell” is not enough.

BACKGROUND, DATA, AND CASE STUDY

Prompted by the corpus analysis described earlier, we entered a 3-year collaboration with mathematics teachers in Atlantic Canada who expressed interest in considering the way authority works in their classrooms. After interviewing each teacher at the outset, we recorded 15 consecutive sessions of a mathematics class they each chose. The group of teachers met with us about once every six weeks during the research. Further classroom recording was done when they wanted to try new things related to authority. In addition to video recording, we used voice recorders to capture more local audio of group work. We also interviewed the participant teachers periodically and sometimes interviewed students who were in the classes that were recorded.

The teacher in this case study, Mark (pseudonym), had taught mathematics and physical sciences for 4.5 years prior to this study. He was teaching grades 9-12 mathematics in a rural high school with approximately 150 students. Mark chose a grade 12 classroom for us to observe and record. The student's families generally had incomes lower than average, compared to others in the province, and even lower yet compared nationally. Many parents worked in the forest industry and/or commuted about 1-1.5 hours to a larger centre for work. After the first year of our work together in this research project, Mark took a position in an urban school with well over a thousand students. Now instead of being the only mathematics teacher in the school, he was one of many. He taught multiple sections of grade 9 mathematics and grade 11 physics. Students did not know him so he had a sense of having to establish his authority both mathematically and as a teacher who cares for his students. Mark's situation provided a setting in which we could explore the case of how a teacher considers and enacts authority in changing contexts (i.e., from a familiar context where he was comfortable and established in a small school to an unfamiliar context with different demographics in a much larger school).

As is common in case study research, the data and analyses were interwoven. We began with talking to the teachers about authority, were able to observe them teaching, and had continued conversations with them about their considerations. We iteratively sought and discussed the patterns we observed and modified the interview questions and observations as needed (Yin, 2006), e.g., we recognized that changing schools could allow particular aspects of authority to surface and thus agreed to observe almost every day as his school year began. We realized that Mark's situation was an interesting case of a teacher grappling with authority in two different contexts over a period of time. Thus, we present this longitudinal case study in chronological sequence (Yin, 2006).

CONSIDERING AUTHORITY AS CONTEXT CHANGES: THE CASE OF MARK

Talking with Mark about Authority in the Familiar Context

In the initial interview with Mark, he was asked about his role as a mathematics teacher, to which he replied, "The students look at you as their sole source of knowledge, very few [take] the initiative to go and find answers on their own. […] Like, if you run through investigations with them, by the time you get to the end they look at you and go 'Why didn't you just tell us that?' […] They're quite reluctant to accept the authority really." His use of the word *investigations* connected with "Investigations" in the textbook he was using, which described them as "a situation in which students explore a new skill or concept [and include] questions designed to lead students to a more thorough understanding" (Barry et al., 2001, p. viii). Mark's conceptualisation was quite focused on authority and was, of course, skewed by participation in this research.

When asked more focused questions about authority, Mark's attention moved toward the "Focus" and "Check your Understanding" work following investigations in the class textbook. When asked, "What or whom do your students see as authorities in their classrooms?" he said:

Mark: I don't think they look beyond [us math teachers]. They feel like we should have all the answers. And sometimes they don't realize that sometimes we have to go look for answers as well. So even though we demonstrate that the authority is found in other places, like textbooks and other colleagues and things like that, they still, … they're focused right in on their teacher. Their teacher must have all the knowledge.

When asked what would happen if he were to disagree with the textbook, he stated "they'd have a hard time believing me over the textbook." He recalled, however, situations in which he went through answers with his students who were then convinced that there was an error in the textbook. Nevertheless, Mark's focus in this interview somehow switched from developing understanding to "getting answers."

When asked, "How do students know what to do in mathematics?" Mark did not seem to understand the question. Perhaps the idea that students do what their teacher tells them was hegemonic and, thus, the question did not make sense. When we clarified the question as asking about how students decide what to do when addressing a problem, he said, "Some of them that have actually remembered previous teachings will just… automatically go to the rules they've previously learned." They would look at the examples he gave, but "some will just constantly ask you, 'What do I do now?', 'What do I do now?', 'What do I do now?'" Mark's frustration with students' dependence was palpable.

Mark was asked to draw a diagram that illustrates the way authority works in his classroom. To start the diagram, he was given a blank piece of paper with a dot in the middle, which he was told represented him. Mark completed his diagram (Figure 1)

with a physical representation of the classroom, showing the arrangement of students, who are smaller dots, the blackboard (the straight line), a bookshelf with texts (also authoritative dots) that students can refer to, and his desk at the back of the room. Some students have larger dots because they were recognized as having more authority than the others.

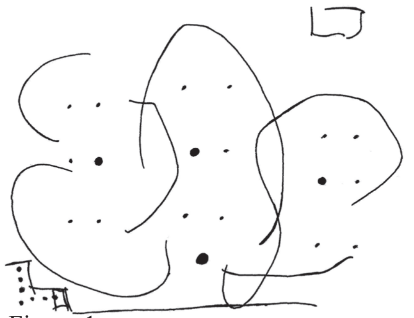

Figure 1

When drawing, Mark talked about balance and said that authority should be spread throughout the classroom. Thus he arranged seating plans to spread the students regarded as authorities around the room, and he himself moved around to avoid fixing authority in one place. His elaboration is interesting and compelling. It relates to positioning, but unlike most scholarship on positioning that uses physical relationships as metaphors for interpersonal relationships, his conceptualization recognizes the effect of physical positioning. In addition to being a schoolteacher, Mark is a coach who runs sports camps. His conceptualization reminds us of play sheets, and he talked about the need for every student (like every player) to follow the directions of the "coach" as they make decisions within the coach's system.

Observing Mark Teach in the Familiar Context

Mark's classroom had examples of each type of authority described earlier from the findings of the corpus analysis. Thus it would be hard to characterize positioning of students in his classroom in one way. He positioned himself as having personal authority by asking students to do things without giving reasons for them to do these things. For example, in our first observation, when Mark turned on his projector, a student asked, "Are these notes?" Apparently, this boy relied on Mark's authority when deciding what to write and what not to write in his notes. Later on, Mark had the students graph $y = x^2$ by saying, "What I need you to do now is just to sketch this graph, okay?" Because the students did not know why he was asking them to draw the graph (except that he needed them to), significant decisions about how to sketch the graph were difficult for them to deal with. Thus a boy asked Mark how to scale the axes.

Mark also positioned the authority of the discipline of mathematics as being transcendent. He marked the discursive power of the discipline in the same session by referring to vocabulary definitions coming from outside the classroom: "We're starting exploring having to find instantaneous rates of change – how fast things are changing at that very moment. So what is it? Technically speaking, it is the change in the dependent variable over an infinitely small change in the independent variable."

In addition to direct reference to outside authority, Mark deferred to a transcendent authority more subtly by using language that suggested their work was entirely predictable – for example, before allowing students to work out a problem, he said, "So we're going to get 4188." There was no doubt what would happen, thus the actions of people in the classroom were deemed redundant.

We were most interested in the ways in which personal latitude was evident. Students asked Mark questions often. For example, in the first session we observed, Mark said, "We calculated average rates of change over various periods or various intervals, right? But what if we start bringing that interval closer and closer and closer together?" and a boy responded by asking, "What happens if they touch?" The boy showed authority to direct the conversation by asking about things that Mark did not appear to be addressing. Mark responded with a show of his mathematical authority by answering the question, saying, "Then you get an instantaneous rate of change." As Mark continued, he positioned the mathematics as being responsive to their intentions: "If we want to find the instantaneous rate of change on a particular graph…" Perhaps this acknowledgement was rhetorical because finding such rates of change was demanded by the curriculum

Also in this same class session, a girl asked Mark if there was an easier way to write about the interval $0 \leq x \leq 4$. A boy asked if the method being discussed would always give the rate. In the first half hour of class (all whole class discussion), 5 of the 11 students took initiative to ask questions. Mark set the agenda (following the curriculum) but students exercised their agency by thinking about what eventualities they might face and asking Mark for clarification that might help them face these eventualities.

The personal latitude expressed by students in Mark's class could be attributed to various factors. Most importantly, Mark was responsive to their questions and thus encouraged more, but there were other factors. Intimacy had a chance to develop with the small class that comprised a relatively stable cohort over twelve years.

Negotiating Authority in an Unfamiliar Context

The circumstances that supported the discourse that Mark and his students grew into did not follow him to his new school. In the new context, Mark had to negotiate with his students their positioning with no personal history. We agreed that recording the initial classes could be revealing, and noticed that Mark and his class settled into a positioning structure that was significantly more reliant on him as an authority. Although there was a perceived need for him to establish his authority, he continued to desire a situation in which the students would develop their own authority. Having to adjust to a new large school themselves, these grade 9 students may have felt lost and thus more reliant on their teacher.

Each of the forms described above from the previous context appeared in this class – personal authority, disciplinary authority, and personal latitude – but this group was much more dependent on him. In order to change the dynamic, Mark chose to devote time to challenge students with questions about authority. Approximately two months into the term, he started a class telling students about the research participant teachers' interest in authority. The following excerpt comes from near the beginning of the class session:

17	Mark:	We're looking at [authority] not necessarily the way that you guys probably think of authority. We're not talking about necessarily who's in charge, per se. That kind of authority. Like police kind of authority. Now that does play a little bit of a role in a classroom obviously. But we're looking more at authority as to the holder of knowledge. Who is the holder of knowledge? Am I?
18	Students:	No
19	Mark:	Okay
20	Girl:	It's us.
21	Mark:	Okay. Good. There're lots of sources of authority. Right? If we're talking about mathematical authority, there're lots of sources. Correct?

> I am, I guess. I consider myself **a** source of mathematical authority in the classroom. But, I also consider each and every one of you guys a source of mathematical authority. […] the whole idea is to disburse the authority a little bit more so that it's not just one big source, and that's the only place where you can get information, the only place you can think of as being a source of knowledge, a source of information. The idea is to make yourself your own source of authority.

Mark then displayed with his projector 2 + 3 = 5 and 2 + 3 = 7. He asked which expression was true and why. Many students seemed frustrated. At first, students said that they know 2 + 3 is 5 because teachers said so, but eventually a girl explained why it has to be five; she grouped two fingers on one hand with three on her other hand, and said, "We learned it when we were younger – the counting numbers. We used our hands to count, and adding numbers. Through the years you kind of adapt to it being five."

Next, Mark displayed two further equations, 2 + 3 × 5 = 25 and 2 + 3 × 5 = 17. One boy said, "It depends on how you do BEDMAS" (Brackets, Exponents, Division & Multiplication, Addition & Subtraction). Mark revoiced this statement and the class erupted. One voice stood out saying, "If you do it right you get 17, if you do it wrong you get 25." When Mark asked who decided on this order of operations the students guessed names: you (i.e., Mark), Stephen Hawkings, Albert Einstein. The students concluded that the convention was passed down through generations, but were vague about how the convention started. Someone suggested "the beginning of time."

After this, Mark was no longer following a plan and he was speaking about as much as the students (usually he spoke much more than the students). Significantly, the students began exercising agency by making demands of him. 31 minutes into the conversation a girl said, "You are asking a hard question. An example would be really helpful." Mark responded with a scenario in a game and another girl interrupted, "No, a *real* life example." Then Mark started using an example from when he built his deck, but students argued for an example from *their* real life, not *his*. When he used the example of choosing mobile phone packages, the class was finally content with that example.

When Mark challenged his students with questions about authority, they exercised authority by telling him how to teach them. Reflecting on the conversation, one student said to Mark, "You asked all these questions but they didn't have answers." The conversation was about 44 minutes, evidencing the students' interest and Mark's dedication to developing a different authority structure in class.

Mark's Reflection

We were curious about how this exchange would change the classroom dynamic. Again, it was not possible to characterise the class as fitting one authority structure because personal authority, disciplinary authority and personal latitude continued to appear. Mark noted, however, that the students began to ask questions. They were becoming like the class in his previous school. Some of this change may have been related to the passage of time with him, but we think the conversation he had about authority made a difference, too.

In a formal presentation to teachers later in the year, Mark characterized these students as "very frustrated," "not engaged in their own learning," and "passive participants," at the beginning of the year and he recognised a move to "students questioning," "asking for alternative methods," "demanding explanations," and "giving their own examples of problems they wish to know." He also gave an example from five months later: a student asked him to demonstrate a certain kind of problem, and others gave further directions to him about what they wanted demonstrated, and even posed their own problems. He saw this as a shift toward students taking more authority for their own learning.

OUR REFLECTION

In this case study we can see how authority is central to mathematics classrooms and to Mark's position as a teacher. This position is especially significant in the establishment of classroom routines at the beginning of a semester (or year), and even more so when the teacher is new to the community. The case raises questions for us. First, in relation to Mark's positioning in the classroom, we wonder how much of his perceived need to establish authority was necessary. It is important to be *an* authority in mathematics and to be *in* authority to some extent as a teacher, but it is also important to establish a routine in which each student sees him/herself as *in authority* of his/her own learning so that s/he too could become *an authority* in mathematics. The students themselves probably had similar struggles – wanting to be independent of Mark while depending on him for guidance in various ways. Yet, little is known about the dynamic.

Second, we reflect on Mark's explicit discussion about authority with his students. The importance of mathematics teachers to "step out" or have meta-conversations about norms has been demonstrated (Cobb, Yackel, & Wood, 2003; Rittenhouse, 1998). Authority is central to these norms, so we argue that meta-conversations about authority in mathematics classrooms can help students come to terms with their mathematics. Further research on teachers using such strategies is needed.

ACKNOWLEDGMENT

This research was supported by the Social Sciences and Humanities Research Council of Canada, as part of a grant entitled "Positioning and Authority in Mathematics Classrooms."

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