(RE)DISTRIBUTION OF RESOURCES = EQUITY?

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In this presentation I will discuss part of an ongoing research project with teachers studying a Further Diploma in Education (FDE) in Mathematics Teaching at the University of the Witwatersrand (WITS). The aim of the research is to investigate the relationship between formalised INSET and the quality of teachers' classroom practices. Its value for MEAS proceedings is that it illuminates the pervasive development-democracy tension in South Africa, particularly as regards the provision and redistribution of educational resources.

In the South African context, educational resources are not only seriously limited, but also differentially distributed. A central educational challenge in South Africa is thus the provisioning and redistribution of human and material resources for learning and teaching in schools. At the start of the 1998 school year, the Sunday Times newspaper (January 18, p.9) interviewed pupils from four different secondary schools. Two pupils from historically advantaged schools described how books were distributed, classes organised and formal work begun on the first or second day. In contrast, pupils from historically disadvantaged black township schools said that at the end of the first week, they were still waiting for textbooks and stationery, and formal classes had barely begun. In the word of one pupil: 'They blame pupils when we fail but they (government) forget that they fail to give us resources early enough'.

Behind common and prevalent laments on the 'lack of resources' across many schools is firstly, the history of inequity in provision in South Africa. There are numerous schools that still do not have basic resources like water and electricity, let alone sufficient classrooms and learning materials. More generally, is the assumption that the quality of learning and teaching in school is related to availability and use of learning resources. Recent studies of 'effective' or 'successful' schools - schools with good matriculation results (Christie, P. et al, 1997; Naidoo, 1998) have attempted to identify elements of school and classroom practice that could account for their effectiveness or success. The interesting point is that effectiveness does not seem to correlate directly with human and material resources available in the schools. There are rural and/or under-resourced schools that manage to achieve good results. A controversial point is that the practices in some of these schools are not all to be valorised - authoritarianism, narrow, theoretical orientations to scientific knowledge and rote learning were in evidence. What these studies nevertheless confirm is that learning occurs in a range of contexts. Effectiveness is not simply a function of availability of resources.

With a commitment to contributing to the democratisation of education in South Africa, the Wits FDE programme has (among other aspects) paid attention in most of its courses to accessing and using resources. A key question for the research project has thus been *what resources teachers recruit into their practices, whether and how these change over time and with what effects*. Elsewhere I have argued for an elaborated view of resources (Adler, 1998a, 1998b). In this project, resources include cultural and social resources like language and languaging.

The unit of study in the project is the 'teacher-in-context'. A purposive sample of 11 primary and secondary math teachers in the FDE programme was drawn from three different resource-based contexts: urban or semi-urban township schools in that have basic resources, and rural and under-resourced schools, some of which have a close supportive relationship with a local education NGO. In the presentation I will draw on cases from these diverse and unequal resource-based contexts in which teachers work to illustrate the argument that, in general, and in relation to resource use in particular, ***change in classroom practice is always partial. Moreover, it is neither linear, nor uniform, but uneven, personal and contextual.*** This is in contrast to a great deal of research and development work in teacher education that has tended to homogenise and decontextualise teachers.

Briefly, in secondary schools, textbooks and use of chalkboard remained strong, but incorporated new uses to support elements of learner-centred practice. At the primary level, where knowledge boundaries and legitimators (like the matriculation examination) are more remote, there were more disruptions, greater inclusion of additional material resources and more risk-taking by teachers trying out new ideas with additional resources. The potential effects of these were uneven and sometimes worrying as attention focussed more on form - on the resource itself - than on supporting subject learning. A critical theoretical point to emerge interactively from data analysis *dialectical recontextualisation*. Simply, resources shape and are shaped by their contexts of use. Resources that are brought into the classroom do not necessarily have educational meanings built into them. Nor do educational meanings shine through them. The meanings of the resources emerge in their use in the context of classroom practices and the subject knowledge being learnt. There is a dialectical interaction between the bringing in of a new resource or using an existing resource in a new way (like the chalkboard) and the shaping of classroom practices. Contrary to taken for granted assumptions, more resources make greater demands on teachers.

The point for discussion at this conference is that it appears that in contexts of greatest need the effects of recontextualisation were most worrying, perhaps exacerbating inequality. There are teachers whose context and/ or personal disposition seriously mitigates against pedagogic innovations. And so the question: (re)distribution of resources = equity? Elsewhere (Adler, 1998b) I have argued for a reconceptualisation of resources as a verb - where in the context of mathematics teacher education, the resourceful teacher needs to be understood as one acting with resources-in-practice-in-context. More resources is not a quantitative issue, nor a decontextualised panacea for improvement.

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Literacy, Matheracy and Technoracy - The New Trivium for the Era of Technology

Ubiratan d'Ambrosio

Introduction

In the taped lecture he prepared for ICME 9, Paulo Freire recognised that Mathematics is intertwined with all forms of human behaviour and that there is a mathematical way of being in life. He essentially recognises that his program of Critical Literacy cannot be complete without the recognition that mathematics underlies human and societal behaviour. This goes much beyond the acquisition of mathematical skills.

In this Paulo Freire Memorial Lecture I will discuss, under the inspiration of his ideals, the role of Mathematics in building up a new civilisation which rejects inequity, arrogance and bigotry.

Mathematics, history and education

The nature of mathematical behaviour is not yet clearly understood. Although in classical Philosophy we can notice a concern with the nature of mathematics, only recently the advances of cognitive sciences have probed into the generation of mathematical knowledge. How is mathematics created? How different is mathematical creativity from other forms of creativity?

From the historical viewpoint, there is a need of a complete and structured view of the role of Mathematics in building up our civilisation. For this we have to look into the history and geography of human behaviour and find new paths in the measure we advance in the search. History is a global view in time and space. It is misleading to see History only as a chronological narrative of events, focused in the narrow geographic limits of a few civilisations which have been successful in a short span of time. The course of the history of mankind, which can not be separated from the natural history of the planet, reveals an increasing interdependence that crosses space and time, of cultures and civilisations and of generations.

Education is a strategy created by societies to promote creativity and citizenship. To promote creativity implies helping people to fulfil their potentials and rise to the highest of their capability. To promote citizenship implies showing them their rights and responsibilities in society.

Educational systems throughout history and in every civilisation have been focusing on two issues: to transmit values from the past and to promote the future. In other words, Education aims equally at the new (creativity) and the old (societal values). Not irresponsible creativity – for we do not want our students to become bright scientists creating new weaponry – neither docile reproduction – for we do not want our students to accept rules and codes which violate human dignity. This is our challenge as educators, particularly as mathematics educators.

My role as a mathematics educator

The strategy of educational systems to pursue these goals is the curriculum. Curriculum is usually organised in three strands: objectives, contents and methods. This Cartesian organisation implies accepting the social aims of educational systems, then identifying contents which may help to reach the goals and developing methods to transmit these contents.

To agree on objectives is regarded as the political dimension of education. But very rarely have mathematics contents and methodology been examined under this dimension. It is generally accepted that contents and methods in mathematics have nothing to do with the political dimension of education. Since mathematics is the imprint of the Western thought, our responsibility as mathematicians and mathematics educators is a major one.

I see my role as an Educator and my discipline, Mathematics, as complementary instruments to fulfil those commitments. In order to make good use of those instruments, I must master them, but I also need to have a critical view of their potentialities and of the risk involved in misusing them. This is my professional commitment.

The proposal

It is difficult to deny that Mathematics provides an important instrument for social analyses. Western civilisation entirely relies on data control and management. Social critics can not be proposed without an understanding of basic mathematics. But regrettably the term "basic" has been abusively identified with critical skill and drilling.

The proposal of this paper is a reorganisation of school curricula in three strands: **Literacy, Matheracy**, and **Technoracy**.

Literacy - Clearly, reading has a new meaning today. We have to read a movie or a TV program. It is common to listen to a concert with a new reading of Chopin! Also, socially, the concept of literacy goes through many changes. Nowadays, "reading" includes also the competency of numeracy, interpretation of graphs, tables and other ways of informing the individual. And also understanding the condensed language of codes. These competencies have much more to do with screens and button than with pencil and paper. There is no way for reverting this trend, the same as there was no successful censorship in preventing people to have access to books in the last 500 years. Getting information through the new media precedes the use of pencil and paper and numeracy is dealt with calculators. But, if dealing with numbers is part of modern literacy, where has mathematics gone?

Matheracy is the capability of drawing conclusions from data, inferring, proposing hypotheses and drawing conclusion. It is a first step towards an intellectual posture, which is almost completely absent in our school systems, Regrettably, even conceding that problem solving, modelling and projects can be seen in some mathematics classrooms, the main importance is given to numeracy, or the manipulation of numbers and operations. Matheracy is closer to the way Mathematics was present both in classical Greece and in indigenous cultures. The concern was not with counting and measuring, but with divination and philosophy. Matheracy, this deeper reflection about man and society, should not be restricted to the elite, as it has been in the past.

Technoracy is the critical familiarity with technology. Of course, the operative aspects of it are, in most of the cases, inaccessible to the lay individual. But the basic ideas behind the technological devices, their possibilities and dangers, the morality supporting the use of technology, are essential questions to be raised among children at a very early age. History shows us that ethics and values are intimately related to technological progress.

The three together constitute what is essential for citizenship in a world moving fast into a planetary civilisation.

The move towards a new civilisation

It is an undeniable right of every human being to share all the cultural and natural goods needed to her/his material survival and intellectual enhancement. This is the essence of the *Universal Declaration of Human Rights* (1948), to which every nation is committed. The educational strand of this important profession on mankind is the *World Declaration on Education for All* (1990), to which 155 countries are committed. Of course, there are many difficulties in implementing the effectiveness of the United Nations' resolutions and mechanisms. But as yet, this is the best instrument available that may lead to a planetary civilisation, with peace and dignity for the entire mankind. Aren't these the most fundamental principles to which we subscribe? Regrettably, these documents are short of being unknown to most mathematics educators.

It is an unrelinquishable duty to co-operate, with respect and solidarity, with all the human being, who have the same rights, for the preservation of all these goods. This is the essence of the **ethics of diversity**: respect for the other (the different); solidarity with the other; co-operation with the other. This leads to quality of life and dignity for the entire mankind.

Quite unusual as a piece on Mathematics Education, many will say. But if we do not accept, very clearly and unequivocally, our general and global professional commitments subordinated to a global ethics such as the proposed ethics of diversity, it is very difficult to engage in a deeper reflection of our role as mathematics educators.

It is impossible to understand the process of exclusion of large sectors of the population of the world, both in the developed and undeveloped nations, without a deep reflection on the colonial period. It is not the case of putting the blame in one or another, neither to attempt to redo the past. But to understand the past is a first step to move into the future. To persist in former paths and styles is irrational and may lead to disaster. Maybe the real threat to humanity are not people looking for aliens coming in UFOs, but are the earthlings nostalgic of a fading order anchored in inequity, arrogance and bigotry. Mathematics has everything to do with this past. A new world order is urgently needed. Our hopes for the future depend on learning – critically! – the lessons of the past.

Ethnomathematics programme in history and epistemology, with its intrinsic pedagogical action, is a proposal motivated by the commitment to fulfil these responsibilities. With the growing trend towards multiculturalism, ethnomathematics is recognised as a valid school practice, which enhances creativity, reinforces cultural self-respect and offers a broad view of mankind. In everyday life, ethnomathematics is increasingly recognised as systems of knowledge, which offer the possibility of a more favourable and harmonious relation in human behaviour and between humans and nature.

As History of Mathematics goes, there is need of a broader historiography. History of Mathematics can hardly be distinguished from the broad history of human behaviour in definite regional contexts, recognising the dynamics of population exchanges. This is a way of identifying the origin of exclusion of populations and entire civilisations through denial of knowledge, which allows for the proposal of corrective measures. By looking into the bodies of knowledge which have been integrated in the syncretic evolution of Mathematics, Ethnomathematics allows for a better understanding of the cultural dynamics under which knowledge is generated. The proposed historiography can be seen as a transdisciplinarian and transcultural approach to the History of Mathematics.

The denial of knowledge that affects populations is of the same nature as the denial of knowledge to individuals, particularly children. To propose directions to counteract ingrained practices is the major challenge of educators, particularly of mathematics educators.

Cultural conflicts and social change: conceptualising the possibilities and limitations of mathematics education

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1. The personal journey

In some sense this talk is a reflection and critique of a personal journey. Like many who conduct research in mathematics education, I come to this conference with a background in mathematics, mathematics education, and the psychology of mathematics education. In 1985 I started what became the Social Psychology of Mathematics Education Working Group within PME because of my concerns and those of others about the way that the social aspects of learning mathematics were being ignored in research. Following on the 'Mathematics for all' sessions and D'Ambrosio's plenary lecture on ethnomathematics at the ICME conference in Adelaide in 1984, and after much lobbying, we were finally able to persuade the ICME organisers for 1988 to put some focus on social aspects and they agreed to have a special day on Mathematics, Education, and Society (Keitel, et al.,1988).

That day, and that experience, not only met a need, it also created others. As a result of the understandable frustrations which several people felt with that compromise of a day, a conference was held on the Political Dimension of Mathematical Education (Noss et al., 1990) Since that time we have seen other significant developments, particularly in relation to ethnomathematics, technology, and critical mathematics education.

My own journey since that day in 1988 has been one of exploring further the cultural terrain and of developing my knowledge and critique of the anthropological perspective. I explored mathematical enculturation as a metaphor for mathematics education (eg. Bishop, 1988), provoking some challenging but helpful critiques from Connors (1990) and Chevallard (1990). For various reasons however, I decided for that book to just focus on the enculturation metaphor, but since that time I have been drawn more and more into exploring the ideas of acculturation, mainly through working with some interesting colleagues in challenging situations in their countries.

A first excursion was with an article in 1994 which tested the idea of a research agenda in relation to cultural conflicts (Bishop, 1994). Several papers, several projects, several PhD students, and several readings later, I am hoping that this conference gives me, and all of us who attend, an opportunity to discuss our journeys and through our papers to develop some new perspectives in research with which to confront the meaningless and oppressive mathematics education which many students still have to suffer today around the world.

The cultural metaphor

Interrogating the cultural metaphor has revealed for me some insights as well as some challenging gaps. As a first example, the metaphor of enculturation assumes that there is a cultural consonance between the culture of school and the culture of home, that enculturation is somehow a natural process, and that home and school experiences are symbiotic. However detailed research on the notion of 'home culture' challenges those assumptions (eg. Abreu, Bishop and Pompeu, 1997) and shows that in certain situations the home culture and the school mathematics culture can be conceptualised as mutually exclusive.

The research literature generally indicates that some members of all the following groups have suffered in some way from conflicts with what I have called the Mathematico-Technological Culture (MTC) (Bishop, 1988):

- Girls in 'Western' societies
- Ethnic minority children in 'Western' societies
- Indigenous 'minorities' in Westernised societies (where I include black students in South Africa who are numerically in the majority)
- Western 'colonial' subjects
- Non-Judeo-Christian religious societies
- Rural learners, particularly in developing countries
- Physically and mentally impaired learners
- Children from lower class (caste?) families
- Adult workers in lower status jobs who are in training or re-training.

For many learners around the world the educative experience within schools and other institutions is clearly not consonant with their home, or outside, experience. The situation is one of cultural dissonance and the process is an acculturative one.

Cultural conflict

Conflict is a construct referring to affective aspects of a particular situation, which involves antagonists. My observations have led me to suggest that conflict fits within the following table of emotional and affective states in mathematics classrooms, which can vary from time to time:

Feeling/State	**Interaction**	**Consequences**
Comfort	Discussion	Stability, agreement
Tolerance	Easy negotiation, debate	Small change, assimilation
Concern	Hard negotiation, contestation	Large change, accommodation
Conflict	Hostility and confrontation, or non-communication	Rejection or acquiescence

Further it is likely that a student will experience greater conflict in their MTC classrooms if s/he:

- comes from a strongly supported outside-school minority culture with which s/he strongly identifies, or
- lacks strong role models in the dominant culture, or
- identifies with a counter-culture unrepresented and unaccepted in school, or
- identifies with a counter-culture represented by significant peers in the school.

I have been working with the construct of 'cultural conflict'. I will argue that a young person's mathematics education is necessarily an acculturation experience, with its accompanying emotional states and cultural conflicts, which need to be understood and tolerated.

Mathematical acculturation

I intend to interrogate this acculturation metaphor further. For example, if education is considered to be an intentional process on the part of society, and carried out by the teachers, what then can be understood by 'intentional acculturation'?

The period of school education can be a time of turmoil for both learners and parents. In relation to mathematics education it can arguably be even more of a turmoil. Some parents, and other people also, despite (or perhaps because of) their limited understanding of the role of mathematics in formatting modern society, hold onto the myth that mathematical knowledge is crucial for gaining 'success' in that society.

Often their children undergo various forms of mental and personal anguish in order to gain this knowledge, and of course many reject it, or are excluded by 'official' methods. The irony is that having been through this experience themselves, and presumably having seen the myth exposed, they then seem to even more vehemently exhort their own offspring to go through the same anguish!

But there are good psychological and sociological reasons why these parents and their teachers cannot deny or expose the myth, of course, including the fact that the mathematical qualifications which are obtained through examinations, selection mechanisms, streaming in school etc. are the gates and hurdles on the way to 'nirvana' in this technocratic society.

5. The culture of Mathematicians

Another crucial question concerns into what culture are these young people presently being acculturated? The critical perspective makes us look beyond the reified nature of Mathematics and ask: where does this "culture of Mathematics" come from; who owns and sustains it; who is empowered by it, and who has the most to lose if it is attacked? The answer to all these questions is simply 'Mathematicians', but it is also necessary to unpack that construct further. (I am using the capital 'M' where I feel the words refer to the Western hegemonic mathematical tradition, which underlies the MTC above.)

Abraham and Bibby (1988) already showed one way by considering how the 'certified Mathematicians' at university level become what they called "the industrial trainers" who lobby for a certain kind of Mathematical acculturation via their work in industry, government research establishments, defence industries, businesses etc. These, plus of course Mathematicians in university departments of Mathematics, would be one way of identifying the group of Mathematicians who together sustain "the culture of Mathematics". Restivo's (1993) 'mathematical workers and mathematicians' are very similar to these constructs.

What the learners come into conflict with in the classroom then is the whole Mathematical knowledge and affective environment which is:

- socially constructed by the teachers and their peers,
- embedded in a society which is increasingly being formatted by Mathematicians and industry spokespersons,
- demanded by parents, administrators and society's leaders,
- controlled by the institutions and frameworks of a vast education industry manifested in the books, materials, calculators and computers with which they currently engage,
- underpinned by a culture whose values are implicit, and
- destined to select a minority and fail a majority of the students.

6. Values in mathematics education

To what extent do the teacher/acculturators share the values and knowledge of the culture into which they are acculturating the young? Do mathematics teachers consider themselves part of the MTC cultural group? Perhaps this depends on whether they consider that they are mathematics teachers, or teachers of Mathematics? I assume that society in any event thinks that they are part of the dominant cultural group. But in my experience there are few teachers of Mathematics, and even fewer mathematics teachers, who consider that they are themselves Mathematicians, or that they 'know' Mathematics in the same way that Mathematicians do, or that they teach the Mathematicians' values, or even that the Mathematics which they teach has any particular values.

I am not of course saying that they think that the Mathematics they teach has no value. They may not think it has any value in fact, but that it has value is one of the values that they are actually teaching, by merely teaching the subject! There are others also; rationalism, objectism, control, etc.(see Bishop, 1988, 1991) They could be acculturating their students into believing that they are learning Mathematics for its benefits as applicable knowledge, or to have pleasure in its finest or most intriguing discoveries or inventions, "for its own sake", or even to train their minds.

For me understanding more about values is the key to generating more possibilities for mathematics education. I am very sympathetic to the ideas of critical mathematics education, and would align myself with its goals (Skovsmose, 1994).

Sadly very little is known about the values which teachers think they are imparting, nor about how successfully they are imparting them, although the evidence from students' opinions suggests that they are certainly imparting values. Moreover little is known about how teachers and others change the values they are teaching, or even if they are able to consciously do that.

Why do mathematics educators know so little about values in this context? Is it because we/they too are part of the acculturator group, and as such have been fostering the same myths and the same ignorance?

7. Social change

What then is the role of mathematics education in social change? How does any of the above analysis help with facing this challenge? Here then is the key problem. Mathematics is such a strong formatter of society that it *should* be a key area to focus on if one is seeking to change the social order. However, the fact that the MTC is so entrenched, and accepted in modern society makes it so difficult to affect.

In my view 'Mathematics educators' is the group whose views need to change to make the wider change happen. But to what extent can this group bring about change in the Mathematicians' culture from within mathematics education? (See Breen, 1993)
Are there other sources of resistance? What about the learners themselves?

So how could we all proceed? Here are some questions which I hope will provoke some discussion at the conference towards achieving social change.

Should we:

- examine more closely the myths which society currently holds about Mathematics?
- examine our own roles in sustaining these myths?
- consider whether we are mathematics educators, as I have optimistically written it, or Mathematics Educators, or perhaps Mathematics educators?

- be engaging in more research which informs our ideas about the relationships between emotions and values in mathematics education?
- be researching more about the mathematical acculturation experience, about the powerful institutions of mathematical culture which shape and control our mathematical minds, and about critical pedagogy?
- consider whether the institutions, organisations, and ways of working of mathematics educators are more, or less, powerfully placed to influence the use of Mathematics as a critical tool, because of their close links with the institutions of the Mathematicians?
- consider whether MEAS should, or should not, link itself with ICMI because of the latter's Mathematical affiliation?
- be linking with educators in other subjects to work to develop more movement towards critical education in our overall curriculum?
- recognise the limitations of individual mathematical educators in changing this culture, but also recognise that as a group there can be more chance of effecting some change?

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Thinking about Mathematical Thinking - heterogeneity and its social justice implications

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School curricula persistently demonstrate an unresolved conflict between developing the mathematicians of tomorrow, equipping them with the knowledge and skills deemed necessary by university mathematicians, and providing an entry for everyone into mathematical culture adequate enough to meet the demands of society. Most societies fall between the two and, certainly in the UK, both industrialists and mathematicians have been articulate in their claims as to the 'failure' of school maths. From these it would appear that young people are neither knowledgeable and skilled enough to cope with the mathematical demands in the work place, nor ready to pursue further study at university. But the studies which were done for the Cockcroft Report, 1982, (particularly the Bath and the Nottingham studies) to some degree contradicted this in demonstrating that workplace-based mathematics was *not* the same as school-based mathematics and that employees developed an on-the-job facility with workplace-based mathematics (also borne out by the work of Mary Harris, see Harris, 1990). Nonetheless, many adults are quick to identify school mathematics as an area of failure for them. As far as degree level work is concerned, students have been voting against mathematics with their feet for some time and alongside this must be put a persistent critical theme emerging from studies done in universities listening to the voices of students (see, for example, Crawford et al., 1994) and experiences of attempting to interest mathematicians in innovative styles of teaching and learning (see, for example, Burton & Haines, 1997).

I believe that much of this confusion is exacerbated by a teaching obsession with content and, at the same time, an ignoring of the impact of epistemology *and* pedagogy on the mathematical experiences of learners. To take this further, I generated an epistemological model to describe the process of coming to know mathematics (Burton, 1995) and I have recently undertaken a study of research mathematicians to ascertain to what degree my model matches how they describe their own activities. The model understands coming to know, in mathematics, in terms of:

- its person- and cultural/social-relatedness i.e. it locates knowing rather than regarding it as 'objective' and free of influence from the individual or their society;
- the aesthetics of mathematical thinking it invokes i.e. how coming to know and knowing is described in terms of feelings;
- its nurturing of intuition and insight i.e. how the pathway to knowing is understood;
- its recognition and celebration of different approaches particularly in styles of thinking i.e. how the knowing is achieved;
- the globality of its applications i.e. not only applicable maths but what I have come to call the connectivities both within and across mathematics.

In 1997, I undertook a study of 35 women and 35 men in career positions as mathematicians in universities in England, Scotland, Northern Ireland and the Republic of Ireland. The female participants in the study were found by invitation and through snowballing (one person involving another) and each female was asked to find a male "partner" preferably in the same institution, to pair her for the purposes of the study. All 70 participants were individually interviewed, 64 face-to-face the remainder by telephone. The interviews ran from an hour to the longest which was two and a half hours. Most were between one and a quarter and one and a half hours.

My purpose in doing this study was to try and find out how mathematicians understand their researching practices in order to try to map the disjunction between mathematicians as learners, and mathematicians as teachers. I believed that the practices of mathematicians might be closer to the learning practices that many of us have been promoting in formal mathematics education for a very long time against a critical backdrop of some powerful university mathematicians.

I am convinced that the classroom experiences of mathematics learners are a result of a complex relationship between epistemology, pedagogy and the discipline of mathematics. I do not see any evidence that teachers have clarity of vision on any one of these three even though policy makers attempt to provide such clarity at least on the third. I believe that we damage both the learners, the discipline and ourselves as teachers when we fail to take this complexity into account by operating as if only one is important (usually the mathematics itself) and do not recognise that the mathematics itself is permeated by our epistemological and pedagogical perspectives. Hence the assertion of an epistemological model which attempts to include the who, and where with the what of the mathematics as well as invoking the senses with the cognitions. It was my belief that research mathematicians would use these categories in speaking about their working practices and that it might then be possible to relate them to mathematics teaching and learning experiences.

Heterogeneity

Homogeneity was not revealed by my participants when they spoke about mathematics, how they understand mathematics, how they think about mathematics, how they work in mathematics. Many public stereotypes were overthrown.

There is not ONE:

- mathematics - depending upon the research area, it was differently understood as
    - a 'rigorous' proof process;
    - empirical;
    - uncertain;
- way of understanding mathematics
    - as well as the invention/discovery split, I found socio-culturalists and those who understand mathematics as a language;
    - role of intuition/insight - there were those who denied and those who affirmed the importance of intuition and those who wanted to talk about the meaning of the different terms;
- way of thinking about mathematics
    - three, not the conventional two, different thinking styles were described;
- way of working in mathematics
    - individual/co-operative/collaborative were all described with an emphasis on the latter two;

Only when it came to discussing ways of experiencing the world of mathematics, and the impact of sex, 'race' and class, was there any singularity of experience. A world of power was revealed and that power had all the overtones demonstrated in the literature (see, for example, Acker & Feuerverger, 1996, Aisenberg & Harrington, 1988, Morley & Walsh, 1996, Lie & O'Leary, 1990 and Seymour & Hewitt, 1997). It was acted out in their research and career patterns, in research supervision, and in the women's reporting of aspects of:

The inner battle that professional women fight [which] is particularly difficult because its terms are rarely clear. Unpredictably, women will encounter trouble that looks like a knot of circumstances that they seek to pull loose, not recognizing at its center - except possibly in retrospect - a profound conflict concerning their own identities.

(Aisenberg & Harrington, 1988: 8)

The role of Oxbridge in the training of future mathematicians was a particularly powerful factor in the allocation of access to influence.

Social Justice Implications in Classrooms

The celebration of heterogeneity is, I believe, something about which to be exceedingly joyous. For me, this study has provided evidence to support a mathematics, and styles of learning and teaching the subject, which emphasises humanity, vision, creativity. It has also provided evidence of where and why the teaching of mathematics has failed many generations of learners. To cling tightly to one 'true' mathematical path might provide a sense of security but it is also stifling and unreflective of the world we all experience. To open mathematics to multiple interpretations, multiple possibilities, provides opportunities for learners to experience what the mathematicians whom I interviewed described:

"When I think I know, I feel quite euphoric. So I go out and enjoy the happiness. Without going back and thinking about whether it was right or not, but enjoy the happiness. When I discover something, I just enjoy the feeling."

"You can do all these interesting and exciting things without having to go out and do things with them. Whether what you are thinking about is new, research, known things or not, for you it is all new. When you understand a new proof, it becomes your own. Internally, it is as though you did it."

"It is just fun."

In the presentation, I will explore the implications of this approach for potential classroom confusion! In particular, I will look at:

Challenging teachers' assumptions	- doing it "my" way
	- respecting the many different routes
	- giving time to making mathematical maps
The celebration of differences	- pedagogy
	- meaning making
	- links between informality and formality

Re-writing the mathematical experience

What are the powerful teacher questions to ask?
What about learning time? What should be acceptable and how long is given?
In the light of curriculum constraints, what is possible? What is feasible?
Can we re-write the mathematical experience? What is *your* emphasis?

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Why *The Sociology of Mathematics Education*? Activity, Strategy and Dialogue

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The position that I want to develop in my lecture is, at least in part, a tactical response to what I perceive as an impending crisis in educational studies in the UK. Arguably, the crisis has its roots in the new forms of governance, which have shifted away from the state provision of services and towards the surveillance and regulation of services that are increasingly out-sourced. The efficiency and scope of this surveillance and regulation is achieved via the recruitment of the rapidly developing and expanding technologies of information and communication. These technologies, of course, include those that are commonly referred to as information and communication technologies. However, their general characteristic entails the minute precodification of the objects of their scrutiny. On a day-to-day basis, we feel the impact of such technologies in, for example, the automatic switchboards, which, in some cases, allow us no route to communicate at all other than through the touch-tone keypads on our telephones. These new technologies of bureaucratic informatics are now being turned on us so that the intellectual field in which we operate is increasingly penetrated and dominated by them. The effect of this new informatic accountability is to urge us to exchange methodological and theoretical rigour for fast-track, quick-fix remedies that must make extravagant claims to act directly on the improvement of teaching and learning in schools. The result would appear to be the dissolution of the languages which have hitherto constituted the disciplines of educational studies and so the potential for the productive interrogation of educational practices. These languages once also constituted the visible guarantees of competence of the academic: to be a sociologist of education might be taken to entail familiarity with, say, a canon of texts (admittedly fuzzily defined and, to a certain extent, of dynamic composition) and their associated terminologies and principles of description. The academic, now, is urged to abandon this now redundant expertise and seek authority in their diplomas and official affiliations which, we know only too well, were never any guarantee of anything (well, not much).

The bureaucratic dissolving of the academic languages is of course being mirrored in schooling itself, which is also increasingly subject to informatic fragmentation through regulation and surveillance. However, and at the risk of sounding controversial, some of the supposedly critical responses to such curricular denaturing are, arguably, themselves contributing to the same process. In particular, I am referring to the liberal democratising of education powerfully proposed by Piaget and taken up by successive generations of pedagogic constructivism. For Piaget, culture is relative. Therefore, the authoritative imposition of a cultural product, in the form of a discursive of practical schema, is, in his terms, socio-centric. As such, Piaget claims that it must inhibit the development of rationality which can occur effectively only where relations are non-authoritative. Authority, for Piaget - power for others - is dispensable and with it the voice of the pedagogue as subject of the discipline as the content of pedagogic action. I contend that this form of liberal constructivism concurs with informatic fragmentation in the dissolution of academic expertise, if in nothing else.

My position, along with Marx, Freud, Foucault and others, is to construe power not as a dispensable condition, but on the contrary, as a *sine qua non* of subjectivity: power constitutes rather than (or, shall we say, in addition to) constraining the subject. My tactical response, then - tactical in de Certeau's sense of the strategies of the subaltern - is, firstly, to present my own sociological language which, through its explicitness and (it is to be hoped) its coherence can attempt to validate its own utterances. In this validation, it also attests to the competence of its speaker. In its deployment, the facility of the language is the production of sociological analyses of texts, the term 'text' being interpreted in its broadest sense to refer to any closed corpus of data. The language was inaugurated through a dialogue within the theoretical field which constitutes educational studies and through a dialogue with the empirical field of educational practice. The choice of school mathematics as the focus of my work was motivated by virtue of its own highly explicit grammar and because of my own professional investment in the activity. My original empirical setting was the secondary school mathematics scheme, *SMP 11-16*, although in this paper I shall also refer to one or two other mathematical settings which will serve as illustrative overtures to the presentation of the main structure of the language itself.

The language that I shall introduce is constituted as a cultural product - a discursive schema. In order for it to develop beyond the status of idiolect, I must attempt to apprentice others into it pedagogically. I am therefore constituting a conception of pedagogic action as authoritative.

Issues and questions

- I have already raised the issue of my perception of an impending crisis in educational studies, which is leading to the dissolving of academic languages within the field. To the extent that my perception is shared, this is clearly a major problem for educationalists generally and for those of us involved in mathematics education, in particular. My first question, then, is strictly a political one: *In an era of academic barba/erism, how can we/should we identify, develop, deploy, disseminate and institutionalise existing and new languages of description—coherent theoretical frameworks—in order to produce and market rigorous and coherent analysis of the empirical field of mathematics education?*

- I contend that a text is constituted as a tactical or a strategic recruitment of available resources in the production and reproduction—(re)production—of the social structure of the activity within which the text itself is produced. I further contend that the social structures that characterise research activity are, in general, quite distinct from those characterising the practices of schooling. This being the case, there is no simple transferability of products between research and professional practice (or, incidentally, between mathematics and its non-mathematical public domain incorporating domestic and working practices and so forth). The relationship between mathematics education research, on the one hand, and the professional practices of mathematics education in schooling and other settings, on the other, is therefore one of dialogue; any attempts to collapse the distinction between these two fields can only confound the dialogue. My second question, then, is in two parts: *Are educational research and professional practice in mathematics education (and, correspondingly, school mathematics and the non-mathematical practices which it recruits) properly conceived as distinct activities or as a single field of practice; if the former, how can we achieve a dialogic relationship which is productive within both?*

- I define a pedagogic activity as entailing the transmission and acquisition of a privileged discourse, narrative, skill, or comportment under conditions whereby the principles of evaluation of texts are located with the transmitter. I want to hypothesise that *apprenticing transmission* is properly conceived as textually oriented. That is, transmission constructs an apprenticed acquirer position to the extent that it makes explicitly available the esoteric domain principles that generate valid texts. However, this can be achieved only in the presentation of texts for interpretation by the acquirer. *Apprenticed acquisition*, on the other hand, is essentially grammar-oriented. That is, the apprentice is confronted by texts that they must interpret in terms of their generative principles. To simplify: the teacher deploys principles in the generation of texts, which are presented to the student; the student must read the text with a view to accessing the teacher's principles. This is because it is the principles rather than or, at least, in addition to the texts themselves which constitute the content to be transmitted. My third question is: *To what extent does this interpretation of pedagogic activity enable reconciliation between the notion of pedagogy as a transmission, on the one hand, and pedagogic constructivism, on the other?*

- My understanding of sociology is that it is concerned with patterns of relations between individuals and groups—the social—and their production and reproduction in cultural practices. My analysis of mathematics education, in particular, describes this particular set of cultural practices as constituting a range of myths concerning the relationship between the mathematical and the non-mathematical and of distributing these myths such as to (re)produce an intellectual/manual hierarchy. My fourth question is: *(How) can mathematics education in itself or in its incorporation into alternative curricula ever be anything other than (re)productive of a hierarchical division of labour?*

The Critical Mathematics Educators Group (CMEG): Attempting to Connect Anti-Capitalist Work with Mathematics Education

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In "Scenes from the Inferno", Alexander Cockburn (1989) wrote about the reality behind, the so-called triumph of capitalism. One of his illustrations is particularly relevant for a critical mathematics education: in Chile, where in some Santiago neighborhoods, "the diet of 77 to 80 percent of the people does not have sufficient calories and proteins… to sustain life", Pinochet's regime measured malnutrition in relation to a person's weight and height, in contrast to the usual comparison of weight and age. "So a stunted child is not counted as malnourished, and thus is not eligible for food supplements". (p 510) This talk will explore the connections between understanding the outrageousness of collecting such statistics, and acting to change the outrageousness of such situations.

Broadly speaking, the Critical mathematics Educators Group hopes to connect critical mathematics educators' work with economic, political and social movements towards a just, humane society. We share the concerns of humanistic mathematics educators to respect our students and to teach mathematics in such a way that understanding is emphasized over memorization and students actively participate in their own learning. We share the concerns of ethnomathematics educators to counter the Eurocentric models of the development of mathematical knowledge, to consider the interactions of culture and mathematical knowledge, and to start the learning process from our students' mathematical knowledge. We add to those humanistic and ethnomathematical concerns an attention to how the power dynamics of society result in the situation where "the intellectual activity of those without power is always characterized as non-intellectual". (Freire & Macedo, 1987, p 122) We view mathematics as one area of knowledge constructed by humans in order to understand and learn about our world. We believe that major objectives of all education are to shatter the myths about how society is structured; to understand the effects of, and interconnections among racism, sexes, ageism, heterosexism, monopoly capitalism, imperialism, and other alienating, totalitarian institutional structures and attitudes; to develop the commitment to rebuild those structures and attitudes; and, to develop the personal and collective empowerment needed to engage that task.

This talk will outline the organizational roots and the organizational connections of the Critical mathematics Educators Group; discuss the non-static definition we have proposed and the intellectual currents that underlie it; review the activities in which we are involved; and raise various political and research questions for exploration.

Those questions include:

- What is critical mathematics education in contrast to excellent, humanistic teaching, project-based curricula, and so on?

- What are OUR politics – the politics that underlie "the politics of mathematics education" that we are discussing?

- How do we work within our own organisations/conferences to change our internal power dynamics that result, for example, in the First International

Conference on Mathematics Education and Society having 8 out of 8 white keynote speakers?

- What connections/collaborations/organizational structures do we want to build among MEAS, PDME (Political Dimensions of Mathematics Education), CmEG, TSGEm (International Study Group on Ethnomathematics), Humanistic Mathematics Network, etc? (And what relationships do we want with the larger mathematics education organizations, such as ATM, NCIM (National Council of Teachers of Mathematics, USA) AMESA (Association of Mathematics Educators of South Africa), etc.?)

- How can we connect critical mathematics education in the classroom and in the community with political struggles and, social movements for a just, equitable society?

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Why children fail and what the field of mathematics education can do about it: The role of sociology

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Abstract

We welcome the opportunity provided by the organisers of this conference to engage with issues in mathematics education from the perspective of the social. For more than 20 years psychology has offered many researchers in mathematics education a scientific paradigm, some illusion of certainty about the individual and about what the problems are concerning learning mathematics, but it has not given adequate answers to why children fail. Sociology, in particular Bernstein's model, gives a very precise description of the pedagogic mechanism through which educational and social inequality is reproduced in schools. We will argue that a systematic reading of Bernstein's theoretical frame, and also of the research done using this frame, can help: to understand the nature and the character of traditionally organised schools; to understand whether initiatives which change the way mathematics is taught, in England and elsewhere, are the appropriate ones when judged with pedagogical and especially with sociological criteria and arguments; and to articulate a place of intervention towards greater equality and access.

I. Introduction - Starting points and basic assumptions

We will speak from the point of view of sociology and its possible contribution to the question mentioned in the title of this paper: why children fail in school mathematics, in particular, and what the field of mathematics education can do about it. We will, mainly, draw on the work of Basil Bernstein (1971, 1990, 1996), which is concerned with showing how the pedagogic text is constructed, distributed, acquired and assessed. We will also consider empirical research studies in mathematics education and related areas that have been influenced by his work, and which, in Davies' (1995) words, will make Bernstein's work relevant to the analysis of the classroom.

As mentioned, in the light of Bernstein's work, to deal with the question of the systematic failure of certain categories of pupils is to engage with the processes through which the pedagogic (mathematical) text is produced, acquired and assessed. The starting points of such an investigation are as follows:

1a. School knowledge, and therefore school mathematics is different from both everyday and academic knowledge. This is because school knowledge and school subjects are constructed through a social process, what Bernstein calls a process of recontextualisation. We will refer to it as recontextualisation 1.

1b. One can talk about constructions of school knowledge at different levels. For example we can talk about the level of curriculum construction (e.g. a national curriculum), the production of textbooks, or the interactions in the mathematics classroom. Bernstein's model deals with them as different expressions of school knowledge.

1c. Independently of the level we are analysing, the three basic elements of school knowledge - what Bernstein calls message systems - are present. These message systems are: content; pedagogy; and evaluation. Therefore, when constructing or analysing school knowledge, all three message systems and their interconnections are at issue.

1d. To be able to analyse, i.e., specify principles of construction of school knowledge, at any level, one can use one common conceptual framework, and one common research tool.

We will mention briefly a few instances of empirical research whose Bernsteinian treatment of message systems emerges. In mathematics education the work of Dowling (1995, 1998) will be discussed below. In science education Fontinhas, Morais and Neves (1995) use different values of classification and framing to construct different types of classroom practice. Their aim is to find pedagogic practices that are more appropriate to all children. Morais & Abtunes (1994) explored the issue of regulative pedagogic practices that are formed by changes in the values of classification and framing, concluding that this influences children's achievement.

Regarding assessment, Morais & Miranda (1996) studied the extent to which students understand the evaluation criteria, and more specifically, teachers' marking criteria and procedures, that is, the extent to which students have recognition and realisation rules in the assessing context. Cooper, Dunne & Rogers (1997; see also Cooper & Dunne, 1998) explored the national system of testing in Britain, and more specifically the way different items are structured (differentiation in classification and framing values). The results show that certain categories of children do not recognise the context of the question and therefore their answers draw on everyday resources rather than the specialised resources of mathematics. Singh (1993, 1995) examined discourses of computer contexts in primary school classrooms.

Indeed, the empirical research which we have located are amongst the best combination of theoretical and empirical approaches to the question of success and failure of pupils, and exemplify aspects of Bernstein's conceptual framework (see also Davies, 1995b).

The construction of specialised educational discourses, such as mathematics education, science education, etc. involve the recontextualisation of discourses from the field of knowledge production, such as psychology and sociology. We will call this recontextualisation 2. This is a second plane within the field of recontextualization, which affects the construction of school knowledge. An interesting tension can be discerned already here in classifying academics working in specialised educational discourses. Are they producers or recontextualisers of knowledge?)

Mathematics education as an area of study plays an important role in the construction of school mathematics. This happens either directly through its participation in the processes of recontextualization for the construction of school mathematics and school textbooks, or indirectly through the dissemination of research findings in the education community (e.g. through in-service teacher training courses). Therefore, in what follows, we argue that the analysis of the field of mathematics education (of its assumptions and its directions) is an important part in the analysis of school mathematics.

II. The internal structuring of mathematics education discourse

Lamnias & Tsatsaroni (1997a & 1997b; cf Bernstein 1996, ch.9) in attempting to analyse the internal structure of education discourse (the discourse of pedagogy, or of didactics) have argued that there might be a possible relation between specialised education discourses and the type of school practices that prevail or are made available to teachers. More specifically, it has been argued that the structure of the existent education discourse seems to consist of a series of language games that attempt to establish local hegemonies within the discourse and to create pedagogical knowledge and forms of school practice.

One distinguishing feature of these language games is a sharp distinction between theory and practice. This means that each of these language games produces and disseminates, mainly, theory, this way attempting to influence the construction of curricula, textbooks or forms of school practice. Therefore, because of the way each of these languages operates, and because of the power relations between different agents in the field of recontextualization 1 (i.e., between official state agents, textbook producers, academic community) its influence at the level of recontextualization 1 is probably limited or produces many contradictions; though this is a matter of empirical investigation. However, we wish to point out the similarities between the typology to which we refer in the text below, which has been constructed with reference to specialised education discourses and Dowling's (1998) typology which analyses forms of school knowledge in school mathematics textbooks in England)

Thus, when one attempts to analyse the types of school knowledge/school practice that each of the language games in the specialised discourse of education proposes, one comes up with the typology below. The typology and the revealing of the implicit social assumptions of each type of practice were developed by Lamnias & Tsatsaroni (1997b) by making a systematic reading of Bernstein and attempting to assess the usefulness of his theoretical framework in analysing the most general trends of the discourse of pedagogy in Greece and beyond.

A TYPOLOGY OF SCHOOL PRACTICES			
1	C+	F+	Cr Formal explicit
2	C+	F-	Cr Informal
3	C-	F-	Cr implicit

Diagram 1

Comments on the typology
a. definitions
C - Classification, in Bernstein's framework, refers to relations between categories (e.g. between contents), and defines what counts as valid contents.
F - Framing refers to relations within categories (e.g. pedagogical relations) and defines what counts as valid pedagogical transmissions (teaching and learning)
Cr - Criteria refers to what constitutes valid communications from the point of view of the pupil, i.e, refers to evaluation.

b. Types of pedagogic practice
* **Type 1** relates to a mode of education discourse which was once hegemonised by behaviourism, and, now, probably, by neo-behaviourism. It was imposed and is still the predominant type in many traditionally organised schools. It is characterised by strong classifications, strong framing and explicit, formal and concrete criteria of evaluation.

*** Type 2** relates to a mode of education discourse that is hegemonised by developmental and other areas of psychology. It is characterised by strong classifications, weak framing and informal criteria of evaluation.
*** Type 3** relates to a mode of education discourse which has had some important influences from sociological critiques of the neglect of pupils' everyday knowledge and the hierarchisation between everyday and school forms of knowledge (Young, 1971) influenced by critical sociology. It is characterised by weak classifications, weak framings and as a matter of principle rejects the idea of testing and evaluation. However, as Lamnias & Tsatsaroni (1997b) have argued, and empirical studies have shown (Cooper, 1998), the criteria of evaluation are in fact an implicit feature of this type of practice.

c. Sociological assumptions implicit in the typology of pedagogic practice
Bernstein's theoretical framework, the main concepts of which are shown in the diagram below, connects the macro- with the micro-level of analysis and purports to describe the pedagogical mechanism that is responsible for social and educational inequality.

Source: Bernstein, 1990, Figure 1.8, p. 42

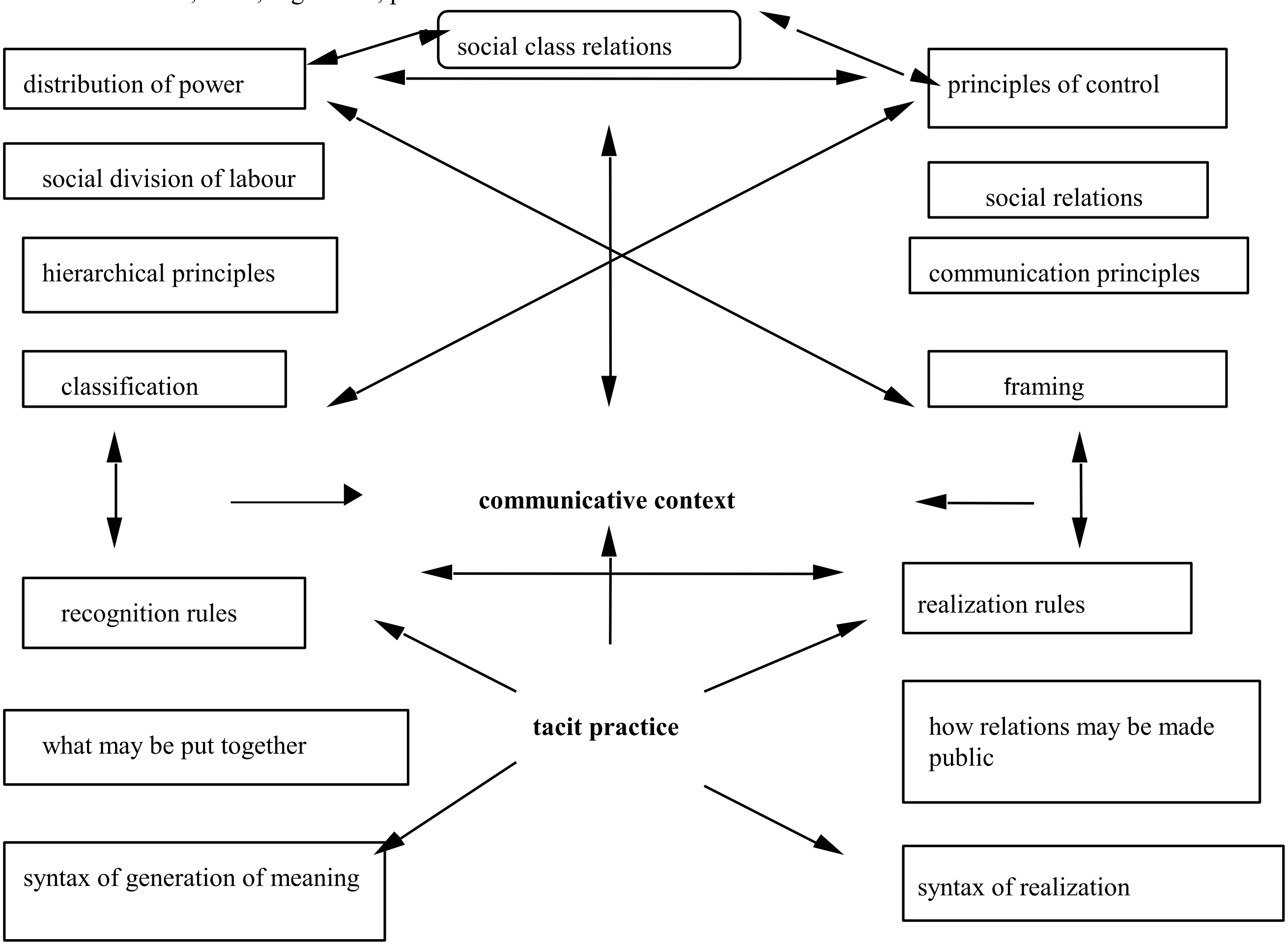

Diagram 2

Using the diagram above, we can make the following points:

1. Bernstein considers the relationship between the socio-economic background of pupil, indicated here and the notions of relations of power and principles of control; classification and framing, specific values of which produce forms of knowledge and types of practice; and pupils' forms of consciousness and pedagogic identity, indicated by the concepts of recognition and realization rules. This means that a form of school knowledge, constructed by certain values of classification and framing, will produce different recognition and realisation rules to different categories (e.g. social class, gender, ethnicity) of pupils. Though not systematically and rigorously addressed (at least not with the appropriate concepts) sociology has long been arguing that the traditionally organised curriculum excludes from the educational process certain categories of pupils, almost by definition.

Thus:

2. When the pedagogic communicative context of the classroom is structured by traditional practices, of the type [C+F+], with formal criteria of evaluation, working class pupils are likely to fail.

3. The type [C+F-], when used to structure the pedagogic communicative context, is unable to destabilise the pedagogic mechanism, responsible for social and educational inequality. Indeed, it suggests no changes at the level of the recontextualization for the construction of school knowledge. Thus it focuses on processes of teaching and learning and on informal criteria of evaluation and emphasises acquisition rather than transmission. It attempts to achieve a change in the quality of communication between teachers and pupils, without however addressing the question of where and how the complex communicative competencies and skills are going to be acquired.

4. When the communicative context of the classroom is proposed to be structured by the type [C-F-] of school knowledge the traditional pedagogical mechanism, again, cannot be destabilised. More specifically, this type recognises the value and even validity of pupils' everyday knowledge. However, it assumes that between the two there is a continuity. This way, the many and important differences, epistemological and sociological in character, between the two forms of knowledge are ignored. Therefore this type uses implicit criteria of evaluation, and assumes that the ability to recognize the context of everyday and distinguish it from the scientific (to have the recognition rules), and to use resourses from the latter to construct specialised answers to the questions addressed to him/her in the classroom, are givens for all categories of pupils.

In our presentation we will show the current stage of an analysis we are making of articles and papers in the specialised discourse of mathematics education over the last decade, using this typology.

III. Pedagogic practice as a methodological choice in the analysis of the school classroom.

In order to appreciate from a methodological point of view the turn to the analysis of pedagogical practice, either at the level of recontextualization 1 or 2, of the research studies mentioned in the preceding sections, we will refer to Bernstein's basic model which attempts to synthesise the macro- with the micro- level of analysis.

Bernstein analyses the traditional school and the traditional pedagogic mechanism and constructs a model based on the pairs of concepts: Power and social control, classification and framing, recognition and realisation rules (see diagram 2). These concepts link the macro- with the micro-level of analysis. At the communicative and interactional level of the school classroom the interrelationship of these concepts is expressed in the construction of the type of pedagogic practice and reveals the social and educational determinants of the processes of knowledge acquisition. The diagram is the outcome of Bernstein's "... attempt to link the societal, institutional, intrapsychic realms and to demonstrate how the microprocesses of schooling relate to complex institutional and societal forces" (Sadovnik, 1995, p.25).

Therefore, with the synthesis of the macro-/micro-level (Bernstein, 1990, ch. 5; Lamnias & Tsatsaroni, 1997a & 1997b), the model succeeds in describing, in a systematic way, the pedagogic mechanism responsible for the reproduction of educational and social inequality. In particular, the specialised communicative context of the classroom is structured with:

a. Structural elements of the macro-level, in particular elements that are linked with the socially determined practices of pupils. These social practices construct the socially determined "position" of the subject, which on the basis of the dominant classifications and framings of the society in which he/she develops, constructs particular kinds of recognition and realisation rules. Thus he/she produces meanings which can be interpreted by reference to the social category (social class, gender, ethnicity) to which she or he belongs.

b. Structural elements that are produced in the recontextualization and the construction of the school form of knowledge. Thus, as already argued, the values of classifications and framings of school knowledge are, partly, determined by the concerns and hegemonies that prevail in the relevant research community (mathematics education, science education, English language teaching, etc).

Therefore the model describes the pedagogical mechanism which is a function of these two kinds of structural elements and which explain the reproduction of social inequality. Thus it gives a very cogent answer to the question of why certain categories of children fail, and at the same time leaves open the possibilities for conceptualising change through an intervention in the constructions of school knowledge/type of school practice. More specifically, we have argued that the model can: help to reveal the unthematised, implicit social assumptions that are included in certain constructions of school knowledge; and show the role and the importance of (maths) education discourse in the formation and projection in schools of types of school knowledge, as well as its consequence.

At this point we should state, with Ladwig (1997), that: "Scientific sense will suggest that it is wise to guard against over zealously defending a core theoretical framework that may well require fundamental reconsideration.", though it is obvious that we are suggesting to push this framework as far as it will go. Indeed this has been suggested by Bernstein himself (1998), when, in self-criticism, he confesses that he adopts the method of "productive imperfection" in his writings. The conceptual tensions thus generated have to do the following:

a. The implicit assumption that a type of practice can regulate the communicative context of the classroom. However, the communicative context is shaped not only by structural elements refered to earlier, but aslo the dynamics that the use and function of language introduces; to express it in a more technical language, as a consequence of the fact that a type of school practice is, above all, a discursive practice.

b. The assumption that the pedagogic subject, though it is formed through social processes, it remains in its core a rational, unified and coherent subject.

c. The structural linguistic assumptions concerning language.

In many cases in published texts, talks, interviews Bernstein compares his model to other theories of cultural reproduction (especially Bourdieu & Passeron, 1977) and emphasises that these theories are more concerned with how the external unequal social relations are relayed and legitimated by the education system. They make a diagnosis of its pathology, focusing in particular on the ideological messages of the pedagogic text and not on how the pedagogic text (e.g. school maths as a text) is produced, distributed, acquired and evaluated.

The notion of the (pedagogic) text shows the influence that linguistics had on Bernstein's work and on the model. A series of other terms such as discourse and context, as well as the concern with "a language of description" at the methodological level, confirm these influences. And if the criticism that sociology borrows these terms from linguistics without any attempt to incorporate them in the sociological vocabulary (Chalaby, 1996) cannot hold in Bernstein's case, still the structural linguistics on which his model relies, makes his approach a structural one. This approach is able to construct typologies (of pedagogical practice) but it is a theoretical and empirical question whether it can read the dynamics of classroom practice and the actions of concrete pupils.

One could argue that, for analytic purposes, a distinction could be made between a structural and a textual analysis, which can probably function in a complementary way. This question is indeed important in assessing the conceptual framework and the model that Bernstein's work has developed. That depends a lot on theoretical (and empirical) work that is required to define the notion of discourse, of context, of text, the "subject" and "reading".

Dowling (1998, ch. 6), commenting on the language of description, which relies considerably on Bernstein's work, writes that the typology is a tool for a better, constructive description of social reality. The product, the description, is not facts, and not representations of reality, but "the description produces systematic order". However, the attempt to thematise the presuppositions about language (text, context, discourse, etc.) that are implicit in the model and the typology of pedagogical practices is linked directly to this tendency to produce systematic order (Derrida, 1988), the emphasis more on typologies than on reading and more on subjects regulated by all powerful social, institutional and linguistic structures, rather than on the subject as a fully blown textual category, as fluid as the fluidicity assumed in post-structuralist conceptions of language.

We conclude that, in view of the move to synthesise the macro/micro-level of analysis of educational processes and its important consequences in thematising the implicit assumptions in the constructions of the pedagogic-math text, it might now be the time to deconstruct "The Sociology of Mathematics Education", i.e., the philosophical assumptions about language implicit in it. This would help to open the field of research concerned with the analysis of school practices, though in our view not without an opening, at the same time, to other fields of inquiry, and indeed to an inter-disciplinary research programme.

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MATHEMATICS, MIND, AND SOCIETY: AN ANARCHIST THEORY OF INQUIRY AND EDUCATION.

Sal Restivo

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> *[J]ust as the oppressor, in order to oppress, needs a theory of oppressive action, so the oppressed, in order to become free, also need a theory of action.*

*Paulo Freire*This abstract sketches the basic objectives of the plenary lecture. The lecture is based on the sociological theory of mathematics outlined in my paper, "Mathematics, Mind, and Society (MMS)". In my lecture, I will briefly summarise my theory of mathematics, and then clarify the basic terms of my argument: mathematics, mind, society, and anarchist theory. My objective in this lecture is to begin the process of extracting, refining, and developing a politico-theoretical framework and agenda that is at least implicit in MMS and has been slowly emerging in my work over the last twenty years or so. By "politico-theoretical" I mean to link theory, practice, and power. This is tricky in the sense that properly understood, theories are or engage worldviews, so they are or are integral with forms of discourse and practice, that is politics and power.

> *At the moment in which you say, Look, but now I invite you to be responsible!, immediately they think in opposition that your hypothesis is not rigorous....we have to fight with love, with passion, in order to demonstrate that what we are proposing is absolutely rigorous. We have , in doing so, to demonstrate that rigor is not synonymous with authoritarianism, that 'rigor' does not mean 'rigidity.' Rigor lives with freedom, needs freedom. I cannot understand how it is possible to be rigorous without being creative.*
>
> *For me it is very difficult to be creative without having freedom.*
>
> *Without being free, I can only repeat what is being told me.*
>
> *Paulo Freire*

Mathematics represents and embodies human labor; and human labor is always social labor. Even when I sit and think alone, I am performing social labor because the language of my thoughts and emotions is given to me by my society and culture, and even the very self and consciousness I experience in this (as in every other) situation are social because given to me and sustained in and for me by everyday social interactions. This principle of the pervasiveness of the social is very little understood. It is the basis for understanding mind and consciousness as socio-cultural products and processes. Even the brain is socially constructed. The significance of the social fact that minds and brains are not independent, free-standing entities and that independent, free-standing individuals are illusions has not yet reached into the social worlds of education (although some progress in this direction has clearly been made among those attending this conference).

> *By perpetuating the school as an instrument for social control and by dichotomizing teaching from learning, educators forget Marx's fundamental warning in his third thesis on Feuerbach: "The educator should also be educated." Paulo Freire*

Society is symbolically useful in my title, but does not convey the central idea I want to emphasise, that our selves are structured and re-structured, produced and re-produced, in moment-to-moment social interactions during the course of our everyday, everynight lives. These interactions are, in fact, ritualised and linked (in what Randall Collins has called "interaction ritual chains"), and these rituals and ritual chains are the crucibles in which we make and re-make our selves and our cultures. We could, then, say that mathematics, like language, and like any symbolic system, represents the product(s) of sets of interaction ritual chains.

> *Just as there is no such thing as an isolated human being there is no also no such thing as isolated thinking. Paulo Freire*

My conception of theory reflects my anarchist objectives. The craft or practice of theory is widely misunderstood.. It is, properly practiced, a subversive activity; indeed, it may be the most subversive activity humans are capable of. From an anarchist perspective (and here I follow Brian Martin), "Ideas are central to social struggles. Most of the intellectual work in government, corporations and universities is too technical or obscure to be of any value for popular use, or else, like advertising, it is manipulative. Are there ideas and methods of thinking that are specially suited for developing insights and strategies to challenge hierarchical systems? How can "theory," thinking systematically, become a popular pastime rather than an elite pursuit?" The sociologist Charles Lemert has in fact argued that "Everyone can do [theory]. Everyone should do more of it. Responsible lay members of society presumably would live better - with more power, perhaps more pleasure - if they could produce more social theories." We need to help ourselves and others understand the power - the critical and subversive power - of theory, and to help eliminate the idea of theory implied in such statements as "It's only (or merely) theory," "It's fine in theory, but not necessarily in reality," and the idea that somehow theories worth the label are constructed in vacuums out of nothing, without any grounding.

> *Dialogue in any situation (whether it involves scientific and technical knowledge, or experiential knowledge) demands the problematic confrontation of that very knowledge in its unquestionable relationship with the concrete reality in which it is engendered, and on which it acts, in order to better understand, explain, and transform that reality.*
>
> *Paulo Freire*

Finally, I need to explore the anarchist agenda. To begin with, I follow Peter Kropotkin's conception of anarchism as one of the sociological sciences. For the moment, I can only outline some of the basic ingredients of the anarchist agenda. In my lecture, and in the paper that will generate that lecture, my objective will be to integrate this agenda with the general sociological theory that has guided and grown out of my work on mathematics and science. This integrated agenda will form a foundation for reforming and rethinking mathematics and mathematics education. Fortunately, I have the advantage of being able to draw on a recent issue of Social Anarchism in which several contemporary anarchists outlined their versions of the anarchist agenda. I have adapted their program as follows:

The Anarchist Agenda

1. Human and ecological contexts for human survival with dignity and integrity.
2. The self is a social structure, community dependent and inter-connected.
3. Promoting diversity in selves and communities.
4. To transform bureaucracies into worker organised and operated organisations.
5. To strengthen popular involvement in and control over mass media.
6. Demarchy: local networks of volunteer based functional groups, dealing with various community functions including education.

7. Anarchafeminism: bringing the anarchist movement to bear on male domination and the oppression and suppression of women.
8. To search for and implement alternatives to state-market political economies.
9. Developing networking into a strategy for social action.
10. Challenging taken-for-granted ideas about material and intellectual property, and promoting non-ownership and collective usage; the rejection of property, consumerism, and commodification.
12. Facilitating organised non-violent action in and by communities.
13. Promoting science and technology for the people, alternative technosciences.
14. Theory as a subversive activity
15. Intellectually and theoretically, the rejection of transcendence, immanence, and psychologism.
16. The complexity of the world requires that anarchists avoid become enclavists, and instead work in consort with other activists for social change.
17. The anarchist tool kit should be part of a larger variegated toolkit of strategies, skills, tactics, and technologies for social change.
18. Anarchists should practice heterodox borrowing of ideas, perspectives, strategies, theories, and technologies.
19. Anarchists should avoid dogma in theory and practice.
20. Anarchism is a form of life.

In my lecture, I will begin the process of developing and applying an anarchist sociological theory to the problem of rethinking mathematics and mathematics education as social constructions. Some of the questions participants might care to consider are:

1. Reuben Hersh has written a recently published book titled *What is Mathematics, Really?* That question could (pre)occupy us for a bit.
2. Is mathematics invented and/or discovered?
3. What does mathematics represent?
4. What is a mathematical object?
5. What is the relevance of the sociology of mathematics and mind to mathematics education? In particular, what are the implications of the strong social construction conjecture as formulated by David Bloor, and by Sal Restivo and Randall Collins for designing relationships, structures (including the use of space and architecture), and pedagogies in mathematics education?
6. What are the contributions that we can anticipate from philosophy of mathematics and sociology of mathematics to issues in problems in mathematics education and society/culture?

Aporism, and the problem of democracy in mathematics education.

Ole Skovsmose

The following notes are based upon two papers 'Aporism: Uncertainty about Mathematics' and 'Linking Mathematics Education and Democracy' which both will appear in *Zentralblatt für Didaktik der Mathematik*. The notes may serve as an introduction to my lecture.

A Paradox

"In the last 100 years, we have seen enormous advances in our knowledge of nature and in the development of new technologies. ... And yet, this same century has shown us a despicable human behaviour. Unprecedented means of mass destruction, of insecurity, new terrible diseases, unjustified famine, drug abuse, and moral decay are matched only by an irreversible destruction of the environment. Much of this paradox has to do with the absence of reflections and considerations of values in academics, particularly in the scientific disciplines, both in research and in education. Most of the means to achieve these wonders and also these horrors of science and technology have to do with advances in mathematics." This is how Ubiratan D'Ambrosio, in 'Cultural Framing of Mathematics Teaching and Learning', introduces a section about mathematics and society.

According to the Enlightenment, scientific development and human progress are closely related. Therefore it seems a paradox that science can be related to human destruction. This paradox questions the optimistic assumption, that science also sustains progress in an economic and political sense. Has science come to play a dual role? Is science related not only to human progress but also to human disaster? Does mathematics play a double-role, representing both reason and unreason in social development?

Aporism

The Greek word *aporeo* means 'being in a loss' or 'being without resources'. *Aporism* represents an uncertainty about how to understand and criticise the 'social agency' of mathematics. Aporism is an expression of a concern for decoding also the horrors that might be associated with applications of mathematics.

Aporism acknowledges the possibility that pure reason may turn into perverted forms, meaning that the ideal harmony between reason, scientific development and human and social progress is broken. As part of the rationalistic perspective, reason ensures the progressive qualities of knowledge, but aporism accepts the possibility that pure reason develops pathological cases, and that some of these are connected to the development of mathematics.

Aporism elaborates on the paradox mentioned by D'Ambrosio. On the one hand, mathematics is a condition for technological wonders, on the other hand, mathematics appears to be part of a destructive force also associated with technology. Reason, in the shape of 'instrumental reason', becomes problematic.

The Formatting Power of Mathematics

If mathematics is interpreted as language, the speech act theory of language will raise the question: What can be done by means of mathematics? Mathematics can be interpreted not only as a descriptive tool, but also as a source for action. This brings into focus the notion of 'symbolic power' and the theme of 'knowledge and power'. Mathematics as a possible source for technological action and we may consider the thesis of the *formatting power of mathematics*: Social phenomena are structured and eventually constituted by mathematics.

In *Descartes' Dream: The World According to Mathematics*, Davis and Hersh provide a long list of examples of prescriptive use of mathematics which leads to some sort of human or technological action: "We are born into a world with so many instances of prescriptive mathematics in place that we are hardly aware of them, and, once they are pointed out, we can hardly imagine the world working without them. Our measurements of space and mass, our clocks and calendars, our plans for buildings and machines, our monetary system, are prescriptive mathematisations of great antiquity. To focus on more recent instances ... think of the income tax. This is an enormous mathematical structure superposed on an enormous pre-existing mathematical financial structure. ... In American society, there are plentiful examples of recent and recently reinstated prescriptive mathematisation: exam grades, IQ's, life insurance, taking a number in a bake shop, lotteries, traffic lights ... telephone switching systems, credit cards, zip codes, proportional representation voting ... We have prescribed these systems, often for reasons known only to a few; they regulate and alter our lives and characterise our civilisation. They create a description before the pattern itself exists." This illustrates the scope of the thesis of mathematical formatting.

However, my claim is not that the thesis of the formatting power of mathematics is true. The only claim is that the thesis expresses a possible truth, and that this possibility is important to consider when mathematics and mathematics education are investigated from a social and political point of view. Nor is the claim that the thesis is simple. Naturally, it does not make sense to claim that mathematics *per se* has a formatting power. The thesis concerns mathematics in context. Social, political and economic interests can be pursued by means of the powerful language of mathematics. In this way the thesis of the formatting power of mathematics becomes a thesis of the existence of an interplay between mathematics as a source for technological actions and other sources for social development.

The Vico-Paradox

According to Giambatista Vico, the rationalist idea that it is possible to come to understand nature and the whole universe, expresses a blasphemy: How can humankind imagine that, by its limited resources, it could come to understand the creations made by an almighty and omniscient God? Each individual human being has only a limited knowledge and a limited power. God, as the creator of the universe, can understand how it works, but only the creator will be able to understand his work. What human beings can hope to understand is what they themselves have been able to create.

The Greek *techne* refers to human creation. Following Vico's line of ideas, we should expect it possible for the human mind to grasp technology which is the paradigm of human creations. But when we consider the functions of technology we are lost. Humankind is not in control of technology, not even from a conceptual point of view. We are unable to express effects of technology, whether intended or unintended. This, I want to call the *Vico paradox*: Not even what we ourselves have constructed are we able to grasp and to understand.

We no longer live in 'nature'. Our environment is structured and organised into a 'techno-nature'. Science has provided us with means for describing and predicting natural phenomena, which can be used for technological inventions. But when we face techno-nature, which includes our own constructions, then natural sciences fail. Scientific knowledge of nature is not sufficient for interpreting the totality of nature and human construction. Neither sciences nor 'critique of culture' do provide us with means for clarifying the effects of science.

Mathematics Education

Does mathematics education produce critical readers of mathematical formatting? Or does mathematics education prepare a general acceptance of the formatting, independent of the critical nature of the actual formatting?

Essential functions in the technological society depend on how a competence in mathematics is distributed by means of the educational system. Mathematics education can serve as a 'blind' instrument for providing the mathematical competence in a form that is 'adjusted' to the present technological development. The structure of the educational system can make sure that the mathematical competence is distributed in such a way that, for instance, the 'adequate' number of people, needed in developing the information technology, in fact receive a sufficient mathematical competence.

Mathematics education can also make sure that the 'inverse competence' is in place and distributed in a functional way, meaning that a sufficient number of people come to understand that mathematics is not their business. Excluding a certain number of people from a competence can also be 'functional'. (A potential group of critics are eliminated.) In 'Mathematics by All', John Volmink writes: "Mathematics is not only an impenetrable mystery to many, but has also, more than any other subject, been cast in the role as an 'objective' judge, in order to decide who in the society 'can' and 'cannot'. It therefore served as *the* gate keeper to participation in the decision making processes of society. To deny some access to participate in mathematics is then also to determine, *a priori*, who will move ahead and who will stay behind."

Mathematics education is facing a problem of democracy. In my lecture I want to discuss aspects of this problem.

TEACHING MATHEMATICS IN LEBANON
A POST-WAR EXPERIENCE

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ABSTRACT

This paper presents the author's experience in teaching of Mathematics at the Lebanese American University in Beirut, Lebanon, in the aftermath of the civil war that ravaged the country from 1975-1990. A comparison of students' preparation as well as behavior with that before the war indicates patterns that can be attributed to the war that lasted from 1975 until 1990.

In this paper, I describe my experience of teaching mathematics at the college level at the Lebanese American University in the aftermath of the civil war that waged all over Lebanon in the period from 1975 and 1990. The Lebanese American University, (LAU), formerly a women's college, is a private university following the American system of education. Initially, it was a small college dedicated to the education of women in Lebanon and the Arab world, and as such it attracted a large number of female students from all over the region. In 1973, it went coeducational, and started introducing new disciplines, to meet the changing needs. Later on, a new campus was developed in Byblos with a school of Engineering, Arts & Sciences, Pharmacy and a Business school. More recently a new campus in Sidon was established to primarily accommodate a school of Agriculture. In addition, graduate studies in a number of disciplines were introduced to meet the growing needs of the market place. Thus the name of the institution was changed from "Beirut University College" (BUC), to "The Lebanese American University" to reflect the changes that had taken place as well as the history of the university. The Beirut campus, where I teach, includes about three thousand students, and most of my students are majoring in Computer Science, Math Education or Engineering. In addition to teaching mathematics courses, I also teach computer science courses.

My experience before the war included teaching at LAU from 1973-1976 when it was still a small college and had just started admitting men. I also taught at the Faculty of Sciences of the Lebanese University, the national university, this period being from 1977-1984, entirely a war period. In the 70's, the Faculty of Sciences had developed an excellent program with outstanding faculty and was able to attract excellent students from all over the country. However, the Israeli invasion of Lebanon put an end to the aspirations of many people of having an outstanding national university. The university was first occupied by the Israelis, abruptly terminating the academic year. After their withdrawal at the end of 1983, it was occupied by Lebanese militia groups, making the whole area a front line inaccessible to civilians . In 1984, due to the worsening security situation, the university was unable to function at all. That year instead of teaching we started a study group to explore the field of Computer Science, and it was then that my professional interest began to shift in that direction. At that time, few computers were available in Lebanon.

During that year, the security situation continued to deteriorate, with the fighting not being restricted to the front lines, but just about everywhere, not sparing residential areas or civilian life. "Normal" life was no longer possible. At that time, the adversaries were all Lebanese and the fighting took on a fratricidal character and often between militias belonging to different groups competing for control of the streets, and the neighborhoods. I do not intend to delve into the political reasons of the war, or to identify the interests of the various groups within Lebanon, or countries in the region or internationally, my purpose is to lay the background for this paper.

Finally in the summer of that year, I decided to escape and leave to the US, until things calmed down. My absence from Lebanon lasted ten years in a state of waiting for the war to end. During that period I studied Computer Science and taught at a university in Washington D.C. Finally in 1994, I was able to return to Lebanon, when I accepted the teaching position that at LAU, that I currently hold. In this paper, I compare my teaching experience on the one hand, before and during the war as well as my experience in the US.

Even though the war officially ended in 1990, the scars of the war were still there, the whole downtown district of Beirut as well as other areas was completely destroyed. Numerous villages were ravaged. There were thousands of people killed, and many more injured or permanently maimed. The south of Lebanon was and is still occupied by the Israeli forces and the entire infrastructure of the country completely destroyed. Electric power was still being rationed to 12 hours a day, the telephones for the most part not functional, roads in a very backward state, and the economy in shambles. In addition, there were still hundreds of thousands of families that were still displaced and have not been able to return to their villages or to the areas they lived in before the war. Even though the war was over, the great majority of the people suffered great hardships. A large number of families who had fled the country because of the war decided to stay in their adopted homes in the US, Europe, Australia or elsewhere. Now, that the reconstruction effort has been in full force the last four years, life is almost going back to normal, at least as it appears on the surface. However the problems of the displaced have been only partially solved and still thousands of people have not been able to return to their homes. In addition, life has become very expensive, unemployment is rampant, and the shelling in the south is ongoing on a daily basis, and the future often seems to be precarious and uncertain.

I will now describe briefly the educational system in Lebanon. Historically Lebanon was under the French mandate until it got its independence in 1943; hence there is a strong French Legacy reflected by the number of educational institutions using French as the language of instruction and adopting the French educational system. On the other hand, missionary schools were created by various denominations each defining its own system. As a consequence there are schools using French, English, Arabic and even German as the language of instruction. Public schools exist, but lag behind in quality partly because of the war and partly for inadequate support.

Lebanese students wishing to enroll at any university have to pass the Lebanese Baccalaureate, a national examination prepared by the Lebanese Ministry of Education and administered at the end of each school year. Students planning to go to the sciences either opt to take the "Mathematics" section or the "Experimental Sciences" section of the exam. All students majoring in Computer Science or Mathematics Education come from this kind of background.

The high school curriculum has not changed in the last 25 years, and many educators have put the validity of the Lebanese Baccalaureate into question. Had it not been for the war, the high school curriculum would certainly have been restructured to take into consideration changes in society, as well as changes in technology and education. However it was only last year that a new curriculum was put into place and now the National Center for Educational Research is preparing the instructional materials needed. Hence, most of our students have followed a high school program that is completely outdated, and none have any experience with computers or mathematical software.

The Lebanese American University being a private university still commands high tuition fees and as a result most of the students come from either middle class or upper class. A consequence of the war is the devaluation of the Lebanese pound[1], which impoverished the middle class, formerly substantial and making the poor still poorer. That leaves a tiny minority of very wealthy people. As a result many of our students require financial aid, and actually 20% get financial aid in the form of work-study program or loans[2]A lot of students whose parents have limited means aspire to have them enroll in an American institution, whose diplomas guarantee a good job for the graduate, whereas a degree from the Lebanese University does not necessarily insure the future, especially in the sciences. The Lebanese University which at one time had a lot of promise, now has been partitioned and weakened but still educates about 50,000 students, by far the largest number of students compared with any other university.

The student body at LAU (Beirut) is 80% Lebanese, 13% citizens of other Arab countries and 7% non-Arab[3]. The Lebanese students belong to all the existing sects of Lebanon, with students coming from all parts of the country even though the majority is from Beirut. Amongst students majoring in computer science, about 15% of the students are female. A large number of students come from schools using French as the language of instruction and some are from schools using Arabic as the language of instruction. In addition, the students coming from other Arab countries are used to having Arabic as the language of instruction. The problem of the English language causes a great deal of difficulty for the students, especially in the beginning years.

During the last three years, I have been teaching Calculus III, a course in Abstract Algebra, one in Linear Algebra and some computer science courses. In the first session of the Calculus III class, I start a dialogue with the students trying to assess how much they know of Calculus, and their understanding of the concepts involved. The majority of the students had already studied Calculus for one year, as this is a major topic of the Lebanese Baccalaureate exams. Students seem to have good algorithmic knowledge, such as knowing how to compute derivatives and integrals, but when asked about the meaning of the derivative very few would be able to articulate it as the rate of change, and none would be able to define it. When asked about the difference between the definite and indefinite integral, all identified the definite integral with the process of evaluating it. They have absolutely no idea that the relationship is a consequence of a profound theorem. When asked whether they had studied the Fundamental

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1 In 1982, the US dollar used to be around 3 L.L., and later on it fluctuated between 1500 L.L. and 2,500 L.L. Now it is stable at around 1500 L.L.

2 This data is taken from the administrative offices of LAU.

3 This data is taken from the Registrar's office at LAU.

Theorem of Calculus in school, the answer was that the teacher told them that all they had to know was how to evaluate the integral, and that the other matter was not important.

The mathematical skills of our students are confined to routine algorithmic problems such as calculating the derivatives or integrals. When given problems that require some reasoning or even understanding of the problem, they fail miserably. In addition, they have poor reading and writing skills. After having studied Calculus for one year in high school, they show that they have almost no understanding of the concepts underlying the Calculus. What they seem to know quite well are the steps that are involved in routine problems, but have little understanding of the relationship between the various steps. They have no idea of why a particular topic is interesting or why they are doing it. When asked to do some reading, or asked some problems that require thinking, their main worry about whether it is will be on the test, and if reassured that it is not, they will not give it any importance.

During the war years, the primary concern of individuals and institutions was mere survival. During a typical year of the war, there was fighting and random shelling not only on the front lines but in the residential sectors of Beirut as well as other towns. For at least one third of the year, no one was able to venture outside their homes, or the shelters. The only people who moved around were the militiamen and the fighters. Frequently, a day would start with relative calm, however in the middle of the day, fighting would erupt and parents would be in a desperate state trying to get their children back to safety. We have no exact statistics on how many school days were lost during those years. However going back to the account of the war for the year 1986[4], there were 300 days of war activity in various parts of the country. As to the security situation in and around Beirut, there were about 100 days, when there was random shelling, street fighting, and instances of car bombs or sniping. During that year, there were 2592 people killed, and 7250 people injured. There were 42 individuals who were assassinated that year; these were leaders, intellectuals and politicians. In 1987, there were 142 days of fighting, shelling, bombing and for 93 of these days, these acts were taking place in Beirut and within 20-km radius around the city.

For a short period some schools tried to organize classes at teachers' houses for those students who lived in the immediate neighborhood. Since schools are not organized around neighborhoods, that experience was unsuccessful and did not last. We can give a conservative estimate that schools opened for less than two thirds of the required time. When classes were held, material had to be quickly covered, and there was very little time and opportunity for exploration and experimentation to develop the higher level cognitive skills. Thus, the luxury of exploring, learning concepts and problem solving had to be subordinated to the goal of finishing the curriculum and to prepare the students to sit for the Baccalaureate exams in spite of the war and the continuous interruptions. During a number of years, the Baccalaureate exams could not be held, and students were then given certificates attesting that they had completed their high school studies in lieu of the official diploma, even though they had not finished the curriculum. This happened in 1980 when students were admitted on an open basis to the Lebanese University. It was a disaster, students did not have the necessary background. Classes were held in an amphitheater; it was chaos with students going in and out as they pleased. Those serious about studying had to come early in the morning to guarantee a seat in the front rows. Because the university was free, students enrolled in large numbers, as they had nothing to lose, and had no other alternative. In other

[4] This data is compiled from [3] and [4]

years when the exams were held, the questions were predictable, being largely restricted to algorithmic knowledge and students as well as teachers knew how many questions to expect on each of the topics included. In these sessions the rate of success was almost total, and the rationale was not to ruin the future of the students and to enable them to continue their studies or to allow them to pursue their life as they wished.

According to a study by Zein El Din (1997), an analysis of 10 sessions of the official exams (in the Mathematics Section) showed that 91.1% of the questions tested algorithmic knowledge, whereas only 1.8% of the questions tested understanding of concepts and only 7.1% of the questions involved problem solving. Similar statistics were obtained on the "Experimental Sciences" sections. Considering the situation in Lebanon, it made sense for examiners then not to be tough on the exams, to take into consideration that life and schooling was not "normal". Often, while teaching at the Lebanese University, we would be hearing the bombing and the shelling and we would be wondering, which areas have been hit, and worrying whether we would be able to make it back home safely. I would agonize whether class should be dismissed or not, because we were trying desperately to keep a semblance of a normal life. Not to mention the psychological state of the students who for the most part came from the south, or lived in areas subjected to constant bombardment and shelling. They lived in a constant state of insecurity and fear. When they went back home, there was overcrowding, often there was no electric power, and sometimes they had to go to shelters or to the safest area of the building.

The ages of the students that I teach at LAU vary from 18 until 22. So, these students spent a big part of their childhood, knowing nothing but a state of war. In a study by J. Abu Nasr (1981) about the effects of the war (1978-79) on children, a sample of 548 Lebanese children, from different parts of the country, and belonging to various social economic classes. This study shows that 61% of the children were directly exposed to the war, while 53% were displaced at some point from their homes and 14% had their homes completely destroyed whereas for 11.7%, their homes were partially destroyed. 15.5% had lost a member of the family whereas 5.3% had lost more than one member. We observe in our students the lack of the ability for concentration and perseverance in their study habits. In class, they do not have the ability to pursue a line of reasoning that would take more than a few steps. At home, they do not have the perseverance to solve non-routine problems. Their attention span is extremely limited and they can hardly stop from talking to their neighbor in class. One might say that this was a cultural characteristic, but certainly, this behavior was not prevalent before the war. Students had more discipline and accepted the authority of the parents and the teachers. Students had certain amount awe being at the university and felt a great deal of responsibility towards their parents and their teachers.

Another strange matter, is the students' inability to assess the seriousness of a particular situation. For example, we may be in the midst of a difficult proof and all students seem to be involved when a student will interrupt and ask permission to ask a question. When allowed to speak, assuming that his question to be relevant to the point under discussion, it turns out the contribution is a funny story or something totally irrelevant to the topic such as "When are we having the next test"? It might have been the student's way of expressing his frustration and trying to break the tension that was becoming unbearable.

There are also economic considerations. Before the war, a university education was a promise for a better future. Whereas now, students are very cynical about their future, justifiably so, since they know that when they graduate, if they are lucky to get a good job, the salary of 500 US dollars would be insufficient to have them rent an apartment, let alone get married and raise a family. A few of them aspire to find a job in the Gulf region where the jobs pay well, and some of the better students hope to do graduate work and try to emigrate to the US or Canada. However for the majority of the students from the middle class, the future looks bleak. The role models in society that they see now are the successful businessman, who made a lot of money very quickly, or the powerful warlords and politicians. During the war, the hero was the militiaman, the macho character carrying the Kalashnikov[5] and demonstrating his power in the streets. In a sense, the young militiamen were expressing their rebellion against a patriarchal authority and traditionalism, unfortunately they were unable to replace it with anything qualitatively better. On the contrary, the net result of the war that ordinary people became poorer, and the society more corrupt and decadent, with democracy lessened and people losing power, rather than being empowered.

Neither here before the war, nor in the US, have I experienced similar behavior on the part of university students. No doubt, the times are different and values have changed. Whether we can attribute all the above-mentioned symptoms entirely to the war, may be put into question. Some observations we make here about students understanding of mathematical concepts and problem solving skills seem to be widespread as has been documented in the math education literature. But we can certainly say that the war exacerbated matters related to the learning environment, making it extremely unhealthy, and not conducive to real learning. Amongst LAU students, there are some students whose parents had fled the country during the war, but returned when life went back to "normal". These students spent their childhood and did their schooling in another country (mostly in Europe, The US and Canada) and we observe that they do not manifest the same behavioral symptoms that other students exhibit. They exhibit a higher level of maturity. They have better reading and writing skills, and more discipline and perseverance in pursuing a subject. The students who have studied in Lebanon have been conditioned very strongly to have to know the next step, and their primary concern is the test. They rarely ask why! Nor will they explore for knowledge's sake.

Another matter that creates serious problems for students is English, the language of instruction at LAU. Even for those students who have studied in schools using English for the sciences, their language skills are quite weak, so they are unable to communicate effectively, let alone read or write. The textbooks used are mostly books published in the US and written to native English speakers, and all the examples taken are culture specific. As a result, textbooks are used only as reference to look up how specific problems are solved so that these can be mimicked in solving homework problems. When students ask questions in class, they use a combination of English, Arabic as well as gestures, and are unable to convey what they mean. For those students coming from schools using French or Arabic as the language of instruction the problem becomes much worse. Even though the educational system has not changed, the level of the students has certainly gone down

In conclusion, the students at the Lebanese American University observed in the period from 1994 to the present exhibit behavior and learning difficulties that were not prevalent before the war or in the early part. Other learning difficulties such as weak problem solving skills or lack of understanding of mathematical concepts seem to be more universal.

[5] A Soviet automatic weapon that was widely used in the war.

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Beginning Women Teachers Using Action Research towards Inclusive Mathematics

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Making mathematics more inclusive is a stated aim of many curriculum documents in Australia and overseas. The achievement of the inclusive agenda is problematic to many beginning teachers struggling in the so called "survival stage" of transition into teaching. This paper discusses the learnings of a group of beginning women teachers in multi-cultural, Aboriginal and non-English speaking background classrooms. The model used is participatory action research. The paper highlights the learnings of the teachers about inclusive mathematics and presents a critical reflection about the use of action research with teachers in wide geographical locations.

The National Statement on Mathematics for Australian Schools (Curriculum Corporation, 1991) argues that "access to and success in school mathematics should be independent of gender, social class or ethnicity" (p. 8). It adds that "we are now beginning to understand some of our past curriculum practices in mathematics which have disadvantaged groups of students. For example, many of the contexts in which mathematical concepts were developed, applied and assessed were more likely to be central in the lives of boys than in the lives of girls. ... In a similar way the mathematics curriculum has tended to emphasise values and concerns which are more middle class, and to draw on experiences which are more relevant to children of Anglo-Celtic descent than those of Aboriginal descent and those from non-English speaking backgrounds" (p. 9). The National Council for Teachers of Mathematics (NCTM) (1989) asserts that "the social injustices of past schooling practices can no longer be tolerated. ... Mathematics has become a critical filter for employment and full participation in our society. We cannot afford to have the majority of our population mathematically illiterate. Equity has become an economic necessity" (p. 4).

In discussing the needs of students from different backgrounds who may not be achieving as well in school mathematics, the National Statement states that "sometimes such students are regarded as lacking in mathematical ability when they are actually experiencing problems with the formal language of the mathematics classroom" (p. 9). In the NCTM yearbook on equity issues and inclusive mathematics, Trentacosta and Kenny (1997) argue that "in order to create an equitable learning environment among a growing diverse student population, it is important for teachers to understand the relationship between learning mathematics and the linguistic and cultural background of the students .... Teachers who understand the interrelatedness among topics of mathematics and acquire the facility to operate using different mathematical world views can help students develop their ability to understand mathematics and to build on their own mathematical world views" (p. 5). Likewise, Frankstein (1997) demonstrates how the use of critical mathematics from the real context of the student can assist in making mathematics more equitable and accessible for the students. Other voices in the yearbook have stressed the importance of involving parents in the decision making process to increase availability of mathematics to students from underrepresented and/or underachieving backgrounds (Peressini, 1997; Strutchens, Thomas, & Perkins; 1997).

Women Primary Teachers of Mathematics

A recurring issue raised by the various reviews and reports (Curriculum Corporation, 1991; Department of Employment Education & Training, 1989) is that the teaching of mathematics is for many primary teachers an area of major concern. Among the reasons for the concern are inappropriate teaching and learning practices that teachers have themselves experienced in their own schooling and preservice courses, and many primary teachers have limited content backgrounds in mathematics and little interest or confidence to teach it.

These problems are more prevalent among women teachers who dominate the teaching profession in the primary and early childhood school years. Historically, the lack of content background in mathematics and science among women teachers may be accounted for by traditional perceptions of women as nurturers, and thus more suited to primary and early childhood teaching and subjects such as English, home economics and biology. These perceptions has been detrimental to women wishing to enter such fields as the physical sciences and mathematics. Although policies for change have been initiated (e.g., Clark, 1990; Kenway & Willis, 1993), and despite encouraging indications that women have moved into mathematics and the sciences (Willis, 1989), reports (e.g., Cobbin, 1995) continue to note that many women preservice teachers remain weak in mathematics, and have little interest in teaching the subject. Arguably, if the teaching of mathematics in many primary and early childhood classrooms remains problematic because of these factors we will continue to fail to address deficiencies in girls' education and access to nontraditional subjects higher up.

Needs of Beginning Teachers

The early classroom experiences of beginning teachers may either inhibit or catalyse a lasting commitment to effective mathematics teaching. Successful early experiences may contribute to a positive sense of self-efficacy and hence instil confidence in the teaching of mathematics. Therefore, it is a crucial aspect of teacher professional development that we seek ways of fostering the professional growth of beginning women teachers so that they can acquire the confidence to be effective teachers of mathematics in the long term.

First year teachers enter the teaching profession with varying levels of skills, content knowledge, and pedagogical knowledge. Because of the lack of employment opportunities, many apply for, or are posted to isolated schools or schools with students who are culturally unfamiliar to the teachers. While first year teachers are attempting to overcome difficulties faced in the new school environment, Veenman (1984) suggests that these teachers "need both pedagogical assistance and psychological support." Katz (1972) describes four stages of teacher development: survival, consolidation, renewal and maturity. It is suggested that the first two stages characterise the first two or three years of teaching. The survival stage is distinguished by self interest and self concern, for instance, getting through the day and planning for a short period of time. In the consolidation stage, concerns move beyond self, and towards children.

Fuller (1969) describes three major phases in teacher development: pre-teaching, characterised by non concerns phase; early teaching phase, characterised by concerns for self; and a late teaching phase, characterised by concerns for pupils. This model was revised by Fuller and Bown (1975) to three stages of concerns of an inservice teacher's development. The stages were characterised by concerns for survival, the teaching situation (e.g., content, methods, materials), and pupils (e.g., students' learning and emotional needs). Other models have been reported in the literature, for example, Vonk (1983) and Burden (1980). Common to all of these models is the initial survival stage.

The Project

The study reported here is part of the ***E**nhancing the immersion of beginning women teachers into **M**athematics and **S**cience **T**eaching through participatory **A**ction **R**esearch networks* (EMSTAR) project, a collaborative participatory action research (PAR) among nine first-year women teachers and university researchers. For the university staff, one of the main aims of the project was to investigate the support needed to enhance the transition of teachers from their university course into the profession, and the use of action research for facilitating such transition. For the participating teachers, the project allowed them to collaborate with each other and the university staff to deal with specific aspects of their teaching of mathematics in their schools. The focus of this paper is on the learnings of one group of three teachers and an academic investigating problems and issues in inclusive mathematics. Other papers consider the findings from the action research cells investigating issues in assessment (Suhrbier, Moman, Fitzgerald, & Ginns, 1997) and catering for the gifted and talented (Watters, Andrews, Henderson, & Everett, 1997).

Action research in Education

Atweh and Heirdsfield (1998) have identified several approaches to cater for the needs and support for beginning teachers. The methodology adopted in this project was PAR. Kemmis and Wilkinson (1998) discussed the following characteristics of PAR. First it is a *social* activity in that "it deliberately explores the relationship between the realms of the individual and the social." It recognises that "no individuation is possible without socialization, and no socialization is possible without individuation" (Habermas, 1992, p. 26). PAR is also *participatory* in that "it engages people in examining their knowledge (understandings, skills and values) and interpretive categories (the ways they interpret themselves and their action in the social and material world)." It is also participatory in the sense that people can only do action research "on" themselves - individually or collectively. It is *not* research done "on" others. PAR is also *collaborative* in that "[a]ction researchers aim to work together in reconstructing their social interactions by reconstructing the acts that constitute them. It is a research done "with" others. PAR is *emancipatory* in that "it aims to help people recover, and unshackle themselves from the constraints of irrational, unproductive, unjust, and unsatisfying social *structures* which limit their self-development and self-determination." PAR is also *critical* in that "[i]t is a process in which people deliberately set out to contest and to reconstitute irrational, unproductive (or inefficient), unjust, and/or unsatisfying (alienating) ways of interpreting and describing their world (language/discourses), ways of working (work), and ways of relating to others (power)." Finally PAR is *recursive (reflexive, dialectical)* in that "it aims to help people to investigate reality in order to change it (Fals Borda, 1979), and to change reality in order to investigate it ... It is a process of learning by doing - and learning with others by changing the ways they interact in a shared social world."

Participants

The teachers participating in this group were three beginning women primary teachers who came from a four year BEd course at the Queensland University of Technology. During their final year in their course, these teachers had participated in a study on Women Trainee Teachers in Mathematics (Atweh & Burnett, 1997; Atweh, Kyle, & Burnett, 1996). The teachers were joined by a university lecturer who facilitated the project supported by one research assistant, and an experienced teacher and author, as a critical friend, who has worked in Aboriginal contexts.

Procedures

In 1996, the three teachers were interviewed during the last year of their preservice course regarding their life histories in studying mathematics and their teacher preparation course. Special attention was given to the perceptions of these teachers about their confidence in the content of mathematics and in their ability to teaching it in the primary school. At the end of the year the teachers met to discuss issues related to action research and to plan the overall structure of the project for 1997.

The main activities of the project were conducted in 1997. At the conclusion of the first school term in April, the participants finalised the decision on the specific areas on which they would concentrate in their action research. Because of the great geographical distance between the participants, the main proposed means of communication among the participants were teleconferencing and email.

In May, 1997, the participants had their first one hour teleconference. During the meeting, the participants briefly discussed their experiences in their respective schools as well as agreed on some of the processes for conducting the project. General issues such as the need for establishing good contact with parents and for making the context of activities relevant to the student background were discussed. The participants agreed to write situational analyses of their schools stressing the specific problems they were encountering. These were to be distributed to each other for discussion in future meetings. The possibility of writing a paper on the project for presentation at national educational conference was met with support by the participants. The participants agreed that this paper should be written collaboratively and not by the university staff on behalf of the teachers.

The second telephone conference was held in August. The discussion on the situational analysis could not proceed since not all participants had received each others papers. It was agreed to have another meeting within a week where each of the situational analyses would be discussed in turn and each participant would attempt to provide some critical comments and suggestions on each other's situational analysis. Further planning for the writing of this paper was done at this meeting. The concepts of critical mathematics were discussed and some examples given. Likewise the principles of "culturally relevant context" vs "individually meaningful context" were discussed. In terms of planning action research projects within the school, the meeting discussed the need for projects to start very small - rather than attempting to change the whole classroom context. It was suggested that participants should concentrate on a small manageable aspect over which they have control in changing. The next meeting occurred a week later where the situational analyses were discussed.

One of the final activities of the group in 1997 consisted of writing the conference paper. The participating teachers were asked to submit their reflections on the first year of teaching and their reflection on the project. The intention was that these would be distributed to each other and further meetings would be devoted to compilation of the individual stories toward the writing of a cohesive paper. This could not happen due to delays in submissions of the final reports from the teachers. Christensen and Atweh (1998) have discussed in detail the problematics of collaborative writing in action research projects. The authors have identified different processes used for development of writing. This paper was written by the university research team members based on material supplied by the participating teachers.

Findings from the Project

The Three Teachers

Lisa's career choice of primary teaching was perhaps by elimination rather than deliberate planning. She felt rather confident in certain areas in mathematics such as operations and measurement, while she was least confident in algebra and fractions. Similarly, Lisa expressed some lack of confidence towards teaching mathematics, because in her mind "you can not afford to make a mistake in mathematics, because the kids will pick it up - once they have learnt something wrongly, it is very difficult to unlearn it." In her first year of teaching, Lisa was appointed to a remote Aboriginal community in the Northern Territory, about 300 km northeast of Katherine. Many students missed a large portion of the school year because families move toward the out-stations in the dry season. Kriol is the spoken language outside school by all students. However, at the community’s request, the school's focus is learning to speak, read and write English. Lisa taught a multi-age group consisting of grades 3 and 4, covering a wide range of educational achievement levels.

Gabrielle also "fell into" general primary school teaching, with her initial preference being special education. She considered herself rather confident in mathematics in general, though not confident enough to teach it at a secondary level. However, Gabrielle was not as confident about teaching mathematics. In her first year of teaching, she was appointed to a regional primary school about 500 km north of Alice Springs. The school of 420 students had a mixed student population, consisting of Asian, Europeans from various national origins and Aboriginal students. She taught a combined class of years 4 and 5. However, in mathematics she took the grade 5 students only. Students were streamed in mathematics classes based on ability. There were no Aboriginal students in the upper stream class.

Janette worked full time for three years before deciding to pursue a teaching career. Of the three teachers, Janette is the only one who studied Mathematics I (a medium level mathematics course in Queensland) at senior high school. She did this because of the perceived opportunity that this subject would provide in selecting university options, even though she lost interest in the subject by Year 11 when it became "too scientific" and not applicable to real life. Like Gabrielle and Lisa, Janette was also disillusioned by the lack of support from her mathematics teachers who seemed to devote more attention to the high achieving students. She did indicate however, that she was confident that her mathematical background was sufficient for teaching primary school mathematics. Janette taught Year 5 in an independent school, which primarily caters for students with a non-English speaking background.

Problems Identified by the Teachers

Through their involvement in the project the teachers have had several opportunities to reflect on the difficulties that they faced in teaching mathematics in their diverse contexts. In her situational analysis, Lisa identified several factors hindering inclusive mathematics. Lisa was aware that the world view of traditional Aboriginals may be incompatible with aspects of Western mathematics. She wrote: “The teacher is confronted continually with Aboriginal world view of these concepts which are vastly different from and more complex than non-Aboriginal concept of time, measurement and space.” Further, the day to day experiences of students in isolated areas is quite different from those of urban students "as [their] mathematics usage is usually only necessary for purchasing goods at the store, ... money is the only mathematics concept that is used frequently. Usually mental computation of adding or subtracting money is a strong point [with many of these students]". Further, language difficulty compounded these factors. She wrote: " As English speakers, we have a variety of words that may describe one mathematics concepts, for example, the concept of addition [i.e.,] add, how much all together, plus, etc. For Aboriginal children [whose first language is not English] they find it difficult to learn all the different language varieties for the concept of addition".

Other factors were identified by Gabrielle. First there are difficulties related to the teacher. She wrote that she had the "[t]endency to teach as I was taught: [being] teacher directed and prescriptive. I feel that I understand how mathematics should be taught but because my [own] schooling is so embedded in my way of thinking, I find it difficult to change." Similarly, there are certain problems with the mindset of the students. "Most students see mathematics as an isolated subject area and don't understand the connections with the real world or the language used". Other difficulties arose because of the time limitation available for a beginning teacher to master a variety of tasks. She wrote "I have so much to plan all the time and other school wide commitments that I find I have little time to spend on re-reading university notes or other professional development materials". Other contextual factors she identified related to the rigid structured of the school timetable and curriculum plans that did not allow her flexibility to dwell on the content that has not been mastered well by the students.

Lastly, Janette identified the diverse background of her students as a major problem that she had to deal with. Within her class there were "7 children with behavioural problems; 2 visually impaired and 1 with a speech impairment. Nearly all of these children are from Arab countries, and have English as a Second Language; ... 2 of these children have been in Australia for less then 1.5 years". As a beginning teacher, one of the main problems she faced was to cater for the diverse needs. She wrote: "The ability levels within the children are diverse, therefore I need to acquire good management routines and techniques to cater for more children within the classroom. [For example], one child in particular is unable to communicate effectively and I find it very difficult to help her when I have so many other children within the classroom who are also in need". Further it was difficult for the teacher to "create an interesting curriculum for all ... children [who] have different interests, backgrounds and educational history". Finally the lack of adequate school facilities such as playground, power, place for books and bags, and classroom space, create a difficult work environment for classroom management. Within this context, student behaviour was a very serious problem identified by Janette. Lack of parental support and the existence of "family feuds" between some students compounded the problems within the school.

Learnings about Inclusive Mathematics

There is varied evidence that throughout the first year of teaching, and through their involvement in the project, the teachers have been able to develop significant learnings towards inclusive mathematics. Lisa concluded that "making mathematics more inclusive in not an easy task". She realised that learning about and from the student background of the students is a "first and vital step". She related how toward the end of the year she was able to "get acceptance both from the community and the children". She also demonstrated an ability to be critical of curriculum material that are available within the school. In trialing one curriculum series developed specially for Aboriginal students she reflected how inappropriate it was because it "was so basic. It underestimated how much previous knowledge these children have." She concluded:

> The process of making mathematics more relevant/inclusive has helped me to identify and acknowledge that culture is a key indicator in different processes of learning and understanding. As teachers we fall into the trap of placing our own cultural and social expectation on other cultures that have a unique worldview different from our own. The most important aspect of teaching Aboriginal children mathematics is that they need a lot of modelling and hands on experiences with concrete and real world experience that relate to them. I tend to have more enthusiasm and positive responses from the students when introducing a new concept. Students are able to experience success.

Likewise, Gabrielle has reflected on her learning from the first year of teaching and from her involvement in the project in ways that show she has gained in her understanding of inclusive mathematics. She wrote that "the project ensured I was thinking critically about my methods of teaching when planning specific lessons or units of work", adding "I gradually learned to use the outside world more in my teaching" and overcoming the structural barriers on the school curriculum. One particular area she points out as particularly benefiting was in the area of assessment where she was able to make it more integrated with learning. However, realising the possibility that more may be needed she raises the question "Is this an appropriate way of assessing? Perhaps [this is] a start!"

Finally, Janette has been able to identify the social context of first year teaching as needing to change to achieve the inclusive mathematics. She wrote "None of my previous practical teaching experience gave me the appropriate skills to deal with a high concentration of multi-cultural, non-English speaking students. I believe that some additional training and/or support should have been afforded during my first year of teaching, and also on an ongoing basis." In discussing the preparation of teachers to work in such contexts she suggested that the university preservice courses had not adequately prepared her for being a first year teacher.

Reflections

All three teachers were quite aware of the great gap between their cultural background and that of the particular school context that they found themselves in during their first year of teaching. The Aboriginal background of the students at Lisa's school, the multicultural background of the school at Gabrielle's school, and the non-English speaking background at Janette's school have presented great challenge to the three teachers to make mathematics more meaningful to the students and for finding a pedagogy that is culturally appropriate. The discussions at the teleconferences often expressed these concerns. Has the project assisted them in dealing with these concerns?

At the conclusion of the first year of the project, Lisa talked about gradually becoming accepted by the school's parent community. This was discussed during the project meetings. Rightly, she concluded that the process to make mathematics more inclusive is "not an easy task". Arguably, it is a much more difficult task for a beginning teacher who herself is being enculturated into the dominant school culture and often lacks confidence and experience. Further, in her reflection on teaching and learning of mathematics she re-discovered lessons that she has learnt from university lectures of how to teach mathematics through emphasis on mathematical language and using techniques such as big book and learning cards. These seemed to have helped her teaching; however, their cultural relevance is not directly clear.

Similarly, Gabrielle was self critical of her own approach to the teaching of mathematics that is based on her own experience in being taught mathematics and of the practices of streaming that her school used that were counter to her beliefs about inclusive mathematics. As a new teacher, she showed signs of overcoming these limitations. Yet, at the end of the year, she concludes that overall her practices were still not catering for the students' specific needs.

While Janette did not address the meaning of inclusive mathematics in her reflection, her overall discussion of the failure of the pre-service program in preparing her for such a job seemed to imply that she too had misgivings about her achievement in that area.

Does this mean the aims of the teachers at the beginning of the year with regard to learning about making mathematics more inclusive have failed? We do not think so. It is clear that the three teacher had shown a great ability in becoming reflective about the difficulties in their classrooms - and one of the main reasons being the cultural background of the student, and more importantly the difference between the culture of the teacher and that of the school. Identifying the problem is the first (and important) step toward its solutions. Arguably, as first year teachers having to develop the many survival skills in the new culture of the school, perhaps the skills needed to make mathematics more inclusive have to take a second priority. However, as we argue below, this project has assisted the beginning teachers to develop concerns and learnings beyond the mere “survival stage” discussed above.

Learning about transition

In becoming involved with this project we were interested in learning about the problems that teachers face in the transition from the university to the workplace. The experiences of these three teachers, although not typical of all beginning teachers, are not unique. The support that teachers are supposed to have is not always available in schools. Smaller schools, more isolated schools and less affluent schools often do not have programs in place to induct the beginning teacher into the profession. All three teachers indicated that the most valuable thing about the project was the chance to discuss their concerns with others in somewhat similar situations. This made the sense of isolation felt by the teachers a little less acute. Veenman (1984) argued that beginning teachers need psychological as well as pedagogical support. All three teachers identified the gain in confidence as a major outcome of their involvement in this project.

As discussed above, the first year of teaching has often been described as a survival year with major concern of the teacher is about the self. It is true that during many of the initial deliberations in this project the teachers were expressing their needs for teaching strategies and resources to use with students constructed as “weak” in mathematical knowledge and motivation. It also could be argued that elements of their final reflections also reflect the deficit model in describing the type of students that communities that they have. Yet, within these reflections are also elements of becoming critical in raising general questions about their practices and context.

Learning about action research as professional development

Traditionally, there has been somewhat of a demarcation between the responsibilities of the university and that of the employer in the professional development of teachers. Universities are often seen as responsible for the initial training that ceased at graduation. Induction programs are often seen as the responsibility of the employer. The funding arrangements for universities and the school sectors are consistent with this division of responsibilities. However, this does not mean that the two stages of professional development need necessarily be separated. This project is an example where funding from both sectors (the University internal research grants and the Queensland Broad of Teacher Registration) has allowed people from the university to work with school teachers in this important stage. However, the point that we want to stress here is that the involvement of the university in the provision of support during this transition period is useful in connecting what has been learnt during pre-service training with what is happening in the school. This leads to lessening the divide between the "ivory tower" and the "real world" and increasing the nexus between theory and practice.

In many ways the experience in this project has been uncommon for action research projects. I have not been involved in action research projects that consisted of people isolated by huge geographical distance. One of the basic components of participatory action research is the collaboration and negotiation among participants to develop a shared understanding of and change the practice under consideration. This has been difficult to achieve in the project. The project used teleconferencing as a means of direct communication with the participants. Due to the high cost of this medium, only few and short meetings were possible. Further, the limitations of email communication included lack of availability of personal email for the teachers, unreliable telephone lines for communicating with some isolated cites and lack of experience with email culture on the part of the teachers. However, this geographic isolation is the precise reason why this type of activity is important. Perhaps the combination of mentoring and action research into a community of learnings is useful.

Participatory action research aims at empowering participants. Has this project achieved this aim? Undoubtedly, the three teachers have found that their involvement in the project of some use to them. It allowed them to reflect on their practice and gain confidence in meeting the demands of teaching mathematics in a multicultural context. It also helped them feel less isolated in their practice. Their experiences and learning from the project have varied as portrayed in their statements above. We believe that their involvement in the project has been an enriching experience for the teachers. However, we would hesitate to use the term empowerment to describe the outcome for the teachers. Perhaps this is an aim that requires time. As one of the teachers has indicated that "next year if this project continues we may learn more from it."

Our involvement in the project has highlighted to me once again, the problems arising from the increasing demands that the university and the workplace are placing on the lives of teachers and academics. For all participants, the involvement in this project was in addition to an already busy schedule with many competing responsibilities. Often it was difficult to arrange meetings, communication was slow and feedback delayed. Little time was available for reflection on practice. Arguably for academics, as well as teachers, empowerment would include the ability to take control of one's time and setting one's own priorities.

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Mathematics as social practice: implications for mathematics in primary teacher education.

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Abstract
"Public" concerns about standards of attainment in mathematics in UK primary schools have led to the development of a National Curriculum for ITT. Such a curriculum is founded on the belief that teachers' subject knowledge is an essential ingredient for successful teaching. Simplistic responses to teachers perceived lack of mathematical knowledge include provision of more inputs of the same kind. This article questions whether the adoption of a social practice model with its explicit acceptance of maths education as a socio-cultural, ideologically constructed process and acknowledging the complexities of learning, teaching and schooling is an attractive alternative worthy of further exploration with potentially significant pedagogical implications both for student teachers and their future classroom practices.

Introduction

Teaching and teacher education is more inspected, measured, analysed and publicly judged than ever before. The frightening centralisation of control instigated over the past 20 years shows no sign of abating. Structures which expose and weed out teaching failure and attempts to provide remediation programmes for what is perceived as national educational under achievement have become powerfully dominant.

At the same time there are pockets of educational research both within the UK and in other countries with similar socio-economic contexts where attempts are being made to understand the complexity of the educational endeavour rather than to respond with simplistic rhetoric. Two such are in literacy and numeracy education. Mathematics holds a powerfully privileged position in the order of higher status knowledge, a position reflected in the UK's National Curriculum for schools and now also in new curricula for initial teacher training (sic). It is also the most incontestable and autonomous discipline of the school curriculum. "Mathematics and science are the two areas of the curriculum where the effects of the educational system outweigh the effects of home background" (Reynolds 1996, p 2).

It is in this climate and context that an opportunity arose at an HE teacher education institution to apply a socio-cultural model of mathematics to modules addressing concerns about subject knowledge. Such a model could have significant pedagogical implications both for student teachers and their future classroom practices. This paper seeks insights into the current situation in mathematics education through an implementation of a social practice approach. It explores student responses to challenges to their models of knowledge and beliefs about teaching and learning. It addresses issues that arose for me as a tutor implementing a social practices model in a hegemonically dominant discipline. Interpretation of the emerging data and the implications for both teacher education and pedagogical practices are discussed in the light of an analytical framework developed during the research, (cf. acknowledgements).

Background

> " Maths crisis diagnosed. One in three English children leaves school unable to do simple sums, a failure which drives them into an underclass of young people unable to get jobs."
>
> (The Times Educational Supplement July 18, 1997)

This claim is based on data in TIMSS (Keys et al, 1996 and 1997). It seems that children in England tend to score poorly on international numeracy tests both in comparison to similar countries and in comparison to earlier surveys. Reynolds et al (1996) in their survey of educational achievement in England state that:

> " Performance in maths in England is relatively poor overall but has considerable weakness in arithmetic ... There is a greater proportion of low achieving children in England "
>
> (Reynolds, 1996, p. 52)

It is interesting to note that a major international report (Beaton *et al,* 1996) claimed that 13 year olds in England are particularly weak in the areas of "fractions and proportionality". Although there is debate about the validity of the data and of the conclusions drawn, the frequency of appearance of such statements over the last few years reflects a substantial and growing concern about achievements in numeracy that warrants careful consideration and response, (cf. Brown, 1997). However, in both Beaton's and in Reynolds's work low achievement is seen only in relation to specific areas of mathematical content without any reference to, or siting in, context, culture or ideology. This fits with my analysis that current practices are viewed and researched from an implicitly neutral socio-cultural position , which links readily to the theoretical basis of this paper and hence to its framework of analysis. It is also worth noting concerns about primary teachers' subject knowledge in mathematics which is seen as a possible contributor to low achievements in numeracy, (OFSTED, 1994, 1996). This is clearly a complex issue (Askew *et al*, 1997; Jeffery et al, 1995). The former claims that it is the interconnectedness between, and the beliefs about, mathematics, which are important rather than the quantity or security of that knowledge.

I was concerned, therefore, to question dominant approaches to the subject knowledge issue in mathematics which sees the provision of more content courses for student teachers as <u>the</u> solution. Instead I wanted to consider more complex social practice analyses. Here students would no longer be considered "in deficit", pathologised by earlier failures but, in confronting mathematics in new ways, would be enabled and encouraged to reconstruct their own pathways into understanding. Through interrogation of the dominant canon of knowledge in mathematics they would understand complexities of curriculum, epistemology, pedagogy and ideology which would increase the chances of deepening their own understanding and confidence with benefits to their classroom practices.

Theoretical framework

I need first to explain the theoretical framework which provide a different perspective from which to view and understand education practices. Recent work in literacy theory (Baker & Street, 1994; Street, 1995) has developed a cultural model which conceives of literacy, and subsequently numeracy, as social practices. Traditionally, 'numeracy' concerned a technical capability in understanding and manipulating numbers. These concepts were not seen as social practices but rather as mechanistic skills to be acquired and in which one's competence could be objectively measured. This conceptualisation is described by Baker & Street (1994) as 'autonomous' and is characterised in terms of simplicity, singularity and without explicit ideology. In this autonomous model, mathematics would be perceived as a unified, determined and legitimated body of knowledge, a set of conventions and procedures, abstract in nature, value free and universal - a dominant view of the subject. For example, Singh (1997), says "that mathematics relies solely on absolute, undeniable, logical proof, and therefore remains true forever". In a social practice model of literacies and numeracies, however, it is seen as highly complex regions of human activity within the social arena. It is described as 'ideological'. Knowledge is conceived as socially constructed; the model acknowledges that the contexts, values and beliefs and the power relationships in which knowledge is sited affect both ways of making meaning and ways of knowing. The autonomous model does not acknowledge the ideological nature of knowledge whereas the ideological model exposes the ideological, cultural, pluralistic and contextual nature of that knowledge. I do not view these as dualisms but as representing different ways of making meaning and knowing. There is considerable evidence that the autonomous model is currently dominant in formal education, and not just in the UK (Baker, 1996; OFSTED, 1994; Keys et al, 1995). Challenging this has significant implications for both models of 'knowledge' as well as for teaching, learning and classroom practices.

This paper challenges the dominant 'autonomous' model of knowledge. I am drawing on the ideological model, the theoretical framework of mathematics as social practices, which provides space to explore the socio-cultural embeddedness of mathematics. I am drawing also from the notions of 'critical literacy' (Street, 1995), and 'critical numeracy' (Johnston, 1996). Thus, knowledge within the ideological model is considered to be created within the historically possible, to be culturally and ideologically sited. It is constructed by, through and for, interested and exclusive groups in society. Such an approach will construct knowledge with an element of critique of the hegemonic canon. It will reveal and confront underpinning ideologies; it will expose relations of power within the domain; it will ask questions about the legitimation of the selected body of knowledge, its conventions and procedures; it will seek exposure of the partial nature of the legitimated way of knowing, its limitations, implications and consequences.

At the same time, my experiences with, and considered observations of student teachers, led me to design a two-dimensional analytical framework. This described the student teacher from 'compliant', through 'reflective' to, 'interrogative' and evolved from a framework previously designed by Baker (1994) along with the currently dominant model of 'reflective practitioner' originating in the work of Schön (1987) and developed further by Miller (1996). One critique of Schön foregrounded his lack of any acknowledgement of the socially constructed nature of knowledge itself (Smyth, 1991) and thereby inhibiting any sense of the dialogic nature of reflection. In other words, and most importantly for my thinking, the "reflective practitioner" is still an autonomous rather than ideological thinker. What was needed was a progression beyond the idea of reflection into the critical, or what has been termed the 'interrogative', (Miller, 1996). I have done this in response to increasingly centralised systems of education where compliance is demanded by educational authorities as an essential characteristic of the educational professional. In my view, this compliance is retained in the concept of 'reflective practitioner' whose drive is towards finding optimal teaching approaches and strategies within the given educational structures, systems and curricula. It is "pedagogical polishing" (Baker, 1996). Interrogation, on the other hand, opens spaces to challenge the models of knowledge framing school curricula, underpinning values and beliefs, relations of power and pedagogical practices.

The project

The project involved designing and implementing an element in curriculum mathematics module in a teacher education programme from a social practices approach. This approach survived considerable opposition from colleagues in an essentially autonomous module where workshops were provided to help students "fill gaps" in their knowledge. Thus the rationale for the module was that students, "extend their own knowledge and understanding of mathematics in order to teach it more effectively". (University of Brighton, 1996). The beliefs underlying the module, supported by TTA frameworks (TTA, 1997) and OFSTED standards, were that increased subject knowledge was necessary for more effective teaching. In contrast, other workshops based on a social practices approach were set up. In these, rather than getting students to identify "gaps" in their knowledge and then to cover these "deficits", students were asked to work in a particular way on their mathematical knowledge, both on their strengths and their concerns. Working on and re-framing areas of strength gave them a positive starting point and an opportunity to reconstruct their existing knowledge whilst encouraging them in making connections between different aspects of their knowledge. The approach involved four phases, describing, informing, confronting and reconstructing, derived from Smyth (1991). Firstly, they had to describe their position in an area of mathematics in terms of both content and context; secondly, they informed themselves about the reasons for that position, uncovering hidden beliefs and values behind it; thirdly, in confronting the area, seeking to make power relationships explicit, they discussed why knowledge about the area was important, was valued and had status, and what their relationship was to that knowledge; finally, they worked on reconstructing their pathway into knowledge through their own research, working with others or seeking activities or help from a tutor. This I have identified as a social practices approach because it makes underpinning power relations, content, context, values and beliefs explicit.

The analytical instrument

The investigation sought three outcomes: insights on current concerns in mathematics; students' reactions to a social practices approach and my own observations on implementing such a model. An ethnographic style of methodology was chosen with a case study approach (Hammersley & Atkinson, 1993; Hitchcock & Hughes, 1989). I selected a small group of students as source of data, which was collected through individual student diaries, group interviews and individual interviews at the start and again towards the end of the semester. The selection was made to provide the most telling cases from their acknowledged weakness or strength in classroom **practice.** This characteristic was expected to expose the greatest range of difference in terms of compliance, reflection or interrogation.

Drawing on the theoretical framework, an analytical tool was developed to represent student teachers as a two-dimensional matrix. The first axis represents a model of student teacher as practitioner using the concepts of compliant, reflective and interrogative. The second axis represents dimensions of the student teacher's practices. These dimensions were derived from data obtained in this study and refinements from previous theoretical analyses of social practice models (Baker, 1996; Miller, 1996). They are subject knowledge, beliefs and values, power relations and pedagogical practices. They should not be viewed as discrete but rather as representing elements of student teachers' epistemological, cultural and ideological positions and pedagogical practices. The cells contain descriptors of these dimensions in relation to the notions of compliance, reflection and interrogation which are thus differentiated. Appendix gives a distillation of the evidence from student interviews and diaries. This is interpreted next in the context of the three elements of the investigation.

Interpretation of data

The first concern was with students' epistemological models, that is to say their perceptions of, attitudes towards and relationship with mathematics knowledge. The evidence (appendix) suggests the 'compliant' student teacher perceives of the knowledge as context and value free, that she 'gets it' and that, " it is having that knowledge to give to other people", a simple sufficiency of understanding from the given canon. The 'reflective' student teacher has considered the relationship of school-sited mathematics and the everyday, " I could do capacity and volume in school. But I suppose you don't use them". She is aware of the differences in the two practices but also of a real boundary between them. She is aware of different understandings, different ways of knowing. Identifying behaviour that is seen as the 'interrogative' is harder to find. Elements are demonstrated by the student who observed that basics relate to other practices, "to help you survive the demands of society", thus showing an initial awareness of the social construction of mathematics knowledge. Data provided us with telling evidence of underlying beliefs and values. The compliant student teacher's interest in the subject extends only to her teaching needs, hence, " I don't want to develop my understanding of it in any great detail. Just enough so that I can actually teach it". Evidence of reflection on practice appears in the comment, "some will get it before others, and some will be able to do it", implying a child-centred approach. The student moving towards the interrogative position begins to question the role of the social in education processes ".. you've got their social and cultural background. The home that they come from", implying a belief in teachers as mediator between child and knowledge as well as the social context of that knowledge and of schooling. Awareness and explicit acknowledgement of power relations are a crucial indicator in a social practice model and evidence was apparent of a clear range here. Compliance is expressed as, " ... they say I've got to teach it, so I will"; the reflective student is aware of the gatekeeping role played by maths, "... if you haven't got English and maths you aren't going to do anything"; in moving towards the interrogative a student expresses disquiet about the dominating role of SATS, the consequent importance of memory in learning mathematics: "you've got to revise this, remember that. I didn't like it at all."

The evident range of models, understandings and beliefs appear to result in different pedagogical practices. Compliance is demonstrated in the student who accepts transmission from the teacher, "gives the knowledge to others", and for whom, " everyone should have a good grounding in the basics". The reflective student rates helping the individual as important, "you can try and help children of lower ability". The interrogative is again not so evident but some acceptance of multiple ways and of valorising children's work appears in " ... why should we dictate your adding in your head, your way of doing it?"

This two-dimensional analytical tool has proved useful in enabling the three models of student teacher to be differentiated. Evidence from the group of students suggested that they tended to be either compliant or reflective. There was little evidence of the interrogative in the sense of genuine challenges to or questioning of accepted practices. As the group presented the widest range of classroom skills, it indicated that student teachers tend to be compliant and reflective rather than interrogative, certainly in their *epistemological* models. The implications of this will be discussed in terms of the research foci, insights into current concerns, student responses and observations on the application of a social practice model.

Implications

In terms of gaining insights into current concerns, the implications of the study of student teachers are that their understandings and practices maintain autonomous approaches to both subject knowledge and pedagogical practices - they accept what they are told to do and how they are to do it - or that they seek best ways of teaching given content. Pedagogical practices, from transmission of knowledge to mediated exploration dominate their classroom approaches. This means that they take little account of the sitedness of children's knowledge or of children's practices and when difficulties arise they continue to pathologise the children or to see themselves, their knowledge and attitudes as the problem. Students as reflective teachers continue to try to mediate between the children and curriculum. In many cases these attempts are directed towards motivation of the children through a veneer of the everyday and not genuinely situated teaching and learning. Further, problems some children have in crossing boundaries between different practices, (*cf.* Baker 1996), are not acknowledged nor genuine attempts made to ameliorate the problems whilst neither curriculum nor pedagogy are interrogated. The reflective practitioners are therefore providing at best a marginal improvement in children's access to mathematics.

My conclusion is that while this autonomous model persists so will the status quo of the failing state of mathematics education. Adoption of a social practice model with its explicit acceptance of education as a socio-cultural, ideologically constructed process with critical epistemology and pedagogies interrogating the status quo and acknowledging the complexities of learning, teaching and schooling is an attractive alternative worthy of further exploration.

The views expressed by the small research group on subject knowledge and their revealed beliefs, values, perceptions of power relations and their classroom practices were significant in one or two important aspects. A perception of a personal "deficit" in mathematics subject knowledge in many cases resulted from continual perceived failure over years of formal schooling. Images of the subject as hard, as abstract, as gendered and with negative relevance to them persisted in terms of their articulation of the formal subjects themselves. Although they expressed needs relating to both subject knowledge and pedagogical knowledge, when challenged about their formal mathematics knowledge on, say, fractions, they accepted that they had the subject knowledge and what they really need was knowledge of a variety of ways of teaching it, i.e. pedagogical knowledge. Responses to mathematics seemed mainly compliant. I contend that this is in part due to the abstracted, de-contextualised nature of dominant formal mathematics practices. The students saw mathematics as important but often hard and irrelevant. One said:

> "I think maths is important because I suppose it's socially accepted for whatever reasons by employers for whatever you're doing. ....[but] ..... I've never used it since school. ... Maths is hard.... I also think it is perhaps quite more abstract than - it's a lot of the things you can't physically see or grasp".

Compliant or reflective student teachers will retain this image of mathematics. It is only the interrogative who will have the drive and means to confront their position and then to reconstruct both a more positive image and a more secure and confident approach to mathematics in schools.

Finally, what lessons have been learned about introducing a social practice model? The severe resistance to these ideas from outside the School of Education is evident in official documents and statements. What was surprising was the extent to which these attitudes and resistance were evident not only within the School but also had been internalised by individual academic colleagues. This bodes ill for the development of the reflective practitioner let alone the interrogative learner in the present educational and political climate. Yet the development of social practice approaches to teacher education modules in mathematics like the ones briefly described here might make for more intellectually assertive and confidently interrogative teachers. Developing beyond the reflective practitioner, student teachers would be able to unpack underpinning ideologies, become teacher-as-researcher in a critical manner to move into new ways of knowing, valorising learners and so on. It will provide them with a depth of understanding and a fuller range of strategies to deal with the demands on their subject and pedagogical knowledge that will be placed on them as they develop from NQT to "expert teacher". Resistance to these approaches will result in continuing concerns about mathematics education, continuing difficulties in recruitment to mathematics teacher education courses and prevent complex, radical attempts to address concerns in mathematics education. Instead there will be a reliance on the quick and straight forward responses to the concerns that will have no more than a marginal effect.

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School and surplus-value: contribution from a Third-World country

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Abstract

The paper accepts Vinner's [1997] conception of school as a credit system and addresses two questions inspired by Chevallard and Feldmann [1986]: why do teacher and students' interests diverge necessitating negotiation of the knowledge to be taught and the legitimacy of exam questions? Why do these interests converge and both parties pretend that exams are a measure of acquired knowledge and that negotiation does not exist? The answers evoke educational and political circumstances of a Third World country and rely on the conception of school as a place of production of qualified labour force. A pass/fail criterion – to each one according to his/her work – is hinted at.

Introduction

Marshall and Thompson [1994] surveyed six recent books on assessment [Niss, 1993A, 1993B; Romberg, 1992; Lesh & Lamon, 1992; Leder, 1992; Gifford & O'Connor, 1992]. In the 1994 pages surveyed, I could not find studies about the implications of assessment for social promotion and selection. The authors seem to believe in the existence of a real object to be measured. They are mostly concerned with the search for *satisfactory, valid and reliable* methods of evaluation. My conjecture that that they generally believe that social selection is a natural consequence of the various evaluation processes incorporated in society. They seem to feed the hope that trustful evaluation procedures in mathematics could contribute to the edification of a just society: to each according to his/her merit. In fact, an ideology of justice and an implicit validation of instructional objectives is observable at the basis of most research about evaluation.

An omission is also notable in 1673 answers obtained by Rico et al [1995] from a questionnaire addressed to 59 teachers: none referred to the implications of assessment to social promotion/selection. After declaring that "*one of the axes of research in didactics consists in extracting the constraints which influence the didactic* system" Laborde [1989] lists six of the "*most important*" constraints: the characteristic of the knowledge to be taught, the social and cultural constraints that determine the teaching content, the linearity of syllabi, the pupils' concepts, the teacher-learner asymmetry, and finally the teacher's knowledge. Pass/fail criteria is not cited. It is beyond the sixth magnitude, invisible to the naked eye. This

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symptomatic blindness was finally broken by Shlomo Vinner in his plenary address in the PME meeting of Finland. He brought the theme sharply to light: *"the educational system is, above all, a credit system*" [Vinner, 1997: 68]. Four years before, William Thurston had also simillarly characterized mathematical scientific production itself: "*More than the knowledge, people want* peronal understanding. *And in our credit-driven system, they also want and need* theorem-credits" [Thurston, 1994: 174].

The importance of the credit system for mathematical education had been hinted at in a previous monograph by Chevallard e Feldmann (1986). These authors propose

> "*(...) a new look at didactical facts of evaluation. The marks assigned by graders are not a measure but a message that intervenes in a negotiation; this is a transaction that validates a power relation between teacher and students about the knowledge to be taught*" [Chevallard and Feldmann, 1986, preface, my translation].

According to the authors, the didactical contract is negotiated in a climate of opposition between teacher and students.

> "*Students try, if not constantly, at least in a systematic and regular way (...) to diminish*[2] *the teacher's requests about the kinds of competencies to be acquired about the elements in question – if we cannot avoid such a concept, at least let us avoid certain of its uses*" [Chevallard & Feldmann, 1986:105, my translation].

However, in the language in which the negotiation occurs, in spite of being in opposition about what is legitimate to demand from students, the interests of teachers and students converge on another point: both collaborate in making believe that the negotiation does not exist.

> "*The discourse (about the negotiation) proposes (to the outsiders as well as to the participants) an image that can be agreed upon; that is, an image that makes the modalities of negotiation socially acceptable. It is a rationalisation discourse whose function (if not the intention) is to defend and enhance the rationality of the enterprise (...). In precise terms, what such a discourse says (or at least what it indicates) is that, here, negotiation does not exist (...)*" [Chevallard & Feldmann, 1986: 70, my translation].

The questions that I intend to address in this paper are the following: What values are at stake in the negotiation? Why do teacher and students' interests diverge, necessitating negotiation? Why do they converge and both parties pretend that

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[2] *Péser à la baisse*, literally *weigh down.*

negotiation does not exist? For the first and third questions, the particularities of negotiation in Third World countries may shed some light on phenomena barely observable but surely present in First World countries too. The concept of surplus value will help me to answer the second question.

Subsidiary promotional criteria.

Rationalisation failures in the speech about evaluation are more easily observable in classrooms of third world countries since here, school practices depart more widely from their face values. What can be said about negotiation if there are students already majoring in mathematics, who need five two-hours sessions of individual tutoring with plastic cubes in order to account for the relation of $\binom{5}{2}$ with 5-factorial? How does this student participate in a negotiation about knowledge? What about another one who needs one hour of assistance in order to enlace two sets of logic blocs to form a Venn's diagram such as the "reds" and the "squares"? What about the one who faces a chessboard supposed to be perfect and is not sure whether the prolongation of the diagonal of a particular square will cover the diagonal of a square at the periphery of the chessboard? What about a student who, after minor changes of the expression of an indefinite integral, repeats the same mistake $\frac{1}{a+b}=\frac{1}{a}+\frac{1}{b}$ three times in a period of twenty minutes. Such cases are not exceptional. They are rapidly becoming the general rule. For these students, learning seems an impossible strategy to pass. Insofar as they end up getting credit and certificates, we might ask what do they actually negotiate about?

Teachers also participate in the negotiation. They foresee that learning may be an impossible passing strategy for many. They take care not to assign a number of failing grades beyond convenience. One of them told me: *I make two easy questions for those who know very little and two difficult ones, to detect the good ones*. From another teacher I got: *A certain amount of rote is not harmful. I tell them that I am going to ask one of these twenty integrals.* I open a classroom door, and I see that the students are taking a written final. They are sitting on arm-to-arm chairs. The teacher is reading a newspaper... Some teachers make really hard questions but they supply a considerable amount of help during the exam. Others get the students together the day before the exam for a "last review" and make clear, at least to those who develop a certain ability to understand it, what is going to be asked the following day.

These are well known facts, not mentioned in studies on evaluation. It is hardly seen how all such instances of negotiation, directly connected to promotion, can be analysed from the strict point of view of didactical contract about knowledge. What is really at stake in the negotiation are the *subsidiary promotional criteria*. These criteria validate non-learning strategies to get credit, to the benefit of those students for whom the learning-based strategy is impossible. Subsidiary promotional criteria keep the output of certificates at a level compatible with the investment made in the school system. Knowledge becomes an alibi for educational credit practices. A certain amount of faking is present in different degrees throughout the school system: "*But don't we want to be deceived, especially when it comes to our student's achievements?*" [Vinner, 1997: 73].

School and surplus-value

Why is credit so important to people? Because it leads to certificates and certificates imply higher salaries. Any one who has ever looked for a job knows it. An economical value is at the base of the negotiation occurring in school. Let us consider this.

For political economy, salaries pay for work. School produces work of higher quality: managers, supervisors, executives. The problem of accounting for the presence of different degrees of work quality in the economy was established by Adam Smith: "*Different degrees of effort and ability should be taken into account*" [Smith, 1776, Book I, Ch. V[3]]. However, subsequent political economy has discarded this problem: "*If one day's work of a jeweller is worth more than one day's work of a simple worker, this relation has been adjusted long ago and placed in its right position in the scale of values*" [Ricardo, 1821, Ch I, Sec. II]. Even Marx has refused the problem: "*(...) for the process of creation of surplus value, it does not matter whether the work seized by the capitalist is simple work, average work or a more complex work, of an up-per specific weight*" [Marx, 1890, Ch. V, 2].

Classical political economy, up to Ricardo, considered that the salary paid for *all the work* done by the worker. A difficulty arouse: in steady-state economies, all exchanges occur between merchandises of equal values: linen for iron, iron for gold (money), gold for work. If so, where does the increase of the national product come from? The answer produced by Marx was the following: the salary only pays for that part of the work necessary to reproduce and replace one special commodity used in the production process, namely, the *labour force*. Its owner is the worker and its use has the property of increasing the value of all the others, because the rest of the work done by the worker, beyond the work necessary to reproduce his/her own labour force, remains unpaid. This unpaid work constitutes the surplus-value.

"*(...) the product representing the work that the worker does for himself, what this work brings him, his income, constitutes only the salary; it is the fraction of (created) value that expresses his salary. If salary-paid work and work coincided, the salary would coincide with the (total) product of work (...)*" [Marx, 1890, Ch. XLVIII, 1].

Marx proposed the problem of determining the prices around which the market adjusted the exchange rates of commodities, but he could not solve it. More recently,

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[3] Quotations from Smith, Ricardo, Marx and Sraffa are my translations from Brazilian editions.

Piero Sraffa [Sraffa, 1960] proposed a system of linear equations from which these prices could be determined. Each equation corresponded to the production of one commodity. However, there is no equation expressing the production of the labour force. He does not consider it a commodity like the others that must be produced somewhere. He also explicitly discards the problem posed by a higher quality labour force. "*We assume that the work is uniform in quality; in other words, we suppose that any differences in quality have been previously reduced to equivalent differences in quantity, so that any unit of work gets the same salary*" [Sraffa, 1969, Ch. II, Sec. 9].

It seems that political economy has refused to speak about school. Indeed, historically, universities have been more closely associate with churches than factories. It is perhaps time to look at school as a place of production. I shall retake an Adam Smith idea: "*Salaries vary according to the cost necessary to learn the profession*" [Smith, 1776, Ch. X]. The bulk of my argument will be that, just as simple labour force is produced in the families, a *higher quality labour force* is produced at school. At least two extra equations should be added to Sraffas's system: one for the family and one for the school. However I shall not go into these here. I will simply say that, in the social practices that occur at school, students, teachers and the administrative staff participate in a process of transformation of students' labour force, initially simple and unqualified, into a commodity of higher value, to be sold in the future for a higher salary, expected to pay off the investment of time and effort.

In this process of raising the quality of their labour force, students occupy a double position: while actively engaged in the work of raising quality, they occupy the position of *workers*; while owners of the commodity in process of increasing quality, they occupy the position of *capitalists*. This remark will help us to understand their behaviour.

The student's simple labour force is deposited as a reserve of capital necessary to guarantee the production, just as in any capitalist enterprise: a certain amount of money, real estate or land is registered as the company's capital and prevented from being used for other purposes. Otherwise, one cannot participate in the process of seizing surplus-value. The school system strictly controls the students' presence in class in order to guarantee that, during that time, they are not selling their deposited simple labour force. It is assumed that during this time students are *studying*, that is, working to increase the value of their deposited capital. The ritual marking the stripping of the students' simple labour force is well known: freshmen hazing, shaving heads, etc. The ritual marking the recapture of the now qualified labour force is the solemnity of graduation, dressed up by formal clothes and panache...

Future salaries are the price of the higher quality labour forces. They depend on three values. The *use-value* is the know-how, the sum of all abilities developed by the students during school time, necessary for their future professions. The *exchange* *value* is the total amount of work of students, teachers and staff, incorporated in the higher quality commodity. The *sign-value* [Baudrillard, 1972] is the importance that society assigns to the particular certificate, considering its duration, difficulty, social status, tuition level, etc. All students and their families collaborate in the formation of the sign-value. They build the reputation of the school system. They make their general behaviour (class attendance, boast about the course's importance in social meetings, etc.) signify how much we should praise their efforts.

However, only students who get certificates recapture their labour force. This labour force embodies the work done by all, by those who flunked, by those who abandoned the course, by those who could not buy a higher education and remained at the lower levels of the pyramid. Graduates get higher salaries because their labour force embodies more value, more work done by themselves but, mainly, by others who were left behind. Hereby we can find an answer to Althusser's question: why the school apparatus has become the dominant ideological state apparatus [Althusser, 1976]? It is because at school the student learns, above all, to participate in and accept the conditions of production and seizure of surplus value, the work done by one's fellow men.

Insofar as students participate in the process as workers, their goal is to use their student's energy, their active labour force, the little as possible, with the least possible effort. Insofar as they participate as capitalists, owners of the reserved labour force, their goal is to increase its value to the maximum.

However, students will only get their increased capital back if they reach the certificate. Hence, it is necessary to pass, but, with the minimum effort, if possible, without having to adopt the strategy called learning. In order to perpetuate the process of production/seizure of surplus-value, a certain amount of failure is necessary. The adequate levels of production and the average guarantees of exchange have to be determined just as in any sales process. Hence, it is necessary to negotiate.

The cynical[4] consciousness

Why is it necessary to pretend that negotiation does not exist? Here again, the examination of situations clearly visible in Third World countries, but surely present to some degree in central ones as well, can help us to find an answer. Recent Brazilian political events provide some helpful clues. A congressman explains the origin of his fortune: *"God helped me win the lottery four hundred times*". Society takes this with only a smile. Another congressman is videotaped confessing that he has faked the signature of a mayor and a payment order of a governor: "*I did not do that as a Congressman*", he explains. "*There are no proofs.*" His peers seem prepared to accept the logical contradiction. Three upper middle class boys throw one quarter of gallon of alcohol on a sleeping person at a bus-station and set fire. "*We thought it*

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4 From French *cynique*: feelings and opinions contrary to accepted moral.

was a beggar; we did not know it was an Indian. We had no intention to kill". The blind justice accepts it. They will not face a jury. Another videotape shows a policeman shooting at a car after having beaten and extorted its five passengers. One person in the rear seat is hit and dies. *I did not kill him*, he protests. *My gun was loaded with fake bullets*.

In vain, we hope that somebody will admit a crime and confess. We get the impression that these people are attached to some kind of seriousness that they never abandon.. We cannot repeat with Christ: "*Forgive them because they do not know what they do*". Nor can we repeat with Marx: "*They will do it best insofar as they do not know what they do*". Apparently people are well-informed that they have been discovered; nevertheless, they continue to sustain their innocence. Apparently this is a new form of ideology, resistant to unmasking and immune to assailing. It can be properly called the *cynical consciousness*[5].

"*Should we say that with the cynical consciousness we surpass the ideological field and enter the post-ideological universe where an ideological system reduces to a simple means of manipulation that is not believed even by its inventors and preachers?"* [ZIZEK, 1990: 75]

Would there be a kind of cynical conspiracy immune to the classical methods of struggle, revelation and assailing? The answer is no. We simply have to take into account the listener as well as the speaker. Pay attention to the demanding ears, not only to the speaking months.

In order to hear the criminal a façade of seriousness is put up; people in white-collars, mahogany tables, direct TV cameras at every corner. The criminal is not scolded or mistreated. He deserves respect because, in spite of the film that shows him beating, extorting and killing, we "do not know" yet if he is guilty. He has not been judged! The judge asked this criminal with a voice as tender as the voice of a mother: "*So you declare that you did not do anything violent?*" The tone of the question unbalances the criminal: "*At least not in the way they accuse me*". The scene was shown on Brazilian TV in April 11, 1997.

The problem is not to know whether the people who participate in these rituals believe in what is being said there. What has to be noted is that the State, Justice and Ideology, in one word, what Lacan calls the big-Other, believe so. Lawyers do not defend criminals; they defend a thesis before a society. Society believes in what it wants to hear, in spite of nobody personally believing in what they hear. Official truth breaks apart from personal truth.

How can this contradiction persevere? Why does society need an official version of facts covering up what everybody knows? Because the basis, the ground of this

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5 Expression of Peter Sloterdijk, according to ZIZEK, 1992: 74.

social formation is an imposture whose revelation threatens to throw the whole society into an abyss. It is the basic imposture of equality in work contracts, the assumption that employers and employees freely meet together in the market where they exchange commodities of equal values: salary for work. This is a lie. Not only is the salary worth less than the work, but also the worker sells his/her labour force because s/he has nothing else to sell. S/he sells it to remain alive, this when s/he can find a job. Third World countries make this picture very clear. Remember that 15% of the Brazilian labour force is unemployed in March, 1998. The price of labour force in the "market" depends on all tricks set by globalisation of capital controlled by bankers and managers of multinational companies. In order to be brave enough to go on his/her daily search for a job, the workers has to feed the fantasy of equality and believe that the exchange will be between equals.

Unfortunately, it is the same kind of seriousness that is present among teachers and students when they negotiate the didactical contract. They know very well that the learning-based strategy is impossible for most students and that the success of some depends on the failure of many. Therefore, they tend to make believe that negotiation does not exist and that grades reflect the measure of acquired knowledge.

> "*At this point the distinction between "symptom" and "fantasy" made by J. Alan Miller shows all its weight (...) the "cynical" person, who "does not believe it", who knows very well the uselessness of ideological propositions, does not know the fantasy that structures social reality itself (...) What individuals do not know, what they do not realise, is the fetishist illusion that guides their own effective activity*" [Zizek, 1992, 75].

A possible way out: solidarity assimilation groups

If we accept that this analysis is well-founded, how can we continue to act in our classrooms? If we cannot answer this question we risk being immobilised by the analysis. Since the world is structured in this way, there is nothing that we can do. How can we solve this problem? In fact, I have developed the solution simultaneously with the analysis. What I did was to explore the weak point of cynical consciousness:

> "*The cynical one lives out of the distance between the announced principles and the general practice − all his/her wisdom consists in legitimising such a distance. The most unbearable thing for the cynical position is to watch an open, proclaimed transgression of the law, that is, the enhancing of transgression to the condition of an ethical principle*" [Zizek, 1992: 75].

The solution takes the form a didactical and pedagogical proposition called Solidarity Assimilation Groups (SAG) [Baldino, 1997]. It consists in the introduction of an explicit subsidiary promotional criterion based on assessment of group work quality, measured by duration of work. This criterion is put on the negotiation table of the didactical contract – this table around which everybody is trying to make believe that negotiation is not there, or that only requirements about mathematical competencies are at stake. Cynical consciousness knows very well that subsidiary promotional criteria have always been present in the list of transgressions, as the hidden counterpart of the nice façade principles. Cynical consciousness classifies these subsidiary criteria together with the raw material for future public scandals. All its wisdom consists in hiding such material where, for a long time, desire has learned to look for its objects and has always found the nice face of a possible non-learning strategy to capture the other's work and get credit. All the wisdom of cynical consciousness consists in legitimising the choice of such strategies.

Suddenly, on the negotiation table of didactical contract, emerges the proposition according to which someone who has worked hard should get credit even though s/he has not learned the so called necessary minimum: *credit for work, not for mathematical ability*! The cynical consciousness panics, because this looks very much like what it has always done. It feels like a vampire in sunlight. The danger is double. The proposition threatens the functioning of the school apparatus but, furthermore, it threatens to shed light on the founding contradiction of the capitalist mode of production; namely, the capture of plus-value. We refer the reader to Baldino [1997] for more information about SAG.

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Analysing power relationships in collaborative groups in mathematics

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This paper describes the development of a framework for analysing power relations in small groups of students working on collaborative activities, and is based on an approach to power derived from the work of Michel Foucault. Student-student interactions in two classrooms were observed and videotaped. Key features that emerged were techniques used to control the flow of the discourse in the group and behaviours which influenced the mathematical knowledge constructed. Other factors included gesture and the use of resources.

Introduction

In recent years, collaborative learning has been widely recommended as a strategy to enhance mathematics learning for all students (e.g., NCTM, 1989) and especially girls (Cordeau, 1995; Jacobs, 1994; Solar, 1995). As part of a study of collaborative learning, I am developing a framework for the analysis of power relationships among students working on mathematical tasks, in small groups, with shared goals. This paper reports work-in-progress on this project.

My study has as its main focus students' experiences of collaborative learning and the ways in which gender impacts on, and is affected by, these experiences. In particular, I am investigating patterns of interaction among students working collaboratively. In addition I aim to discover how they perceive themselves as learners of mathematics. I chose to focus on senior students around the stage when they make key course choices affecting their post-school options, and their future relationship to mathematics. These decisions may be mediated by the students' evolving constructions of themselves as learners of mathematics.

Feminist theory and my own experience both suggest that a study exploring gender effects in collaborative learning needs to take account of the exercise of power within groups, and the potential of this to influence learning outcomes. A framework for analysing power relationships could also be useful in studies involving class and/or race where power and status differences may be salient.

Power and knowledge

What is power and how can it be investigated?

My starting point is an understanding of the nature of power, and an approach to analysing power relations, proposed by Michel Foucault. The questions most often asked about power deal with its nature and sources, (the "What?" and the "Why?" of power). Foucault, on the other hand, chose to ask about the "How?" of power. The shift in focus, from theorising about the sources of power to asking how it is exercised, opens up the possibility of empirical investigation.

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Questions of the nature and sources of power are not ignored. On the contrary, empirical evidence of the exercise of power may provide insights into these questions and so help us understand how power functions in different contexts.

Foucault claimed that power in modern society is not a commodity, which some possess and others do not. Rather, it is a structure of relationships, jointly constructed, which shapes people's actions. "Power exists only when put into action." (Foucault, 1982 p. 219). Furthermore, the effects of power are not all negative: "it induces pleasure, forms knowledge, produces discourse. It needs to be considered as a productive network which runs through the whole social body, much more than as a negative instance whose function is repression." (Foucault, 1980 p. 119). This is particularly important in studying classrooms, where the formation or construction of knowledge is the object of the enterprise.

Power relationships and the construction of knowledge

Systematic observation is needed to clarify the operation of power in pedagogy (Gore, 1997). It is particularly important in a study of collaborative learning, which involves a shift in traditional classroom power relationships. By relinquishing some control over classroom interactions, the teacher shares power with the students. I claim that the exercise of power among students working together on a mathematical activity can influence the construction of knowledge by the group—both the personal understanding of mathematics constructed by each individual, and the knowledge which is "taken-as-shared" within the group.

The extent of a student's influence on a group's discussions has the potential to affect their self-perceptions of mathematical competence and of ownership of the mathematics constructed; and also how their capabilities are perceived by others. Thus the exercise of power within small groups is potentially important in a study of how students construct themselves as learners of mathematics.

Studies of teacher-student power

A Foucauldian view of classroom power sees it as a relationship between the participants, claiming that there can be no power relations without the possibility of resistance. Manke (1997) adopted an interactive conception of power, taking into account actions of students as well as teachers. But her focus was on the struggle for power between teacher and students, and strategies which teachers adopt to achieve their objectives in the classroom. In a current project, Gore (1997) is using categories derived from Foucault's work to analyse the practice of power in a variety of educational settings.

Power relations have not been an explicit focus of most studies of interaction in mathematics classrooms, including those dealing with gender issues (Koehler, 1990; Leder, 1990), but some of the findings of these studies suggest the exercise of power by male students. Leder, for example, found fairly consistent differences in teachers' interactions with male and female students, and noted "the pervasiveness of males' domination of teacher attention" (Leder, 1990 p. 165). Jungwirth (1991) found gender-related modifications of "typical" teacher-student interaction patterns, and argued that their effect was the interactive constitution of boys' mathematical competence and of girls' mathematical incompetence. Interactions between students were not analysed in these studies.

Power relations within collaborative groups

Forgasz (1995), studying groups working together in two Year 7 classrooms, observed disruptive behaviour by boys, occasional abusive behaviour by boys towards girls, and work-avoidance tactics by boys who left the girls in their group to do most of the work for which all group members would receive credit. These could all be interpreted as ways of exercising power. Forgasz, however, did not explicitly address power issues, choosing to focus instead on autonomous learning behaviours and attributions for success and failure.

Procedures

The present study used naturalistic inquiry methods, in order to disturb normal classroom processes as little as possible. Case studies were conducted in two government high schools in large Australian cities, involving Year 11 classes working on elementary calculus. The teachers of both classes used collaborative learning methods, but they implemented them in very different ways. In one class, the teacher first introduced and explained the topic, and then the class worked on a variety of short collaborative activities in which they applied the ideas they had learned. I call this *collaborative practice*. In the other class, groups worked, without prior instruction, on carefully chosen open-ended problems, and in the process developed the new mathematics they needed. At intervals, groups reported progress to the whole class, so that ideas and methods could be discussed and shared. I describe this as *collaborative inquiry*.

Data included videotapes of lessons, field notes from classroom observations, and copies of student worksheets. I prepared "rich" transcripts of the videotapes, including descriptions of actions, gestures, facial expressions or voice intonations which I judged relevant. To validate these judgements, a colleague was asked to view a sample of the taped lessons and comment on the information included in, or omitted from, the transcripts.

Indicators of the exercise of power

My aim was to develop a set of criteria for identifying the exercise of power which could be applied to rich transcripts by someone with no information about the gender, class or ethnicity of the participants. This would make it possible for a colleague working from transcripts alone to verify the reliability of my analysis, and so provide a safeguard against any unintentional bias on my part.

I sought a framework for analysis grounded in the data, which could be applied to groups using either collaborative inquiry or collaborative practice. I began by studying and reflecting on the transcripts of the lessons, and re-viewing the videotapes, trying to gain a feeling for the power relations involved. I was able to classify significant influences under two main headings: control of the flow of discourse, and influence over the construction of knowledge. Use of resources, body language and voice emphasis were also used, but in subsidiary ways.

Control of the flow of discourse

A student can control the discourse in a group by influencing the topic to be discussed, including the timing of transitions from one topic to another. A study of the resolution of uncertainty (Clarke & Helme 1997) provided a useful approach to this. Clarke and Helme proposed that a mathematics lesson can be divided into episodes, each defined by a consistent purpose such as the solving of a particular problem. An episode is made up of one or more negotiative events involving identifying a sub-goal, and attempting to resolve it. These are usually initiated by an expression of uncertainty such as the asking of a question.

I found that the transcripts I was analysing did not divide neatly into sequences of negotiative events, each satisfactorily resolved before the group moved on to the next. Discussions were often inconclusive, or interrupted by off-task talk. Nevertheless, negotiative events appear to be a key unit for analysis, because transitions from one to the next mark the progress of a group's work on an activity. A student who enacts closure of a negotiative event by initiating a new one is exercising considerable control over the discourse.

The transcripts revealed the following ways in which students act to control or influence flow of group discourse: initiating a negotiative event; initiating off-task talk; and rejecting or ignoring off-task talk (by continuing the negotiative event, or initiating a new one). Examples of these are given below:

Explanation of symbols used in the examples:

–>	the significant turn in an excerpt	[ ]	observations from tape or field notes
(…)	an indecipherable utterance	=	"latching", i.e., no perceptible gap between speakers, usually experienced as an interruption.
(by)	the best guess for an indistinct utterance		
why	emphatic speech		

Initiating a negotiative event.

The person who initiates a negotiative event is attempting to take control of the discussion. Initiation is frequently, but not always, signalled by a discourse marker like "Okay", "So", "Now", "Well" or "Right". In both examples below, the initiation was followed by several turns of discussion of the question.

Example 1:

–> D: Well, let's think of things we can do, like what?

Example 2:

–> D: Okay, this graph tells us … —no it doesn't tell us—what does it tell us?

Initiating off-task talk

Changing the subject is also a way to control the discussion, but as the examples below show, the effect depends on the context and frequency. Occasional off-task talk can provide a break after a period of intense engagement, but the sustained or repeated introduction of irrelevant topics disrupts the group effort.

Example 3:

B: Mm yeah. I think we plus it together.
–> L: Oh, shit. [Looks up and smiles to T (sitting opposite).] I'm so tired.
T: Go to sleep then.

This was the beginning of a series of 13 off-task turns, terminated only when L herself brought the group back on task by initiating a new negotiative event.

Example 4:

P: How about 2.23, I mean 2.25?
–> G: [laughing] I can't be bothered. It's too hot. It's too hot!

This exchange occurred when the group had been engaged on the problem for some time and had effectively solved it. Immediately afterwards, they returned to work and completed the solution. Thus G's aside did not disrupt the group effort, and so did not function as an exercise of power.

Rejecting or ignoring off-task talk

Example 5:

L: I'm so tired, man.
–> B: Okay, what's the rest of the question?

Example 6:

P: I reckon that=
G: =What?
P: We should all go to the beach.
–> G: Okay, I need paper. Okay, so the basic formula. What did we have?

In both examples, one student controlled the group's discussion by ignoring the distraction, and in a business-like manner drawing attention back to the topic.

Terminating a negotiative event may also seem to be a powerful move. But it is never clear that a negotiative event has ended until the next event, or discussion of another topic, is established. Until then, although a group may appear to have reached agreement, one member can always have a change of mind and resume negotiation. The sequence of events is thus interactively constituted by all participants. One member can only control the proceedings if the others allow it.

Construction of knowledge

The sequence of topics discussed tells only part of the story. In a mathematics lesson, it is the mathematical ideas that are important, so we need to look at the influence of different students on the knowledge constructed or negotiated. For this, we must pay attention to individual turns within negotiative events. A study of the transcripts revealed that the following types of moves could be significant: introducing a new idea or making a suggestion about solving the problem; rejecting an idea or suggestion; endorsing an idea or suggestion; asking for an explanation or justification; giving an explanation or justification; correcting or questioning an error; and assigning tasks to the group.

Introducing a new idea, or making a suggestion about solving the problem.

Example 7:

–> A: Could I suggest that … we choose one variable to work it around, and then work from the lowest to the highest one, using integers in the table, in that way we get a really good pattern, you know, that we can see.

Example 8:

L: How do you do that?

–> B: Sub one in as x.

Following example 7, A's group adopted and used his idea. After example 8, B was challenged about his suggestion, expressed uncertainty, resorted to looking up notes, and the pair's work on the problem ground to a halt. I suggest that these different responses derive from differences in the students' sources of authority for their ideas, and differences in the type of problem. A's authority came from himself, and he was able to give a reason for his suggestion. B's authority derived from remembering a procedure, which he was later unable to explain. The depth and open-endedness of the problem A's group were working on gave opportunities for the use of original ideas. The more routine question B's group were tackling did not provide such opportunities.

Rejecting an idea or suggestion

Example 9:

[R and N are working on a task involving matching functions and derivatives]

R: So, which one is this?

N: Negative six. [Moves a card forward, but does not put it in place]

–> R: Negative six. [Looks at the card suggested] No, it should have only x.

This sequence is similar to the initiation-response-feedback (I-R-F) pattern common in teacher-led classroom dialogue (Stubbs, 1983). Although R did not at that point know the answer to the question, he echoed the "teacher" role by first initiating, and then evaluating N's response. Throughout the whole time that the pair worked on this task, R continued to exercise control in this way.

Example 10:

M: It's half. Because, like, it seems like (pause) you see how here, this is half of the base. [pointing to the model they have made].

–> D: Is that, just a coincidence?

Following this remark, nothing more was said about M's suggestion for 10 minutes, while the group fruitlessly pursued other approaches to the problem. Thus D's incorrect rejection had more influence than M's correct suggestion.

Example 11:

I: Hey, what if you graph it?

–> G: Yes, but then, this will give us the exact volume.

V: Oh no, the graph gives the exact volume.

–> G: Yeah well, if someone else wants to draw a graph …

G seems to have had her mind fixed on the trial-and-error method she had been using with success, and so rejected the suggestion of drawing a graph. Although she did suggest that the others could draw a graph if they wanted, the fact that none of them attempted to do so illustrates her dominant influence in the group.

Endorsing an idea or suggestion

Example 12:

M: I just thought it's got to be half of x, so it will fold up.

D: It go—hang on, is this x? [studying the model]

–> You're right, it is. [Looks towards M and nods.]

This sequence took place 10 minutes after example 10 above. It constituted a significant breakthrough for the group concerned, changing the track of their work on the problem. The key to the breakthrough, however, was not M's suggestion, which had been made earlier without effect, but D's endorsement of it. This underlines D's power and M's lack of power within the group.

Example 13:

R: What do you think this one is?

N: [Points to a card] This?

–> R: [Moves into place the card N was indicating.]

This again has the form of an I-R-F sequence. N's suggestion was tentative, expressed as a question rather than a statement. R endorsed it wordlessly by moving the card into place, and the pair moved on to the next stage of the task.

Asking for an explanation or justification

Asking a question can be productive, or counter-productive.

Example 14:

D: Because we can't have three variables in an equation.

–> A: Why can't you?

A's influence here was productive. His question helped the group to focus on the next step in solving the problem—using substitution to reduce the number of variables and obtain a function of one variable which they could differentiate.

Example 15:

–> P: That’s your base? Is that your base? Is that going to be your base?

G: Yes.

–> P: And that’s going to be a side there?

G: Those are the sides.

Three members of this group had worked together on their problem, while P pursued his own ideas independently, and fruitlessly. When the three had completed the first stage of the problem, P began asking G to explain what they had done. He persisted with questions like these, interrupting other remarks and requiring each step to be clarified, halting the group’s progress until eventually the solution had been explained to him three times. Thus P exercised power obstructively, first by non-participation and then by persistent questioning.

Example 16:

–> L: Substitute x is one yeah. Wouldn’t that still be the same thing?

B: I don’t know. I think I’ll go check my notes.

This exchange happened soon after example 8. B had seemed confident, but had a poor understanding of the concepts. L’s question undermined his confidence. He turned to a reliable source of authority, while progress on the problem halted.

Giving an explanation or justification

Explanations and justifications are powerful if the rest of the group find them convincing, whether or not they would be regarded as correct by a trained mathematician. Gestures and manner of speech can help to direct the attention of the group to the explanation, but it appears that the status and power of the speaker are crucial. An argument presented by a student of lower status may not be found convincing, or may not even be attended to, as was seen in example 10.

Example 17:

D: … we’ve got three variables, that’s what I don’t=

A: =We don’t have three variables=

∅ D: =Oh, because we can do x over two. Look what we can do! [with great excitement]

Here, D’s excited voice and emphatic speech commanded the group’s attention for the detailed explanation which followed, but the commanding role which he had taken in the group from the start of their work may have played a key role.

Correcting or questioning an error

Example 18:

∅ R: Minus n <u>minus</u> 1, which is minus three, oh yeah.

R recalled the differentiation rule for powers of *x* (although expressed in a rather confused way), realised they had misapplied it to a negative power, and made the correction, without reference to his partner. In so doing he retained control.

Example 19:

∅ I: How come you multiplied here?
V: Because [pause] I don't know.

When V found that he could not answer the question, he reassessed, and later amended, what he had done. Thus I's question was challenging and productive.

Example 20:

∅ L: Should it be eight x squared from four x and two x?
B: Nuh.
L: Oh nah, yeh, yeh yeh yeh.

In this case, L correctly pointed out an elementary algebraic mistake that B had made, but acquiesced immediately when B rejected her correction. She appears to mistrust her own correct reasoning. For her, B's rejection carried more power.

Assigning tasks to group members

A group may sometimes decide to share out parts of the work among members. The student who makes this decision, and the one who allocates the tasks (not necessarily the same) exercise power—if the rest of the group accept their direction. But, as example 11 shows, such suggestions may not be followed.

Other factors

Other factors seen to be important included gestures, body language, and use of resources. A gesture such as pointing to a model, diagram or graph can draw attention to what one is saying, so can be an exercise of power. But gestures may also help to explain one's ideas or clarify one's thoughts, and so can support the construction of knowledge without necessarily exercising power.

Observation revealed that resources such as worksheets, textbooks, models and calculators are used in a variety of ways to support both the control of discourse and the construction of knowledge. If there is a single copy of a worksheet or other resource, the student who has it is at an advantage in controlling the discourse. Conversely, passing a worksheet to another student can be a way of handing over control. Occasionally, one student hands over a worksheet to another but then dictates what to write, in this way maintaining control.

Discussion

The examples demonstrate that power is interactively constituted: the influence of an utterance cannot be determined until its reception by the rest of the group is known. The most important indicators of the exercise of power seem to be the initiation of a new topic, either a negotiative event or off-task talk; and the endorsement, rejection or challenge of a statement. Gestures and the use of resources can act to intensify or moderate the effects of an utterance.

The objective of a negotiative event is the resolution of uncertainty. The key to this may be contained in an idea suggested by one group member, but what really counts is its reception by the rest of the group. No matter how good the idea, it will not advance the group's endeavour if it is rejected, so a successful rejection move is powerful. Being ignored by the rest of the group is a form of rejection, and signifies the individual's lack of power within the group.

Similarly, endorsement of a suggestion resulting in its adoption by the group, is clearly an exercise of power, whereas uncritical acquiescence is not. A good idea may be accepted because of the status of its originator, without all group members understanding its significance. Or a misleading idea may be accepted, and cause a time-wasting digression, or failure to complete the task successfully. In such cases, the originator of the idea exercises power. Discrimination between weak and powerful acceptance moves needs careful interpretation. The manner of saying and doing things can be as important as what is said or done.

Asking for an explanation or justification can be an important and powerful move, but again this depends on the manner of asking, whether it is interpreted as a challenge or a threat, or simply a request for help or clarification. Finally, giving an explanation or justification can be powerful, but only if it convinces the hearers. It will be important to take into account the nature of the authority to which the respondent appeals, and which the group find convincing. Do they, for example, rely on an external authority like the teacher, a textbook, or an established formula or rule, or on the internal authority of a rational argument.

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Students' experiences of ability grouping —disaffection, polarisation and the construction of failure

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Introduction and background

In the UK there is a long tradition of grouping by 'ability' - a practice founded upon the idea that students have relatively fixed levels of ability and need to be taught accordingly. In the 1950s almost all the schools in the UK were 'streamed' - a process by which students are grouped by 'ability' in the same class for all subjects. A survey of junior schools in the mid-1960s (Jackson, 1964) found that 96% of teachers taught to streamed ability groups. The same study also revealed the over-representation of working-class students in low streams and the tendency of schools to allocate teachers with less experience and fewer qualifications to such groups. This report contributed towards a growing awareness of the inadequacies of streamed systems, supported by a range of other research studies which highlighted the inequitable nature of such systems. Studies by Hargreaves (1967), Lacey (1970) and then Ball (1981) all linked practices of streaming and setting (whereby students are grouped by 'ability' for individual subjects) to working-class under achievement.

The late 1970s and early 1980s witnessed a growing support for mixed-ability teaching, consistent with the more general public concern for educational equality that was pervasive at the time. But in the 1990s, concerns with educational equity have been eclipsed by discourses of 'academic success', particularly for the most 'able', which has meant that large numbers of schools have returned to the practices of ability grouping (Office For Standards in Education, OFSTED, 1993). Indeed ability-grouping is now widespread in the UK, not only in secondary schools, but also in primary schools, with children as young as 6 or 7 being taught mathematics and science (and occasionally other subjects) in different classrooms, by different teachers, following different curricula with different schemes of work. This phenomenon may also be linked directly to a number of pressures from government. The 1988 Education Reform Act (ERA) required schools to adopt a national curriculum and national assessment which was structured, differentiated and perceived by many schools to be constraining. Research into the effects of the ERA on schools has shown that a number of teachers regard this curriculum as incompatible with mixed-ability teaching (Gewirtz, Ball, & Bowe, 1993). The creation of an educational 'marketplace' (Whitty, Power & Halpin, 1998) has also meant that schools are concerned to create images that are popular with local parents and 'setting' is known to be popular amongst parents, particularly the middle-class parents that schools want to attract (Ball, Bowe & Gewirtz, 1994). The White Paper 'Excellence in Schools' (DFEE, 1997) revealed the new Labour Government's commitment to setting:

> *'... unless a school can demonstrate that it is getting better than expected results through a different approach, we do make the presumption that setting should be the norm in secondary schools.' (p. 38)*

In mathematics however, relatively few subject departments have needed to change *back* to ability grouping as the majority have remained faithful to practices of selection, even when they have been the only subject department in their particular school to do so. An OFSTED survey in 1996 reported that 96% of schools taught mathematics to 'setted' groups in the upper secondary years (The Guardian, 1996). This has non-trivial implications for students' learning of mathematics. Despite this, our understanding of the impact of ability grouping practices upon mathematics teachers' pedagogy and, concomitantly, students' understanding of mathematics, is limited.

Previous research in the UK has concentrated, almost exclusively, upon the inequities of the setting or streaming system for those students who are allocated to 'low' sets or streams. These are predominantly students who are also disadvantaged by the school system because of their 'race', class or gender (Abraham, 1989; Tomlinson, 1987; Ball, 1981; Lacey, 1970; Hargreaves, 1967). These research studies predominantly used qualitative, case-study accounts of the experiences of students in high and low streams to illustrate the ways in which curricular differentiation results in the polarisation of students into 'pro'- and 'anti'-school factions. Such studies, by virtue of their value-based concerns about inequality (Abraham, 1994), have paid relatively little attention to the effects of setting or streaming upon the students' development of *subject* understandings (Hallam & Toutounji, 1997). Furthermore, they have tended to concentrate on 'streaming', in which students are allocated to the same teaching group for a number of subjects—what Sorensen (1970) termed a wide *scope* system, rather than on 'setting' which is carried out on a subject by subject basis (narrow scope).

Research in the USA has provided a wealth of empirical evidence concerning the relative achievement of students in academic, general and vocational tracks. Such studies have consistently found the *net* effects of tracking on achievement to be small (Slavin 1990), with evidence that tracking gives slight benefits to students in high tracks at the expense of significant losses to students in low tracks (Hoffer, 1992; Kerchkoff, 1986). However, such studies have given little insight into the way that tracking impacts upon students' learning of mathematics, the processes by which it takes effect or the differential impact it has upon students. This is partly because quantitative methods have been used almost exclusively, with no classroom observation and no analysis of the mechanisms by which tracking influences learning. Many of the studies into tracking have also focused upon differences in group means, masking individual differences within groups (Gamoran and Berends, 1987; Oakes, 1985).

This paper will report upon interim data from a four-year longitudinal study that is monitoring the mathematical learning of students in six UK schools. This follows on from a study of two schools that offered 'traditional' and 'progressive' approaches to the teaching of mathematics (Boaler, 1997a, b, c). Although ability grouping was not an initial focus of that study, it emerged as a significant factor for the students, one that influenced their ideas, their responses to mathematics, and their eventual achievement. One of the schools in that study taught to mixed-ability groups, the other to setted groups, and a combination of lesson observations, questionnaires, interviews and assessments revealed that students in the setted school were significantly disadvantaged by their placement in setted groups. A year group of students was monitored in each school over a three year period (n ? 300) from the beginning of year 9 until the end of year 11 (ages 13-16). The disadvantages affected students from across the spectrum of setted groups and were not restricted to students in low groups. The results of that study, that related to setting, may be summarised as follows:

- Approximately one-third of the students taught in the highest ability groups were disadvantaged by their placement in these groups because of high expectations, fast-paced lessons and pressure to succeed. This particularly affected the most able girls.
- Students from a range of groups were severely disaffected by the limits placed upon their attainment. Students reported that they gave up on mathematics when they discovered their teachers had been preparing them for examinations that gave access to only the lowest grades.
- Social class had influenced setting decisions, resulting in disproportionate numbers of working-class students being allocated to low sets (even after 'ability' was taken into account).
- significant numbers of students experienced difficulties working at the pace of the particular set in which they were placed. For some students the pace was too slow, resulting in disaffection, while for others it was too fast, resulting in anxiety. Both

responses led to lower levels of achievement than would have been expected, given the students' attainment on entry to the school.

A range of evidence in that study linked setting to under-achievement, both for students in low *and high* sets, despite the widely-held public, media and government perception that setting increases achievement. Indeed the evidence was sufficiently broad ranging and pronounced to prompt further research in a wider range of schools.

Research design

In our current study we are working with six state schools that have been chosen to provide a range of learning environments and contexts. The schools are located in five different local education authorities. Some of the school populations are mainly White, others mainly Asian, while others include students from a wide range of ethnic and cultural backgrounds. The performance of the schools in the national school-leaving examination (the General Certificate of Secondary Education or GCSE) ranges from the upper quartile to the lower quartile, nationally, and the social class of the school populations range from mainly working class, through schools with nationally representative distributions of social class, to strongly middle class. One of the schools is an all-girls school and the other five are mixed.

All six schools teach mathematics to mixed-ability groups when students are in year 7 (age 11). One of the schools puts students into 'setted' ability groups for mathematics at the beginning of year 8 (age 12), three others 'set' the students at the beginning of year 9 (age 13), and the other two schools continue teaching to mixed ability groups. The students in our study have just completed the end of year 9, which has meant a change from mixed ability to setted teaching for three of the cohorts. There are approximately 1000 students in the study. Research methods have included approximately 120 hours of lesson observations, during years 8 and 9, questionnaires given to students in the six cohorts (n=943 for year 8, n=977 for year 9, with matched questionnaires for both years from 843 students) and in-depth interviews with 72 year 9 students. This has included 4 students each from a high, middle and low set in the setted schools and students from a comparable range of attainment in the mixed ability schools. We have also collected data on attainment, social class, gender and ethnicity. This paper will draw upon questionnaire responses, lesson observations and 72, 30-minute interviews to illustrate the ways in which ability grouping practices have impacted upon students' learning of mathematics.

Research Results

When students moved from year 8 to year 9 in our study, it became clear from questionnaire, lesson observation and interview data that many students in the setted schools began to face negative repercussions as a result of the change from mixed-ability to setted teaching. Students were chosen for interview by asking teachers of high, medium and low setted groups to select a pair of girls and then a pair of boys who would be relaxed and happy to talk. Forty of the forty-eight students interviewed from setted groups wanted either to return to mixed ability teaching or change sets. The students reported that teaching practices emanating from setting arrangements had negatively affected both their learning of mathematics and their attitudes towards mathematics. Three major issues that were raised by students are discussed below:

A - High Sets, high expectations, high pressure

In Boaler's previous study (Boaler, 1997b) at least one-third of the students taught in the highest set were disadvantaged by their placement in this group, because they could not cope with the fast pace of lessons and the pressure to work at a high level. The students that were most disaffected were very able girls, apparently because able girls, more than any others, wanted to understand what they were doing — in depth — but the environment of set 1 classes did not allow them to do this.

We chose to observe set 1 lessons and interview set 1 students in this follow-up study to determine whether the environment of set 1 lessons in other schools was similar and whether students were disadvantaged in similar ways. Early evidence suggests that this is the case. Every one of the 8 girls interviewed from set 1 groups in the current study wanted to move down into set 2 or lower. Six out of eight of the set 1 boys were also extremely unhappy, but they did not want to move into lower groups, presumably because they were more confident (although no more able), than the girls, and because of the status that they believed being in the top set conferred. Observations of set 1 lessons make such reactions easy to understand. In a range of top-set classes the teachers raced through examples on the board, speaking quickly, often interjecting their speech with phrases such as 'come on we haven't got much time' and 'just do this quickly'. Set 1 lessons were also more procedural than others — with teachers giving quick demonstrations of method without explanation, and without giving the students the opportunity to find out about the meaning of different methods or the situations in which they might be used. Some of the teachers also reprimanded students who said that they didn't understand, adding comments such as 'you should be able to, you're in the top set'. Before one lesson the teacher told one of us (JB) that about a third of his class were not good enough for the top set and then proceeded to identify the ones that "were not academic enough", with the students concerned watching and listening. The following are descriptions of 'top set' lessons, from students in the 4 setted schools:

School R: Mainly white, working class school with low attainment

> *Lessons are difficult and if you can't answer he says, "You won't be in set 1 next year — you are the set 1 class you shouldn't be finding this difficult". (school R, boys, set 1)*
>
> *He wants to be successful, better than set 2, so he goes really fast, but it's over the top. (school R, boys, set 1)*
>
> *He explains work like we're maths teachers — really complex, I don't understand it. (school R, boys, set 1)*
>
> *I want to get a good mark, but I don't want to be put in the top set again, it's just too hard and I won't learn anything. (school R, girl, set 1)*

School W: Mainly Asian, middle and working class school with average attainment

> *She says, "You have to do this quickly", so you just rush and write anything. (school W, girls, set 1)*
>
> *Practically all the time you are rushing through and not understanding. (school W, girls, set 1)*
>
> *I want to go down because they do the same work but they do it at a slower pace, so you can understand it better, but we just have to get it into our head the first time and that's it. (school W, girls, set 1)*

School A: Mainly white, middle and working class school with average attainment.

> *It's too fast, I can't keep up. My friends are in different groups and you can't ask them for help, because you're the top set and you're supposed to know it all. (school A, girl, set 1)*

Most of the difference is with the teachers, the way they treat you. They expect us to be like, just doing it straight away, like we're robots. (school A, boy, set 1)

School F: Mainly White, middle class school with very high attainment:

I preferred it in years 7 and 8, you felt more sort of comfortable, you didn't feel you were being rushed all the time (school F, girl, set 1)

I used to enjoy maths, but I don't now because I don't understand it —what I'm doing. If I was put down I probably would enjoy it. I'm working at a pace that is just too fast for me. (school F, girl, set 1)

These are just a small selection of the complaints raised by students in top sets, who characterised their mathematical experiences as fast, pressured and procedural. The four schools that are represented by the comments above were not chosen because of the way that they taught mathematics and the schools are quite different in many respects. Yet the students' perceptions of set 1 lessons were similar in each of the schools. In a previous paper Boaler (1997b) argued that teachers change their normal practices when they are given top set classes to teach, appearing to believe that being a 'top set' student entails a qualitative and meaningful difference from other students, rather than simply being in the highest-attaining range of students in the school. Top-set children, it seems, do not need detailed help, time to think, or the space to make mistakes. Rather they can be taught quickly and procedurally because they are clever enough to draw their own meaning from the procedures they are given. In questionnaires students in the six schools were asked, 'do you enjoy maths lessons?' set 1 groups were the most negative in the entire sample, with 43% of set 1 students choosing 'never' or 'not very often', compared with an average of 36% of students in other sets and 32% of students in mixed ability classes. Students were also asked whether it was more important "to remember work done before or think hard" when answering mathematics questions. The set 1 groups had the highest proportion of students who thought remembering was more important than thinking. In the set 1 classes 68% of students prioritised memory over thought, compared to 56% of students in the other setted groups and 51% of students in mixed ability groups.

In the same paper, Boaler also argued that the fast, procedural and competitive nature of set 1 classes particularly disadvantages girls and that the nature of high set classes contributes to the disparity in attainment of girls and boys at the highest levels. Despite media claims that girls are now overtaking boys in all subjects (Epstein, Maw, Elwood & Hey, 1998), boys still outnumber the number of girls attaining A or A* grades in mathematics GCSE by 5 to 4. As the vast majority of able girls are taught within set 1 classes for mathematics in the UK (The Guardian, 1996) and the four schools in this study are unlikely to be particularly unusual in the way that they teach set 1 lessons, it seems likely that the under-achievement and non-representation of girls at the highest levels is linked to the environments generated within top set classrooms.

B - Low sets, low expectations & limited opportunities

Students in low sets at the four schools appear to be experiencing the reverse of the students in high sets, with repercussions that are, if anything, even more severe and damaging. Indeed, the most worrying reports of the implications of the setting process for students in our sample came from students in low groups. These students reported a wide range of negative experiences, substantiated by observations of lessons. These included a frequent change of teachers (in one school the 'bottom' set had been taught by 3 different teachers in the first 9 months), the allocation of non-mathematics teachers to low sets and a continuous diet of low-level work that the students found too easy. For example:

It's just our group who keeps changing teachers.

JB: Why?

'Cause they don't think they have to bother with us. I know that sounds really mean, but they don't think they have to bother with us, 'cause we're group 5, so if they have a teacher who knows nothing about maths, they'll give them to us, say a PE teacher. They think they can send anyone down to us, they always do that, they think they can give us anybody. (school R, set 5, girls)

We come in and sir tells us to be quiet and gives us some questions then he does them on the board, we want to do it ourselves but he does it.

Even though we're second from bottom group, I think it would be much better if we didn't have the help with it.

JB: Why does he write the answers on the board?

I don't know, he thinks we're stupid.

He thinks we're really low. (school A, set 6, boys)

Students were particularly concerned about the low level of their work and talked at length about teachers ignoring their pleas for more difficult work, making students who had finished the work in the first 5 minutes of the lesson sit and wait with nothing to do for the remaining 55 minutes and in some cases students being told "you can't have finished, you're set 5" (school R, set 5 girl). In some low set lessons the students were not given any mathematics questions to answer — only worked solutions to copy off the board.

You just have to come in, sit down, there's stuff on the board and he says copy it.

It's too easy, it's far too easy.

JB: What happens if it's too easy?

You just have to carry on and do it, and if you don't he gives you detention.

Last year it was harder, much harder. (school R, set 5, boys)

He just writes down answers from the board, we tell him that we can do it, but he just writes down answers anyway.

JB: And what are you meant to do?

Just write them down. That's what we say to him, 'cause people get frustrated from just copying off the board. (school A, set 6, girls)

We do baby work off the board — stupid stuff that we already know, like 3 times something equals 9, it's boring and easy. (school R, set 5 girls)

In questionnaires 27% of students taught in the bottom *half* of the setted groups reported that work was too easy, compared with 7% of students in the top half of the setted groups and 14% of students in the mixed ability schools. Students in low groups were upset and annoyed about the low level of the work they were given, in addition to finding lessons boring, they knew that their opportunities for learning were being minimised:

> *Sir treats us like we're babies, puts us down, makes us copy stuff off the board, puts up all the answers like we don't know anything.*
>
> *And we're not going to learn from that, 'cause we've got to think for ourselves.*
>
> *Once or twice someone has said something and he's shouted at us, he's said — well you're the bottom group, you've got to learn it, but you're not going to learn from copying off the board.' (school A, set 6, girls)*

The students' reports were consistent with our observations of low-set lessons, in which students were given answers to exercises a few minutes after starting them or required to copy work off the board for the majority or all of lessons. In response to the questionnaire item 'how long would you be prepared to spend on a maths question before giving up?' 32% of students in the bottom half of the setted groups chose the lowest option — 'less than 2 minutes' compared with 7% of students in the top half of the setted groups and 22% of students in mixed ability groups. The polarisation in the students' perceptions about mathematics questions in the setted schools probably reflects the polarisation in their experiences of mathematics. We have not yet interviewed teachers to talk to them about the choices they make about the level of work but the students were convinced that teachers simply regarded students in low sets as limited:

> *Sir used to say — you're the bottom group, you're not going to learn anything.*
>
> *JB: He says that to you?*
>
> *Yeh.*
>
> *JB: Why?*
>
> *I don't know, I don't think he's got faith in us, or whatever, he doesn't believe we can do it. (school A, set 6, boys)*

All four schools that use ability-grouping have told us that the system is flexible and that students will change groups if they are inappropriately placed, but the students in low groups believed there to be little hope of moving to higher groups. This was partly because they did not believe that teachers were aware of the work students could do:

> *I can get high if I'm pushed as hard as I can to get up there, but it's not easy when you just do the same things over and over again.*
>
> *JB: How can you move up?*
>
> *There is nothing you can do, he has no idea how we're doing, he hasn't taken our book in once. (school R, set 5 girls)*

The students also believed that they were trapped within a vicious circle — to move up they needed good end of year test results, that were comparable with students in higher groups, but they could not attain good results because they were not taught the work that was assessed in the tests:

> *The SATS were hard because our classwork is so easy, so we hadn't done it.*
>
> *I want to be brainy and go up and be good at maths, but I won't go up if the work is too easy. (school R, set 5, boys)*

In the same way as the 'top set' teachers had fixed ideas about the high level and pace of work students should have been able to do, the low set teachers had fixed ideas about the low level of work appropriate for 'bottom set' students. The students reported that teachers continued with these ideas, even when students asked them for more difficult work:

> *If you tell the teacher I've done this before, he'll say — well you can do it again—he doesn't set you up with any harder work, nothing like that. (school R, set 5, girls)*

> *The work is far too easy, but if we try and complain he says, "Be quiet", and then, "Detention", because we tried to explain it to him. Today he sent Mark out, 'cause he told him it was too easy, so he just sent him out. (school R, set 5, boys)*

The students were clearly disadvantaged by the diet of low-level numeracy work that they were given. This problem seemed to derive partly from the teachers' perceptions about the level of work appropriate for low-set students but also from an idea that is intrinsic to setting policies and will be discussed in the final section—that students in setted groups have the same mathematical capabilities and learning styles and may be taught accordingly.

C - Restricted pedagogy and pace

In mixed ability classes teachers have to cater for a range of students whose previous attainment varies considerably. Most teachers respond to this challenge by providing work that is differentiated either by providing different tasks for different students within the same class (sometimes called 'differentiation by task'), or by giving all students a task that can be attempted in a variety of ways and at a variety of different levels (sometimes called 'differentiation by outcome'). Teachers often let students work 'at their own pace' through differentiated books or worksheets. In setted classes students are brought together because they are believed to be of similar 'ability'. Yet setted lessons are often conducted as though students are not only similar, but *identical* — in terms of ability, preferred learning style and pace of working. In the setted lessons we have observed, students have been given identical work, whether or not they have found it easy or difficult and they have all been required to complete it at the same speed. This aspect of setted lessons has distinguished them from the mixed-ability lessons we have observed. The restrictions on pace and level of work that are imposed in setted lessons have also been a considerable source of disaffection, both for students who find the pace of lessons too fast and for those who find it too slow.

In interviews students talked at length about the restrictions imposed upon their pace of working since changing to setted groups, describing the ways in which they were required to work at the same speed as each other. Students reported that if they worked slower than others they would often miss out on work as teachers moved the class on before they were finished:

> *People who are slow they don't never get the chance to finish because she starts correcting them on the board already and you don't finish the module. (School A, set 4 boy)*

Students also described the ways in which teachers used a small proportion of the students as reference points for the speed of the class (cf Dahllöf, 1971), and the detrimental effect this could have on their learning:

> *Sometimes you can do it fast, but you don't really know it. But if she knows people have finished, she tells you have even less time to do the work, she says, "Look at these 5, they have finished, hurry up!" (school W, set 1 girl)*

Students also reported that if they worked quickly they were disadvantaged as teachers made them wait for the rest of the class:

> *Now in year 9, we're sort of — people can be really far behind and people can be in front. The people who work fast have to wait for people at the end to catch up. Like I finished before and I had a whole lesson to do nothing. (School A, set 4, boy)*

Again the students linked these restrictions to the norms generated within setted groups:

> *Last year it was OK 'cause when you finished work miss would give us harder, more to do, but this year when you finish you've just got to sit there and do nothing.*
>
> *It's different 'cause in sets you all have to stay at the same stage. (school W, set 3, boys)*

Such problems were not caused by teachers simply imposing an inappropriate pace upon their groups — some students found lessons too fast whilst other students in the same groups found the same lessons too slow. The two boys in school W, quoted above, described the problem well — in mixed ability classes students would be given work that was chosen for them, if they finished the work teachers would give them harder work; in setted lessons "you all have to stay at the same stage". Being able to teach the whole class as a single unit is the main reason that teachers put students into 'ability' groups, and it was also one of the main sources of the students' disaffection. The students also described an interesting phenomenon — that some teachers seemed to hold ideas about the pace at which a class should work that were independent of the capabilities of the students who were in that set. For example:

> *If you're slow she's a bit harsh really, I don't think she can really understand that some people aren't as fast as others. If you say — I don't understand the work, I'm slow— she'll just say you're in the middle set, you had to have got here somehow, so you've got to do middle set work. (School A, set 4 boy).*

The teachers of the top sets also exemplified this phenomenon with the frequent remarks they made to students in the vein of:

> *"You are the set 1 class, you shouldn't be finding this difficult" (school R, set 1 boy).*

It seems that the placing of students into 'ability' groups creates a set of expectations for teachers that over-rides their awareness of individual capabilities. This is a particularly interesting finding given that the main argument that the Prime Minister, Tony Blair, and other government ministers have given for supporting setting is that children need work that is at an appropriate pace and level for their particular 'ability'.

But the process of ability grouping did not only appear to initiate restrictions on the pace and level of work available to students, it also impacted upon the teacher's choice of pedagogy. Teachers in the four schools in our study that used ability grouping responded to the move to setted teaching by adopting a more prescriptive pedagogy and teachers who offered worksheets, investigations and practical activities to students in mixed-ability groups concentrated upon chalk-board teaching and textbook work when teaching groups with a narrower range of attainment. This is not surprising given that one of the main reasons mathematics teachers support setting is that it allows them to 'class teach' to their classes, but it has important implications for the learning of students. When students were asked in their questionnaires to *describe their maths lessons*, the forms of pedagogy favoured by teachers in the schools using ability grouping were clearly quite different from those in the schools using mixed ability teaching. Some of the students' responses to this question were given the code 'lack of involvement' because students wrote such comments as 'lessons go on and on' or 'maths lessons are all the same'. Twelve per cent of responses from students in setted groups reflected a lack of involvement, compared with 4% of responses from students in mixed-ability groups. An additional 12% of students from setted groups described their lessons as 'working through books', compared with 2% of students in mixed ability groups; whilst 8% of setted students said that the 'teacher talks at the board', compared with 1% of mixed ability students. Fifteen per cent of students in setted groups described their mathematics lessons as either "OK", "fun", "good" or "enjoyable", compared with 34% of mixed ability students.

In a separate open question students were asked how maths lessons could be improved. This also produced differences between the students, with 19% of students taught in sets saying that there should be more open work, more variety, more group work, maths games or opportunity to think, compared to 9% of mixed ability students. Eight per cent of setted students said that lessons should be slower or faster, compared to 4% of mixed ability students and 4% of setted students explicitly requested that they return to mixed ability teaching.

The influence of ability grouping upon teachers' pedagogy also emerged from the students' comments in interview. The following comments came from students across the spectrum of setted groups:

> *JB: What are maths lessons like?*
>
> *Rubbish — we just do work out of a book.*
>
> *It was better in years 7 and 8. We did all fun work (school R, set 1, girls)*

> *I would like work that is more different. Also when you can work through a chapter, but more fun.*
>
> *Could do a chapter for 2 weeks, then something else for 2 weeks, an investigation or something —the kind of investigations we used to do. (School R, set 5, girls)*

> *Last year it was better, 'cause of the work. It was harder. In year 8 we did wall charts, bar charts etc, but we don't do anything like that. It's just from the board.*
>
> *I really liked it in year 7, we would work from books and end of year games — really good. This year it's just work from the board. (School R, set 5, boys)*

> *In year 8, Sir did a lot more investigations, now you just copy off the board so you don't have to be that clever.*
>
> *Before, we did investigations, like Mystic Rose, it was different to bookwork, 'cause books is just really short questions but those were ones Sir set for himself, or posters and that, that didn't give you the answers. (School A, set 4, boys)*

> *In year 7 maths was good, it was alright. He got us thinking for ourselves and we did much more stuff like cutting out, sticking in, worksheets. Now, everyday is copying off the board or doing the next page, then the next page and it gets really boring. (School A, set 6, girl)*

The change in teaching approach that appeared to be initiated by setted teaching could simply reflect the increase in students' age and progression towards GCSE, but similar changes did not take place in the mixed ability schools. The implications of such changes for students' learning of mathematics will be discussed below.

Discussion

The students interviewed from our setted schools create an image of setted mathematics lessons, broadly substantiated by our observations of lessons and by questionnaire data, that is one of disaffection and extreme polarisation. It seems that when students were taught in mixed-ability groups, their mathematics teachers gave them work that was at an appropriate level and pace. When the students were divided into ability groups, students in high sets came to be regarded as mini-mathematicians who could work through high-level work at a sustained fast pace, whereas students in low sets came to be regarded as failures who could cope only with low-level work — or worse — copying off the board. This suggests that students are *constructed* as successes or failures by the set in which they are placed as well as the extent to which they conform to the expectations the teachers have of their set. In particular, within top sets, students are constructed as successes and failures according to the extent to which they can cope with the highly procedural approaches adopted by teachers of those sets. Other notions of success in mathematics, such as those which emphasise depth of understanding, which are arguably much closer to the concerns of professional mathematicians (Buxton, 1981, Burton, 1997) are ruled out.

The requirement to work at an inappropriate pace is a source of real anxiety for many students, particularly girls:

> *I mean I get really depressed — it really depressed me, the fact that everyone in the class is like really far ahead and I just don't understand.*
>
> *Yeah 'cause like especially when everyone else understands it and you think 'Oh my God I'm the only one in the class that doesn't understand it'*
>
> *If you don't understand something, then it's just like, you know, it really depresses you. (School F, set 3, girls)*

These students were not talking about minor feelings and peripheral details but issues that go directly to the heart of their experiences, and which have a profound impact both on their attitudes towards, and their achievement in, mathematics.

The major advantage that is claimed for ability-grouping practices is that they allow teachers to pitch work at a more appropriate level for their students. However, while ability-grouping practices can *reduce* the range of attainment in a class, within even the narrowest setting system, there will be considerable variations in attainment. Some of this will be due to the inevitable unreliability of mechanisms of allocating students to particular sets, and even if the average attainment of students in a set is reasonably similar, this will mask considerable variation in different aspects of mathematics and in different topics, as the students were well aware. Indeed the students held strong beliefs that individuals have different strengths and weaknesses and that it is helpful to learn from each other and to learn to be supportive of each other:

> *I prefer groups when we're all mixed up— like in form groups. 'Cause all mixed up, a variety of clever and dumb. So the dumb learn from the clever and then sometimes the clever can't do it, so they'll learn from people who aren't as good, 'cause sometimes they're good at some things but not others. (School W, set 3, boys)*
>
> *Classes should be mixed, then everyone can learn from everyone, it's not like the dumb ones don't know anything, they do know it, but the atmosphere around them in lessons means they can't work and they just think to themselves — well, what's the point? (School W, set 3, boys)*

Students' experiences of ability grouping

Perhaps the most surprising finding is that setting did not appear to accomplish the one thing that it was designed to do—to allow teachers to match the work set to the strengths and weaknesses of individual students. When the students were asked if work they were given was at "the right sort of level", the proportion of those taught in mixed-ability groups who said that the work set was 'usually about right' for them was actually higher (81%) than that for those taught in ability-groups (77%).

Another consequence of setting that emerged in Boaler's previous study, and which is beginning to emerge in the current study, is the consequence of set allocation for students' entry to the GCSE. The report of the Committee of Inquiry into the Teaching of Mathematics in Schools (1982), generally known as the 'Cockcroft report', argued that it was unacceptable that the majority of students entered for the school leaving examination would gain less than 40% of the available marks. The report recommended that school-leaving examinations in mathematics should be differentiated, so that students would take only those papers appropriate for their attainment. For the mathematics GCSE, there are currently three 'tiers' of entry, with different syllabuses. Because schools find it difficult to operate with students in the same class following different syllabuses, most schools in the country (and all the four schools using ability-grouping in our study) enter all the students in a particular class for the same tier of the examination. The effect of this is that students in the lower sets will be entered for an examination in which the highest grade they can achieve is a grade 'E', whereas the only grade that is ever specified for recruitment or for further study is a grade 'C'.

In Boaler's previous study, the students did not become aware of this restriction until their final year of schooling, year 11, and this discovery caused considerable resentment and disaffection. In the current study, only a few students (exclusively in the top sets) are aware of the effects of tiering, but it is already a significant issue for those beginning to understand the implications of the tiering system:

> *I was reading from the maths literature that if you get put in the middle group for maths, that means they are aiming for a B for GCSE. But I don't think that is fair, it's like saying you can't go higher than a B sort of thing. I think they should give you the work and what you get is what you get. They shouldn't try and aim you for something, because you never know, you could get an A. They put you in separate groups next year and you stay there for 2 years and set you work for a B and I don't think that's fair. (School F, set 1, girl)*

There were, of course, some students in our sample (one-sixth of those students we interviewed) who were comfortable with being taught in sets. The majority of these were those taught in intermediate groups, who did not want to move up (interestingly) or down and worked at a pace and level that was appropriate for them. Another benefit of setting for these students was the opportunity setting provided for bringing together a large group of students working on the same areas of mathematics. However, none of these students knew about the restriction of grades in the GCSE, and it is doubtful whether they would continue to be happy to remain in an intermediate set if they discovered that the school had decided that they would be entered for a tier of the GCSE for which the maximum grade they could achieve would be an 'E'.

As we have noted above, many of the disadvantages of setting that we have described are contingent rather than necessary features of ability-grouping, but we believe that they are widespread, pervasive, and difficult to avoid. The adoption of ability-grouping appears to signal to teachers that it is appropriate to use different pedagogical strategies from those that they use with mixed-ability classes. The best teachers are allocated to the ablest students, despite the evidence that high-quality teaching is more beneficial for lower-attaining students (Black & Wiliam, 1998, p42). Curriculum differentiation is polarised, with the top-sets being ascribed qualities as mathematicians, not as a result of their individual qualities, but simply by virtue of their location in a top set. In order to ensure that the entire curriculum is covered, presumably to suit the needs of the highest-attaining students within the top set, the pace of coverage is both increased and applied to the whole class as a unit, and teachers seem to make increased use of 'transmission' pedagogies. For some students, who are able to conceptualise the new material as it is covered, the experience may be satisfactory, but for the remainder, the effect is to proceduralise the curriculum until it becomes a huge task of memorisation. The curriculum polarisation results in a situation in which upward movement between sets is technically possible, but is unlikely to be successful, because a student moving up will not have covered the same material as the class she is joining. Finally, because of the perversities of the examination arrangements for mathematics GCSE, the set in which a student is taught determines the tier for which a student is entered, and thereby, the maximum grade the student can achieve, and, for most students, this decision will have been made three years before the examination is taken.

Of course, we are not advocating that schools should dispense with ability-grouping immediately—that would clearly be disastrous—but we do believe that the features of the practices adopted by the schools who have maintained mixed-ability teaching with older students provide important suggestions as to how schools can reduce their dependence on between-class ability grouping as the primary strategy for dealing with the diversity of attitudes, capabilities and attainments of students in mathematics. We would also suggest that government ministers should be promoting research and inquiry into mixed ability teaching, and supporting those schools that use such forms of grouping succesfully, rather than discriminating against these schools and exerting pressure upon them to change (Boaler, 1997c).

Because all of the schools in ur study make some use of mixed-ability grouping in the earlier years, all the teachers in our sample have some experience of teaching mixed-ability classes, for which a variety of strategies are used. Some make substantial use of independent learning schemes which allow a teacher to give each student an individual programme of work. They also use within-class grouping, with students on different tables working on different materials and at different speeds. Most of the teachers in the sample also made some use of more open tasks, which can be tackled at a variety of levels. Although these more open tasks were used infrequently with setted classes, it was surprising how favourably these were regarded by the students. When the students who were taught in sets were asked for the best lesson they remembered that year, almost every student described a lesson where the whole class had worked on an investigation or a problem that could be tackled in different ways

Within-class grouping, a system which is used by some of the teachers in one of our 'mixed ability' schools, is much more flexible. It allows opportunities for whole classes to do the same work and allows students that are regarded as weaker on some areas to shine. One student, regarded by her teacher as the 'weakest' in her mixed ability mathematics class, described her best lesson thus:

> *It was last week, we were doing bar charts and pie charts and all that and I think I was the 3rd person in the class who got it properly — we had to make it into a graph, it was good. (School C, mixed-ability, girl)*

Some degree of within-class grouping also allows teachers to ensure that students are given appropriate work, and, importantly, that the level of assigned work is altered if and when this becomes appropriate:

> *We have different books — high books, medium books, low books, so everyone has the right amount of work — no-one's doing nothing too hard or too easy. If you think that it's too hard or too easy you just tell miss and she gives you the right level. (School C, mixed-ability, girl)*

Of course, within-class grouping does often result in a situation in which the teacher ends up explaining the same idea to different groups at different times, but this seems a small price to pay compared to the alternative. As one boy remarked:

> *In my primary school we weren't in groups for how good we were in subjects we were just in one massive group and we did everything together. You got some smart people and some people in our class, so well, we all sort of blend in, so you don't have to be that good and you don't have to be that bad. (School A, set 4, boy, original emphasis)*

Indeed, this student captures eloquently what we found to be the most important, and previously unreported feature of ability grouping — it *creates* (McDermott, 1993) academic success and failure through a system whereby students "*have* to be that good" or they "*have* to be that bad".

Conclusion

We are aware that this may seem like a one-sided report, but we are confident that our findings fairly represent the data that we have collected, and that our data collection methods, while not unproblematic, have captured a reasonably faithful picture of the day-to-day realities of the classrooms we are studying. We are also confident that the schools in our sample are not untypical of the generality of schools in Britain.

Although there are substantial problems in interpreting the results of international comparisons (Brown, 1998, Wiliam, 1998), there is little doubt that, in a variety of respects, the performance of primary and secondary school students in the United Kingdom is modest by international standards (Beaton, Mullis, Martin, Gonzalez, Kelly & Smith, 1996; Mullis, Martin, Beaton, Gonzalez, Kelly & Smith, 1996). Kifer & Bursten's (1992). Analysis of data from the Second International Mathematics Study (SIMS) suggests that the two factors that are most strongly associated with growth in student achievement in mathematics (indeed the only two factors that are consistently associated with successful national education systems) are *opportunity to learn* (ie the proportion of students who had been taught the material contained in the tests) and the degree of *curricular homogeneity* (ie the extent to which students are taught in mixed-ability, rather than setted, groups).

While Bennett, Desforges, Cockburn and Wilkinson (1984) found that teachers using within-class ability grouping tend to *over-estimate* the capabilities of weaker students, and set insufficiently challenging work to the most able, the evidence that we have found in the current study suggests very strongly that between-class ability grouping produces the opposite effect. Indeed, the strength of the curriculum polarisation, and the diminution of the opportunity to learn that we have found in the current study, if replicated across the country, could be the single most important cause of the unacceptably low levels of achievement in mathematics in Great Britain. The traditional British concern with ensuring that *some* of the ablest students reach the highest possible standards appears to have resulted in a situation in which the vast majority of students achieve well below their potential. As one student poignantly remarked:

> *Obviously we're not the cleverest, we're group 5, but still— it's still maths, we're still in year 9, we've still got to learn. (school R, set 5, girl)*

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The South African New Mathematics Curriculum: People's Mathematics for People's Power?

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Abstract

At the height of the internal fight against apartheid, People's Education for People's Power became one of the key action-fighting plans. People's Mathematics for People's Power was one of the products of the strategies. The significance of mathematics emanated from its role as a gatekeeper. In this paper focus is directed towards the place of People's Mathematics for People's Power and its place in the new South African mathematics Curriculum. In particular attention is given to one aspect of the People' way of life botho (African Humanism) that enable blacks to sustain togetherness among the people, through serious economic hardships, leading to the people's regaining of political strength. Questions are raised with regard to the extent to which the framework of the new curriculum for South Africa provides room for the previously disenfranchised and whether they will be enabled to gain access to economic and Political Power, mainly through engaging the strength of botho.

Introduction

The South African slogan, 'People's Mathematics for People's Power', arose as part of 'People's Education for People's Power'. One of the principles underlying the concept of People's Education, according to Taylor *et al.* [1991], is the need to democratise knowledge. Taylor et al. went on to point out that the essential aspects of such democratization are that:

1. Education must be accessible to all the people of South Africa, and
2. Education must be relevant to the economic, social and political activities of its participants. (p.21)

To some extent mathematics served as a gatekeeper in terms of people needing to follow different careers. The mathematics that was taught and stills being taught had very little relevance to the people's economic, social and political activities.

Mathematics was largely seen as one of the major stumbling blocks between mediocrity and excellence. Mathematics was presented as a body of knowledge that consisted of truths that could not be challenged. What is following next is an outline of the views of People's Mathematics. The rest of the paper will focus on the degree to which the new South African Government Mathematics Curriculum has taken on board People's Mathematics. The analysis of People's Power, for the purpose of this paper, is discussed under Economic and Political Power as well as power derived from embracing People's cultural philosophy – *botho/ubuntu* [*Botho* and Ubuntu are synonymous words, *Botho* used largely by the Sotho speaking group (Bapedi, Basotho and Batswana), while *Ubuntu* is largely used by the Nguni (Xhosa, Zulu, Ndebele; Swazi) group]. In this paper I will use *botho* for my own convenience.

Botho is an aspect of the African people's culture. It has to do with concepts such as: *"Motho ke motho ka batho*" literally translated as: a person is a person through people; which can be further translated to mean that it is through support from other people that a person is able to achieve set goals, a statement that may be related to the *collectivism*, i.e.; better outcomes are achieved through working as collective. It can also be argued that it is within this background that the concept of stockfels* originated. [* Stockfels are people's social schemes, which involve regular contribution of fixed amount of money to one member of a scheme at a time.] Through stockfels, members of the stockfel are for example, among other benefits, able to pay goods at cash price and thus save on hire purchase costs.

People's Mathematics

Rote learning constituted one of the main approaches to mathematics teaching during the apartheid era. Adler [1991] pointed out that with only 12% of black secondary school teachers having a degree, mathematics teaching by and large was tackled bravely by teachers barely one step ahead of their students. As a result, Adler continues, authoritarianism and rote-learning methods predominate. Such approaches nurtured the view that mathematics was an unalterable body of truths, that all what the learners had to do was to memorize formulae and theorems and reproduce them whenever asked for in tests and examinations. To a large extent the questions in tests and examinations remain a meaningless set of questions that have not much bearing on students' lives, or that of the people in the environments in which they find themselves. According to Julie [1991], "the view of mathematics as a human construction to address, describe and solve problems facing society at particular moment is suppressed and obscured", and the question that is still remaining to be answered is "Whose interest is this suppression serving?" Is this a universal phenomenon or is this typically South African? One fact that stands out is that South Africa has its own peculiar problems that have drawn the attention of the international community. As of now, not much has happened in the opinion of a number of South Africans that have previously been disenfranchised in a number of ways. Very little relief has thus far been realized by the few that managed to go through their mathematics studies. Mathematics continues to be used as a gatekeeper, but those who manage to pass through these gates do not seem to make an impact in terms of ploughing back into communities from which they arose.

Slammert [1991] argues that:

> The kind of mathematically educated person that our society produces is usually conservative in outlook, timid in thinking and uncritical about life in general. Further they hide behind the symbolism of their discourse and they regard themselves as neutral, studying only absolute and eternal truths. And so we are witnessing a mystification of ideas and an antagonistic attitude, which gives rise to a completely unacceptable situation. A situation which we are now focusing on in order to transform and build a new maths, a new Science, a new Education and a new technology for a new South Africa. Since we are the makers and actors of such a movement, it is up to us to identify, explain, understand and provide better ways of approach. This calls for a framework with a contextual basis in which people, both inside and outside of the discipline, can work from, identify with, and develop a maths for liberation. The ideas for using the term 'contextual is that our education, our culture, our whole way of life have been pervaded with imperialist ideology.
>
> (p 73)

It is ideas such as expressed by Julie in the previous paragraph and Slammert above, questioning the kind of mathematics that was taught then in South Africa, and the kind of people that the kind of teaching brought about, that gave rise to the concept of People's Mathematics for People's Power.

Adler [1991] also gives the background towards the establishment of the concept People's Mathematics for People's Power, which according to her, was also informed by earlier developments that focussed on mathematics education for democracy in South Africa. The view expressed in the foreword of Julie [1989], that which Frankenstein [1991] reiterates, is that People's Mathematics arose as part of People's Education, a counter - hegemonic movement to remedy the crises in education in South Africa brought to world attention by the school boycotts since the 1976 Soweto schools riots. In teaching People's Mathematics it is expected that some of the outcomes will be the ability of students to be able to integrate knowledge from different mathematical topics; it is also expected that students will be able to perceive mathematics as a human construct, not confined to a particular species of people based on race.

The process of enabling students to integrate knowledge from different mathematics topics is a necessary step towards demystifying mathematics. People's Mathematics takes critical account of how mathematics was and continues to be taught. As a constituent part of People Education, People's Mathematics highlights issues that tend to reinforce mathematics education as a gatekeeper. Among contributions by Slammert [1991] we have:

> I once listened to a group of preschool, primary school and high school teachers complain about their problems with maths teaching. They said that they were convinced that the drilling method of teaching is still the best in that their pupils will then know their work better and by heart. And that teaching children how to tell the time and investigate geometrical figures was quite difficult. Most teachers said that they did not really know why they have to deal with specific topics and not with others. Some even questioned the existence of other topics. For them the syllabus is about the only maths they've seen. What is more alarming is that a lot of progressives also think in this way, leaving very little hope for the subject to be included in the discourse of struggle. So already students, teachers and researchers of maths cannot really be political, unless of course they endeavour to take part in debates of a more sociological nature. In short, mathematics is regarded as an abstract, irrelevant and esoteric discipline having only meaning to those who understand it.
>
> (p 69)

The statement above brings to light the degree of absence of a critical approach to mathematics education particularly at our colleges of education. What is alarming is that since the presentation of the paper by Slammert, outcomes in school mathematics, as judged by the matric results in South Africa, have deteriorated. A number of reasons can be attributed to this fact. One of these is that we are only now beginning to know the truth of the actual products we have in our schools - the truth having been hidden through the moderation of marks during the apartheid era. Now that South Africa is in the process of implementing the new curriculum which can be said to be a product of inputs from among other areas, People's Mathematics, one really wonders how much in service teacher training will be needed to bring them to the level of being able to counteract the imbalances of the past, partly brought about through the style of mathematics teaching.

Drill work is still largely considered as the best way of teaching mathematics; the investigative approach is considered as time consuming and delaying the process of completing the syllabus. The pressure of covering the syllabus is sometimes so much on teachers that they at times do not mind or are not aware that they 'cover' the syllabus so well that students ultimately are not able to 'see' the syllabus at all. Still there is very little room for student teachers at colleges of education to engage critically in the inclusion of some of the topics in school mathematics. What People's Mathematics aspires to achieve is to 'uncover' the syllabus so that students can be able to see among other issues, links between various aspects of the syllabus as well as links between these aspects of mathematics and their own lives and future plans.

Adler [1991] talks about her experience from dealing with students from 'white' South African schools. She describes this group of students as a reflection of a presentation of mathematics as '... a body of knowledge that must be absorbed: questions, problems have only one answer and the object of study is to get each answer right. This technicist approach to scientific knowledge, produces students who are expert in memorising and applying rules, but who struggle to step out of this narrow frame to make meaning of their 'knowledge''. Fasheh [1990] on the other hand correlates most of the graduates of the formal education system within the Palestinian community with:

> ... the Israeli hen: their survival depends on external support, and their values are based on artificial, induced, or symbolic qualities. Such graduates live on a special mixture of courses and curricula that are "scientifically and rationally" planned and prepared for them by experts, mainly from abroad. Further, such graduates are in general alienated from their own environment and are mostly blind or insensitive to its basic problems and needs. When the surrounding conditions change, or when real-world situations must be dealt with, such graduates become confused: the "correct" answers and ready solutions they learned in the schools and universities suddenly become useless and meaningless.
>
> (p 80)

There is a similarity between students described by Adler and those by Fasheh. However the difference ends when the students ultimately graduate. In the case of Fasheh these are people who have not yet attained full political autonomy while in the case of Adler's group, the ultimate graduates are mainly a subset of the economic hegemony. As to what happens in terms of turning these students into the minority elite that dominates the running of South Africa's economy is a matter of consideration elsewhere. What is of interest for the moment is that both pictures, of students who see mathematics as facts that need to be absorbed and memorised, clearly resembles the situation I went through at the University of the North and that which is described by my colleagues from other historically back universities. What is also clear is that the historically Black universities were specifically designed to produce graduates who were destined to perpetuate the effects of apartheid.

In the light of the above it is evident that People's Mathematics faces serious challenges, in the sense that the current mathematics passing rate is deteriorating at an alarming rate. This is happening while the new curriculum, which, as will be shown later, has embraced a great deal of the People's Mathematics philosophy, is being introduced. Unless drastic improvement at grade 12 level occurs between now and the implementation of the new curriculum happens at grade 12, most parents will be clamoring for the "good old days" approaches of mathematics teaching - which was mainly based upon the same uncritical rote learning that is now responsible, ultimately, for the current crisis. Or is apartheid education solely responsible for these outcomes? What has to be remembered in trying to answer this question is to borrow from the same text of Fasheh the message that, '... the ideological environment serves to mark "the boundaries of permissible discourse, discourage the clarification of social alternatives, and makes it difficult for the dispossessed to locate the source of their uneasiness, let alone remedy it"'.

Going Beyond the Dry Facts – The Calculations!

People's Mathematics has to do with going beyond just dry mathematical manipulation of figures. Consider a table presented by Mathonsi [1988] (p.23).

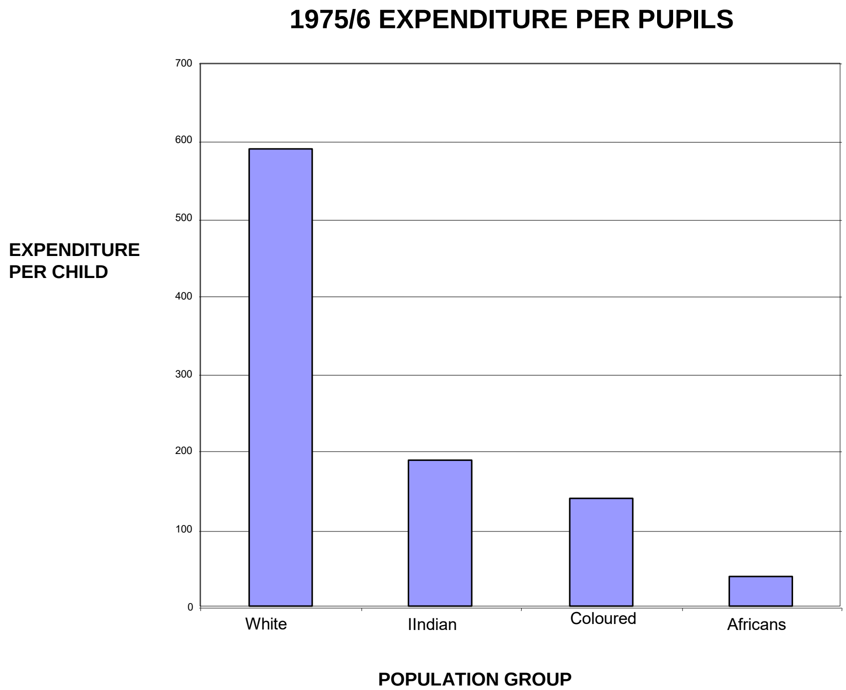
Table 2
Expenditure per Pupil (Rands)

YEAR	RACE	UNIT COST PER PUPIL
1975/76	White	590
	Indian	190
	Coloured	140
	African	40
1979/80	White	1190
	Indian	390
	Coloured	234
	African	78

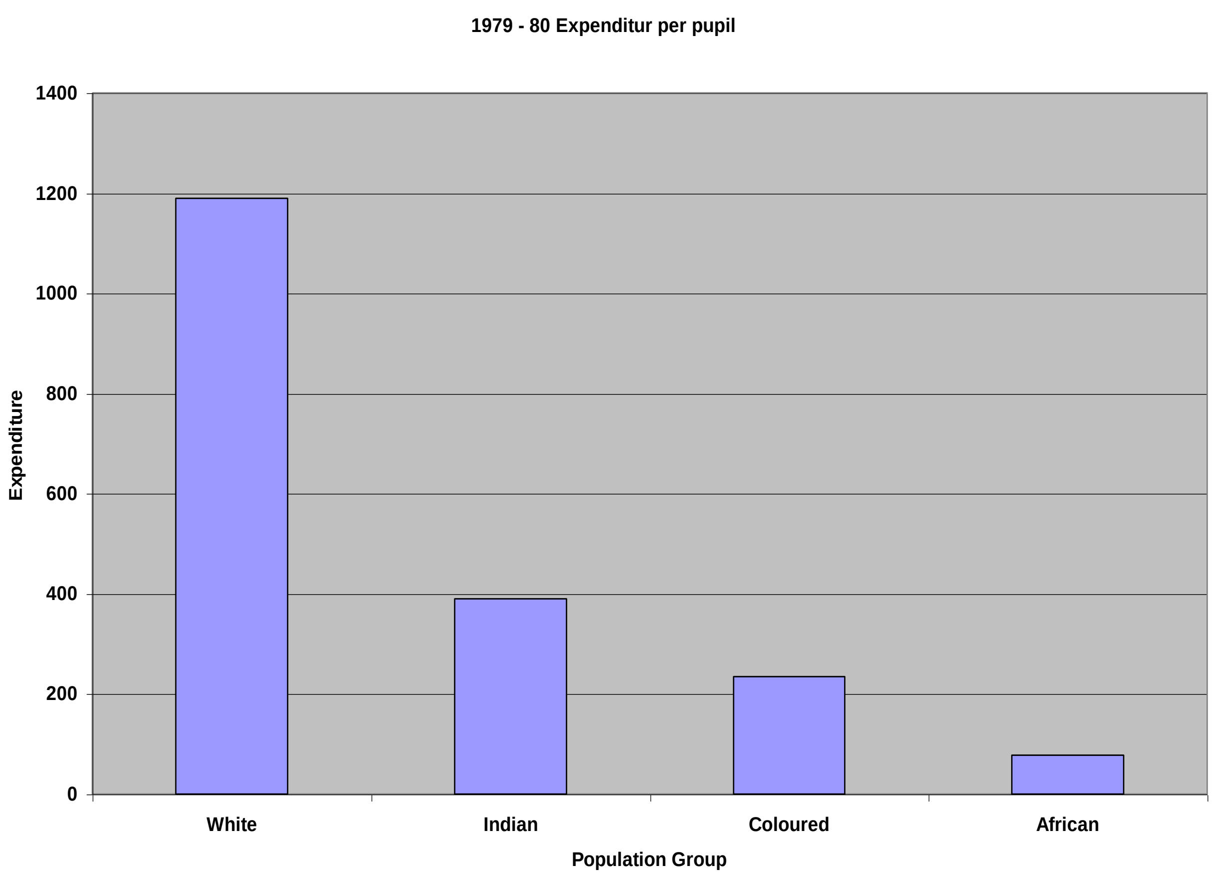
Source: PASCA Factsheet 13

In some of our contemporary mathematics textbooks the question would be: Present the information in the above table graphically. The answer could then be the picture as presented on page 23 presented as follows:

Graph A

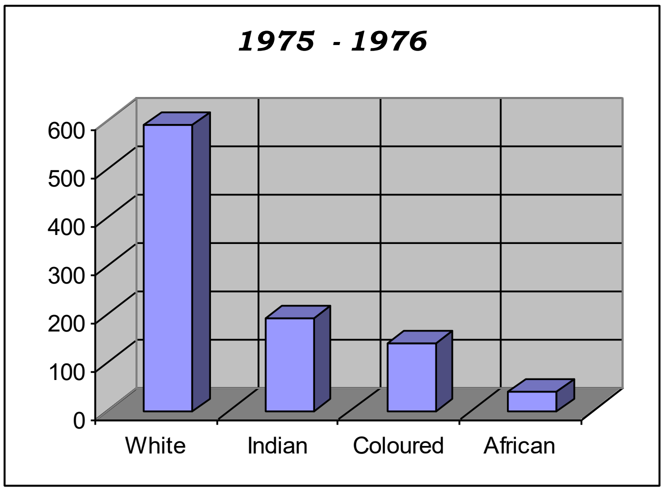


Graph B

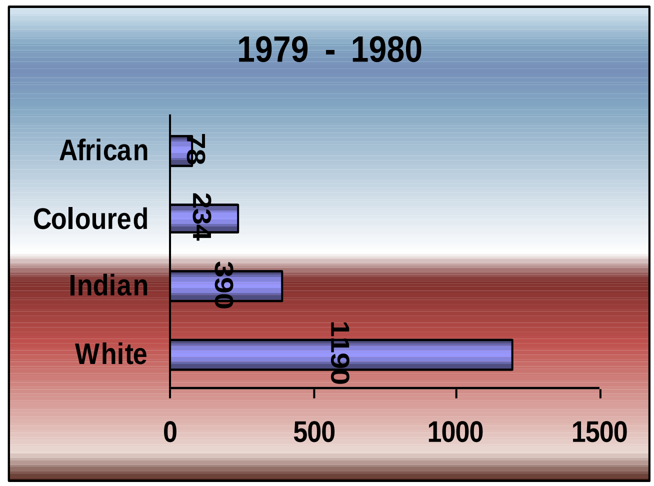


What People's Mathematics demands is that students should look beyond the information provided - the story behind the figures. Students should pose questions beyond the teachers'. These are questions such as why was there such a disparity? How come that the government of the day allowed such state of affairs to happen? What are the implications in terms of the level/quality of education of children of different race groups? What impact could such a difference have when such students went to the university? What should be done by the government? What must we do as mathematics students?

Graph C

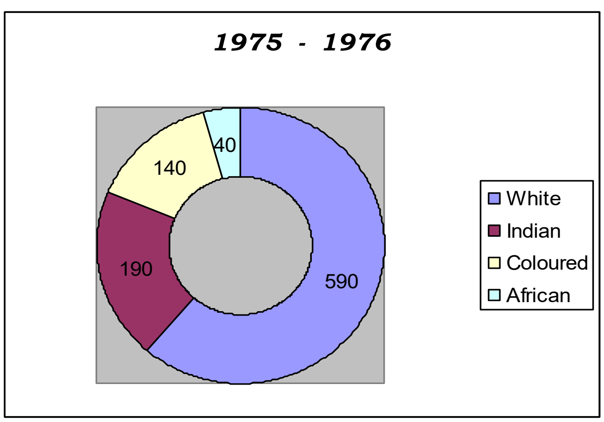


Graph D

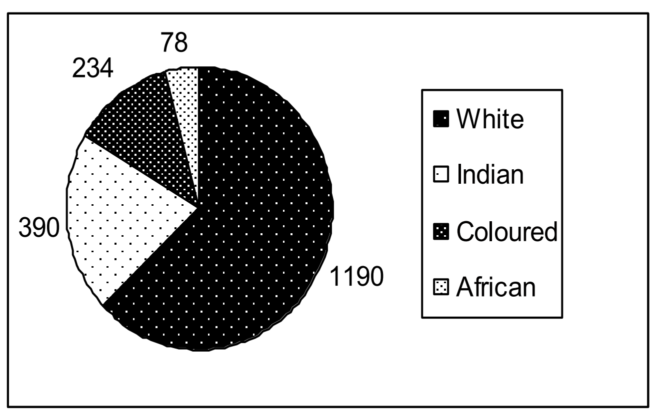


A pie graph for the above would present the following picture:

Graph E



Graph F



How do the two representations affect your perception of the information provided? Is yet there yet another graphical approach that you could think of? How can you present the cumulative effect of the figures presented? And what story does such a picture present to you? Comparing graphs A to G what would normally guide the choice of any one of the given graphs?

Through such questions students may begin to relate mathematics to their daily lives and begin to look at newspaper graph with greater interest. What is of particular significance is the kind of questions that students are encouraged to pose. The essence of People's Mathematics is realized through the creation of the conducive environment for students to be able to ask critical questions. This demands a great deal of time and patience.

It is appropriate to state at this stage that People's Mathematics is actually an interactive approach. It is not only the teachers that have the privilege of asking probing questions. The students as well have a right to pose questions to other students as well as to the teacher, to inquire further about the mathematics that is being taught. Students have to find out what mathematics exactly has to be learned, how this mathematics should be learned and the reasons why the particular mathematics or mathematical activity has to be done. In responding to questions students have the freedom to answer in a manner that they feel appropriate, as a means of expressing the understanding rather than only attempting to remember what the teacher said or what the text book states. This form of interaction provides the teacher with a better conception of how students perceive mathematics and the world around them. The realization of this People's Mathematics goal of having students enjoying the freedom of interacting with one another will take some time to be achieved. As pointed by Taylor et al. [1991], it is a long process, which needs to be worked out through before any definite answers concerning People's Mathematics can be arrived at. By definition, People's Education should be formulated through democratic discussions amongst as wide a spectrum as possible.

People's Mathematics is, as also correctly viewed by Frankenstein [1991], one of the groupings in the recent international efforts to organize critical mathematics education. Working against methodologies that indirectly nurture oppression was one of the critical areas of operation of the Mathematics Commission of the NECC. People's mathematics, has to do with bringing the critical perception of mathematics and its teaching together. Focus in the teaching of People's Mathematics is not only on skills, but also mainly on application. Various authors such as Taylor et al. [1991] present examples of this. In this article attention is drawn to facts such as mathematics as a response to particular problems and that all cultures borrow from all cultures to suite their own needs. The tendency, as Breen [1991] outlines, is to move towards mathematical modeling and mathematics in the real world. In contextualising reality Breen does however caution that greater effectiveness will be realized if it is the students who take greater responsibility in problematising their own reality.

People's Mathematics, as has been said of People's Education, takes a dynamic outlook. It has to be relevant to the current students'/society's needs as well as their future directions, i.e. foregrounds. As a result, it offers choice, freedom to decide and responsibility over the choices made. Through its critical nature, these choices are informed by developments within and around the communities. People's Mathematics embraces democracy, thus fosters liberation of people from all sorts of oppression from a mathematical perspective.

People's Mathematics also stresses that mathematics is a human creation and that people over the years have been able to create mathematics to suit their needs of the time. This makes a close link of People's Mathematics with Ethnomathematics. This link of People's Mathematics with the people's mathematical daily lives renders it accessible, thus eliminating the gate keeping nature that characterizes the alternative mathematics.

Through the learning of mathematics there is a great potential that we as a nation can get to know South Africa better so that we can benefit from its multiplicity of resources. Mathematics provides a foundation to access different study fields for the purpose of exploiting these resources. However, as has been alluded to earlier, mathematics has been, and still has a potential of being, used as a gatekeeper. The nation is still at risk of being kept out or being disempowered through the negative approach to mathematics teaching as well as lack of resources to address the subject accordingly. Volmink [1990], addresses some of these issues as follows:

> To know and to understand is a basic human right. Mathematics, maybe more than any other subject, explains things and helps us come to know our world. It provides us with the means to think thoughts and to create and examine ideas that we otherwise could not. It also helps us to articulate these ideas and images, which would not be expressible in any other way. Mathematics is therefore a significant means of empowerment. To deny some students access to the process of mathematics is also to predetermine who in society will move ahead and who will stay behind. But at the same time mathematics as it is taught in schools has been disempowering
>
> (p 98).

The implication here is that a great deal of power has been lost through the manner in which mathematics was and still is taught. The devastation of apartheid has left deep wounds within the people's education system. The after effects of this scourge, declared as heresy against humanity, is still evident even now, four years after the gaining of political power by the people.

BOTHO

Before looking into the extent to which the new curriculum has incorporated People's Mathematics and the extent to which the philosophy of *botho* was encompassed in the curriculum, it is necessary to look further into the *botho* concept.

Our freedom in South Africa has come with it a greater revelation of disparities between blacks and whites. The *botho* aspect of People's Mathematics call for a 'collective human response to an oppressive situation', which in this case is general poor outcomes from schools (as exemplified by matric results), poverty and crime - some of the by-products of the apartheid regime in the land of plenty. The collective human (*motho ke motho ka batho)* exercise must 'reclaim people's lives, their sense of self-worth, and their ways of thinking from hegemonic structures, and facilitate their ability to articulate what they do and think about in order to provide a foundation for autonomous action'. (Fasheh [1990]). Working as a collective, working towards upliftment of fellow human beings, and working towards equitable distribution of our national resources is necessary if we have to reclaim our lives from the shackles of the past. Working through mathematics education is one of the main routes to success as mathematics has in the past, more than any other subject does, served as a gatekeeper towards the 'green pastures' for which we were not meant, according to Verwoerd.

Adler [1991] in her analysis of Breen's [1991] work makes reference to Humanistic mathematics. There are clear similarities between this approach of mathematics and a *botho* perspective. Reference is made to combating elitism, racism and sexism. Under *botho* this is addressed under the concept of cooperative action (operating as a collective, in solidarity - addressed earlier under 'motho ke motho ka batho'). Adler goes further to say that:

> The social organisation of the classroom is seen as a fundamental part of this work and involves small groups who work together on the task at hand. The skills developed through this kind of practices are: specialising, pattern-seeking, generating, conjecturing, and importantly, communicating. Children are encouraged to develop ways of communicating their findings verbally and symbolically, so that they are intelligible to those not involved in the task. It is argued that these involve learners in mathematical thinking. In addition because the work is done in groups, and because there is no single way of progressing through the task, children can learn to co-operate, share ideas and discuss amongst themselves what they think and why. Experience has further shown that children of ranging 'ability' can become effectively and constructively involved in an investigation and so develop positive attitudes to themselves and their ability to do 'maths'. Gender domination is also undermined since research suggest that girls function positively in co-operative learning situations. The social reality so constructed is non - authoritarian and non- elitist. It is a far cry from the passivity and alienation currently produced in South Africa.
>
> (p 55).

Talking of co-operation or solidarity, one of the *botho* expressions in Sepedi is that *"Tau tsa hloka seboka di fenywa ke nare e hlotsa*". Literally explained this means that limping buffalo can beat lions without unity. Figuratively what this means is that unity is strength or simple tasks may remain impossible unless there is cooperation. Aspects that are embraced in the process include: social organisation; cooperation; communication; sharing of ideas; solidarity. Because of collegiality children or members of the cooperative group are able to ask the 'why' and 'how come' questions. Subsequently participants are able to come out of the groups with a better understanding of underlying concepts. Given room to explain each member is then able to verbalise or transmit the message differently though similarly.

Reading through a 'Dialogue' between Ascher and D'Ambrosio, [Ascher and D'Ambrosio [1994]], one identifies clear links between Ethnomathematics and *Botho*. In response to one question on educational aspects of ethnomathematics, this was D'Ambrosios' response:

> " .... I see school as a kind of meeting place where people with different experiences come together to socialize their experiences. Thus they begin another experience, which is to put their capabilities together to function at a common task. Ethnomathematics is a most suitable pedagogy for this kind of school, an institution which addresses not individual action but cooperative action. Because ethnomathematics is not passive it is loaded with critical components. But most importantly, the gains and advancements will be collective and not individual. While keeping capabilities very individualized (each individual is different from the other) we have to generate, through this socializing school, respect for the other with all his/her differences, solidarity with the other in his/her pursuit of

> satisfying the needs of survival, and transcendence of their material and spiritual needs, learning how to act in cooperation with others, putting together physical and intellectual resources to reach common goals. These three components: respect, solidarity, and cooperation, constitute an ethics for a global civilization and serve as the basis for my model of the school of the future.
>
> (p 43).

Respect, solidarity and *cooperation* are some of the main aspects of *botho*. This is interesting in the sense that ethnomathematics is also associated with the mathematics that engages cultural aspects that in some cases have grown unfamiliar to the current generations. On the other hand there is also an outcry that a good number of young South Africans have lost their culture, the young have lost respect for the elderlies, people have turned too individualistic!

Outcomes Based Education [OBE] Curriculum and People's Mathematics

Outcomes Based Education and Training [OBE] is the new South African system of education that has replaced the apartheid education system. This system is being phased in, in stages as a result of training implications. There is a need to outline the context in which some of the terms are used in OBE:

Specific Outcomes:

> These refer to the specification of what learners are able to do at the end of a learning experience. This includes skills, knowledge and values, which inform the demonstration of the achievement of an outcome or a set of outcomes. The focus of OBE and training is the link between the intentions and results of learning, rather than the traditional approach of listing of content to be covered within a learning programme
>
> (DoE:1997c, p.17)

It can be argued here that an attempt is being made to clarify the learning activities as well as to enhance teaching, by way of emphasizing that at the end of a teaching process, the success should be judged by the objectives realized. To further help with the assessment of the success of teaching, another expression that forms part of the new curriculum discourse is **Range Statements:**

> Range statement indicate the scope, the depth, and parameters of achievement. They include critical areas of content, process and context, which the learner should engage with in order to reach an acceptable level of achievement.... The range statements provide direction but allowance is made for multiple learning strategies, for flexibility in the choice of specific content and process of a variety of assessment methods.... The range statements have the additional function of ensuring that balance is maintained between the acquisition of both knowledge and skills and the development of values.
>
> (DoE 1997c, p 16).

Provision of a room for 'development of values' here may be seen as creation of space for *botho* in OBE. While this may be so, it must be remembered that the South African situation is rising up from a period of dominance. Ideologies have in more than one way relegated African values to the 'back seat'. What this means is that values will only be seen as those that are in line with the dominant culture - the western culture. The place of *botho* in enhancing the learning of mathematics will for generations be regarded as an imposition of something that does not belong to the African culture.

Botho within the OBE

While it may be argued that *botho* contributed towards allowing the foreigners to invade the African soil, in some circles it could also be argued that it is the same *botho* that sustained the people during the dark years of oppression. Attention is now being given to the extent to which the unifying South African main world view, or philosophy of life, *botho* is embraced within OBE. Ndungane, the Anglican archbishop of Cape Town in his introductory article on *botho* in the Mail and Guardian [February 20 to 26 1998], implores that:

> Ubuntu [Botho] should be embodied in all that we do: the big act of society and the little acts of kindness of the individual. ..... We therefore face the enormous challenge of teaching people about ubuntu[botho]. This is the responsibility of the government - to introduce it in the curriculums of our schools and universities, for example - business, the media, religious institutions and parents.

(p 34).

Ndungane also makes a point that it is important to remember that the values that are involved in botho place a strong emphasis on the respect paid to ancestors and traditions and to various religious mores. Of significance here is coincidence of D'Ambrosio's [1994] reference to *respect* as an aspect of the ethnomathematics that has to be taught in schools. What is of immediate concern at this stage is the teaching and learning of mathematics. and how OBE is being implemented, and the extent to which *botho* is being integrated in the mathematics curriculum.

In some respects, the disempowering, lack of collectivism, regimentation and compartmentalization approach that is being applied in mathematics teaching has much to do with the absence of *botho* in our teaching approach. The empowering effect of mathematics is lost as a result of lack of the essence of togetherness. People's Power comes from organised people. *Botho* does neither feature directly in the mathematics curriculum framework nor in other learning areas such as Human and Social Sciences. This is regrettable if one consider the extent to which people's culture has been interfered with by the western 'civilisation', particularly through apartheid policies. The fact that *botho* may be incorporated under *Specific Outcomes* such as, "*Demonstrate understanding of the historical development of mathematics in various social and cultural contexts"* or "*Critically analyse how mathematical relationships are used in social, political and economic relations"* is not enough. *Botho* is a unifying concept within South African people culture and thus deserves prominence in the curriculum in no uncertain terms.

People's Mathematics and Cultural Affirmation in the OBE?

For the large section of our mathematics teaching community, using artifacts of European origin still remains the only way for providing teaching aids. The beautiful pebbles of the South African oceans and rivers never find a way into the class rooms in the teaching of counting, colors, sizes, mass, etc. Can culture be used as a tool of oppression and at the same time be used as a tool of liberation? Through religion Africans have to a large extent been advised to look down upon their own cultural values and beliefs. This state of affairs has led to a loss of sense of direction, lack of self-esteem, values etc. Those Africans who embraced Christian faith did however remain embedded within the African culture- thanks to apartheid. During this period majority of the blacks never found an opportunity of linking up the education they received with their own culture.

One of the mathematics specific outcomes in DoE [1997c] is *"Analyse natural forms, cultural products and processes as representations of shapes, space, and time"*. Acknowledgment is also made in the OBE document that those mathematical forms, relationships and processes embedded in the natural world and in the cultural representations are often unrecognised or suppressed. Learners should be able to unravel, critically analyse and make sense of these forms, relationships and processes. Among the range statements (that is statements that indicate the scope, depth, level of complexity and parameters of achievement on a particular specific outcome) we have:

* Observe nature, cultural products and processes;
* Explain use and value of cultural products and processes;
* Analyse different cultural products and processes at different epochs;
* Represent artifacts in various mathematical forms - 2D and 3D;
* Critically analyse the misuse of nature and cultural products and processes.

(DoE 1997c, p. MLMMS -16)

Given the OBE framework such as outlined above on aspects of culture, what more would we like to have as mathematics teachers to ensure that people's cultures are taken on board in our daily teaching? The geometric techniques used in the design and the building of thatched roof houses by some of the people in the country were never considered in the teaching of Pythagorean theorem and other areas of geometry. In some cases some of the very good builders never had an opportunity to attend any formal mathematics class. All their trade they got from their predecessors. Inclusion in a formal way of these aspects of cultural artifacts or products, as referred to in OBE, should contribute towards restoration of self pride among communities whose culture were hitherto looked down upon. Hopefully this can also bring the sense that education is not only for those who have gone through the formal schooling.

This background information is essential for our contemporary mathematics students to note that there is and there has always been some degree of mathematics among the people even those who never had an opportunity to learn mathematics in formal mathematics classes. The OBE specific outcome, *"Demonstrate an understanding of the historical development of mathematics in various social and cultural contexts"* presents a framework within which the teaching of mathematics can then be linked with various communities' developments. Some of the range statements related to this specific outcome are:

- Show knowledge of counting styles in different cultures;
- Demonstrate knowledge of ways of pre-colonial counting in Africa;

- Critically analyse mathematics as a predominantly European activity;
- Analyse mathematics ideas from own culture.

That the framework for addressing the educational imbalances exists is one thing, but the actual critical analyses of these imbalances and correcting them is another. The level of contamination of people's thinking as a result of many years of domination has to be taken up seriously in the implementation of the OBE. The extent to which people have tended to look down upon themselves is an issue that demands special attention.

Focus also has to be directed to the compatibility of the mathematical methods that were applied then and environmental conservation. In outlining the *Rationale* in the mathematics curriculum, Mathematics Literacy, Mathematics and the mathematical Sciences as domains of knowledge are viewed as significant cultural achievements of humanity. Thus our students have to appreciate that they can also create mathematics as their predecessors have done in the past and that their creation has to be compatible with our current environmental needs. Indeed, as Volmink [1990] outlines:

> For so long, learners in mathematics classrooms have been socialized to believe that their own experiences, concerns, curiosity and purposes are not important. Mathematics is seen as being devoid of meaning, bearing no relevance either to their every day experience, or to the pertinent issues in their societies. Learning mathematics for these students partakes more of the nature of obedience than of understanding.
>
> (p 98)

As result of obedience being one of the main aspects of *botho,* it then becomes problematic for students to challenge some of the issues encountered at school level. Mathematics taught in a traditional way, as a result, becomes a tool for training students to be submissive. This may be seen as the dark side of *botho* - that the young ones are not to question the wisdom of the older members of the society. Subsequently the element of critical outlook of issues is then undermined.

On the other hand, the Learning Outcomes for Teacher Education in DoE [1997a], under Areas of Learning - Life Orientation, does make provision for students to question issues that in the past would only be accepted as facts that need no questioning. This provision is covered in the statement: 'The learner will demonstrate the ability to: Exercise a critical and informed understanding and the nature of discrimination and barriers of learning.'(p.87). On the same page it is also stated that learners will demonstrate the ability to 'Show knowledge and appreciation of, and respect for, the beliefs, practices and cultures of the communities of South Africa.' While this provides room for *botho* consideration, one interesting aspect under these 'Life orientations' is that students will not blindly fall into the *botho* culture, but will do so with some degree of critical outlook.

People's Mathematics and Economic Power in the OBE?

Economics is the science of the production and the distribution of wealth. Production and distribution are both mathematical terms. The extent to which our school mathematics teaching addresses the empowering nature of economics as well as the disempowering nature of lack of wealth is an issue that warrants some explanation. What does it mean to say that a country is economically strong? What links does this have with People's Mathematics?

> *Mathematics is used as an instrument to express ideas from a wide range of other fields. The use of mathematics in these fields often creates problems. This outcome aims to foster a critical outlook to enable learners to engage with issues that concern their lives individually, in their communities and beyond. A critical mathematics curriculum should develop critical thinking, including how social inequalities, particularly concerning race, gender and class, are created and perpetuated.*
>
> (DoE1997c, p. 9 MLMMS)

The specific outcome linked to the above statement reads as follows: " Critically analyse how mathematical relationships are used in social, political and economic relations". The overt linking of mathematics and economics issues such as income distribution in South Africa clearly covers areas that People's Mathematics calls for.

The fact of the matter is that disparities are still so vast that the situation tends to threaten the newly born democracy. The prevailing violence is widely attributed to these vast economic disparities. In fact, according to Phinda Madi, as reported in the "Sunday Independent Business" of the 11th January 1998, " There is now serious risk of a new kind of economic apartheid where the recently unbundled organizations feel that they have now earned the right to be left alone." For the disenfranchised, it is necessary to look closely at the mathematics that is being offered in the curriculum currently and that which should be in the new curriculum, to ensure that the content and implementation contribute towards their well being economically. On the other hand if we all want to be the united "rainbow nation", it is the responsibility of those who are better off also, to ensure that necessary steps are taken to facilitate the mathematics learning that contributes towards the better life of all South Africans.

Conditions under which mathematics is being studied is an area of concern that demands action linked to People's Mathematics for People's Power. Julie [1991 (p.38)], made reference to the NECC acceptance of the resolution : ... *teaching practice, which helps people to be creative; to develop a critical mind; to analyze* . He went on to refer to the ' *...replacement of rote learning methodology of Bantu Education with a methodology that develop an inquiring and critical mind...'* and that reflective thinking and inquiry method were nothing new. That may have been true to the audience that was being addressed. Unfortunately for the large section of our current teaching force such concepts are still very new. Creativity still remains a rare commodity. Whilst provision of resources over the years was not equitable, the maintenance of the meager resources also left much to be desired. This tendency has not improved in most of the areas since attainment of freedom. Some of the schools are not habitable at all, not because there are no facilities but simply because of lack of some degree of creativity and minimal improvisation. The concept of collectives as an aspect of the people's education becomes an issue of great relevance in this regard. The struggle for resources needs to take a different format. Understanding of the range statement *" Demonstrate importance of social service charges, pensions, etc."* may seem common for those who grew up in democracies. For a number of young and old South Africans, inclusion of such statement in the curriculum framework is essential. Over the years the understanding and the observation was that such taxes were mainly used to benefit one section of the population, the privileged whites.

One of the mandates of the People's Mathematics Commission was to contribute towards the development of new educational materials. Very few materials for schools are available at this stage. The development of these materials has to be accompanied by a rigorous exercise of providing teachers support programmes into the effective utilization of the new materials. The range statement "Compare the financing of education under apartheid and after 1994" is very relevant. Comparing the expenditure per child for the 1975 and 1976 and 1979 and 1980 expenditure per child, a number of questions arise. What is the cumulative total of the disadvantage that built up to 1994? How does this affect the post 1994 budget for the previously disenfranchised, and why so? Critical mathematics education calls for response to such questions. The legacy of the past has created a problem to the current generation. A full understanding of the past is essential so that thorough critical approach to the solutions to our current educational problems can be resolved. Engagement of students now is essential to ensure that these problems are addressed now and not postponed.

In 1975 expenditure per child was over R500-00 more on white a white child than on an African child. In 1979 the difference was more than R1000. What is the value of a rand now as compared to the rand then? How does this provide a white child with an advantage in life? What are the critical areas where funding should be focussed in terms of addressing the backlog? How should this reallocation of funding impact on unemployment? Critical analysis is of the essence, and this has to be done in the context of the RDP demands, the 'culture of entitlement', the prevailing culture of teaching and learning. Have we grown wasteful over the years? What impact has the culture of 'lack of ownership' made on our respect for community property?

People's Mathematics and Political Power in the 0BE?

Power has to do with the ability to act; to influence; to exercise authority or authority to exert force. We need to understand how People's Mathematics relates to political power. Do we have a full understanding of political power? How does this relate to mathematics teaching and learning? Can we clearly outline the role of teachers, students and parents in this context? In South Africa clear definition is still necessary for people to find their feet in the fields. The power to discuss community concerns, to make recommendations with regard to steps to be taken on the basis of our mathematical understanding of issues still needs some development. Recommendations have to be considered meaningfully before people can claim to have some power over their lives. How does mathematics relate to political freedom and empowerment? How is this captured in the new curriculum? How do we achieve in our learning and teaching?

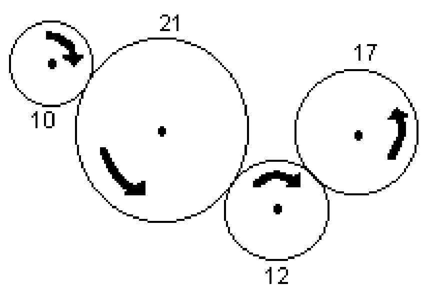
Despite the fact that South African people have attained political power there are still signs that indicate that this power has not sunk to the level of some members of the community. How mathematics is taught in the classroom now and in the future can have some impact in the way people perceive themselves in life - either as independent or as dependents. The extent to which people are and will be able to make choices in areas related to the choice of mathematics as a subject; or its contents as a field of study; or how it should be taught or studied, will determine the extent of the people's freedom and power.

It is important to look carefully at the South African new curriculum proposal and compare it with ideas raised during the days of attempting to bring about or laying the foundations for the alternative mathematics curriculum, the People's Mathematics. In one of the papers presented towards this ideal, Taylor et al. [1991] argued that:

> The way in which curriculum materials give meaning to mathematical ideas is crucial to the shaping of pupils' conception of mathematics and the world around them. Examples, illustrations and exercises contained in the textbooks and other teaching materials should be constructed so as to break down race, class and gender barriers and to foster critical and inquiring attitudes.
>
> (p. 30)

The rationale for studying mathematics and other related mathematical fields, as outlined in OBE [1997], is, among other issues, *to provide skills to analyse, make and justify critical decisions and take transformative decisions, thereby empowering people to participate in their communities and in the South African society, as a whole* in a democratic, non-racist and non-sexist manner.

The nature of the problems given to students has bearing on performance of students as well as how teachers perceive mathematics teaching. Looking at the 1986 Mini Mathematics Olympiad question papers [MASA .:1988],one finds questions such as:



> The illustration shows part of a fixed-axle gear mechanism, which consist of four cog-wheels in mesh. The largest cogwheel has 21 teeth and provides the driving power to rotate three smaller cog-wheels, which have 10,12 and 17 teeth respectively. If the gear mechanism starts from rest, how many revolutions will the large cog-wheel have to turn before one full cycle is completed and all four cog-wheel are in the identical position from which they started?
>
> (p 10)

For some this problem may seem like a real life problem. For most non-English speaking students such a problem may be difficult right from the beginning, consideration being only on the language used and in the mathematics involved. Unfortunately under-achievement based on language difficulty is most of the time not much given consideration. This is a very disempowering experience, mainly through the curriculum materials being made available to learners and teachers. The exercise of People's Mathematics for People's Power takes such issues much into consideration to ensure that people are not kept out of the mathematical arena through non-mathematical issues.

Taylor et al. [1991], points out that the ways in which curriculum materials give meaning to mathematical ideas is crucial to the shaping of pupils' conceptions of mathematics and the world around them. Very little during 1998 can therefore be expected from the implementation of the new curriculum at grade 1 level. 1998 marked the beginning of the implementation of OBE. Teachers at grade 1 level have only begun to familiarize themselves with OBE. The new shaping that we hope to achieve through the new curriculum can only be felt, in terms of matric outcomes, much later. However, the contents of this framework do not stop teachers from implementing some of the ideas right away. Engaging with political organizational systems and socio-economic relations is already taking place. The question could be the extent to which this engagement is taking place or how the processes link up with mathematics teaching and learning.

The product of working as collectives in and outside maths classes must have a bearing on how teachers and students develop trust in working as a group. It is within group dynamics that people's power is generated. One area where collectives can be effective is on professional subject associations or organisations. Over the years people were discouraged to participate on the basis of colour. An effort is essential by all that have some understanding of the baggage that the majority of black South African teachers are still carrying. This baggage unfortunately continues to be transferred to younger generations. Participation by staff members from colleges of teacher education has over the years remained very limited. What this means is that very little is known by teacher trainees, and subsequently the vicious circle is being perpetuated.

The extent to which *botho* has been incorporated in the mathematics curriculum will remain subject to individual judgment. It should however be noted that according to the introductory statement to the policy document [DoE.1997c]:

> The curriculum is at the heart of the education and training system. In the past the curriculum has perpetuated race, class gender and ethnic division and has emphasized separateness, rather than common citizenship and nationhood. It is therefore imperative that the curriculum be restructured to reflect the values and principles of our new democratic society.
>
> (p 1)

It is in the above spirit that issues such as *botho,* in the eyes of the author, were not expressed as explicitly as one would have expected. It could be in line with this spirit of reconciliation that concepts such as power to the people have been played down, if not totally excluded from the text. What remains a fact is that for many of us who have survived the apartheid regime, the little economic strength that remained with the people was large due to *botho* that prevailed among them. The extent to which this exclusion will impact upon the actual empowerment of people remains to be seen.

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SOCIAL INTERACTIONS AND MATHEMATICS LEARNING*

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Abstract

In the 70s Doise, Mugny and Perret-Clermont (1975, 1976) underlined for the first time the essential role played by social interactions in cognitive development. Since then many authors have been studying social interactions and their mediating role in knowledge apprehension and in skills acquisition. Inspired by Vygotsky's theory, many contextualized researches were conducted and they showed that social interactions, namely peer interactions, were a main facilitator factor for pupils' socio-cognitive development, their performances in Maths tasks and their school achievement in this subject. Contextualized studies also underlined the power of peer interactions in promoting pupils' social integration and participation. Interaction and Knowledge is a research-action project that is implemented in several Maths classes (5th to 11th grade) and whose main goal is to promote peer interactions in the Maths class as a way of changing the didactic contract and facilitating pupils' socialization and school achievement. A deep analysis of peer interactions shows how important the social and cultural aspects of learning mathematics are in pupils' performances in the Maths class. The examples that we are going to discuss underline the role of many psycho-social factors such as the situation, the social and school status of the peer, the work instructions that are given and the didactic contract. Our data stress the importance of this kind of analysis if we want to promote more positive attitudes towards Maths, as well as better school performances.

Theoretical background

Social interactions play a fundamental role in knowledge apprehension and in skills acquisition as well as in socio-cognitive development. The first studies by Doise, Mugny and Perret-Clermont (1975, 1976), still using a *quasi-experimental* design and based on piagetian tasks, clearly underlined the power of social interactions. In these studies children showed more progress when they were interacting while solving the tasks then when they did them individually. But even stronger than that, the promotion of their cognitive development remained stable in time, as they maintained their performances when asked a long time later and when working individually once again.

Vygotsky's (1962, 1978) theory shed light on the essential role played by social interactions, namely when we think about scientific knowledge. The importance of contextualized researches became apparent and school classes have been a privileged stage for research during these two last decades. Tasks were no longer piagetian, but were directly related to curricula contents and psycho-social factors such as the situation, the task, work instructions, the actors involved in the situation, the contents, all of which were deeply analysed. Performances were no longer seen as independent of these psycho-social factors and so social interactions played a significant role in the way they mediated pupils' relations with school knowledge as we may see in a recent literature review (Liverta-Sempio and Marchetti, 1997).

Peer interactions were often studied and they seemed to be quite effective. In the two last decades many studies have stressed their positive effect on pupils' performances and in their school achievement, namely in Mathematics (Perret-Clermont and Nicolet, 1988; Perret-Clermont and Schubauer-Leoni, 1988; Schubauer-Leoni and Perret-Clermont, 1985; Sternberg and Wagner, 1994). Peer interactions were often associated to the socio-cognitive conflict and were seen as a way of implementing a co-construction of knowledge. They seemed to be a powerful way of confronting pupils with one another's solving strategies and that made them decentralize from their own position and discuss the one another's conjectures and arguments.

Knowledge was then conceived as a social construction. Mathematical knowledge was seen as exterior to and pre-existing in the subject and so one of the pupil's tasks was to find out meanings of that knowledge in order to apprehend it. Facing the social dimension of mathematical learning obliged us to conceptualize learning as a much more complex process, in which teachers and pupils played dynamic roles. The relations established among them, the intersubjectivity they were (or were not) able to build (Wertsch, 1991) were main points in pupils' performances and school achievement. Peer interactions promoted better relations among pupils, an increase in their self-esteem and in their ability to construct a common intersubjectivity. Thus, implementing peer interactions within the Maths classes proved to be an effective way of promoting pupils' performances and school achievement (César, 1997; César e Torres, 1997).

The importance of the situations was illustrated by several studies that compared school performances with daily life performances in tasks that were equivalent in their degree of complexity and in the contents they were related to (Carraher, Carraher and Schliemann, 1989; Saxe, 1989; Wistedt, 1994). In all these studies subjects had much better performances in their daily life activities than in school tasks, probably because daily life activities were meaningful to them. But performances may also be different when we change only the work instructions even without changing the situation nor the task, which is extremely important in pedagogical terms (César, 1994, 1995; Nunes, Light and Mason, 1993) and when the didactic contract changes (César, 1997; César and Torres, 1997; Schubauer-Leoni and Perret-Clermont, 1985). It is the didactic contract that legitimates what both pupils and teachers expect from each other and so it plays a most important role in the way pupils behave, in their self-esteem, in their persistance when they are solving a task, in their performances and in their school achievement.

Vygotsky (1962, 1978) introduced the notion of zone of proximal development (ZPD) and argued that teaching would be much more effective if teachers were able to work with their pupils in that zone. Vygotsky believed that social interactions were powerful, but he thought they were only an efficient way of promoting students' learning and socio-cognitive development if students were interacting with a more competent peer. Recent studies showed that peer interactions are much more powerful in themselves than what Vygotsky conceived, as both in assymmetric and symmetric dyads pupils are able to progress and, more important still, in assymmetric dyads they both progress. This means that there is no need for a more competent peer in order to facilitate better performances, interaction itself is enough.

All these findings can be very important in teachers' practices as it becomes clear that pupils' performances and school achievement are very complex processes. Discovering that the more competent peer could also progress was an essential step to believe that implementing peer interactions in the Maths class during a whole school year or several school years could be a good way of dealing with the degree of underachievement we have in this subject. Pupils' attitudes towards Maths are often very negative, they usually don't believe they can learn this subject and be successful. Thus, implementing a new didactic contract and innovating practices was fundamental if we wanted to promote school achievement in Maths.

To understand the role played by peer interactions in the promotion of a positive self-esteem, more positive attitudes towards Maths, better performances and school achievement we need to carry out a deep analysis of the interactions that take place within the Maths class. Some authors had already done this kind of work but only from a didactic perspective (Brun and Conne, 1990). We decided to do this work in an interdisciplinary team as we wanted to gather different ways of looking at the same reality. This analysis provides information about the mechanisms involved in pupils' communication and performances, about how they build their conjectures and argumentations, about the role played by self-esteem and emotions in all that work, about how they are able to regulate different ideas and solving strategies related to a given problem, about how they negotiate within the dyad and the way they play (or don't play) a leadership role (César, 1997; César e Torres, 1997).

We all live in a social world. Social interactions play a most significant role in our lives. Why should this be forgotten when we are in a school class?

Analysis of a peer interaction

CASE 1 - And if I don't really believe you, can I still learn with you?

Problem - A grocer sold half a cheese, then a quarter and finally a sixth. He then checked that 125gr. were left over. How many kilos did the cheese weigh in the first place?

[V. starts drawing a circumference and then stops to read the problem again.]

M. - What's that?
V. - It's a cheese.
M. - A cheese?... What for?
V. - Now I'm going to draw what he sold...
M. - But I think you do this with sums...
V. I don't know how to do it with sums... so I'm going to see if it works this way...
M. - Then do yours, I'll do mine and then we'll explain.

[Each one uses his own solving strategy. V. uses a graphic representation strategy connected to an arithmetic strategy; M. uses an arithmetic strategy from the start. V. finishes first.]

M. - How's yours?

V. - I drew the cheese, then I divided it into 6 equal parts... so as to be easier... Get it?
M. - More or less... I understand what you did, but I haven't yet figured out why you divided the cheese in 6 parts and not in 2... first he sells half...
V. - I know... but I had to know how to mark a half, a quarter and a sixth... a half and a quarter is easy... the hardest one is the sixth, so I started with that... or else I wouldn't know how to go about it, after marking the half and the quarter I wouldn't see where a sixth was...
M. - Sorry?
V. - Draw a circle! [M. draws the circle]
V. - Now, mark half, which is what you sold. [M. does as he tells her to]
V. - Now mark another quarter, which is the other bit you sold. [She does]
V. - Now mark a sixth, which is the third bit you sold. [M. stops, pencil in hand and says]
M. - Oh! I see! It's much harder like this... do yours! It must be better.

[V. draws the cheese again, divides it in six parts, traces half, then a quarter and a sixth. M. follows his steps carefully and says the numbers out loud as he goes along. Then he says]

V. - See what's not traced?
M. - Yes.
V. - I think it's half of a sixth... so it's 1/12. If 1/12 is 250gr, then the whole cheese is 250gr x 12, which is 1500 gr [He had done the sum on the calculator]. **That's 1,5 kg.**
M. - But I didn't get that!
V. - How did you do it?
M. - With sums. I added 1/2 + 1/4 + 1/6 and ended up with 3/12. That's what he sold. One cheese is 12/12. So I subtracted 3/12 from 12/12 and got 9/12, which are 250gr... but now I don't know how to go on.
V. - I don't understand your sums because I don't know maths... but you've got that wrong... because you say he sold 3/12 and that's a quarter of the cheese...
M. - Don't be dumb! No it's not... It's the sum of all that...
V. - You wish, but that's not what your sum did... See... [He draws another cheese, divides it into 12 parts and marks 3. Then he looks at M.]
M. - What a mess! I can't understand why... the sums should also give...

Teacher - How are we doing here?
M. - He says he's right, and when he does the drawing he seems to be, but I think this is maths, we should do it with sums!... [V. explains what he did]
Teach. - Do you understand how he did that?
M. - Yes.
Teach. - How about you? Do you understand what she did?
V. - I just understand that her sums are wrong... I drew it here and it's only 1/4... but I think it should also be possible that way... but I don't know any of this, I don't know how to do it that way...
Teach. - So, M., how did you think this out?
M. - That I had to add everything he sold to see how much I got... 1/2 + 1/4 + 1/6
V. - So far I agree.
M. - And I got 3/12...
Teach. - 3/12?
M. - Yes... 1 + 1 + 1 = 3 and 2 + 4 + 6 = 12
Teach. - And how do you add fractions? [Silence]
Teach. - What do you need to do to be able to add fractions?
M. [Hesitating] **- Reduce them to the same number here?** [She points to the denominators]
Teach. - Of course!
M. - Oh, then I know!... It's 6... No, it's 12. 6/12 + 3/12 + 2/12 = 11/12
V. - Right... so you were left with 1/12 after all, like me! [He's visibly happy]
M. - Yes... then you just have to do the same sums you did.
V. - After all I was the one who was right this time! [Victorious] **I think I'd never got anything right in maths before... by myself.**

The students who established this interaction were in the 9th grade. M. was considered a good student in Mathematics, while V. had repeated failures in this subject (mark 1, which is the lowest possible), thinking it wasn't even worth trying, because he knew nothing and was incapable of learning, as he explained at the start of the year. This interaction takes place about two weeks after classes began and this pair had been formed because M. normally used an analytical reasoning, grasped previous years' contents better, but had difficulty whenever problems implied geometrical reasoning or called for a good mathematical intuition. Besides, she was convinced she was one of the best students of her class, and that she was damaged by some of the students' slow rhythm and by others' lack of interest. V. had an extremely low self-esteem in the beginning of the year, but had reacted positively to the few successes he had already had in the short period of classes until then. He had demonstrated that, if stimulated, he could have great ideas about problem solving, his mathematical intuition was very good, and found it easy to visualise situations that required this, but lacked a great deal of knowledge content-wise. Our expectation was that both, quite suspicious of each other at the start, would discover what interaction with one another could offer them in terms of personal progress.

When the problem is set forth, one of the features we had identified in V. becomes apparent straight away: he does not know how to solve the problem through calculations, so he uses a graphic representation - ***he draws the cheese***. Naturally at this stage of the school year, M. feels very sure that she knows the right path towards the correct solution. ***"A cheese?... What for? (...) But I think you do this with sums... (...) Then do yours, I'll do mine and then we'll explain "***. She is the leader, in the sense that she is who decides that what V. is doing is no good and it is also she who decides that it's best that they work separately and only interact afterwards. At this point his lack of faith in his ability to solve the problem is still overwhelming. But we can see that V. is starting to gain some confidence in his abilities and is knowing how to deal better with the limitations that come from not knowing a lot of the contents. ***"It's a cheese... (...) Now I'm going to draw what he sold... (...) I don't know how to do it with sums... so I'm going to see if it works this way..."***. A week ago, V. would have erased everything and stayed still, doing nothing, as soon as he heard M. say: ***"A cheese?... What for?..."***. Now, he already knew it was worth it to keep trying.

So, this brief moment of initial interaction, when they decide how they are going to work, is followed by a moment with no interaction, during which each one follows the solving strategy he/she picked. However, as soon as she has finished, M. turns to V. and asks him how he did it. We are not sure, but we believe she was impressed by the fact that he finished first and by the happy expression on his face. But she also knew the work instructions set by the teacher, so she might just be doing her part as the obedient student she was.

V. begins his explanation, concerned that M. understands all that he did, each decision he made. However, their understanding is not always easy: she wanted him to follow the elements in the problem step by step: ***"(...) I understand what you did, but I haven't yet figured out why you divided the cheese in 6 parts and not in 2... first he sells half..."***; as V. could visualise easily, he quickly understood that the difficulty lay in tracing 1/6, not one half: ***"(...) a half and quarter is easy... the hardest one is the sixth, so I started with that... (...)"***. In this manner, he had decided to begin his graphic representation according to what he had understood... which was not at all obvious to M., hence her exclamation: ***"Sorry?"***.

Since M. is not managing to follow his reasoning, V. changes his strategy and decides to tell her to do as she pleases, step by step, so as to be confronted with the final difficulty. At this moment, V. clearly takes charge of the process: he is the only one giving instructions and M. obeys. The strategy chosen by V. works in full for, as she arrives at 1/6, M. becomes still, holding her pencil in the air, not knowing how to continue. This has the effect V. intended: she understands he had some reason in what he did and decides to listen to him carefully, instead of trying to convince him that only she knows best. She even praises him for the first time***: "(...) Do yours! It must be better."*** Here we find an interesting interactive process: the most competent element of the two, in terms of previous years' contents, loses the dyad's leadership; and it is V.'s enormous mathematical intuition and his ability to visualise which lead him to an increasingly important role during interaction with his pair.

The way V. continues his explanation is thrilling, especially for a student who says he ***"doesn't know maths"***. He looks at the diagram he drew and says ***"I think it's half of a sixth... so it's 1/12. If 1/12 is 250gr, then the whole cheese is 250gr x 12, which is 1500gr. That's 1,5Kg."*** V.'s ability to visualise is, indeed, extraordinary. He does not know how to work with fractions but, when he looks, he can see that what is left is half of 1/6 and, just by looking at the cheese he drew, he can immediately see that that would correspond to having divided it into 12 parts and taking one of them.

It is amazing to see that a student with these abilities has always failed at Mathematics since the 5th grade and that last year's teacher described him as ***"incapable of mathematical reasoning and totally ignorant"***. In fact, at the beginning of the year, V. was convinced of this himself... but he quickly began to change his opinion.

The same does not apply to M., who seemed willing to listen to him and collaborate with him, as long as her wisdom was not questioned. Therefore, when she saw that V.'s result was different from hers, she hurriedly exclaimed ***"But I didn't get that!"***, despite not having been able to even finish solving the problem.

V. asks her what she did and she answers ***"I added 1/2 + 1/4 + 1/6 and ended up with 3/12. That's what he sold."*** V.'s answer is quite revealing: ***"I don't understand your sums because I don't know maths... but you've got that wrong... because you say he sold 3/12 and that's a quarter of the cheese..."***. That is, he presumes - and continued to be completely certain of this, at that time of the school year - that he does not know Mathematics, but he has made considerable progress: he no longer believes that he does not know how to think. Therefore, he does not know how to correct M.'s sums, but he is sure they are wrong. Through the graphic representation he draws, he knows very well that 3/12 are the same as 1/4, so her sums cannot be right.

But M. does not readily admit that she is wrong. After all, she is a good student, the one who usually knows the answers, and is not willing to let a couple of cheese sketches defy her wisdom. So she hastens to reply***: "Don't be dumb! No it's not... It's the sum of all that..."***. And V., who does not want to get mad at her and probably knows all too well how frustrating it is to make a mistake when you think you are right, answers back without arguing, but with extreme subtlety: ***"You wish, but that's not what your sum did... See...".*** And he goes back to his graphic representations to prove to M. that what he is saying is correct. Faced with this proof, M. becomes really confused and all she can say is she does not understand what happened and that her sums should also do.

At this crucial moment the teacher, who has been going around the room looking at what each pair has done and is not aware of what is happening between V. and M., arrives. It would have been interesting to have seen what they would have done if the teacher had not turned up then, how they would overcome this dilemma. But a contextualised investigation is just that: it happens on a stage which is the classroom, in a dynamic social climate that does not always develop in a manner that allows us to observe all that we wish, how we wish.

It is important to note that, as soon as the speaker is the teacher - the most competent person in the classroom, the one who gave the work instructions and who evaluates, which is still a big concern for M. - it is M. who talks to him, trying to make him back her idea that ***"This is maths, we should do it with sums!..."*** It is worthy of notice that the teacher does not support M.'s claim, but does not criticise it either. He is more concerned about finding out if each one managed to understand the strategy used by the other. Only when the teacher speaks directly to him does V. reply, taking part in the dialogue, which is now between the three of them. V. reveals a great humbleness: ***"but I don't know any of this, I don't know how to do it that way..."*** because he does not know how to work with fractions. However, he does know that his ***"drawings"*** are right and that M.'s ***"sums"*** are wrong. But since V. has no past history of success in Mathematics, he is perfectly willing to accept that there are alternative strategies. On top of this, he does not know Mathematics, as he often states, but he is already capable of finding certain strategies to solve the tasks proposed. For him there is no question that Mathematics can be done through sums, but he thinks - and rightly so - that in that case the result should be the same.

Once again, the teacher avoids any sort of judgement and asks M. how she thought things out again. She explains that: ***"1/2 + 1/4 + 1/6 (...) and I got 3/12 (...) Yes... 1 + 1 + 1 = 3 and 2 + 4 + 6 = 12"***. From this the teacher asks how fractions are added. Since he receives no answer from either of them, the teacher decides to reword the question, which works, for M. can remember that it is necessary to reduce fractions to a common denominator, although her language is not very rigorous. At this point of interaction, the teacher has called for knowledge from previous years and, as might be expected in this dyad, M. was the one who answered. But it is important to highlight that V. kept on listening with all his attention.

As soon as M. does her sums and reaches the result of 11/12 for all that has been sold, V. looks visibly happy and exclaims: ***"Right... so you were left with 1/12 after all, like me!"*** To which M. adds: ***"Yes... then you just have to do the same sums you did."*** And finally, with a victorious look on his face, he said ***"After all I was the one who was right this time!*** □Victorious□ ***I think I'd never got anything right in maths before... by myself."***
After this episode, V. began to participate more and more. He did not just copy from the blackboard during the stage of class debate, or when the teacher explained something. He would ask for more explanations until he had understood. For a while, he would keep apologising and stating that ***he "knew nothing about Maths...*** □he□ ***just wanted to understand"***. Sometimes he would say ***"When I see things, I can do them"***. But at the same time he revealed a great ability to grasp knowledge he did not have. In this case, he managed to learn how to add fractions. And he never forgot again that it was necessary to reduce them to a common denominator. In order to learn how to divide, we set him a challenge: he would go home and think that if, as he put it, ***"1/6*** □ ***2 = 1/12"***, then what was the rule for dividing fractions? To our amazement, V. did not go home to think. He stayed there, during breaktime, and asked with a suspect look whether ***"you could swap downwards and upwards"*** and, as I did not reply but simply smiled, he said: ***"I don't get any of this... but maybe it's like this... the first one stays as it is... this one*** □pointing to the second fraction□ ***rolls upside down... and then what's really weird... because it seems that instead of dividing you multiply... nah... can't be... but I don't see any other way"***. And he also never again forgot that in order to divide fractions he had to turn the second one upside down, and was more and more excited by the fact that Mathematics could be learnt by seeing. After all, learning Mathematics was much more fun and much easier than he had ever imagined!

Concluding remarks

To promote peer interactions in the Maths class it is not enough to sit pupils side by side - we also need to define the criteria for choosing the peers. When we put M. and V. in a dyad we were expecting them to be suspicious of each other, but we also knew they had different kinds of solving strategies and abilities and we hoped they would discover how useful their interaction could be for their performances and their school achievement. Anyhow this would only function if we were able to implement a new didactic contract within the class and if they accepted it.

Although suspicious - each one follows his/her own solving strategy in the beginning - they followed the rules implemented by their teacher: they had to be able to explain both solving strategies and any of them could be the one who was going to present the dyad work during the general discussion. So, as soon as they finished each one's solving strategy, they were interested in explaining and understanding what their peer had been doing alone. And they don't merely hear what the other one is saying, they are really listening carefully and they ask questions and argue each time they don't agree or don't understand the reasons why their peer solved the problem that way.

This didactic contract enables them to show each other their abilities and their difficulties. It makes each of them take the role of leader in different parts of the interaction and so M. is confronted with the fact that sometimes she fails too and that V. may have good ideas, while V. becomes more and more confident about his abilities. Peer interaction is a fine way of stimulating pupils' autonomy, which is quite visible in this case. Pupils work by themselves for a long time and they are able to regulate their ways of solving the task. Even when the teacher approaches them they know he wouldn't give them answers, he will mostly ask them questions and they are ready to answer him.

This is just one case chosen among many others, but after four years work in 26 different classes, from the 5th to the 11th grades, we are convinced that peer interactions are an effective way of promoting pupils' positive attitudes towards Maths, their self-esteem and socio-cognitive development, better affective relations in the class and their achievement in Maths. It is also a way of exploring each pupil's abilities and of making their past underachievement and their socio-cultural differences less determinant. Pupils accept each other more easily and they can profit from their differences instead of being penalized by them. Their representations about Maths change and they become more deeply engaged in their school activities. But, above all, for the first time many of them are able to have a future life project in which school has a role to play.

Note

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Common Sense or Good Sense?
Ethnomathematics and the Prospects for
a Gramscian Politics of Adults' Mathematics Education

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Abstract

This paper looks at Antonio Gramsci's concepts of 'common sense' and 'good sense' in the light of recent research and practice in adults learning mathematics, focussing particularly on issues around ethnomathematics, and explores the prospects for a Gramscian politics of adults' mathematics education.

Introduction

Mathematical knowledge is socially powerful: it enjoys high prestige and being 'mathematically knowledgeable' is often treated as an indicator of general intelligence, as evidenced by the widespread use of mathematics in entry tests for employment and employment training; mathematics is precise, rigorous, a powerful discipline in its own right. Common sense, by contrast, is regarded - or rather, it is often disregarded - as a low-level, practical, 'everyday' phenomenon, hardly noticed, except when its absence is suddenly revealed in the actions of an otherwise apparently intelligent, capable adult. What then is the connection between mathematics and common sense and where does Gramsci come into the picture?

For me, the connection between mathematics and common sense has long been a source of interest. As an adult numeracy tutor in the 1970s and 1980s I was aware that there seemed to be a strong connection in students' minds between those elements of mathematics they felt comfortable with and what they called common sense and my later research, with Gillian Thumpston, on adults' mathematics life histories, bore this out (Coben and Thumpston 1995, 1996). We found that some adults consistently undervalue the mathematics they can do, dismissing it as 'just common sense', while regarding as mathematics only that which they cannot do. Mathematics was thus effectively rendered invisible, a phenomenon also noted by Mary Harris (1997).

And Gramsci? Antonio Gramsci and Paulo Freire figure as 'radical heroes' in my research into political theory in relation to adult education (Coben 1998) and I became intrigued by the possibility that the concept of 'common sense' developed by Gramsci in his prison notebooks (Gramsci 1971) and his distinction between 'common sense' and 'good sense', might shed light on adults' learning and practice of mathematics. I began to explore aspects of the complex relationship between adults' mathematical knowledge and understandings and conceptions of common sense (Coben 1997).

In this paper I want to focus on Gramsci's concepts and their relevance to current debates around ethnomathematics, but first, a brief review of recent research and practice in adults learning mathematics may help to set the scene.

Recent research and practice in adults learning mathematics

As a field of practice, adult mathematics (numeracy) education in the UK flourished from the mid-1970s largely without benefit of research and underpinning theory in the wake of the adult literacy campaign of the 1970s, with interest periodically re-fuelled by reports of the parlous state of the nation's numeracy. In January 1997 two reports were published in Britain to coincide with the BBC numeracy campaign, 'Count Me In'. The first was a survey of the numeracy skills of adults in seven countries undertaken by the Opinion Research Business (ORB) (Basic Skills Agency 1997) and the second looked at the impact of poor numeracy on adult life (Bynner and Parsons 1997). In the first report, adults in the UK came bottom of the international league: on questions covering the addition and subtraction of decimals, simple multiplication, the calculation of area, calculating percentages and using fractions, only 20% of people tested completed all twelve tasks accurately. The authors of the second report conclude that the impact of poor numeracy on adult life is profound and severe:

> *People without numeracy skills suffered worse disadvantage in employment than those with poor literacy skills alone. They left school early, frequently without qualifications, and had more difficulty in getting and maintaining full-time employment. The jobs entered were generally low grade with limited training opportunities and poor pay prospects. Women with numeracy difficulties appeared especially vulnerable to exclusion from the clerical and sales jobs to which they aspired. Men's problems were less clearly differentiated between occupations. (Bynner and Parsons 1997:27)*

A series of reports in the 1980s revealed an equally sorry picture (ALBSU 1987; ACACE 1982; Sewell 1981). For example, Brigid Sewell's (1981) enquiry into adults' use of mathematics in daily life, commissioned for the Cockcroft Committee (Department of Education and Science 1982), together with the associated national survey (ACACE 1982) revealed that approximately 30% of those questioned could not handle simple subtraction, multiplication, division or percentages, or understand a simple graph; almost half the adult population could not read a simple timetable and over half did not understand the meaning of the rate of inflation (ACACE 1982). The cost to British industry of poor basic skills was estimated at £4.6 billion a year in 1993

(ALBSU 1993).

Both the 1997 reports discussed above are based on somewhat restricted notions of numeracy, a concept which is deeply contested and subject to shifts of definition (Baker and Street 1994). The international survey (Basic Skills Agency 1997) is based on a notion of numeracy as a set of computational skills, while Bynner and Parsons' (1997) research employs a functionalist notion of numeracy. Both the computational and the functionalist models of numeracy ignore other important aspects of mathematics, such as its use as a means of communication. Also, as mentioned above, research on adults' mathematics life histories (Coben and Thumpston 1995, 1996) indicates that some adults undervalue the mathematics they can do, dismissing it as 'just common sense', while regarding as mathematics only that which they cannot do. It seems likely that this phenomenon may have a bearing on adults' poor performance in numeracy tests.

Whether or not there is a real crisis of adult innumeracy, there is certainly a perception of one in many countries and it may come as no surprise that research in adults learning mathematics has emerged in recent years as a growing field of international interest after a long period of neglect. It is an exceptionally diverse field which defies precise categorisation. What practitioners and researchers have in common is a serious interest in adults learning mathematics, whoever and wherever they are, however and for whatever purpose they are learning. For example, contributors to the Working Group on 'Adults Returning to Mathematics Education' (WG18) at the eighth International Congress on Mathematics Education (ICME-8) focussed on a wide range of issues, from adults studying mathematics for an engineering degree in the UK to mathematics education and the struggle for land in Brazil (FitzSimons, ed.1997).

The ICME-8 Working Group was one of a series of separate conferences in the 1990s, focussing on different aspects of research and practice in adults learning mathematics. These included the first UNESCO International Seminar on adult numeracy, which was held in France in 1993; UNESCO held a further conference, on literacy, which included adult numeracy, in 1996 in Philadelphia, USA. The third *Political Dimensions of Mathematics Education* (PDME-III) conference took place in 1995 in Norway, and included sessions on the political dimensions of adults' mathematics education (Kjærgård et al., eds, 1996).

Meanwhile, researchers and practitioners have come together to form national, regional and international organisations, such as the Adult Learners Special Interest Group of the Mathematics Education Research Group of Australasia (MERGA), and, in the USA, the Adult Numeracy Network (ANN). Other international groups include the UK-based Philosophy of Mathematics Education Network (POME), the US-based Criticalmathematics Educators Group (CmEG) and the International Study Group on Ethnomathematics (ISGEm) founded in 1985 under the guidance of Ubiratan

D'Ambrosio, who coined the term ethnomathematics to indicate the influence of sociocultural factors in the creating, teaching and learning of mathematics.

Adults Learning Maths (ALM)[1], established in 1994, is an international research forum in which researchers and practitioners come together and share experience. ALM has undoubtedly been an important factor in the worldwide development of interest in this area as a field of research as well as a field of practice and the Forum now has members in Argentina, Australia, Austria, Belgium, Brazil, Canada, Denmark, Finland, Germany, Ireland, The Netherlands, New Zealand, Slovenia, South Africa, Spain, Sweden, Uganda, UK, USA and Zimbabwe. Research topics discussed at the fourth international ALM conference, ALM-4, reflect the diversity of the field, including: mathematics in work, such as the use of calculators by nurses and the development of a 'mathematics profile' of adults undergoing training as crane operators; the assessment of numeracy skills of adults with special needs; adults' independent learning in mathematics; and Paulo Freire's legacy for adults learning mathematics (Coben and O'Donoghue, comps, 1998).

Other relevant research includes that at the Institute of Education, University of London, on adults' mathematising at work (Hoyles, Noss and Pozzi, in press); Mary Harris's research on - and celebration of - the 'invisible' mathematics in women's work (Harris 1997); Juan Carlos Llorente's investigation of ways in which adults constitute knowledge (including mathematical knowledge) in their work (Llorente 1997); work on situated cognition by Jean Lave (1988) and others; and developments in the field of ethnomathematics (Powell and Frankenstein, eds, 1997), discussed below.

Some of the above research engages implicitly (and occasionally explicitly) with notions of adults' common sense in relation to mathematics, including my own (Coben 1997, Coben and Thumpston 1996). Research on common sense *per se* (although not necessarily in relation to adults) is discussed by contributors to 'Mathematics Education and Common Sense' (Keitel *et al.,* eds, 1996). On the whole, however, common sense has not been the subject of a great deal of research in relation to adults' mathematics learning and education. This is strange when one considers that, in the West, at least, common sense is one of the features that distinguishes an adult from a child, in that adults are expected to demonstrate common sense whereas children are regarded as in need of adult care and protection at least partly because, paradoxically, they are not expected to have what is also called 'the sense they were born with'.

But first, to return to Gramsci: what are his concepts of 'common sense' and 'good sense' and what distinction does he draw between them?

Gramsci's concepts of 'common sense' and 'good sense'[2]

Common sense

For Gramsci, common sense comprises the "diffuse, unco-ordinated features of a general form of thought common to a particular period and a particular popular environment" (Gramsci 1971:330n). It contains "a healthy nucleus of good sense" which, he argues, "deserves to be made more unitary and coherent" (Gramsci 1971:328). Gramsci states that:

> *Its most fundamental characteristic is that it is a conception which, even in the brain of one individual, is fragmentary, incoherent and inconsequential, in conformity with the social and cultural position of those masses whose philosophy it is. At those times when a homogeneous social group is brought into being, there comes into being also, in opposition to common sense, a homogeneous - in other words coherent and systematic - philosophy.*
> *(Gramsci 1971:419)*

He emphasises the chaotic and contradictory nature of 'common sense', describing it as "a chaotic aggregate of disparate conceptions, and one can find there anything one likes" (Gramsci 1971:422). It is "an ambiguous, contradictory and multiform concept". Nonetheless, although it is "crudely neophobe and conservative" (Gramsci 1971:423), it contains truths.

Gramsci insists that both 'common sense' and 'good sense' are historically and socially situated: "Every social stratum has its own 'common sense' and its own 'good sense', which are basically the most widespread conception of life and of man" (Gramsci 1971:326, n5).

Good sense

For Gramsci, 'good sense' is exemplified by the 'philosophy of praxis' (a term he uses throughout the notebooks for Marxism, partly as camouflage to deceive the prison censor). 'Good sense' is analogous to 'philosophy', in that it is inherently coherent and critical. As he says, "Philosophy is criticism and the superseding of religion and 'common sense'. In this sense it coincides with 'good' as opposed to 'common' sense" (Gramsci 1971:326). Good sense is thus an "intellectual unity and an ethic in conformity with a conception of reality that has gone beyond common sense and become, if only within narrow limits, a critical conception" (Gramsci 1971:333). In order for 'common sense' to be to be renewed, i.e., to become 'good sense', one must start with

> *a philosophy which already enjoys, or could enjoy, a certain diffusion, because it is connected to and implicit in practical life, and elaborating it so*

that it becomes a renewed common sense possessing the coherence and sinew of individual philosophies. But this can only happen if the demands of cultural contact with the "simple" are continually felt. (Gramsci 1971:330n)

Good sense, for Gramsci, may be created out of common sense through an educative Marxist politics. This process does not entail "introducing from scratch a scientific form of thought into everyone's individual life, but of renovating and making 'critical' an already existing activity" (Gramsci 1971:331). So how do Gramsci's concepts of 'common sense' and 'good sense' relate to issues in adults' mathematics education, and in particular, to ethnomathematics?

Ethnomathematics[3]

Ethnomathematics seems to me to offer several parallels to Gramsci's concepts. Ethnomathematics problematizes dichotomies between different 'knowledges', formal and informal, academic and popular, and questions the allocation of power to preferred forms of knowledge, in ways that find parallels in Gramsci's concepts. Gramsci's broad conception of culture encompasses mathematics as a cultural phenomenon, a core conception for ethnomathematicians. Gramsci, crucially, regarded common sense as something to be worked with and transcended rather than rejected. This implies an educative process rooted in, and respectful of, people's lived experience. It is the nature of people's experience in relation to mathematics that Gillian Thumpston and I sought to explore in our mathematics life history research, and the exploration of the nature of people's mathematical experience lies at the heart of the ethnomathematics enterprise.

Thus far, it seems that a Gramscian and an ethnomathematics approach may have significant elements of congruency, at least in terms of method and approach, though not necessarily in terms of political purpose. Gramsci was, after all, a major figure in twentieth century Marxism; I am not suggesting that all ethnomathematicians share his political perspective and commitment. Ethnomathematics is a diverse movement, bringing together researchers with different political perspectives, as is evident in Arthur B.Powell's and Marilyn Frankenstein's edited collection, Ethnomathematics (Powell and Frankenstein, eds, 1997). Such diversity is not only historically inevitable in these postmodern times but probably also essential for the intellectual health of the movement. Instead of a shared political commitment, it seems to me there is in ethnomathematics a shared ethical commitment: most ethnomathematicians would presumably echo Ubiratan D'Ambrosio's humanitarian concern with "an ethics of respect, solidarity, and co-operation" (D'Ambrosio 1997:xx).

But there is another problem in assuming congruency between a Gramscian conception of good sense and common sense and ethnomathematics, and it is a problem that strikes at the heart of the ethnomathematics project, as I understand it. This is the question of whether Gramsci's distinction between good sense and common sense is predicated on an irredeemably hierarchical conception of knowledge, as Gelsa Knijnik has argued (Knijnik 1996).

To an extent, this must remain an open question, since the distinction between good sense and common sense is not fully worked out in the prison notebooks. Certainly, it may be interpreted as a distinction between true and counterfeit knowledge; between order (unified knowledge), and chaos (fragmented knowledge); between higher and lower forms of knowledge. If so, it would seem perverse and irrational to prefer the counterfeit to the true, to celebrate people's inchoate lived experience and practice of mathematics, whether effective and accurate in mathematical terms or not. In educational terms, such a view of common sense would imply that it should be rejected in favour of academic rigour. If ethnomathematics were aligned with common sense conceived in this way, that would be to relegate ethnomathematics to the status of a non-academic practice and an anti-science theory, connotations which it already holds for many observers, as D'Ambrosio points out (D'Ambrosio 1997:xxi). It would also be to align Gramsci's concept of good sense, in this context, with academic mathematics, privileging what Gramsci would call the 'traditional intellectual' and the traditional intellectual's knowledge over the simple. It would be tantamount to accusing Gramsci of educational conservatism (as Harold Entwistle does in his 1979 book) and acknowledging that ethnomathematics sacrifices concern about issues of accuracy and effectiveness in mathematics on the altar of a sentimental relativism.

I believe to make such an alignment would be to misread Gramsci and to distort ethnomathematics. On the question of misreading Gramsci, as I argue in my book,

Radical Heroes,

> *Gramsci's distinction between good sense and common sense is both epistemological and sociological: both a distinction between different forms of knowledge and a distinction between the 'knowledges' characteristic of different social groups. But the distinctions are not mutually exclusive in either case. In epistemological terms, common sense includes elements of good sense. In sociological terms, good sense is not the preserve of an elite, and common sense is common to us all. (Coben 1998:213-4)*

Gramsci problematizes both common sense and good sense. He makes a conceptual rather than an empirical distinction between common sense and good sense, since the categories are not mutually exclusive.

On the question of distorting ethnomathematics, I defer to D'Ambrosio (1997:xx-xxi) who insists that ethnomathematics is "an holistic and transdisciplinary view of knowledge", "a research program", "a comparative study of the techniques, modes, arts, and styles of explaining, understanding, learning about, and coping with the reality in different natural and cultural environments" and "an analysis of the generation of knowledge, of its social organization, and of its diffusion". If it is all these things, then the question is not whether ethnomathematics should be equated with a Gramscian conception of mathematical 'common sense' but instead how ethnomathematics might contribute to our understanding of common sense and good sense and deepen and enrich our conception of mathematics and our commitment to radical democratic principles of adult mathematics education.

If mathematics life histories research tells us that adults tend to dismiss the mathematics they can do as "just common sense", and if ethnomathematics helps us to engage with adults' chaotic, fragmented 'common sense' in an educational context, and to understand better the relationship between adult students' - and our own - 'common sense' and 'good sense', then we have the beginnings of some fruitful lines of research and practice in adult mathematics education. We have the possibility of research into 'adult numeracy' that makes better sense of adults' mathematical strengths as well as their weaknesses and we have the prospect of more effective and socially and politically sensitive practice in adults' mathematics education. In terms of the development of a radical, democratic politics of adults learning mathematics, it seems to make 'good sense' to start with adults' 'common sense'.

Notes

1. For details of *Adults Learning Maths - A Research Forum* (ALM), please contact me in my role as ALM Secretary, address below.

2. I explore Gramsci's concepts of 'common sense' and 'good sense' in greater depth my book, Radical Heroes (Coben 1998) and especially in my chapter in a forthcoming book on Gramsci and education in preparation under the auspices of the International Gramsci Society (IGS). For details of IGS, contact the International Gramsci Society, Secretary: Joseph A. Buttigieg, Department of English, University of Notre Dame, Notre Dame, Indiana 46556, USA.

3. This section draws on a paper I presented at the International Study Group on the Relations between History and Pedagogy of Mathematics and the International Study Group on Ethnomathematics Conference in Honor of the 65th Birthday of Ubiratan D'Ambrosio, in Baltimore, USA, 6 January 1998.

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Social class, gender, equity and National Curriculum tests in maths

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Abstract

The paper draws on data from a recent ESRC project which has explored children's interpretation of and performance on the English National Curriculum tests in maths at Key Stages 2 and 3. Using data concerning more than 100 test items taken by 10-11 year-olds the paper shows that 'realistically' contextualised test items produce greater social class and gender differentiation than 'esoteric' items (i.e. those not contextualising mathematical operations in 'everyday' settings). The results are employed in a simulated selection process of children for secondary school places (either for a selective school or for a top set within a school). This simulation shows that, on the basis of our data, a test consisting of 'esoteric' items might be expected, all other things being equal, to select many more working class children than a test consisting of 'realistic' items.

Background

The 1988 Education Act introduced a national curriculum (NC) and assessment of children at the end of four Key Stages (KS) for England and Wales. There has been considerable debate and conflict about the nature of the curriculum and its assessment. As a result, the initial proposals for assessment mainly by teachers in their classrooms have been replaced by a stress on testing via group tests (DES/WO, 1988; Brown, 1992). While there has been a restatement recently of the importance of teacher-made assessment (Dearing, 1993), England now has an institutionalised pattern of annual national testing of children in maths at ages 7, 11, 14, and 16. In 1997 the first national league tables were published for 11 year olds[2].

One other key point must be made. Maths, though it centrally concerns number, space, measure, etc., is not fixed and unchanging. During periodic re-negotiations of what counts as school maths the cognitive demands made on children change (Cooper, 1983, 1985a, 1985b, 1994a). These demands have typically been differentiated by measured 'ability' and/or social class in England, as the case of SMP well illustrates (Cooper, 1985b; Dowling, 1998). In England in recent years, such re-negotiation has led to an *apparent* weakening of the boundary between 'everyday' knowledge and 'esoteric' mathematical knowledge both in the curriculum and in its assessment, perhaps especially so for children deemed 'less able' (e.g. Dowling, 1998). While in the 1960s and early 1970s the preferred version of school maths tended to favour

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[2] For fuller accounts of the policy background, see Ball (1990, 1994); Brown (1992, 1993); Cooper (1994a, 1994b).

‘abstract’ algebraic approaches[3], the dominant orthodoxy since the time of the Cockcroft Report of 1982 has favoured the teaching and learning of maths within ‘realistic’ settings (Cockcroft, 1982; Dowling, 1991; Boaler, 1993a, 1993b). This preference within the world of maths educators has been reflected in the national tests (Cooper, 1992, 1994b). It has been argued, drawing on the work of Bernstein (1990, 1996) and Bourdieu (1986), that test items contextualising mathematical operations within ‘realistic’ settings might be expected to cause problems of interpretation for certain students. Working class children may experience more difficulty than others in choosing ‘appropriately’ between using ‘everyday’ knowledge and ‘esoteric’ mathematical knowledge when responding to items (Cooper, 1992, 1994b). This may lead to underestimation of their mathematical capacities in cases where a rational ‘everyday’ response is ruled out as ‘inappropriate’ by the marking scheme but is ‘chosen’ by the child in place of an alternative ‘esoteric’ response (Cooper, 1996, 1998a&b). Similar arguments have been advanced in respect of gender, with girls seen as likely to be disadvantaged by ‘realistic’ assessment items (Boaler, 1994). In summary, performance on ‘realistic’ items may not reflect underlying competence[4]. It is upon this possible threat to valid and fair assessment that our research has focused.

While the assessment literature has many useful discussions of item bias and differential validity (e.g. Wood, 1991, p.177; Gipps & Murphy, 1994) these tend not to draw on relevant sociological insights concerning the relation between culture and cognition (e.g. Bernstein, 1996; Bourdieu, 1986, 1990a,b&c). Discussions of bias are frequently technical if not empiricist in tone (e.g. Camilli & Shepard, 1994). While purely quantitative methods can identify items, or classes of items, which some groups of test-takers find more or less difficult than other groups, they are less good at increasing our understanding of *why* such items ‘behave’ in the way they do. To advance our understanding in this area, a more qualitative concern with children’s cognitive strategies and processes is needed, coupled with the use of relevant theoretical insights from outside the area of assessment itself[5]. It is this more explanatory problem to which our research has been addressed - in the belief that a better understanding of the ways culture, cognition and test performance interact should inform test design (e.g. Cooper, 1998b; Cooper & Dunne, 1998). It would then be *possible* to avoid more easily those items which cause unnecessary and construct-irrelevant difficulty to some test takers (Messick. 1989, 1994). However, here we intend to show how differently contextualised NC item types are associated with different relative performances by certain social groups. Our focus will be therefore on some of our quantitative data.

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[3] Though, at the same time, dominant versions of school maths also incorporated newer applications of maths (Cooper, 1985a).

[4] For discussions of similar issues in the case of science see Morais et al (1992) for class; and Murphy (1996) for gender. For a useful discussion of the competence/performance distinction see Wood & Power (1987). The distinction clearly begs many theoretical questions. It does, however, allow a particular critical perspective to be taken on the validity of test items.

[5] For earlier examples of this strategy see Mehan (1973) and Bourdieu (1984).

Research Focus and Methods

Our research has explored children's interpretation of and performance on the NC tests, relating this to social class and gender. A subsidiary goal of the research has been to explore what might follow from taking the work of Bernstein seriously in analysing children's responses to maths test items. This has become a central concern of the research as it became clear that the 'appropriate' negotiation of the boundary between the 'everyday' and the 'esoteric' is difficult for many children. The work of Bernstein and his collaborators is very helpful here, especially the theorising of 'recognition and realisation rules' in relation to children's cognitive strategies (Bernstein, 1990, 1996; Holland, 1981). For an account of how his (and Bourdieu's) theoretical concepts might be put to use in this area see Cooper (1996, 1998b).

We have employed both quantitative and qualitative methods. The basic strategy has been to use initially statistical analysis of children's performance on items in test situations to generate insights concerning broad classes of test items (e.g. items which embed mathematical operations in 'everyday' and 'esoteric' contexts respectively). This has involved coding test items on a number of dimensions[6]. Analyses of the relationships between social class, gender, measured ability, item type and performance have been carried out. Some of these use the child as the case for analysis, others use the item itself (see below and Cooper, Dunne & Rogers, 1997). Alongside this we have used more qualitative analyses of children's responses to particular items in both the tests and subsequent clinical interviews to generate understanding of *why*, for example, 'realistic' and 'esoteric' items seem to be differentially difficult for children from different socio-cultural backgrounds (e.g. Cooper & Dunne, 1998). This has involved the coding of children's responses on various dimensions, especially the child's use, whether 'appropriate' or not, of 'everyday' knowledge in responding to items. In parallel, informing and being informed by this work, a model of the way culture, cognition and performance on 'realistic' test items interact has been developed (Cooper, 1996, 1998b).

In each of three primary and secondary schools, Year 6 and Year 9 children took three group tests in maths. Two of these were the actual May 1996 Key Stage national tests. The third, taken some four months earlier, comprised a test put together by us, drawing on previous NC items. Our tests were designed to cover a variety of item types and four Attainment Targets (ATs)[7]. Our secondary test, like the May 1996 test, was tiered by NC level. Our tests were marked according to the NC marking schemes. Between

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[6] These have included type of contextualisation, 'wordiness', difficulty levels, attainment target, type of response required, and use of pictorial representation.

[7] It is important to note that, in order to maintain comparability across the years 1992-1996, we have worked within the 1991 NC framework for maths which comprised 5 Attainment Targets one of which, *Using And Applying Mathematics* was not assessed via the tests. The other 4 were *Number, Algebra, Shape And Space, and Handling Data*. For the same reason, and because we wished to throw light on the use of Statements of Attainment (SoA), we have also used the SoAs allocated to items where appropriate and, in some analyses, have allocated SoAs to items in years where the official rubric had not (mainly coded by examination of comparable previous items).

the administration of the first test and May 1996 we interviewed all of the Year 6 children and a 25% sample of the Year 9 children while they worked individually through a selection of items from the first test. This allowed access to children's interpretations of the items and their methods. Furthermore, *and this has been a crucial part of our approach*, it was possible to allow children to reconsider their approach and answer in cases where they had initially chosen an 'inappropriate' 'everyday' reading of the meaning and requirement of the item. This has allowed us to explore the ways in which the use of a certain class of 'realistic' item can lead to the underestimation of children's actually existing knowledge and understanding (Cooper, Dunne & Rogers, 1997; Cooper & Dunne, 1998). In order to allow an examination of social class effects we have also collected information on parental occupations. The issue of parental occupations was a sensitive one, especially in the secondary schools. Two of the three schools required parental permission before children were allowed to supply this information. The third required that the question go directly to the home with the result that we gained this information for only 43% of the sample in this school[8]. We also have children's scores on the three Nelson Cognitive Ability tests. We have also interviewed teachers, concentrating on the school's approach to maths, and on teachers' perspectives on NC assessment and the pupils in their schools. The nature of the samples and the project's activities are set out in Table 1.

Table 1: The primary and secondary school samples

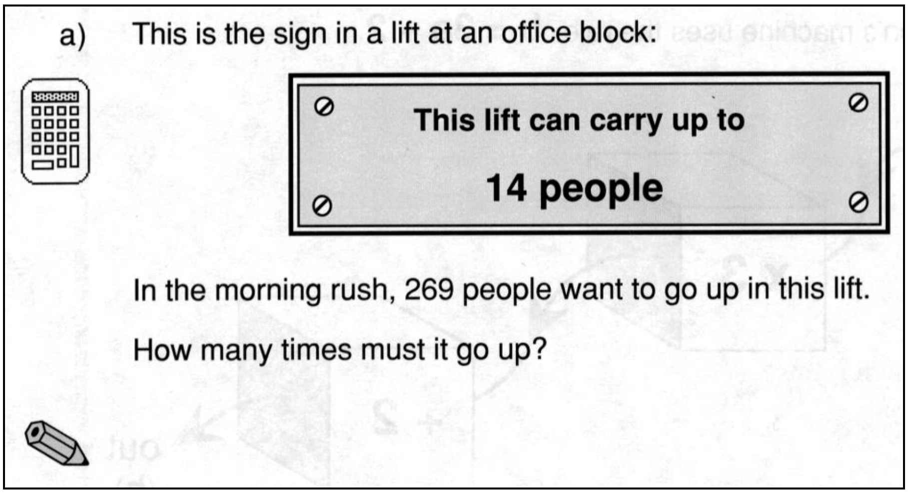
	Children Tested (n)	*Children Interviewed (n)*	*Teachers Interviewed (n)*	*Lessons Observed (n)*
Key Stage 2				
School A	63	63	4	4
School B	44	44	3	4
School C	29	29	6	5
Total KS2	136	136	13	13
Key Stage 3				
School D	254	50	6	10
School E	102	37	5	5
School F	117	36	4	5
Total KS3	473	123	15	20

Results

In this paper we will report on Key Stage Two[9]. We begin with an intrinsic discussion of one item in order to illustrate the type of issues which arise when mathematical operations are contextualised within 'realistic' settings. This happens to be a KS3 item, though similar items appear at KS2.

[8] For discussion of issues of definition and coding re class, see Cooper & Dunne (1998).
[9] Work on Key Stage Three is reported in the Project Report to the ESRC (October 1997, currently being peer-reviewed) and forms part of a book currently in preparation.

Figure 1: ‘Realistic’ items and ambiguity: an illustration (from SEAC, 1992).



Statement of Attainment: "Solve number problems with the aid of a calculator, interpreting the display" (2/4d).

The Marking Scheme (Band 1-4, Paper 1) gives as "appropriate evidence" of achievement: "Gives the answer to the division of 269 by 14 as 20, indicating that they have interpreted the calculator display to select the most appropriate whole number in this context. Do not accept 19 or 19.2".

The item in Figure 1 is one of a type much discussed in mathematical education circles (e.g. Verschaffel, De Corte, & Lasure, 1994).The key point is that the child’s answer must not be fractional. The lift can not go up (and down) 19.2 times. The child is required therefore to introduce a ‘realistic’ consideration into his or her response. In fact the child must manage much more than this. S/he must introduce only a small dose of realism - ‘just about enough’. S/he must not reflect that the lift might not always be full; or that some people might get impatient and use the stairs; or that some people require more than the average space - e.g. for a wheelchair. Such considerations - ‘too much realism’ - will lead to a problem without a single answer, and no mark will be gained[10]. There is a certain irony here. Many reformers have argued for the use of ‘ill-structured’ items in maths teaching, learning and assessment contexts (e.g. Pandey, 1990). This item, however, is *unintentionally* ill-structured. Children’s and schools’ interests now hinge on managing the resulting ambiguities in a legitimate manner.

The child is asked to exercise some ‘realistic’ judgement and, in doing so, might be presumed to be undertaking a ‘realistic’ application of some mathematical (or at least arithmetical) knowledge. But on whose account of ‘applying’? The lift item essentially concerns queuing behaviour. A mathematics of queuing exists. We might turn for some insight to an elite disciplinary source. Let’s try *Newer Uses of Mathematics*[11], edited in 1978 by Sir James Lighthill, FRS, then Lucasian Professor of Applied Maths at Cambridge[12]. This edited collection includes a paper on methods of operational

[10] See Cooper (1992, 1994b) for a fuller discussion.
[11] In the preface, Lighthill says, “…we want to outline some of the many ways of using mathematics for significant practical purposes …”
[12] Other holders of this post have included Sir Isaac Newton and Stephen Hawking.

analysis by Hollingdale (former Head of Maths Dept at the Royal Aircraft Establishment) which discusses queuing. An edited extract follows:

> Everyone, nowadays, is only too familiar with queues - at the supermarket, the post office, the doctor's waiting room, the airport, or on the factory floor. Queues occur when the service required by customers is not immediately available. Customers do not arrive regularly and some take longer to serve than others, so queues are likely to fluctuate in length - even to disappear for a time if there is a lull in demand…. The shopper leaving the supermarket, for example, desires service; the store manager wants to see his cashiers busy most of the time. If customers have to wait too long, some will decide to shop elsewhere; … The essential feature of a queuing situation, then, is that the number of customers (or units) that can be served at a time is limited so there may be congestion. …. Queuing problems lend themselves to mathematical treatment and the theory has been extensively developed during the last seventy years. …The raw materials of queuing theory are mathematical models of queue-generating systems of various kinds. The objective is to predict how the system would respond to changes in the demands made on it; in the resources provided to meet those demands; and in the rules of the game, or queue discipline as it is usually called. Examples of such rules are: 'first come, first served'; 'last come, first served', as with papers in an office 'in-tray'; service in an arbitrary order; or priority for VIPs or disabled persons. To analyse queuing problems, we need information about the *input* (the rate and pattern of arrival of customers), the *service* (the rate at which customers are dealt with either singly or in multiple channels), and the *queue discipline*…. (Hollingdale, in Lighthill, pp. 244-245)

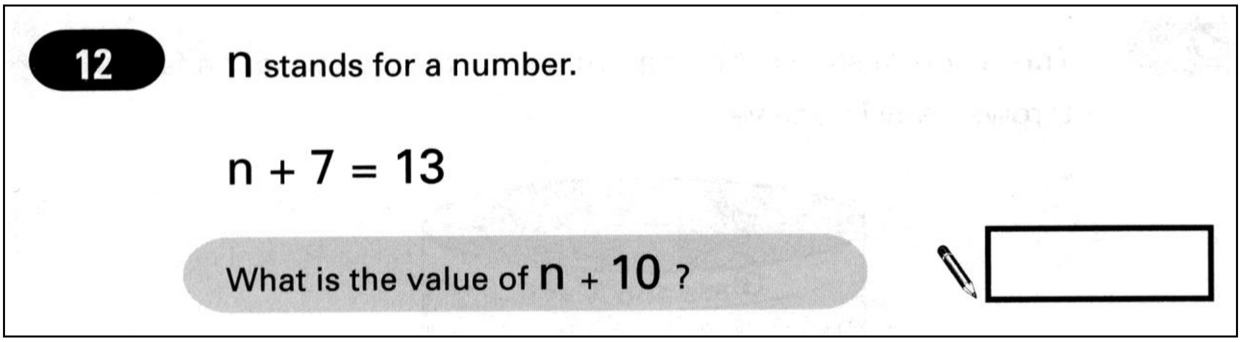
The question which arises then is would any of these models deliver the correct answer according to the producers of marking schemes for National Curriculum tests[13]. If not, why not, and what approach does? Can the ‘required’ approach be specified via teachable ‘rules of engagement’ for such items? If not, why not? Should they be?

Various writers have employed the notion of educational ground rules to capture what is demanded of children in cases like the lift item (Mercer & Edwards, 1987). There is clearly some affinity between this concept and those of recognition and realisation rules as employed by Bernstein (1996). However, it can be seen that it would be quite difficult - if not impossible - to write a set of rules which would enable the child to respond as required to the lift question. Certainly, the rule - in the sense of a mandated instruction - to employ ‘realistic’ considerations would not do, since ‘how much’ realism is required remains a discretionary issue. It is this problem that has led to a range of attacks on the use of rules to model human activities (e.g.Taylor, 1993) and, in particular, has led Bourdieu to reject a rule-based account of cultural competence (see Bourdieu, 1990a). His concept of *habitus* aims to capture the idea of a durable socialised predisposition without reducing behaviour to strict rule-following (Bourdieu, 1990c). Bourdieu sometimes describes what habitus captures as ‘a feel for

[13] These producers of the items and marking schemes exist in a field distinct from that of Hollingdale, of course.

the game' and we can see that this describes fairly well what is required by the lift problem and others like it[14]. Both Bernstein and Bourdieu have shown that members of the working class are more likely to respond to test-like situations by drawing on 'local' and/or 'functional' rather than 'esoteric' and/or 'formal' perspectives[15]. We have shown elsewhere that this can lead to the relative underestimation of these children's mathematical capacities when test items are superficially 'realistic' but actually demand an 'esoteric' response (Cooper, 1996, 1998b; Cooper & Dunne, 1998). Because of lack of space, we will not present any findings concerning the lift item here, nor will we be able to present the explanatory perspective. We move instead to present a statistical overview of children's relative performance on 'realistic' and 'esoteric' items at KS2. We have already described our simple coding of 'realistic' and 'esoteric' items. The lift item can serve as an exemplar of the former[16]. The following is an example of the latter:

Figure 2: coded as 'esoteric' (Key Stage 2: SCAA, 1996)



Quantitative Analysis: an Overview

Each separately marked item or sub-item[17] of the three tests taken by 10/11 year olds was coded on a variety of dimensions including a two-fold division into what we have termed 'realistic' or 'esoteric' items[18] using a rule which is simple to state though not always easy to operationalise. An item has been categorised as 'realistic' if it contains either persons or non-mathematical objects from 'everyday' settings[19]. Otherwise it is coded as

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[14] For a qualitative comparison across a range of test items of two children who differ markedly in their 'feel for the game' see Cooper (1996, 1998b).

[15] For examples of such research, see Holland (1981), and Bourdieu (1986).

[16] Clearly, 'realistic' items differ amongst themselves in numerous ways. In particular, some require 'realistic' considerations to be taken into account; others do not. The latter typically embed a 'hidden' mathematical structure in the 'noise' of the 'realistic' (see Cooper, 1992 & Cooper & Dunne, 1998, for discussion of some examples).

[17] In a few cases there is some dependency of one sub-item on another.

[18] Given that in some cases a person appeared just to *introduce* the item we experimented with a threefold category system, putting such items into a category we termed 'ritualistically' 'realistic'. However, in the end, we decided not to pursue this as we felt unable to judge, when coding items *prior* to analysis of data, whether what to us might seem 'ritualistic' might seem the same to a child.

[19] Clearly, it is possible to raise questions here about *whose* 'everyday' and *whose* 'esoteric'. We wish 'everyday' *here* to refer to such activities as shopping, sport, etc. of which we can assume most children have some knowledge and personal experience.

‘esoteric’. For each child the percentages of total marks scored on the two categories of items[20] were calculated, giving a ‘realistic’ and an ‘esoteric’ percentage for each child. Then, for each child a ratio was created by dividing the ‘realistic’ by the ‘esoteric’ percentage’ achieved. Table 2, Table 3 and Table 4 show the distribution by social class and sex of the two percentages and the resulting ratio for the primary school children for whom we have full relevant information[21]. Our social class categories are set out in Appendix 1.

Table 2: Percentage score achieved on KS2 ‘realistic’ items on the three tests by class and sex

	Female	*Female*	*Male*	*Male*	*Total*	*Total*
Class	Mean	Count	Mean	Count	Mean	Count
Service class	57.74	26	60.33	34	59.21	60
Intermediate class	55.68	13	55.04	17	55.32	30
Working class	47.34	13	51.07	20	49.60	33
Total	54.62	52	56.46	71	55.68	123

Table 3: Percentage score achieved on KS2 ‘esoteric’ items on the three tests by class and sex

	Female	*Female*	*Male*	*Male*	*Total*	*Total*
Class	Mean	Count	Mean	Count	Mean	Count
Service class	71.07	26	70.10	34	70.52	60
Intermediate class	70.35	13	69.98	17	70.14	30
Working class	65.71	13	64.69	20	65.09	33
Total	69.55	52	68.54	71	68.97	123

Table 4: Ratio of KS2 ‘realistic’ percentage to ‘esoteric’ percentage by class and sex

	Female	*Female*	*Male*	*Male*	*Total*	*Total*

Solving $x^2 - 3 = 6$ might well be describable as ‘everyday’ by reference to some group’s behaviour in some setting, but we assume here that such items are recognisably different from those which embed maths in shopping etc. The purpose of our distinction is not to legislate on what ultimately counts, in some universalistic way, as ‘everyday’ or ‘esoteric’, but to enable empirical analysis of important issues to get off the ground.

[20] The final handful of items from our ‘mock’ test were omitted from this analysis in order to only include items which all or very nearly all children had definitely attempted. 110 separate items or part-items entered the analysis. Two-thirds of the items come from the 1996 tests and a third from earlier incarnations of the NC tests.

[21] We are providing tests of significance in footnotes, though we have some doubts about their value. Our samples are not the sort of simple random samples which the maths of significance testing generally assumes (e.g. Hoel, 1971). Neither are the members our samples selected independently of one another, given the decision (the only practical one) to select schools as our basic unit. We tend to see the relationships discussed as features of these particular groups of Year 6 and Year 9 children. Whether the relationships are likely to generalise to larger populations is, for us, as much a matter of theoretical plausibility as of the application of significance testing to the data.

Class	Mean	Count	Mean	Count	Mean	Count
Service class	.81	26	.88	34	.85	60
Intermediate class	.79	13	.79	17	.79	30
Working class	.71	13	.79	20	.76	33
Total	.78	52	.83	71	.81	123

Ratios such as these have properties that can make them difficult to interpret. In particular, a ratio of percentages will have an upper bound set by the size of its denominator. If, for example, a child scores 50% as their 'esoteric' subtotal then their highest possible r/e ratio will be 100/50 or 2. If another child, on the other hand, scores 40% as their 'esoteric' subtotal their highest possible ratio will be 100/40 or 2.5. Since service class children, on average, do better than others on the 'esoteric' subsection of the tests their potential maximum r/e ratio is lower than that for the working class children who score lower on the 'esoteric' subsection. Notwithstanding this, Table 4 shows that the service class children have the highest ratios of any group.

There is a clear relation of this ratio to social class background, with its value ranging from 0.85 for the service class, through 0.79 for the intermediate grouping, to 0.76 for the working class for boys and girls taken together[22]. Service class children as a whole have a better performance on 'realistic' items in relation to 'esoteric' items than do working class children. The relation of the ratio to class is particularly clear in the case of girls. Looking at sex, the r/e ratio is higher for boys in both the service and working class groups, though it is identical for girls and boys in the intermediate grouping[23]. The class effect is illustrated in Figure 3, where two linear regression lines have been fitted to capture the 'realistic'-'esoteric' relation for these two class groupings. *What this finding suggests is that, all other things being equal, the higher the proportion of 'realistic' items in a test, the greater will be the difference in outcome between service and working class children*.

It is important to stress that these class differences are not ones of kind. There is much overlap in the three distributions of these ratios by social class. The differences in Table 4 are differences 'on average' not of kind. The charts in Cooper et al (1997) demonstrate this clearly. However, it is also worth noting that, given the many other dimensions on which these test items differ *within* the categories 'realistic' and 'esoteric', it is also possible that these results underestimate the importance of the effect of 'realistic' versus 'esoteric' contextualisation. It is perhaps surprising that the effect appears at all amidst all this 'noise'[24].

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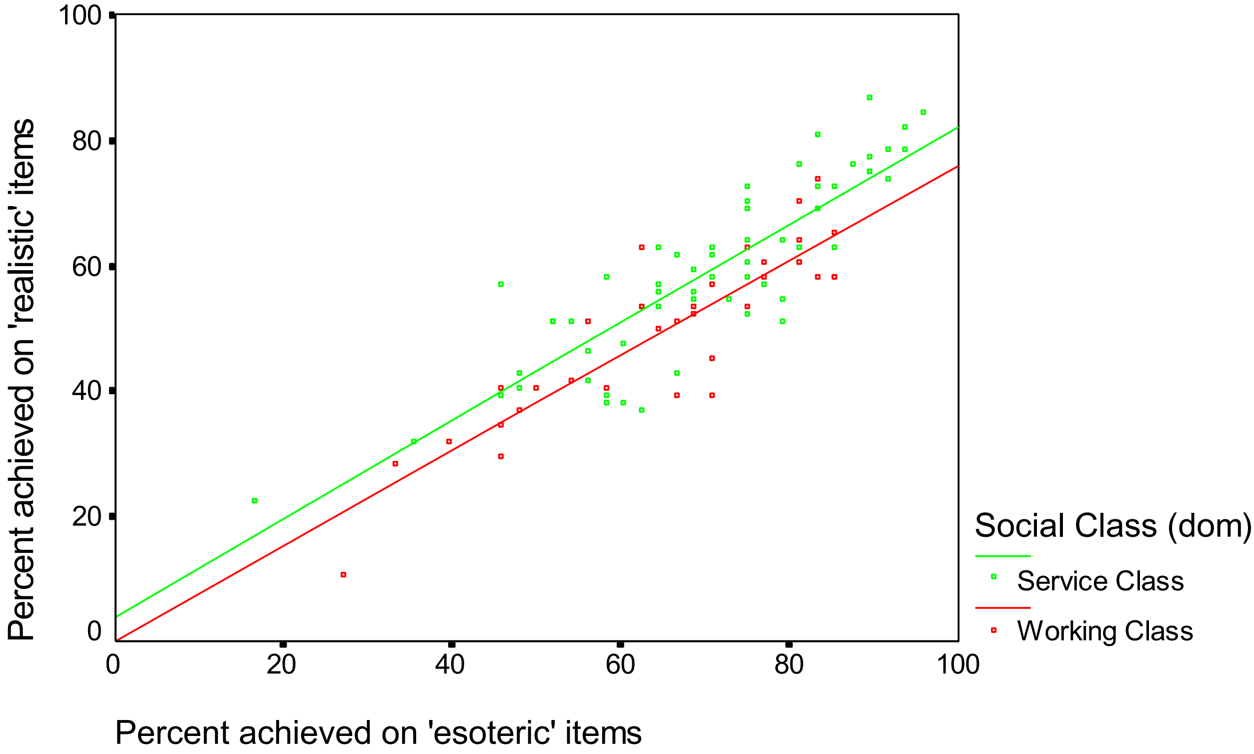
[22] An analysis of variance (simple factorial) of the r/e ratio by social class finds the differences between classes to be statistically significant (p=0.005).

[23] An analysis of variance (simple factorial) of the r/e ratio by sex finds the differences between sexes to be statistically significant (p=0.027). A further analysis of variance including both class and sex finds both independent variables significantly related to the r/e ratio (class: p=0.003; sex: p=0.048) and finds the class/sex interaction to be non-significant. R-squared is 13.7% (adjusted R-squared 10%).

[24] Furthermore it is a common error of empiricism to move from the absence of an effect, or the small size of it, to the absence of a mechanism, forgetting that the effects of a real mechanism may be hidden by other factors at work. See, e.g., Bhaskar (1979).

Figure 3: The distribution of KS2 ‘realistic’ percentages by ‘esoteric’ percentages by child (service class and working class only[25])



Social class may, of course, only *appear* to be a causal factor here. It might be the case, for example, that ‘ability’, some concomitant of school attended such as curriculum coverage, and/or systematic differences in the easiness of the ‘realistic’ versus ‘esoteric’ items are the real underlying causes of the results in Table 4. We have tried to approach these problems from two directions. First, we have used logistic regression to examine the associations between school, ‘ability’, sex, class and the ratio. Secondly, concerning curriculum topic/area we have looked at how the ratio varies within Attainment Targets. The regression analysis (Cooper, Dunne & Rodgers, 1997) with our ‘realistic’/ ‘esoteric’ ratio as dependent variable and social class, sex, school and non-verbal ‘ability’ as independent variables, suggests that class and sex are statistically significant here and that school and non-verbal ‘ability’ are not[26]. Details of the analysis by Attainment Target are set out in the following section of the paper.

[25] The line for the intermediate class falls between these two with a similar slope.
[26] Logistic regression, employing backward elimination. It should be noted, however, that statistical significance is difficult to interpret when procedures such as logistic regression are applied to samples such as ours which are not simply random. See, e.g., Gilbert, (1993) pp.77-78.

The differences within Attainment Targets

How do these class and gender differences in the r/e ratio behave within attainment targets, i.e. in relation to broad topic areas within maths[27]. In fact, Tables 5-7 show that the class and gender differences continue to appear within ‘number’, ‘algebra’ and ‘shape and space’.

Table 5: Ratio of ‘realistic’ percentage to ‘esoteric’ percentage by class and sex (number)

	Female	*Female*	*Male*	*Male*	*Total*	*Total*
	Mean	Count	Mean	Count	Mean	Count
Service class	.79	26	.83	34	.81	60
Intermediate class	.82	13	.81	17	.81	30
Working class	.78	13	.79	20	.78	33
Total	.79	52	.81	71	.80	123

Table 6: Ratio of ‘realistic’ percentage to ‘esoteric’ percentage by class and sex (algebra)

	Female	*Female*	*Male*	*Male*	*Total*	*Total*
	Mean	Count	Mean	Count	Mean	Count
Service class	.69	26	.88	34	.80	60
Intermediate class	.66	13	.71	17	.69	30
Working class	.57	13	.56	20	.56	33
Total	.66	52	.75	71	.71	123

Table 7: Ratio of ‘realistic’ percentage to ‘esoteric’ percentage by class and sex (shape & space)

	Female	*Female*	*Male*	*Male*	*Total*	*Total*
	Mean	Count	Mean	Count	Mean	Count
Service class	1.17	26	1.17	34	1.17	60
Intermediate class	1.04	13	1.13	17	1.09	30
Working class	1.04	13	1.19	20	1.13	33
Total	1.10	52	1.17	71	1.14	123

The patterns are less clear than they were in Table 4 but are nevertheless there. In each case an overall service/working class comparison of the r/e ratio favours the service class against the working class. In parallel with this, an overall male/female comparison of the r/e ratio consistently favours the boys. These differences are particularly marked in the case of algebra. It is also interesting to note that, in the case of ‘shape and space’, the children found the ‘realistic’ items generally easier than the ‘esoteric’ ones.

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[27] Early versions of the English national curriculum assumed that each test item could be associated with one statement of attainment - a form of behavioural objective within each of the Attainment Targets of ‘number’, etc. We are not believers in the idea that an item can assess just one statement of attainment from within an attainment target. However, we are following the early ‘official’ practice of the national curriculum assessors in coding each item (or part-item) as belonging to one AT. Clearly, any item is likely actually to demand a cluster of skills and understandings for its solution. More recently, the National Curriculum test papers have dropped the labelling of each item by one statement of attainment. We have taken the ‘official’ coding where it exists and have tried to simulate it in the case of more recent items where it does not. Some of these codings are difficult for the very reason mentioned above.

Nevertheless, the r/e ratio remains highest in the case of the service class taken as a whole, and boys have a higher ratio than girls. We are not able to present a table for the case of data handling since all of the items under this heading have been coded as 'realistic'. However, some idea can be gained of the 'behaviour' of the latter items in relation to class by examining their position in Table 8. Here we show how children from each class group performed on each of the seven *attainment target - context* coding combinations. Table 9 shows comparable calculations for boys and girls. Comparing the service class with the working class, and boys with girls, there appear to be similar class and gender effects across attainment targets, suggesting that the differences in the r/e ratio in Table 4 are not 'spurious' topic effects.

Table 8: Mean percentage scores by class for each existing attainment target/context combination

	Service class	*Inter-mediate class*	*Work-ing class*	*Total*	*service mean/ working mean*	*number of separately coded items & sub-items*
Number - 'esoteric'	78.81	77.94	75.61	77.74	1.04	21
Number - 'realistic'	64.05	63.57	59.96	62.83	1.07	22
Algebra - 'esoteric'	68.46	66.92	61.54	66.23	1.11	10
Algebra - 'realistic'	50.60	44.05	30.74	43.67	1.65	11
Shape & space - 'esoteric'	66.79	66.19	57.58	64.17	1.16	11
Shape & space - 'realistic'	72.50	66.33	60.00	67.64	1.21	9
Handling data - 'esoteric'	n/a	n/a	n/a	n/a	n/a	0
Handling data - 'realistic'	62.86	55.42	49.34	57.42	1.27	26
n (children)	60	30	33	123		110

Table 9: Mean percentage scores by sex for each existing attainment target/context combination

	Girls	*Boys*	*Total*	*Boys' Mean / Girls' Mean*	*number of separately coded items & sub-items*
Number - 'esoteric'	76.63	78.27	77.56	1.02	21
Number - 'realistic'	60.71	63.88	62.51	1.05	22
Algebra - 'esoteric'	68.23	64.36	66.03	0.94	10
Algebra - 'realistic'	42.06	44.37	43.37	1.05	11
Shape & space - 'esoteric'	63.49	64.08	63.89	1.01	11
Shape & space - 'realistic'	65.19	68.87	67.28	1.06	9
Handling data - 'esoteric'	n/a	n/a	n/a	n/a	0
Handling data - 'realistic'	55.67	58.19	57.10	1.05	26
n (children)	54	71	125		110

Another possibility which needs to be addressed is that it is because the 'esoteric' items are, in general, found easier in this data set, coupled with class related differences in typical educational achievement, that the r/e ratio patterns by class are as they are. Perhaps working class children just perform less well on harder items? In fact, however, statistical analyses employing items rather than the child as the case have shown that broad social class differences in a relative of this ratio remain (though are reduced in importance[28]) when examined within four categories of items ordered by average difficulty levels[29]. The means in Table 10 derive from a variable constructed by dividing, *for each item*, the service class mean score by the working class mean score[30]. It can be seen that, within each category of items, from the most easy to the most difficult, the service class children perform relatively better than working class children on 'realistic' items as compared to 'esoteric' items[31].

Table 10: Ratios of service class mean score to working class mean score for an item by observed item difficulty and nature of item (count is of items)

	Realistic Items		*Esoteric Items*		*Total Items*	
Item difficulty levels	Mean	Count	Mean	Count	Mean	Count
1. Most difficult quartile	1.62	21	1.37	6	1.56	27
2. Second quartile	1.42	18	1.20	9	1.35	27
3. Third quartile	1.21	14	1.10	12	1.16	26
4. Most easy quartile	1.06	15	1.03	15	1.04	30
Totals	1.35	68	1.14	42	1.27	110

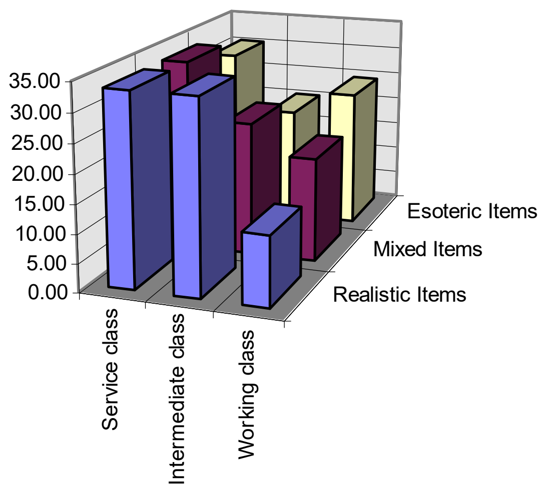
These effects may appear small. However, in the world of educational practice, where decisions are often taken on the basis of thresholds being achieved or not by children, differences of this size can have large effects. To illustrate this, we have developed a simulation of what would happen to children from different social class backgrounds if a selection process were to occur on the basis of three differently composed tests: one comprising items which behave like our 'esoteric' items, one of items which behave like our 'realistic' items, and one comprising an equal mixture of the two[32]. This process might be realised as a selection exam for secondary school or for set placement within the first year of secondary school. A summary of the results is shown in Table 11 and Figure 4. It can be seen that, using our results as the basis for predicting outcomes, the proportion of working class children in this sample who would be selected by an 'esoteric' test is double that which would be selected by a 'realistic' test. The two tests lead to quite different outcomes, mainly for intermediate and working class children[33].

[28] An analysis of variance of this service/working class ratio by difficulty level and nature of the item ('realistic' v. 'esoteric') finds both independent variables significant (difficulty: p=0.001; nature of item: p=0.051), with the interaction term non-significant. R-Squared is 25.7% (adjusted R-Squared is 20.5%).
[29] Similarly, the findings hold when the 'wordiness' of items is controlled for.
[30] Differences in measured 'ability' are automatically controlled for in this approach, as in the use of the realistic/esoteric ratio earlier.
[31] The nature of this ratio is such that it is constrained to be smaller as difficulty level falls.
[32] Because of ties in the data, it has been necessary to select very slightly different overall proportions of children in the three cases: 26% for the 'esoteric' simulation, 27.6% for the 'realistic' simulation, and 26.8% for the mixed test. These are small differences in relation to the size of the resulting effects.
[33] The findings would also have implications for any comparison of schools via league tables based on the three simulated tests discussed here. We have not attempted to apply significance testing to these models. It should be recalled that they employ social class

Table 11: Percentage outflow selected from classes under three simulated testing regimes (KS2)

	Esoteric Test (26% selected in total)	*Mixed Test (½ & ½) (26.8% selected in total)*	*Realistic Test (27.6% selected in total)*
Percentage selected			
Service Class	30.0	33.3	33.3
Intermediate Class	20.0	23.3	33.3
Working Class	24.2	18.2	12.1

Figure 4: Percentage of children selected from each social class under three simulated testing regimes (KS2)



A similar simulation for sex does not show such large effects, reflecting the smaller differences in the realistic/esoteric ratio in Table 4. While in the case of class, a move from 'realistic' through mixed to 'esoteric' composition linearly increases the proportion of working class children selected, any pattern for sex is less clear (see Table 12 and Figure 5).

Table 12: Percentage outflow selected from sexes under three simulated testing regimes

	Esoteric Test (26% selected in total)	*Mixed Test (½ & ½) (26.8% selected in total)*	*Realistic Test (27.6% selected in total)*
Percentage selected			

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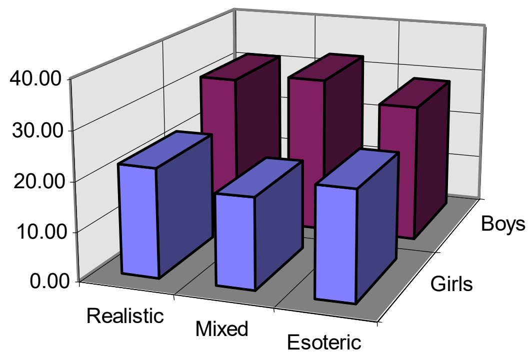
and sex differences previously shown to be statistically significant in the treatment of the 'realistic'/'esoteric' ratio.

Girls	22.2	18.5	22.2
Boys	28.2	32.4	31.0

Figure 5: Percentage of boys and girls selected under three simulated testing regimes



Given the small cell sizes which would result, we will not present a simulation for the six sex/class groups.

Discussion

Considering the marked class effect, a key issue begs to be explored. Is there any evidence that ‘realistic’ items, for various reasons, are *underestimating* working class capacities relatively more than those of children from other class backgrounds? Might they be differentially valid in general? Or is it the case that ‘realistic’ items happen to demand ‘legitimately’ some mathematical capacities which are more social class-related than those required by ‘esoteric’ items? We have presented evidence elsewhere suggesting that part of the social class effect found is due to the social class distribution of children’s ‘choice’ of an ‘illegitimate’ and ‘inappropriate’ ‘everyday’ response mode rather than to their lack of mathematical capacity per se (Cooper, 1996, 1998b; Cooper, Dunne & Rogers, 1997; Cooper & Dunne, 1998). We had hoped to address these explanatory issues here, but space makes this impossible. We have discussed the use of Bernstein and Bourdieu’s ideas to make sense of these findings in these papers and we must refer the reader to these. We are, in a current ESRC project, exploring similar arguments concerning gender. However, whatever the best explanation of these findings is, one thing is clear. Serious equity issues seem to be raised, probably unintentionally, by the continuing emphasis on the ‘realistic’ contextualisation of maths, especially when this is carried over into national test contexts. Darling-Hammond (1994), amongst others, has raised similar concerns about performance assessment in the USA. Whether these are problems that can be addressed successfully by teachers remains to be established.

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Appendix 1: Occupational Groupings

(combined from Goldthorpe & Heath, 1992 & Erikson & Goldthorpe, 1993)

1. Service class, higher grade: higher grade professionals, administrators and officials; managers in large industrial establishments; large proprietors.
2. Service class, lower grade: lower grade professionals, administrators and officials; higher grade technicians; managers in small industrial establishments; supervisors of non-manual employees.

3. Routine non-manual employees
4. Personal service workers
5. Small proprietors with employees
6. Small proprietors without employees
7. Farmers and smallholders
8. Foremen and technicians

9. Skilled manual workers
10. Semi- and unskilled manual workers
11. Agricultural workers

We have collapsed 1&2 into a service class, 3-8 into an intermediate class, and 9-11 into a working class.

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Pupil entry for National Curriculum Mathematics tests: the public and private life of teacher assessment:.

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Abstract

This paper brings together data from two research projects concerned with assessment in school mathematics. It is an attempt to combine data and analysis focusing on the selection of pupils for certain mathematical experiences within school classrooms and their subsequent test entry levels. Highlighting the teacher, connections are made between the external system of national testing and internal formative assessment in continual process during schooling. Data from a recent ESRC is used to examine different school practices for pupil test entry. Behind the resulting official statistics of examination performance - the public, lie the everyday practices in the maths classroom - the private. Insights from an ethnographic study of secondary school mathematics classes are used to elucidate some aspects of this private life. In bringing these two elements together this paper highlights both the overt and covert influence that teachers still have on the test results of their pupils.

Introduction

The historical context of the two research studies in this paper is a period in which a conservative agenda, initially sponsored by the New Right was set up to gradually displace liberal progressive educational ideology (Ball, 1990; 1994). The emphasis behind this effort to reconstitute state educational provision has continued into the late 90's despite a change of national government. The incremental establishment of this conservative educational agenda has been effected by changes in the detail and emphases of teachers' work, a changing balance of activities and priorities (Brown, 1992) supported by the introduction of a different educational discourse around the problematics of everyday life in schools (Dunne, 1994). The pressures on teachers to make the transition from progressive to conservative are both coercive and obvious in

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terms of particular innovations and legislation eg the National Curriculum (NC) or the Teachers Pay and Conditions Act 1986, but they are also pervasive and subtle.

The National Curriculum (NC) and Key Stage (KS) Assessment at ages 7, 11, 14 and 16 were introduced through the Education Act 1988. This has led to the establishment of annual national testing of pupils at each of these four Key Stages. School and teacher accountability measures, other features of the educational reform, have given rise to enormous interest in school effectiveness. The publication of school league tables is an important element in the government efforts to raise standards and make schools more effective. With the emphasis on measurable school outcomes, these tables report the relevant KS test results for all state schools. As the teachers in our research studies observed, this focus on the national paper and pen test results has affected their pedagogy and teacher assessment practices. In relation to the latter, the teachers noted a diminished contribution of teacher assessed attainment levels to each pupils final KS result.

In this context, in which teachers appear more distanced from the formal processes of pupil assessment, this paper is concerned with the ways in which teachers still have overt and covert influence on their pupils' test results. Constraints of space permit only a glossing of some of the most significant elements of this argument which otherwise might be more elaborated and developed. After a brief overview of the two research projects informing this paper, I first look at the implications of particular school practices for pupil entry to the different NC mathematics test levels at KS3. Following this, I examine the ways in which teachers describe their formative assessment practices taking place continually during mathematics classes with their pupils. The focus here will be an analysis of mathematics teachers' explanations of the processes by which certain pupils are selected for particular classroom experiences and ultimately particular levels of the National Curriculum tests. The final picture that emerges shows how, despite major educational reforms, teachers still have a highly significant influence on the public grading of mathematics performance of their pupils.

Research overview

The data used in the next section derives from an ESRC study concerned with pupil interpretation and performance on KS2&3 NC tests in mathematics (Cooper and Dunne, 1997). At the KS3 level we collected three Nelson Cognitive Ability test scores (CATs) and the 1996 Mathematics National Curriculum test scores of 473 Year 9 children from three secondary schools. We also interviewed 15 teachers, concentrating on the school's approach to mathematics and on teachers' perspectives on the NC assessment and their pupils in their schools. The selection of schools was based upon providing a cross social class sample of children and a willingness of the school and mathematics teachers to be involved in the research project.

The second research project was an ethnographic study undertaken from between 1991 - 1994 in four state secondary schools. The data and analysis presented here were obtained from a year of intensive school-based field work, followed by continual periodic contact with four mathematics teachers from each school. The use of formal and informal interviews were central to the data collection. In the larger study a wide variety of data collection methods were employed. The data presented and analysed in this paper derive predominantly from the individual and group interviews with four mathematics teachers. Transcriptions of each of these interviews were circulated to the teachers for comment. The selection of the participating schools was based upon providing a broad range of school contexts, in terms of social class and ethnic mix. All school were co-educational to provide a gender mix. The final selection was made to maximise the representation of different sex and ethnic groups among the teachers. Both the schools and the teachers were volunteers.

Public performance

The tables and figures presented below are derived from a section of a recently completed ESRC project on mathematics assessment (Cooper and Dunne, 1997). In this paper I want to draw attention to school practices in relation to pupil KS3 mathematics test level entry. Decisions about examination entry are informed by mathematics teachers' assessment of their pupils' abilities which in turn are mediated by the mathematics department and/or school policy (whether formal or informal) in this regard. There are several factors above and beyond a concern for the individual child that might influence school policy regarding pupil entry. School examination results have implications for school / teacher accountability, the position of the school in the league tables and in the educational market place. Given that a pupils' NC level is capped at the top of each test's level range a school's attitude to risk as well as its expectations of children will affect test entry decisions. Whatever the practice within each context, decisions on test level entry are underpinned and rationalized by teacher assessments of their pupils' mathematical abilities relevant to the test.

It was evident from this research that schools had different practices in relation to these pupil entry decisions. Taking measured ability as an heuristic baseline, schools differ in their allocation of children with given ability scores to levels of the May 1996 tests. The example of non-verbal ability is shown in Table 1 where it can be seen that the first school ‘requires’ a higher CAT score for entry to the three major test levels than the other two.

Table 1: Mean non-verbal CAT score by school and tier of entry for May 1996 test

	SAT 3-5 taken		*SAT 4-6 taken*		*SAT 5-7 taken*		*SAT 6-8 taken*	
School	Mean NV CAT score	Count	Mean NV CAT score	Count	Mean NV CAT score	Count	Mean NV CAT score	Count
D	94.74	84	106.70	110	113.65	51	120.67	10
E	85.46	39	99.60	50	109.09	11	126.50	2
F	89.30	54	98.31	35	107.25	28	n/a	0

It is possible to see the effects of this from another perspective. First a variable is constructed by defining 3-5 entry as 4, 4-6 as 5, 5-7 as 6, and 6-8 as 7. A ratio is then created by dividing SAT level *actually achieved* by this measure of SAT level *taken*. The ratio obtained which will give some idea of the differences between the schools in respect of levels of entry. The distributions of this ratio by school, are shown in Table 2[2]
The higher ratios in the first column bear out the tendency of School D to ‘under-enter’ children relative to the others in our sample.

Table 2: Ratio of SAT level achieved / SAT taken: Means by school

	School D		*School E*		*School F*	
All pupils	Ratio	Count	Ratio	Count	Ratio	Count
	1.03	255	0.85	102	0.93	117

It is possible to push this analysis further through an exploration of the considerable overlap of scores on baskets of common items across neighbouring levels (e.g. 3-5, 4-6) of the May 1996 tests. As an illustration, the distribution of marks on the 61 common items[3] are shown for the two groups entered for 3-5 and 4-6 in Figures 7 and 8. Ten children appear at the right of Figure 7 who were entered for the 3-5 test but

[2] Note this is a bounded ratio. Also its maximum value is a function of tier taken, with the maximum possible ratio falling as tier entered rises. For further detail please refer to Cooper and Dunne ,1997. It is interesting to note that the school best positioned in the school league tables is apparently the most cautious in its test entry policy.

[3] These are drawn only from the May 1996 tests.

who score better than the mean achieved by children entered for the 4-6 test (52.1). . This difficult area of level placement, critical to both school and pupil profiles, raises a threat to valid assessment inherent in the testing arrangements for KS3

Figure 1: Distribution of children's scores on items common to the May 1996 3-5 and 4-6 tests for children who took the 3-5 tests in May 1996

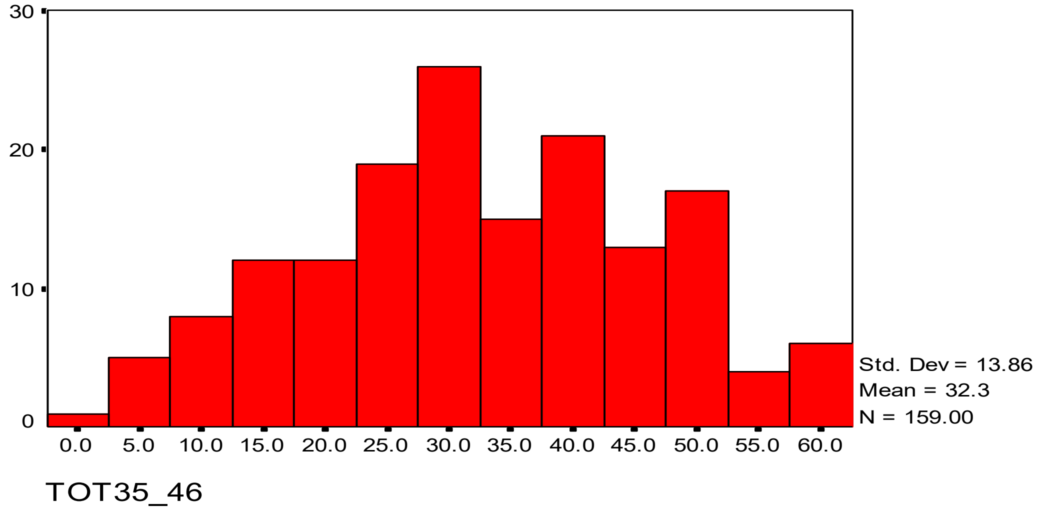
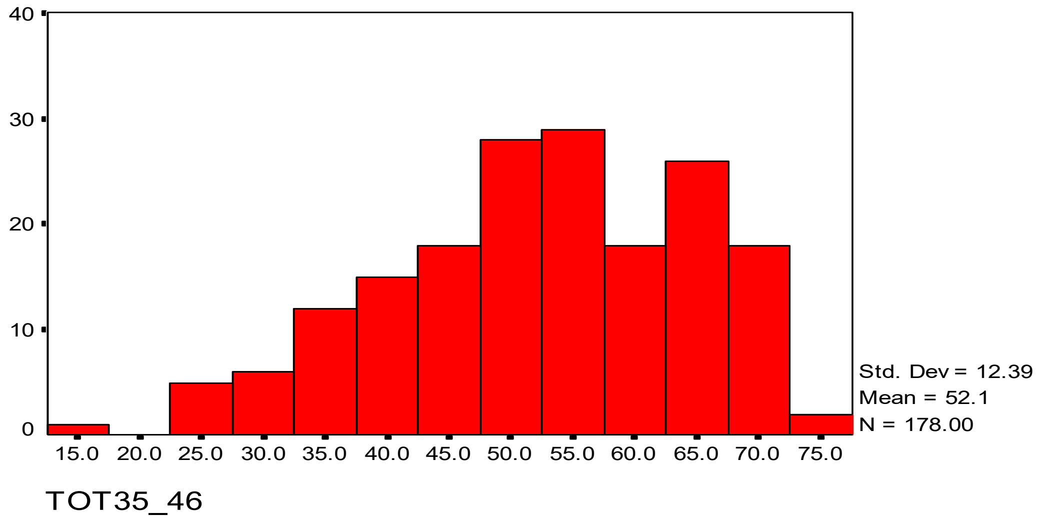


Figure 2: Distribution of children's scores on items common to the May 1996 3-5 and 4-6 tests for children who took the 4-6 tests in May 1996



Although they can hardly be expected to get selection for test levels 'just right', teacher judgements are key to the limits of their pupils' possible KS level attainment. It should be noted, however, that these judgements are circumscribed by structures for the national testing from which, on the whole, mathematics teachers are distanced. Mediation at the institutional level, through school and mathematics department policy/practices are processes in which mathematics teachers have more direct but variable influence. Nevertheless, at the fundamental level, in the classroom, teachers clearly have an overt and pivotal role in the entry of their pupils to specific test levels.

Having touched upon some of the problematics of the public life of test entry, I now move, in the next section, to consider the private life of the classroom as the context within which pupils are framed in terms of their mathematical ability and teachers have a more covert role in their pupils' NC test level entry.

The private life of teacher assessment

An element of the educational reforms was the introduction of systems of school accountability, one net effect of which has been to highlight examination results as a measure of school effectiveness. Interviews with teachers from both projects elicited descriptions of resultant changes in pedagogy and assessment. The overwhelming majority of these teachers reported increased tendency to teach whole class lessons in a formal style, give tests and continually to reinforce basic mathematical operations. They also noted the diminished importance and independence of the Teacher Assessment level recorded for each pupil in the official NC test results. Despite these latter observations concerning the greater circumscription of teachers' control over the mathematics curriculum and its assessment, there are still significant ways in which teachers make crucial decisions about the school mathematics experiences of individual pupils and the limits of their achievement level in public examinations. The previous section considered this influence in terms of the apparent school policy for examination entry. The rest of this paper will look at the processes of teacher assessment through a preliminary exploration of the social relations of the classroom. At issue here is an attempt to understand how teachers explain their part in grading their pupils. It is important to note here that this not a claim that mathematical ability is only constructed within the confines of the classroom or to suggest that there are no extra-situational components at work in the shaping of individual mathematical achievement.

In the initial phases of this research with teachers it became evident that despite the divergence in their responses to the changing material and ideological conditions of their work, brought on by the educational reforms, there were also implicit continuities. Hidden sets of social relations, part of the covert curriculum, structure the teaching and learning context. The local conditions that frame school teaching and learning settings are assumed across different educational ideologies and often remain unproblematic. As such they represent certain continuities underlying ideological conflicts emergent at different junctures in educational debate. This basic argument is made succinctly by Davies et al (1990) in reference to the most recent educational reforms.

"The broad dichotomy between traditional and progressive education can serve to hide the variability which exists within the framework of progressive education. Forms of progressive education can be class, race and gender biased and depress the performance of working class children, blacks and girls. These biasses intrude from a

range of sources - the middle class assumptions upon which schooling itself is predicated, teacher expectations, the impact of hidden curriculum - which operate in both traditional and progressive educational environments. " (Davies et al, 1990: 26)

The evident poor achievement and participation in mathematics of the same particular social groups (Apple, 1992; Dowling, 1991; Ernest, 1991; Dubberley, 1988) despite educational and pedagogical reform lends support to the assertion of fundamental continuities. The assumptions teachers make about pupils and schools are more part of the hidden agenda than the official rhetoric of an educational or political party line. Importantly, these assumptions directly influence the assessments teachers' make of individual pupils' mathematical abilities.

In efforts to understand the process of teacher assessment, this study began by exploring how teachers described the ways they made judgements about their pupils' mathematical capacities. The initial discussions revealed the teachers shared confidence around the way they assess the 'ability' of pupils.

MD: What will happen next year to this group?
R: . . . we're going to have setting within two streams - an A and B stream. For maths a top, middle and lower group in both streams. . . .
MD: Do you already envisage where people in this class will be?
R: If I went through a list I think I'd get 90% right without looking at marks. I haven't given it any thought. But I'd probably be able to say what group they'd be in. . . . I've always felt I've known kids. I've known them well enough and a lot of my assessment is mental, stored in my braincells and not down on paper.

Such confidence in an implicit and largely unarticulated process was also extended in some cases to pupils who were not in the teachers' mathematics classes

R: . . . Looking at them, just knowing them as people, I would have said that half of the kids that put their hand up were in the lower half of the group.
MD: As you don't teach them what gives you that impression?
R: I would have said if we weren't talking about this, that they are the less able kids within the group.
MD: How do you know?
R: Their written work is not very good, they are not the quickest at thinking if you ask them questions. Making decisions, not very quick at making decisions, not always the best decision. The comments I hear from other staff.
MD: One pupil I talked to felt she could never be good at maths because she was shy.
R: Well, I mean certainly that would be true of somebody whose left my group. You would think that he was very able because he'd talk a lot.

Through critical reflection attempts were made to make explicit those factors that informed teacher assessments.

H: As a teacher of maths . . . the balance of what I do as a teacher changes with different groups. . . . I don't think it's an active decision . . . . It just so happens if individuals talk to you in a particular way, you respond in a particular way so if you think they're being particularly aggressive then you in turn, might be aggressive back and therefore you've automatically got a confrontation situation which means that you're not gonna get as far as you should. Whereas an individual that will ask you a question and just seems to be what a teacher perceives as the way children ask questions, then you answer them and you think this is a good pupil because he's doing what I expect.

In an account of his own experience of classroom social relations from the other side of he desk, albeit in a selective school, Furlong (1991) uses a notion of 'class cultural affinity' to describe how some of his class mates had closer relations with their teachers. “It was a common value system that was both intangible and powerful, producing a bond which transcended the day-to-day conflicts of classroom life." The mathematics teachers recognised this:

H: I've seen pupils in the street some usually say 'Hello!', others just walk past. I think it depends on the individuals or the class. There are some that you do talk about your home especially when there is some common ground between you. There are some individuals that their home life is so alien to what I've experienced it would be so inappropriate. I'm not saying they can't talk to us [teachers] because there is no common ground and I suppose you do say a few things.

Teacher P traces a connection between this cultural affinity and teacher assessment, explaining how such inter-personal relations influence teacher judgements of pupils capabilities.

P: Again they're all labelled aren't they, just depending on where they live or their family background. We do that all the time. I think a lot of the time, even at school, we treat pupils differently because of their background. You can pick out straight away which ones are sort of from a background who's parents are going to support your actions, lets say. . . . And you as a teacher do that straight away. By doing that you've already limited their success in your subject haven't you? I don't know if you do it consciously or unconsciously but if you look I think you do it.

However, as Teacher H describes below, the structurally ascribed power positions of pupils and teachers is not sufficient to describe classroom social relations. Experience inside schools will quickly demonstrate social interactions as a complex of resistance and collusion of pupils and teachers and between them.

MD: So when we go on to that automatic pilot stage what tells you that it's okay to behave in that way?
H: Because you get away with it and there's no one else around to say that that's not acceptable. It's only when you get an adverse response from the pupils that you know that you've done something particularly wrong.

The significance of the pupils in the school and the classroom has been acknowledged within several studies that have focused upon pupil perspectives or responses within social institutions (See for example Mac an Ghaill, 1992; William's, 1988; Griffin, 1985 and Willis, 1977). The various ways in which the pupils' identity affects teachers responses are described by the mathematics teachers by focusing upon how gender structures social interactions in their classrooms.

R: I do think there are quite a lot of boys comments and girls comments, hurtful comments and again do the staff help at times? I don't know whether we do. I hope kids realise I'm joking, but that is just as bad. . . . I certainly heighten it rather than letting it sleep. I think they are very different in the way they react and boys get me to react a bit silly. Comment more regularly whereas the girls don't get me like that, they don't make the same comments.

MD: I wonder with people like Sofna for example, if she were a boy, how different would the school or teachers response be to her?
A: Yeah, because she's very demanding yet. . . . If it was a boy I wouldn't let them nag me so much, I'd just say go away. I say go away enough to Sofna, but I'd say it more to a boy.
MD: In a way, talking about her being very demanding, she's actually always on task.
A: Yeah, she's always trying, she tries but yeah, I mean she is always on task. That is amazing, she never really strays off. . .

These teachers' comments clearly indicate largely unexamined sets of their inter-subject relations, integral to school and classroom routine, that inform the process of teacher assessment. Culturally and contextually specific expectations and codes of behaviour, although highly significant, remain unarticulated and hidden. Other research has focused on some of these factors in relation to mathematics test items (see Cooper and Dunne 1998; Cooper, Dunne and Rodgers, 1997) and school mathematics texts (see Dowling, 1998). In the case here, of the mathematics classroom, the routinisation of teacher pupil interactions, in practice, acts to normalise, de-personalise and de-politicise these processes. Appeals to fairness and professionalism are strategies that distance teachers personally from their assessment decisions, even though they are fundamentally influenced by interpersonal interactions (Avis, 1994: Grace, 1987). Such objectification of a highly interactive arena, camouflages the ways in which classroom social relations constitute teachers' assessments of their pupils. The routinisation of these assessment activities works to normalise and even neutralise their covert power, rendering it extremely difficult to make the complexity of these relations visible.

The deeply personal effect of what is a routine teacher task is clearly described by two Year 9 pupils talking about being moved down a mathematics set:

MD: So what did you feel like when you had to go [down] to that class?
Andrea: I was really angry. So was Daniella. Well she was in there before so it's worse for her. That means she hasn't done no good during the year, so it's worse. At first I said I wasn't gonna come to school. When I just found out I told Miss, 'How come I'm in that class.' and she didn't say nothing. I don't feel like going to maths anymore. I used to love maths when I used to go to the other group.

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MD: You've just been put into that group. Why did that happen?
Harvinder: I don't know. I mean I was doing really okay in Miss Stanton's class. I thought I was really good and I was proud that I'm okay and then when I just went down there, I just felt ashamed of myself and didn't want to live any more. . . . When I found out. Disappointed. . . . Felt stupid. I feel dumb.. . . It's just that the work that we do now we're supposed to do it in the first year. And that's what really disappoints me.

These pupils have been subject to unexplicated judgements about their mathematical capabilities and future performance, with the explicit expectation that they accept these - interpellation. The effect of these experiences is not only immediate and limited to the specific context of the mathematics classroom, it is likely to be carried with that individual through school, into other curriculum areas and beyond. Teacher P recognises the potential effect of her assessments upon her pupils,

P: It just takes one teacher to say that you're stupid and that's it. That's practically your whole life gone. For most people it is. For other people, yes it's something for them to challenge. But there aren't a lot of people that are going to challenge it. For most, l think, it just destroys them there and then and they think 'What can l do now?'

Teacher judgements at the classroom level not only contribute to the official documented mathematics level attained by each pupil, but they also inform decisions about the test tier entry and/or appropriate mathematics class set. The subsequent differential treatment of pupils by teachers is justified predominantly by reference to individualised and essentialised notions of ability (Dowling, 1991; Dubberley, 1988; Ruthven, 1987). The reduction of ability to only a personal attribute is superficial, it diminishes the significance of the classroom as an arena for inter-subjective interaction, and ignores the constitution of an individual pupil's and teacher's subjectivity by relations of, for example, age, gender, class and ethnicity. The personally interactive context within which teacher assessment takes place is depoliticised and depersonalised through a normalisation which conceals the complexities of classroom social relations and the dominance of particular cultural forms. Such reference to individual qualities naturalises and neutralises (O'Loughlin, 1992) the 'cultural affinity' that teachers enjoy with certain pupils and the covert, though perhaps not conspiratorial, power they have over each pupils' schooling in mathematics. The simple and well recognised deference to a clinical notion of ability conceals the social relations which are the substance of schooling. (Delpit, 1988; Dowling, 1991; Connell et al, 1982). Indeed the dominant cultural codes in schools against which individual behaviour is assessed, are made invisible or neutral.

Conclusion

This paper has attempted to highlight the various ways in which the daily work of mathematics teachers is fundamental to the limits and possibilities of their pupils mathematics education experiences. The increase of external regulation on the teaching profession has coincided with the institution of national testing arrangements that contradictorily, depend upon the teachers' professional attitude to their work. Indeed, the validity of national KS testing rests on assumptions of such teacher attitudes. of Despite the displacement of teacher assessment in favour of national paper and pen tests it is evident that teachers retain powerful influence over the processes integral to national assessment. This influence is overt in the public sphere through pupil test entry and more covert, in the private context of the mathematics classroom. Teachers connect and mediate between the local arena of classroom mathematics and the department, school and national results. Interests in the school outcomes and effectiveness have tended to focus on the public part of these processes of assessment. More research on the hidden structuring of teacher decisions about the mathematical capabilities of their pupils would undoubtedly provide greater understandings of the social relations of the classroom and the assessment process. Such developments would clearly contribute also to associated issues concerned with social justice in education

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RESTORING DISCIPLINE TO THE CLASS: THE NEW NATIONAL CURRICULUM FOR PRIMARY MATHEMATICS TEACHER EDUCATION

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This paper attempts to identify the underlying influences acting on and ideologies detectable within the new national curriculum in mathematics for initial primary teacher education. The analysis uses the model of mathematics education ideologies in Ernest (1991). The paper concludes that reactionary perspectives dominate this curriculum. The tone is autocratic, directive, managerial, and assertive, redolent of the imposition of discipline on an unruly and untrustworthy class. The new regulations specify an unbalanced curriculum that will lead to one-sided, utilitarian and technicist teachers and pupils.

This paper analyses the underlying ideology of the new National Curriculum for Initial Teacher Training in Primary Mathematics (DFEE 1997). For this project it is necessary to have a theoretical framework. Various models of ideologies have been proposed, including Meighan (1986) and Hill (1991), but this paper uses the model in Ernest (1991), because of the special attention it pays to the role of mathematics in education. It distinguishes five historical groups contesting for control of the curriculum. The model is summarised in Table 1.

Table 1: Summary of the Five Ideological Groupings (adapted from Ernest 1991)

Interest group	Industrial Trainer	Technological Pragmatist	Old Humanist	Progressive Educator	Public Educator
Politics	Radical 'New Right'	meritocratic conservative	Conservative	Liberal	Democratic socialist
View of mathematics	Set of truths and rules	Unquestioned body of useful knowledge	Body of structured pure knowledge	Process view: personalised maths	Social constructivism
Set of values	Authoritarian values choice, effort, work	Utility, progress, expediency	Objectivity, rule-centred, hierarchy	Person-centred, 'Romantic' view	Social justice, critical citizenship
Theory of society	Rigid hierarchy, market-place	Meritocratic hierarchy	Elitist, class stratified	Soft hierarchy, welfare state	Reform inequitable hierarchy
Theory of ability	Fixed and inherited, realised by effort	Inherited ability	Inherited cast of mind	Varies, but needs cherishing	Cultural product: not fixed
Mathematical aims	'Back-to-basics': and social training in obedience	Useful mathematics and certification (industry-centred)	Transmit body of maths knowledge (maths-centred)	Self-realisation, creativity, via maths (child-centred)	Critical democratic citizenship via mathematics
Theory of learning	Hard work, effort, practice, rote	Skill acquisition, practical experience	Understanding and application	Activity, play, exploration	Active, questioning, empowerment
Theory of teaching mathematics	Authoritarian transmission, drill, no 'frills'	Skill instructor, motivate through work-relevance	Explain, motivate, communicate, pass on structure	Facilitate personal exploration and prevent failure	Discussion, conflict, questioning content and pedagogy.
Theory of resources	Chalk and talk only, anti-calculator	Hands-on and microcomputers	Visual aids to motivate	Rich environment to explore	Socially relevant, authentic data
Theory of assessment in mathematics	External testing of simple basics, avoid cheating	External tests and certification, skill profiling	External exams based on knowledge hierarchy	Teacher led internal assessment, avoid failure	Various modes. Use of social issues and content
Theory social diversity	Hierarchic by social class, Eurocentric	Vary curriculum by future occupations	Vary curriculum by ability only	Use local culture to humanise maths	Accommodate social / cultural diversity

This model was proposed to analyse the contestation between groups in the development of the National Curriculum in mathematics in Britain (Ernest 1991). At the heart of this contest was the diametrical opposition in ideologies between the traditional authoritarian Industrial Trainers and the Progressive Educators. Ironically, the outcome was an unstable equilibrium in which elements consistent with both ideologies coexisted. The external testing, hierarchical view of knowledge and assessment system, assessment driven curriculum, emphasis on basic skills, and warnings of the dangers of calculators were outcomes consistent with the Industrial Trainer ideology. Progressive mathematical activity and pedagogy were introduced through the Trojan horse of relevance, utility and applications, because many of those involved were committed to a utilitarian Technological Pragmatist vision of an industry-centred technological education. It is no accident that the attainment target under which progressive, creative mathematical activity is legitimated is *Using and Applying Mathematics,* its title emphasising utility and application. This coalition of the Progressive Educators and Technological Pragmatists is confirmed by Brown (1993: 13) who agrees that the outcome resulted from the "collusion of industrialists and educationists".

The other two groups played lesser roles. The Old Humanists formed a partly effective alliance with the Industrial Trainers on the basis of a shared commitment to hierarchical and elitist views of ability, knowledge and society. But they were only partly successful in getting more rigorous and advanced mathematical content into the curriculum. The Public Educator ideology had no impact. If it had, it would have been opposed by all other groups, especially the Industrial Trainers, for politicising the curriculum and for challenging the dominant absolutist view of mathematics. There is support for this analysis of the varieties of reactionary groups (Lawton 1988), for their influence on the mathematics curriculum (Noss 1989, 1990), and for the pattern of contestation over the mathematics curriculum (Brown 1993, 1996).

It is widely agreed that the National Curriculum resulted in the centralised regulation and control of two aspects of the curriculum, content and assessment.. A third area, pedagogy, remained free from direct regulation, although crowded content and new assessments have an indirect impact. It is my contention that the imposition of the present proposals represents a move to control this last remaining area of self-regulated professionalism in teaching.

THE BACKGROUND TO REFORM

In 1993 there were new regulations for initial primary teacher training (DFE 1993). Their novelty lay primarily in the recasting of requirements into the language of competences.

> Higher education institutions, schools and students should focus on the competences of teaching throughout the whole period of initial training. The progressive development of these competences should be monitored regularly during training. Their attainment at a level appropriate to newly qualified teachers should be the objective of every student taking a course of initial training. (DFE 1993: 15)

These regulations specified that 150 hours must be devoted to mathematics, including 50 hours on the teaching of arithmetic. They specified that the time spent on practice teaching in schools should be significantly increased. Otherwise, the document is non-directive on a variety of issues including pedagogy. However, the proposals downplayed traditional specialist subject expertise. There were proposals suggesting both a reduced emphasis on traditional specialist subject expertise by having a six subject BEd degree, and by reducing the length of courses from 4 to 3 years. These proposals would make it impossible to reach honours degree level in mathematics. Overall, DFE (1993) emphasises competences and increased practical training, including training in basic mathematics and language pedagogy. Disciplinary expertise is downplayed in favour of practical skills, and the overall proposals suggest a strong Technological Pragmatist influence emphasising learning by apprenticeship, and expediency in addressing the teacher supply problem.

The 1990s has seen pressure for the reform of teacher education from a number of quarters. Industrial Trainers have criticised teacher education on the grounds that progressive teaching methods, attention to irrelevant and modish theory and neglect of basic skills is driving school standards down. Thus, Lawlor (1990), claims that teacher education is “too bound by theory; with too little emphasis on the subjects to be taught or on the practical activity of classroom teaching” (Lawlor 1990: 9). Marks has argued it is necessary to “ensure that all primary school children are taught arithmetic using traditional methods and practices similar to those found on the Continent” (Marks 1996: 6). The chief HMI claims that the results of inspections show that “better lessons include: the effective use of exposition, instruction and direct teaching” (Woodhead, 1996: 164)

Old Humanist mathematicians have also criticised progressive pedagogy, and the lack of both core mathematical skills and higher mathematical content. Thus London Mathematical Society (1995: 9) offered the following criticism. “In recent years English school mathematics has seen a marked shift of emphasis, introducing a number of time-consuming activities (investigations, problem-solving, data surveys, etc) at the expense of ‘core’ technique.” This report also claims that school leavers suffer from “a serious lack of essential technical facility – the ability to undertake numerical and algebraic calculation with fluency and accuracy.” (LMS 1995: 2). The President of the Mathematical Association criticised “the mathematics education establishment – who continue to impose their pet half-baked ‘initiatives’ on ordinary punters” (MA 1997: 4). Thus the mathematics establishment seems to want more basic skills and advanced mathematics, more traditional pedagogy and less educational theory in initial teacher education.

Technological Pragmatists appear also to have been swayed by the argument that pedagogy needs to be reformed because it has caused poor attainment in international comparisons of attainment in mathematics, especially number (Keys *et al.* 1996). Thus Reynolds (1996: 21) claims that other more successful countries use “High quality interactive whole-class instruction”

> poor performance in maths may be linked to the way the subject is taught in primary schools. … Observations of classes in Taiwan suggests that teachers might do better by dropping group and individual work and teaching the class as a whole. (Hackett 1996: 1).

A government White Paper promised “reforms in teacher training to raise the standard of literacy and numeracy teaching” (Whitehead 1996: 11).

These shifts of emphasis reflect a change in the political ideologies and social pressures informing the debate on the proposed curriculum for teacher education. One may see this as a move away from the influence of Progressive Educators and the Technological Pragmatist support of progressive teaching styles towards that of Industrial Trainers, Old Humanists, with a new Technological Pragmatist emphasis on efficiency and international competitiveness.

ANALYSING THE NATIONAL CURRICULUM FOR INITIAL TEACHER TRAINING

This paper attempts to identify the underlying influences in the new primary teacher education curriculum, especially Annex C. concerned with mathematics. An interpretive approach is adopted, using the model described above as a tool for analysis. The model provides indicators of different perspectives on the aims, content, pedagogy and assessment of mathematics, and other significant features. More or less the same groups which the model claims were active in contesting the National Curriculum in mathematics are equally active in contesting this new curriculum, so application of the model is justifiable in terms of relevance.

DFEE (1997) is a slim document of 46 A4 pages divided into 5 sections including Annex C: Initial Teacher Training National Curriculum for primary mathematics, 15 pages in length.

Assessment

The introductory section focuses heavily on standards and targets in literacy and numeracy, and indeed the document mentions standards 47 times in the first 13 pages. Furthermore, the first four references in the introduction are part of a rhetoric of 'raising standards' or 'higher standards', implicitly criticising teachers and teacher educators. Throughout, the proposed standards are emphasised strongly as an essential assessment yardstick against which all newly qualified teachers must be measured. In Annex D the first five criteria specifying types of courses permitted concern standards of compliance, content, assessment, attainment, and student profiles.

In Annexes B and C, there is a treble emphasis on assessment. First, trainee teacher knowledge, understanding and skills in mathematics and English must be 'audited', i.e., assessed against the National Curriculum and the new requirements. Second, the courses of initial teacher education must cover the extensive sets of knowledge, facts and skills specified in these annexes and only those who attain the targets (i.e., master the content) are allowed to gain qualified teacher status. Third, the content itself emphasises the assessment of pupil learning as one of the standards to be achieved, both for mathematics and English.

The emphasis on strictly regulated assessment monitored by external authority (Ofsted and TTA) is indicative of an Industrial Trainer influence, although the additional emphasis on practical skill acquisition and teaching, i.e., employment relevance, also suggests Technological Pragmatist influence. A further strong emphasis on the mastery of mathematical content suggest an Old Humanist influence at work. These three groups have compatible views favouring strictly regulated assessment standards and two of them (Industrial Trainers and Old Humanists) do not trust the producers (i.e., teacher educators) to be self regulating.

Aims

There is no overall statement of aims for Initial Teacher Education, but there is mention of

> particular priority on early years and on raising standards of literacy and numeracy ... to underpin higher standards and effective teaching in schools. …
>
> The standards are intended to ensure that, before taking responsibility for their own classroom for the first time, every new teacher will have proved his or her ability in a wide range of knowledge, understanding and skills including effective teaching and assessment methods, classroom management, discipline and subject knowledge. (DFEE 1997:. 3).

The repeated rhetorical emphasis on 'raising standards' is open to at least two interpretations. The first is that there is something wrong in teacher education which needs correction in order to raise school standards. The second is that the efficiency of teacher education needs to be improved to raise standards. The first of these interpretations suggests an Industrial Trainer influence focussing on mastery of basic numeracy skills and the critique of the perceived liberal or radical influences of teacher educators. Likewise the focus on the transmission of mathematical knowledge is indicative of an Old Humanist influence. The second interpretation suggests a Technological Pragmatist influence, with its emphasis on skills and *efficiency* with regard to teaching. In support of this second interpretation, there is reference to improved effectiveness, and indeed 'efficiency' and 'effective teaching' are mentioned 23 times in the document. Both these interpretations seem to hold, given the overwhelming emphases of the document which fit the aims of these groups.

The emphasis in the quotation and the document overall on *effective teaching and assessment*, on *management and discipline* and on *subject knowledge*, and the exclusion of any mention of children, their experience, the community, the social context of schooling, and aims or values, support the analysis given above. After all, the document is primarily specifying the National Curriculum for Initial Teacher Education for early years and primary school teaching, and thus might be expected to reflect some of these sensitivities widespread in the profession.

Content

An examination of the overall balance of content in DFEE (1997) gives a powerful further indication of the aims implicit in the document. Clearly mathematics/numeracy and English/literacy dominate. This emphasis contrasts with the treatment of other primary school curriculum subjects. These are mentioned altogether in DFEE (1997) with the following frequencies: science (9), religious education (9), information technology (7), physical education (3), design and technology (3) times; whereas history, geography, foreign languages, dance, drama, and music are not mentioned once. The message is clear: basic skills dominate the initial teacher training National Curriculum, and other subjects which appear useful in preparing future employees are also given space. Thus science and information technology appear, presumably because they are understood to be technologically and useful and hence economically valuable. Religious education is presumably intended to inculcate moral values to develop the law abiding future citizen. Each of these subjects thus serves (or is perceived to serve) a socially useful function. Design and technology and Physical education are only mentioned in the context of an optional “few hours of … safety training in PE and/or design & technology.” (DFEE 1997: 9, 42, 44), which while evidently utilitarian does not really concern the content of these two subjects.

In contrast, the ‘non-utilitarian’ creative and cultural foundation subjects are not mentioned at all, although every primary school teacher must teach them. Presumably this reflects the back-to-basics agenda of the Industrial Trainers, and the utilitarian agenda of Technological Pragmatists. Only those skills which appear immediately useful for work are given any attention.

Annex C of DFEE (1997) specifies the mathematical content in great detail. This primarily covers Attainment Targets 2 to 4 of the National Curriculum in mathematics for schools, although he match is not exact. An overwhelming part of the section is devoted to number and arithmetic (6 out of 15 pages). The approximate share of space devoted to the different elements of mathematical content is Number and arithmetic 40%, Total mathematical content excluding number 33% (Data handling 7%, Algebra and pre-algebra 7%, Shape and space 7%, Measurement 4%, Problem Solving 4%, Proof 2%, Information Technology in mathematics 2%). Given the emphasis on the other content areas in the National Curriculum their neglect is unwarranted, especially since primary student teachers can be expected to have mastered basic number skills before entry to university. The treatment of number does not include number theory or other advanced content, but is focussed on basic number concepts and skills, shown in Table 2.

Table 2: frequency of occurrence of arithmetical terms in Annex C (DFEE 1997)

Arithmetical terms	**Frequency of occurrence**
Numbers, numerals, counting, numeracy	80
Calculating, computations, operations, algorithm	51
‘+’ used arithmetically (not algebraically)	33
tables, multiplication, ‘×’ used arithmetically	42
Decimals, place value, decimal point ‘.’	25

Thus elementary numeracy and arithmetical operations are overemphasised, while other aspects of mathematics are underemphasised. Using and Applying Mathematics is neglected with problem solving occupying only about 4% of Annex C. This is an important part of primary maths, and is an area in which teachers have expressed concern about being under-prepared (Koshy 1997, Stoessiger and Ernest 1992). Another 2% is devoted to Proof, but this plays little part in Using and Applying Mathematics in primary school. Instead, proof suggests attention to rigour, correctness and strictness in reasoning, consistent with the absolutist epistemology and values of both the Industrial Trainers and Old Humanists.

The language of Annex C reveals very little attention to open problem solving. Although ‘problem’ and ‘solving’ are used about 12 times each, only two or three instances refer to non-routine problems. The term ‘strategy’ occurs three times, but in connection with choosing a mode of calculation. Terms related to ‘applying’ or ‘application’ occur 8 times, but only three of these relate to Using and Applying Mathematics. Instead, the discussion is dominated by skills and standard methods (11 mentions), practice (2 mentions), and basics and facts (9 mentions).

In conclusion, it can be said that the mathematical content is dominated by a concern with basic arithmetical skills, and that the treatment of other topics is proportionately much less, and the Using and Applying element of mathematics is only touched upon in a very limited way. There is also some treatment of higher mathematics (algebra and proof). Overall, this fits with the aims of the Industrial Trainers and Old Humanists. The discussion or treatment of practical application of mathematics to non-routine, non-text book situations is limited but utilitarian in emphasis suggesting in addition a Technological Pragmatist influence, for supporters of the Industrial Trainer ideology left to their own devices would eliminate this type of activity altogether.

PEDAGOGICAL CONTENT

The pedagogy specified is largely teacher-centred with whole class teaching, direct instruction, and explaining, mentioned four or five times each, and other teacher-centred terms like demonstration, consolidation, and review also mentioned. Discussion is mentioned only once, and this is in the context of whole class questioning and teaching. There is a striking contrast between the number of references to teaching (72) and learning (5). Thus the pedagogy is teacher centred and directive. The child centred, facilitative model which has long been the orthodoxy in primary education is rejected. There is also a strong managerial element with progress and progression repeatedly emphasised (17 mentions) and pace, stages, and review, mentioned two or three times each. Assessment and testing are also stressed (18 mentions) as well achievement, qualifications, and standards (14 mentions). Thus the emphasis is on teacher direction, control and surveillance.

In addition to the explicit pedagogical elements there is also a hidden autocratic dimension to the tone of the document. There are 34 commands using the word ‘must’, as well as repeated emphasis on the strictly regulated assessment of trainee teachers’ knowledge and skills. Both the tone of the document and the explicit avowal of teacher-centred instruction suggest a traditionalist ideology of the type shared by Industrial Trainers and Old Humanists.

One element which undercuts this is the recommendation concerning the use of practical apparatus and real-life materials (made twice) in primary school. This fits better with a Technological Pragmatist ideology (and also in part with Progressive and Public Educators), so the ideology is complex and multi-dimensional. Further support for this modified reading can be found in the emphasis in information technology in Annex C. Calculators are mentioned 3 times and computers, information technology and software 9 times. This is significant, because calculators have traditionally been anathema to Industrial Trainers, and it is the Technological Pragmatists and other progressives who support their use. However the emphasis on having “a working knowledge of information technology (IT) to a standard equivalent to Level 8 in the National Curriculum” also fits with Industrial Trainer concerns with basic skills for employment.

VIEW OF LEARNING

In the treatment of pedagogy, learning is very much dominated and overshadowed by teacher-centred instruction. Instead of learning, measures of learning, i.e., assessment and assessment outcomes, dominate the discussion. There are in addition indicators of which learning outcomes are valued. These include knowledge (21 mentions), understanding (63 mentions), and skills (10 mentions). There is also the claim that the connected nature of mathematics should be understood, mentioned twice. Affect is mentioned but only marginally. Thus there is no recognition of the importance of pupils' engaging in active, participative learning to develop their understanding. The focus is not on learning processes but on their external products, scores gained in assessments. This is typically Industrial Trainer in emphasis (and Technological Pragmatist) . There is, however, some emphasis on the acquisition of a structured and well connected body of knowledge. This fits well with the Old Humanist view and aims of learning.

EPISTEMOLOGY

There are frequent references to exactness and precision (16), correctness and certainty (10), whereas less stress is devoted to approximation and estimation (9). Of itself, these references do not indicate an absolutist epistemology, for mathematics is widely celebrated for its precision and exactness. However, there is also a great deal of emphasis on errors and misconceptions (17) with no mention of alternative conceptions or the necessary role of errors in learning and coming to know, which is widely recognised in the literature (Askew and Wiliam 1995, Novak 1987). In addition, Annex C is written in the language of compulsion and autocracy. This combination of emphasis on certainty, on knowing labelled as correct or erroneous, and on authority as the arbiter of knowledge suggests an absolutist epistemology. The frequent reference to error which needs rectification suggests the Industrial Trainers. Absolutism also fits with the Old Humanists, and to a lesser extent the Technological Pragmatists, but they are less punitive in their attitudes to error.

SOCIAL DIVERSITY

Annex C ignores social diversity. Special educational needs, under-achievement, and the very able are referred to three times in total, but in each case the concern is with assessment issues. There is no discussion of curriculum differentiation or other measures to meet special educational needs in the teaching and learning of mathematics. There is no mention of other elements of social diversity including race, multiculture, or gender. These are perceived to be irrelevant to primary mathematics teaching. Once again, this is consistent with the Industrial Trainer ideology, which strongly repudiates any issues of social diversity, as well as with the Old Humanists.

ROLE OF RESEARCH

The role of research in the preparation and practice of teaching is acknowledged, but only in a limited sense. The term research occurs 3 times in Annex C, but only one mention concerns the utility of a research knowledge base for professional teachers. One of strengths in the document is the identification of misconceptions in the learning of mathematics and attention to their avoidance. Unfortunately this is presented in an autocratic way and no indication of the research evidence is given on the nature, causes, frequency or possible means of remediation of the 15 errors and areas of misconception listed. Although it is due to the impact of research in mathematics education that the naïve view that errors are random or careless has been overturned, the role of research is not credited.

UNDERLYING MANAGERIALISM AND MARKET METAPHOR

A dominant theme is the presence of a technicist, efficiency-orientated managerialism, as well as an underlying market place metaphor. There is repeated reference to trainees (49 mentions) and training (6). These suggest an underlying market and business training model, but not too much should be inferred from this use of 'official-speak'. Throughout the document the TTA presents itself as an independent regulating agency mediating within an education market between producers and consumers. This is very much a free market model, one which detaches the education service from the state and treats it as just one more enterprise in a skills market. There is also a stress on efficiency and the managerial imposition of value judgements, which is more unambiguously ideological. Thus in Annex C efficiency is mentioned 10 times, and the assumed effectiveness or appropriateness of the proposals is mentioned 25 times. As mentioned above the compulsive 'must' occurs 34 times, and other terms such as 'to secure', 'command', and 'monitoring' occur another 10 times altogether. The overall result is the imposition of a technicist, efficiency-orientated managerialism and the associated values and ideology. This fits with a number of perspectives, including the Technological Pragmatists and Industrial Trainers.

CONCLUSION

This paper attempts to identify the underlying influences acting on and detectable within the new national curriculum in mathematics for initial primary teacher education (DFEE 1997). The different factors combine to suggest that an Industrial Trainer ideology is dominant, because of the back-to-basics numeracy and social regulation aims, the autocratic teacher-centred pedagogy, the market place values, the absolutist and error focussed epistemology, the strict, imposed assessment system, and the rejection of social diversity and very restricted attention to research.

There is, in addition, evidence of an Old Humanist influence in the focus on both basic mathematical skills and higher mathematical content and proof, in the attention to understanding of the connected nature of mathematical knowledge and on an hierarchical model of mathematics and school mathematics, in the transmissive pedagogy with some emphasis on understanding, and in the strict assessment system and repudiation of research and social diversity with the exception of attention to the more able pupils.

Lastly, there is evidence of a Technological Pragmatist ideology influence in the emphasis on utility and efficiency and on a business-mentality, on basic skill content plus applicable mathematics, on a training view of learning but with the use of information technology, practical pedagogical elements and relevant applications encouraged, and on the limited attention to relevant or useful research which remains in the document.

Overall, the proposals should not be seen as a conceptual unity, but instead as resting on a plurality of competing and overlapping ideologies. There appears to be a compromise between the major contesting interests and viewpoints which contributed to and influenced its development.

> The embodiment in the curriculum of the values and practices of any particular group is the result of a process of struggle, and represents the apotheosis of the power of that group, although it is always related to the broader field of power in society at large. This is a precarious position, which needs to be defended by continuous struggle. Thus every description, redescription and canonisation represents a site of struggle where rival groups battle control of the transaction of knowledge/power. (Taylor 1993: 315)

The ideological underpinnings of the new Initial Teacher Training National Curriculum are very significant. Some elements may have a positive effect. However most of the innovations are likely to have a negative impact. Teachers are being regarded as skilled operatives rather than as reflective professionals, and teacher knowledge, and intellectual skills are being 'dumbed down'. A restricted and restricting view of mathematics is embodied in the proposals, one which will fail to deepen and extend student teachers' understanding of mathematics as a whole. An autocratic and insensitive pedagogy is both promoted and embodied in the new regulations, and if successfully implemented might bring back the fear and negative attitudes traditionally associated with school mathematics. These negative responses seemed to arise for many when arithmetical skills were taught in an authoritarian way, and have been receding since the 1980s (Assessment of Performance Unit 1991, Ross and Kamba 1997).

METHODOLOGICAL REFLECTIONS

Finally, it is necessary to critically evaluate the text analysis methods used from the perspective of their validity and the trustworthiness of the results. Electronic versions of the various sections of the document were processed in various ways to derive word and phrase frequencies. These were then grouped into clusters which seemed to have a shared meaning. Subsequently, in writing this account, terms were chased back to their original locations to check their sense in relation to the context of occurrence, for this sometimes resulted in variations of meaning and interpretation. Clearly there are methodological difficulties in the selection and interpretation of the terms in the text, following by the interpretation of their ideological significance. This depend on the judgement of the researcher which cannot be neutral. The use of the model of ideologies helps insofar as it provides a consistent reading of the values attached to concepts and terms, from the theorised ideological perspectives. Nevertheless considerable problems of interpretation remain. There is systematic ambiguity concerning the terms used in education and teacher education. Askew (1996) has reported on the distinct interpretations of key terms in curriculum documents and reforms. Grenfell (1996: 289) argues that "teacher education takes place in a field in which there is a struggle for the very language used to express it". Related methodologies have been employed widely, both in and out of education. Meighan (1986) and Stubbs (1976) describe the 'hidden curriculum of language' in which both written and spoken language convey covert and often unintended messages. Detailed analyses of word use, as in Brown and Gilman (1972), have related specific patterns of terminology and use to differences of power and ideology. Postman and Weingartner offer a method of ideological analysis which involves the interrogation of a text to answer questions including: "What are some of its critical, underlying assumptions? What are its key words?" (Postman and Weingartner 1969: 119). What is offered here is thus the deployment of a widespread method of text analysis. However, problems of interpretation and ambiguity and the risk of subjectivity and distortion in interpretation inevitably remain.

There are also weaknesses in the Ernest (1991) model utilised here. Ideological perspectives could in theory be charted multi-dimensionally along several continua, and the simplification of this down to the five discrete positions used here immediately risks stereotyping patterns of belief. It also closes off the possibility that ideological elements may be observed in more complex combinations, overlapping several of the five positions. In an earlier project applying this model to empirically classify teachers' espoused and enacted belief systems it was found that the most accurate tabulation of observed indicators sometimes involved elements from more than one of the five positions (Ernest and Greenland 1990, Greenland 1992). Of course no claim is made that individuals can be fitted into the five ideological positions, rather they define 'ideal types'. Nevertheless, the potential risks and weaknesses of the model of ideologies and of its use as a research tool is acknowledged. What this paper offers is one reading

Finally, it is worth remarking that in comparison with DFE (1993) the tone of DFEE (1997) is much more autocratic, directive and assertive, redolent of the imposition of discipline on an unruly and untrustworthy class. The new regulations specify an unbalanced curriculum that will lead to one-sided, utilitarian and technicist teachers and pupils, not the well rounded, creative and flexible teachers and citizens that society needs. There is a real risk that the new ideologically driven regulations will damage teacher education, teaching and hence learning in schools.

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You Are as You Read: the role of texts in the production of subjectivity

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Abstract

In this paper, we discuss an approach to the development of subjectivity that is social and based in social practices constituted by discourses (rather than on concepts of the individual such as "personality", "attitudes", "characteristics"). We shall discuss the way in which contexts and the material might be said to be "textualised", and hence the importance of intertextuality. We consider how these ideas (in relation to those of competing theoretical frameworks) can aid the understanding of the subject's "positioning", the basis of their readings of a mathematical problem, and their related thinking, affect and "performance".

Our theme is how texts produce subjectivity. We are interested in adults' numeracy and how their thinking about problems and their affect in particular situations is specific to the context. We argue that subjects' cognition, affect and the context are all based in or constituted by practices.

So we need to talk about texts first of all. The point that we want to develop is that the text is not something given: multiple readings of the text are possible; there isn't one privileged reading.. There are several ways in which the text is fluid even though it looks like the words stay the same on the page. First, its meaning is not constituted once and for all, it's susceptible to change over historical time and it's susceptible to change over life of a subject. If we reread a text 10 years after, its meaning has changed; see for example, the menu in Fig.1. Second, at any given moment it is not unambiguous, it can be read in different ways.

The third thing is that the text is potentially open: there are going to be links outwards in all kinds of directions that are very important to understanding how someone understands the text and what they do with it (or what the text does to them) . Therefore, if multiple readings of the text are possible, if there isn't one privileged reading, then it seems reasonable to accept the post-structuralist position that the text is not the letters on the page, but it is that which is produced in a reading

(Derrida, 1976).

What about context? People have been tempted to think about a context as something material, something outside of the text, as background. However, the problem is that this suggests that there is a "pre-discursive" context that exists prior to language, or outside of it. There are a number of ways to deal with this problem.

Jean Lave (1988) presents a two-sided approach in studying adult shoppers. In one sense the context of their shopping, and of the sorts of numerate or mathematical things they do, is the supermarket as an *arena* with certain objective properties. At the same time she talks about it as a *setting* which is different for different shoppers. So in her work there is a tension between the supermarket as an arena, an objective site of shopping - which can therefore be studied as an object in political economy terms (cf. Lave, 1988, Ch.8) - and the supermarket as a setting which has different meanings for different shoppers.

Valerie Walkerdine (Walkerdine et al., 1989, ch.11) seems to play down the arena as objective, when she says "No easy materiality exists outside the practices through which it is read" (p.192). But this does not mean that she is seeing the setting as subjective, dependent on the individuals working or shopping or whatever. Rather, reality / materiality is read, interpreted, through practices, which are organised (regulated) by certain discourses and which therefore are themselves social: discursive practices. Thus we can perhaps offer a synthesis between the objective and the subjective, between the individual and the social.

We could say that the context itself has to be seen as, in some sense, *textualised in* the same way as we argued for text. In other words, for the context to be grasped, I need to be able to present it to myself. So there is no way that there is a pre-discursive context: to describe it and to engage with it is already to be engaged in certain kinds of language and discourse.

Although we often talk about contexts where texts are produced through readings, the idea of intertextuality points to the inseparability of text and context. As Derrida writes, "The phrase...there is nothing outside the text...means nothing else: there is nothing outside context.......the notion of text/context embraces and does not exhaust the world, reality , history. The text is not the book, it is not confined in a volume itself confined to the library. It does not suspend reference to history to the world, to reality, since these things always appear in a movement of interpretation which contextualizes them according to the network of differences and hence of referral to the other"(1988, 136-137).

Intertextuality is discussed by Fairclough (1992, Ch.4). Bakhtin shows the ways in which texts or utterances are shaped by prior texts and therefore can be seen as a response to them. So today's news reports on the Middle East relates to last night's and previous ones, and also - this is slightly harder to imagine - they relate to subsequent texts, in a sense they anticipate them, because those subsequent texts will be related back to today's. Thus, in David Lodge's *Small World* (1984), one of the main characters causes a stir in an international conference by claiming that T. S. Eliot's writing has affected Shakespeare's work. On the face of it, that is absurd - because how could Eliot writing in this century do that? But what he is saying is that when we go back and read Shakespeare, we re-read him through ideas of the 20th Century, which include those of Eliot.

A slightly more social, more material view of intertextuality comes from Kristeva (1986), a disciple and translator of Bakhtin. She talks about intertextuality being the insertion of history or society into a text and insertion of this text into history.

So what is the importance of all this, for learning mathematics? It is that the meaning of a mathematical operation or a mathematical signifier like a numeral or taking a percentage is not universal. It is in principle ambiguous, and the way it gets its meaning is from that meaning being constituted by, being supported and shaped within, the discourses and practices in which it is inscribed or used. So if I am shopping, a unit price calculation has certain characteristics because it has come up while shopping. It would have different characteristics - perhaps in terms of the accuracy that would be required - if I were doing it as a problem in school maths. So, we argue, what appears to be "the same calculation" is not the same - because it is part of a different practice, which uses different terms which make different kinds of distinctions, and which represent different goals and values. When you are doing a calculation in shopping, you have different purposes and constraints than when you are doing it in the maths classroom. The calculations have to be more accurate in the classroom, because that is what is required, or what it takes to keep the teacher happy.

For example, in JE's interview (Evans and Tsatsaroni, 1994; Evans, 1999), there were two questions where a 10% calculation was required for both (for most respondents). The earlier question was Qu.2: What is 10% of 6.65? given to them simply on a sheet of paper, whereas the later one, Qu.4, was put in a broader everyday context; see Fig. 1. I (i.e.JE - see NOTE 2) led into the task by asking them ... in a restaurant with a menu like this , how they would tip (if at all), and I asked them to choose a dish.... So here is another example of text - a calculation of 10% - that has quite different meanings. So for Qu.4, the same piece of text, the menu, generated at least four different rules and practices for tipping.

To put this in a more general way, the way we understand text and context must be reevaluated, "textualized", since "Every sign, linguistic or non-linguistic, spoken or written, in a small or large unit, can...break with every given context..engendering and inscribing itself, or being inscribed, in new contexts" (Derrida, 1988, p. 79). Thus, contextual transformation remains an always open possibility.

So let us begin to try to describe what that "context" was like.... First, it clearly had a lot to do with language, with discourse. Second, there were also different social relations in a restaurant, or in an interview about being in a restaurant, than in an interview with a maths teacher, if that is how the students saw it. You tend to have different "resources" - you may have a calculator with you, or some restaurants actually work out the 10% tip for you! But above all they are different calculations in the whole sense of what is going on. Because they are inscribed in, or they form part of, different activities or practices where different things are at stake. So we look at the context as something based in, formed by, or constituted by, practices - indeed practices based in language so discursive practices. (eg everyday activity, a school maths activity, etc.) - each with its own set of social relations and resources.

That leads us on to the question how is subjectivity produced within these discursive practices. There are available a number of theoretical frameworks in maths education for investigating how subjectivity is produced, how a person's understanding of the meaning of a mathematical object is produced, how their thinking and feeling about maths has developed. First, are two sorts of cognitive approaches, one relatively transitory, the other more durable: on the one hand, we could look at their cognitive processes of thinking and their misconceptions. Or on the other hand we could look at something a bit more durable such as cognitive style or a notion of an "ability" - perhaps in percentages, or in fractions. Third, we might look at affective characteristics, e.g. affective barriers to doing maths as a fairly permanent characteristic of a person, maths anxiety, and beliefs in mathematics as a male domain. Or affective barriers in problem-solving processes (e.g. McLeod and Adams, 1989). Then there are social structural determinants for example social class, or gender itself. Finally, there is the notion of "experience": with adults; it is tempting to explain differences in performance as being due to experiences they have had as a waitress or experiences they've had as somebody who had to eat out a lot because of a job. Again, it can be tempting to think of this experience as based on a "reality" - which is somehow simple, pre-discursive, not depending on language in the way we argue the text and context are. However again we can make the same points we made in connection with context. Experience in order to have any effect must be perceived by the subject, it must be presented and represented to itself and again that is done through the medium of language. It cannot be done pre-discursively, on non-discursively. As with the term reality, "experience" appears in a movement of interpretation that contextualizes it.

What we argue, on the question of how is subjectivity produced, is that texts put subjects into positions, subject positions, so school maths texts, say, may put girls into the position of somebody who can't really do anything very mechanical, or somebody who's a helper in the classroom, or somebody who's really not concerned with the more intricate or powerful matters of business in the office but is relegated to a sort of subordinate role. However, the problem about positions, or subject positions, is that it may feel somewhat determinist: if texts position someone, there isn't much space left for their freedom or their agency. For example, the early work of Foucault focussed a lot on how institutions like the mental asylum, the clinic, the prison in 18th and 19th century France produced certain kinds of subjectivity in the people working in them, not just in the inhabitants but also the warders.

As an alternative view, we consider a way to acknowledge the subject's agency, their choice: "Man makes his own history but not in the conditions of his own choosing," propounded by Marx in his Theses on Feuerbach (1845 / 19xx). We read that as an attempt to balance the constraint and partial determinism in the historical conditions in which people find themselves, with the possibilities of freedom (for a class, at least) to make their own history. Therefore, in trying to avoid the over-determinism entailed in investigating subject-positions only, when we were looking at how JE's students approached any of these problems, we postulated a two-stage process. First of all we tried to analyse the situation in a way similar to the way Foucault did (or Walkerdine, 1984) although not in such detail, looking at what discourses or what practices are in play in this situation of the interviews. We thought that we could analyse the discourses that might possibly be positioning subjects in general here. In this general analysis, we thought for all problems that there was more than one practice at play in the situation.

Second, JE then read the transcripts, and tried to make a decision on which of those practices at play, or mix of practices, was actually positioning that particular person - and therefore was related to all the ways of producing subjectivity - making available certain ideas, giving an emotional charge to the problem, giving them possibilities for critical reflection, having certain aims and certain things at stake.

So a two stage process: the first stage involves setting out in general the possible practices at play, acknowledging the partial determinism; and the second stage is judging the particularity and variation across different subjects, looking at the possibility for partial freedom. And this two-stage process is analytical: we are not claiming this is the way the subject sees it.

Thus when I showed people the menu in Qu.4, when I came to the point where I asked them about a 10% tip, the practices in play at that moment could be either of two: school maths or eating out. That is the general stage of the analysis. Then I looked at each particular transcript (including their worksheets, because they had paper to work on), to see what they had actually said and done, so that is the particular stage of my analysis. For example, some of them got out a pencil and used the paper to calculate: for me that was an indicator that they had their positioning in school maths because you don't normally use those resources when you're in the restaurant. But mostly, the analysis of transcripts was done by looking for the use of key signifiers, terms (or symbols) that I thought were located, or had meaning, in one practice but not another. Part of making sense of any situation for a subject is deciding - or recognising - what activity, what practice this is and therefore what discourses they are going to be able to draw on, to make conversation or solve a problem or whatever.

The result of that two-stage process we called positioning, to distinguish it from a more determinist view of this whole process which would tend to talk about positions or subject-positions, to mark that this is an attempt (at problematizing the distinction between structure/agency, or determinism / freedom). This positioning in our view both supports and constraints subjectivity: it gives resources for thinking for example, but constrains the kind of critical reflection you can do - in the sense that it puts some limits on the play of signifiers, therefore on the production of meaning.

What else there might be that might have an effect is a very interesting question. Might there be something 'residual' in the subject, something that the person carries with him/herself, rather than its being provided by the discourse out there - that might affect what is called up, which signifiers are activated (i.e. are inscribed in other contexts)? We certainly feel that calling up is going to depend on memory. However, again, like experience and like context, memory needs to be understood as something subjects presents to themselves using language, discourse. Memory is also textualized.

Applying these Ideas

As maths educators, we are particularly interested in the practice of school maths although, as argued here, it relates to many others (see e.g. the work of Merttens, Vass, Brown et al. on the relations between home and school practices). What can we say about how subjectivity is constructed in a maths classroom?

We could begin with a structuralist analysis of maths in a maths classroom, as an activity which is organised by classification and framing (e.g. Bernstein, 1996) whose strength depends on the way it is recontextualsed as a subject of the curriculum. Therefore, first it has to be analysed as such, in terms of what type of domain of practice is constructed - maths as an esoteric, a public domain, etc.(e.g. Dowling, 1998). Then, we need to consider how the way school maths is constructed as a practice affects the way different subjects (seen structurally as middle-class and working-class, boys and girls, etc) perceive it differently, how they are differently positioned; there is a range of studies of maths and science done within this approach (e.g. Dowling, 1998; Cooper, Dunne and Rodgers, 1997; Morais et al., 1995).

However, the idea of "positioning" points to something beyond the structuralist analysis, using social categories. The argument about textualisation means that every signifier can be inscribed in new contexts; it cannot be absolutely controlled or confined within the context of its supposed original production, nor within the context to which the school activity attempts to confine it, and which structuralist approaches attempt to study.

Therefore, these issues can be studied better with interview material, because in such a setting, the elements of the context seem to be more fluid.

Now let us look at an interview transcript from one of JE's subjects. These were students at the end of their first year in the social science degree. They all had to take maths as about 1/6th of their study time. I had given all students (almost 1000) a questionnaire at the beginning of the year. In addition, at the end of the year, I did semi-structured interviews with a subsample of 25, using stratified sampling to try to ensure a certain representation of men and women, pre-21 and mature students, and middle-class versus working-class students. The interviews included both life-history questions, e.g. on what had been their experience with maths, and a number of practical problems (see e.g. Fig.1), many of which were designed by Bridget Sewell (1981), in her study of the use of maths by adults for the Cockcroft Report. What was different about JE's use of these questions was his use of what I call "contexting questions", asked both before showing them anything mathematical, and afterwards - to get an idea of what the text of the problem meant to them in terms of their everyday activities and if any memories were brought up. So that led to a lot of fruitful material as well as (normally) an answer of some kind to a mathematical question.

The contexting question that was asked before was of the form: "Does this remind you of anything you currently do?" And the one afterwards was: "Does this remind you of any earlier experiences?" They were commonsensically formed but they led to a lot of very interesting material.

When we analysed in general terms the practices in play in the interview one was the practice of school or college maths and in that the two positions that are normally available are teacher and pupil. The other practice at play was an interview practice where I was a researcher, they were an informant or respondent, and that clearly involved different power positions. It involves a different relationship to knowledge. So those were the two basic practices at play in general in the interview.

We thought that the research interview practice would lead to the student calling up other practices and in the case of the menu (Qu.4) for example, the other practice we call eating out at restaurants. There are quite a number of related positions available when you eat out: you can take somebody out or you can be their guest - that's one kind of pairing which has different power and other implications; you can "go Dutch" where on the face of it you're going out as equals, certainly in terms of paying. And also in restaurants there is a customer -- waitress/ waiter relationship. So all of those depending on which of those you've been positioned in, in the past, might affect how you would respond to this question.

So basically, in our general analysis, we saw those two practices were positioning the people in the interview. And the research interview in particular opened up space for them to call up other practices such as eating out or shopping or baking cakes, depending on the kind of (often everyday) activity that was recontextualized in the material used in the interview problem.

What indicators do we have for positioning? First, we had my scripted talk: I tried to talk about research rather than a test, and I tried to talk about using numbers rather than maths and so on. I tried to use terms that were more consistent with the research interviewer practice than with the academic maths. Also very important is, second, the unstructured talk of the subject; for example, in Qu. 3 on the changing gold price, if they talked about 'graphs' and 'gradients', I saw that as an indication that they had their positioning in college maths because they were the terms we used. If they talked about 'charts' and 'trends', I took those as equivalent terms but indicating that s/he was thinking about it from within what we might call business maths or business practices. Besides, third, describing the interview setting, there were, fourth, what I call general reflexive accounts, for each of the two year groups I interviewed. Fifth, I wrote a particular reflective account of my relationship with each individual student because I had different relationships with each.

Let us have a look at "Donald" (not his real name). I started off the interview with a few questions about how he had used numbers in the past and so on. He was a man in his 40s, he'd worked in the London money markets before he came to the college. In the first semester or two, he got very interested in philosophy and also maths, though he hadn't been very good in maths before, and he decided he was going to do a social science degree so he was doing town planning. This was almost the end of the third term, within a four-year degree.

Let us consider Question 3; see Fig.2. I ask him my first contexting question: "Does that remind you of anything that you do these days or that you've done recently?" and he says "some of the work we did in Phase One [the first two terms] but, if you ask me straight out of my head, what it reminds me of I worked once with a credit company and we had charts on the wall trying to galvanise each of us to do better than the other and these sodding things were always there and we seemed to be slaves to the chart". He goes on a few more lines and he says: "That's what that reminds me of, a bad feeling in a way. I felt that a human being was being judged by that bit of paper." What I want to do here is to see if we can judge what has been called up. The first thing he says is that it reminds me of the maths course that we did. But then he pulls right away and he says that if you ask me, what it reminds me of, is working with this credit company and the charts, and the bad feeling.

The "graph" as signifier links with "chart" and pulls him in two directions but we think more towards business maths: he remembers both practices, but what is more strongly called up is the business maths. A minute later or so, I go back and ask: "Does it remind you of phase one?" and he says "yeah , well, we did some questions like this and it was the run over the rise and that kind of thing...." Then, after a hesitation, "Trends, I suppose if you were judging a trend it would remind me of that", and then a few lines later "I like the fact that I can do a chart now, but even to do a chart like that now, I couldn't sit down and do it straight away....". He talks a bit more about his feelings about maths. "With maths I have to go back to the basic things all the time." What's interesting here is again it's almost as if he's called up both practices. He's gone back to college maths, he talks about "the run over the rise" (NOTE 3).

Our conclusion is that he moves back and forth between the two practices; he is positioned in both and he is able to use terms - and ideas - from both. So let us see what happens when I ask him to try these problems: "May I ask you which part of the graph shows where the price was rising the fastest?" He says, first of all, "if I was to make an instant decision I'd say that one" and he points to the time before lunch, "but obviously I'd want to make it on a count of the line wouldn't I, I'd count a line"; he measures along the line and the vertical rise - and comes to the right conclusion that it's going up fastest in the first part of the slope. I ask, "What was the lowest price that day?" and he says this one here and points to the graph and says 580 and then, just as he finishes he says, "For some reason the price went higher at the close...." That is, his positioning in business maths leads him to seek an *explanation* for the situation depicted in the graph.

Thus he seems able to address this problem within either business maths as an "instant decision", or within school maths, by "counting a line", because that's what he's learned to do in college maths. (He was probably the only one of 25 interviewees who took the trouble to do the latter: most of the others made an instant decision but didn't mark it as such). He can think within both discourses about this problem and he's called up rather more strongly the business discourses than the college maths, although the latter is fluent, comfortable.

Is he just drifting in and out of the two discourses, or more deliberately "crossing the boundaries"? Or is he in his mind closing the gap, in some sense, between his experiences in the money markets and the college maths because these confrontations with the text, the graph, are allowing him to create "bridges" or "channels" that may be somehow more permanent? We might say that the graph signifies within both practices, and hence facilitates intertextuality, by opening a "channel" between the two practices. (NOTE 4)

A number of people including Donald were making distinctions that might be analysed via Bernstein's concept of classification (Bernstein, 1996). For Bernstein, classification is an expression of power relations, which organises an activity and constitutes practices. It leads one to apply a structural analysis to define how practices constitute the subject, according to its social position (class, gender, etc.) Bernstein can be seen as making a "continued attempt to link the societal, institutional, interactional and interpsychic realms and to demontrate how the microprocesses of schooling relate to the complex institutional and societal forces" (Sadovnik, 1995, p.25). Yet as others have argued (Ladwig in Atkinson et al., 1997), he maintains some notion of a coherent, unified rational subject.

My interviewees were making distinctions which appeared to be based to a great extent on feelings and what they liked and disliked and what they didn't want to have anything to do with or did. We find this in about half of the 25 interviews. Donald's distinction is captured by his talking about 'figures' which is what you use in business, and maths or 'formulas and rules' on the other hand; this distinction is related to the two practices - work maths versus college maths - "the figures were a job" which certainly had strong positive components for him, whereas maths "was there to trip you up". Figures have a point, were meaningful, whereas maths was divorced from reality. He was confident with figures, he was still frightened by maths. These are distinctions he makes throughout the interview.

Other examples: "Jean" articulates her distinction between the two exam courses - CSE Maths and CSE Arithmetic on the basis of 'usefulness', and whether it should be optional or compulsory; she relates this to a similar division in topics in the 1st year course. "Peter" uses the dimensions numerical / not, useful / not, and pressure / relief, in a somewhat fluid way to distinguish mathematics and physics, from the sorts of subjects he took at A-level and now college.

So we can see a very interesting thing about affect here. At the social level, as Bernstein argues, it is experienced as values, perhaps sacredness or profanity, wonder, identity, commitment to rules - whereas at the individual level, it is experienced as feelings such as anxiety, emotion, liking. In between, perhaps spanning the two 'levels', we have attitudes and beliefs. These two levels also correspond roughly to the two fields of sociology and psychology. Perhaps social semiotics is what can link them together.

Concerning the question of an element of freedom or agency for the subject. Scott

(1991), looking critically at the notion of experience, argues that 'experience' cannot be seen as separated from the discourses that are the basis for the subject's positioning. She, like us, argues that subjects are positioned discursively but that doesn't mean they're completely determined. There are several reasons for this. First, certainly to the extent that subjects are positioned in more than one practice, there's a possibility of conflict between the practices in terms of aims, in terms of the way you calculate, the values, the feelings attached to signifiers. Donald's interview shows very well that the first possibility for escaping determinism is conflicts among the multiple discourses one is positioned in - or spaces or gaps between them. Secondly there may be contradictions within a discourse. Valerie Walkerdine (Walkerdine et al., 1989) has written about double-binds that girls find themselves in as far as doing well in maths:.you're supposed to do well in maths but it's quite dangerous, you may be seen as less feminine. It's not good as a girl to excel at things that aren't girls' subjects. Third, we've seen that signifiers can have multiple meanings and indeed they can be connected in chains that go outside the immediate text. Here is where the idea that discourses are not closed, is important. So Donald sees the graph / chart, and thinks about his work practices in the city. Another subject, Fiona, thinks about her father who is a stockbroker and there are all kinds of meanings come up that get attached to what seems to be clearly a mathematical object but it isn't simply that - since words have to be used in order to describe it or even to present it to ourselves. The words clearly have a shared cultural meaning but may also, to some extent, mean different things to different subjects. For example, the graph problem is also about *gold*: that might have a particular meaning for some subjects.

These are three bases for the determinism to be broken up. As a result, we can say that subjects do have agency, they do have freedom, they can choose to some extent, but they are not indivisible unitary individuals that are exercising their free-will. Their agency is created *within* the situations and statuses or positions that are conferred on them.

So to come back to Marx and the earlier quote: "Man makes his own history but not in the conditions of his own choosing". What we can say in parallel is: "Subjects have their own agency but not through discourses and positioning of their own choosing." So within the frame of the discourses that position us that we don't choose, we have a certain element of agency and choice, but it's not complete in the same way that Marx's historical actors had some freedom of action but not in conditions of their own choosing.

Thus "how you position yourself" - as an interviewee, for example - is certainly important. It depends on how you read the elements of a situation, and you may negotiate a change in this rule, and/or you may learn to change that positioning. However, the analysis here suggests that we should more precisely say: "how you adopt a positioning within the activity that positions you". As is clear now your intentions, rather than being simply freely chosen are shaped within the discourse(s).

An important issue, which is central to the possibility of social change, is: Are there variations of a particular text, which might allow the reader to be repositioned in a different subject-position, hence, producing a basis for a different subjectivity? For example, is it possible to change the texts in such a way that the double bind is lessened for girls?

A first response is it's not so easy: feminists have tried to produce changes in language, but apparently there is a lot of resistance to change in discourses producing gender inequality. This resistance comes from individuals, of course, but also from society in general - that is, all of us to the extent that we repeat and reproduce the discourses. We do it all the time unconsciously, especially in areas that are very important to us, that are effectively highly charged as sexuality and gender are. (Discourses of gender are related, for example, to feelings about mothers and fathers.) Thus it is very difficult to produce radically different texts, as it is harder to change highly charged discourses than we wish. (NOTE 5)

This raises the further question of what is effective in the long term. The interview (like most research) just sampled briefly subjects' lives: all we could really show was a transitory characteristic. If Donald was angry or feeling bad, we can't be sure whether that was only that day - although of course, as the interview allows him to talk about different times of his life and these things come up, then we can have a sense that they are based on *repetitions*. Now, th```ese are not repetitions of "behaviour" which is rewarded, in any simple behaviourist sense - but rather, a repetition of engagement with certain strings of signifiers, i.e. certain texts. So that for us is an important consideration in understanding the establishment of long-standing characteristics. Teachers (and others) repeat the same text that positions the learner in the same way; they see a certain idea of being used again, they have certain feelings about it. The subject judges him/herself and evaluates him/herself. Repetition will establish some chains of signification - and limit the construction of others - better than if it there is no repetition.

Conclusion

The approach discussed here is based on the view that context, memory and the material may be said to be textualised, and hence texts play a crucial role in the production of subjectivity. We think this view is an important supplement to studies of the mathematics learning, affect and classrooms, which use either a more structural approach or interaction in the classroom.

Notes

1. An earlier version of this paper was given by JE at a University of Exeter School of Education Seminar in Nov. 94. The presentation in this paper has been influenced by the dialogic structure of the seminar, and we especially acknowledge the contributions of Dennis Almeida and Paul Ernest.
2. Since the interviews and the original analysis were done by JE, the use of 'I', in a number of places in the paper, simply indicates this.
3. Though that is not quite right for the gradient: it should be "rise" over "run".
4. A nice example of intertextuality is provided by Donald's use of "chart" within business discourses, which may have anticipated the later use of "chart" instead of "graph" in computer graphic packages - where terms have been chosen, presumably, so as to appeal to businessmen.
5. However, powerful institutions like the Conservative Party in the 1980s had as a political strategy, "discursive shifting", for example working on words like 'enterprise' (Fairclough, 1992; Lerman 1990).

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Figures

Figure 1 Details of the 10% questions used in the interview

Qu. 2: "abstract" 10%
(C) Does this remind you of any of your current activities?
What is 10% of 6.65?
(R) Does this remind you of any earlier experiences?

Qu. 4: 10% tip on selected restaurant meal
(The problem was introduced by reading out several contexting questions - CA, CB and A)
[Show the "menu" (see Fig. 1)]
(CA) Do you ever go to a restaurant with a menu anything like this?...
(CB) Would you please choose a dish from this menu?...
(A) Suppose the amount of "service" that you leave is up to the customer: what would you do? ...
(B) Could you tell me what a 10% service charge would be?...
(R) Does this remind you of any earlier experiences?

The "menu" for Qu.4 in the interview.

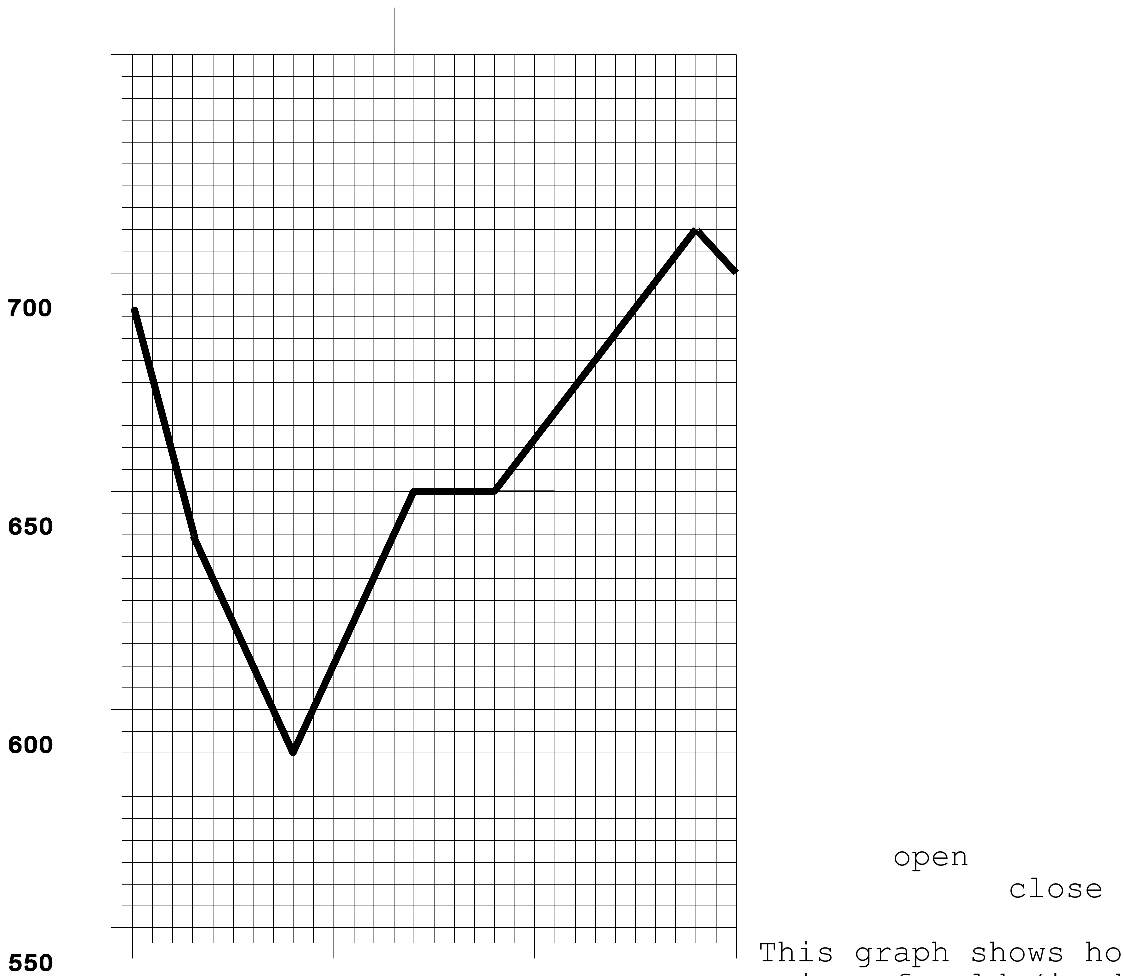
CHICKEN MARYLAND	Served with sweet corn, banana fritter, bacon, fresh tomato, whole French beans, jacket baked potatoes with sour cream and chives or French fried potatoes. Roll and butter. Ice cream, or a selection from our cheese board, biscuits and butter.	
		£3.75
SEA FOOD PLATTER	Served with tartare sauce, whole French beans, jacket baked potatoes with sour cream and chives or French fried potatoes. Roll and butter. Ice cream, or a selection from our cheese board, biscuits and butter.	
		£3.53
GRILLED TROUT 10 OZ	Served with tartare sauce, whole French beans, Jacket baked potatoes with sour cream and chives or French fried potatoes. Roll and butter. Ice cream, or a selection from our cheese board, biscuits and butter.	
		£3.81
Coffee	Special blend black or with cream	**27p**
Connoisseur Coffees	Served in large goblet glass with cream: Irish (*Irish Whiskey*), Caribbean (*Rum*), Russian (*Vodka*), Parisienne (*Brandy*), Calypso (*Tia Maria*), Highland (*Scotch Whisky*), Mine Hosts (*Cointreau*) Connoisseur coffees include sugar unless otherwise requested	**67p**

Figure 2 Question 3 in the interview

[3]

This graph shows how the price of gold (in dollars per fine ounce) varied during one day's trading in London. Which part of the graph shows where the price was rising fastest? What was the lowest price that day?

The London Gold Price - January 23rd 1980



This graph shows how the price of gold (in dollars per fine ounce) varied during one day's trading in London.

Source: Evans (1998), based on Sewell (1981)

Reading the World with Maths: Goals for a Criticalmathematical Literacy Curriculum

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Abstract

In this paper I suggest ways that teachers can introduce mathematics as a tool to interpret and challenge inequities in our society. The teaching methods suggested also make maths more accessible and applicable because it is learned in the context of real-life, meaningful experiences. This suggestion is particularly useful for teachers who are creating an interdisciplinary math and social studies curriculum. The examples are based on my work teaching at the College of Public and Community Service, Umass/Boston. My students are primarily working-class adults who did not receive adequate mathematics instruction when they were in high school. Many of them were tracked out of college preparation. Therefore, the ideas presented in this article can be applied to the secondary classroom.

When my students examine data and questions such as the one shown in the box on the next page they are introduced to the four goals of the criticalmathematical literacy curriculum.

1 Understanding the mathematics.

2 Understanding the mathematics of political knowledge.

3 Understanding the politics of mathematical knowledge.

4 Understanding the politics of knowledge.

Clearly, calculating the various percentages for the unemployment rate requires goal number one, an understanding of mathematics. Criticalmathematical literacy goes beyond this to include the other three goals mentioned above. The mathematics of political knowledge is illustrated here by reflecting on how the unemployment data deepens our understanding of the situation of working people in the United States. The politics of mathematical knowledge involves the choice of who counts as unemployed. In class, I emphasize that once we decide which categories make up the numerator (number of unemployed) and the denominator (total labor force), changing that fraction to a decimal fraction and then to a percent does not involve political struggle - that involves understanding the mathematics. But, the decision of who counts where does involve political struggle - so the unemployment rate is not a neutral description of the situation of working people in the United States. And, this discussion generalizes to a consideration of the politics of all knowledge.

Example: Unemployment Rate

In the United States, the unemployment rate is defined as the number of people unemployed, divided by the number of people in the labor force. Here are some figures from December 1994. (All numbers in thousands, rounded off to the nearest hundred thousand).

. In your opinion, which of these groups should be considered unemployed? Why?
. Which should be considered part of the labor force? Why?
. Given your selections, calculate the unemployment rate in 1994.

1 101,400: Employed full-time
2 19,000: Employed part-time, want part-time work
3 4,000: Employed part-time, want full-time work
4 5,600: Not employed, looked for work in last month, not on temporary layoff
5 1,100: Not employed, on temporary layoff
6 400: Not employed, want a job now, looked for work in last year, stopped looking because discouraged about prospects of finding work
7 1,400: Not employed, want a job now, looked for work in last year, stopped looking for other reasons
8 60,700: Not employed, don't want a job now (adults)

For discussion: The US official definition counts 4 and 5 as unemployed and 1 through 5 as part of the labor force, giving an unemployment rate of 5.1%. If we count 4 through 8 plus half of 3 as unemployed, the rate would be 9.3%. Further, in 1994 the Bureau of Labor Statistics stopped issuing its U-7 rate, a measure which included categories 2 and 3 and 6 through 8, so now researchers will not be able to determine "alternative" unemployment rates (Saunders, 1994).

In this article, I will develop the meaning of each of these goals, focusing on illustrations of how to realise them in their interconnected complexity. Underlying all these ideas is my belief that the development of self-confidence is a prerequisite for all learning, and that self-confidence develops from grappling with complex material and from understanding the politics of knowledge.

Goal #1: Understanding the Mathematics

Almost all my students know how to do basic addition, subtraction, multiplication and division, although many would have trouble multiplying decimal fractions, adding fractions or doing long division. All can pronounce the words, but many have trouble succinctly expressing the main idea of a reading. Almost all have trouble with basic math word problems. Most have internalised negative self-images about their knowledge and ability in mathematics. In my beginning lessons I have students read excerpts where the main idea is supported by numerical details and where the politics of mathematical knowledge is brought to the fore. Then the curriculum moves on to the development of the Hindu-Arabic place-value numeral system, the meaning of numbers, and the meaning of the operations.

I start lessons with a graph, chart, or short reading which requires knowledge of the math skill scheduled for that day. When the discussion runs into a question about a math skill, I stop and teach that skill. This is a non-linear way of learning basic numeracy because questions often arise that involve future math topes. I handle this by *previewin*g. The scheduled topic is *formally* taught. Other topics are also discussed so that students' immediate questions are answered and so that when the *formal* time comes for them in the syllabus, students will already have some familiarity with them. For example, if we are studying the meaning of fractions and find that in 1985, 2/100 of the Senate were women, we usually *preview* how to change this fraction to a percent. We also discuss how no learning is linear and how all of us are continually *reviewing, recreating*, as well as *previewing* in the ongoing process of making meaning. Further, there are other aspects about learning which greatly strengthen students' understanding of mathematics:

(A) breaking down the dichotomy between learning and teaching mathematics;

(B) considering the interactions of culture and the development of mathematical knowledge; and

(C) studying even the simplest of mathematical topics through deep and complicated questions.

These are explained in more detail below.

(A) Breaking Down the Dichotomy between Learning and Teaching Mathematics

When students teach, rather than explain, they learn more mathematics, and they also learn about teaching. They are then empowered to proceed to learn more mathematics. As humanistic, politically concerned educators, we often talk about what we learn from our students when we teach. Peggy McIntosh (1990) goes so far as to define teaching as "the development of self through the development of others." Certainly when we teach we learn about learning. I also introduce research on math education so that students can analyse for themselves why they did not previously learn mathematics. I argue that learning develops through teaching and through reflecting on teaching and learning. So, students' mathematical understandings are deepened when they learn about mathematics teaching as they learn mathematics. Underlying this argument if Paulo Freire's concept that learning and teaching are part of the same process, and are different moments in the cycle of gaining existing knowledge, re-creating that knowledge and producing new knowledge (Freire, 1982).

Students gain greater control over mathematics problem-solving when, in addition to evaluating their own work, they can create their own problems. When students can understand what questions it makes sense to ask from given numerical information, and can identify decisions that are involved in creating different kinds of problems, they can more easily solve problems others create. Further, critical mathematical literacy involves both interpreting and critically analysing other people's use of numbers in arguments. To do the latter you need practice in determining what kinds of questions can be asked and answered from the available numerical data, and what kinds of situations can be clarified through numerical data. Freire's concept of problem-posing education emphasises that problems with neat, pared down data and clear-cut solutions give a false picture of how mathematics can help us "read the world". Real life is messy, with many problems intersecting and interacting. Real life poses problems whose solutions require dialogue and collective action. Traditional problem-solving curricula isolate and simplify particular aspects of reality in order to give students practice in techniques. Freirian problem-posing is intended to reveal the inter-connections and complexities of real-life situations where "often, problems are not solved, only a better understanding of their nature may be possible" (Connolly, 1981). A classroom application of this idea is to have students create their own reviews and tests. In this way they learn to grapple with mathematics pedagogy issues such as: what are the key concepts and topics to include on a review of a particular curriculum unit? What are clear, fair and challenging questions to ask in order to evaluate understanding of those concepts and topics?

(B) Considering the Interactions of Culture and the Development of Mathematical Knowledge

This aspect is best described with the following example.

Example: When we are learning the algorithm for comparing the size of numbers, I ask students to think about how culture interacts with mathematical knowledge in the following situation:

> Steve Lerman (1993) was working with two 5-year-olds in a London classroom. He recounts how they "were happy to compare two objects put in front of them and tell me why they had chosen the one they had [as bigger]. However, when I allocated that multilinks to them (the girl had 8 and the boy had 5) to make a tower ... and I asked them who had the taller one, the girl answered correctly but the boy insisted the he did. Up to this point the boy had been putting the objects together and comparing them. He would not do so on this occasion and when I asked him how we could find out whose tower was the taller he became very angry. I asked him why he thought that his tower was taller and he just replied 'Because IT IS!' He would go no further than this and seemed to be almost on the verge of tears."

At first students try to explain the boy's answer by hypothesising that each of the girl's links was smaller than each of the boy's or that she built a wider, shorter tower. But after reading the information, they see that this could not be the case, since the girl's answer was correct. We speculate about how the culture of sexism - that boys always do better or have more than girls - blocked the knowledge of comparing sizes that the boy clearly understood in a different situation.

(C) Studying Mathematical Topics through Deep and Complicated Questions

Most educational materials and learning environments in the United States, especially those labelled as "developmental" or "remedial", consist of very superficial, easy work. They involve rote or formulaic problem-solving experiences. Students get trained to think about successful learning as getting high marks on school or standardised tests. I argue that this is a major reason that what is learned is not retained and not used. Further, making the curriculum more complicated, where each problem contains a variety of learning experiences, teaches in the non-linear, holistic way in which knowledge is developed in context. This way of teaching leads to a more clear understanding of the subject matter.

Example: In the text below, Sklar and Sleicher demonstrate how numbers presented out of context can be very misleading. I ask students to read the text and discuss the calculations Sklar and Sleicher performed to get their calculation of the US expenditure on the 1990 Nicaraguan election. ($17.5 million - population of Nicaragua = $5 per person.) This reviews their understanding of the meaning of the operations. Then I ask the students to consider the complexities of understanding the $17.5 million expenditure. This deepens their understanding of how different numerical descriptions illuminate or obscure the context of US policy in Nicaragua, and how in real-life just comparing the size of the numbers, out of context, obscures understanding.

> On the basis of relative population, Holly Sklar has calculated that the $17.5 million US expenditure on the Nicaraguan election is $5 per person and is equivalent to an

expenditure of $1.2 billion in the United States. That's one comparison all right, but it may be more relevant to base the comparison on the effect of the expenditure on the economy or on the election, i.e. to account for the difference in per capita income, which is at least 30/1 or an equivalent election expenditure in the United States of a staggering $30 billion! Is there any doubt that such an expenditure would decisively affect a US election? (Sleicher, 1990).

Goal #2: Understanding the Mathematics of Political Knowledge

I argue, along with Freire (1970) and Freire and Macedo (1987), that the underlying context for critical adult education, and criticalmathematical literacy, is "to read the world". To accomplish this goal, students learn how mathematics skills and concepts can be used to understand the institutional structures of our society. This happens through:

a. understanding the different kinds of numerical descriptions of the world (such as fractions, percents, graphs) and the meaning of the sizes of numbers, and

b. using calculations to follow and verify the logic of someone's argument, to restate information, and to understand how raw data are collected and transformed into numerical descriptions of the world. The purpose underlying all the calculations is to understand better the information and the arguments and to be able to question the decisions that were involved in choosing the numbers and the operations.

Example: I ask students to create and solve some mathematics problems using the information in the following article (*In These Times*, April 29 - May 5, 1992). Doing the division problems implicit in this article deepens understanding of the economic data, and shows how powerfully numerical data reveal the structure of our institutions.

Drowning by numbers
It may be lonely at the top, but it can't be boring - at least not with all that money. Last week the federal government released figures showing that the richest 1 percent of American households was worth more than the bottom 90 percent combined. And while these numbers were widely reported, we found them so shocking that we thought they were worth repeating. So here goes: In 1989 the top 1 percent of Americans (about 934,000 households) combined for a net worth of $5.7 trillion; the bottom 90 percent (about 84 million households) could only scrape together $4.8 trillion in net worth.

Example: Students practice reading a complicated graph and solving multiplication and division problems in order to understand how particular payment structures transfer money from the poor to the rich.(2) *The Rate Watcher's Guide* (Morgan, 1980) details why under declining block rate structures, low-income citizens who use electricity only for basic necessities pay the highest rates, and large users with luxuries like trash compactors, heated swimming pools or central air-conditioning pay the lowest rates. A 1972 study conducted in Michigan, for example, found that residents of a poor urban area in Detroit paid 66% more *per unit* of electricity than did wealthy residents of nearby Bloomfield Hills. Researchers concluded, "approximately $10,000,000 every year leave the city of Detroit to support the quantity discounts of suburban residents".

Example: Students are asked to discuss how numbers support Helen Keller's main point and to reflect on why she sometimes uses fractions and other times uses whole numbers. Information about the politics of knowledge is included as a context in which to set her views.

Although Helen Keller was blind and deaf, she fought with her spirit and her pen. When she became an active socialist, a newspaper wrote that "her mistakes spring out of the ... limits of her development". This newspaper had treated her as a hero before she was openly socialist. In 1911, Helen Keller wrote to a suffragette in England: "You ask for votes for women. What good can votes do when ten-elevenths of the land of Great Britain belongs to 200,000 people and only one-eleventh of the land belongs to the other 40,000,000 people? Have your men with their millions of votes freed themselves from injustice?" (Zinn, 1980).

Example: Students are asked to discuss what numerical understandings they need in order to decipher the following chart. They see that a recognition of how very small these decimal fractions are, so small that watches cannot even measure the units of time, illuminates the viciousness of time-motion studies in capitalist management strategies.

Samples from time-and-motion studies, conducted by General Electric. Published in a 1960 handbook to provide office managers with standards by which clerical labor should be organized (Braverman, 1974).

Open and close	**Minutes**
Open side drawer of standard desk	0.014
Open center drawer	0.026
Close side drawer	0.015
Close center drawer	0.027

Chair activity	**Minutes**
Get up from chair	0.039
Sit down in chair	0.033
Turn in swivel chair	0.009

Goal #3: Understanding the Politics of Mathematical Knowledge

Perhaps the most dramatic example of the politics involved in seemingly neutral mathematical descriptions of our world is the choice of a map to visualise that world. Any two-dimensional map of our three-dimensional Earth will, of course, contain mathematical distortions. The political struggle/choice centres around which of these distortions are acceptable to us and what other understandings of ours are distorted by these false pictures. For example, the map with which most people are familiar, the Mercator map, greatly enlarges the size of "Europe"(3) and shrinks the size of Africa. Most people do not realise that the area of what is commonly referred to as "Europe" is smaller than 20% of the area of Africa. Created in 1569, the Mercator map highly distorts land areas, but preserves compass direction, making it very helpful to navigators who sailed from Europe in the sixteenth century.

When used in textbooks and other media, combined with the general (mis)perception that size relates to various measures of so-called "significance", the Mercator map distorts popular perceptions of the relative importance of various areas of the world. For example, when a US university professor asked his students to rank certain countries by size they "rated the Soviet Union larger than the continent of Africa, though in fact it is much smaller" (Kaiser, 1991), associating "power" with size.

Political struggles to change to the Peter's projection, a more accurate map in terms of land area, have been successful with the United Nations Development Program, the World

Council of Churches, and some educational institutions (Kaiser, 1991). However, anecdotal evidence from many talks I've given around the world suggest that the Mercator is still widely perceived as the way the world *really* looks.

As Wood (1992) emphasises:

> The map is not an innocent witness ... silently recording what would otherwise take place without it, but a committed participant, as often as not driving the very ats of identifying and naming, bounding and inventorying it pretends to no more than observe.

In a variety of situations, statistical descriptions don't simply or neutrally record what's out there. There are political struggles/choices involved in: which data are collected; which numbers represent the most accurate data; which definitions should guide how the data are counted; which methods should guide how the data are collected; which ways the data should be disaggregated; and which are the most truthful ways to describe the data to the public.

Example: Political struggle/choice over which definitions should guide how data are counted. In 1988, the US Census Bureau introduced an "alternative poverty line", changing the figure for a family of three from $9453 to $8580, thereby preventing 3.6 million people whose family income fell between those figures from receiving food stamps, free school meals and other welfare benefits. At the same time, the Joint Economic Committee of Congress argued that "updating the assessments of household consumption needs ... would almost double the poverty rate, to 24 percent" (Cockburn, 1989). Note that the US poverty line is startlingly low. Various assessments of the smallest amount needed by a family of four to purchase basic necessities in 1991 was 155% of the official poverty line. "Since the [census] bureau defines the [working poor] out of poverty, the dominant image of the poor that remains is of people who are unemployed or on the welfare rolls. The real poverty line reveals the opposite: a majority of the poor among able-bodied, non-elderly heads of households normally work full-time. The total number of adults who remain poor despite normally working full-time is nearly 10 million more than double the number of adults on welfare. Two-thirds of them are high school or college-educated and half are over 33. Poverty in the US is a problem of low-wage jobs far more than it is of welfare dependency, lack of education or work inexperience. Defining families who earn less than 155% of the official poverty line as poor would result in about one person in ever four being considered poor in the United States" (Schwartz and Volgy, 1993).

Example: Political struggle/choice over which ways data should be disaggregated. The US Government rarely collects health data broken down by social class. In 1986, when it did this for heart and cerebrovascular disease, it found enormous gaps:

> "The death rate from heart disease, for example, was 2.3 times higher among unskilled blue-collar operators than among managers and professionals. By contrast, the morality rate from heart disease in 1986 for blacks was 1.3 times higher than for whites ... the way in which statistics are kept does not help to make white and black workers aware of the commonality of their predicament" (Navarro, 1991).

Goal #4: Understanding the Politics of Knowledge

There are many aspects of the politics of knowledge that are integrated into this curriculum. Some involve reconsidering what counts as mathematical knowledge and re-presenting an accurate picture of the contributions of all the world's peoples to the development of mathematical knowledge. Others involve how mathematical knowledge is learned in schools.

Winter (1991), for example, theorises that the problems so many encounter in understanding mathematics are not due to the discipline's "difficult abstractions", but due to the cultural *form* in which mathematics is presented. Sklar (1993), for a different aspect, cites a US study that recorded the differential treatment of Black and White students in math classes.

> Sixty-six student teachers were told to teach a math concept to four pupils - two White and two Black. All the pupils were of equal, average intelligence. The student-teachers were told that in each set of four, one White and one Black student was intellectually gifted, the others were labelled as average. The student teachers were monitored through a one-way mirror to see how they reinforced their students' efforts. The "superior" White pupils received two positive reinforcements for every negative one. The "average" White students received one positive reinforcement for every negative reinforcement. The "average" Black student received 1.5 negative reinforcements, while the "superior" Black students received one positive response for every 3.5 negative ones.

Discussing the above study in class brings up the math topics of ratios and forming matrix charts to visualize the data more clearly. It also involves students who are themselves learning mathematics in reflecting on topics in mathematics education. This is another example of breaking down the dichotomy between learning and teaching, a category discussed in the above section on *Understanding the Mathematics*.

And, of course, Freire (1970) theorises about the politics of "banking education", when teachers deposit knowledge in students' empty minds.

Underlying all these issues are more general concerns I argue should form the foundation of all learning, concerns about what counts as knowledge and why. I think that one of the most significant contributions of Paulo Freire (1982) to the development of a critical literacy is the idea that:

> Our task is not to teach students to think - they can already think, but to exchange our ways of thinking with each other and look together for better ways of approaching the decodification of an object.

This idea is critically important because it implies a fundamentally different set of assumptions about people, pedagogy and knowledge-creation. Because some people in the United States, for example, need to learn to write in "standard" English, *it does not follow* that they cannot express very complex analyses of social, political, economic, ethical and other issues. And many people with an excellent grasp of reading, writing and mathematics skills need to learn about the world, about philosophy, about psychology, about justice and many other areas in order to deepen their understandings.

In a non-trivial way we can learn a great deal from intellectual diversity. Most of the burning social, political, economic and ethical questions of our time remain unanswered. In the United States we live in a society of enormous wealth and we have significant hunger and homelessness; although we have engaged in medical and scientific research for scores of years, we are not any closer to changing the prognosis for most cancers. Certainly we can learn from the perspectives and philosophies of people whose knowledge has developed in a variety of intellectual and experiential conditions. Currently "the intellectual activity of those without power is always labelled non-intellectual" (Freire and Macedo, 1987). When we see this as a political situation, as part of our "regime of truth", we can realise that all people have knowledge, all people are continually creating knowledge, doing intellectual work, and all of us have a lot to learn.

Marilyn Frankenstein *is one of a group of scholars and activists in the field of mathematics education from a critical perspective. She is co-founder, along with Arthur B. Powell and John Volmink, of the Criticalmathematics Educators Group (CmEG) and the author of numerous articles and books on criticalmathematics (see references below). She is a Professor of Applied Language and Mathematics, College of Public and Community Service, University of Massachusetts-Boston, Boston, MA 02125. E-mail: Frankie@ umbsky.cc.umb.edu*

Notes

(1) Thanks to my friend, Umass/Lowell economist Chris Tilly, for this problem.

(2) This situation has changed in Massachusetts, which now has a flat rate structure, and my reference did not contain real data for Michigan. So although the context-setting data are real, the numbers used to understand the concept of declining block rates are realistic, but not real.

(3) Grossman (1994) argues that "Europe has always been a political and cultural definition. Geographically, Europe does not exist, since it is only a peninsula on the vast Eurasian continent." He goes on to discuss the history and various contributions of geographer's attempts to "draw the eastern limits of 'western civilization' and the white race" (p.39).

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Starting a research project with immigrant students: constraints, possibilities, observations and challenges.

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A project was commissioned by the Ministry of Education in Catalonia, in northern Spain, concerned with mathematics teaching in schools with large numbers of immigrant students. This paper describes the preliminary stage of the project and focusses on the political, socio-cultural, and educational aspects of starting such a project. It presents a number of challenges that the project team has to face.

Introduction

In recent years, there has been increasing immigration into the north-eastern region of Spain known as Catalonia (capital Barcelona) which has led to significant quantitative, and qualitative, changes in the school population. Most of the immigrant pupils come from North Africa (Magreb), but also from other countries in Africa, North and South America, Asia and East European countries. This new situation has focussed attention on the inadequacy of the educational provision in schools and classes that can be thought of as highly multicultural.

The project described here began in 1997, is funded by the Catalan Ministry of Education, and has the support of a Catalan private foundation devoted to education, Fundació Propedagògica. The project is concerned with finding more appropriate ways to teach mathematics to immigrant students both in primary and secondary schools. This paper is a report of the preliminary stage of the project highlighting the constraints, possibilities and challenges of doing research in this highly politicized social-cultural context.

The context of the project

The project will be contextualised firstly in the legal and institutional framework where it takes place, and then in the social framework.

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The legal situation of foreigners in Spain is at present regulated by a law promoted in 1985[2], and its subsequent modifications and developments which took place in 1986, 1992, 1995 and 1996[3]. Taking into account the fact that the previous regulations about immigration were created in 1968 and 1978, one can see that the development of these laws is a response to the “need” to control immigration and particularly the numbers of people coming from underdeveloped countries. In addition to the immigration laws, nearly every year regulations have been developed to normalize the situation of illegal immigrants, by fixing the number who would receive residence and work permission. It is also important to point out here that even if the law considers the regrouping of the family as a possibility, the practical and legal obstacles needing to be overcome to achieve it lead to the presence of many unstable, unhappy, and unstructured families.

By the end of the 1980s, both state and regional governments with educational power in Spain (as in the case of Catalonia) promoted a broad reform of the educational system. This new reform is currently being implemented. The New Educational Act, LOGSE[4], organises compulsory education into two stages: primary education (6-12) and compulsory secondary education (12-16). We can claim thereby that for the past decade, education to age 16 has been regarded as a right accessible to most children in the country.

The structure of the curriculum in all compulsory education is common to all fields of knowledge. Each curriculum description contains global objectives for the whole educational stage and for each field (including mathematics), general goals, three sets of contents (knowledge, skills and attitudes), final aims and teaching guidelines. The official regulations are very general, because there is an explicit intention that every school should adapt the curriculum to the actual needs of their pupils, and to its own possiblities for meeting those needs.

Generally speaking, the new system meets current educational needs much better than the previous one, although there is much to be done in the area of teaching in multicultural situations, which is one of the weakest points of the implementation of the reform. Firstly, even if the “treatment of diversity” is considered in the teaching guidelines, it is addressed in a quite naiive way. The difficulties which many teachers encounter today are far beyond their initial expectations, and they can find no solutions in the official documents. In a word, even the “good” teachers do not know how to work with culturally diverse classes. Moreover, although the changes suggested in the official documents are meaningful, old beliefs have not yet been abandoned by most of the educational community, including the administration.

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[2] Ley Orgánica 7/1985 de 1 de Julio sobre derechos y libertades de los extranjeros en España.

[3] Real Decreto 1099/86 and R D 1119/86, modified later by R D 766/92, RD 737/95 and RD 155/96.

[4] LOGSE, Ley de Ordenación General del Sistema Educativo.

In addition, in-service teacher education programs dealing with multicultural education are scarce, and consequently the teachers and schools do not yet receive enough support to carry out any teaching innovations. As a result of that, the implementation of the curriculum still takes place, for the most part, by teachers following the guidelines of the textbooks, which have become the interpreters of the new curriculum. Although there are many innovative textbooks, in general they fail to facilitate any approaches to the teaching of mathematics which take into account cultural diversity.

Concerning the social framework in which the project takes place, the immigration context in Spain until the seventies was that of a country where most of the foreigners were tourists, a fact which made our citizens able to claim that they were tolerant of foreigners. The relatively small number of working immigrants of that decade have already "stabilized" within the society, and their children are already well integrated. The working immigrants of the eighties have not yet stabilized and the process of their integration is still going on; thus they keep strong not only their languages but also their life-styles. Since the beginning of the nineties, the illegal entrance of immigrants has become an accepted fact. This fact, together with the economic and structural crisis, and the concentration of the immigrant working population in certain areas of the country, no longer creates the illusion that our society is a tolerant society, when the individuals concerned feel that their integrity or their status is at risk.

At the beginning of the project, and probably as a partial result of the facts described above, we are acutely aware of one crucial social issue. Many parents of Catalan children have removed them from the schools that have "too many" immigrant children. Besides that, the schools that have committed themselves in the past to working seriously for immigrant and cultural minorities have attracted even more of these children. Consequently many of the schools which initially did good work with a culturally diverse population of students have become "ghetto" schools. However, it must also be said that in some towns the educational administration has taken a strong position to address this situation, by for example changing the rules for school registration, so that the first criterion for allocating a child to a certain school is no longer the place where the child lives.

Negotiating the aims of the project

If we consider the three approaches to ethnomathematics research as outlined by Bishop (1995): anthropological approach, historical approach, and social psychological approach, our starting point was to consider the project from its third meaning. We are not dealing with how research can help rebuild a country after many years of colonialism; neither are we dealing with how the different cultures have contributed to the history of mathematics; but we are dealing with the mathematics education of culturally diverse students who live in the same country.

We have to deal with students whose parents belong to a culture different from the one that hosts them, but we regard the students themselves as being at a certain point on a continuum between their parents' culture and the host culture. Therefore we believe that the main educational approach should be to help them create their own psychological and social identities. In line with this belief, we want our research and its implications to take into account the students' out-of-school knowledge, including their values, beliefs and expectations. But the question for us is how to take this knowledge into account? We see no point, for example, in trying to teach aspects of the Moslem history of mathematics when the students make explicit in class that they want to become fully integrated Catalan adults. (Even if they are faithful Moslems, some students change their names to disguise their family and cultural origins.)

A further point is that the research team strongly believes that such an inclusive approach to both content and methodology in mathematics classroom, will be beneficial not only to children who are "culturally different" but also to the children of the Catalan communities, because it will make them aware of learning in a non-ethnocentric context, which has respect for other cultures and which also enlarges their understanding of mathematics as a cultural product.

Even if the project was the result of a request from the administration, the team's understanding of the multicultural fact in schools goes far beyond that of the educational administration. The team has negotiated strongly, and continues to do so, to change what initially was a policy-driven "research" project into a research project with no inverted commas. The team does not see cultural differences as a "problem to be solved" nor as a "diversity to be treated" but as a potentiality, to help ALL students to learn from the contrasting experiences and out-of-school knowledge of their peers.

As the result of the negotiation the aims of the project include, among others:

- To know more about the "out-of-school knowledge" connected with mathematics that immigrant students have, and how this knowledge can be linked with the curriculum content and its development in class.
- To uncover the values immigrant students associate with school and out-of-school mathematics and how these can help or interfere with the teaching of mathematics.
- To develop both proposals and practical examples of how to adapt the school curriculum to the multicultural situation of the classroom.
- To help make teachers aware of the different cultural backgrounds immigrant students have and their implications for the teaching of mathematics from a non-ethnocentric point of view which could help both immigrant and Catalan students.

- To promote and synthesise research in the field which could shed light on the previous points.

The research team

Having received the request to address an issue which is mainly connected with schools, the author, as a researcher and university lecturer, argued for the necessity of having a team which included different members of the educational community, to work in the project. The result of this negotiation resulted in a collaborative team, whose other members are two secondary mathematics teachers (Núria Planas and Xavier Vilella) a primary teacher (Montse Fontdevila) and a psychologist (Montserrat Benlloch). Núria is a research student and teaches in a secondary school in a very socially-deprived area of Barcelona, which has both immigrant and very poor Catalan pupils. She has got a partial release from her teaching hours to devote time to the project. Xavier normally teaches in a secondary school in an agricultural area near Barcelona, where cultural diversity occurs but is not necessarily linked to poor Catalan pupils. He has been given one year's leave to devote to the project. Montse, the primary teacher, has also partial leave, and is an expert on producing materials and on in-service teachers training. Montserrat, the psychologist, is teaching at the University of Vic, a city that has a strong commitment towards solving the problem of Catalan parents removing their children from the schools which have immigrant students. As part of her job she can work as a researcher and also with her in-practice pre-service students teachers in the schools in Vic.

With such a team we consider that not only do we have the support and the expertise from the university level[5] but also knowledge and expertise from school practitioners. However, as all the members of the team consider themselves as "outsiders" to the immigrant situation, we are now trying to involve in the project a primary teacher who has been trained in Catalonia, but who comes from Morocco, the country from where most of the immigrant students come. It is also our intention for the future to try to involve in the project some adults from the different communities, and also to extend the number of collaborating teachers.

Research procedures

The research procedures being followed by the team are similar to those used by Abreu (1995) and Presmeg (1997), and consist of the following:

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[5] The members of the team want to take this opportunity to thank Guida de Abreu, Alan Bishop, Ken Clements and Norma Presmeg, for their support and their advice on starting the project and its development. We want also to thank the Centre de Recerca Matematica, Institut d' Estudis Catalans, for having funded the TIEM98 project that gave us the opportunity to work with them during their stay in Barcelona.

- Collecting examples of classroom situations and incidents that exemplify cultural conflicts.
- Interviewing teachers to find more about their understanding and beliefs of cultural conflicts in mathematics classrooms, with examples of classroom situations and incidents being the motivation for the discussion during the interview.
- Observing students systematically during mathematics lessons to identify their potentialities as well as weaknesses regarding mathematical content.
- Interviewing students to investigate mathematical possibilities in their environment and their expectations about their learning mathematics.
- Documenting social practices that immigrant and Catalan students could understand as social activities involving mathematical knowledge.
- Creating classroom activities which are rich both from the point of view of research and learning, and which could be used as models for other teaching situations.
- Creating classroom activities that could lead the students to project their values and beliefs to identify diferent cultural practices.
- Documenting procedures which could be used by other teachers in other situations.

Constraints and possibilities for action

When first defining the project the research team made some assumptions, maybe too optimistically, regarding the contribution we could expect to receive from the educational administration. Throughout the negotiation of the project's aims and the resources to develop it, we have realized the existence of various constraints which we are unable to affect. In particular we have found that it is impossible:

- to change any aspects of the intended curriculum,
- to have release time for any other teachers to collaborate in the project,
- to involve adults of the different communities in the actual teaching in the classroom to help with communicating with students who have language difficulties,
- to allow teachers to teach in any other language different from Catalan, the oficial language of the schools, even though the students understand Castilian (Spanish) much better,
- to get any global data regarding the level of attainment of both immigrant and Catalan pupils,
- to obtain any information about the previous school history of the pupils before coming to our country.

Conscious of the constraints surrounding the project, the team is however clear that there are several possibilities for action to achieve the goals of the project and to implement its implications for teaching, such as:

- to hear the opinion of the different communities about the research project and their understanding of the educational situation,
- to involve someone from the different communities to work with the research team,
- to find ways to overcome the language barrier in the classroom, in particular to analyse the effects of letting the pupils work in small groups using their home or preferred language,
- to work with some schools and analyse comparatively the data they have about the level of attainment and difficultites that both immigrant and Catalan pupils have.
- to influence the implemented curriculum by offering models of rich activities which take into account the out-of-school knowledge and the practices of the different communities and which build on the students' potentialities,
- to participate in the in-service teacher education programs, and to promote reflection among the teachers' professional associations, to educate the awareness of the teaching community about the multicultural situation, to change their perceptions of difference, to present them with strategies which would allow them to know more about their students' previous knowledge and potentialities, and to give them clues as to how to change the social dynamics of the classroom.

Observations and immediate challenges

During the first year's work we have observed many aspects of the situation, most of which have convinced us that it is important to keep working on the project. In particular, we have data which documents that the understanding of the multicultural situation by teachers is far from what we believe it should be. For many of the most sensitive and sensible teachers we have interviewed, the cultural differences are reduced to language differences, and they believe that once the language barrier is overcome, if one ignores the colour of the skin, there are no more differences among the students.

Another observation concerns important differences between primary and secondary schools regarding their immigrant students. In primary schools the immigrant children are mostly of a second generation (but still legally considered immigrants) and therefore have had all their schooling here. In secondary schools there are many students who have just arrived in our country (many of them illegally) and who therefore received most of their schooling abroad. Moreover, there is another important distinction between primary and secondary schools, due to the characteristics of their teachers. Primary school teachers are more concerned about individuals than secondary teachers who are more concerned about content. All these facts mean that the way we address the two communities of teachers must be different.

Differences between the characteristics of schools in relatively small towns and big urban areas have also been observed. The cultural conflict in schools receiving immigrant children in big urban areas is associated with social conflicts and economical deprivation, while in rural areas or small towns, the cultural conflict is only that.

The above points have to do with general aspects and do not only refer to the teaching of mathematics. Regarding the curricular mathematical content, we have observed that the actual mathematical content is far away from the students' interests, and in some cases, students' needs. Therefore all the efforts we have made to know more about out-of-school and social practices involving mathematical knowlege promise to be worthwhile. Also regarding the mathematical content, we have observed, to a certain surprise, that what is taught in mathematics lessons in Catalonia is very close to what is taught in the students' countries of origin. This fact reaffirms the need to know more about students' needs and interests and to promote changes not only in the mathematical content, but also in the way it is to be taught, and in the social dynamics of the classroom and of the school.

If we had to say which has been the most significant observation after this year's work we would certainly say that it is about differences in the social dynamics of the educational situation. When talking about differences in a social situation one means differences from the "normality", where this is defined according to the assumptions and expectations of the individuals concerned. Thus, teachers find immigrant students to be "different" from what they expect their pupils to be. Immigrant students find their teachers "different", as they do the dynamics of the classroom and the school they are in. The interactions among students and between students and teachers are also culturally "different" as are the relationships between parents and the school system. We strongly believe that if we want any educational act to be positive both for the individuals and their communities it is crucial to make explicit to everybody the social dynamics of these "differences", and to begin to consider them as a source of richness, rather than problems, in the educational context.

Together our observations during the first year have shown us that there are some real and immediate challenges which we face. These include:

- to achieve a full acceptance of the project by the educational politicians, by convincing them that our project is not only "politically correct" but a real social need,
- encouraging the involvement of school principals and inspectors; in a word, "selling" our project to those who see it as a threat to the well-established

tradition, and convincing them that the education of children within a multicultural framework is, at the present time, a priority,

- seeking support from groups of teachers within schools and overcoming teacher-researcher conflicts, and, in particular, teachers' reluctance to participate in the project,
- gaining the trust of students, and establishing the credibility of the research team with immigrant parents and with significant members of the different communities, in order to extend the ownership of the project from the research team to the communities involved,
- overcoming practical matters for gathering and interpreting information, especially language barriers, by involving a language specialist who could help the team not only with translation, but also with the interpretation of the different language structures which can interfere with the teaching of mathematics in Catalan.

To end this paper we raise two sets of questions which are of concern not just to us, but we suspect to anyone involved in such work. They reflect the political sensitivities of the situation:

(1) Who is responsible for the growth and development of a project like ours? The researchers of course, but who else? What is the responsibility of the administration which has funded the project?

(2) Who is responsible for any implementation of the ideas for the classrooms and the schools? The teachers certainly, but not as isolated individuals. And then who else?

If we have the possibility of orally presenting the paper, more details and examples will be given, but we mainly look forward the oportunity to have the reaction of the ones that already have expertise on the field.

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Tales of power: Foucault in the mathematics classroom

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Abstract

In this paper I offer some of my work on power and how mathematics teachers talk about it. I am intrigued by the work of Michael Foucault on the circulation of power and attempt here to use this notion to consider the formation of the interactions and organisation of the mathematics classroom. I move on to explore a number of concerns including the constitutive and constituted nature of teachers' practice and teacher ideology. and how power, in this sense, might legitimate certain classroom actions.

I will start with a brief account of an incident that I hope will help locate the concerns of this writing. I have joined a Y8 class where a PGCE student is working with the class teacher. The teacher approaches me and says,

> *'You can see the enormous ability spread in this class. The difficulty is the books. They are supposed to be a set 2 so we use these red books - but some really need the examples and reinforcements that are in the green books. Some need extra support to work on even the basic examples in the red books. How am I supposed to teach them as a class? I'm finding it so difficult.'*

Incidents such as this have caught my attention and generated an interest in how teachers talk about what empowers them or limits them; an interest in what they say is controlling or disempowering or enabling in their professional lives. In particular I have become interested in the power relation between mathematics teachers and the descriptions of mathematics teaching and the school mathematics curriculum that seem to abound in current documentation and guidance. My intention here is to develop an account of how that power relation works.

I have been dissatisfied with theoretical frameworks that construct the actors in the education arena as oppressor or oppressed and that discuss power as something wielded by a few over the many. This does not reflect for me what it feels like to be a teacher (or be a human being). I have found that the theories of Michel Foucault on the operation of power within groups and institutions provide me with tools with which I can develop a more recognisable account. I have used those 'tools' to give me a way of looking at teaching and learning interactions, and particularly, teachers' talk about how they plan their work and how they view their practice and I have found that the careful use of these tools can highlight previously unexamined areas for me.

I want here to introduce some of the ideas from Foucault's work on discursive practices and power relations that I have found helpful and consider in some depth their applicability to the mathematics education context. I have arranged these under 4 headings: Power is the relation, Power is productive, Discursive practices, Power conceals itself.

Later, by drawing on the discussion that followed a presentation of these notions to colleagues at the BSRLM conference in June 1997, I will give a example of how these tools might be used to work on a particular account of a lesson and start to see what (new ?) meanings might be made of the teaching and learning interactions. What is here is very much a beginning. My future work will extend into a fuller analysis of the circulation of power in the mathematics classroom.

Theoretical Tools:

Power IS the relation

I start by investigating Foucault's (1972a) statement that '*Power is in the relation. It is not exercised in a repressive sense from outside the individual'* Foucault starts from the premise that, primarily, knowledge and power work through the language; that as we learn to speak we pick up the basic knowledge and rules of our culture at the same time. He extends this to see that all human sciences define human beings at the same time as they describe them, and that these organised forms of knowledge working together with their associated institutions, have significant effects on people (particularly in terms of positioning them and determining their possible actions). He talks about this in a variety of places and in various ways; relating specifically to power he writes:

> *'Individuals are the vehicles of power, not its points of application.'*
>
> *Foucault in Gordon (1980)*

> *'What characterises the power we are analysing is that it brings into play relations between individuals (or between groups)... In effect, what defines a relationship of power is that it is a mode of action which does not act directly and immediately on others...The exercise of power consists in guiding the possibility of conduct and putting in order the possible outcome'*
>
> *Foucault in Dreyfus and Rabinow (1983 p217/220)*

Foucault makes a powerful shift here away from individualistic ways of viewing power to a notion of power that is not outside of an individual or invested in one individual to exert over another. Here power is in/of the relation between people and of the institutions that they work in. It is this sense of power that I argue can be appropriately applied to my context as a member of the mathematics education community and can offer a fresh view of our environment. But I am aware that it is powerful but not an easy shift for me (or other educationalists) to make.

Maggie McBride (1989) tries to apply this notion of power for her mathematics classroom. She writes: *'Foucault claims that power does not act directly on people but on their actions.' and 'Power is made and exists in every social interaction and classroom....the site of power is within individual students and teachers.'*

I want to argue that power brings into play the relation at the same time as it is constituted through the relationship itself. In other words, in accepting the constitutive nature of power and knowledge I need to be careful (in using power in Foucault's sense) not to pretend to the possibility of separating out the entities - people, their actions, their relations, their institutions and power itself.

Power is productive.

For Foucault power is internally contradictory. It oppresses and enables.

> *'If power were never anything but repressive, if it never did anything but to say no, do you really think one would be brought to obey it? What makes power hold good, what makes it accepted, is simply that fact that it doesn't only weigh on us as a force that says no, but that it traverses and produces things, it induces pleasures, forms, knowledge; it produces discourse. It needs to be considered as a productive network which runs through the whole social body, much more than as a negative instance whose function is repressive.'*
>
> *Foucault in Rabinow (1990)*

Referring back to Foucault's words in Dreyfus and Rabinow (1983), the exercising '*of power consists of guiding the possible conduct, putting in order the possible outcome'.* This introduces the notion that when power is circulating it determines, to some extent, what are possible ways of acting and limits in some way what can be done and enables one to act. The productive and determining effects of power are important for my analysis. To use this appropriately I need to remain aware that I might tend to look for the inhibitory nature and limiting effects of power and that this might lead me to a skewed and incomplete account.

Discursive practices

Foucault considers how knowledge is actually produced and induces power through what he calls 'discursive practices' in society. 'Discourse' is a central term for him. In its broadest senses it means anything written, said, or communicated using signs. A connection can be made here with Structuralism (see for example Levi-Strauss) and its dominant focus on language. He describes a discursive practice as

> *'a body of anonymous, historical rules, always determined in the time and space that have defined a given period'.*
>
> *'There is no knowledge without a particular discursive practice and any discursive forms'*
>
> *Foucault (1970, p117, p183)*

A discursive practice is constituted by the actions of the members, their interactions with each other and the texts and communications and artefacts from within that practice and Foucault argues that these discursive practices have profoundly shaped the structure of our society.

In the context of my researches these theoretical notions allow me to look at my classroom (set within the wider mathematics education culture) as a discursive practice, and I can consider how the actions and reactions of the people in it are constituted by the discourse and, at the same time, the discursive practice is actually constituted by their actions. This enables me to consider how power is formulated within teaching practices and interactions with curriculum texts.

What might be my research task ?

Power conceals itself

In looking at aspects of classroom practice one task (and that task was set out by Foucault (Foucault and Deleuze 1972 p 208) is to try to reveal the power that is operating there. He argues that one of power's characteristics is that it is often invisible, hidden. And one of the tricks of power is that it makes things look natural, obvious, and unquestionable. One task of a researcher then would be to try to reveal that hidden power.

The foregrounding of language and texts marks a shift away from seeing cultural norms as formed and perpetuated through traditions and practices to seeing a process of normalisation coming about through descriptions and a particular way of ordering things. The constitution of the National Curriculum for Mathematics in the UK is a example of mathematics described in categories (ordered into levels and (5, no it should be 4) attainment targets) - a 'constructed naturalness'. The nature of mathematics is somehow changed as a result, this categorisation becomes a cultural norm, regulation through descriptions that come to be taken as natural and obvious. The way that mathematics is constructed, the determination of mathematicians' and mathematics teachers' consequent actions and the power relation itself can all remain hidden.

I find this notion of power as hidden and best exercised when people are unaware of it key in education researches. On an electronic mailing list dedicated to the work of Foucault (A posting on the subject of Foucault and Habermas to the Foucault electronic mailing list @jefferson.village.virginia.edu), Sam Binkley (1995) remarked

> *'In fact as in Nietzsche's critique of morality, he (Foucault) would say that the mere pretence to operate on a realm free of power relations is perhaps the most cunning and sinister play of power there could be, which masks itself behind a benign facade of liberal (Christian) generosity, a mode of domination specific to the period of modernity.'*

In his work looking at medicine for example Foucault was not interested in what characteristics (personal characteristics) doctors had. He was interested in the role that those who practised medicine must follow to maintain their position and be seen as doctors. In the same way, an appropriate classroom research strategy would be to look at what roles, what ways of talking, what ways of behaving the teacher, the pupils, the text, the activity must take in order to be able to stand in that position.

Revealing the Power

What follows is the transcript of a video extract taken from 'From The Trouble with Numbers' broadcast on BBC2 Thursday 30th January (further information available on http://www.bbc.co.uk/education/cmi/tblnum.htm) that formed the basis of the discussions with maths education colleagues mentioned earlier. The task I set to focus our viewing of the video was to reveal the power that may not initially be on view and to consider the questions –

- How do you have to act, what do you have to say be seen as a teacher of mathematics or a learner of mathematics ?

and

- In what way are you limited or enabled by the power that circulates ?

(You may want to read this transcript with these questions in mind and see whether you recognise in the discussion that follows the themes that took our attention in that session.

Curriculum project leader comment: *In order to convince our teachers that they can actually do this we felt it important to provide them with precise lesson plans, so that we have in our document quite clear-cut lessons that, in fact, indicate to the teacher what they should be doing, what they should be covering in each of the lessons with their classes.*
Commentator: *Following the example of Hungary every lesson starts with the previous day's homework. Any problems children had are sorted out in front of all the class and only then do they move on to that day's lesson.*
Teacher comment: *The first day we introduced this particular scheme into year 10, one of the children said 'This is the hardest day's maths I've ever done in my life'. and that was because she knew that for the whole lesson she had to keep on task and couldn't have 2 minutes rest while I wasn't watching her while I was attending to somebody else.*
Scenes from the start of a lesson
Teacher: *Right I want now to go through any of the questions people had difficulty with. Who would like to come up and put any angle in there on B?*
Teacher comment: *It's very demanding for the teacher because you are in control for the whole lesson and you are having to answer questions that are unexpected questions where as if you set an exercise for children to do for 10 minutes say you have 10 minutes thinking time to get yourself organised for the next bit and you don't have that kind of break in these kind of lessons.*
Pupil A comment: *If you are up at the board then you don't have to know exactly what you are on about but you've got to have a good idea 'cos you've got the whole of the class that are there to help you. So you've got like 32 teachers instead of one.*
A right-angled triangle is drawn on board with sides marked 6 and 8.
Pupil A goes up to board and writes $c^2 = a^2 + b^2$
Teacher: *Excellent*
Pupil A: *Your hypotenuse is always opposite the right angle*
Teacher: *Excellent*
Pupil A: *so...* (and writes on board marking one side as 'hyp')
Teacher: *Do you want to put your values in now, into your equation?*
Pupil A writes on board $c^2 = 8^2 + 6^2$
Derek: *no shouldn't it..*
Teacher: (to Derek) *go on ...yeah... tell her*
Pupil A: *so 8 squared is 64* (and writes on board $c^2 = 64 + 36$)
Derek: *It's not c squared - shouldn't that be c equals 64 + 36*
Pupil A: *that's c squared*
(affirmative mutters of c^2, c^2 from other pupils)
Pupil A now writes $c^2 = 100$ on board
Teacher: *I think that you are out voted on that one, Derek*
Derek: *no...*
Teacher: *Just a minute. We'll talk about it in a minute.*
Commentator: *Teaching thirty or so students is always a challenge for the person in charge.*
Pupil A continues by writing something we don't see on the video then c = 10
Teacher: *do you all agree with that...?*
Class chorus: *yeah*
Teacher: *right, there's just one little thing wrong with it and that's that we shouldn't have had that square root of 100 there. We write....*
She rubs a line off the board and writes √] c = 10

The video extract can be treated as a piece of text. There are different ways in which we may read it. Multiple discourses can be drawn out and we each choose the one/s with which we work. The strategy we used was to look at the event in the detail, at the acts and the setting, and see what is highlighted for each of us. We have to bear in mind 'the trick of power/knowledge to conceal itself' and so have to work carefully with the 'microphysics of power' (Foucault 1977) wherein forms of licence come about. It is this care that gives value to working with a video clip in this way (without denying the many things that we do not and cannot know, such as whether the teacher moved on to some quite different style of working, and that new accounts may have altered our readings). The task is not to try to establish the truth, to tell the one story that fits.

I give here a brief collage of power themes - it is uneven, incomplete, fragmentary and overlapping - taken from colleagues' contributions to the discussion after viewing the video clip. These select detailed acts and utterances that may initially seem unimportant or slight but could reveal the workings of power and the conditions of existence of the relations here. The extracts point to what meaning 'power ... brings into play relations' might bring to this context and give some formulations of the exercising of power and the creation of knowledge.

I have included further reference to Foucault's work as seemed appropriate.

(Extracts from the (transcribed) discussion are formatted and inset like this)

Maths as right or wrong

> *'The nature of the mathematics contained in the video clip is one where there's a right answer and a wrong answer and we might say from what the teacher was writing on the board at the end, there was one correct method as well. That isn't how mathematics has to be seen. It is just one formulation of mathematics. The teacher said that there is just one thing wrong with it*
>
> *- it deviated just a tiny bit from 'the right way'.*
>
> *That adds emphasis to there only being one way to do it.'*

This draws our attention to the constituting nature of discursive practices. As McBride (1989) says, *'(Mathematics) needs to be understood as a constructed discourse that, with its rules and practices, effects our concept of truth, accepted methods of learning, and many attitudes within the classroom'*

Handling the questions

> *'The teacher said it's difficult to answer (unpredictable) questions. We could ask 'What questions?' To me there were none asked in the extract. Nor were the pupils faced with any questions.*
>
> *The teacher's and pupils' form of speech was 'this is a right angled triangle', 'this is opposite to this', 'this one is the hypotenuse'. Consideration of questions like 'what if it wasn't a right angled triangle?' were not legitimate. What sort of questions would have been legitimate?*

It might be important to ask what generated this anxiety about questions for the teacher. What might the pupils have asked (of her) in that situation that she wouldn't have understood or been prepared to deal with?

Pathologising the pupil

The phrase '(questions that) people had difficulty with' came up in the video. This can be seen to refer to a common-sense and obvious notion that some pupils do 'have difficulties with' their homework. This positions the pupil as the one who has difficulties.

The difficulty is not with the teacher or the homework. Perhaps I should consider my teaching as poor if not all the pupils couldn't understand. But, this way of thinking does not appear to be entertained.

Moreover, the curriculum devisors appear as legitimating her position. She seemed to feel she was absolutely right in what she was doing because she appeared to have been told that this was the right way. Is that odd? She wasn't responding to the students; she was responding to the curriculum.

This removed from her the possibility that there could be difficulties with her teaching. 'They're not understanding, and so I need to consider how I can do my job differently to improve this' wasn't a legitimate part of her thoughts.

There is an issue for us as teachers to work on here: what is it about what we say and which of our actions position the pupil as the ones with difficulties and our teaching as unquestionable. How can we talk and act differently and still maintain working well in our classrooms?

Ideology and discursive practices

A technical note: It can be seen that where Foucault uses the term 'discursive practice', Lacan and Marx use the term ideology. Their construal of the notion of ideology is portrayed by Zizek (1989) in his work 'The Sublime Object of Ideology'. In line with this I use a definition of an ideology as a framework made up by set of nodal points, tenets, principles which when held together generate further consequences beyond their statement or articulation. Agreeing with these tenets puts you in a position of finding that you have subscribed to much more, and are unable to deny some argued consequence. This also runs parallel with Althusser's (1994) definition of the joy of ideology as the pleasure, the non-critical tautology, in saying 'yes, the way I see the world IS the world.' From my reading of Foucault I would claim that he invests 'discursive practices' with the same meaning, particularly his sense that discursive formations operate so that the power is exerted, people are positioned and their actions defined. There is a compulsion, a necessity, it could not be other. In this sense we have no choice.

The intended audience (for the programme from which this video extract was drawn) may not have been teachers or researchers but a wider audience of television. The message of the piece could be summaries as 'traditional teaching is best'. Traditional teaching here is signified by the teacher's and the pupils' roles, signified by the fact the pupil is using chalk on blackboard, signifying by the use of Pythagoras as an example of mathematics being done. The teacher's commentary and the pupil's commentary act to support that particular message. The teacher's teaching and pupils' learning are part of an exemplification of a traditional mode of teaching.

Earlier in the video the project leader described the class scene as an example of whole class interactive teaching. That expression and others like 'going back to traditional methods' are currently heard in the discursive practice within our society. The curriculum project leader's use of thes,e allow him to occupy the position of technical expert. He is constructed (by the media, by the discourse) as someone who knows something about these issues. Not because of what he knows but because of what he says and how he says it.

Surveillance

Much of Foucault's work is focused on the construct of 'normalisation'. He claims examination

> *'is a normalising gaze, a surveillance that makes it possible to quantify, to classify and to punish. It establishes over individuals a visibility through which one differentiates them and judges them'*
>
> *Foucault (1977)*

There was a phrase 'if I wasn't watching her'. In the language of surveillance - As a teacher I have to watch people to make sure they're doing things. Learning happens when you are watched.

Also is there an implication that if the teacher is occupied, if demands are made on her for the whole lesson, then the pupils learn more?

32 teachers

One of the pupils said that there are 32 teachers.

How do I have to act to be a teacher? What element of her peers' actions made them occupy the position of teacher for her? The pupil could have seen herself as teacher as she was at the board. Her peers were sitting behind their desks in straight rows which was facing her and yet she still looked at the whole room as teachers.

They are teachers because they are seeing if something is right or wrong- an arbiter's role. The teacher said, at one point, to a boy, Derek "Go on, tell her." There are 32 teachers because they can all tell you what you should be doing, what you should be writing.

..Or/and...

The pupils' perception could also be that 'the teacher is there to help us'. In that particular clip the teacher did say 'we'll talk about it afterwards'. Otherwise her comments were of the form 'excellent, excellent', 'just one mistake' etc. In a sense what the teacher was doing was directing. Is this guiding seen as helping them? Is this guiding seen as a legitimate role for a teacher?

Children can be very emotional about 'teachers are there to help us' and they get very angry when teachers do not help them.

The process of 'normalisation' is the mechanism that categorises people into normal and abnormal (Foucault 1977). Linking the notions of normalisation and power as a productive network allows us to see the process that determines what is considered to be knowledge in the classroom and how that knowledge can be expressed and by whom. It is the process of normalisation that determines who is included and who is excluded in this discourse. Those who conform to these roles are likely to have their voices heard.

> *'Discursive practices are characterised by the interplay of these rules that typically are not written out; nor can people usually articulate them. Rules determine the possibilities and limitations for the content of a discourse. They determine the conditions under which discourse is used, who can speak, how individuals must speak or write, and who speaks the 'truth'. Rules also determine what can and cannot be talked about. Another group of rules has to do with the form that theories must take in order to be seen by people as truth within any discourse. These rules determine the vocabulary that must be used in stating the truth. Even the arrangement of statements follow these rules.'*
>
> *McBride (1989)*

More on teachers' role: what teachers do

> *We should be careful not to generalise and say that telling someone something or leading someone through something is in itself wrong or bad teaching. And, it is interesting to here see how the pupils view it.*
>
> *But knowing what to teach is important. The pupils couldn't teach it - they don't know the key points to draw out, they don't know how to guide.*

I dwelt a little longer for myself on the theme of '32 teachers' because it points to enabling and silencing characteristics of power that are of particular interest to me. At this stage this produces mostly (more) questions. My future work will need to recast what I do as a teacher when I try to enable my pupils to speak, try to allow questions and comments and to bring out in the open children's' own ideas, and in this forum enable them to be mathematical. This will involve a search for new terms and descriptions. I am caught by pre-existing ones: they are never neutral.

In the video extract the class survey a pupil whilst the teacher surveys - what? the rest of the class ? the symbolic representation of knowledge appearing on the blackboard ? the integrity of the classroom scene ? If I take a theatrical metaphor then who is/are the audience, the actors, the chorus ? A pupil is 'out at the front' involved in some form of enactment that depends on some form of surveillance for its meaning. This enactment is observed and interpreted. There is arbitration and correction:- '32 teachers', 'outvoted', 'affirmative mutters', 'you have to have a good idea', 'just one little thing wrong'. This is a public performance of learning, something held up as an exemplar. Am I pushing this too far if I consider this theatre stage to be everywhere about teaching and nowhere about learning ? Can I ask where is and what is the learning that takes place ? In the wings ? Absent from the scene ? Waiting for a costume change?

Where does all this get me?

> *Each society has its regime of truth, its 'general politics' of truth: that is, the types of discourse which it accepts and makes function as true.....The problem is not changing people's consciousness - or what's in their heads - but the political, economic, institutional regime of the production of truth. It's not a matter of emancipating truth from every system of power (which would be a chimera, for truth is already power),*

but of detaching the power of truth from the forms of hegemony, social, economic, and cultural, within which it operates at the present time

Foucault in Rabinow (1991)

It is not to awaken consciousness that we struggle but to sap power, to take power; it is an activity conducted alongside those who struggle for power, and not their illumination.'

Foucault and Deleuze (1972 p208)

I said that I intended 'a struggle aimed at revealing and undermining power where it is most invisible and insidious' (Foucault, Deleuze (1972)). Have I revealed that power? I think through discussion of a range of power themes related to the video extract that I have at least started. More importantly, I have identified some tasks to engage in as part of that struggle. I have identified some areas that will reveal the discursive practice(s) of mathematics education and the ways this positions people within the classroom, limiting their actions and determining what each has to say to be heard.

'The work of an intellectual is not to mold the political will of others; it is, to re-examine evidence and assumptions, to shake up habitual ways of working and thinking, to dissipate conventional familiarities, to re-evaluate rules and institutions and starting from this re-problematization ...to participate in the formation of a political will

Foucault (1989)

The formation of a political will involves enabling people to act in society. There is a need, as Foucault shows, to identify tasks and questions that work 'to sap power' and so enable me to act. He does indicate to me that such re-problematisation is hard but imperative. Our discussions around the video extract point to important, and I would argue, not easy steps I can take to reproblematise aspects such as teachers' role. I admit that it is challenging to articulate these tasks in practical terms

There is also a tension - Foucault talks strongly about how actions are limited and controlled, whilst in the quotation above he moves to talk about how I can act with some autonomy and not be entirely caught in that controlled situation. Whilst I am a little uncertain about the exact nature of the move that I can make to work on this tension, I believe the strategy of working with newly devised terms and new conceptualisations will be empowering. My future researches and analysis of teachers' descriptions of their practices aim to tell a previously untold and silenced story and to make me more aware of the workings of discourse practices in the generation of power and truth. This moves me towards being able to take more considered action in my classroom practice.

I would greatly welcome others' responses to this article and to hear of others' experiences of using such notions to research their own practice. I am at Tansy@kalliste.demon.co.uk

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MATHS AND GENDER: GIVEN OR MADE?

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Much of the literature on maths and gender takes either maths or gender, or both, as given. I use the work of R W Connell to argue that frameworks for describing and accounting for gender relations - particularly sex-role theory and categoricalism - have weaknesses as well as strengths. I adapt a framework developed by Connell for analysing gender relations within a theory of practice, and suggest that it can also be a useful tool for understanding mathematics as practice. I conclude by indicating how, after employing the feminist methodology of memory-work to collect data, I used the framework to analyse the mathematical practices of a group of women.

Much of the literature on maths and gender takes either maths or gender, or both, as given, unquestionable, categories. I want to consider here a framework for describing and accounting for both gender and mathematics that does not make this assumption, seeing them instead as historically constructed, changing and changeable, practices.

GENDER: GIVEN OR MADE?

> Sexual difference is necessary for the continuation of the human race. Gender difference is not .... Social processes of differentiation and separation serve power, whether that of a class, a race or a sex. They are universal devices of oppression. (Cockburn, cited in Lee 1985)

The general consensus of the research on maths and gender strongly rejects biological explanations of difference (Willis 1989; Bellisari 1989; Hyde, Fennema & Lamon 1990) or at least rejects their usefulness in constructing interventions (Fennema et al 1985; Leder 1992), although some explanations slide into essentialist beliefs. If biological explanations are

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not very helpful, then we must look to social explanations, explanations which in turn are not unproblematic. Connell, for example, argues that underlying many of apparently social analyses of gender is a persistent 'assumption that a transhistorical structure is built into gender by the sexual dichotomy of bodies' (1987: 64) - an assumption that implicitly gives priority to biology, making social analysis marginal. Against this view, Connell argues that research has shown cultures where gender is not necessarily binary or assigned according to biological characteristics: that 'gender is accomplished by social practice' (1987: 76) and our bodies 'grow and work, flourish and decay' (1987: 86), within institutions and relationships, at a material as well as a symbolic level - within history .

There has been a vast array of social analyses of gender in the literature of the last two or three decades, and a number of ways of categorising them. It is my intention here, not to explore these analyses and their logics very deeply, but to indicate my position in relation to this array, using Connell's approach as my basic framework. Two distinctions that I would like to follow up are:

> ... between [accounts] that focus on custom and those that focus on power; [and] within power theories, between those that see categories as prior to practice and those that see categories as emerging from practice. (Connell 1987: 42)

The main accounts that he explores are sex role theory - in relation to custom - and categorical theory - in relation to power. It is in attempting to hold on to the strengths and to overcome the weaknesses of these positions that he outlines the framework of a theory of practice.

Sex role theory is very prevalent in the literature on maths and gender and clearly has contributed insights into the relationship between the two. Most of the many different versions of sex-role theory have in common: a distinction between the person and her position; a set of actions belonging to the position - in keeping with the metaphor, an actor and a script; and accompanying expectations and sanctions from others. Role theory is an important shift away from biological to social explanation, and sees individuals in relation to social structures - although in the end these structures depend in fact on a biological dichotomy. Because sex role theory focuses on norms and gives explanatory priority to will, choice and attitudes, avoiding questions of power and social interest, it has no way of understanding the historicity of gender and is limited in what it can offer.

Categorical theory enables more attention to be given to power, to social institutions and social structures. In such theories, the social order is seen in terms of a few major categories - often only two - that are related by power and interests; the focus of the argument is on the whole category, rather than its particular members or how it is made up; and there is a close identification of opposed interests with specific groups of people.

> Analysis takes the categories for granted while it explores the relationship between them. Categorical theories of gender differ from each other mainly in the accounts they give of this relationship (Connell 1987: 55).

Such theory is useful for mobilizing political action, as for instance 'a first approximation' in describing sex inequalities in such fields as income, health - and mathematics education - and struggling for more equal access. However in painting the broad picture, its universalising view of gender - 'all women against all men' - leaves out 'the element of practical politics; choice, doubt, strategy, planning, error and transformation' (Connell 1987: 61).

Gender as social practice

Valuing both categoricalism's recognition of power, and the 'practical politics' of daily life, Connell calls for a theory of practice that would help us get a 'grip on the interweaving of personal life and social structure' (Connell 1987: 61). Work by Juliet Mitchell, Adrienne Rich, Jill Matthews and David Fernbach are cited as examples of studies that use such a framework for analysis. Practice, says Connell, is 'what people do by way of constituting the social relations they live in' (Connell 1987: 62), and he sees human action as involving 'free invention (if "invention within limits", to use Bourdieu's phrase)' (Connell 1987: 95). Such a position presupposes both the person as agent, and the existence of structure. Practice is specific, historical and constrained by structure. Connell argues that to describe the constraints of the particular situation to which the active individual responds is to describe structure:

> ... structure is more than another term for 'pattern' and refers to the intractability of the social world. It reflects the experience of being up against something, of limits on freedom; and also the experience of being able to operate by proxy, to produce results one's own capacities would not allow. (Connell 1987: 93)

To explore gender in this framework is to ask how gendered practice is 'organised as a going concern' (Connell 1987: 62). It is to work from the assumption that social structures are not given but historically constituted, an assumption that implies the possibility that different social interests and constraints may result in different organisations of the practice of gender.

Such a theory of practice would seek to avoid categoricalism, and so would be interested in differences within categories as well as between. It would try to locate these differences historically. It would ask how differently gendered practices of moral judgement, of knowing and thinking, had grown out of differently gendered experience, and how they had been structured by power and labour relations, by emotional signification and symbolization. From such a position, the 'givenness' of gender would melt into thin air and 'practical politics' would become possible (Connell 1987: 62). From such a position, we would be able to generate 'partial perspectives' of the complexity of gender construction, gender movement and gender relations (Haraway 1991).

Feminist writers have suggested other frameworks for the analysis of gendered practice. Sandra Harding, for instance, conceptualizes gender as 'individual, structural and symbolic - and always asymmetric' (Harding 1986: 52). Donna Haraway in supporting this view of gender, suggests that science as well as gender could be usefully analysed using Harding's three dimensions - as 'gender symbolism, the social-sexual division of labour and the processes of constructing individual gendered identity' (Haraway 1991: 250). Haraway however adds two extra dimensions of her own - material culture, and the 'dialectic of construction and discovery'. Joan Scott (1988: 42) conceives of gender as both a 'constitutive element of social relationships' and a 'primary way of signifying relationships of power'. The former is based on perceived differences between the sexes and involves four main elements: cultural symbols, subjective identity, normative concepts, and social organisations and politics.

These proposals overlap to a greater or lesser extent with each other and with Connell's elaborated framework (1996b), which allows a systematic view of questions that could be asked about the interplay between different levels of practice and different structures. Gender studies suggest that four structures currently play an important part in influencing practice: power relations, production and consumption, symbolization and emotional commitment or cathexis (Connell 1996a: 10). Connell suggests that practice can be explored at different levels: the level of personality, social relations or institution. He proposes a simple grid which cross-classifies structure by level as a way of systematically assessing the dynamics of change in gender relations; a parallel grid might also be useful in setting up an overview of other questions about gender - for example, its relationship to science - or mathematics.

MATHS: GIVEN OR MADE?

> Some attitudes and assumptions are so basic to how we think about and experience the world that it is difficult to consider them critically. For anyone educated in an 'advanced' technological society it is practically impossible to imagine that our ideas of *objectivity* and *factual accuracy* and the basic place of *numbering* or *quantification* in our world-view, are historical products rather than eternal principles of analysis. However, stress has been placed on these as part of an experimental, investigative methodology only since the late sixteenth and seventeenth centuries. It was in that period that modern capitalism and its way of knowing nature - modern science - were developing as a new and unified socio-economic order with a new way of defining reality and knowledge. (Young 1979: 63)

It is not only gender that appears as a clear and given category in much of the maths and gender research: it is also the other member of the pair, mathematics. Young's emphasis in the quote above on the historicity of quantification is still a startling challenge for many students of mathematics, and some educators.

Doing mathematics, learning mathematics, teaching mathematics, teaching teachers how to teach mathematics - pervading all these situations has been for many of us the image of mathematics as given, fixed, to be passively received and unquestioningly transmitted. How have such 'truths' come to be accepted? What effects do they have? These would be fascinating threads to unravel, but for the moment I want to explore an alternative possibility: that mathematical practice is constructed, by active people, that it is structurally connected with other practices and involved in the constitution of social interests.

This is not an idea that has come easily - in relation to maths. In relation to almost any other field it became increasingly clear to me over years of learning and teaching that knowledge was contextual, made by people and implicated in relations of power. The realisation of how this is true also of mathematics is a relatively recent achievement.

I have mentioned above Haraway's suggestion that Harding's framework ('individual, structural and symbolic') would be useful for exploring science as well as gender. A parallel proposal is that of adapting Connell's framework to explore mathematics. To do this is to ask how mathematical practice - its discourses, its institutions, its ideologies, its social relations and its practitioners - is 'organised as a going concern' (Connell 1987: 62). It is to work from the assumption that social structures, in maths as elsewhere, are not given but historically constituted, with the concomitant possibility that there might be more than one mathematics, more than one mathematical practice. To explore practice properly would involve looking at not just what maths is done, and what Connell calls its 'historicity', but the institutions within which it is practised, the ideology that informs it, the discourses it produces and is produced by, the way it shapes its practitioners and how they relate to one another.

Maths as a social practice: towards a systematic analysis

Using the methodology of memory work (Haug 1987), I have begun to explore the idea that mathematical practice is actively constructed and to examine how the mathematisation of the everyday world constructs us and how we construct or resist it.

I was one of a group of women involved in this exploration. We found that much of our mathematical experience had been of a dominating practice that alienated us from our own knowledge and the everyday world, separating mind, body and emotion, and prioritising abstraction and generalisation over meaning. Understanding both maths and gender as practice, however, allows us to see them as produced by humans, in specific historical circumstances, and therefore as able to be different from the constellation of arrangements at any particular moment. These hegemonic practices are therefore not the only ones possible; the work that is required to maintain them, in fact, demonstrates the degree to which they can be seen as denials of alternative experiences.

In order to demonstrate how Connell's suggested grid (1996: 7) can be used to systematically document practice, I will indicate how I used it to analyse one dimension - the structure of power - of the dominating practice that we experienced. The other dimensions can be similarly analysed.

It is important to remember in using the grid that the categories are not discrete, that overlap occurs between structures - for instance between symbolization and power - and that, for instance, most social relations take place within institutions. The grid is simply a tool that allows us to obtain an overview of important issues in a particular field of practice, recognizing the different structures that affect that practice, and the different levels of social reality at which it is manifested. It also allows us to identify which areas have been well served and which ignored by research.

		LEVEL OF PRACTICE		
		PERSONALITY	INTERACTION	INSTITUTIONS
STRUCTURE	POWER	capacities for power	practices of dominance	distribution of rights, office and influence
	LABOUR	capacities for labour	work relations	distribution of income, systems of ownership
	SYMBOLIZ-ATION	capacities for meaning	social relations generating meaning	public & organizational representations
	CATHEXIS	capacities for emotion	social relations enacting emotion	maths as truth

Table 1: Mathematical practice as it interacts with gender practice. The entries in the cells of the table identify (possible) core issues relating to the interaction between maths and gender.

At the level of personality, practice - 'as seen from the perspective of life-history' - can be analysed in terms of capacities for power, work, meaning and emotion, according to the different major social structures of mathematical practice. At the level of interaction, practice - from the perspective of social relations - can be analysed in terms of face-to-face enactments of power relations, work relations, relations generating meaning and relations enacting emotions. And at the institutional level, practice - from the perspective of the whole mathematical order - can be analysed in terms of its embodiment in organisational structures of power, work, symbolization and cathexis. In general, my study contributes to this last level only from the perspective of an individual's location within an institution: larger ethnographic or statistically based analyses are beyond its scope.

For example, power

Alison, Colleen, Kath, Louise, Maggie, Marie, Sophie, and Viv, mentioned below, were eight of the fourteen women involved in the memory-work study.

Capacities for power concern here the ability to participate in the decision-making processes of a democratic society as numerate citizens. Kath and Colleen illustrated different sides of our inadequate preparation: Kath in a mathematically refined but socially naive understanding of the process of cost benefit analysis, Colleen in a mathematically inept, but socially critical response to the teacher-student ratio 'formula'.

Practices of dominance abounded, in control through measurement - the pressure of time in exams, in school, at home, at work; through language - a chain of meaning linking wrong with error with sin; through physical class arrangements - Maggie was shifted from the back of the room with the 'good' ones, to the middle with the ones who were not clever; through inclusion and exclusion - Viv was not one of the chosen few; and through humiliation and a veiled threat of force in Marie's encounter with a teaching nun, Sister Peter. Teachers and fathers were prominent actors in these dramas; in particular, the measurers in our stories were men.

The distribution of what Viv called educational ’goodies’ often uses mathematics itself as a filter, and almost always uses some form of quantification. The power of number, in both these senses, was clear in Louise’s struggles as a teacher, around the entrance exam for the selective High School where she taught. Measurement - of time, of ability, of worth - was an organising and often violent strategy permeating our lives. We had been categorised, compared and measured from such an early age that measurement had become our normality: cost benefit analyses, student-staff ratios, merit, accountability, selection, productivity, disability were just a few of the contexts in which it emerged.

It could be argued, as Sophie and others did, that to be a success in maths as we knew it you had to be satisfied with a 'thin' maths, a maths that was stripped of context, of layers of meaning - of critique, of history, of everyday use; a maths that separated mind from body, proof from intuition, ourselves from our own knowledge, symbol from sense.

Other stories, other practices

To stop at that point, however, would be to deny the very theory of practice that I want to use. It would be to suggest that the only active parts we played, our only inventions, were those of complicit partners in a hegemonic drama, reading scripts we had been given. But Kath, agreeing with Sophie about what was missing in maths, continued:

> If you've had any other experience at all you can't accept the thinness of what you're told in maths, and you actually want it to make more sense.

You want to experience it as meaningful, making wide ranging connections which challenge separations. You want to own it, imagine it and feel it. You want to retrieve it from formalism, to acknowledge its real, human, clay feet, to play with it in a variety of semiotic systems, to make connections with its history, its questions, its power, and not to ignore either its elegance or its crimes.

And of course there are other stories, less visible at first and not so easily told, of practices that contest the dominant tales. They are often stories we had to invent words for, or stories that we apologised for as being irrelevant or embarrassing. Within the constraints of specific situations we and others did do things differently, we acted to weave different relations with others, we invented different lives.

In relation to structures of power, for example, Alison told how her experiences with 'arbitrary' teachers and their 'incompetent labelling' had helped her distance herself, making the relation with the teacher less powerful. Louise was able to use the memory-work group itself as a platform not only for exploring issues about gender equity at her school, but also for intervening actively in the struggle to change the system.

Conclusion

The theory of practice outlined here offers an alternative both to an ahistorical role theory, with its explanatory preference for norms, choice and attitude, and to a categorical theory, which acknowledges power but leaves out 'practical politics' and the concern with difference within categories. A theory of practice is eager to explore those very differences, to understand how people 'invent' their lives - gendered or mathematical - within the constraints of specific, historical, social structures. It is keen to tease out the implication that different constraints and interests could result in different organisations of practice. Like the method of memory-work that I used in my study of practice, it fosters possibilities for action and change.

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Researching 'Hard to Reach' Groups: Some Methodological Issues.

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The aim of this paper is to discuss some of the methodological issues which have arisen in the context of two research projects focussing on the education of Somali children in London. The projects are concerned with the links between children's learning of numeracy at home and at school. In both projects the research is designed to collect data from parents, children and the schools. One focussed on children in Reception year, the other on children in Y7. Many of the Somali immigrants in the U.K. have entered the country relatively recently and many of the parents are only just beginning to learn English. For these reasons the group who form the focus of the research can be considered to be a 'hard to reach' group (Toomey, 1996). Tomlinson (1993) takes issue with this term, seeing it a euphemism for those working class groups regarded as problems, but recognises that teachers

> 'have difficulty informing themselves about the lives and backgrounds of ethnic minority parents and continue to resort to stereotyped beliefs about ethnic minority homes.'

<u>Views of the parents' role.</u>
The role of parents in their children's education has long been recognised as a significant factor in educational success and school improvement (Epstein, 1996, Safran, 1996). In recent years we have reached the stage where certain educational organisations and international conferences concentrate almost entirely on the issue of partnership between schools and parents (E.g., Parents in Education Research Network, European Research Network about Parents in Education. Education is Partnership Conference, Copenhagen, November 1996). Within this area of interest lies a vast spread of concerns and purposes.

Parents clearly have rights in terms of their children's education. Hughes (1994) provides interesting insight into the notion of 'parents as consumers', a view promoted in the U.K. under the conservative government and continued under the present government. Within this view is the associated idea of education as a commodity. Parents are given the right to 'shop' for their children's education in the school of their choice. Vincent and Tomlinson (1997) suggest that this view of parent power, together with the notion of schools' partnership with parents is little more than rhetoric. In reality there is little opportunity for parents to exercise an individual or collective 'voice' which will have an effect on the children's school experience.

An alternative view of parents is that they can be seen as a 'problem' for teachers. In particular children are frequently judged to come from 'poor backgrounds', from a home environment which is unsupportive to the school and unsupportive of the educational process. e .g. Tizard and Hughes, (1984), p18. refer to,

> 'a widespread belief amongst educationalists that working class parents do not stimulate their children adequately and in particular do not develop their language.'

Such views of parents were radically challenged by researchers such as Tizard and Hughes (1984) and Walkerdine, (1988) who recognised the substantial contribution made to their children's pre-school education by parents from working class as well as middle class backgrounds.

However, these views persist and very recently David Reynolds, in a discussion about the introduction of a national campaign for family numeracy was attributed with the remarkable statement that,

> 'At present most children get no support at home. … Many parents would feel able to help with their children's reading, but it's clear from everything we know that parents find it much harder to engage with numeracy or maths.
> At home there is almost zero provision. If we were able to generate some provision, some parental skills where at the moment there are low levels, the family background would have an enormously increased effect.' (T.E.S, 14.11.97)

Many projects have sought to develop work with parents starting from this 'deficit' view of the family background and set out to educate the family in order to help to educate the child. Even where the deficit model is less apparent projects are designed to suggest activities which develop school methods at home (e.g. Merttens and Vass, 1990, National Numeracy Project, 1997). However, as argued elsewhere (Jones, 1996) this appears to offer the parents the chance to participate in the culture of the school, but offers no opportunity for them to recognise the contribution of their own knowledge and social background to their children's education. More recently projects have developed in which there is a more equal notion of partnership developed between the school and the community and in which the richness of the home environment is recognised. (E.g. Bouchard, 1996, Civil, 1996, Macbeath, 1996) Findings from the first Somali project mentioned above suggest that these children were indeed receiving support from the family at home and that the oral tradition was particularly helpful in helping children to acquire numeracy. (Jones & Farah, 1995)

It was thus with a view of parents as contributors to their children's education that the projects were designed. We knew that parents and older siblings took an active part in developing young children's number knowledge, but that the parents of young children had expressed concern about their ability to support children as they progressed through KS2 and into KS3.

<u>Somali children in the U.K.</u>
However, reaching parents of particular groups continues to be problematic. In particular it can be problematic and expensive to work with ethnic groups in England, where English is not spoken fluently in the home. It is acknowledged amongst researchers into family involvement that there are groups who are regarded as 'hard to reach'. It was with such groups that the present studies were conducted. Many London Boroughs work with

children who come from ethnically diverse backgrounds. It is not unusual to find children from 20 different ethnic groups in one school, some of whom will have been in England for only part of their life and whose language skills are still developing.

There is clear evidence that children in the U.K. are underachieving in major aspects of mathematics in comparison with their counterparts in other countries (Reynolds & Farrell, 1996) and that within this pattern is a very mixed experience for children of different ethnic backgrounds (Gillborn & Gipps,1996, Smith and Tomlinson, 1989). It is difficult to locate precise information about specific ethnic groups, but information gathered through the first project (Jones and Farah, 1995) suggested that the education of Somali children was a concern to their parents when they reached the age of entering secondary school. Evidence from this project suggested that the Somali culture, with its rich oral tradition, offered very useful learning experiences in the home environment, which could prove very helpful to children in their development in mathematics in school. The focus of the current project then became to identify factors which influenced these children's attainment as they progressed through the U.K. school system. In particular we wanted to identify significant home numeracy practices, so that the findings could be disseminated to teachers and used to inform their practice. The view of learning taken in the research is one which does not separate the psychological from the social aspects. (Cotton and Gates, 1996) The research was to focus on one particular cultural group and to attempt to track down significant cultural practices which supported or otherwise affected school experience. However, it was anticipated that it would be possible to extend the findings to other groups and find effective ways for parents to develop and continue their own cultural practices in a way which would support the children's work in school.

The research methodology was designed as an ethnological case study, originally intending to focus on children in year 6, but eventually using a small group of children in Year 7 in two single sex schools. Data was collected by participant observation within mathematics classes, informal interviews with children and teachers at school and semi-structured interviews at home with parents and children. The appointment of a research assistant fluent in the Somali language was considered an essential prerequisite to the success of the project.

Methodological issues.

Locating the target group

In both projects there were great difficulties in initially locating a target group of pupils. Comparatively large numbers of Somali pupils were identified in the two London Boroughs concerned. Staff in the education office or at the ESOL centre gave assurances that there were large numbers in particular schools. However, despite these reassurances, in both projects we had the experience of tracking down a school which sounded as if it contained a sizeable sample of Somali pupils, only to find that the families had recently moved and the numbers of Somali pupils in the school diminished. Somali families, it seems, retain their nomadic lifestyle on arrival in London. Eventually a decision had to be made which changed the target age from Y6 (final year in primary school) to Y7 where there was a sufficiently large group of pupils to make the research viable.

Making Contact with the Homes

In the first project we worked with children of Reception age. The research assistant was convinced that initial contact by letter would not be helpful. We did not know whether the adults in the family could speak or read English or Somali. We solved this problem by 'hanging around' the school gate or the classroom door and making direct contact with the parents or the older children who collected the infants. This personal contact worked well and guaranteed a warm welcome when we arrived at the homes at the appointed times.

In the second project we had the advantage that the research assistant was working within one of the schools and knew a number of the Somali families through his additional community work. It was clear was that here again personal contacts were the most effective and it was through this method that the first home visits were arranged. It was clearly an advantage to have prior contact with the community, and this prior knowledge of the families helped to build an ethnographic picture of the children in their home circumstances.

Gender and Culture

Neither of these projects could have taken effect without the services of an interpreter. In each project a research assistant was appointed and a major part of their role was to interpret and communicate with the families. In the first project the research assistant was a Somali woman who had previous research experience, but no previous experience within the English school education system. In the second project the research assistant was a Somali man who was working in a state school and in a community school, but had no previous research experience.

A new issue which emerged in the second project was the question of gender and culture in relation to the research staff. In the previous project one of the authors (female) had worked with the female research assistant and conducted interviews in the homes with the mothers of the families. These had generally been conducted when the fathers were not at home, but when the men were home they had remained in a different room. In Muslim society it is not appropriate for an adult male to spend time with an adult female outside the family. In the new project the engagement of a male research assistant led to very different arrangements and relationships within the research process, particularly in the home context. We were able to interview men at home, which had not previously been the case and the men were more fluent and confident in their spoken English. This meant that the research assistant did not need to translate and it was easier to maintain a flow of conversation during the interview. There were times in the interviews with the women in the homes when the male research assistant would represent the interviewee's views briefly, then give elaborate interpretations on his own behalf. In a society where gender roles are clearly defined and very different from each other the areas of expertise and interpretation of events seemed to be very different according to the gender of the parent. Each interview was conducted with only one parent from the family and in each case this meant there was a set of interactions from which the researchers were excluded and another set of interactions which we were permitted to enter.

In this kind of research the researchers are necessarily learning a great deal about the culture and background of the target group as they proceed. The researcher who acts as interpreter is also mediating your inculcation into the culture. Recognising your interest in all things connected with the culture, he/she enthusiastically 'fills you in' on many aspects which help you to build up the ethnographic picture which enables you to appreciate the central aspects of the research. For this reason we introduce the term 'Inside Researcher' to refer to the person who is working from inside the cultural framework. Their role becomes that of an enthusiastic tourist guide wanting you to love and appreciate their country and customs as much as they do. Where, then, in all of this, does reflexivity stand, as a research ideal? The 'Outside' researchers, in this case, white, English, middle class and female recognise (hopefully) the limitations of their educational and cultural background in the context of the present research. By the nature of the ethnographic design of the research their aim must be to enter into the cultural framework as far as possible. The Inside Researcher, who has been trained in research techniques and encouraged to allow the interviewees to speak for themselves, finds it hard to break free of his/ her cultural and gender role. The irony of the research methodology is that whereas the 'outside' researcher needs to get inside the culture and to be able to see the world through the eyes of the group being researched, the 'inside' researcher needs to break free of his/her cultural paradigm in order to be able to represent the views of the interviewees, particularly if they belong to a different sub-group.

Certain key issues emerge. Siraj Blatchford and Siraj Blatchford, (1997) refer to, 'Powerful social actors who are able to impose their meanings on others'. They suggest that 'researchers who are members of socially dominant groups will be unable to work effectively with certain respondents' and that 'In some projects it will be desirable to recruit and train interviewers drawn from the respondent's own age, 'race' gender or ethnic group.' Our findings suggest that of these factors, gender is a major consideration when working with a culture in which there is clear gender differentiation within its structures. However, the very nature of social dominance means that it is from the group who are dominated that it is most difficult to recruit researchers. One result is that for a generation, 'hard to reach' groups remain hard to reach.

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"OCCUPATION OF OUR MINDS" : A DOMINANT FEATURE IN MATHEMATICS EDUCATION IN SOUTH AFRICA

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ABSTRACT

As South Africa moves forward with new curricular initiatives which are aimed at the elimination of many disparitie, questions about what needs to be done in order to adress the inequities to mathematics arising from the education system under the apartheid regime are also being asked. Such disparities certainly extend to mathematics. I believe questions about ***what*** is/has been taught in various subjects should be an important consideration. Reconstruction of the educational system must be in accordance with national policy that education should be non-sexist, non-racist and committed to equal access.

In this article, I make the point that the way South African society was organised during apartheid contributed a lot towards disadvantage in mathematics for Blacks. Redress in mathematics will require some serious commitment to ending 'occupation of our minds'. I begin by providing a brief description of the notion: 'Occupation of our minds' as it is explained by Munir Fasheh (1996). I then make some attempts to understand 'occupation' by also providing examples of some places elsewhere in the world which have made some means toward ending
`occupation.´ I end by raising the question whether current reforms in mathematics teaching are adequately addressing the problem of occupation in South Africa. I also offer some suggestions as to how 'occupation of our minds' can be eliminated.

1. WHAT DO WE MEAN: *"Occupation of our minds?"*

I first came across the phrase : "Occupation of our minds" from an article by Fasheh (1996) entitled *The main challenge : ending the occupation of our mind, the main means: building learning environments and recontextualising knowledge*. He also indirectly refers to the same concern of 'occupation' when he raises the question : *Is mathematics in the classroom neutral or dead?* Fasheh (1997:24). I am using this idea of 'occupation' in this paper for two reasons : Firstly, I find it both an interesting and fascinating notion and secondly, I have found it to be relevant in providing me with answers to some of the question I'm currently grapling with in my research of *The history of mathematics education in South Africa : 1948-1994*[1].

[1]I am currently participating in a collaborative Ph.D. project between Aalborg

An attempt to understand the context in which Fashesh uses the notion: "Occupation of our minds" would seem to require two thing, viz:

i. The knowledge of who Munir Fasheh is and also
ii. An understanding of the Israeli-Palestine conflict and the effect it is having on their education.

I cannot, however, claim for a moment to be an expert in answering the question of who Munir is or to be able to provide a detailed explanation about an Israeli-Palestine conflict. Perhaps, the best I can do is to say what Munir has written about himself in the past.

In introducing himself, for instance, Munir Fasheh writes:

> *I was born in Jerusalem, Palestine. Except for a few years when I had to go out and study, I have lived all my life in Palestine. When I was born, the British were occupying Palestine and thus the British system of Education was used. After 1948, the Jordanian syllabus with Israeli intervention became the curriculum in schools. Since 1993 a so called " Palestine syllabus"is being developed* (Fasheh, 1997:24) .

Below is another glimpse of Munir:

> *When the 1967 Israeli-Arab war broke out, I was 26 years old, already with a masters degree and four years experience of teaching math at various levels ... I was formally involved for six years (1972 - 1978) in math instruction at several levels and in different ways in the schools of the West Bank.*(Fasheh, 1997:57) .

Fasheh's writings seem to suggests that he has spent some considerable time working for Birzert University and a small learning institution, Tamer Institute in Palestine. The region had been under British rule since 1948, Jordanian rule from 1967, and Israel occupation from 1972. In writing about *ending the occupation...*, Fasheh acknowledges the role Palestinians have always played in waging a war against the occupation of their land and resources. He is, however, critical of the Palestine curriculum which he claims has been *meaningless and not built on aspects and issues of the Palestinians reality...*(Fasheh, 1997:27). He is even more critical of the

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Univeristy (Denmark) and University of Natal (Durban). My study is on the history of mathematics education in South Africa.

insensitive and unresponsive nature of the math curriculum and he then concludes that : *the maths we teach and study, at least in the schools and universities in Palestine is basically like a corpse that doesn't feel anything of its surroundings* (ibid : 24). He believes that although the "occupation" of their land is an extremely serious issue,

> *The biggest danger Palestinians currently face is the confiscation of our last possessions as people: our history, our voice, our experience, our vision, our hopes, our unity, our sense of belonging, our rights, our ability to learn and create and our means of survival* (Fasheh, 1996:14) .

He raises questions, therefore, about the silence of mathematics during the time when changes that have accompanied the various realities in Palestine were so drastic. He therefore sees the struggle for ending the "occupation of our minds" as important because*:The most potent weapon in the hands of the oppressor is the mind of the oppressed (*ibid. p.14).

What most characterises the "occupation" according Fasheh, are:

> *First, the belief that Western cultures are superior to all others, and that the path followed by Western nations was the only path to be followed by others; hence the belief that knowledge and solutions can only come from the West via experts, plans, etc.*(ibid p. 14).
>
> *Second, preventing our voices, histories and ways of living, thinking and interacting with one another and with nature from surviving and flourishing.*

Fasheh's use of "occupation..." emanates from the fact that Palestinians have been living under Israeli occupation of their land and resources for a long time. He is concerned, though, that the struggle waged by the Palestinians has focussed largely on the occupation of their land and not enough was done in resisting "occupation" of their minds. He mentioned, for instance, that:

> *Palestinians like most other peoples in the Third World have been critical of almost everything related to the Western domination except science, math technology and research. We have considered them an ideal to be in critically imitated and followed* (ibid p. 15).

In this paper, I am attempting to look at what "occupation" means in mathematics in general and also how "occupation" is reflected in the history of mathematics education in South Africa. My particular attraction to the notion of "occupation" lies in the fact that our minds have been controlled in South Africa by limiting the options and alternatives in how mathematics was taught and learned in the past. I am aware of this fact because of my past experience as both a student and teacher of mathematics in South African schools. We have been blinded so that we are unable to see the alternatives in both our teaching and learning of mathematics. The nature of mathematics teaching has been such that students are not encouraged to realize that there are different points of view and to respect the right of every individual to choose his/her own point of view.

A question can be asked with regards to mathematics education, namely: of what relevancy is the Fasheh's notion of the 'occupation of our minds' to the way mathematics was taught and learned in South Africa? Perhaps a second question can be: How widely can the same notion of the "occupation of our minds" be applied to other situations elsewhere in the world?

The third question is : if the battle for ending the "occupation" is won, who is the beneficiary? The last (but not least) question is: can we have a system free from occupation of some sort? If yes, what would the problems be? I will make attempt to deal with all these questions I have raised.

2. SOME ATTEMPT TO UNDERSTAND "OCCUPATION"

A number of questions has been raised above. I need to point out that there doesn't seem to be direct answers to all these questions. They were raised in order to make an attempt to come closer to the understanding of the notion of "occupation". I begin with the first question:

My own study of the history of mathematics education in South Africa during the apartheid years has convinced me that the curriculum was largely a technique curriculum. I see the important function of a techniques curriculum as having to do with providing the teacher with the 'tool box' full of techniques which are drilled in the minds of students and which they have to master in order to be successful in their learning. Part of my data for this study was gathered through interviews with individuals who I believe were prominent and successful in mathematics teaching during the apartheid years. Opinions (from the interviews) were often expressed that the mathematics curriculum for Black children, for instance, should be preparing them in techniques that they will need in order to take over the key positions within their community and also techniques which will take the traditional cultural background of the Bantu child into account [see for example, Wilkinson's study (1981) and Groenewald study (1976)]. Teachers in turn became the victims of a system which assigned to them the task of implementing a ready made curriculum and of testing students on techniques. This problem is highlighted in Julies' (1991/1992 : 4-5) detailed account of *the production of the South African school mathematics curriculum*:

> *Mathematics teachers thus receive a syllabus describing goals and aims, some comment on methodological aspects, the content to be covered per year and the evaluation procedure to be followed. The intended curriculum reaches teacher after it has been designed elsewhere. Who designed the curriculum, the process that is being followed and the underlying motivations for the curriculum are unknown to those who must implement it.*

The control factor seems to be very strong in the above quote by Julie. I am arguing here that there was no space available in allowing people to "see the alternatives" and in turn, their minds were highly controlled and thereby "occupied". This top down production of a technique curriculum (illustrated by the quote above) was in contrast with the current progressive thinking which later developed amongst the community of mathematics educators. For example, Bishops in **Mathematical Enculturation** (1988) is critical of a technique curriculum for it:

> *... cannot help understanding, cannot develop meaning, cannot enable the learners to develop a critical stance either inside or outside mathematics... a technique curriculum cannot therefore educate... For the successful child it is at best training, for the unsuccessful child it is a disaster" (Bishop, 1988 8-9).*

In South Africa, like in Palestine (also true in many places especially the Third World Countries), we have fallen victims of a widely accepted conception of mathematics as a science that is always true and cannot make mistakes. We have considered mathematics to be neutral subject that should be followed blindly and uncritically.This view of mathematics - as a perfect system, as pure, as an infallible tool if well used - contributes to political control (Borba M.C. and O. Skovsmose, 1997:17). This view was in accordance with apartheid conception of education. Viljoen in Beard and Morrow (1981:109) for instance, says that :

> *Education is a science. It revolves around definition, substantion, logical reasoning, experimentation etc ....*

It comes as no surprise, therefore, that whilst subjects such as history were often criticized for being biased in South Africa, mathematics remained untouched for many years. Although profound changes were made to subjects such as history, geography and languages to rid them of their primarily British character, during the early 80's mathematics was kept intact. It was only in the mid-80's that this ascertion was challenged when new curriculum initiatives were proposed and the role of mathematics within Peoples Education for Peoples Power was investigated (Breen, 1986, for instance). Taylor et al (1986) mentioned, for instance that:

> *Mathematics not in itself but in the way it is constituted, taught and applied... contributes specifically to cultural, class and gender discrimination and to the authoritarian technocracy which dominates all aspects of life in South Africa.*

The above quote indicates that the way South African society was organised during apartheid contributed a lot towards disadvantage in mathematics for Blacks . The research by Broekman et.al. (1994), for instance, which aimed to *investigate to what extent gender differences in access and progression are underpinned by regional, departmental, economic and social differences* (p 4) was able to show how the patriarchal and race relations underpinning the apartheid system favoured white boys, providing them with a strategy for obtaining a qualification in mathematics.

Also typical of the technique curriculum is its restrictive approach which denies people the opportunity to see alternatives which I regard as the opposite of “occupation”. Wilkinson (1981) in her study of the problems experienced by pupils in mathematics of standard 5 level mentions as one of her recommendations, that :

> *...the Department of Education should provide each classroom with a day-to-day teachers guide (preferably linked to a textbook)* (p. 123).
>
> She goes on further to suggest that *:*
>
> *... this text book should be accompanied by a day-to-day teachers' guide which provides every possible guidance regarding the teaching method, setting and marking of tests and examination papers, revision and enrichment work, etc...* (ibid. 1981: 125).

The lack (or even absence) of flexibility allowed to teachers is clear from Wilkinson’s suggestion above. Any creative work with the learners is denied, thereby denying the opportunity for both the teachers and students to "see alternatives" in their work. Perhaps, some people can find some justification in Wilkinson's suggestion if it is taken into consideration that South Africa has always lacked teachers who are qualified in mathematics and science (see for example, a Report for the Department of Education and Training and the Department of Arts, Culture, Science and Technology, 1997). My uneasiness, however, with the above suggestion stems from my belief that the situation cannot be improved by further disempowering teachers by having a system that encourages passiveness, rote learning, obedience to authority and discourages intellectual risk taking, curiosity or independence of thought.

The fact of the matter is that the above suggestion was strictly adhered to by the Department of Education (I am, however, not suggesting that this came about as a direct result of Wilkinson's suggestion). I am convinced, though, that this tendency (as reflected in Wilkinson's suggestion) in turn severely limited the options and alternatives in peoples' minds. Let us consider the issue of textbooks : That teachers and learners should rely heavily on a mathematics textbook is problematic when one examines the nature of mathematics textbooks we have had in South Africa during the apartheid rule. Most of them resemble the description given by Volmink (1994 : 61) :

> *Most school textbooks are written in a style which emphasises drill and practice or routine exercises. At the end of these exercises, some space is given for problems for which a standard recipe for obtaining the answer cannot be used. These problems are generally decontextualised. At best, they are applications of a previously learnt principle or concept. So application problems in school textbooks are used mainly to provide exercises in or to illustrate one or other mathematical technique...*

Now, I have made an attempt to partly answer the question I raised earlier which is largely about the relevance of Fasheh's notion of "occupation" to the way mathematics was learned and taught in South Africa. I now turn to the second question namely: How widely can the notion of "occupation" be applicable to other situtions elsewhere in the world?

3. **"MATHEMATICS BOTH WAYS" : AN EXAMPLE OF A MEANS TO END "OCCUPATION OF OUR MINDS"**.

I believe that there are othe examples of countries which have made attempts to end "occupation"[1]. Stanton's (1994) Australian example of 'Mathematics Both Ways : A Mathematics Curriculum for Aboriginal Teacher Education Students" is one such example.

The notion of a "both ways" education is described by Stanton (1994:15) as *an education that recognises the validity of the knowledge bases from the Western and Aboriginal traditions*. This contrasts with the curriculum of white schools in which the focus is on "one way" "Western traditions". In an attempt to decolonise their schools, the Remote Area Teacher Education (i.e. Rate) "both ways" pedagogy place emphasis on problem posing/problem-solving approaches to learning, curriculum negotiation and integrated curriculum planning supported by appropriate assessment strategies including criterion referencing descriptive reporting and non-completive assessment. It is community - based and community focused and aims to have a role in developing its students' skills in the defence, maintenance and further development of Aboriginal culture.

In mathematics, "both ways" education does not see mathematics as the mastery of classroom techniques which have no real-world relevance, and are 'owned' by the dominant culture. The most obvious way in which this mathematics curriculum looks different from the 'standard' techniques oriented curriculum overview is that it does not consist of a list of techniques, sequenced in terms of an arbitrary hierarchical structure. Instead the techniques may be found subsumed under the notion of six component symbolic "activities" that Bishop (1988) proposes are found across all cultures (ibid. p.19). These symbolic activities are 1) counting, 2) locating, 3) measuring, 4) designing, 5) planning and 6) explaining. Stage one of the 'both ways' course followed at Batchelor College Teacher Education, for instance, has as its central theme the social contexts of mathematics learning. The focus is on "What is this thing called mathematics?" and "How is mathematics used in my community" (Santon, 1994:19). Other curriculum activities centre around:

> *... ways which demystify and make mathematics accessible to the Aboriginal teacher and child alike, ways which allow the Aboriginal community to co-opt mathematics, its symbolic technology and machines ... (ibid. p. 19).*

I cannot describe everything about the mathematics curriculum at the Batchelor

[1]In my M.Ed dissertation I considered some countries which have made attempts to put mathematics in its social and political context.

College Teacher Education but it can be said (from my description above) that an attempt to go some way towards ending "occupation" is realised in this example. Through the "both ways" approach opportunities are created for students to "see the alternatives". The mathematics curriculum students use is not imposed from above but is negotiated between lecturer and student. Most importantly they are able to deal with Western mathematics which is described by Bishop as *one of the most powerful weapons in the imposition of Western culture* (Bishop quoted in Stanton 1994 :5). They, for example, engage in activities which provide focus on issues for community research that have implication in the development of Aboriginal mathematics pedagogy (ibid :20).

I need to point out here that my choice of examples I have used in this paper (i.e. South Africa, Palestine and Aboriginal Australians) could be problematic in that the reader may believe that only the marginalised, the oppressed and the powerless are "occupied". The problem could be that during "occupation", the "occupier" and the "occupied" may not be aware of it and also the often privileged position of the "occupier" may further contribute towards this negligence. In South Africa, for example, many White people did not believe there was anything wrong with the education of their children. This was largely due to their privileged position and the fact that their education was well resourced compared with the education of Black children. This was, however, a big mistake. Wedekind, Lubisi, et. al. (1996), citing Sprocas, mention that:

> *The White education system has been severely criticized for its authoritarian, teacher centred teaching and management approach and the overly academic and uncontested view of knowledge that is presented to the pupils. This was often coupled with overt programmes of indoctrination which attempted to justify apartheid and prepare the 'white' youth for their role in defending and maintaining the system (Pretoria :TED : 432).*

There seems to be strong element which characterize "occupation" in the above quote such as "authoritarianism", "indoctrination" "teacher-centred teaching", "uncontested view of knowledge" etc. There is clearly no "seeing of alternatives" under such circumstances. So "occupation of the mind" is a condition which attacks both the advantaged and disadvantaged.

The majority of those teachers in the White Education system had also come out of an education system which was based on Fundamental Pedagogics (FP), an authoritarian philosophy associated with Christian National Education and apartheid (Enslin, 1990).

Perhaps the most serious charge that can be laid at the door of FP which dominated apartheid education, is that it discouraged the very qualities regarded as essential for sustainable development and success as new millennium approaches : risk taking, a sense of adventure, curiosity, a critical and unquestioning attitude, self motivation and reflection, inventiveness and independence of mind; in a phrase : creativity and innovation (Arnott and Kubeka 1997:6).

Instead, South Africa had a system of education that encouraged passiveness, rote learning, obedience to authority and discouraged intellectual risk taking, curiosity or independence of thought. This was in accordance with a view held by the proponents of FP, which viewed education as a science based on definitions, logical reasoning, experimentation etc.

The proponents of FP also see the role of the teacher as authoritarian. Gunter (1974 : 144) notes, for instance, that in FP:

> *... the educator is invested with authority and as such he has the right to prescribe to the educand what he must do, and how or what he must not do, while the educand has to respond to his being addressed by the educator by accepting what he says*.

The point I am making here is that Fundamental Pedagogics was a powerful weapon used by apartheid ideologues to "occupy" not only the minds of Blacks but also Whites. So, during "occupation" both the "occupier" and the "occupied" could be victims of "occupation" in a number of ways.

6. ENDING OCCUPATION : WHO BENEFITS?

The question I raised earlier in this paper, namely: who benefits if an attempt is made to end "occupation?" would seem to raise other questions : For example, can the mind be "un-occupied?" At what stage (of ending "occupation") can such a claim be made? I cannot claim to have answers for these questions. I, however, believe that an attempt to free people from "occupation" is both a worthwhile and a just course and is also to the benefit of everybody. The answer to the question :who benefits is therefore (naively) : everybody! If we are concerned about the education of our children we need to think seriously about acquiring the means to end “occupation”. This should begin with the realisation that a threat posed by “occupation” always exists for both the victim and the perpetrator. An important consideration should be to allow people to “see the alternatives” and to re-contextualize knowledge. This is an important message for South Africa.

There is no doubt about the extent of damage caused by many years of apartheid in our education in South Africa. The interviews I conducted during different stages of my research taught me one thing, and that is : the dominant feature of mathematics teaching in South Africa (1948 - 1994), has been the “occupation of our minds” (see also the paper I presented at Amesa Conference July 1997). This was demonstrated by a number of comments mentioned by my interviewees. Some of the comments are the following:

- *... mathematics was just purely academic, I do not remember during my days if teachers ever did anything ... We were brought up in a highly academic atmosphere. In fact, it was heavily British oriented...*

The person who commented above is a Black South African teacher. He is a teacher who has been extremeley successful in establishing a name for himself in the field of mathematics both inside and outside South Africa since 1946 (when he began his teaching career) to date. He is dubbed a mathematics wizard and is still involved in the teaching of mathematics both inside classroom even thoug he is formally retired now.

- *... as a matter of fact, right up to the degree level mathematics to me was nothing else but something to be memorized ... It was a matter of rote learning to a point that before exams in matric, one would memorize specific sums with a hope that they would come up in the examination paper!*

The person who also made the above comment is currently a senior mathematics subject advisor for KwaZulu-Natal Province. For many years he was the only mathematics subject advisor for the entire Natal Province under the former KwaZulu Department of Education. Throughout his teaching career he has been concerned with the teaching of mathematics.

The comment below was made by the same person as in the first comment.

- *... when I was learning my maths, ... our mathematics teacher used to make us recite the theorems. We did not even understand them. It was quite common for the teacher to say: Will you enunciate theorem 49? He would not actually lead you to the theorem. We knew the theorems according to numbers. If you had caught me in standard nine or eight and made me to prove the theorem and you put it up side down, I would not be able to prove the theorem and you put it up side down, I would not be able to prove it. I knew the theorems as letters were, that is, A, B, C and if you put X, Y, Z. I would not be able to prove the theorem. It was just like that and we passed with high marks!*

The above quotes are examples which demonstrate some strong presence of "occupation" in how maths has been taught and learned in South African schools.

Can we have a system free from "occupation?" Perhaps it is possible to have a school system which is free from "occupation of the mind". It would be a system which respected the voices of all pupils, all groups in society, all sides of every power relation, etc. Now, how is this done in a centralised system such as we currently have in South Africa? If it is achieved, do we not risk introducing a perspective, where each group is taught from within its culture. This maintains status quo, at best, at worst contributes to the widening of the power gap. Thus education must allow for a multitude of voices including critique of these voices and of this perspective which promotes a certain culture, a certain set of values, where all experiences and all knowledge are always open to critique.

9. A CLOSER EXAMINATION OF THE NOTION : "OCCUPATION" IN BOTH THE SOUTH AFRICAN AND INTERNATIONAL CONTEXT

Fasheh's idea of the "occupation" can also be linked to the work and writings of Paulo Freire in the late 1950's and 1960's in Brazil. Freire was concerned with the development of radical pedagogy which was able to contribute to progressive social change. He was convinced and believed that in order to achieve freedom, people must actively wage a war and struggle against those stereotypes made of them by their oppressors.

He also criticized traditional narrative forms of education as oppressive and likened them to a system of 'banking'. He says that education which follows this mode :

> *...becomes an act of depositing in which the students are the depositories and the teacher is the depositor. Instead of communication the teacher issues communiques and 'makes deposits' which the students patiently receive, memorise and repeat. This is the 'banking' concepts of education in which the scope of action allowed the students extends only as far as receiving, filing and storing deposits.* (Freire, 1985 : 53).

The banking concept of education encourages the form of teaching which is one-way dependence of the student upon the teacher. The memorising and regurgitation of facts are important characteristics of banking education. Freire argues that such a process is anti-dialogical and therefore anti-educational on the grounds that 'dependency' presents a contradiction and an obstacle to 'authentic free thinking and real consciousness." (Freire, 1985, p.85).

Freire's critique of banking education can therefore be seen as an attack on all forms of "occupation".

In a situation where students are 'depositories' and the teacher is the 'depositor' there is a lack of diversity and standardized thinking is encouraged. There is definitely no 'seeing of alternatives'. 'Occupation' is therefore, reinforced. Fasheh, points out that the key idea to ending occupation involves, inter alia:

> *"The need to stress in schools the means that help children learn much more than stressing a ready content put forward by experts who have lost their integrity and their senses* (Fasheh, 1996 : 21).

I have already pointed out in this paper that the system of education in South Africa encouraged passiveness, memorization and rote learning and discouraged curiosity and critical thinking. This was made evident by the persons I interviewed when I conducted a study of the history of mathematics in South Africa during the years

1996 and 1997. These interviews (see some statements given earlier in this paper) demonstrated to me the extent to which learners were denied the opportunity to do creative work. For Freire this could be described as mere manipulation, rather than education.

10. ARE CURRENT REFORMS IN MATHEMATICS TEACHING ADEQUATELY ADDRESSING THE PROBLEM OF OCCUPATION IN SOUTH AFRICA?

I believe that one of the important challenges facing all those who are involved in mathematics education (and education generally) in South Africa today, is to begin to explore ways and means of ending the "occupation of our minds" brought about by years of apartheid education. The following extract from the White Paper on Education appropriately sums up what our attitude should be towards the reconstruction of South African Education:

> *"It is time to declare that a new era has dawned... The efforts of all South Africans will be needed to reconstruct and develop the national education and training system so that it is able to meet the personal and social needs and economic challenges that confront us as we build our democratic nation. The Ministry of Education invites the goodwill and active participation of parents, teachers and other educators, in bringing about the transformation we all seek" (Chapter 3 : 20).*

This is indeed a strong plea by our National Minister of Education. The challenges facing curriculum developers in South Africa are particularly daunting, given the heritage of passivity, prejudice, ignorance and resistance. Fasheh has an important message for those who are committed to ending the "occupation" :

> *... ending the occupation of our minds is a personal task, its continuation depends solely on our acceptance of it. So is its termination. Since ending it is crucial for ending other*

forms of occupation, and for building our future it is a main challenge... (Fasheh , 1996 : 25-26).

In an effort to redress the educational imbalances and inequalities of the past in South Africa, the Department of Education has introduced (since the beginning of this year 1998) a new outcomes-based curriculum (OBE) also known as Curriculum 2005. This is seen as a major paradigm shift in education, a shift from learning and teaching which focused primarily on content to learning and teaching focused on outcomes. The development and maintenance of a national, outcomes-based education is aimed at :

- creating opportunities for all South African to become *life learners;*
- removing artificial boundaries between education and training by *integrating* theoretical and practical learning and teaching etc. (Malan, 1997 :3).

As South Africa moves forward with new curricular initiatives which are aimed at the elimination of many of the disparities of the past, I believe the education directed at ways and means of ending our occupation should be an important consideration. I also believe OBE is an exciting challenge, one which presents itself as a good platform from which to tackle the problem of "occupation". I see OBE as a revolutionary good move which attempts to address some of the concerns I have raised in this paper. One positive example is that the OBE curriculum makes it obligatory for mathematics teachers to teach the history of mathematics (which could go some way in addressing the problem of the 'occupation'). Out of the ten specific outcomes for the learning area: *Mathematical Literacy, Mathematics and Mathematical Science* specific outcome 3 states that children *should be able to demonstrate understanding of historical development of mathematics in various social and cultural contexts.*

I am citing specific Outcome 3 in particular, because I believe if we are serious (as mathematics educators) about improving the quality of our teaching and making mathematics a living subject, we must include its history. One explanation of the fact that so many people - particularly children and young people in schools and college - find mathematics dull, boring, uninteresting, even hateful, could be that they were taught - or are now being taught - mathematics without its history, that is mathematics as if it were dead (Heide 1996). In a country such as South Africa, where our history has been distorted for years, I believe the incorporation of the history of mathematics could be an important step in an attempt aimed at addressing the problem of occupation. In South Africa we are advantaged by having at our disposal a number of examples that are a result of our rich history. Ending 'occupation' in the case of mathematics can also include contextualizing its teaching. This is encouraged in OBE. Other Specific Outcomes (SO) such SO4 and SO5 would also favour an approach that would help the learners be creative and analytic. SO4 and S06 respectively state:

> *Critically analyse how mathematical relationship are used in social, political and economic relations* (SO4).
>
> *Use data from various contexts to make informed judgements* (SO6).

My only fear though is that the kind of progressiveness and some of the good intentions demonstrated in the above cited "Specific Outcomes" may not be actualised by most teachers at the classroom level. I see this resulting from the unfortunate lack of debate prior to the implementation of OBE and the fact that it came as a surprise to many people (including teachers) who were/are directly involved in the education system. What we are witnessing instead is that the current reform process is being led and left to "experts" who are perceived to be in a position to "teach" teachers about OBE. There is a danger that if OBE is left only to university academics and other experts, for instance - it may not be suited to teachers' needs. Despite my criticism of the way OBE is being introduced in South Africa, I do believe, however, that OBE does incorporate some elements which can be used as some attempts to end "occupation".

10. CONCLUSION

In conclusion, I believe that Fasheh's notion of "occupation of our minds" is not only relevant to our past history and present but I also think that it has an important message for our attempts to transform mathematics education in South Africa for the future (and for the better). I also believe any meaningful curricular reform requires that we start by:

> *... shaking off the dirt that has been accumulated in our minds,*

mainly through formal education (including science) television, and other killers of cultural diversity and of human societies... (Fasheh, 1996 : 19-20).

In South Africa we need to think carefully about how we are going to be "shaking off the dirt that has been accumulated in our minds" and which has been left by long years of Fundamental Pedagogics which dominated apartheid education. We can do this in more than one way : We can for instance, do this by incorporating some ideas of "planting the seeds" mentioned by Fasheh (1996:23) and also incorporating some elements of the "Rate Both Ways : curriculum for Aboriginal Teacher Education Students" model (see Stanton; referred to earlier in this paper). The kind of 'seeds' Fasheh mentions are, for instance, the following:

- Recognizing and supporting teachers who use innovative ways of teaching mathematics
- Recognizing and supporting students who are involved in some interesting problems
- Integrating the teaching of art and mathematics
- Integrating the teaching of language and mathematics
- Producing materials that could improve the learning process in the traditional classroom
- Integrating the teaching of mathematics with art, language and nature
- A 'contest' encouraging teachers (as individual or groups) to write very inspiring and relevant questions that relate mathematics to local conditions etc.

We should however consider what is suited and relevant for our needs. Most importantly, we should guard against importing. For our curriculum reforms to have meaning it should be based on aspects and issues of the

South Africa reality. One disturbing reality, for instance, is that although most mathematics teachers have professional qualifications, less than half have accredited training in the subject they are teaching and fewer than 50% of mathematics teachers have at least one of specialised training in mathematics (Arnott and Kubeka, 1977:1). We need also to take seriously the following points raised in the Report (1997 : 6-7) for the Department of Education and Training, and the Department of Arts, Culture, Science and Technology .

Lip service to decontextualising science education (and mathematics education) is no longer enough. It must take local needs into account. It must be relevant, while at the same time sensitising learners and teachers to the practical benefits that can accrue to the community, which benefits cannot simply be regarded in terms of immediate material rewards ...

Although a well trained workforce, thoroughly grounded in mathematics and science is essential for a competitive edge in the age of the globalized economy and the information technology revolution, it is not enough. Workers need to be able to think independently and creatively and have the courage to act on their understanding of the challenges that confront them. They need to be enterprising and innovative if they cycle of intellectual and material poverty is broken ... .

Interactive mediated learning and teaching styles need to be developed in colleges of education, technikons and universities which take into account the multi-cultural and multi-lingual composition of the population, so that these institutions can fulfil their function as role models for the wider teaching community. Such approaches are particularly important for the teaching of mathematics and physical science where curiosity and intellectual risk taking are essential ... pupil centred, mediated and

> *interactive strategies are vital to the demystification of mathematics and physical science which have developed an aura of unintelligibility and remoteness from the daily lives of ordinary people.*

The above quoted statements from the Report and which are aimed at developing students who are able to “see the alternatives” demonstrate the extent to which the quest for ending occupation should also be a concern for South Africa.

Last (but not least) : We should be careful that out endeavours to end occupation (through the introduction of Outcomes-Based Education, for instance) are carefully considered. The greatest mistake we can commit is to replace apartheid type of “occupation” by other forms such as having a curriculum that is top-down and expert driven. Importing a curriculum with disregard of existing realities to our situation would not only result in a waste to our limited resources but would also be disastrous for the new South Africa.

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Discourse as a Problem Solving Strategy

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Abstract

Drawing on research with students in a summer program for disadvantaged youth, this paper illustrates that students bring a variety of resources beyond mathematical expertise to the problem solving process and may be engaged in tasks quite different from and with different purposes from those intended by the researcher or teacher. In particular, the paper describes several routines jointly produced by Sam and the researcher that mimic traditional mathematics classroom routines, but which are not particularly productive to either solving mathematics problems or gaining insight into Sam's mathematical understanding. Failure to take into account these kinds of interactions between "expert" and student during teaching or research can lead not only to invalid judgments about what is occurring, but to misappropriation of responsibility for failures.

Introduction

Drawing on both social interactionism and radical constructivism, Bauersfeld (1995) suggests an orientation to analyzing the processes of communicating and their impact on personal development in the mathematics classroom. He argues that language cannot be thought of as a medium to be used or as an object. He maintains that a speaker's utterances can function for the listener only as directing the focus of attention, whereas the construction of what might be meant, that is, the construction of references, is with the listener. The speaker's utterances and intentions then can have no direct access into the listener's system. What the listener's senses receive, undergoes spontaneous interpretation. Bauersfeld further claims that such interpretations have emerged from many social interactions, encounters through which the person has tried to adapt to the culture by developing viable reactions and trying to act successfully. Bauersfeld describes how communication takes place:

> *The focus of attention, our "looking at," cuts something out of the perceived diffuse continuum and makes an object of it. This process does not necessarily result in the same consequences with all persons, because it is a historical, situated, and individual process. Only across social interaction and permanent negotiations of meaning can "consensual domains" emerge, so that these "learned orienting interactions" can lead an observer to the illusion of some transport of meaning or information among the members of the social group*
>
> *(p. 275)*

This position differs from that of traditional teaching, which is based on the assumption that the teacher teaches some objective version of mathematics, using language as a representing object and means. This latter view, which holds that language can exist on its own objectively and thus serve as a verbal transport of knowledge and direct learning and teaching, is in direct opposition to individual construction of meaning. In the face of such beliefs about teaching, what do students do? Bauersfeld (1995) claims that without simple transmission of meaning through language, the students often develop routines in which they learn to say what they are expected to say in certain defined situations. Students have learned, for example, certain routines that occur in mathematics classrooms that help them to get through the lessons successfully, sometimes without engaging with the mathematics.

This paper, using research interviews with students, gives evidence for the validity of Bauersfeld's arguments by exhibiting three such routines. One might argue that research interviews are not classrooms. They do, however, share the attribute that there is an "expert" in a power position who is directing the flow of the activity. The conversation between researcher and student is very similar to that of teacher and student in a traditional classroom, and in this case, the researcher was an experienced mathematics teacher.

The Context for Discussion

In a recent study, I sought to explore high school students' conceptions and representations of elementary function classes. Students were participants in a summer program for disadvantaged youth who were interested in science and mathematics. Data consisted of interviews of four focal students, pre- and posttests, written homework, and fieldnotes and videotapes of classroom observations. Students had five 45-60 minute interviews. Questions varied from simple tasks, such as translation of a linear function from tabular to symbolic form, to more complex tasks such as comparing two quadratic functions situated in an economic application context and critiquing the correctness of conjectured mathematics. As the interviews progressed, there were many instances where it appeared there was a mismatch between the orientation of students and the researcher. Both the classroom teacher and I, the researcher, were oriented by our views of the mathematics, setting tasks and looking for evidence of student understanding. Students, however, often seemed oriented less toward the mathematics per se, but more toward what it means to be in school and in the successful completion of the tasks. In particular, they evoked patterns of interaction with me that mimicked school routines, but could be described directly as problem solving strategies. The problem to be solved, however, was not simply mathematical, but rather, getting through the interview successfully.

Each routine involved questioning and answering. The strategies students used were: (1) responding to questions in a way that provoked a funneling pattern in the researcher's questioning (Bauersfeld, 1988), (2) determining how to proceed with a problem through questioning the researcher, and (3) gaining clues about how to proceed from the researcher's comments. All focal students participated in these kinds of interactions with the researcher, but one student, Sam, regularly used all three, so I use interviews with him to illustrate the points.

Sam is a boy who comes from an economically poor family in a small Midwestern city in the USA. At the time of data gathering, he had completed two years of algebra, receiving an A for the first year, and was reading above grade level. He had a strong sense of humor and reported a career interest in zoology. During an awards ceremony at end of the program, Sam was chosen by his teachers as outstanding student in his section in biology, physics, and anatomy. In class, Sam was often a lively participant, offering suggested solutions, asking questions, or perhaps commenting on difficulties with his calculator computations.

School Routines

Narrowing the Focus of the Questioning

One pattern in Sam's responses in interviews was that he often paid only lip service to my questions until they moved from being conceptual to requiring a highly specific piece of information. This interaction relied on a routine common in classrooms. In order to help a student who might be stuck to be successful and to arrive at the answer the teacher wants, the teacher continues to ask more and more specific questions or to give hints until a student succeeds in giving a "correct" response.

For example, in the last interview Sam was engaged in a card sorting task and had decided that the graph of a rule on card B, Profit = $-300p^2 + 3600p - 1500$, was a parabola, but when he graphed the function on his graphics calculator, he got a vertical line because the y-dimensions for the viewing window were too small relative to the specific x-dimensions chosen. (Note that: “Trace” moves the cursor along the points on a graph (both within and outside the viewing window) and displays the values of the x and y coordinates of the current point. If the point is off the screen, the cursor itself is not displayed. The image scrolls left or right if one traces beyond the x-values or the window currently displayed.)

> *S:* *Is it a parabola? [turns on TRACE and moves the cursor along the graph] It don't look like it's ever going to go down.*
>
> *G:* *You're on TRACE and looking at (values of) the x and y? [S: Um-huh.] What are the numbers doing?*
>
> *S:* *Keep going up.*

Sam correctly thought that this graph was an inverted parabola, and he expected to find the vertex by finding a place where the y-values reversed directions. This strategy could have helped him find an appropriate viewing window on his calculator by locating a maximum function value or, through the pattern in the data, confirmed his thinking that the graph was a parabola. However, since the vertex was to the right of the cursor and Sam was moving the cursor to the left, the cursor failed to reach the vertex. As he moved the cursor down the left-hand branch of the parabola, both the x- and y-values continued to go down. Sam thought because the magnitude (absolute value) of his negative numbers was going up, the cursor on the calculator ought to go up as well and thus ultimately reach the vertex of the parabola. He was trying to coordinate a change in magnitude (size) with an upward direction along the y-axis and thus with an upward change in position of the cursor along the graph. Although these three kinds of change are ordered (a math convention) in the same way for positive numbers, the direction of change in magnitude for negative numbers is exactly backwards from that of both y-values on the axis and the relative vertical position of the cursor.

As we continued this problem, my efforts to straighten out the number confusion led to continuing difficulties with the viewing window, in part because of Sam's style of responding. I started by deciding to help Sam discover his error and asked if the numbers were positive or negative. He said they were both negative. Then I tried to call his attention to the distinction between the magnitude (absolute value) of a number and the relative order of two signed numbers:

> *G:* *Are they going up in absolute value or are they going down? Are they going up if it's negative?*

S: *If it's absolute value, it's goin' to be positive all the time.*

G: *I meant the size of the number, is the size of the number going up?*

S: *Down. [He continues tracing.]*

G: *It's going down. If I said -200 is going to -300, is it going up or going down?*

S: *Say that again.*

G: *If I went from -200 to -300, would you say that's going up or going down?*

S: *Up.*

G: *It's going up in size, but the number itself is actually going down, isn't it? Isn't -300 less than -200?*

S: *Um-huh.*

During this exchange, Sam was very busy tracing on the calculator. He appeared to be paying only cursory attention to me. When I raised absolute value, he gave a "textbook" answer about absolute value being positive all the time, not attempting to answer either of the questions I had asked. Thinking perhaps that the difficulty was my poor questioning, I reconsidered and asked only about the *size* of the number. I had hoped he would understand this informal description that is sometimes used with absolute value, but he answered my question as if it were about the order of *signed* numbers when I had asked about *unsigned* numbers. I echoed that he had said "down," thinking perhaps he had figured out the discrepancy between magnitude and direction. To confirm which question he was answering, I asked a more focused question about how he perceived change between two specific numbers, from -200 to -300. His answer confirmed that he had not sorted out the order concept, but perceived these numbers as increasing, rather than decreasing.

The question Sam avoided was a conceptual question about the magnitude of numbers and how numbers are ordered. In the passage above on questioning whether numbers were increasing or decreasing, Sam continued to give me partial or misleading answers. His responses to my questions had an effect on the interviewing process. They caused me to narrow the questions from one about a general concept to one involving the relationship between a single pair of numbers. Although Sam might have been ignoring me somewhat as he concentrated on the problem, the net effect of the interaction was that ultimately I asked a question that he could readily answer and finally told him a "correct answer." This style of delaying or deflecting responses was a reasonable problem solving strategy on his part that shaped my behavior as an interviewer to simplify the task and move the interview forward. It did not, however, really help Sam, nor did it give me much insight into his understanding of the task at hand It simply confirmed the point that he viewed -300 as larger than -200.

Seeking Clues

Another style of interacting with me that seemed to advance Sam's progress in the interview was his use of questions to gain clues for how to proceed. Sam began 7 of the 17 problems he worked on during interviews with a sequence of questions that sought additional information about the problem. On two of these occasions, his questions were only to find out what I expected of him, much as a cooperative student would ask a teacher in class. For example, at the beginning of the fourth interview he asked if I wanted him to think aloud and whether I wanted him to write his response. Sometimes Sam wanted clarification of terms or of the problem statement that might be necessary to solve the problem, or after I had explained their meaning, to check if he had understood correctly.

At other times, however, Sam seemed to be seeking hints for how to proceed. In problem 2A-2 (Figure 1 below), he asked a number of questions:

S: *You want me to draw this?*

G: *I want you to read what this problem is and see what you can do with this problem.*

S: *Now, I'm going to look over this again.*

G: *Okay.* *[he reads the problem]*

S: *This is like that x + 1 stuff equal y.*

G: *You got it.*

S: *So that's what the function rule is?*

G: *The function rule is like what you were doing in class when you were trying to find the function rule.*

S: *[pause] Okay, now I remember. It'd be like x [pause], two x equal [pause] y. [writes 2x = y] I don't know. I don't know if I'm right. I can't remember.*

For the table below, what is the function rule? What is the output for input 15? input 30?

input	output
1	10
2	5
3	0
5	-10
10	-35

Figure 1. Problem 2A-2

Sam had seemed to connect this problem to graphs in a previous problem in the same interview, so he asked if I wanted a graph, a clarifying question. After I told him to just *read* the problem, he checked his understanding that a function rule was an equation written y equals an expression involving x. When I agreed with him, he asked again about what the function rule was. At the time I sensed he was fishing for a direction to proceed, and so simply mentioned we had done function rules in class and gave completely redundant information. My response no longer added anything useful to the problem solution. Sam then made a guess, but his announcement that he did not really know if his guess was right indicated he was stuck and hinting for additional help. This strategy was wise since when students were stuck, I usually provided some question or bit of information that would help them get going again. We continued the problem with my giving him help by asking if his rule fit the table. By that time Sam had figured out to look for a pattern of change in the data and immediately computed the pattern of change. Our question and answer session had given him enough information, or possibly just time to think, about the problem to get him started.

In the Profit Problem in interview 3, a long complex, situated problem involving comparison of two polynomial functions, Sam also engaged in a long sequence of questions and comments before he chose any plan of action. In short, Sam's use of a legitimate school routine, asking questions to clarify the problem statement, served both to advance the interview in a non-threatening way and to gather information to make progress toward producing what was expected by the researcher.

Researcher's Comments and Questions Triggering Responses

Sam seemed attuned to my questions as well, sometimes taking them directly as hints. At other times they seemed to trigger a change in the direction of his problem solving. As we continued problem 2A-2 above, one of my questions helped him again. He had just described the pattern of change in the table and conjectured a rule, $y = -5x$, which matched the change pattern in the dependent variable, but not the table as a whole. Sam quickly scratched it out. I told him to just put an x through his work so I had a record of his thinking. Then because I wanted him to verbalize his thinking, I asked him why he scratched out the $-5x$. He explained that when he had substituted 1 for x, the result did not match the table entry. Unbeknownst to me, this question caused him to reconsider his choice and quickly add the necessary 15 to verbally give the correct rule. As he continued, he made a number of recording errors, but finally wrote the rule correctly as $y = -5x + 15$ and checked its accuracy with $x = 2$ and $x = 3$. When I asked Sam if he was sure about it, he checked the rest of the table entries in his rule. He took my question, which was to find out how certain he was of his answer, to indicate the possibility that the rule could be wrong. At the end of the problem he announced that he did the problem with "that hint" from me. When I asked what hint, he said, "When you said why did you scratch out that five, negative five." This question became a hint not to abandon $y = -5x$. Sam had started this problem with several questions that gave him additional information. When he was stuck, he got a suggestion about how to proceed. He was explicit about how my question caused him to reconsider his first choice. Finally, when I asked if he were sure about a response, it triggered additional checking for a correct solution with the table values. Until I had asked, Sam had omitted checking the remaining table entries in the rule, a step that was necessary unless he gave some other argument to confirm the match between the equation and all table entries.

Sometimes my questions about whether he was sure about a response triggered checks that were not necessary. In one problem to choose either Alice's, Bea's, or Carolina's graph to match a table, Sam had used the units from a sheet of graph paper to plot the points in the given table on the axes for Alice's graph. He then chose Bea's graph as correct, and when I asked why, he explained his choice well:

S: *Because the form right there follows the pattern that she got. (shape of the graph)*

G: *The pattern of the dots that you drew [T: um-huh] looks more like Bea's pattern?*

S: *Yeah. So I'm going to say her chart is more accurate than theirs. Now, I'm gonna recheck my stuff to see if I'm right.*

Here when I simply echoed his response, substituting "Bea" for "she," he repeated the point plotting with the same units, this time on Bea's axis. Even though the units he used were different from those actually on the graph, he had judged the match correctly by the shape of the graph, so there was no need to repeat this plotting. My question had seemed to trigger the check. Sam's being tuned into my responses was a frequent occurrence, and as an interviewer, I had to work very hard to refrain from giving clues to Sam inadvertently.

I assume Sam had learned a strategy of paying careful attention to teachers' responses. It is likely that asking a large number of questions or getting a problem rephrased could result in subtle differences in a problem statement that made it clearer. Answers could also clarify "what the teacher wanted." In addition, it is likely students are familiar with a style of questioning in mathematics classes where, when students give wrong or incomplete results, their teachers echo students' responses or ask if they are sure of an answer specifically to indicate the students' answers are wrong without saying so directly. Thus, they give students an opportunity to think harder about the problem by extending the time. I had the strong sense in working with Sam that sometimes his questions were "buying time" to think, as well as to get hints, while he thought about how to proceed.

Discussion and Conclusion

Each of these interaction patterns can be associated with similar routines in school where the teacher attempts to advance the discussion of mathematics, diagnose student's difficulties, and attend to students' self-esteem so they can continue to participate. Even though I was acting in the role of researcher and asked questions for different purposes than in a classical classroom, I am also a mathematics teacher, am familiar with the routines discussed., and was strongly motivated to keep the interactions moving.

Assume a teacher in a traditional classroom is motivated to help students understand what she takes as shared by the mathematics community and at the same time likes to feel effective in her work. If a student is motivated to cooperate, as Sam seemed to be, he attempts to do what is expected of him, that is, solve mathematics problems as defined by giving correct answers. Yet he expects not to lose face while doing so. Each of these stances require that any interchange between teacher and student not reach an impasse; otherwise, one or the other, or both, is viewed as deficient in some way. The teacher is viewed as unsuccessful or ineffective at teaching because she has failed to convey the appropriate information to the student or her ability to ask appropriate questions is suspect. The student is seen as not knowing the appropriate information or means of stating it and, thus, is judged unable to do the mathematics or possibly simply dumb. Thus, both teacher and student are vested in making the interaction routines work. Given a classic style of instruction where the student is to arrive at a correct answer, it is natural that to help a student save face after several impasses, the teacher gives more and more focused tasks. The student then must divert the task from a focus on solving mathematics problems toward successfully completing the interchange--with or without progress in the mathematical task.

Most dedicated and perceptive teachers are well aware when a student is not viewing mathematics as they do and would welcome assistance in being successful in their classrooms. In a traditional classroom, however, teachers may not even be aware that although these kinds of interchanges seem to be open to student's views, they are actually a form of direct instruction where students attempt to fill in the blanks with the "correct" response. Further, they are likely not to be aware that students are involved in a task with an entirely different purpose than the problem solving intended by the teacher; rather, students see their task as completing the interaction without becoming stuck.

The consequences of orienting teaching solely by the mathematics and not recognizing that how the ways in which we think of mathematics and instruction impacts the social interaction in the classroom are dire. Teachers might question their own dedication or ability and become frustrated with all sorts of ugly results. Students often simply assume they cannot do mathematics, sometimes with devastating results to their self-esteem and, and certainly by being cut off from many rewarding life paths because they do not have the required mathematics.

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CULTURAL CONFLICT: A PRESERVICE JAPANESE STUDENTIN AMERICAN SCHOOLS

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There is a considerable difference betweenwhat goes on in American mathematics classrooms and what goes on inJapanese ones (Jones, 1997). As Reyes & Stantic (1988) stated:

> Clearly, we live in a society where racist, sexist, and classistorientations exist in individuals and in institutions. What is not clearis how such ideas are transmitted to and through schools, how the ideas aremediated by the democratic ideals of equality and equality of opportunity, and the extent to which teachers andstudents accept and resist the ideas. More specifically, we do not yetfully understand how these ideas affect the teaching and learning ofmathematics (p. 27).

Culture, a popular catchword, is often used byeducational researchers without clarifying what the term means orconsidering its relevance to mathematics education. Nickerson (1992)recommended coming to someunderstanding of culture within the classroom before considering how wideraspects of culture impinge on that classroom. This paper will present aJapanese student's cross-cultural perspective of American educationand mathematics education and will provide insight into the classroom culture and the effects of thewider aspects of culture and society in it.

Keiko, a graduate student in an elementaryand middle childhood teacher certification and Masters of Education programin a midwestern university, has lived in the midwestern United States for12 years and is 36 yearsold. Her American husband and his family are from the Appalachian culture;their two sons attend first grade and fourth grade at an alternativeprivate school which is not associated with any religious group. Theschool's philosophy is based onBruner's ideas of discovery learning, and is more aligned with theeducational environment Keiko desired for her children. Keiko is activelyinvolved in the children's education both at school and at home.

In the American culture, Keiko isviewed as a 'super-mom' and a 'super-student'because of her exemplary parenting and outstanding achievements ineducation. However, Keiko takes responsibility for the family and childrenas she would in the Japanese culture where her role is to "prepare the child for life, to help provide a bridge between the home andthe outside world" (White, 1987, pp. 37-38). In the Japanese culture, the family is thewoman's source of influence and value, and this is embodied in thevertical ties of parent and children rather than in theWestern nexus of husband and wife (White, 1987). In Japan, any kind ofwork requires 100 percent effort and a person who tries to combine differentwork lives, such as Keiko does, is seen as lacking in a fixed groupidentity (White, 1987).

While completing her undergraduate degree,a B.Ed in elementary education with areas of concentration in mathematicsand social studies, she completed a *non-required* Honor'sResearch project that investigated the effects of language and culturaldifferences on a Japanese eighth grade student in the mathematicsclassroom**.** During the project, Keiko's mothercame to the US to helpfulfill the mother role in her family. At other times during hereducation, however, Keiko gave 100 percent to both roles without anyadditional family support. Keiko's cross-cultural perspective ofthe elementary classroom offers a newperspective about how the wider aspects of culture and society influencestudents' learning of mathematics. Indeed, it suggests that thesewider aspects have a direct effect on the mathematics learning in theclassroom. Reyes andStantic (1988) said that in mathematics education there is little researchdocumentation of the effects of societal influences on other factors suchas school mathematics curricula,teacher attitudes, students attitudes and achievement related behavior, andclassroom processes. Moreover, they suggested that documenting theseconnections is the most difficult and the most necessary direction forfurther research on differentialachievement in mathematics.

Raymond (1997) offered a model describingthe relationships between mathematics beliefs (nature of mathematics andmathematics pedagogy) and mathematics teaching practices (mathematicaltasks, discourse, environment, andevaluation) which included the factors identified by Reyes and Stantic(1988). In this model, past school experiences (successes in mathematicsas a student and past teachers) had a strong influence on theteacher's mathematics beliefs. These mathematics beliefs and the immediateclassroom situation (students' abilities, attitudes, and behavior;time constraints; the mathematics topic at hand) had a strong influence onthe mathematics teachingpractices. In turn, teaching practices strongly influenced future teachingpractices.

Brown & Borko (1992) found that beginning elementary teachers whoentered the teaching profession with nontraditional beliefs about how theyshould teach tended to implement more traditional classroom practices aftertheywere faced with the constraints of actual teaching in the American society.However, Keiko entered the teaching profession with nontraditional beliefsabout mathematics andteaching that were developed in the Japanese culture and were reinforced bythat society. When she faced the constraints of actual teaching intraditional American classrooms and the classroom management practices usedin this society to externally'control' students' classroom behavior, sheexperienced cultural conflict.

. Studies concerning teachers'beliefs about mathematics and mathematics pedagogy, found thatteachers' beliefs are not always consistent with their teachingpractices (Kaplan's study; Peterson, Femmema, Carpenter, & Loef's study, as cited inRaymond, 1997). However, the inconsistency was deeply rooted in thedifferences between the cultural and societal expectations in education andmathematics.

While working with Keiko on the researchproject, we developed a close relationship, the type of nurturingrelationship between student and advisor that is expected between a teacherand student in the Japaneseculture (White, 1987). We used the term 'mono-vision' todefine our educational perspective--one eye saw the classroom throughKeiko's cross-cultural view, the other eye saw the classroom throughmy mathematics educator's view, and with both of our eyes focused, we more clearly saw the effects of the culture and language on the student's mathematics learning.

Keiko's philosophy of education wasdeveloped in the Japanese culture and her values influenced her teachingand learning in many ways. Moreover, she learned mathematics thinking andreasoning in Japanese schools.Keiko's beliefs about mathematics mesh with the goals of NCTM (1989,1991, &1995) but not with the traditional classroom and ways of teachingmathematics in American schools.

Sugiyama (1993) stated that the higherachievement of Japanese students depends chiefly on the social conditionsand the general educational environment in Japan and believes that this,rather than excellentmathematics education, accounts for the higher achievement of Japanesestudents on international studies. These factors include: the Japaneseeducational level is higher in general, including mathematics; thesituation is caused by the belief that education is the basis of social development of the country and of theprosperity of the individual; the Japanese educational system has norepeaters because effort, rather than ability, determines success; theJapanese educational system hashigher quality teachers who are expected to teach higher quality lessonsthat emphasize the development the students' ability to solveproblems.

During the first two quarters in the graduate levelteacher preparation program, Keiko's value system conflicted withwhat was emphasized as important in some of the methods courses, in the elementary classrooms she visited and taught in, and in her work in theuniversity Math Lab. By the end of her two week field placement, Keikoconcluded that she would not teach in public schools because the culturaland societal differences were toogreat; they demanded that she give up her identity. Indeed, this wasanother way to state what she already knew about education in this regionalmidwestern society -- students from different cultures attend privateschools with the exceptionof the African American students who mainly attend the urban schools; thepublic schools are homogeneous (Sugiyama, 1993)

The data presented in this paper wasgathered primarily from daily e-mail discussions. Other data formsincluded: videotaped lessons, folders of Keiko's coursework and reflections, and field observation reports and notes.Pseudonyms were used in the paper; it presents preliminary findings relatedto the cultural and social values that were in conflict and that arerelated to mathematicsteaching and learning.

Discussion

Keiko's beliefs about mathematics andmathematics pedagogy were formed in the Japanese culture and wereinconsistent with traditional beliefs about mathematics and the practicesof teaching in the American culture. Thisconflict was apparent to Keiko in the elementary schools and in her work inthe university Math Lab.

Many students who use the Math Lab services weretaking remedial level mathematics courses focused on middle school and highschool mathematics and others were students taking the required courses forthe elementary teacher certification program. As lab tutor, Keiko attended themathematics class so that she could meet the students and encourage them tocome to the lab for assistance. However, she was concerned about theteaching in these classes, and shedifferentiated between teaching for mathematical understanding and teachingprocedures. She learned mathematics in Japanese schools and had aunderstanding of why the procedures 'worked.'

> It was busy in the lab, and my visits toMrs. M's classes were fun. I see your point, after looking at thetextbooks, that they only teach procedures in those classes. But why, though? Those kids are in the classes becausethey probably have missed something on the way learning math in theirprevious school experiences, and they don't have the concepts down. Then,they are taught the procedures allover again without any help in understanding the reasons for it??

Keiko related what she observed in a 5th gradeclassroom to the teacher's knowledge and understanding ofmathematics and then to what she knew about the college level studentstaking the Mathematics for ElementaryTeachers courses. She began by talking about the 5th grade teacher andsaid:

> For math, she uses the manualsexclusively, but when she teaches Language Arts, her lessons are great. Icould see she was enjoying teaching the subject, too. I guess it is safetosay that math is not her strong subject area. I was helping a few mathstudents in the lab earlier this afternoon who are taking math 105 thisquarter, also, but it was amazing howlittle they understand the material. I can see when those students finishthe program and go out to teach, they might be tempted to rely on manuals.Even though they might be exposed with how exciting math learning is/shouldbe while they are in the (education) program, once they are out in theschools and when they feel the time-crunch, it is easy for them to spendmore time preparing for other subject areas which they are interested inand put off math until the lastminute. And meanwhile, the students suffer, don't they?

In an elementary classroom, she observed that teachingmathematics was often avoided and was concerned about the message that gavethe students. As a primary subject, Keiko thought that it should not onlybe taughtdaily, but also be taught when the children were ready to do their bestthinking.

> You know, as many days as I have been in theclassroom, so far I have not seen one math lesson there. I am beginning towonder if she ever teaches it! My co-op definitely likes to replace her"math period" with other busywork period. I'm afraid this is giving the kids an impression that math isnot a desirable or important subject in the relationship to the others. Itwill be interesting when I take it over; we'll see how the kids react to the change. I plan to move math to themorning, whenever possible, as the kids' are ready to work the best then.

Keiko observed how little the mathematics methodscourses in the education program affect the teaching practices in theclassroom. Several teachers she worked with were graduates of the sameteacher preparation program shewas in, yet, in the classroom the philosophy of education and thepedagogical practices in mathematics were not evident. Indeed, some of theteachers advised her to ‘play the game’ at the university, and you can join the real world of teaching in theschools after you graduate. This advice disturbed Keiko greatly, becausemost of what she learned about teaching at the university in themathematics methods course was consistent with her cultural view.

The teaching differences between Japanese andtraditional American teaching in mathematics are well documented in theliterature. In traditional American classrooms, mathematics is a unrelatedcollection of facts, rules, and skills, while in nontraditional classrooms,mathematics is dynamic, problem driven, and continually expanding. Thenontraditional view of mathematics is aligned with the beliefs Keiko hadabout the nature ofmathematics and mathematics pedagogy, and inconsistent with the traditionalview that is prevalent in this society.

Keiko questioned how some professors ‘helped’ studentslearn in the university setting. For example, she talked about how aprofessor ‘gave her the right answers’ and wondered how thathelped her learning. Keiko believed that as a student it was *her* responsibility to solve problems and that the professor wasthere to provide guidance rather than answers. Again this is consistentwith theroles of student and teacher in the Japanese culture but is not always thecase in the American culture. She said:

> When I went to see Dr. P to go over my lessonplan Wednesday afternoon, I was surprised when she started editing my plan.She had a pencil in her hand and wrote what I should say word forword.

Reflection is a part of Keiko’s life; shedesired strongly to understand the events of her life. She remembered thatwhen she was in school, there was a time period at the end of each day whenstudents reflected about the day’s events, and she noticed this type of reflection was not a part ofthe task-oriented American classroom life. Moreover, she recognizedindividual differences. She said:

> But at the same time, I know some Japaneseadults who just cannot reflect at all, so there is certainly an individualdifference there. Through reflecting and thinking about what happened, youcanlearn to recognize that there are variety of reasons for a thing to gowell/not to go well. It's not "you" only who is responsible for theoutcome, usually.

In a message she shared another discovery that helpedher understand more about the conflict she was experiencing related togeneral pedagogy and its implication for her as a teacher. She said:

> As I was working on a paper, I came across abook on my shelf. As I read through it, it made a lot of sense to me, now,that I understand those educational terms.It talks about why Japanese teachers teach the way they do, and theJapanese students learn the way they learn. What I found interesting wasthat it gave me insight as to why I disagreed with so many things Dr. Psaid in the pedagogy classes. Myexperience in schools has been so different that now it is understandablewhy I couldn't agree with what she said proven effective in Americanclassrooms.

Keiko can think and speak fluently in bothJapanese and English. We found that there were some differences intranslating mathematical ideas between English and Japanesethat were not easily noticed   For example, while teaching second grademathematics, a question arose about 2 dimensional shapes, and Keikodiscovered a difference between the languages.

> Rectangles must have four right angles. Iguess I was thinking about quadrilaterals, and also thought that inJapanese there is not such a word as "rectangles"---there is a term forquadrilaterals, though, to generally include all four-sided figures. Isn'tit interesting? So, culture and language DO matter when it comes tolearning mathematics.

After discussing this difference, we pondered whyin the American culture, primary students are taught the general wordtriangle to name all 3 sided shapes, and yet, more specific names, such assquare and rectangle, aretaught for 4 sided shapes, rather than the general word quadrilateral.

This society's ideas of competitionand perfectionism were ones that Keiko often reflected about and discussedwhile trying to understand her place in the cohort group. Her locus ofcontrol isinternal, and her confidence, creativity, humor, and openness, come fromthis feeling of personal control. However, as she struggled to understandcompetition and perfectionism, she observed that when the locus of controlis external, a negative sense of competitiveness, helplessness, unwillingness to be open to new ideas,fear, etc. were developed as personal traits in her peers. Keiko shared adiscussion she had about one peer in the cohort (Jeanette) with a fieldadvisor (Katie).

> I told Katie that I am a perfectionist, too, butmy way is so much different from Jeanette's. I said that Jeanette'sperfectionism depends heavily on others' approval. She is always soconcerned about how others perceiveher, and she can get pretty annoyed when things do not go her way. I thinkher perfectionism comes from her uncertainty of herself. She has to doeverything better than the others because, otherwise, she thinks thatothers will notrecognize her. My perfectionism comes from my confidence. I know I can doit well, whatever I do, and I try my best to go up to (live up to) my ownexpectation(of myself) because I am confident about my capability. Perfectionismcomes with some negative sense in this culture, while it does not in Japan.If you consider perfectionism in Jeanette's terms, I think it is prettynegative.

Keiko also talked about sensitivity, a character traitthat has a positive meaning in the Japanese culture, but she found it hadan opposite meaning in this culture. Indeed, it was a character trait shefostered in herself and in her children.

> I always thought the word "sensitive" waspositive, but when he said to me, "you are too sensitive abouteverything!", it shocked me. That was a culture shock---I thought the moresensitive the better.

She related these ideas about character to therelationships adults have with children in the classroom environment andpower struggle that she observed between the teachers andstudents. This power struggle, often under the guise of classroommanagement or discipline, was in conflict with the mores of Japaneseculture. In Japan, the teacher is a respected member of the classroomcommunity, where hard work and fun were one inthe same thing. Moreover, teachers and students develop close bonds sincethey stay together as a community of learners for the first six years ofschool.Power was not an issue in the classroom because the societal and culturalexpectation of education were different.

> I was thinking that the message I want tosend to the kids most is "you don't have to be overwhelming/strong & bigphysically/pushy/verbally abusive/etc. to be respected. I want them toknow that quiet and sweetperson can be just as strong or stronger, mentally, than those people whotry to exhibit their strength on the surfaces. What I am trying to do heremight be to change the cultural norm, and I know it will not be doneeasily.

During the first week of full time field placementteaching, Keiko was enthusiastic about teaching second grade, especially,teaching mathematics.

> I interviewed the kids about shapes usingtangrams today. You know, they really amazed me. I feel that I can justteach Geometry all year like this---they should be able to understand"infinite planes" by the end ofthe year. It's such a joy to work with them. Those surprises(students' thinking) I encounter in the classroom are what makesteaching so interesting! I feel that I want to do more and more when theygive me their new ideas. It's just soooowonderful:-).

However, at the end of the first week Keiko explainedher feelings about teaching in the public school. She was overwhelmed bythe realization of the differences between the culture and society:

> One of the major roles schools play ineach culture is to teach children their own societal values so that theyfit in the culture. In a homogeneous community setting such as this, itseems that this role becomes even more important. Teachers, schools, and parentshave the same value system so that children are socialized in the dominantculture. Because the cultural belief is so rigid and not accepting ofdifferences, every child is forced to fit in. I am from a differentculture, and I will not be very effective in that role.

On the same day, the field supervisor gaveadvice during the final seminar that summarized the cultural conflict sheobserved and experienced in the classroom. The field supervisor said:

> ‘The most important thing for a classroomteacher to have is a strong control over students.’

At the end of the second week, Keiko recognized the‘big dragon’--conflict between cultures.

> As strongly as I feel about my philosophy, I amso sick of the way they are treating the kids at school out there, and Ifeel powerless because I cannot do anything about it. It bothers megreatly that the kids are notrespected as "kids" at all. And teachers' role in it is so significantthat I really do not want to have any part of it. So, in Autumn quarter, Iwas upset because I felt that I was fighting against "the cohort", but nowIgave up because I know I am fighting against "the culture", and I know Icannot do much about it at all.

The first week of full-time field placementteaching was exciting for Keiko, but during the second week, culturaldifferences about how to deal with immediate classroom situations causedKeiko to reconsider her desireto obtain teaching certification for public education. She felt that shewould have to give up her identity to fit in the classroom and schoolculture, and to meet the societal expectations of public schools.

Conclusion

Clearly, Keiko was in cultural conflict andthe causes of her conflict were embedded in the differences between thecultures and the societal values fostered in the educational system.Although she was knowledgeableabout mathematics, could teach mathematics effectively for understanding,and loved teaching and her students, she decided that she would notcontinue in the certification program and teach in public schools. Shebelieved that: the classroom was a community of learners, with the teacher at the center of the learning; theresponsibility for learning rested with the student; the role of the teacher was to challenge and nurture the students growth; the interactionsbetween students and teacher were based on a loving relationship ratherthan a power struggle; and, hard work and fun were one and the same thing.Instead, she will teach mathematics in a private school where the philosophy and goals of education are morecompatible with her beliefs.

White (1987) said that the children inJapanese classrooms are good as well as happy because no one taught themthat there is any contradiction between the two. As early as in 1919, John Dewey observed the absence of overtdiscipline in Japanese classrooms.

> They have a great deal of freedom there,and instead of the children imitating and showing no individuality -- whichseems to be the proper thing to say -- I never saw so much variety and solittle similarity in drawings and other

handwork, to say nothing of itsquality being much better than the average of ours. The children were under no visible discipline, but weregood as well as happy; they paid no attention to visitors É I expectedto see them all rise and bow. (White, 1987, p. 122)

Implication

Reform in mathematics curriculum, teaching,and assessment have been espoused since the National Council of Teachers ofMathematics Standards documents (NCTM, 1989, 1991, & 1995) were written, however, the philosophyand spirit of the reform has had limited influence reforming mathematicsteaching and learning. American students were ‘at-risk’ in mathematics when they were compared to Japanese students in the ThirdInternational Mathematics and Science Study (TIMMS) report. AlthoughAmerican students’ scores were in the average range in elementaryyears, middle grade students and high school students scored poorly. The report has generated muchconcern about the teaching and learning mathematics in American middlegrade and high schools, through its comparisons between the curriculum andmathematics pedagogy in the two countries. Indeed, the traditional mathematics teaching and learning in theelementary grades is a factor affecting students future poor schoolperformance in middle and high school mathematics. However, this is notthe case in the Japanese schools where the emphasis in elementary mathematics is on developing mathematicalthinking by exploring, developing, and understanding concepts, ordiscovering multiple solutions to the same problems.

In both cultures, the teachers’beliefs about mathematics are formed during their personnel experiences inmathematics learning, and these beliefs are reinforced by societalexpectations. Moreover, American teacher preparation programs have little effect on reforming teachingpractices in the classroom because they are in opposition to the culturalnorms in the classroom and in the broader society. Indeed, many of theteachers are as ‘at-risk’ in mathematics as their students are. How can *allstudents* learn mathematics when culture and societal norms conflictwith the desired outcomes of the reform? To answer this question, consideration need to be given to the effects ofthe culture and societal expectations on students performance inmathematics.

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ETHNOMATHEMATICS AND POSTMODERN THINKING: CONVER/DIVERGENCES

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The paper establishes connections between post-modern thinking and the field of Ethnomathematics, using as an argumentative resource some critiques that have been presented by mathematics educators regarding theoretical perspectives of Ethnomathematics. In discussing convergences and divergences of postmodernism and this field of mathematics, the centrality of introducing the category of power when analysing the social and the cultural is presented.

The time we are living in is marked by profound and frighteningly rapid changes: access to a virtual world that simultaneously brings us closer together and keeps us irretrievably apart, due to the exclusion of most of us from an increasingly sophisticated technology; the economic globalization process which simultaneously makes us feel "at home" in any supermarket of the world and fixes us definitely in our own places, due to the impossibility of transiting borders that are increasingly demarcated politically; massive access to the media that almost instantaneously and simultaneously shows us the robot arriving on planet Mars and poverty and death in the Congo, but that says little of the "misfortunes" that happen very

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close to home. All these changes have caused contrasting and irretrievable changes in our everyday worlds, broadening their very meaning.

Our everyday life is no longer geographically as definitively circumscribed (Has it ever been?), since it is precisely this geography that is blurred, without definitive borders as though it were always moving between here and there, wandering through times that are also undefined, between the past that is present and is already itself also tomorrow.

It is in this time so strongly marked by the "new", by relativity and by uncertainties that Education is rethinking itself, taking a fresh look at its own trajectory to account for the multiple processes that have come into our lives and with which a still puzzled school has dealt now with contempt and now with stubbornness. Even the so-called hard sciences such as Mathematics have been inexorably affected by all of this turbulence that is the hallmark of the end of this century. The rise of Ethnomathematics — an area of Mathematics Education that looks at the connections between mathematics and culture — must be seen in this globalised, cross-cultural scene, where times and spaces mix and mingle.

In this paper, it is my intention to establish connections between post-modern thinking and the field of Ethnomathematics, using as an argumentative resource some critiques that have been presented by mathematics educators regarding theoretical perspectives of Ethnomathematics. Let us begin by looking at Paul Dowling's discussions (1993). The author's purpose (1993) is to show that it is only apparent the non-affiliation of Ethnomathematics to the thinking of modernism. In constructing his reasoning, he refers to the existence of an "ideology of monoglossism" in the field of mathematics education of which constructivism is one of its forms. According to Dowling (1993, p. 36) a second manifestation of the "ideology of monoglossism" in mathematics education is “plural monoglossism” and here Ethnomathematics is the example par excellence. In this form of monoglossism, emphasis changes from the individual subject to the cultural subject; society is seen as constituted by a plurality of cultural communities, since in them there is complete absence of monoglossism. Thus society is considered heteroglossic, but the communities that constitute it are monoglossic. Following this rationale, Dowling concludes that Ethnomathematics is a discourse impregnated by the project of Modernity.

Using different arguments, Nick Taylor (1993) also emphasises the strong connection between Ethnomathematics and modernity, when speaking about a "profound ambiguity " in the Ethnomathematics discourse. He takes as one of the focuses of his reasoning the work of Walkerdine, whom he considers the "pre-eminent theorist of Ethnomathematics", stressing the relevance of the post-structuralist approach of the studies performed by this author in discussing questions of context and transference in the learning process. He shows the density of what he calls Walkerdine's "revolt" against the role of mathematics in the modernist project, reinforcing the concepts that

> (...) there is only one objective answer to any question; that the world can be represented mathematically and controlled rationally; that universal canons exist in matters such as art, government, sexual preference and cultural norms; and that there is one mathematics which originated in Western Europe and the boundaries of which continue to be patrolled by liberal-democratic, middle class, phallocentric standards" (Taylor 1993, p.132-133).

For Taylor it is from this standpoint that Walkerdine provides theoretical support to Ethnomathematics. However, he identifies the "dilemma of Ethnomathematics" precisely in the approach used by Walkerdine when she discusses context and transference. Criticising her he says:

> (...) the end goal in working from a specific bit of local knowledge — one metaphorical manifestation — to the underlying metonymic principle, is formal mathematics. It is hard to square this teleology with Walkerdine’s devastating attack on the role of formal mathematics as a central repressive mechanism of modernity. It is hard to reconcile the connection she draws between the metaphorical and metonymic elements of knowledge, with her postulation of a disjuncture between problems of practical and material necessity versus problems of symbolic control (ibidem, p.132).

Taylor criticises Walkerdine's approach because it is eminently political and pedagogical and not epistemological, which he would consider more central. Even acknowledging that it is not easy to separate the pedagogical, political and epistemological fields in educational discourses, Taylor is interested in establishing differences between them, at least in order to problematise ethnomathematical studies. Taylor’s intention is to show that "whereas the Ethnomathematics ostensibly concerns epistemology, much of the debate revolves around the relationship between pedagogy and politics in mathematics education” (Taylor, 1993, p. 130).

The critiques by Dowling and Taylor could be problematised from different standpoints. The first of them concerns the emphasis placed by Ethnomathematics on the political and pedagogical dimension of

Mathematics Education, in detriment of an epistemological approach, an argument presented especially by Taylor.

In fact, the heart of the discussion on Ethnomathematics has not been epistemology. There are at least two distinct perspectives to analyse this fact. The first of them can be summed up in the argument by D'Ambrosio (Knijnik, 1996, p. ix) when he states that the concepts of knowledge, science and education involved in Ethnomathematics have constituted alternative epistemologies and that these are the ones which must be further analysed. On the other hand, in the post-structuralist approach into which Walkerdine fits, the epistemological question is viewed from a different angle. Walkerdine (1990a, 1990b) stresses the distinction between her approach and those eminently epistemological or empirical ones connected to the thinking of Modernity. Thus, Taylor's critique that Walkerdine's approach is more political than epistemological shows above all that the he did not understand from what "theoretical position" Walkerdine is speaking. She refuses to enter the field of epistemology as understood by Modernity. In her eyes such an epistemological approach does not make any sense because this is exactly the approach that she is opposing. If we consider Walkerdine (as Taylor suggests) the "most prominent theoretist of Ethnomathematics", we see that concerning epistemological matters, there is a close relationship between Ethnomathematics and the Post-structuralist perspective and, in a broader sense, Post-modernity.

Ethnomathematics presented by the authors concerns its affiliation (or not) to the thinking of modernity. What I want to stress here is that Ethnomathematics centrally problematises the "great narrative" which modernists consider to be academic mathematics. Indeed, in a modernist view, Academic Mathematics is the language *par excellence* to describe the distant as well as nearby Universes. In problematising this metanarrative, Ethnomathematics introduced a discussion that had thus far been absent from debates about Mathematics Education. Indeed, this is a most valuable contribution of Ethnomathematics to Mathematics and Mathematics Education. By legitimising as Mathematics more than just the intellectual products of academe and by considering as forms of other, non-hegemonic ways of knowing and producing mathematics, Ethnomathematics relativizes the "universality" of (academic) Mathematics and, moreover, questions its very nature.

Walkerdine (1990b, p.6) presents a significant argument when she says that:

> mathematics provides a clear fantasy of omnipotent control over a calculable universe, which the mathematician Brian Rotman (1980) called Reasons Dream; a dream that things once proved stayed proved for ever, outside the confines of time and space.

It is this Dream of Reason that appears to be coming to an end at the close of this millennium. The "promises" of a better life for most women and men on this planet, "promises" promised by scientific advances, by the dominance of reason over the "universe" have definitely disappeared together with the hundreds of thousands of us who continue to be persecuted by hunger, poverty, disease, death. This is a time of the Death of a failed Modernity, because it did not fulfil what it had promised since the French Revolution: fraternity, equality, liberty. In these death throes, "science is part of the problem, not its solution" (Silva, 1996, p. 144).

However, this is also a time of life to take another view of science, to redefine what is after all going to be called science, to think about its place in society, its destiny, which is, in fact, our own destiny as humanity. Ethnomathematics is centrally concerned with problematising such issues and, from this standpoint it converges with discussions proposed by post-modernity. Nevertheless, these convergences can not be constructed without paying attention to some important aspects which must be questioned, as well demonstrated by the authors I mentioned before. Let us take Dowling's arguments concerning Ethnomathematics as an exemplar of plural monoglossism. This is a key-point to be discussed. What is at stake here is the uniformity, the homogeneity with which the Ethnomathematics perspective has often treated the culture of different social groups, "the identifiable cultural groups".

From the theoretical perspective of Ethnomathematics, the concept of culture moves away from a traditional view that expresses culture as "humanity's cultural heritage". Considering that this cultural heritage is a social construction resulting from the effort of all of us, the expression "humanity's cultural heritage" thus supports the argument that humanity as a whole has the right to access and use knowledge created by humans.

Nevertheless, from a traditional view, the expression "humanity's cultural heritage" is usually identified only with academic mathematics. It is exactly this identification that masks power relations that, in turn, legitimise one very specific mode of producing meaning — the Western, white, male, urban and heterosexual one — as "the" cultural heritage of humanity. In contrast, by providing visibility to other mathematics besides the academic one, Ethnomathematics problematises precisely this apparent "consensus" as to what counts as "humanity's cultural heritage". But, in moving away from this position it also ends up by producing a sort of homogenisation, although many-hued; for Ethnomathematics, each "identifiable" group is identified by what is the same in its members, and, in this sense, similarly to the previous perspective I have just mentioned, the differences, the asymmetries which after all provide the kernel of this very group are silenced.

In ethnomathematics approaches there has often been a "forgetting" of the power relations produced within each cultural group, such as those configured, for instance by gender, race and ethnic relations. Everything happens as though each "identifiable" group were "free" of such power relations, which certainly points to a simplified, not to say naive view, of how social identities are constituted, how people organise their daily lives and give meaning to their life experiences. This is precisely what Skovsmose & Vithal (1997) argue about the lack of discussion regarding relations between culture and power in the research developed by Ethnomathematics. The authors write:

> The ethnomathematical practice, generated by a particular cultural group, is not only the result of interactions with the natural and social environment but also subjected to interactions with the power relations both among and within cultural groups. ethnomathematical studies have demonstrated how this has been played out between the Eurocentrism of academic mathematics and the mathematics of identifiable cultural groups, but have not equally applied this analysis to an analogous situation that occurs within an identified cultural group (Skovsmose & Vithal, 1997, p.11).

As long as ethnomathematical studies do not pay careful attention to these issues, Ethnomathematics will fluctuate ambivalently between Modernity and Post-modernity. My most recent efforts have been directed at developing empirical projects with the Landless People which lead me to theorise more emphatically the power relations produced *within* that social movement, projects that allow me to examine the processes established in the dispute to define what knowledges among those practised by their members are instituted as regimes of truth, in the words of Foucault. To analyse such power relations will provide us with better conditions to politicise the field of Ethnomathematics in a post-modern approach.

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Facing Exclusion: the student as Person

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Abstract

Brian Rotmans's semiotic model of mathematics is applied to examine the nature of the students' problematic relationship with word-problems. It is shown that word-problems are examples of the formal discourse of mathematics, therefore excluding the "Person", as semiotic figure, from mathematical activity.The investigation concludes that a student mathematical performance may be linked with the problematic of cultural encounters that exist between the society of practitioners of mathematics and those who do not belong (yet or never) to this community.[*]

1. Introduction

During my professional activity as a mathematics teacher I was always surprise with the way that "medium-ability" students almost immediately decide to give up from a mathematical task that did not require a different pattern for its solution than one handled well previously by the student. It was this observation that motivated my interest in knowing the "unconscious" facet of the mathematical performer: I wanted to know what students think about when they think that they do not know how to solve a mathematical problem.

2. Rotmans's semiotic model of mathematics

Rotman's semiotic model of mathematics (1993) presents a theoretical framework to examine the doing of present-day mathematics. This model was conceptualized with mathematicians in mind, and is developed through an elaborated analysis, wherein the features of mathematical discourse are essential.

This semiotic model of mathematics discerns three different figures or agencies - the Person, the Subject, and the Agent - that jointly process any mathematical activity. To better introduce each one of the semiotic agencies, as well as to explain their different functions, it is necessary to consider at first the major discursive features of mathematics.

According to Rotman, mathematicians have two distinctive dimensions of discourse: the formal mode, or "Code", and the informal mode, or "metaCode". The formal discursive mode of mathematics is related to the objective and rigorous aspects of mathematics. It is what one finds in mathematical written texts, and it consists of the symbolic notation used in mathematical texts as well as its precise rules. The

[*] Parts of this article were first published in Moreira, D. (1994).

utterances made in natural language which are mixed in the symbolic notation are also a part of the Code (Rotman.1993:69).

The informal discursive mode, or metaCode, concerns the different ways whereby the Code is contextualized and linked to historical, empirical, social, psychological, or cultural realities. In Rotman's words (1993) it consists of:" drawing illustrative figures and diagrams; giving motivations: supplying cognate ideas; rendering intuitions; guiding principles, and underlying stories; suggesting applications; fixing the intending interpretations of formal and notational systems; making extra-mathematical connections" (p:69-70).

Although the construction of mathematics embraces these two discursive dimensions, the metaCode is traditionally viewed as unrigorous and epiphenomenal when compared to the Code. Consequently, what is established by the Code (mathematical definitions, procedures, enunciations, problems, and demonstrations, as well as its discursive means) is what is promoted as mathematics, and considered as the "true" corpus of mathematical knowledge.

In considering that mathematics is done by humans to humans, Rotman examines the "recipients" of mathematical discourse, and sets up the three semiotic agencies. Briefly summarizing they arise as follows:

> The Person - that operates within the metaCode, and consequently is embedded in psycho-social, and historic-cultural references. The Person has access to dreams, motivations, narratives, and speaks with the personal pronoun of natural language.
>
> The Subject - an idealized person that is addressed by the Code and functions within it. The subject cognitively interpret signs, and mathematical texts, according to the rules and conventions of the Code.
>
> The Agent - a machine, that mechanically manipulates the signs of the Code, without concerns of meaning and interpretations.

As Rotman points out, the Subject is the most "visible and palpable" (Rotman.1993:81) of the three semiotic agencies. It is the Subject that is addressed by mathematical texts. However, the Subject's activity is expropriated from the excluded yet proximal presence of the Person, and it impels the Agent to produce.

The processes that accomplish the changes from the Person to the Subject correspond to the obscurement of reference and corporeality, and the processes that accompany the metamorphoses whereby the Subject gives rise to the Agent mainly coincide with the vanishing of "meaning and sense" (Rotman.1993:92).

As Rotman (1993) argues, mathematical activity demands the simultaneous presence of the three semiotic agencies; only in the discursive dimension of the metaCode can mathematicians interpret the wholeness of a mathematical proof that is not contained in each one of its logical steps; and only the Person is allowed to search for the "idea behind the proof" that gave rise to such a proof (Rotman.1993:80). Ultimately this means that the metaCode is the place where debate and eventually creation of the Code takes place.

3. Analysis of word- problems' texts

As I have mentioned before it was the "unconscious" aspects of the mathematical performer that interested me mostly. Thus, I started to meet personally with students and asked them to make some assignments from mathematical text-books that they were familiar with. By this way, during my mathematical encounter with students, several tasks were done without mathematical obstacles. Despite this sucess, some word-problems appeared that provoked failure. Two of the word-problems that students firstly, did not know how to solve, or were not sure about their solution, are enunciated by the following texts:

TEXT 1 - "Mr Joaquim has a rectangular land with 50m by 20m. He wants to put four rows of barbed wire in the fence. How many rolls of 50m of barbed wire each does he need to purchase?"(Neves & Monteiro, 1996:117).

TEXT 2 - "Aunt Helen is making a 90-in-wide by 105-in-long bedspread. She wants to add fringe to the two long sides and one short side of the bedspread. How many yards of fringe should she purchase?" (Burton, Hopkins, Johnson, Kaplan, Kennedy & Schultz, 1992:411).

The above problems, collected from two different contemporary mathematics text-books, present similar discursive characteristics.[1] For example, an overall configuration emerges from the texts above; it is possible to break each of them into two distinct sections. In the first section a situation is introduced and described. The second section poses a question or questions, regarding the situation previously described.

The textual organization in each one of the above texts, because they clearly separate off the description of a situation from the posing of a question about the situation previously described, suggests that the above texts have the same conceptual characteristics, and two clear distinct moments in that conception.

The language pattern of the above problems' texts is similar. The first sections mainly use the indicative, in its declarative form. Thus, the language is used mainly to convey information, and each time that new information - new data conditions for the problem - are presented, it is "signaled syntactically and lexically".[2] Thus, no redundancy exists in the text, as each sentence with its new information is

[1]According to Gee, J. (1990) in order to analyze a text from a discursive point of view, it is necessary to consider five interconnected sub-systems, each one accounting for certain characteristics of language, that altogether confer to a text the possibility of being understood, interpreted and located among the social network who produces it (p.104). The five sub-systems are: prosody, cohesion, contextualization signs, thematic organization and discourse organization. Gee's theoretical framework underlies the conception of the following textual analysis.

[2] Expression used by Scollon & Scollon (1981:48).

"explicitly marked".[3] The language pattern in the second sections is the interrogative. Therefore language is used predominantly to request information.

The systematic choice of the indicative either in the declarative form or in the interrogative form, brings on to the reader, as well as to the writer, specific roles. As Rotman (1988:7) noticed by citing (Berry 1975:166), "The speaker of a clause which has selected the indicative plus declarative has selected for himself the role of informant and for his hearer the role of informed". Because, practically, no alternative verbal mode is used in all these texts, nothing that the reader (or the writer) might know already about the situation which is being presented is evoked or recognized from the writer to the reader as relevant to the text.

Moreover, the use of the indicative is not a strange occasion in word-problems. Rather it is in accordance with mathematical more general discursive features. As Rotman (1988) points out "For mathematics, the indicative governs all those questions, assumptions, and statements of information - assertions, propositions, posits, theorems, hypotheses, axioms, conjectures, and problems - which either ask for, grant, or deliver some piece of mathematical content" (p.7).

The information in each one of the texts is factual, punctual and quantitative and they all display socially isolated contexts, with undefined or unknown actors. In regard to time information the verbal mode can be changed without creating constraints to the information presented in the text. Thus, the texts' spacetime references for the actions that are being described, if there, are useless and rhetorical. They just appear to help to set up scenarios where unfamiliar actors mimic actions.

In addition, because the information offered by the writer is no more or less than what the reader will need to answer the question (the which is itself posed by the writer), the function of what the writer mentions in the first section of the text is only discovered in its second section after the posing of a question. And, since the answer of such a question is totally dependent upon the previously presented information, the inflection introduced on the text by the question remits the reader back to already described situation[4], establishing, hence, a closed cyclical type of interaction between the reader and the text that only will end when the question is totally fulfilled. Moreover, such a question is only correctly responded to through the performance of mathematical procedures. Hence, although no allusion is made to mathematics, or mathematical procedures, they are present in a pre-assumed unmentioned context that the text actually demands. Consequently, the contrast reached by the breakdown between given information (first section) and requested information (second section) is more a rethorical distinction than a discursive one,

[3] Expression used by Scollon & Scollon (1981:48). These authors also remarked that texts with such features take the reader as if

"The "reader" is not any ordinary human being. It is an idealization, a rational mind formed by he rational body of knowledge of which the (text) is part. The reader is not allowed lapses of attention or idiosyncrasies" (1981:48).

[4] The reader will look again to problem's text, now, to interpret it from the question's viewpoint.

this is, it is not a different topic that is introduced in the second section, but rather the exhortation of a particular look (a mathematical look) into the text's first section.

In conclusion, the above analysis shows that the texts of word-problems are written in such a way that they deviate the reader from the requisite of direct experiencing of what they present. The texts, because they make use of squeezed, simulated real life based situations with obscure spacio-temporal references, pretend that the reader's experience is already insert in them. In so doing, word-problems induce the mathematical performer to eschew the empirical situation, but to imagine that he/she is acting in it. Moreover, the textual pattern of the above texts is in accordance with the general features of the formal discursive mode of mathematics, in the sense that, their indexality, if there, is fictitious, and they exhort no more or less than the procedures that are required in order to answer mathematically their questions. Therefore, mathematical word-problems only make sense if put within the discursive context where they belong and are embedded. Each text is a microworld that has no existence other than in the mathematical language of objectivity. They can be taken as objects of mathematical discourse.[5]

4. Facing exclusion: the student apprentice as Person

I am going to present, now, the interaction established between the above word-problems and the students who gave up to solve them.

In regard to TEXT 1, the student, a 10 years old 5th grader, prompted calculated the perimeter of the land and stoped after this. Since she did nothing more, I started to ask her what was happening, and, after a while, I knew that all the situation described in the problem was, for her, a completly virtual situation. This is, if the student was not aware that one may purchase barbed wire on rolls, on the other hand, she was trying to figure out a situation that required a fance of barbed wire. After we talked for a while about some situations that eventually would require fances of barbed wire, as the Zoo, for example, and after we both remember materials that are usually sold in rolls, the student did the problem without mathematical difficulties.

In regard to the problem stated by TEXT 2, the student, a 14 years olde 8th grader, told me that she was not capable of doing the problem. This is her first attempt to explain me the reasons.

> They say that she wants to put fringe in... How much did she?....
>
> I do not understand. They do not say...They are not giving any indication...

I asked her: What do they not say? What do you think is missing?

And the student replied:

[5] The features of the formal discourse of mathematics by and large inspire classroom mathematics discourse. See also Pimm D. (1987) for larger aspects of mathematical language

She wants to do a bedspread, and she wants to put a fringe on both

the long side and on one wide.

How many yards of string does she need to buy to do...

It is not coming to my head.

Here, the student looked at me, kept silence for a few seconds and did the problem. I kept asking her what was the missing information in the problem's enunciation that she was looking for at the beginning. She said:

They could have told that in order to do a yard, she would need to buy a certain amount of string

Like that, it would be easier.

Because then, I would know how much was necessary for a longside, and then for all the bedspread.

I think that they did not give enough information about how much string

would she need to do a yard of fringe.

The particular situation presented in this problem - the making of a bedspread - is common among this student socio-cultural community, thus, she was connecting the above problem's information with her experience. However, in the student's community, a bedspread is usually laced, and it includes the lacing of the fringe as well. As the student's commentaries revealed, she wanted to know the total amount of string necessary to do the all fringe, which firstly required that she knew how much string would be necessary to do a yard of fringe. Therefore, the student was looking for missing information which obviously was not there. The data condition given in the problem was neither enough to solve mathematically the reality that she wanted described in the problem, nor to figure out what could be the solution for the correspondent problem's question.

As the analysis of word-problems' texts had shown above, and the interaction between students and mathematical word-problems ilucidate, firstly, the mathematical problem-solver is directed to a hypothetical situation, a scenario, and secondly, she/he is commanded to act within this scenario. The correct performance expected from the problem solver is only possible through the exhibition of mathematical procedures. In this context, implicated with the mathematical reasoning that will lead to the construction of the different steps required for the problem's solution is the semiotic agency of the "Subject", (the idealized suject addressed by mathematical word-problems), who will make sense of the text by reading it according to the conventions of the mathematical Code, and who will launch the process that makes possible the performance of mathematical procedures.

Such procedures are subsequently worked out by the semiotic "Agent" figure, that manipulates signs according to the exact rules of the Code already far removed from concerns of meaning and interpretation.

In all the processes of problem-solving, the "Person" is never invoked to act. It is, rather, excluded in the doing of the mathematical activity proposed by word-problems, since the discursive dimension in which the Person operates, the metaCode, is not engaged by word-problems, written as they are in the formal discursive mode of mathematics.

Nonetheless, the accounts of students' performance in mathematical word-problems' contexts shows that they were involving in their efforts considerations that are not allowed either to the Subject or to the Agent. By doing so, they were trying to produce mathematics by acting with the characteristics of the "Person" semiotic agency, as well. As Rotman says,

> only on the basis "it is like me" is the Subject in a position to be persuaded that what happens to the Agent in its imagined world mimics what would happen to the Subject in the actual or projected world. But it is this recognition, this judgment or affirmation of sufficient similitude, that the Subject cannot articulate, since to do so would require access to an indexical self-description necessarily denied to any user of the Code (1993:78)

It was this kind of authentication, this kind of "it is like me" that students were seeking when trying to figure out possible relationships between the situation that they were encountering in word-problem's text and their own realities. Before trying to discover such a similarity, these students did not allow themselves to be convinced by the proposal of the mathematical word-problem, and consequently they inhibited their passage to the scenarios where the Subject and Agent act in isolation from the Person.

As apprentices of mathematics, students were looking for an "underlying story", some "idea behind the problem"[6] that would acknowledge the mathematical problem, and subsequently empower the imagining of what it proposes, and the acting out of what it demands. Thus, in the particular cases of the above problems, the presence of the Person represent meaningful and necessary mathematical reasoning.

In conclusion, the data show that even mathematical word-problems are a locus of subjectivity. While interpreting the above problems, students invoke the context in which they were designed. By doing so, they are consistently linking their

[6] As Rotman argues, when mathematicians face a new proof, they "often before anything else- ask for and try to find the idea behind the proof. They will want to know the principle, concept or underlying story that organizes the separate logical moves presented to them" (1993:80). In this search, the Person becomes active in their mathematical activity.

mathematical performance with other contexts, and acting as persons embedded in their socio-cultural realities. However, this subjectivity is neither compatible nor meaningful within the contexts proposed by the mathematical problems, and generates conflicts for they as mathematical performers. In addition, what the performer beforehand "unconsciously knows" is that, in mathematical activities, his acting as a subjective, culturally and socially embedded actor is avoided.

The mathematical community already shares, indeed is complicit in, practices that denote the metaCode to a place of subalternity, or even denies its existence, in relation to the Code; the student apprentice, on the other hand, is not yet a member of this community, (and most likely will never be), and is still in the process of being enculturated in its practices. Being so, the student is still developing the "habitus" that will "produce the practice" (Bourdieu 1977:78) of denying the invocations incited by the ambivalent nature of word-problems. Consequently, the unconscious aspect of the practice of abandoning attempts to solve a mathematical word-problem can be interpreted as a demonstration of the implicit knowledge that the student apprentice arrives at: that the revealing of subjectivity in mathematical contexts will never be taken seriously, and even collides with present day dominant culture of mathematics.

Thus, contrary to a person who is already fluent in mathematical discourse, and as such, has already acquired the "habitus" that permits prompt Subject's incarnation in order to enact the scenarios of mathematical word-problems exactly as they demand, the student apprentice as a mathematical problem solver still allows subjective realities to affect their performance. After all, students apprentices are only starting the mathematical training of "forgetting"[7] about themselves, and their surrounding cultural and social realities.

5. Final considerations

What emerges from the present investigation is that a student's mathematical performance can be linked with the problematic of cultural encounters, in its discoursive dimension, that exist between the society of practitioners of mathematics and those who do not belong (yet or never) to this community. The malfunctions of the mathematical problem solver can be interpreted as a broader type of cultural tension that arises when she/he is encountering a discourse that excludes the possibility of her/his participation as an embodied Subject, a Person. Mathematical discourse - and more concretely word problems - alienate the possibility of the student's drawing on her/his own emergent local realities, indeed, own culture, when she/he is performing her/his role as a mathematical problem solver. Feelings of strangeness toward mathematics can be linked with the requirement of the mathematical discourse, that social and cultural references and emotions must be deleted in enacting the imperative orders and the transcultural attributes that characterize the discursive features of the Code. After all, it is the Code's primordial

[7] Rotman's terminology (1993:91).

position within mathematics that forces the student into a position of outsider in relation to mathematics, by not tolerating in its discursive characteristics the student's self expression and socio-cultural embodiment.[8]

Future research should focus on further development of theoretical frameworks that can elucidate the influences of mathematical discourse and perhaps other forms of scientific discourse on student apprentices. This work will have to be accompanied by the development of methods for gathering empirical evidence for demonstrating how mathematical activities are locus of practitioner's subjectivity, as well as how such subjectivity is related to mathematical performance.

[8] As Restivo, S. (1990)has also said, refering to a broader context "technical talk about mathematics (as a system of formal relations among symbols, for example) is not sufficient for a complete understanding of mathematics. Social talk, in contrast to merely technical talk or "spiritualizing" the technical realm, connects mathematics to human experiences, goals, and values" (p.120-121).

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Prospecting Sociology of Mathematics from Mathematics Education

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Abstract

This paper examines relationship between the sociology of mathematics and mathematics education. If in one hand we learn from the sociology of mathematics justifications that mathematics is bounded by the society that produces it, on the other, from mathematics education literature, we know that learning is strongly influenced by social based processes. Being so there are a number of questions that this paper points to as areas for investigation and research.

1. Social bases for mathematical curricula

When looking at recent history of education we find that social issues have been taking a more prominent role in the justification for the inclusion of mathematics in school curricula. By the end of late nineteenth century, mathematics became central in the curriculum of elementary and secondary education. The reasons invoked to determine such a curricular centrality were based on the efficacy of mathematics to exercise the mind, especially reasoning and there was no special place attributed to social aspects. With the increasing of schools' population and with the consummation of industrialization, such a criterion for mathematics curricular inclusion became old fashioned, and, by establishing as more compatible and adequate criteria for the social necessities of that time the cultural transmission and children's accurate preparation for future social performance, started a problematization of the role of mathematics within school curricula, as well as the search for a more utilitarian vision of mathematics education.

Despite all the social changes and the educational debate, the recognition of the need for mathematics for youths and children based upon mentalist criteria remained, i.e., educators continued to believe the virtues of mathematics as a means to effectively develop reasoning and logical power (Stanic, 1987). During the 60s, and in a context of a political race between two blocks, the Modern Mathematics movement sought to increase scientific background of students. Mathematics was then seen as a privileged mean to achieve excellency in a scientific competition. Up to this point the curricular mathematical content was compatible with an absolutist perspective (Ernest, 1994) of mathematics, which views mathematical objects as either independent of human action or as mere symbols.

More recently, the bases for the inclusion of mathematics in the curricula continued to embrace it as a means of cultural transmission and professional performance. The reasons for mathematical curricular inclusion are now more explicitly related to economic and technological dimensions. However, new goals are now added: mathematics incorporates an individual component fostering the possibilities for youth self-fulfillment and the definition of personal goals, and mathematics is perceived as a fundamental tool to understand its role in important social decisions by ensuring a more complete performance of citizenship (NCTM, 1990; Secada, 1990; Skovsmose, 1992).

Another aspect of the social influence over the justification for the inclusion of mathematics in the curricula is possible to be highlighted. On the one hand, there is an imperative for a respect for cultural choice and beliefs. On the other, there is a presence of a global cultural dimension. These two contradictory requirements of modernity, have been generating a new type of problematic that go beyond a local vision of the teaching and learning process of mathematics. There are claims of curricular diversification, made to preserve cultural and individual dissimilarities (NCTM, 1990; Souta, 1996), and we can testify a globalization of Western mathematics (Bishop, 1995). The social dimensions of mathematics education are now visible on the issue of cultural representativity in the mathematical curricula not only in each country and each classroom, but also in the underlying problem of the social status that Western mathematics has been acquiring as a symbol of promotion of the movement toward globalization.

Thus, what is emerging is that the requirements for mathematical curricular inclusion and extension increasingly incorporate a social ground of argumentation, accompanying a shift from the locus of mathematics education's traditional problematic, not anymore simply located in the "mind", or the classroom, or the materials and techniques, or teaching methods to include a broader comprehension of world curricular similarities, as well as curricular diversity to include cultural representativity and the possibility of individual choice.

As mathematics educators, these challenges in curricular perspective prompt us to enlarge our vision of mathematics. This shift is questioning the very nature of mathematics. An absolutist vision has to be replaced by another that incorporates a new account for mathematical activity and a new perspective on the nature of mathematical objects. Claims for diversification and the valorization of students' cultural and personal backgrounds are not compatible with an authoritarian view of the source of mathematical knowledge. It is the purpose of this paper to prospect ideas taken from the sociology of mathematics, as represented mainly by the work of David Bloor and Sal Restivo, looking for a comprehension of mathematics and mathematics education connected with their socially immersion and embodiment.

2. Mathematics as an empirical and social science

We turned first to the issue of the nature of mathematics. Under the denomination of Naturalistic Mathematics, David Bloor (1991) proposes an account of the nature of mathematical knowledge that incorporates contributions both from J. Stuart Mill and Gotlob Frege. Mill proposes that mathematical knowledge comes from experience. We know facts that apply to a wide range of things, namely ways of ordering, sorting, rearranging, etc. These patterns and groupings of physical things provide models for our thought processes, and when we do mathematics, we are tacitly using these models. Mathematics is a set of beliefs about the physical world that arise out of our experience of that world.

Frege criticizes Mill by rejecting the idea that the nature of number has a subjective, mental or psychological component. Moreover, he argues that number is not a property of external things. For him numbers are objects of Reason, or Concepts, which have the important property of objectivity. Frege views objectivity (Manno, n/d) as denoting something that is independent from our sensations, intuitions and imaginations but is not independent from reason. We can call objective to which is subject to laws, what can be conceived and judged, what can be expressed with words.

Bloor argues that, as Frege points out, experience alone does not provide an adequate background for mathematical knowledge. “The characteristic patterns” of objects, as Mill puts it, are not on the objects themselves. These patterns are social, rather than individual, entities and they are at the very root of the objective objects of Reason proposed by Frege. “Mill’s theory only concerns itself with the merely physical aspects of situations. It does not succeed in grasping what it is about a situation that is characteristically mathematical” (Bloor, 1991, p. 100), i.e., it does not do justice to the objectivity of mathematical knowledge, to the obligatory nature of its steps, or to the necessity of its conclusions. This missing component are social norms that single out specific patterns, endowing them with the kind of objectivity that comes from social acceptance. Bloor explains that “the psychological component provide[s] the content of mathematical ideas, the sociological component deal[s] with the selection of the physical models and accounted for their aura of authority” (p. 105).

The connection between the empirical and social aspects of mathematics is also highlighted by researchers in mathematics education. Although mathematics education research has been deeply influenced by psychology in its emphasis on the empirical bases of mathematical understanding, and the role of sensory experience has been at the center of research in mathematics education, specially in the area of mathematics learning, the focus on the external, material aspects of learning has been gradually shifting to encompass other perspectives, as it happened in the shifting of curricular perspectives we pointed out previously.

The perspective that there is an objective student “out there” that can be known has been criticized by researchers from various fields. On the one hand, attention has been drawn to the ways in which our cognition has strong ties to features as basic as our own body (Johnson, 1987). On the other hand, cognition has been shown to rely heavily on cognitive models that have strong social aspects (Lakoff, 1987). In mathematics, in particular, uses of terms like “height”, “basis”, “ten”, “increase” denote an individual cognitive base relating to corporeal features, together with their social acceptance. This blend of individual and social aspects provide the base for the construction of those mathematical concepts as metaphors (Matos, 1992; Pimm, 1990; Presmeg, 1991).

The empirical nature of mathematical knowledge suggests that on the one hand formal drill with written symbols should be discarded in favor of more relevant didactical strategies, and on the other, learning of mathematical ideas should be created out of experience, namely from those very models that underlie mathematical knowledge. The social nature of mathematical knowledge proposes that special attention should be given to the creation of a classroom culture fostering the development of intersubjective means of establishing mathematical truth.

By extending Mill's theory sociologically and by interpreting sociologically Frege's notion of objectivity Bloor opens the door to what he calls "alternative mathematics". Alternative mathematics would look as error to our mathematics. These errors should be "systematic, stubborn or basic" (p. 108) and they should be "engrained in the life of a culture" (p. 109). Bloor presents four types of variations in mathematical thought which can be related to social causes.

Two lines of research in mathematics education have been converging in legitimizing alternative mathematics. Researchers with a constructivist stance have been pointing out that students' own constructions of mathematics may lead them in a natural way to the development of mathematical significances divergent from what counts as school mathematics (Steffe, Cobb, & von Glasersfeld, 1988). Researchers working within the ethnomathematics framework have been pointing out the necessity for school mathematics to take into account the rich mathematical backgrounds that students bring to school (Gerdes, 1996; Powell & Frankenstein, 1997).

3. Mathematics as a social activity

Sociology of mathematics helps us to enlarge our perspective of mathematics activity as a social endeavor. The study of teaching and learning mathematics as a social activity has been a concern of several researchers in mathematics education: social construction of mathematics in classrooms (Walkerdine, 1988), the study of micro-cultures and interactional processes (Bauersfeld, 1980; Voigt, 1992), among others. Analogies between the mathematics scientific culture and the classroom mathematics culture have been drawn. As Cobb, Wood, Yackel and McNeal (1992) point out, both are created by a community and both influence individuals' construction of mathematical knowledge by constraining what counts as a problem, a solution, an explanation, and a justification. To incorporate the social aspects of the creation of mathematical knowledge as a subject with its own specificity, and to articulate its location within a broader social problematic have been a challenge to these studies.

Restivo may provide a way to think about the production of the mathematical knowledge in a more comprehensive view. He proposes (1990) that mathematical ideas, beings, models, etc. are not only social products but also socially constructed, and as such they embody social interests and practices that are inserted and belong to a more vast form of society. As Restivo shows, the organization of the community of mathematicians, what they produce, and context of creation, are all intertwined and are not independent of the larger social organization and interests. Namely, what is generally categorized in our society as mathematical pure ideas are a result of a set of procedures carry out by the community of mathematicians that started at the XIX century strongly related with the process of scientific specialization and professionalization.

In addition, Restivo highlights, that the circumstances that gave rise to the necessity of an ideology of purity are not only linked with the constraints imposed by especialization and profissionalization of science but also with the "imperatives of social conflict" (1990, p. 136), thus, to all this "movement toward 'purity'" (Restivo, 1991, p. 163) is not strange the fact that we live in a society whose social model is built upon a dominant class, and, to the promotion of mathematics in our society are not strange its success yearned by the problem solving of material and ideological question posed by the powerful as for example those linked with technology.

A main challenge to mathematics educators can be taken from Restivo (1991). Mathematical literacy and mathematical education in general can be improved by focusing attention in "revisions, reforms, and revolutions in mathematics always with an awareness of the web of role, institutions, interests, and values mathematics is imbedded in and embodies" (p. 172). Mathematics educators have been paying attention to this area (Frankenstein & Powell, 1994; Mellin-Olson, 1987; Powell & Frankenstein, 1997). For them, mathematics may be used as a means to achieve social change. Special attention has been given to mathematics rooted in specific cultures. However, Western mathematics can also be used as a subject of change and critic since findings from the sociology of mathematics show it as a subject embodied in a scenario of social struggle.

Since mathematics education plays an active role in the design of contents that promotes what counts as mathematics to a larger public, we cannot put anymore the social construction of mathematics out of mathematics education.

4. Mathematics education influences in the social construction of mathematics

Another contribution of the sociology of mathematics to our understanding of mathematics education is related to the role mathematics education plays helping to shape mathematics itself. Researchers in mathematics education have been denominating the process of transformation of mathematics as a science to mathematics as a didactical discipline as "didactical transposition" (Brousseau, 1986). The sociology of mathematics shows that there is also a converse process and mathematics education influences mathematics.

Several authors have put forward the idea that mathematical development, namely that higher levels of abstraction, has been possible because of the special focus put in the transmission of mathematical knowledge to the next generations. For example: "abstraction depends on realizing opportunities for producing, publishing, and disseminating ideas in a specialized community of teachers and students" (Restivo, 1990, p. 136).

The capacity that mathematicians showed to grant a generational continuity of ideas, processes and their own mainstream culture (supported, or not, by social agreement) underlain the social changes that appeared in the beginning of last century in their community, and is highly responsible for the cohesion of the field as well as for themselves to emerge as a new specialized group.

In addition to the idea that the teaching of mathematics was important to the development of mathematics itself there are also references in the literature of mathematics to suggest that the teaching organization and the contents organization have influences of their own. The creation of schools for mathematicians at the beginning of the XIXth century in France and Germany showed, for example, how these multiple aspects relate to each other to sustain social modification of the organizational forms of the community of mathematicians (Restivo, 1990). As Struik points (1986)

> [Attending Felix Klein's conferences] I learned how profoundly the French Revolution had influenced both the form and the content of the exact sciences and engineering, as well as the way they are taught. This was especially due to the establishment of the *École Polytechnique* in Paris.

The actual form of mathematics was also shaped. As Bottazzini noticed,

> The fact that, after the beginning of the nineteenth century, mathematicians also became professors, was to have important results on the character of mathematics, particularly as regards the rigorous organization of the theory for didactic purposes. (...) It also helped to assure for analysis a privileged role among the various branches of mathematics (p.47).

In order to foster especialization and professionalization of mathematics it was necessary to create a body of materials that fixed mathematical knowledge which would be passed from one generation to the next one. In the beginning of XIX century textbooks started to be launched with the specific objective of teaching. The publication of books as *Traité du calcul differentiel et du Calcul Intégral* de Sylvestre Lacroix, *Exercices du Calcul Integral* (1811) and *Traité des Functions elliptiques et des intégrals eulériennes* (1827-32) of Legendre (Bottazzini, 1986; Struik, 1987).

In conclusion, by the late XIX century, the picture of mathematics reveals that the imperatives of mathematics professionalization and especialization lead to an all set of changes in the community of mathematicians that put on teaching an enormous responsibility on its social and internal growing and recognition that conduces to the creation of teaching materials that ultimately configures a particular vision of mathematics. Thus, among the causes that influenced today's vision of mathematics is the way its teaching was organized and promoted: teaching materials of mathematics shaped the body of contemporary mathematical knowledge. An idea of mathematics as a social structured activity has brought out, and by and large implemented in school curriculum, privileging a vision of mathematics as an idealized science.

Mathematics education is ultimately responsible for the enhancement mathematical literacy of a generation. Mathematical knowledge of one generation is a also reflection of what have been taught to them and entails a particular vision of mathematics. Therefore, we may also try to understand additional ways in which we may see what is the social construction of mathematics. Since mathematics education plays an active role in the design of contents that promotes what counts as mathematics to a larger public we cannot put it anymore out of the social construction of mathematics. If previously we argue the importance of the social construction of mathematics for the field of mathematics education, we are now point out a converse conclusion.

5. Final considerations

This paper tries to put forward the idea that work by sociologists of mathematics may help mathematics educators to understand the activity of mathematics, its teaching and learning as a social process embedded in larger social problematics. We began by outlining social requests that are affecting mathematics curricula leading to tensions between a uniformity in mathematics content worldwide and a diversity accounting for distinct cultural backgrounds in students. This issue drove us to the sociology of mathematics, looking for views of mathematics incorporating strong social explanations.

We learned that mathematics is seen more and more as a social dependent field. Although mathematics may have a basis on empirical grounds, social exchanges are vital to the establishment of the "authority" of mathematical objects. Learning of mathematics tends to be broadly understood in a social conjuncture enlarging individualistic or mentalistic perspectives. The understanding of mathematics activity as a social effort was our next concern. Although mathematics educators have been investigating the nature of social exchanges in classrooms, the sociology of mathematics may allow us to deepen this perspective. The point is not that social processes occur in the classroom. They do, and the ways in which mathematics is created in schools influence mathematics learning. But also the very nature of mathematical knowledge embodies a social component. Sociology of mathematics also points to the fact that mathematics is created within a broader social context. It is not possible to account for the complexity of mathematics within the process of its teaching and learning without reference to the complex forces that drive it in the society. A final idea was supported by the sociology of mathematics. Research suggests that the way teaching and content are organized have their own role in the development of mathematics itself.

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Restos

Some questions can be

In this late years Western mathematics appears as a symbol of globalization factors that make mathematics such a world wide generalized field were pointed out (ex, linguagem valor-free). However such values are being posed in causa by recent studies of sociology of mathematics. Being so in what ground may we justify the bases for a unique? What are the main contemporaneous questions and problematics posed by the comunity of mathematiciens that has a broader sociological impact? Don't we have as mathematics educators o dever of telling about them?

The integration of social components of mathematics at all levels of research. As for the curricula, social environment must be taken into account, together with a better appreciation of the value of the classroom environment.

However, what is the role of mathematics education, in order to promote not only the vision that mathematics is a human and social activity but also the vision that mathematics is a rolling class related science (although this may lead to tensions) and that in so doing fosters the view point of the powerfull and the colonizer? What ideologies and biliefs are underlying to such options? Do we have as mathematics educators the possibility of actively participating in the creation of a critical position of mathematics? This context raise us some questions as: what makes mathematics such an important societal field of knowledge? What visions of mathematics has been presented to the younger generations, in the last 50 years that has made it seem so allien?[[Has mathematics education been participating in the social construction of mathematics? Are teachers protagonists of the withspread of a particular idealogical concepcão of mathematics, by veiculando isto in teaching?]]

From this, new challenging are posed to educators in this new social conjuncture.

autenticado

No excuses to command, no excuses to obey, no excuses to ignore
Some data to reflect upon

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Abstract

Starting by looking at the authoritarian school of yesterday and moving on to the inertia of today's school in responding to people's natural need to act, we propose school as a place for conscientization *that may lead to a* sustainable action*. In this paper we also intend to listen to students as they work on a problem situation related to the distribution of incomes. It gives us an opportunity to discuss the role of mathematics education in the development of a democratic competence.*

Introduction – A personal experience of authoritarianism

For seven years, between the middle of the fifties and the beginning of the sixties, six days a week, twice a day, as she climbed the stairs of the high school building, the first author of this paper could not keep herself from facing the sentence placed above the principal's door:

> *If you knew what it costs to command you would prefer to obey all your life.*

It is no longer possible to recall the moment when she realised that obeying is to alienate freedom and how she came to refuse the implied suggestion that such alienation would represent a benefit to the obeying individual. But she can remember having wondered about the motivations that would lead someone to command once it is seen as such a distressing thing. And she found herself discerning the difference between commanding as a sacrifice that one does presumably in favour of others – even if against their own will – and as the thrill of using power over other people.

By the same time she read the Diary of Anne Frank and she was moved by the fate of the young Jewish girl. Only later it came to her knowledge that the executioners of Bergen-Belsen and other extermination camps tried to justify themselves with the excuse that they had strictly obeyed orders. Probably then it occurred to her that obeying is also to give up the responsibility on oneself, it is becoming an object that can be manipulated.

The country – Portugal – was then living under the dictatorship of the man who created the above precept. Obviously it was intended to generate docile, submissive, dependent individuals. School was a place of non-thinking, of non-debating, of non-criticising; school was a place where teachers prescribed and students followed their prescriptions.

Illiteracy reached very high rates among the population but the dictator undervalued schooling, claiming that knowing how to read only facilitated the contact with subversive literature. We can understand what he meant by that, knowing as we do that illiteracy is connected to forms of political or ideological ignorance that work as a matrix of the way we perceive the world (Giroux, 1989) and as a refusal in believing that our acts can change society.

The Portuguese people was then living under the anaesthesia of the trilogy: Fado[1], Fátima[2] and Football. Fado was a convenient a way of making poverty, inequality and injustice look as inevitable. Religion worked as a consolation on such inevitability and football was a form of expelling the dissatisfaction and resentment, in directing them to the referees and the adepts of the opposing team.

But the individuals' development is also influenced by structures which are previous and external to school and shape their interests, their achievements, their believes and their aims (Lobrot, 1992). Some people had the chance of growing up in an environment that favoured the ideals of liberty, equity and social justice. There were those who realised that could be more than just observers of reality; that they could play a role in the transformation of society. This made possible the revolution of April 1974 and the establishment of a representative democracy.

A time when people choose the wrong ways of being protagonists

Twenty four years passed after the revolution, the basic civil rights are assured but there still is a democratic deficit in Portuguese society: income distribution is the most unequal of all the European Community countries (data from 1993), social security is the worse, the public system of pre-school education does not fulfil the needs of the whole country, there are known cases of child work, the number of students abandoning school is disturbing, women are still a minority in posts of leadership.

People are divorced from politicians. The majority is a lethargic mass, resigned to the inevitability of a distant and hermetic power. They could not find ways of extending and deepening democracy both in local communities and in production companies. Therefore they do not participate in the creation of a common destiny. Citizens are confining themselves to vote. But unlike what Schumpeter (1987) argues there is much more to democracy than the regular election of the people's representatives.

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[1] Portuguese folk song, also meaning destiny.

[2] The most important place of worship of Saint Mary.

We live in a time where family has lost importance in the education of the new generations. Young people spend much of the day on their own, partly due to the fact that women left home to embrace professional careers and partly as a result of parents having a more demanding and time consuming rhythm of life.

The room left empty by the family is being occupied by streets or television or computer games or... While people are more and more uncommitted to forms of social intervention they are showing a growing need to take action, to become protagonists. And this happens to be fulfilled in a number of ways: when one is a member of a gang, one is a leading person in a reality show, one is the hero of some exciting virtual adventure.

This thirst of action finds no echo in school. It still refuses students' central role in their learning process. It is not yet a place where students may choose the subjects and issues of their study and where they can be co-enquirers with the teacher, free to question both the curriculum and the pedagogy (Freire, 1972).

For a sustainable action

School is still missing a form of responding to the need of action. Our conviction is that schools must change towards what we are metaphorically calling a *sustainable* action, that is, a form of students action standing on information and knowledge about the society and the world they live in, supported by a critical reflection on that reality and on the way they are being in it, and sustained by the achievement of a consciousness that the situation they are part of is not inevitable but rather can be changed through their action. These are aspects that can be found in Paulo Freire's (1972) ideas on "problem posing" education.

Paul Ernest (1991) has assembled the main features of this perspective under the label of the Public Educators' ideology. For our present concerns we will underline the epistemological and ethical grounds of this perspective. It assumes that knowledge, ethics, social, political and economical issues are strongly inter-related. Therefore, knowledge is connected with forms of empowerment and is tied to possibilities of intervention in reality. In a word, knowledge and real life are not to be viewed as separate but must be integrated. The principles of egalitarianism and social justice also underpin the educational aims of this ideology.

Raising critical awareness or achieving critical consciousness is probably the outmost realisation of school under this kind of philosophy. This means to provide students the tools to become protagonists "here and today", it means no longer postponing the possibility of listening to their voices and valuing their knowledge.

We are thus suggesting the kind of critical pedagogy that, in the words of Giroux and Simon (1989), "takes into consideration how the symbolic and material transactions of the everyday provide the basis for rethinking how people give meaning and ethical substance to their experiences and voices" (p. 237).

In mathematics education, many have argued for such a change in proposing new aims for mathematics education. Keitel (1993) has discussed the need to unveil the implicit mathematics of our everyday routines. Skovsmose (1992; 1994) has pointed out the importance of recognising the formatting power of mathematics in our present society. Both have elected school mathematics as the place for developing reflective knowledge. Once again, we can see the influence of Freire's ideas concerning the way "problem posing" education, of a genuine reflective nature, entails a continuous act of the world's unveiling (1972, p. 99).

Our view of the role of mathematics education includes both the development of a democratic competence and the actual practice of democratic behaviours. School mathematics is foreseen as a place of thinking, debating and criticising. But we are also expecting school mathematics to be the medium through which people can discover what it is they have become and what it is they no longer want to be.

Along with Keitel (1993), we believe that mathematics education should evolve around situations where students and teachers are confronted with open-ended questions that favour the emergence of divergent perceptions of reality and give room to conflicting interests and values.

Sharing with Skovsmose (1992; 1994) the conviction that knowing is a factor of power and simultaneously a possibility of reaction to power, we consider mathematical education as a ground to awake people to social justice. Moreover, as Niss (1994) recalls us, speaking of equity is also speaking of mathematics and mathematical knowledge. There is not a precise allocation for mathematics where we can go and find it. Because it is hidden, mathematics looks as if it is absent. But it is "more like an all-permeating ether" (Niss, 1994, p. 372).

Data to reflect upon

In 1996 we were working on a research project concerning how students produce meanings for mathematical concepts and how real world problems can influence mathematical meanings. The students involved in this project were taking a degree in Business and the research was conducted in the classes of an introductory Calculus course.

Many of the academic subjects included in the curriculum of a Business degree are oriented to the learning of maximising profits while neglecting other concerns of social nature. The development of a democratic competence was not a purpose of the Calculus course. However, some of the problem situations used in teaching mathematical topics were suitable to promote the act of acquiring a new consciousness.

The data upon which we want to reflect were collected during a normal class where students worked in groups on a problem situation that concerned the income distribution of a certain population. A mathematical model for the cumulative income distribution was presented. It was the function $f(x)=x^2$, where x represents the lowest x% of population ranked by income and f(x) is the cumulative percentage of income earned by that part of the population.

One of the tasks suggested to students was to find out the income of the wealthier half of the population and the income of the poorer half. Another request was the interpretation of the outputs of f(0) and f(1).

The resulting dialogue of a group of five students is now transcribed. In this first piece of dialogue some of the students already reveal an awareness of reality in terms of social justice. But others, particularly Cristina, who turns out to be the best of the group in strictly mathematical issues, shows a propensity to keep mathematics and reality apart. At start she refuses to make connections between the mathematical model, with its implications, and the real world.

[1]	Miguel:	Now we just need to compute f(0.5).
[2]	Paulo:	Which gives 25%.
[3]	Miguel:	So 25% is the income of the poorest half of the population. This means that the other half is receiving 75%. It's a striking difference!
[4]	Cristina:	What are you saying? I don't get it.
[5]	Paulo:	He's saying that the first half, which is the worse paid, gets 25% of the total income. Therefore, the second half is receiving all the remaining, that is 75%. These are the better paid, they're the richer people.
[6]	Cristina:	Sure, according to this model...
[7]	Eduardo:	Yeah, I doubt that the income can ever be so unfairly distributed... (Speaking with an ironic tone in his voice).
[8]	Isabel:	Well, you'd better not!
[9]	Eduardo:	OK. Let's move on to this one: how do you interpret the fact of having f(0)=0 and f(1)=1?
[10]	Paulo:	Well, if there's no population there can't be any distribution of incomes.
[11]	Eduardo:	No people, no income.
[12]	Paulo:	The f(1)=1 means that 100% of the people receive the whole income.
[13]	Eduardo:	Exactly.
[14]	Isabel:	Which means that there is no embezzlement of money.
[15]	Cristina:	There's no sense here to speak of an embezzlement of money...
[16]	Isabel:	I mean that there are no false donations, no frauds, no fake payments, no funds deviations and no tax evasions.
[17]	Eduardo:	You're making a good point there...

As students realise the difference between the incomes of the two halves of the population, they start to introduce their own personal judgements based on their conceptions on social justice/injustice.

Miguel finds the difference of incomes between the two halves striking [3]. To Paulo it means the drawing of a line separating the poor people from the rich people [5].

On the other hand, Cristina seems to insinuate that it's only a result drawn from a theoretical model. She tends to detach the mathematical context, which may reflect a common belief that mathematics classes are supposed to deal with mathematical ideas only. At the same time she can be expressing her naiveness in admitting the fact that reality cannot be like the model is suggesting [6].

Isabel and Eduardo who reveal a more accurate and critical perception of the social and economical reality sense much the contrary. Using an ironic tone in his speech, in response to Cristina's ingenuity, Eduardo is perfectly convinced that incomes can be very unfairly distributed in the real world [7].

We may notice that students did not insist in stretching their interpretations further on. This is probably a consequence of the problem formulation, which does not make a reference to a specific situation or to a concrete population. At the moment we are producing this paper we are expecting to see how other students of a similar class will react to the same problem, having this time the income distribution for Portugal, which was in 1993 the most unequal of the European Community (Eurostat, 1997).

Anyway it is still obvious that students brought their views of the world and particular life experiences to the interpretation of the model. It is feasible to admit that such views and experiences are not shaped by the same economical, social and political backgrounds.

The question about the images of the Lorenz function at 0 and 1 would seem to be quite straightforward. Unexpectedly, however, it stimulated another critical look at the meaning of the mathematical model. Again, Isabel manifests her critical consciousness and sharp perception of reality. She integrates in her reasoning what we would call an administrative organisation of incomes distribution: the distribution of taxable incomes. Therefore, as she points out, what seems to be a simple fact, namely that 100% of the population would receive 100% of the income deserves some caution [14].

There are well known phenomena of tax evasion and illicit deviation of funds [16]. Those situations justify Isabel's doubting that the declared income to the fiscal bureau coincides with the true income of the population. Assuming that the model of income distribution is based on the declared income – which, by the way, is realistically the case – the fact that $f(1)=1$ must be interpreted as Isabel does. If there are cases of corruption among the population, then the income declared will not match the real income of the population. That is why the equality $f(1)=1$ really says that the income received by the whole population is the total *declared* income. Although she does not say it explicitly, she may be guessing that the distribution would be much more unequal if the model took into account the real incomes of all the members of the population.

There is strong evidence in Isabel's thinking of her knowledge of reality, and namely of the recurrent stories of financial corruption taking place in Portugal at various levels. For instance there are some political disputes about the introduction of a minimal tax for liberal professionals who are declaring incomes rather below their true value. A few years ago there was also a case of a major fraud involving fake receipts, which is still waiting for a court's decision. Isabel seems to be an informed person who recognises and critically evaluates important current social problems. She is also able to bring them to the foreground as the opportunity comes.

Once more, the mathematical model is put in question as to its reliability; this time what is criticised is its way of hiding part of the truth about the real world.

Having explored the Lorenz function given at the beginning of the task proposal, students were then asked to find the Lorenz curve that would match complete equity. They seemed to get it mostly by intuition, but they looked for a way of supporting their answer. They also discussed what that model could represent in the real world.

[18]	Paulo:	What would be the Lorenz curve in the case where the income is equally distributed?
[19]	Cristina:	Equally?
[20]	Paulo:	Yes, to be equally distributed among all it should be the function f(x)=x. Each share of the population gets the same share of the income. For instance 20% of the population gets 20% of the income: f(0.2) is equal to 0.2.
[21]	Miguel:	If we were to draw a graph, it would be a straight-line and not a curve.
[22]	Cristina:	It must be that. Each point of the X-axis has that very image on the Y-axis. It can only be the bisectrix of the first quadrant: f(x)=x.
[23]	Isabel:	Now, explain me that, will you!
[24]	Paulo:	Mathematically, everyone gets the same, there are no differences. Naturally if we attend to the actual situation of some real country, knowing about its social and economical conditions, we won't believe it can be attained. For instance, in Brazil, 10% of the population gets 90% of the total national income.
[25]	Cristina:	I see. The distribution has to be of one to one. Everyone gets the same amount of money.
[26]	Isabel:	That's what happens in left government countries.

This dialogue provides new interesting elements in revealing how students express their feelings about equality in income distribution. In Paulo's perspective equity is something mathematically conceivable but rather difficult to achieve in a real country [24]. He illustrates his point with a reference to a particular country, Brazil, which he seems to recognise as a paradigmatic case of unfairness in income distribution.

Isabel however does not share this pessimistic opinion. She thinks that some countries – those with left political governments – are examples of economical policies where distribution of incomes fit the one to one pattern of distribution [26]. In her perspective, left ideologies sustain the ideals of social justice. But in choosing those as paradigms of equality, she also seems to forget or to disregard the failure of left policies in what concerns equity.

In due course, we shall see how Isabel will continue to argue in support of her conviction on the possibility of having social and economic justice.

In a later phase of students' activity the issue under discussion is the concept of Gini index as a measure of income concentration. The model to compute the Gini index is given by the formula $2\int_0^1 (x - f(x))dx$ where f(x) is the Lorenz function. Students start to work on the calculations to evaluate the Gini Index for a sequence of income distributions: x^2, x^3, x^4,..., x^n. They use their knowledge on the properties of integration and transform the integral of a difference into a difference of integrals. The most relevant part of their discussions, however, stands on the commentaries they produce as they get their mathematical answers.

[27]	Miguel:	Now, what's the routine? We can do the integral of x minus the integral of x^2. Then we just multiply it by two.
[28]	Paulo:	We get 1/3. The index is 1/3.
[29]	Miguel:	So, in the case of this population the Gini index is 1/3. This tells us already something...
[30]	Paulo:	Indeed, it tells us that there is a deviation. We're founding a deviation from that extreme case where the distribution is equitable.
[31]	Cristina:	Can we say that the Gini index shows the deviation? I mean, when there isn't equity the injustice can be exposed with this.
[32]	Isabel:	It could be used for that purpose. Unions could take advantage of it to claim for raises. They could say that profits are being unfairly distributed...
[33]	Miguel:	Ideally we wish there wouldn't be any deviation. To get that, the distribution had to fit the straight line, but that's very unlikely. In every country this must escape from the straight line.
[34]	Isabel:	But one can try to put salaries a little more balanced... Of course, there's also the unemployment, which doesn't help either. But there should be an effort to make this closer to the straight line...
[35]	Paulo:	That's where the problem is. The deviation looks too large in this case. If the curve is a little smoother, maybe it will approach the straight line a bit more and the deviation can go more unnoticed...

The idea of deviation turns out to be a powerful image in students' understanding of the meaning of the Gini index [30-31]. Having the straight-line y=x as the reference to which they compare other cases of income distribution, they see the parabola as a detour. But there is another sense for the word deviation that may have contributed to their analysis. Deviation has an ethical sense of moving away from the good track, the *right* track. The deviation that is initially suggested by the graphical representation of the Gini index also conveys this idea of moving away from equity. This moral sense seems to be captured by Cristina when she alludes to the Gini index as a portrait of injustice and a form of disclosing it [31].

At last Cristina changes her attitude and starts to relate the model to the real world and she is now taking a stand when looking at that reality. We can see how she considers the index as an instrument to expose injustice.

As we would expect, Isabel approves this idea and extends it by suggesting that the Gini index could be an indicator of the low value of salaries and of the unfairness of profit distribution [32]. Moreover, she pleads for citizens' claiming mechanisms like the unions, proving once more to be aware of such means of intervention.

Miguel describes the straight-line distribution as the ideal situation. But like Paulo thought before he also believes that it fails to be the real case all over the world [33]. Only Isabel refuses to resign herself to the inevitability of the Gini deviations. She recalls the unions' role in demanding better salaries and even though knowing about the negative effects of unemployment she still finds possible to reduce injustice and poverty [34]. To her convincing words she obtains the agreement of Paulo who thinks that differences in people's incomes could be made less flagrant [35]. We can not assure what Paulo is really intending to say. He would prefer to see the gap between the actual line and the straight-line being reduced. But he may just be trying to disguise what he sees as an uncomfortable situation of inequality. Although he is agreeing with Isabel he is probably less committed than she is.

Conclusion

In reflecting upon what these data have shown us, we may notice a clear differentiation in students' attitudes and disposition to critically interpret the ideas involved in the mathematical modelling. In fact, some of the students, especially Isabel, are willing to unveil the reality submersed in the income distribution model, from the very start of the activity.

In contrast, there is also one student – Cristina – who tends to separate the mathematical procedures and results from the real world. She acts as if the model had an abstract nature and she prefers to look at it as a formal entity, which is meant to motivate or illustrate a mathematical exploration.

What happens in the course of the activity reveals a progressive change of attitude towards the acquiring of a critical consciousness. This is a result of an opportunity for students to share information and knowledge, to express their personal points of view on controversial issues, namely on the character, utopian or not, of an equitable distribution of income.

The continuous swinging between mathematics and reality allows for a better understanding of each of them. As a matter of fact, mathematics is working as an instrument to uncover reality in its social, economical and political aspects.

As the activity evolves, more and more elements of a reflective nature are introduced coming down to a critical approach to reality. At a certain point we can see how the entire group is awake to social justice.

The type of task proposal here presented is only a weak example; we must recall that our purpose was just the investigation of students' construction of meanings. Nevertheless it is strong enough to make us understand how mathematics education can contribute to the development of a democratic competence without assuming the role of indoctrination.

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ASSESSMENT OF MATHEMATICAL BEHAVIOUR: A SOCIAL PERSPECTIVE

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In this paper, assumptions underpinning assessment practices in mathematics education are challenged. It is argued that labelling any activity as 'mathematical' is an essentially social act in which power relationships among the participants play a crucial role. The assessment of students' 'mathematical' activity relies on their ability to display certain forms of behaviour appropriate to their assessor's expectations rather than on any necessary underlying 'understanding'. Some implications for research into assessment practices in mathematics education are discussed.

There has been increasing interest in assessment in mathematics education over the last ten years, including, in particular, attempts to match assessment methods to developments in the curriculum. While recent research into teaching and learning has increasingly turned to detailed consideration of its socially situated nature, however, the activity of assessment in mathematics has not undergone such scrutiny. Although we are all familiar, in principle, with the roles that summative assessment may play in regulating learners and reproducing the workforce, this has played little role in research related to assessment at the level of the classroom or the interactions between teacher-assessors and students. There is still an assumption that in some sense assessment actually does measure something that is clearly defined and exists objectively. In this paper I wish to explore this assumption and to raise some issues for future research related to assessment in mathematics education, taking a social perspective.

What may be called mathematics?

The widely discussed question 'What is mathematics?' takes for granted the existence of 'mathematics' as an independent entity or field of activity and generally sees it as more or less well defined (even if there may be disputes about the definition). It is thus possible to make statements such as 'Mathematics is . . . rigorous / systematic / abstract / . . . etc.', to point to various objects and activities in the world and say 'That is mathematical', to point to an individual and say 'She is a mathematician' or 'He is doing mathematics'. I wish to problematise this taken-for-grantedness and to consider how and why particular practices may come to be labelled and accepted as mathematical. This is not a purely philosophical, epistemological problem but has practical significance: assessment (with its multitude of forms and purposes) is pervasive in educational practices, having concrete and often significant consequences for the assessee, and what is assessment in mathematics education if not the labelling of an individual or the action of an individual as 'mathematical'?

The language that I am using should make it clear that I see the question of how we recognise mathematics and mathematical thinking, not as a question about perception of the properties of some independent phenomenon, but as a socio-cultural issue. Any claim that a particular practice is mathematical must be validated, not by reference to some absolute measure, but by acceptance within a community. The questions remain as to what community this is and which participants in this community have the power to accept a practice as mathematical.

Among the least contentious labellings is that of the activities of academic mathematicians. The paper published in the Journal of the London Mathematical Society must be a mathematical text simply because, if it were not, it would not have been published; the scribblings and discussions within university mathematics departments are mathematical activity because that is what happens in such places. Here we may clearly see the role of the community in defining mathematics. The editors, reviewers, department heads and colleagues have the power to include or exclude and, while there may be some areas of contention within the community (witness, for example, the dissension about the status of 'experimental' mathematics), while the boundaries of the discipline are by no means permanently fixed, and some members of the community may have more gate-keeping power than others, no-one from outside the community may challenge the definition. Though outsiders may question the *value* of a particular mathematical activity they may not question the right of the academic mathematics community to call it mathematics.

The identification of mathematics in school is more problematic, perhaps because the community involved is less homogeneous and the location of power within the community is less certain. On the whole the boundaries between activities within the school are clearly marked (strongly classified in Bernstein's terms). That which takes place within the mathematics classroom and is sanctioned by the mathematics teacher *is* mathematics. The mathematics teacher holds the power of definition and may ascribe meaning and value to the activity of students by labelling it as mathematical or not mathematical – in other words, the teacher can declare that a student understands or does not understand that which has been sanctioned as mathematics. There are, however, ways in which the teacher's right to do this may be challenged. Such challenge may come from the academic mathematics community as with, for example, the 'New Math' initiative's attempt to restructure school mathematics to reflect a particular aspect of academic mathematics. More recently in the UK we have seen attacks from the academic community on the use of 'investigation' in the school curriculum on the grounds that it involves and even encourages 'non-mathematical' ways of thinking (e.g. LMS, 1995; Wells, 1993). At the same time, challenge may come from students, parents or other teachers, usually occurring in response to changes in curriculum or teaching styles. For example, the teacher attempting to introduce discussion or problem solving into the classroom may have to face the question 'When are we going to do some real maths?', where 'real' mathematics is seen by students and parents to be pages of written exercises. Each attempt to change the school mathematics curriculum is, in effect, a re-negotiation of what is to be recognised as mathematics. The eventual nature of any particular change is a consequence of which group holds the strongest hand in the negotiation.

Within other disciplines, such as engineering or chemistry, practitioners may also see themselves as doing mathematics on many occasions. It is not always certain, however, that what the engineer or chemist identifies as mathematics within their own practice is the same as what the mathematician sees in engineering or chemistry; this may be one factor in the frequent disputes in universities in the UK about how best to teach ‘mathematics’ to engineering or chemistry students (and whether ‘mathematicians’ should do it). A mathematician involved in designing a course for chemistry undergraduates told me of her difficulty in making sense of a chemist colleague’s articulation of his view of ‘the mathematics’ of Fourier transforms. While both were agreed that students needed to know and understand more than just ‘how to do it’, each had a completely different opinion of the fundamental ideas underlying the theory. In this case, the two colleagues were happy eventually to accept the differences and to recognise both approaches as mathematical and equally valid in mathematical terms. Where communication between practitioners in various academic disciplines is more distant, however, I suspect that mathematicians are likely to be less tolerant of alternatives proposed by non-mathematicians and may challenge their right to label their own activity as mathematical.

Outside academia and the school, the situation is different. Here it is not, on the whole, the participants who identify their practice as mathematical but an outsider (psychologist, anthropologist, sociologist, mathematics educator) who places that label upon it. Indeed, participants may even refute such labelling

(Cockcroft,

1982). It has been pointed out (by, for example, Lave, 1988; Nunes, Schliemann, & Carraher, 1993) that activities in and out of school which bear an apparent resemblance to one another because they both, for example, produce solutions to isomorphic (from the outsider/researcher's point of view) numerical or spatial problems, may in fact be very different for reasons that are structured by the objectives, conventions and tools available within each practice. How then do they both come to be labelled as mathematics? In these cases, the labelling is likely to be irrelevant to the participants but may have great relevance to the outsider/researcher in that it creates a legitimate object for study and an audience for their findings.

In some cases, particularly among mathematics educators, mathematics has been identified in artifacts rather than in practices, for example, the "frozen mathematics" seen by Gerdes

(1986)

or Harris

(1987)

in baskets or knitted socks. Here the labelling tends to be even more contentious as it pushes the boundaries of what may be included as mathematical activity and who may be included in the community of those who do mathematics. In particular, it attempts to include those groups who have traditionally been less powerful and have had less authority (or inclination?) to label their own activity as mathematical. 'Does a spider do mathematics?' Does an individual have to be aware that they are doing mathematics before their behaviour can be labelled as mathematical? If doing mathematics is to be defined as participation in a community of practitioners, it would seem that the answer to that ought to be 'yes', for what is mathematical behaviour if it is not that which is recognised as such within the community?[1]

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[1] Here there is, of course, a question about which community: the community in

Where it is the outsider/researcher who labels behaviour as mathematical, the individual is being recruited into a practice that they may have no awareness of and in which their role is essentially that of an object of study rather than a potentially autonomous or powerful participant.

Recognition of mathematical thinking in school

Within the school context, I am particularly concerned with how children's mathematical thinking may be recognised or, to put this in a way more compatible with the discussion above, how children's thinking or behaviour may come to be recognised as mathematical within the particular community of the school mathematics classroom. Here I turn to my original concern with the nature of assessment – the official recognition of mathematical behaviour in school. Consider the following argument, characterising the common, often hidden, assumptions that underpin assessment practice and much thinking about assessment:

> *Assumption (for the sake of argument):* A child 'has' some cognitive state that, if the teacher could have access to it, would be labelled 'understanding' of some part of what is recognised as mathematics within the classroom.
>
> *But:* How may this 'understanding' be recognised?
>
> *Answer:* Through observing behaviour of particular kinds, especially semiotic production.
>
> *But (a further assumption that, if pressed, most mathematics educators would accept):* There is no *necessary* connection between 'understanding' and behaviour.
>
> *Assumption (necessary in order to make any claims about the validity and reliability of assessment):* A competent participant in the school mathematics discourse who 'understands' *will*, in appropriate circumstances, display the behaviour that will lead to recognition of that understanding by those in positions of authority within the discourse (including in particular the teacher).

This final assumption, of course, raises many further questions about what it means to be a 'competent participant', what might determine 'appropriate circumstances', etc. Ultimately, can we separate knowledge of the 'ground rules'

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which the weaver or knitter is an expert practitioner or that in which they are a powerless object of study for others?

(Edwards & Mercer, 1987) that might be said to constitute competence in the discourse of school mathematics from 'understanding' of mathematical concepts?

The learner thus not only needs to 'understand' a particular piece of school mathematics but also needs to know the forms of behaviour that will lead to recognition of this and how (and when) to display these forms of behaviour. In other words, she needs to have access to the realisation rules of the practice that will allow her to produce a 'legitimate text'

(Bernstein, 1996).

> *Question:* Is it possible to display the behaviour in the circumstances that will lead to recognition *without* ‘having’ the underlying understanding?

At various times, practices within mathematics classrooms such as ‘rote learning’ of multiplication tables have been criticised for producing successful behaviour in learners (i.e. correct, prompt responses to particular questions) without developing ‘the underlying concept’. Such disputes are often presented as being about the relative effectiveness of different teaching methods. Alternatively, they may be seen as competition between different discourses with different sets of meanings for what they both call ‘learning’ and different sets of behaviours recognised as signs of mathematical ‘understanding’ or success.

Competition between 'traditional' discourses of school mathematics, which may be seen to take 'answers' as signs of learning, and 'reform' discourses, which tend to be concerned with behaviours that appear to provide evidence of processes (represented in the US by the NCTM *Standards* and in the UK by the ATM and other advocates of investigation and problem solving), can certainly give rise to apparent anomolies in assessment practices. For example, one teacher, evaluating the work of a student who had presented a generalised algebraic formula as his solution to an investigation, stated:

> *He needs to explore. There's something needed before he could generalise.*

The student was clearly able, at least in the context of this problem, to generalise symbolically. This is, in many mathematical discourses, seen as a sign of high attainment. But, because he failed to provide the teacher with evidence of the expected 'exploratory' behaviour, he was judged not to understand what he had done. The 'need' – expressed in the language of pedagogy, suggesting a process by which the student could achieve 'true' understanding – is actually a need to display a particular type of behaviour that conforms to the expectations of the teacher.

Mathematics teachers and researchers in mathematics education spend a lot of time attempting to identify whether or not children understand or have learnt. In recent years it has increasingly been recognised that this is not a straightforward process and that assessment instruments may be inadequate in a number of different ways (see, for example, Ridgway

& Passey's (1993)) discussion of various types of validity), including those arising from the context of the assessment. Cooper's

(1996)

'Working Model of The Assessment Process in Relation to Culture' identifies some of the 'sources of threat' to the validity of assessment. His immediate concern is with the ways in which the cultural codes and strategies available to different groups of children may lead them to have differential access to the recognition and realisation rules[2] embedded in a system of assessment of mathematical attainment, giving rise to bias in the assessment. Detailed analysis of teachers' assessment practices in the context of GCSE coursework[3] (Morgan, 1996a;

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[2]"Simply, recogniton rules regulate what meanings are relevant and realization rules regulate how the meanings are to be put together to create the legitimate text" (Bernstein, 1996)

[3]The General Certificate of Secondary Education (GCSE) is an examination taken by students aged 16+ in England and Wales. As well as traditional timed

1996c) suggests that there are also differences between the values and strategies of various teachers and assessors that similarly threaten children's potential recognition as having learnt. For example, one teacher may read a succinct algebraic generalisation as a sign of a child's high mathematical ability while another teacher, like the one quoted above, may interpret the same text as a lack of evidence of desired problem solving activity.

The concept of valid assessment itself, however, demands some re-examination within a relativist theory of knowledge and a socio-cultural theory of learning (Galbraith, 1993; Morgan,

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examination papers, assessed by external examiners, it can include a 'coursework' component that is assessed by the student's own teacher.

1996b). It is commonly accepted that 'context' (to put it simply) affects people's performance on apparently similar tasks. This dualist formulation, however, assumes that there is some underlying constant 'understanding' or 'ability' that belongs to the individual, independent of context, and could be measured if only we could gain access to it in some 'pure' way. This assumption, of course, formed the starting point for the characterisation of common thinking about assessment that I outlined above.

But it is possible to see learning as being "not in heads but in the relations between people" (McDermott, 1993). As is often the case, it is illuminating to look at cases in which individuals are identified as *failing* to understand or to learn. McDermott's (Hood, McDermott, & Cole, 1980;

McDermott, 1993) analysis of the behaviour of a child identified as 'learning disabled' suggests that his disability is not his own inherent characteristic but is brought about by the social situations and relationships with other participants which structure opportunities for him to display behaviour that leads to him being labelled 'disabled'. The 'failing' child does not simply fail to learn. In fact she may be being very successful in learning to take up a particular position within a practice and, eventually, within the wider society (Dowling, 1991).

Towards a social perspective on assessment in mathematics education

Current thinking on assessment in mathematics education emphasises a concern for a match between curricular objectives and assessment methods. It is focused on reforming traditional assessment practices that have clearly failed to achieve such a match and, moreover, have often been associated with some obvious lack of equity for some groups of students. The arguments that I have presented in this paper, however, lead me to conclude that thinking at the level of objectives and methods fails to address fundamental issues about the ways in which assessment practices work to distinguish between students and the bases upon which such distinctions are made.

In order to take a critical approach to assessment, we need to examine the behaviours that are recognised as signs of mathematical understanding within the school mathematics community and the ways in which the situations that students find themselves in within the classroom enable them to display these behaviours (or fail to enable them). The values ascribed by teachers to various forms of behaviour are not necessarily obvious or universal. Indeed, the 'same' behaviour in different circumstances may be valued very differently. For example, the presentation of very carefully drawn diagrams may in some situations be valued highly, while in others it is interpreted as a sign of a lack of 'real' mathematical ability (Morgan, 1998). Yet the advice that teachers offer to their students tends to focus on the importance of taking care in drawing diagrams without distinguishing between different situations.

It seems likely that teachers themselves are often unaware of, or unable to articulate precisely, the forms of student behaviour that they will value as evidence of mathematical understanding. There is a fundamental equity issue here that is often ignored. Those students with the linguistic awareness and skills that are generally associated with advantaged, literate backgrounds are more likely to 'pick up' the unspoken distinctions and display the valued behaviour in the appropriate situations. Others, from less advantaged backgrounds, are less likely to come to school with these skills; they must, therefore, rely on their teachers to provide them with the necessary awareness of the forms of behaviour that will be valued. The naive guidance (such as 'draw a diagram') commonly provided by teachers is not adequate for such a purpose. An important task for teachers and researchers who are concerned with equity in assessment, therefore, must be to investigate assessment practices at a level of detail that can identify which aspects of students' behaviour are likely to be recognised as mathematical and valued as signs of mathematical understanding. Such investigations would also need to develop a language to describe these valued behaviours – a language that teachers and students can use both to help students to display the behaviours that will lead to success in the assessment process and critically to interrogate the assessment practices themselves.

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Development of a community of Mathematicians in the elementary classroom

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My presentation on the Development Of A Community Of Mathematicians In The Elementary Classroom is the basis of the research I am conducting for my doctoral dissertation. It is based upon thirty years of teaching and personal experimentation while studying the works of John Dewey, Jean Piaget, and Constance Kamii. While I did not keep formal records of my work during this time, I did get the support of my fellow teachers and my school's administrator. We ran a pilot project for two years prior to my returning to graduate school and beginning to look for the background to support the success we experienced. Early on I found the work of Alan Schoenfeld, Howard Gardner, John Anderson, Thomas Kuhn, Kenneth Boulding, and J. Bronowski. Although their works seemed to support what I was advocating, it was only tangential. It was the work of two people that really excited me, first the research of Alan Bell of the Shell Mathematics Institute who used the term conflict discussion to describe a successful program of confronting misconceptions. My only disagreement with Dr. Bell is that he wanted to identify all the misconceptions and leave the teacher in control. I want peers doing most of the challenging and the teacher educated to respond to any misconception. Second I came across a little known book by Cazden on Classroom Discourse. This tiny book summarises thirty years of research on teaching children to engage in intellectual discourse in the classroom. What little research has been done all points to the incredible power of this method of teaching.

So while there is little direct research support for developing intellectual communities in the classroom, what little there is makes a powerful statement. The fact that more work has not been done is, I believe based upon four things. First it appears to upset the battles over turf, who controls the classroom. It gives too much power to teachers and students. Second, it is hard work and we keep looking for a simple answer. Third, if the teachers at the lower grades change, it will force the issue with upper grade teachers because the children will be much more advanced and used to presenting their thoughts in a coherent manor that is taken seriously. Finally, because this method appears to work with almost all children, it removes the advantage given to the privileged in our society making mathematics education available to all. In other words, true revolution must take place outside the established realms because the establishment has too much to lose to allow major change within.

‘Do triangles exist?’: the nature of mathematical knowledge and critical mathematics education

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Abstract
After referring briefly to the argument that being a critical mathematics educator is associated with a particular epistemological stance with respect to the nature of mathematics, I offer case study data exemplifying this connection in the thinking of beginning teachers. I note that pursuing mathematical subject studies as part of one's initial teacher education course allows students to explore alternative mathematical epistemologies than the ‘common sense’ one currently prevalent and suggest that placing subject studies within teacher education rather than separated from it may therefore have a role to play in supporting the development of critical mathematics educators.

The conventional view of the nature of mathematics, which has an entrenched position in mainstream contemporary Western culture, rests on a taken-for-granted understanding of the nature of mathematical knowledge which accords it the status of absolute truth. The truths of mathematics are certain and unchallengeable, are ‘objective’, given and unchangeable. Mathematics is free of moral and human values and its social and historical placings are irrelevant to its claim to truth (Ernest, 1991, chapters 1.1 and 1.2.).

In a classroom predicated on such an epistemology, the teacher will ‘constitute mathematics as the activity of following procedural instructions’ (Cobb *et al*, 1992, p573), such procedures being regarded as ‘ahistorical, unalterable norms ... [without] a specifiable source ... both fixed and self-evident’ (Cobb *et al*, 1992, p588f). The model of learning will be that of transmission which envisages items of content (or correct strategies reified as content), pre-defined and non-negotiable, being ‘delivered’ to the head of the learner, reinforcing ‘teachers’ allegiance to limited bodies of content... enshrined as ritual’ (Noss and Dowling, 1990, p2). In this ‘banking’ (Freire, 1972, p46) mathematics, knowledge is seen as hierarchical, the teacher's mental models are superior to the learner's and education becomes the process of implanting those models in the mind of the learner. Knowledge is validated by external authority.

An alternative epistemology is based on a view of mathematics as being 'co-constructed' (Cobb, *et al*, 1992, p573) by teachers and students, a product of the human mind and, consequently, historically located, influenced by the knower and mutable. It is an epistemic strategy 'not based on any claims about ultimate truth ... not clothed in guarantees of any kind' (Restivo, 1983, p141). Consequently, learner enquiry and the construction of meaning are valued and the focus shifts 'from teacher delivery of "knowns" to learner investigation of "unknowns" ' (Burton, 1992a, p2). This in turn allows for the development of the capacity to be critical: it allows for the possibility of hope and the belief that things might be otherwise than as they are (Giroux, 1992).

As part of study investigating characteristics of mathematics teachers working for emancipatory change, I asked a small number of beginning teachers to locate themselves on a map of 'one person's way of characterising different outlooks on maths and maths education' (Figure 1) and then to use this and other prompts for a discussion amongst other things of the nature of mathematical knowledge. I report here, very briefly because of space constraints, on the responses from two teachers working for emancipatory change – Frances and Matthew – contrasting them, even more briefly, with responses from other beginning teachers – Janet, Kevin, Simon and Beth. (See Povey, 1995 for a full account).

Social group	**Industrial Trainer**	**Technological Pragmatist**	**Old Humanist**	**Progressive Educator**	**Public Educator**
View of mathematics	Set of truths and rules	Unquestioned body of useful knowledge	Body of structured pure knowledge	Process view: personalised maths	Social constructivism
Mathematical aims	'Back-to-basics': and social training in obedience	Useful mathematics and certification (industry-centred)	Transmit body of maths knowledge (maths-centred)	Self-realisation, creativity, via maths (child-centred)	Critical democratic citizenship via mathematics
Theory of learning	Hard work, effort, practice, rote	Skill acquisition, practical experience	Understanding and application	Activity, play, exploration	Active, questioning, empowerment
Theory of teaching mathematics	Authoritarian transmission, drill, no 'frills'	Skill instructor, motivate through work-relevance	Explain, motivate, communicate, pass on structure	Facilitate personal exploration and prevent failure	Discussion, conflict, questioning content and pedagogy.

Figure 1: Views of mathematics and mathematics education
(Based on Ernest, 1991, p138f)

What is mathematics?
In these first extracts from those discussions, the teachers working for change grapple with the nature of mathematical knowledge. Their approach is mostly tentative but comes within a non-absolutist paradigm. The philosophical problem of the relationship between mathematical structures and the real world

is unresolved but the existence of historical and cultural imprints on mathematics is recognised. Both of them see mathematics as something best described as a human product, either choosing to emphasise its personal construction or its social dimension.

Do triangles exist? [laughs] I started to think about the difference between maths and science ... You always hear it quoted that maths is a tool ... and starting the course ... neatly disposed of that idea and the maths became something in itself but it wasn't science and it wasn't like science so what was it? was it something that was objectively out there and you had to discover it, which I suppose is more like science, so do triangles exist in the universe and human beings have got to find them. Or is it just totally made up by people right from the start. And I think I'd been holding onto the idea that its totally made up by people right from the start for a very long time until I came across, back to pi*, and then I found myself saying this is the only bit of maths that I think is like science because* pi *is just there and people found it and I don't think I think that people made it up at all, it just seems to be there but that makes it stand out from the rest of maths. So maybe it isn't maths, maybe we should get rid of* pi *out of the maths curriculum, it's not allowed to count, so I still think about that because there are other things like* pi *really, geometric things and number patterns. But then I don't know about number patterns because you wouldn't have the number patterns if you hadn't decided on the numbers in the first place which is quite, if you could get people to sit down and think about it, is quite an exciting thing really because if you start off by saying that we're going to count because counting is useful and serves a lot of purposes so we devise a system of counting or tallying in some sense but you then discover that all these strange amazing things happen with these numbers that you've devised and where do the strange amazing things come from? (Frances)*

What I think today, and I might not think it tomorrow, is: it's a game. You play your game with symbols and ideas by moving symbols and ideas around, so I suppose that's seeing it as something that's independent of us but dependent on us as well. So it's obviously got some independence, it's an objective description of something and helps to describe the world around us, a language that helps to describe the world around us and that's objective but at the same time that description has a particular bias and so on (Matthew)

New teachers whom I have not characterised as working for change locate themselves differently with respect to their view of mathematics. Janet's response is emphatically to deny that mathematics is socially constructed. Simon too rejects 'socially constructed': he considers mathematics to be a body of structured knowledge. He adds

I would once have argued more for the social construction of mathematics. I no longer believe this, it seems too wishy-washy (Simon)

For Kevin too mathematics is a body of useful knowledge, a body of structural knowledge and means 'numbers and their manipulation, both in concrete and abstract terms'.

How do beliefs about the nature of mathematics affect pedagogical practice?

The teachers working for change see their understanding of the nature of mathematics as inextricably linked to their aims, motivations and practice. Matthew claims that his view of mathematics as both socially constructed and also as 'a language that helps to describe the world around us' affects what he does in the classroom.

It's not like we have mathematics in there waiting for us to discover it, that isn't the way it works ... the way my ideal about learning is moving ... is much more about kids or individuals building a mathematical structure around them which isn't one that's discovered and it isn't one that's either discovered in themselves or discovered out there but it's a structure which is developed ... [that affects practice], it must do and it does at all sorts of levels. It affects it in terms of trying, where possible, giving problem solving tasks, goal orientated so you've got some problem to solve and you get the maths along the way in the process of solving the goal so in that sense the kids are constructing the mathematics. But at the same time it's to do with describing or trying to solve a problem in the world, the outside world. That's on a very deep level. On a more basic level, I try and get them to puzzle, to think, answering questions with questions and so on (Matthew)

He is also aware that alternative perspectives amongst his colleagues are in turn related to their classroom practice and that changes in the former are necessary if one wants to achieve change in the latter.

... actually in terms of changing people's deep perceptions about, about mathematics, and so on, and about what we should be doing ... (Matthew)

These teachers speak of the need for their students to work in a way which permits the construction of meaning, of not having the curriculum broken down into pre-digested bits, and they relate this to letting the students develop ideas, of allowing the students to have some control over the agenda (Skovsmose, 1994). Flexibility and responsiveness to the students are valued and there is an openness to negotiation.

I've tried to hold on to not to spoonfeed, not give things piecemeal in tiny pieces (Matthew)

I gave them a problem to do with fractions ... and I gave them two weeks to do and they kept coming and talking to me about it and that was a massive change ... One of the things I want to encourage is for them to do longer term pieces of work ... A lot of what they do is disjointed and quick and doesn't come together (Frances)

Start with the topic of say, suppose we're doing some work on circles and I start with the topic of that and have some doing some work on, you know, just drawing circles, circle patterns, others finding out something about pi*, others finding out something about the area of a circle, and have that full range and start with a topic and just go with it ... I think in terms of what I do in the classroom at best there's discussion with them to try and motivate, discussion of targets and goals (Matthew)*

This is linked with the rejection by these teachers of a transmission model of teaching and learning (Burton, 1992, Skovsmose, 1994): Frances says ‘I don't think mathematics is a body of knowledge so I can't transmit it’. Janet too claims to reject a transmission theory of teaching but chooses in its stead ‘explain, motivate, pass on structure’. Kevin also feels happiest with this theory of teaching. He couples this with a statement about ‘discovery learning’;

I feel that, where possible, children should discover mathematics for themselves. Having discovered it for themselves, I then explain that actually someone else discovered this before and here is the appropriate way to write it down (Kevin)

but there is no sense here of a personal and creative act (see Papert, 1972, p236), nor of developing the capacity to criticise and produce classroom meanings. He claims that ‘investigational methods’ are ‘one of his preferred ways of working’ but adds ‘I also like “chalk and talk” a lot’. His overall view might be encapsulated in the following remark: ‘I enjoy starting with a blank class and making it understand’.

All of the teachers working for change place emphasis on and value student discussion and seek to develop it within their classrooms.

within that, conversation and talk are really important ... In terms of teaching and how you actually teach in the classroom you might have what is a textbook exercise and you can turn that into a discussion exercise ... quite simply (Matthew)

Student talk was not valued by all the new teachers. Discussion and questioning were marked by Simon as being definitely not representative of his outlook and Beth said

[Group building?] I don't build any of my groups ... like I don't do much group work at all, the kids talk too much (Beth)

The quest for a curriculum which encourages critical thinking is linked by Matthew to a problem based approach to mathematics (Giroux, 1983):

[I try], where possible, giving problem solving tasks, goal orientated so you’ve got some problem to solve and you get the maths along the way in the process of solving the goal ... the sort of task-orientated, what-do-you-need-to-know-in-order-to-get-to-a-particular-goal which is the way my ideal about learning is moving (Matthew)

and also by Frances who places emphasis on the work being such as to challenge the students and is aware that this is often lacking.

It seems to me that for a lot of them what happens, what their experience of maths is in secondary schools is that they get given things they can already do essentially because they get the satisfaction of doing it right and your classroom control is far far easier, and that for a lot of them they get a diet all the way through of things they can already do, to the extent that the worksheets in year 11 are the worksheets in a sense of year 7 really (Frances)

They want to foster in their students the same confidence and personal authority that they themselves feel where one grapples with difficulties, comes to conclusions which later have to be revised, where getting things wrong and changing one's mind are fundamental to the process.

Some of the things I want to get across to them will be unsettling to them, that writing neatly isn't what it's all about, that I want to see their working, that I don't want it to be hidden, or just the idea that when they actually get things wrong that that process of getting things wrong and then sorting it out is actually when they're learning [laughs] that they're learning by getting things wrong and it's not something that they should tear the page out of their book and try to pretend it never happened ... I often say in the classroom when I'm presented with beautifully neat books usually the girls' with beautifully neat answers written and there's a pile of stuff in the bin, that the stuff in the bin is their maths and I don't want to see the beautifully neat answers so then they get cross with me. Because all the scribblings and the crossings out and the getting it wrong and doing it again is *their maths, it's the process of thinking it through and starting on a track and giving it up and going down a different track (Frances)*

Because they wish to foster autonomy and personal authority in their students, their attitude to classroom management is not disciplinarian.

I'm thinking that as this pressure [of larger classes] increases, you can retreat but it isn't a solution, a more discursive style is needed more *or else you just get into a battle trying to get the kids to do things they don't want to do, you end up rowing and shouting (Matthew)*

I've learnt to avoid confrontation whenever possible, if there's a way to sort things out without confrontation (Frances)

This perspective is not shared by others.

there's the problem of being a new, green teacher who cannot yet anticipate all the fiendish ways students will find to undermine you ... It has made me into a much more suspicious and cynical person than I used to be and I now understand why 'real' teachers on t.p. used to demonise children ... I didn't expect to become quite the authoritarian I am now. It became obvious to me though after a few months of teaching that I needed to feel in control (Simon)

I am more of a disciplinarian than perhaps I thought I might be (Kevin)

I think I am quite an authoritarian person to be honest. I mean I keep saying please and thank you [to the kids] and it really bugs me. I think, I mean [I want to say] 'just do it' you know. Like I'm really nice, I used to be really cold in the classroom on TP, I was told don't tell the kids off so harshly. Like in a school like this, you need positive discipline, but what's *positive discipline or is it just weak discipline? ... There's no school rules, the staff and the pupils agree, you know, agreement that they've come up with, that they shouldn't chew in class, and they shouldn't hit people in class and they shouldn't be offensive to other pupils or staff. But you know there's no real school rules and in terms of sanctions there's no sanctions apart from your own sanctions that you impose. [It's not enough.] ... I don't think the system deals with [the kids] well. Like no-one here will have a confrontation here, there's real avoidance of confrontation in the school, like everything tries to be smoothed over (Beth)*

The role of initial teacher education

The teachers working for change seem to find their initial course of teacher education both relevant and moulding of their views, a perspective not shared by the other new teachers (cf McNamara 1976, Cole 1985). Matthew feels that the course equipped him to resist some of the pressures attendant upon work in school:

the path of least resistance is to revert to tried and tested, how you were taught, all that sort of stuff. I suppose in terms of the course, the course has given me a lot of options in terms of going back and saying hold on a minute this isn't necessarily the best idea in the long run or even in the shorter for that matter (Matthew)

Messages from the hidden curriculum are acknowledged as being central.

[The message from the course was] mathematics is something enjoyable, that it should be displayed ... You get a sense of there being some democracy in terms of the way things are decided ... I think within that, in terms of relationships, with students there was efforts made to give a sense of equality, equal status. [As a teacher] it was motivating and still is in the sense, in the sense that it's important to know that there are, that it's possible to work in that way ... and that general message of valuing mathematics (Matthew)

They talk about the course as though what they most feel they gained from it was that it had opened doors into a new vision for them, particularly a new vision of mathematics. What seems to be a key element in their experience has been pursuing subject studies as part of their initial teacher education. When asked to look back at the course and review it in the light of their experience now as practising teachers, these teachers all immediately speak about *mathematics* sessions and the ways the course has made them see *mathematics* differently.

[The course made me see things differently] certainly in terms of mathematics. A very different way of learning maths. For example the tiling activity we did right at the beginning ... lots of other occasions. You can't rely on that intellectual approach, a more intuitive approach has value. That has personal implications about infallibility and about valuing intuitions and so on. A very strong message "It's OK to be wrong". This did come from the course (Matthew)

I suppose there are times when with particular issues you remember a conversation or whatever ... [but] it's more an opportunity to learn maths and to talk about learning maths with the group of people that I was with who were, in a way, who had a whole range of ideas and backgrounds which made it even better, our reactions to learning maths were very different so we were able to sort of talk about ... and I suppose ... well it did change my view of maths quite a lot and I'm sure that must and definitely does influence what I'm like in the classroom (Frances)

When I embarked upon the PGCE I imagined that the first year would be a straightforward revision of maths; a subject which I considered to be elegant, objective and 'issue free' ... It is impossible to exaggerate the revolution that occurred in my thinking and outlook as a result of the teaching I received in the Maths Education Department ... it was particularly fascinating ... to debate whether maths can be said to have any objective reality ... I now believe that the neglect of the history of ideas is one of the greatest downfalls of current teaching. Children are too often presented with a fait accompli *such as Pythagoras' theorem without any mention being made of its development and significance in a variety of cultures ... Having discovered that maths was more than just elegant I was then to discover that it is far from 'issue free' ... the way we were taught, the resources we used, and the reading/reflecting we were encouraged to do all stimulated thought and I now see how many questions were planted during the first year which I have never stopped considering. (Frances, final Education assignment of PGCE, sent to me after the interviews)*

Their perception seems to be that studying mathematics within the teacher education context, rather than outside it, has been significant in shaping their understandings of the nature of mathematics and of teaching and learning. The claim here is not that pursuing subject studies within initial training guarantees the creation of critical mathematics educators: rather that it supports the effective development of those open to such a perspective and helps them in the process of connecting this with their practice. If these perceptions were to inform teacher education, we would no longer be happy with the mode of preparing to teach, prevalent and often taken for granted in the United Kingdom, of subject studies of an appropriate length and to an appropriate standard being undertaken *before* embarking on professional training. For example, the National Commission on Education writes:

Once trainees have become secure in their subjects as we recommend, classroom skills are at the heart of teacher training (National Commission on Education, 1993, p214)

and the intended National Curriculum for initial teacher education, whilst allowing that some subject knowledge might be taught alongside pedagogical knowledge, nevertheless recommends 'much might be covered through the use of supported self-study and through guided reading prior to the course' (Teacher Training Agency, 1998, p4).

Within courses of initial teacher education which already include a subject studies element (two year PGCE, two, three and four year undergraduate courses), present national structures allow, although they do not require, the integration of professional and subject studies. This can be used to provide rich opportunities for students to rethink their understanding of the nature of mathematical knowledge and their ways of knowing related to teaching and learning based on their own practices as learners. Students can also be encouraged to mine the practices of their tutors for practical help (Sikes 1993) in implementing a liberatory mathematics curriculum.

No such opportunities exist on the one year training route which takes 'subject knowledge' as a prerequisite. The single year is already too full adequately to allow students the opportunity to stand back from their practice and as such seems to discourage student reflection: there is simply no room to expand this curriculum to include a significant element of subject studies. On the data presented, the hypothesis that re-viewing subject studies as important to the epistemological perspective of the becoming teacher seems plausible but, as Leone Burton has noted, if prospective teachers

come from successful experience of didactic learning, very likely for example in the case of the Postgraduate Certificate of Education (PGCE) student who has already gained a degree in mathematics, and their teaching practices are within the same mode, any questioning of its efficacy by the teacher education institution is more likely to fall on deaf ears. (Burton, 1992, p380)

If these observations are sound, then it will not be surprising if initial teacher education through the one year route does not have much success in enabling students to become teachers working for change.

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Society, Mathematics and the Cultural Divide: Ideologies of Policy and Practice 1750 – 1900

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Abstract

This paper explores the social and ideological background that determined the kind of mathematics taught to different groups of people during the industrial revolution in England. These ideologies rise in and are transmitted through institutions that determine choices and decide what is valued as scientific knowledge. The mathematics taught in the universities and in the Public Schools was determined by a classical liberal ideology, whereas the mathematics taught in elementary schools and colleges was driven by a practical ideology of utility, democracy and social justice. The consequences of this conflict can be seen in our current school mathematics curriculum.

1. Introduction

Accounts of the history of mathematics have generally paid little attention to the grass-roots teaching, learning and applications of mathematics in the life of ordinary people.[3] This paper is an a tempt to give a brief overview of some of the significant social and political movements in the eighteenth and nineteenth century which contributed to the development of the mathematics taught outsi e the universities which led to the formation of the ideological divisions within our contemporary school mathematics curriculum.

The population of England grew from six million in 1750 to nine million in 1800, and there was a rapid change from urban to industrial life with all the problems that this brings. With the belief hat industry could improve the conditions of life, the philosophy of utilitarianism was developed intending that society should provide the `greatest good for the greatest number'. By the mid eighteenth century, new theories about the nature of man, of society, and of the acquisition and purpose of knowledge, began to have far-reaching influences on political and educational ideas. At the same time, "laissez-faire" economics was promoted, allowing the free interplay of forces in economics and society and monitoring their effects with minimal legislative interference. These benefits and forces, it was assumed, would be controlled by educated people making the `right' judgements about moral, political, and economic issues, and the principle that education could influence human beliefs, attitudes, morals and conduct was emphasised. In the later eighteenth century we see a gradual development of the professionalisation and institutionalisation of mathematics teaching. Set against the considerable changes in society and economic development, the kind of mathematics, the people who taught it, and the places where it was taught all underwent significant changes. During a critical period from about 1750 to 1850 the gradual isolation of the universities with their classical traditions from industrial growth and technical improvement was evident, and the need for practical applications of science and mathematics was answered by other sources and other institutions. This division was closely linked to the class system and the established English intellectual and social attitudes of the time.[4]

2. Early Traditions in English Mathematics Teaching

In mathematics teaching two traditions can be identified from the sixteenth century. The "Liberal" tradition was based on translations of Billingsley's Euclid (1570) evolving into a formal style which continued until Playfair's Euclid of 1792 became the standard for the next hundred years.

The other "Vocational" tradition is based on Robert Recorde's "Pathway to Knowledge" where the principles of geometry were set out so they might "most aptly be applied unto practise both for the use of instruments geometrical and astronomical and also for projection of plats[5] in every kind, and therefore much necessary for all sorts of men" Recorde 1551).

In this way geometry and arithmetic became popularised in many self-help books where detailed explanations and exhortations to the student accompanied the examples.(Fauvel,1989)

John Dee's "Preface" to Billingsley's Euclid contains a comprehensive description of the "Mathematical Arts" showing their universal usefulness and giving reasons for studying mathematics at all levels. In Elizabethan times mathematics (which included astronomy and astrology) was seen as the key to knowledge and the mysteries of the universe.[6] This tradition continued (Taylor, 1964, 1966) an in the 1780s at Woolwich Bonnycastle was writing alternative treatments of geometry[7] intended for students with different aspirations, and Hutton's "Course in Mathematics" of 1798 was the practical text for the artillery and engineer cadets.

3. The English Radicals: Science as a Foundation for Education.

From the 1790s onwards working people began to read the radical press, attend lectures, and learn by participation in political discussion. Organisations supporting these activities were called "co responding societies", and provided organised and disciplined opportunities for study.

"The Rights of Man" (Paine, 1798), was an attack on the established social order and its exploitation of the poor and working classes. The radicals saw the Church as the main obstacle to political r form in its reinforcement of the strong social stratification, and they replaced this indoctrination with rational education through their own schools, aiming to inform people of the reasons for the r condition and the state of society and industry, and placing instruction within the reach of everyone. Teaching methods encouraged self-confidence, and the capacity for clear self expression, and he organisers realised the importance of combining systematic education with mass political agitation. Books and newssheets were shared: an individual would take a book home, read a passage and prepare a talk for the next meeting; the book would then be passed on, and the process repeated. Many subjects, including some elementary mathematics were learnt in this way. As a result, men and women b came informed and critical leaders of the new working class movement, able to master and comprehend some of the most advanced political thinking. This was recognised as a threat by the establishment and in 1799 an Act of Parliament was passed ".for the more effectual suppression of societies for seditious purposes.." (Simon 1960,p.183)

By 1817 there were popular demands for a rational secular education for all. Paine had demanded the teaching of science which was directly applicable, to be regarded as the cornerstone of a rationalist philosophy. These demands were of great concern, and in 1817 a House of Lords Secret (*sic*) Committee reported on the unprecedented circulation of "publications of the most seditious and inflammatory nature, marked with a peculiar character of irreligion and blasphemy, and tending not only to overturn the existing form of government and order f society, but to root out those principles upon which alone any government or any society can be supported." (Simon 1960 p. 131)

The Stamp Act (1817) required the registration and taxing of all newspapers and journals, and as a result, the radical newspapers were forced underground.[8]

Richard Carlile's "Address to Men of Science" (1821) also demanded a curriculum which contained reading, writing, the use of figures, elements of astronomy, geography, natural history and chemistry so that children may "at an early period of life form correct notions of organised and inert matter, instead of torturing their minds with metaphysical and incomprehensible dogmas about religion" (Carlile 1821 p.22) He believed that science, best studied by observation and experiment, was the key to knowledge and freedom, and promoted a materialist psychology, and demanding social and moral education by example.

4. The Schools.

In the mid eighteenth century some grammar schools existed, but few taught any mathematics; perhaps the first two books of Euclid, and some simple arithmetic. Any other kind of education was locally organised, usually by well-meaning clergymen and public benefactors. Some clergymen took private pupils and this tradition continued well into the next century.

By the late 1780s, to counter the radical political literature that was freely circulating, Sunday Schools were established for the poor, their major purpose being to indoctrinate pupils in the principles of religion and the duties of their state in life. Here, if you were lucky, it was possible to learn reading, writing, elementary arithmetic, and the catechism. However, due to the teachers' concern for the health and welfare of their pupils they unwittingly `created thought in the unthinking masses'.(Simon, 1960 p.183)

In the 1830s we begin to see the establishment of the English Public School system. The amount of mathematics and science taught in these schools was very variable and schools like Eton, Harrow and Rugby[9] did not appoint mathematics masters until challenged by some of the newly founded institutions. Substantial reforms were made to preserve the establishment,[10] by requiring these schools to provide an appropriate education for politicians, civil servants, the clergy, the army and the administrators of the Empire. Since most of the schoolmasters had been educated at Oxford or Cambridge, it was no surprise that the `Liberal' ethos prevailed, and the theorems of Euclid were regarded as part of the corpus of classical literature.

5. Non-Conformist Education and the Mechanics Institutes

From 1766 the "Lunar Society" held informal monthly meetings in Birmingham.[11] This was typical of a number of "Literary and Philosophical' Societies whose members were forward looking scientists o innovators with interests in practical applications of the new ideas of natural philosophy. Later, more radical interests developed, and they also began to encourage social and political education intending to prepare their sons for their place as leaders of the new industries.

The Private or `Dissenting' Academies were the places where Non-conformists could be educated[12] The earliest of these was Warrington Academy, founded in 1757, and the subjects taught had obvious practical applications. Manchester College of Arts and Sciences, founded in

1783, taught sciences and practical arts on four evenings a week. Its syllabus contained classical languages, grammar and rhetoric, mathematics (including trigonometry), mechanics, natural philosophy, (including astronomy and chemistry) English composition, French, commercial and economic geography, history, politics, writing, drawing, book-keeping and shorthand. Subjects like these became the standard curriculum, and most of the important cities of this time developed similar educational institutions. There was a great demand for applied science, and "mixed mathematics".[13] In 1786, the Manchester Academy was established, providing full-time education for students, and a permanent mathematical tutor was appointed in 1787.

The Literary and Philosophical Societies also supported the development of Mechanics Institutes, which became another focus for working class self-education. They introduced science, literature and the arts; deliberately excluded politics and religion, and provided lectures, evening and day classes, and libraries. There was a substantial demand for reading scientific (and clandestinely also political) texts and reading rooms and loan systems were established. The curriculum was based on what was "useful" to workers, and lectures were related to practical applications and local engineering and manufacturing problems.[14] Advanced classes were given in a selection of subjects like Grammar, French, Latin; Science, Chemistry, Electricity; Mixed Mathematics, Algebra and Mensuration. Provision for science also meant that collections of apparatus began to be built up, and lecturers established courses, developed curricula, and wrote texts. (Inkester 1975, Royle, 1971)

6. Military and Naval Schools.

Schools of navigation had grown up in the major ports for merchants and traders, and military and naval academies provided an education for the entrants to the army and navy. Woolwich Academy, where Bonnycastle and Hutton taught, was founded in 1741, and the teachers there were familiar with contemporary continental texts. In 1837 the syllabus consisted of arithmetic: fractions, roots and powers, proportion, interest, permutations and combinations; algebra: arithmetic and geometric progressions, logarithms, simple, quadratic and cubic equations; geometry: plane trigonometry, mensuration, surveying, conic sections; dynamics, projectiles, hydrostatics, hydraulics and fluxions. The syllabus was eventually updated to include the calculus, and other more recent aspects of applied mathematics, and a system of open competitive examinations.(Rice, 1996 p.404)

The Royal Naval Academy, founded at Portsmouth in 1722, (renamed the Royal Naval College in 1806) transferred to Greenwich in 1873. After undergoing similar problems and reorganisations to its military counterpart, from 1885 the Academy taught ballistics for gunnery and torpedo officers, mechanics and heat for engineers, and dynamics for ship construction. Thus it was that by the end of the century clear, practically focused and vocationally relevant courses had evolved for the training of military and naval personnel.

7. The Education of Girls and Women

Sometimes girls attended elementary school, but generally were only taught the most elementary skills. During the eighteenth century a few boarding schools for girls were set up which taught mathematics, science and astronomy, and by the end of the century some women were pursuing their own studies by corresponding with scientists. (Harris,1997 p.37) However, it was not until late nineteenth century that mathematics became firmly established in the curriculum of girls' schools. [15]

No women were admitted to Oxford or Cambridge before the beginning of this century; the Victorian attitude to the mental capabilities of women, and their low social status, together ensured that a y opportunities for further education were severely limited. However, this was to change slowly with the publication of the "Educational Times" in 1847, where subjects like the importance of women in society, and the qualities of women's minds were intelligently discussed. The College of Preceptors, founded in 1846, played a major role in supporting women, and from the 1860s we find a growing movement for the elimination of sex differences in education, particularly in mathematics and science. From the mid nineteenth century, higher education for women began to develop. Queens College[16] was founded in London in 1848, the Ladies College Bedford Square in 1849, and by 1878 University College became the first co-educational institution where women and men were examined together.

8. Changes in the Universities

In 1826 University College was founded with the support of those who were excluded from Oxford and Cambridge, liberal politicians, and Jeremy Bentham, the humanist philosopher. In 1828, after demands to provide a religious foundation in London, King's College was founded. In 1828 De Morgan was appointed the first professor of mathematics at University College. He was a thoughtful, idealistic a d energetic educator whose text books and pedagogical writings show a deep concern for the problems of learning and teaching. His motives for writing On the Study and Difficulties of Mathematics (1 31) are to help `tutorless' students, with the areas of elementary mathematics which give most difficulty, describing their nature without emphasising routine operations. De Morgan takes the view t at mathematics is a necessary part of a liberal education, and that it is useful, being the key to other sciences. Much of his work was serialised through the "Society for the Diffusion of Useful Knowledge" (SDUK).[17]

Meanwhile Whewell at Cambridge, aimed to place mathematics in the curriculum of every student of the university, reinforcing the "Liberal" view: "I believe that the mathematical study to which men are led by our present requisitions has an effect, and a very beneficial effect on their minds: but I conceive that the benefit of this effect would be greatly increased, if the mathematics thus communicated were such as to dissipate the impression, that academical reasoning is applicable only to such abstractions as space and number." (1836, p.44)

As the century progressed, university mathematicians seemed less inclined to spend their time educating the masses; growing professionalism motivated more `pure' mathematical interests and since, from 1850, Cambridge required a knowledge of Euclid for its entrance exams, other universities followed suit.[18]

9. The Ideological and Pedagogical Divide

By the end of the nineteenth century "laissez-faire" economics had given rise to a large number of industrial enterprises each requiring ever more specialised training. The Mechanics Institutes were one way to cater for this need, and they helped to develop ideas of economics, of the idealist possibilities of science and technology to improve everyday life, and an acute awareness of the need f r appropriate training and new teaching methods. A considerable amount of their work was experimental and practical, and the mathematics required to make the machinery work efficiently was being developed alongside the craft skills of manufacture. Thus it became obvious that the traditional mathematical diet was quite inappropriate to the needs of the new industrial community and advocates of practical mathematics were designing new courses and writing new textbooks. Prominent among these was Perry,[19] a significant figure in the reform of mathematics teaching at this time. Reforms in school, however well-intentioned, were hampered by schoolmasters educated in the Oxbridge tradition, and a lack of interest from the universities.[20]

The products of industry shown in the Exhibition of 1851 were based more on the freelance initiatives of innovators than any government sponsored organisation, and it later became clear to government that economic advantage rested not only on technical education but also a good primary education. The 1870 act ensured that education up to age 13 was available to all, and while the attempts to devise differentiated schooling on a class based system had failed, these attitudes prevailed in the secondary, technical and grammar schools that evolved. It is here that we see the ideological divide; the establishment provided for its own in continuing the liberal tradition in the Public Schools and using mathematics to control the `gateway' to Universities where `pure' mathematics flourished at the same time invoking the utility of `vocational' mathematics to train the industrial workforce in technical schools and colleges.

Looking at the more recent past, this conflict has been compounded by issues involving pedagogy as well as style and content, but the expectations of the two ideologies can be detected in the nature and mode of presentation of the curriculum materials of today. Dowling (1998) locates these ideologies in a detailed analysis of contemporary school texts; in this brief presentation I am attemptin to show their social and historical roots.

Notes

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2. Here I give the general background to the argument. Earlier discussion of this subject can be found in Rogers (1979 and 1981). A longer version of this paper will be available on request.
3. There are exceptions; see Taylor's work on the Mathematical Practitioners (1964 and 1966) and Yeldham (1926 and 1936) but there is little attention to social issues.
4. For some general background to this period and its social, economic and political detail see Simon (1960) and Hobsbawn (1968).
5. A "plat" was a plan used for building or surveying; a map or chart for finding direction or navigation; or an explanatory table or diagram.
6. It is interesting to note that the editions of Euclid up to Leek and Serle (1661) all contained Dee's "Preface" but after this it disappears from further editions. Thus Euclid clearly becomes part of the "Liberal" tradition.
7. Bonnycastle (1810 p. ii) carries a list of practical texts of arithmetic, geometry, mensuration, astronomy, plane and spherical trigonometry, some having gone through numerous editions.
8. A significant figure in all this was James Mill (not to be confused with J.S.Mill the philosopher) who was educated at Edinburgh university and came to London in 1802

as a journalist. His politic l and educational theory can be found in the Westminster Review particularly during 1824 - 1826.

9. Leading the reform was Dr. Thomas Arnold, appointed as Headmaster of Rugby in 1828, who reformed the school and created the ideal school for the Victorian upper middle class.

10. The schools concerned at this time are the so-called `nine greats': Charterhouse, Eton, Harrow, Merchant Taylors, Rugby, Shrewsbury, St Pauls, Westminster, and Winchester.

11. Among its early members were Boulton, Watt, Priestly, Galton and Erasmus Darwin (the grandfather of Charles Darwin).

12. Oxford and Cambridge would only admit those who were prepared to acknowledge the King as head of the Church of England.

13. That is, practical geometry, measurement, arithmetic and sometimes fluxions. Nicholson (1823) is a typical text in this genre. See also Cook (1981) 14. The Prospectus of the Sheffield Mechanics Institute (1832) states; "The object of this Institute is to supply, at a cheap rate, to the classes of the community, those advantages of instruction, in the various branches of Science and Art which are of practical appli ation to their diversified avocations and pursuits." (Inkester 1975) 15. Even then, mathematics was not regarded as a subject really suitable for girls neither in the `liberal' sense nor in the `vocational' sense. (see Harris 1997 particularly chapters 3 and 4) 16. De Morgan taught at Queens college, but only for a year, apparently feeling that the ladies were not of a sufficiently high standard; and as a member of the London Mathematical Society, showed an interest in the attempts to reform the teaching of school geometry. (Rice 1996) 17. The SDUK was founded in 1826 by Henry Brougham and other liberal politicians as an alternative to the radical press, and through its publications intended to give a `suitable direction' to working class thinking. The Differential and Integral Calculus (1842) was originally published in the Penny Cyclopaedia in forty two weekly parts.

18. Further discussion of the development of the mathematics curriculum and its pedagogy in the latter part of the century can be found in Price (1983).

19. Perry was an engineer and his syllabus provided a new paradigm that came from outside the school tradition. (DSA 1899) 20. Cayley as chief examiner for Cambridge entrance insisted on keeping Euclid

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Learning about mathematics learning with *ardinas* at Cabo Verde[1]

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Abstract

Among several mathematics education researchers learning is starting to be seen as a social practice (Lave, 1988) and this idea is now being used to look into school mathematics learning (Adler, 1996; Santos, 1997). However, several questions emerge when we intend to think about school learning from this point of view. Lave's results come from studies on adults in situations with relevant differences from schooling. For instance, practices in which adults were involved in Lave's studies were deeply connected to a (chosen) process of becoming. However, it is growing among us a strong belief that, for most of young people (12 to 15 years old) schooling is not explicitly associated to a process of becoming but it is a transitory life-space. Becoming is not the intentionality and purpose of pupils' school practice. Therefore we feel the need to clarify the meaning of learning as social practice, particularly in these aspects that we see as fundamental.

Rationale

Our view of mathematics learning at school draws from the idea that school life is an everyday practice (Lave, 1988). But life at school has its own culture, its history and we must remember that this practice is lived at an institution. One of the goals of our study is to understand which are the elements of this practice and how do they relate to the learning that takes place there. In order to understand what are constitutive elements of that practice, we believe that it is useful to look at an out of school practice. Why do we feel that this is useful? First, because as educators we live the school from the inside and this turns difficult to be sensitive to important aspects of the practice. Looking at and trying to describe a practice which is not familiar is a way to put ourselves as learners about that practice. Second, we searched for a practice out of school that incorporates some common elements to the school practice. These elements are: this practice is lived essentially by youngsters in school age, it has a non professional character, it is part of an institution, it is a transitory situation in the lives of the participants, and we could identify a visible relation to mathematics use.

To describe and analyse this practice we are using an analytical tool proposed by Lave (1996) which we share and believe that is coherent to an approach to learning as a social practice. This analytical tool is "a set of questions for interrogating anything claiming to be an example (...) of learning" (Lave, 1996, p.156):

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"1. *Telos*: that is, a direction of movement or change of learning (*not* the same as goal directed activity),
2. *Subject-world relation*: a general specification of relations between subjects and the social world (not necessarily to be constructed as learners and things to-be-learned),
3. *Learning mechanisms*: ways by which learning comes about" (Lave, 1996, p.156).

However we felt that we need some powerful ideas in order to go deeper in the understanding of this practice. For instance, we found in the work of Wittgenstein (1953) some interesting ideas that seem to be consistent to Lave's approach. We can say that Wittgenstein is concerned with "us as being able to 'go on' with each other" (1953, nos.146-155) "reacting and responding in ways that makes it possible for us to continue our relationships" (Shotter, 1997,p.1). One of the implications of this view is that an important aspect of people life is a concern with how to sustain participation. In the case of the *ardinas*' practice, to learn that practice implies to learn certain things (as mathematics use) that help them to maintain their participation in that practice. Following this line we are working for example on the idea of *rules* and *following rules* from Wittgenstein (1967) in order to make more visible the subject-world relation.

At the moment we are focusing on some examples illustrating how we are using the concepts of *telos* and *learning mechanisms* in the analysis of this practice.

The practice of *ardinas*[2] at Praia, Cabo Verde

The newspaper *A Semana* is sold only on the street once a week at Praia in Cabo Verde. Every Friday (hopefully at morning), at the newspaper office, the newspapers are devilered to an adult (Egídio) who is responsible for the selling and for paying back to the administration. Egídio gives a number of newspapers to each one of the 19 *ardinas* (from 50 to 150 exemplars each). Immediately after that the *ardinas* rush to their selling places in the city and try to sell the newspaper to the costumers as fast as possible during that day. The *ardinas* sell the newspaper at places chosen by themselves according to the rhythm of selling. The newspaper cost 100 escudos (fixed by the newspaper office) and the *ardinas* should pay (at the end of the day) 87.5 escudos to Egídio for each newspaper sold. The *ardinas* are young boys aged from 12 to 20 years (mainly about 15-16-17). Some of them begun to sell one month ago and others are selling for four years. Most of them sell newspapers in order to help their families ("to help my mother"). The level of schooling of these boys goes from 2nd grade to 9th grade. Right now only three of the 19 *ardinas* are studying at the local school as five of them dropped out the school last year and the others left it several years ago.

[2] *Ardinas* is the common Portuguese word for newspaper sellers.

The *ardinas* come from two places. One group is from Praia, the capital of Cabo Verde (from a poor and problematic borough) and the other comes from S. Martinho (a rural small village near Praia). Egídio is the one who invites or accepts boys for the job of *ardina*. Certainly there is an history of these groups. Five months ago the *ardinas* were all from Praia. However problems arrived with some of them and Egídio brought 10 boys from S. Martinho (the place where he also lives) to substitute the others. Egídio gave some hints to this group from S. Martinho about the selling process (the price of the newspaper, some good places for selling). When a newcomer from Praia joints the group, Egídio defines an oldtimer who will be in charge of the newcomer (passing him a small number of his newspapers to sell, protecting him from *piratas*[3] and receiving from him the money to pay to Egídio).

Telos and learning mechanisms

At the beginning the *ardinas* almost only know (i) where to receive the newspapers, (ii) the price of the newspaper, and (iii) where and to whom pay for the newspapers sold. To all of them (newcomers and oldtimers) to be a good *ardina* is to respect and follow the rules (payment rules) and to sell quickly. At the beginning some of their feelings about the selling process are: to be afraid of being roubred or lose money, not having the ability to know how and which people to address in the street and where to go, the big weight they have to carry during the day, the difficulties of getting enough coins and bills to facilitate exchange. The direction of movement of learning (*telos*) to be an ardina draws from these elements. During the process of trying to be a good ardina they use some strategies. From these strategies emerge the use and learning of mathematics. For instance, (i) during the day they check the money they earned according to the number of newspapers already sold — involving estimation or mapping money onto number of newspapers sold; (ii) when giving exchange to the costumers they vary the way they do it in order to keep certain kind of coins and bills — involving linear combinations of numbers; (iii) the preview of the amount they have to give back to Egídio at the end of the day (never through a direct calculation using the value 87.5 escudos) — involving complex processes of calculation using different strategies from additional reasoning to proportional reasoning.

A promising approach?

We are not yet in a position to make conclusions about the usefulness of the approach we are using in the study of learning. However, we feel that what is emerging from the analysis of data (which we tried to give a taste of) show that it is fruitful in order to help us to have a better understanding of Lave's approach of learning as a social practice. This would help us to identify how this approach could be useful to study youngsters' mathematics learning.

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[3] *Pirata* is the word used in Praia to refer to kids who make small robbery in the street.

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CULTURAL CONFLICT: A PRESERVICE JAPANESE STUDENT IN AMERICAN SCHOOLS

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There is a considerable difference between what goes on in American mathematics classrooms and what goes on in Japanese ones (Jones, 1997). As Reyes & Stantic (1988) stated:

> *Clearly, we live in a society where racist, sexist, and classist orientations exist in individuals and in institutions. What is not clear is how such ideas are transmitted to and through schools, how the ideas are mediated by the democratic ideals of equality and equality of opportunity, and the extent to which teachers and students accept and resist the ideas. More specifically, we do not yet fully understand how these ideas affect the teaching and learning of mathematics (p. 27).*

Culture, a popular catchword, is often used by educational researchers without clarifying what the term means or considering its relevance to mathematics education. Nickerson (1992) recommended coming to some understanding of culture within the classroom before considering how wider aspects of culture impinge on that classroom. This paper will present a Japanese student's cross-cultural perspective of American education and mathematics education and will provide insight into the classroom culture and the effects of the wider aspects of culture and society in it.

Keiko, a graduate student in an elementary and middle childhood teacher certification and Masters of Education program in a midwestern university, has lived in the midwestern United States for 12 years and is 36 years old. Her American husband and his family are from the Appalachian culture; their two sons attend first grade and fourth grade at an alternative private school which is not associated with any religious group. The school's philosophy is based on Bruner's ideas of discovery learning, and is more aligned with the educational environment Keiko desired for her children. Keiko is actively involved in the children's education both at school and at home.

In the American culture, Keiko is viewed as a 'super-mom' and a 'super-student' because of her exemplary parenting and outstanding achievements in education. However, Keiko takes responsibility for the family and children as she would in the Japanese culture where her role is to "prepare the child for life, to help provide a bridge between the home and the outside world" (White, 1987, pp. 37-38). In the Japanese culture, the family is the woman's source of influence and value, and this is embodied in the vertical ties of parent and children rather than in the Western nexus of husband and wife (White, 1987). In Japan, any kind of work requires 100 percent effort and a person who tries to combine different work lives, such as Keiko does, is seen as lacking in a fixed group identity (White, 1987).

While completing her undergraduate degree, a B.Ed in elementary education with areas of concentration in mathematics and social studies, she completed a *non-required* Honor's Research project that investigated the effects of language and cultural differences on a Japanese eighth grade student in the mathematics classroom. During the project, Keiko's mother came to the US to help fulfill the mother role in her family. At other times during her education, however, Keiko gave 100 percent to both roles without any additional family support. Keiko's cross-cultural perspective of the elementary classroom offers a new perspective about how the wider aspects of culture and society influence students' learning of mathematics. Indeed, it suggests that these wider aspects have a direct effect on the mathematics learning in the classroom. Reyes and Stantic (1988) said that in mathematics education there is little research documentation of the effects of societal influences on other factors such as school mathematics curricula, teacher attitudes, students attitudes and achievement related behavior, and classroom processes. Moreover, they suggested that documenting these connections is the most difficult and the most necessary direction for further research on differential achievement in mathematics.

Raymond (1997) offered a model describing the relationships between mathematics beliefs (nature of mathematics and mathematics pedagogy) and mathematics teaching practices (mathematical tasks, discourse, environment, and evaluation) which included the factors identified by Reyes and Stantic (1988). In this model, past school experiences (successes in mathematics as a student and past teachers) had a strong influence on the teacher's mathematics beliefs. These mathematics beliefs and the immediate classroom situation (students' abilities, attitudes, and behavior; time constraints; the mathematics topic at hand) had a strong influence on the mathematics teaching practices. In turn, teaching practices strongly influenced future teaching practices.

Brown & Borko (1992) found that beginning elementary teachers who entered the teaching profession with nontraditional beliefs about how they should teach tended to implement more traditional classroom practices after they were faced with the constraints of actual teaching in the American society. However, Keiko entered the teaching profession with nontraditional beliefs about mathematics and teaching that were developed in the Japanese culture and were reinforced by that society. When she faced the constraints of actual teaching in traditional American classrooms and the classroom management practices used in this society to externally 'control' students' classroom behavior, she experienced cultural conflict.

. Studies concerning teachers' beliefs about mathematics and mathematics pedagogy, found that teachers' beliefs are not always consistent with their teaching practices (Kaplan's study; Peterson, Femmema, Carpenter, & Loef's study, as cited in Raymond, 1997). However, the inconsistency was deeply rooted in the differences between the cultural and societal expectations in education and mathematics.

While working with Keiko on the research project, we developed a close relationship, the type of nurturing relationship between student and advisor that is expected between a teacher and student in the Japanese culture (White, 1987). We used the term 'mono-vision' to define our educational perspective--one eye saw the classroom through Keiko's cross-cultural view, the other eye saw the classroom through my mathematics educator's view, and with both of our eyes focused, we more clearly saw the effects of the culture and language on the student's mathematics learning.

Keiko's philosophy of education was developed in the Japanese culture and her values influenced her teaching and learning in many ways. Moreover, she learned mathematics thinking and reasoning in Japanese schools. Keiko's beliefs about mathematics mesh with the goals of NCTM (1989, 1991, &1995) but not with the traditional classroom and ways of teaching mathematics in American schools.

Sugiyama (1993) stated that the higher achievement of Japanese students depends chiefly on the social conditions and the general educational environment in Japan and believes that this, rather than excellent mathematics education, accounts for the higher achievement of Japanese students on international studies. These factors include: the Japanese educational level is higher in general, including mathematics; the situation is caused by the belief that education is the basis of social development of the country and of the prosperity of the individual; the Japanese educational system has no repeaters because effort, rather than ability, determines success; the Japanese educational system has higher quality teachers who are expected to teach higher quality lessons that emphasize the development the students' ability to solve problems.

During the first two quarters in the graduate level teacher preparation program, Keiko's value system conflicted with what was emphasized as important in some of the methods courses, in the elementary classrooms she visited and taught in, and in her work in the university Math Lab. By the end of her two week field placement, Keiko concluded that she would not teach in public schools because the cultural and societal differences were too great; they demanded that she give up her identity. Indeed, this was another way to state what she already knew about education in this regional midwestern society -- students from different cultures attend private schools with the exception of the African American students who mainly attend the urban schools; the public schools are homogeneous (Sugiyama, 1993)

The data presented in this paper was gathered primarily from daily e-mail discussions. Other data forms included: videotaped lessons, folders of Keiko's coursework and reflections, and field observation reports and notes. Pseudonyms were used in the paper; it presents preliminary findings related to the cultural and social values that were in conflict and that are related to mathematics teaching and learning.

Discussion

Keiko's beliefs about mathematics and mathematics pedagogy were formed in the Japanese culture and were inconsistent with traditional beliefs about mathematics and the practices of teaching in the American culture. This conflict was apparent to Keiko in the elementary schools and in her work in the university Math Lab.

Many students who use the Math Lab services were taking remedial level mathematics courses focused on middle school and high school mathematics and others were students taking the required courses for the elementary teacher certification program. As lab tutor, Keiko attended the mathematics class so that she could meet the students and encourage them to come to the lab for assistance. However, she was concerned about the teaching in these classes, and she differentiated between teaching for mathematical understanding and teaching procedures. She learned mathematics in Japanese schools and had a understanding of why the procedures 'worked.'

> *It was busy in the lab, and my visits to Mrs. M's classes were fun. I see your point, after looking at the textbooks, that they only teach procedures in those classes. But why, though? Those kids are in the classes because they probably have missed something on the way learning math in their previous school experiences, and they don't have the concepts down. Then, they are taught the procedures all over again without any help in understanding the reasons for it??*

Keiko related what she observed in a 5th grade classroom to the teacher's knowledge and understanding of mathematics and then to what she knew about the college level students taking the Mathematics for Elementary Teachers courses. She began by talking about the 5th grade teacher and said:

> *For math, she uses the manuals exclusively, but when she teaches Language Arts, her lessons are great. I could see she was enjoying teaching the subject, too. I guess it is safe to say that math is not her strong subject area. I was helping a few math students in the lab earlier this afternoon who are taking math 105 this quarter, also, but it was amazing how little they understand the material. I can see when those students finish the program and go out to teach, they might be tempted to rely on manuals. Even though they might be exposed with how exciting math learning is/should be while they are in the (education) program, once they are out in the schools and when they feel the time-crunch, it is easy for them to spend more time preparing for other subject areas which they are interested in and put off math until the last minute. And meanwhile, the students suffer, don't they?*

In an elementary classroom, she observed that teaching mathematics was often avoided and was concerned about the message that gave the students. As a primary subject, Keiko thought that it should not only be taught daily, but also be taught when the children were ready to do their best thinking.

> *You know, as many days as I have been in the classroom, so far I have not seen one math lesson there. I am beginning to wonder if she ever teaches it! My co-op definitely likes to replace her "math period" with other busy work period. I'm afraid this is giving the kids an impression that math is not a desirable or important subject in the relationship to the others. It will be interesting when I take it over; we'll see how the kids react to the change. I plan to move math to the morning, whenever possible, as the kids' are ready to work the best then.*

Keiko observed how little the mathematics methods courses in the education program affect the teaching practices in the classroom. Several teachers she worked with were graduates of the same teacher preparation program she was in, yet, in the classroom the philosophy of education and the pedagogical practices in mathematics were not evident. Indeed, some of the teachers advised her to ‘play the game’ at the university, and you can join the real world of teaching in the schools after you graduate. This advice disturbed Keiko greatly, because most of what she learned about teaching at the university in the mathematics methods course was consistent with her cultural view.

The teaching differences between Japanese and traditional American teaching in mathematics are well documented in the literature. In traditional American classrooms, mathematics is a unrelated collection of facts, rules, and skills, while in nontraditional classrooms, mathematics is dynamic, problem driven, and continually expanding. The nontraditional view of mathematics is aligned with the beliefs Keiko had about the nature of mathematics and mathematics pedagogy, and inconsistent with the traditional view that is prevalent in this society.

Keiko questioned how some professors ‘helped’ students learn in the university setting. For example, she talked about how a professor ‘gave her the right answers’ and wondered how that helped her learning. Keiko believed that as a student it was *her* responsibility to solve problems and that the professor was there to provide guidance rather than answers. Again this is consistent with the roles of student and teacher in the Japanese culture but is not always the case in the American culture. She said:

> *When I went to see Dr. P to go over my lesson plan Wednesday afternoon, I was surprised when she started editing my plan. She had a pencil in her hand and wrote what I should say word for word.*

Reflection is a part of Keiko’s life; she desired strongly to understand the events of her life. She remembered that when she was in school, there was a time period at the end of each day when students reflected about the day’s events, and she noticed this type of reflection was not a part of the task-oriented American classroom life. Moreover, she recognized individual differences. She said:

> *But at the same time, I know some Japanese adults who just cannot reflect at all, so there is certainly an individual difference there. Through reflecting and thinking about what happened, you can learn to recognize that there are variety of reasons for a thing to go well/not to go well. It's not "you" only who is responsible for the outcome, usually.*

In a message she shared another discovery that helped her understand more about the conflict she was experiencing related to general pedagogy and its implication for her as a teacher. She said:

> *As I was working on a paper, I came across a book on my shelf. As I read through it, it made a lot of sense to me, now, that I understand those educational terms. It talks about why Japanese teachers teach the way they do, and the Japanese students learn the way they learn. What I found interesting was that it gave me insight as to why I disagreed with so many things Dr. P said in the pedagogy classes. My experience in schools has been so different that now it is understandable why I couldn't agree with what she said proven effective in American classrooms.*

Keiko can think and speak fluently in both Japanese and English. We found that there were some differences in translating mathematical ideas between English and Japanese that were not easily noticed. For example, while teaching second grade mathematics, a question arose about 2 dimensional shapes, and Keiko discovered a difference between the languages.

> *Rectangles must have four right angles. I guess I was thinking about quadrilaterals, and also thought that in Japanese there is not such a word as "rectangles"---there is a term for quadrilaterals, though, to generally include all four-sided figures. Isn't it interesting? So, culture and language DO matter when it comes to learning mathematics.*

After discussing this difference, we pondered why in the American culture, primary students are taught the general word triangle to name all 3 sided shapes, and yet, more specific names, such as square and rectangle, are taught for 4 sided shapes, rather than the general word quadrilateral.

This society's ideas of competition and perfectionism were ones that Keiko often reflected about and discussed while trying to understand her place in the cohort group. Her locus of control is internal, and her confidence, creativity, humor, and openness, come from this feeling of personal control. However, as she struggled to understand competition and perfectionism, she observed that when the locus of control is external, a negative sense of competitiveness, helplessness, unwillingness to be open to new ideas, fear, etc. were developed as personal traits in her peers. Keiko shared a discussion she had about one peer in the cohort (Jeanette) with a field advisor (Katie).

> *I told Katie that I am a perfectionist, too, but my way is so much different from Jeanette's. I said that Jeanette's perfectionism depends heavily on others' approval. She is always so concerned about how others perceive her, and she can get pretty annoyed when things do not go her way. I think her perfectionism comes from her uncertainty of herself. She has to do everything better than the others because, otherwise, she thinks that others will not recognize her. My perfectionism comes from my confidence. I know I can do it well, whatever I do, and I try my best to go up to (live up to) my own expectation (of myself) because I am confident about my capability. Perfectionism comes with some negative sense in this culture, while it does not in Japan. If you consider perfectionism in Jeanette's terms, I think it is pretty negative.*

Keiko also talked about sensitivity, a character trait that has a positive meaning in the Japanese culture, but she found it had an opposite meaning in this culture. Indeed, it was a character trait she fostered in herself and in her children.

> *I always thought the word "sensitive" was positive, but when he said to me, "you are too sensitive about everything!", it shocked me. That was a culture shock---I thought the more sensitive the better.*

She related these ideas about character to the relationships adults have with children in the classroom environment and power struggle that she observed between the teachers and students. This power struggle, often under the guise of classroom management or discipline, was in conflict with the mores of Japanese culture. In Japan, the teacher is a respected member of the classroom community, where hard work and fun were one in the same thing. Moreover, teachers and students develop close bonds since they stay together as a community of learners for the first six years of school. Power was not an issue in the classroom because the societal and cultural expectation of education were different.

> *I was thinking that the message I want to send to the kids most is "you don't have to be overwhelming/strong & big physically/pushy/verbally abusive/etc. to be respected. I want them to know that quiet and sweet person can be just as strong or stronger, mentally, than those people who try to exhibit their strength on the surfaces. What I am trying to do here might be to change the cultural norm, and I know it will not be done easily.*

During the first week of full time field placement teaching, Keiko was enthusiastic about teaching second grade, especially, teaching mathematics.

> *I interviewed the kids about shapes using tangrams today. You know they really amazed me. I feel that I can just teach Geometry all year like this---they should be able to understand "infinite planes" by the end of the year. It's such a joy to work with them. Those surprises (students' thinking) I encounter in the classroom are what makes teaching so interesting! I feel that I want to do more and more when they give me their new ideas. It's just soooo wonderful:-).*

However, at the end of the first week Keiko explained her feelings about teaching in the public school. She was overwhelmed by the realisation of the differences between the culture and society:

> *One of the major roles schools play in each culture is to teach children their own societal values so that they fit in the culture. In a homogeneous community setting such as this, it seems that this role becomes even more important. Teachers, schools, and parents have the same value system so that children are socialised in the dominant culture. Because the cultural belief is so rigid and not accepting of differences, every child is forced to fit in. I am from a different culture, and I will not be very effective in that role.*

On the same day, the field supervisor gave advice during the final seminar that summarized the cultural conflict she observed and experienced in the classroom. The field supervisor said:

> *'The most important thing for a classroom teacher to have is a strong control over students.'*

At the end of the second week, Keiko recognised the 'big dragon'--conflict between cultures.

> *As strongly as I feel about my philosophy, I am so sick of the way they are treating the kids at school out there, and I feel powerless because I cannot do anything about it. It bothers me greatly that the kids are not respected as "kids" at all. And teachers' role in it is so significant that I really do not want to have any part of it.*

So, in Autumn quarter, I was upset because I felt that I was fighting against "the cohort", but now I gave up because I know I am fighting against "the culture", and I know I cannot do much about it at all.

The first week of full-time field placement teaching was exciting for Keiko, but during the second week, cultural differences about how to deal with immediate classroom situations caused Keiko to reconsider her desire to obtain teaching certification for public education. She felt that she would have to give up her identity to fit in the classroom and school culture, and to meet the societal expectations of public schools.

Conclusion

Clearly, Keiko was in cultural conflict and the causes of her conflict were embedded in the differences between the cultures and the societal values fostered in the educational system. Although she was knowledgeable about mathematics, could teach mathematics effectively for understanding, and loved teaching and her students, she decided that she would not continue in the certification program and teach in public schools. She believed that: the classroom was a community of learners, with the teacher at the center of the learning; the responsibility for learning rested with the student; the role of the teacher was to challenge and nurture the students growth; the interactions between students and teacher were based on a loving relationship rather than a power struggle; and, hard work and fun were one and the same thing. Instead, she will teach mathematics in a private school where the philosophy and goals of education are more compatible with her beliefs.

White (1987) said that the children in Japanese classrooms are good as well as happy because no one taught them that there is any contradiction between the two. As early as in 1919, John Dewey observed the absence of overt discipline in Japanese classrooms.

They have a great deal of freedom there, and instead of the children imitating and showing no individuality -- which seems to be the proper thing to say -- I never saw so much variety and so little similarity in drawings and other handwork, to say nothing of its quality being much better than the average of ours. The children were under no visible discipline, but were good as well as happy; they paid no attention to visitors ... I expected to see them all rise and bow. (White, 1987, p. 122)

Implication

Reform in mathematics curriculum, teaching, and assessment have been espoused since the National Council of Teachers of Mathematics Standards documents (NCTM, 1989, 1991, & 1995) were written, however, the philosophy and spirit of the reform has had limited influence reforming mathematics teaching and learning. American students were ‘at-risk’ in mathematics when they were compared to Japanese students in the Third International Mathematics and Science Study (TIMMS) report. Although American students’ scores were in the average range in elementary years, middle grade students and high school students scored poorly. The report has generated much concern about the teaching and learning mathematics in American middle grade and high schools, through its comparisons between the curriculum and mathematics pedagogy in the two countries. Indeed, the traditional mathematics teaching and learning in the elementary grades is a factor affecting students future poor school performance in middle and high school mathematics. However, this is not the case in the Japanese schools where the emphasis in elementary mathematics is on developing mathematical thinking by exploring, developing, and understanding concepts, or discovering multiple solutions to the same problems.

In both cultures, the teachers' beliefs about mathematics are formed during their personnel experiences in mathematics learning, and these beliefs are reinforced by societal expectations. Moreover, American teacher preparation programs have little effect on reforming teaching practices in the classroom because they are in opposition to the cultural norms in the classroom and in the broader society. Indeed, many of the teachers are as 'at-risk' in mathematics as their students are. How can *all students* learn mathematics when culture and societal norms conflict with the desired outcomes of the reform? To answer this question, consideration need to be given to the effects of the culture and societal expectations on students performance in mathematics.

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SOCIAL ORIGINS OF SECONDARY MATHEMATICS KNOWLEDGE IN HONG KONG AND MACAU

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ABSTRACT

This paper is a historical study of the infiltration of Western mathematics textbooks into Hong Kong and Macau before the Second World War. By tracing the origin of the textbooks in these two places, different socio-political influences on textbook adoption can be illuminated. European influences are examined first due to the colonial status of both areas; and US influence via the Chinese Kuomintang is then investigated.

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INTRODUCTION

Hong Kong is situated just within the northern tropics of South China. The island of Hong Kong was formally ceded to the British in 1842, and the Kowloon Peninsula in 1860; and, finally, the New Territories were leased in 1898. In 1984 the Sino-British Joint Declaration was signed, in which the British and Chinese governments agreed that Hong Kong would revert to China in 1997. Hong Kong is now a Special Administrative Region of China.

Macau is situated on the western bank of the estuary of the Pearl River at the southern tip of China close to Hong Kong. After several unsuccessful attempts dating back to 1513, Portuguese traders finally secured the right of settlement from the Chinese authorities in 1557. Although the Portuguese were interested in Macau primarily for economic reasons, the territory also played a major religious and cultural role (Cremer 1991) – though after the mid-seventeenth century, the glory of Macau faded due to the decline of Portugal's power. In April 1987, the Sino-Portuguese Joint Declaration was signed and the Portuguese and Chinese governments agreed that Macau would revert to China in 1999. Like Hong Kong, Macau is expected to remain a Special Administrative Region for at least 50 years after the transition.

In order to understand stability and change in school mathematics knowledge, some researchers have adopted a world system perspective by emphasizing worldwide forces dominated by Western culture (Meyer et al. 1992), whereas some others have focused on local shaping forces in developed countries (Young 1977; Hextall and Sarup 1977; Cooper 1985a;

Cooper 1985b; Moon 1986; Stanic 1987a; Stanic 1987b). In this paper on the social origins of secondary mathematics knowledge in Hong Kong and Macau, different source of Western influence was identified but no comparably significant local shaping force was found before the Second World War.

THE ORIGINS OF TRADITIONAL MATHEMATICS TEXTBOOKS

Before the early 1960s, the mathematics curricula in Hong Kong and Macau were textbook-driven. It was a common practice to have separate textbooks, and even separate teachers, for arithmetic, geometry, algebra, trigonometry, co-ordinate geometry and calculus. The theoretical roots of this approach were established more than 300 years ago and its traditional organisation of mathematical content was characterised by the division of the subject into four main branches: arithmetic, algebra, geometry, and analysis, each being considered a closed and separate field of investigation (Fehr 1970, p. 200).

Popular traditional mathematics textbooks adopted by the Chinese-medium schools in both territories were usually Chinese translations of US textbooks of the 1910s (Hong Kong 1955a; Ngai et al. 1987). The most popular algebra and geometry textbooks were *College Algebra* written by Fine, and *Plane Geometry*, *Advanced Plane Geometry* and *Solid Geometry* by Schultze, Sevenoak and Schuyler. Also, Granville's *Plane Trigonometry* and *New Analytical Geometry* by Smith, Gale and Neelley were also widely adopted. All these textbooks were also very well known among teachers in China in the 1920s (Ngai et al. 1987).

In the English-medium schools, UK textbooks were used. For instance, pre-war textbooks such as Mayne's *Essentials of School Arithmetic* and *Essentials of School Algebra,* Durell's *A New Geometry for Schools,* Durell and Wright's *Elementary Trigonometry*, and *Elementary Calculus* by Bowman were included in the approved textbook list (Hong Kong 1955b; Lai 1968) and were widely adopted by the English-medium schools in Hong Kong and Macau from the 1930s. Also, it is worth mentioning that there were a few Portuguese-medium schools in Macau, − which were actually Portuguese schools operating in this enclave in Southeast Asia − and in these, Portuguese traditional textbooks were adopted.

Finding UK textbooks in the English-medium schools and Portuguese textbooks in the Portuguese-medium schools was not surprising due to the influence of colonialism. However, the use of Chinese translations of US textbooks by the Chinese-medium schools in both places deserves further examination.

After the overthrow of the Qing Dynasty and the establishment of the Republic of China in 1911, a new education system adapted from the Japanese and German systems was introduced. However, in 1922, this was in turn replaced by a 6+3+3 system based on the US model. The original English versions of these US textbooks were then adopted by some prestigious schools in Beijing, such as the secondary section of Beijing Normal University. In the 1930s and 1940s, numerous translated versions were published which were very popular among secondary schools in the Republic of China (Ngai et al. 1987).

In 1928, shortly after the establishment of the Nanjing government in China, the Overseas Chinese Education Committee was established under the Ministry of Education. Regulations were issued, and overseas Chinese schools were asked to register themselves with the Overseas Chinese Education Bureau. Details of the curricula and lists of the textbooks to be used had to be submitted in the registration process (Cheng 1949; Chan 1992). The main purpose was to control the structure and curriculum of overseas Chinese schools in order to exclude Communist influence. The strategic considerations behind such a policy might be best explained by Cheng's (1949) examination of the Hong Kong case.

> "Since 1928, the Chinese Government had been trying to control the overseas Chinese schools through its consuls and Kuomintang agents abroad. As there were no Chinese Consuls in Hong Kong and Kuomingtang activities were banned, the attempt of the Chinese Government to influence the Chinese colony must be by very subtle ways....
> One reason why most of the bigger schools had to register themselves with the Overseas Chinese Affairs Committee was that a growing number of Hong Kong students were going back to China for higher studies, and if these schools did not register themselves with the Chinese authorities, their students or graduates would not, as a rule, be recognized in China and therefore could not join any Chinese schools or universities. As registration with the Chinese authorities carried with it an obligation to observe, whether openly or secretly, certain regulations laid down, it was clear that the Chinese authorities had been having an indirect control or influence over a number of the bigger schools" (quoted in Wong Leung 1969, p. 53).

The above strategy was found to be quite successful. In 1928, some schools in Hong Kong started to borrow the American 6+3+3 system, like those Chinese-middle schools in China (Sweeting 1990). Furthermore,

> "[i]n the early 1930s, an increasing number of schools were able to operate with branches on both sides of the border and registered with both governments. In Hong Kong, such schools followed the curriculum

> prescribed by the Nanjing government, used textbooks published in China, mostly at Shanghai, and presented their senior middle graduates for university entrance examinations in China. They engaged teachers trained either in China or in Hong Kong. The colonial government and missionary schools also generally used the Nanjing syllabi and the Shanghai textbooks for the Chinese culture subjects, although they probably followed them less closely. For other subjects, they used textbooks from England or from Shanghai. They also employed teachers educated either in Hong Kong or in China. Hong Kong never developed an autonomous school system before World War II and remained very much a periphery to its dual centers" (Luk 1991, p.661).

In 1929, in order to counteract the Kuomintang influence, a committee was appointed in Hong Kong to draw up a syllabus for private schools to follow. However, the reaction of the Hong Kong government was unsuccessful; in the early 1930s, the vernacular schools always tried, "as far as the Education Department allowed, to follow the curriculum of the schools in China, using the same textbooks, and having the same subjects" (Wong Leung 1969, p. 53).

The Overseas Chinese Affairs Commission was established in Nanjing in 1931. This signified the need of the Nanjing-based government for a more comprehensive and effective policy towards overseas Chinese because their support was found to be more and more necessary with the rise of the Japanese militarism and their attack from the north. A survey was conducted in 1935 by this Commission to estimate the number of overseas Chinese schools throughout the entire world, and about 550 overseas Chinese schools, primary or secondary, were found in Hong Kong and Macau (Chan 1992, p. 256). When Guangzhou was taken by Japan in 1938, however, the number of schools in Hong Kong and Macau increased dramatically as schools moved from the north or from Guangzhou for refuge. The teachers and students of these schools carried with them their curricula and textbooks which had already been adopted by some of the schools in Hong Kong and Macau in the early 1930s (Kong 1995, p. 5).

Another phenomenon of interest is that these translated US textbooks are still very popular among the Chinese-medium schools in Macau whereas they have been replaced by local textbooks in Hong Kong since the late 1960s. It needs another study to investigate the cause of such difference in stability and change in these two places.

CONCLUSION

China had a glorious past in Mathematics and Astronomy. When Matteo Ricci came to Macau in 1582, Western and Chinese Mathematics and Science was very different from each other. Before he was permitted to meet the Chinese Emperor in 1600, he studied study Chinese Classics in Macau; and being a mathematician and astronomer, he also wrote and translated many books about religion and science into Chinese including *Treatise on the Celestial Bodies, Work on Trigonometry and Treatise on Geometry,* and *Elements of Geometry of Euclides.* In the early twentieth century, however, three hundred years after Ricci's stay in Macau, the Western education system and Western mathematics started to infiltrate into the educational institutions in Hong Kong and Macau, partly due to colonization, and partly due to socio-political influence from their motherland. How Chinese people build up their own identity in Mathematics education is and continues to be an important question.

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What if Mathematics is a Social Construction

Why do post-modern reflexive students avoid modern ritualised mathematics

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Today many students avoid math based education within science, engineering and economics. This enrolment problem creates an opportunity to reflect on mathematics and its traditions. Is mathematics part of nature or is mathematics part of culture, a social construction? If mathematics is part of nature, it can be taken for granted, and only mathematics education can be discussed. If however mathematics is part of culture, also mathematics itself can be discussed since an alternative mathematics might be constructed. Asking "What if mathematics is socially constructed", this paper looks at different theories about social constructivism and their consequences for mathematics and mathematics education. The traditional abstract Platonic mathematics is confronted with an alternative contextual nominalistic mathematics.

Mathematics as part of nature

"Are concepts part of nature or part of culture?" was the medieval question of the struggle of universals. "Nature", the realists said. "Culture", the nominalists replied. What are the answers today?

Today's tradition within mathematics and mathematics education is based upon a realist or Platonic paradigm. The concepts of mathematics are part of nature, they are universal and culture free, once discovered they can be taken for granted and taken as objects for investigation, education and application. Within this Platonic paradigm two fractions have emerged: the fundamentalists (in the sense of A. Giddens 1994) and the constructivists.

The fundamentalists are philosophically oriented. There is only one mathematics and it and its teaching traditions must be defended by all means. To the fundamentalists the enrolment problem is solved through finding "teacher-proof" material to implant mathematics in the students' heads, and through more discipline and more mathematics lessons in the schools.

The constructivists are more psychologically oriented emphasising learning issues over teaching issues and stating the point, that students create their own version of mathematics in their head. The constructivists are divided into two directions: the radical constructivists inspired by Piaget and the social constructivists inspired by Vygotsky. Although using the same word, the psychological based social constructivism is quite different from the following sociological based social constructivism.

Mathematics as part of culture

"Mathematics is part of culture" the nominalists said. Concepts are names created by a culture to differentiate between things and thus dependent on the culture. The nominalistic tradition is today represented by different directions such as the

“History and Philosophy of Science”, the “Sociology of Scientific Knowledge” and the “Actor Network Theory”.

In the “History and Philosophy of Science” T. S. Kuhn introduces the notion of paradigms. The paradigms of science reproduces themselves through an authoritarian, dogmatic education, as Kuhn points out: “Even a cursory inspection of scientific pedagogy suggests that it is far more likely to induce professional rigidity than education in other fields, excepting perhaps, systematic theology” (Kuhn 1963).

“Sociology of Scientific Knowledge” or “social constructivism” originates from Great Britain with B. Barnes, D. Bloor and H. Collins as prominent figures. A social construction is a “closure” (Bijker et al 1987), i.e. a victorious solution to a social need. The winner takes it all and rewrites history so that its competitors are forgotten and the closure appears to be the only natural and rational solution. With the passing of time the closure may however become “debunked” by new problems and then the time is in for a “disclosure”, i.e. a reformulating of the social need and a new competition among alternative solution until a new closure takes place.

Actor network theory originates from France with B. Latour and M. Callon as prominent figures. Science is texts, ore rather sets of allied texts referring to each other thus creating a network of discourse fighting other networks with military inspired techniques. The important thing is to maintain and expand the network and to take the power in the network by occupying “necessary passage points”. Peace occurs when one network has become superior. This victorious network then has won the right to posses knowledge (Latour 1987).

One could add that a closure becomes a canonised institution if it succeeds in wiping up the traces to the social need it solved, becoming self-supporting with values of its one, a departure point and not only a passage point, node in the network.

A sociological viewpoint includes a theory of modernity. U. Beck and A. Giddens are talking about reflexive, post-traditional or late modernity (Beck et al 1994) with reflexive individuals having to construct own biographical narratives (Giddens 1991).

J. F. Lyotard and others are talking about post-modernity, which begins with a scepticism toward the grand narratives and the acceptance of a need for locally constructed narratives or stories (Lyotard 1985).

According to A. Giddens, traditions involve rituals, guardians, formulaic truth and a normative content. Rituals are the practical means of ensuring preservation and the continual reconstruction of the past. The guardians are the people believed to be the agents or mediators of the tradition, being not experts but dealers in mystery. The formulaic truth, to which only the guardians have full access, are often formulated in words or practices that the speakers or listeners can barely understand thus reducing the possibility of dissent. The normative content gives tradition a binding character. If tradition is not discursively articulated and defended in dialogues with its alternatives, tradition becomes fundamentalism (Beck et al 1994).

The questions of the new nominalists

With T. S. Kuhn we can ask: What are the paradigms of mathematics and mathematics education? Why does mathematics pedagogy remind of systematic theology?

With A. Giddens we can ask: Is mathematics a tradition ready for de-traditionalization? What are the traditions of mathematics, i.e. its rituals, guardians, formulaic truth and normative content? Is the tradition involved in dialogues with its alternatives or has it become fundamentalistic? What are the alternatives to current mathematics? What does a de-traditionalization of mathematics look like?

With J. F. Lyotard and B. Latour we can ask: Is mathematics a grand story that is suppressing local stories? What can be a possible content of mathematics as local stories, of post-modern mathematics? Who are the allies, and which are the necessary passage points that let to the victory of the current mathematics discourse?

With the social constructivists we can ask: Given today's problems within mathematics such as math anxiety, innumeracy and decreasing enrolment to math based education within science, engineering and economics, can mathematics be considered a closure ready for a disclosure? What is the social need that originated mathematics? To what social question is mathematics an answer?

Possible answers

One possible answer to the last question is: Since numbers and operations are artefacts of our culture we need stories about them. But numbers and operations are themselves socially constructed so we have to ask again: What is the social need that originated numbers and operations? To what social question are numbers and operations answers?

One possible answer could be: In any culture there is a need to differentiate between and communicate about things. To this end a "word-language" is constructed assigning words to things by means of sentences: "This table is high". To differentiate between degrees of many a "number-language" is constructed assigning numbers to things by means of sentences: "The height is ninety cm". Some numbers can only be or are quicker determined through a calculation: "The area is length times width". In abbreviated form these sentences are called equations: h=90, a=l*w.

Traditional mathematics consists of concepts and statements. Concepts are defined from other concepts thus constituting a concept pyramid with the set concept as the mother concept at the top. Statements about the concepts have to be proven or taken for granted. In the former case the statements are called theorems, in the latter axioms. Since theorems have to be proved they are not socially constructed. The concepts however are not proven, they are chosen, they are socially constructed and thus a basis for dissent, closure and disclosure.

In his book "Introductio" from 1748 L. Euler closes the function concept as a name for a calculation. In the following century a disclosure took place producing many competing alternatives. In this century a closure took place, creating the modern abstract definition: "A function is an example of a relation, which is an example of a set product, which is an example of a set". The Euler definition of a function was nominalistic pointing downwards to a universe of examples as a naming of the common features of calculations. The abstract definition of a function is Platonic pointing upwards to the more abstract concept of a relation.

From a given concept in a concept pyramid many arrows are pointing down, but only one up. Pointing upwards there is only one grand universal story about a function, the story of abstract mathematics. Pointing downwards there are many local contextual stories about a function, the stories of contextual mathematics. Substituting contextuality with universality is substituting authenticity with authority, and humanisation with dehumanisation creating problems for today's reflexive post-modern students.

The concepts of mathematics constitute the paradigms of T. S. Kuhn, the guardians of A. Giddens, the subjects of the grand story of post-modernity, the necessary passage points in the actor network of B. Latour, and the closures of the social constructivists.

The concepts create textbooks containing definitions and theorems. Pointing upwards the textbooks of abstract mathematics can only tell its story in one way. The textbook thus constitutes a grand story, a victorious discourse, a ritual, a guardian, a formulaic truth and a normative prescription, a "bible". Alternatively contextual textbooks localise and situate (Lave 1991) mathematics in a context of essential real world problems. Abstract mathematics consider contextual mathematics as just applications of itself and not as another form of mathematics. By avoiding a dialogue with contextual mathematics abstract mathematics becomes fundamentalistic.

The textbooks ensure their own reproduction through echo teaching and through written and oral exams controlled by external examiners. Exams and examiners are perhaps the strongest guardians of the tradition.

The concepts also creates a community of practitioners also being guardians. The mathematicians working at the universities producing new theorems, new mathematicians and new mathematics teachers through an authoritarian, dogmatic education.

The graduates who cannot produce new theorems themselves can become teachers of mathematics in the schools thus becoming the interface between mathematics and the public. Socialised into a Platonic dehumanised paradigm the teachers become not storytellers but echoes of the grand story unable to provide authenticity, only authority. Unable to prepare a defence in a dialogue with e.g. contextual mathematics they are trapped in a “après nous le deluge” fundamentalism.

The mathematics teachers carry the ritual to the classroom, where the students have to memorise the definitions and prove the theorems. Questioning the theorems is considered good manners. Questioning the concepts or asking “What is mathematics” is considered questioning the paradigm. And questioning a taboo might lead to social excommunication.

Also the politicians act as guardians accepting the only way to a number language leading through abstract mathematics by formulating goal statements as: “The purpose of mathematics education is to learn mathematics”. Thus placing abstract mathematics at the necessary passage point of the educational discourse they create the escape of post-modern reflexive students and the following enrolment problems. Changing the goal statement to “The purpose of mathematics education is to develop the student's number language” will allow a free competition between abstract and contextual mathematics. This rehumanisation of mathematics could create a quite different situation with happy students making enrolment problems disappear.

Numbers as social constructions

Numbers are an answer to the social need to differentiate between degrees of many. Not all cultures have this need. Hunters and gatherers count one/two/many and may even lack an abstract standard name for many: flock, bundle, bunch, collection, stack, bouquet etc. Agricultural, industrial and information cultures however share a social need for differentiating between degrees of many. They are immediately confronted with a problem: infinitely many degrees of many suggest the creation of infinitely many names. This problem is solved by bundling, reusing the names when counting pieces, bundles, bundles of bundles, sub bundles etc.

An international closure has taken place using ten as the bundling number, but some cultures had different bundling numbers. In French ninety is called “four-twenty-ten” is and in Danish ninety is called “half-five” with “times twenty” as a silent understanding. Different subcultures and trades used different bundling practices. The farmers used 20, the merchants 12, both numbers being present in the former British monetary system. Although space, mass and money have become standard-ised, time has still kept its own bundling routine.

Operations as social constructions

Where numbers are an answer to the social question: “How many?”, operations are an answer to the social question: “How many in total?”. The Arabs used the name “algebra” for the use of operations. The meaning of this word has since been lost and is today unknown to most mathematicians and math teachers. Algebra means reunite, i.e. uniting what might have been separated. Operations are answers to the social need for reuniting. Addition was an answer to the farmers need to unite unequal numbers. Multiplication was an answer to the merchants need to unite equal numbers. The merchants of renaissance Italy reopening the old trade route to the far east, this time by sea, made so much money they could lend it out. Bankers thus needed to unite interest percentages, to which power was an answer. The British wanting to open a trade route to India sailing without the sight of land, where hostile Portuguese were waiting for an easy catch, had to depend on a way of determining altitude and latitude, which led to a need to unite varying "per numbers" (e.g. meter per second), to which integrating was an answer.

Mathematics as social constructions

Once socially constructed as closures to the social questions of how many and how many in total, numbers and operations themselves become the objects of stories, meta stories, mathematics', just like the sentences of the word-language become the object of meta sentences, grammar. Within mathematics a closure took place in this century resulting in the modern abstract mathematics and its myth: "Good crated the numbers, man created the rest". From now on numbers, operations and calculations were to be understood not as social constructions but as examples of sets, relations, functions and compositions.

Being contextually based the word language is able to differentiate the language from the meta language by the names "language" and "grammar". It also follows the natural order of language before meta language and avoids Russel's type mistakes as: "The predicate is laughing". Being abstractly based the number language is unable to differentiate the language from the meta language using the same name "mathematics" for both. It inverts the natural order teaching meta language before language and makes Russel's type mistakes as: "The function has a value and is increasing".

In modern mathematics education, sets and functions are the fundamental concepts to be introduced as early as possible. And many years pass by teaching children universal facts as 5+7=12 and 5/8+7/16=17/16. Treating numbers abstractly out of context abstract mathematics cannot tell the difference between "per numbers" and "unit numbers" thus having great difficulties explaining why 5+7=12.4 in the case of % interest, why 5+7=705 in the case of cm and meters, or why 5+7 might give many different context dependent results in the case of % parts and meter per seconds. Also with 5 red out of 8 and 7 red out of 16 giving 12 red out of 24, it becomes difficult to rationally defend the statement that 5/8+7/16=17/16 and not 12/24.

When buying 3 kg of 7$/kg, the total price is T = 3 kg*7 $/kg. Abstract mathematics reduces this to the question "3*7=?", i.e. an uncritical calculation without reflecting what is calculated or the denominations of the numbers.

The context dependency and the parallelism between the number language and the word language is the main point of contextual mathematics: always use full and contextual sentences: T = 3 kg*7 $/kg. T is the subject of the sentence and the denominations its predicates determining the calculation. Numbers must not be added uncritically: the question "a+b=?" is answered by the total equation T = a*p+b*q, changing a and b to equal units before adding.

The teaching of contextual mathematics differs from that of abstract mathematics in many fundamental ways. Number language sentences, equations, are introduced from grade one thus making the students appreciate the two ways to numbers: counting and calculating. The main story of contextual mathematics is the original one of algebra, the story about reuniting or totalling: unequal numbers, equal numbers, equal percent numbers and unequal per numbers are united by plus, times, power and integrals. The total equation introduces multiplication before addition (made possible by pocket calculators), and makes addition of fractions superfluous changing fractions to numbers before adding. In calculus the adding of varying per numbers through integration are introduced before differentiating, substituting the pluses with an integral in the total equation.

But first and foremost, contextual mathematics changes mathematics from a "function discourse" pointing upwards into an abstract concept pyramid to a "calculation discourse" pointing downwards to essential contextual real world calculations, emphasising that calculating is quicker than counting, and that the power lies in not doing but setting up the calculation and its equation:

Once upon a time a teacher asked his class: "How many legs of chairs do we have in this room?" A noisy activity began, pupils counting 1, 2, 3..., some 2, 4, 6..., some even 4, 8, 12... Disturbing each other many had to restart. Having written down a variety of different answers the teacher asked little Peter who had been standing in the corner all the time. A roar of protest arose from the class: "He has no say, he didn't take part in the counting". "Lets hear him anyway", the teacher said. "96". "Correct!". "Buuh, luck, you told him", were some of the comments from the classroom. "How did you get that, Peter?". Peter showed his pocket calculator which said: 4*6*4=96. "Anybody can calculate that", the class said. "Precisely", the teacher said. "Anybody can calculate 4*6*4, but who can set up the calculation T=4*6*4?".

The philosopher I. Kant would add: "The statement 4*6*4=96 is analytic, a tautology that can be calculated by technology and only commented with an "I see". The statement T=4*6*4 is synthetic, a statement about reality which must be commented with yes or no".

Presenting modern mathematics to post-modern students

Changing mathematics from a function discourse to a calculation discourse effects the learning of reflexive students. Telling a student: "A function is an example of a relation" is presenting him with a statement where both concepts are outside his horizon and thus only accessible through learning by heart making the student an echo of the function discourse. Echo learning produces encapsulated knowledge, easy to reproduce but hard to apply. Echo learning might keep the ritual alive, but does not extend neither the function story nor the students own story.

Echo learning was no problem in modernity, where students were socialised into arenas of high authority: the home, the school, the army, the workplace etc. Modernity had its grand story of success: "Just follow the authorities and happiness will follow". As indeed was the case: becoming an echo of the grand abstract story of mathematics brought you through the exams and into a nice position as a math teacher where you job was to echo the same grand story.

In late or post modernity robots are taking over production being easily programmed to follow the authorities. Late modernity brings globalisation and individualisation into the culture changing the grand story of happiness to: "Build your own biographical narrative and happiness will follow"(Giddens 1991). A post-modern story builder has to be able to see the subject of the sentence within his horizon in order to expand his story and horizon with the rest of the sentence. To him the grand abstract story of modern mathematics is pure poison, which he quite naturally must try to avoid, creating severe enrolment problems.

Thus late or post-modernity calls for a disclosure and a de-traditionalization of mathematics i.e. of the story about numbers and operations, opening a contest between the grand abstract story and its competing alternatives. One alternative mathematics is the above mentioned contextual story about numbers as social constructions to the social need for differentiating between degrees of many, and operations as social constructions to the social need for reuniting.

In my classes contextual mathematics changes students from turned-off to turned-on, wanting more mathematics until they meet the grand story in the form of an echo teacher following the textbook strictly and just repeating it when asked to explain it. The students then turn off and avoid mathematical based education in the future.

The learning life of Ruth

Ruth is one of my former students from the first year of high school. I asked Ruth to tell about her learning life within mathematics. Ruth began her school in Denmark when she was seven. Four years later she moved with her father to Kuwait. Two years later they returned to Denmark for a year during the war of the Gulf. After another two years in Kuwait she is now back in Denmark, having just finished her second and last year of the special preparatory high school.

At an early age Ruth learned the difference between abstract and contextual mathematics. She also learned that obedience to the ritual and inability to provide authenticity can make teaching an unpleasant job.

> In grade seven we were making graphs with negative and positive scales, how to draw them, and so when we asked why we made them, what purpose it kind of had, well you just had to make them, that's how it was. You didn't get any explanation as to the reality behind this math in Denmark. Our number two teacher, we had two different teachers that year, came in and was drunk as a lord, so we didn't learn very much. Then I went back to Kuwait.
>
> "You experienced a difference on what you had learnt in Kuwait and in Denmark as to explanations?"
>
> Yes a big difference, In Kuwait you got explanations and you always got examples that could be connected to reality, you always got that in Kuwait, whereas in Denmark it was only something you had to do, and apparently they didn't care how it went along. It was as if it was just something to get over and done with.

“There were no connections to reality?”

Not in the examples, they were not connected to reality at all. Mostly the math teacher gave up, he did not take the trouble to explain it to us, he got sour and sulky, and in the end he said: “Why don’t you just leave the <swearword> classroom!”

“What do you mean by the word explain”

To explain, for example a pipe is to tell about its function, what it is used for.

At the Danish high school mathematics is mandatory the first year and optional the second year. The first year Ruth is confronted with contextual mathematics, the second year with abstract mathematics. She notices a difference as to authority.

Back in Denmark I was admitted to the high school, where I had math the first year, and I must say this was just what suited my head, at any case the teaching method was different, one I think should be spread out, for the teacher had a quite different way to explain, one you could understand. You really felt you learned something, even if it was difficult for you, you still learned it along the road. Even if you were a little behind, because first of all, you had a good relationship to the teacher, you felt the teacher was part of the class, not a separate part of the class thinking he has a higher authority. We really felt, the teacher was on the same level, as to authority any way, of course as to mathematics he was at a higher level. I do not know what I can explain about that method, anyway there was something about it that was incredibly attractive. I can compare with mathematics the second year. The method, the teacher used the second year is simply one I find unsuitable and I know that many from the class agree. You felt precisely the opposite, the teacher was not so to speak a part of the class, you felt he was very authoritarian, he used his authority and taught directly from the book, and that helped us very little, when you go home and read the book and prepare your homework and then go back to school and say, that you don’t understand it, the teacher explains it and mostly it helps only little for he explains it directly as it is in the book. He could have turned it, but he didn’t.

At the oral exam, Ruth is asked to tell about quadratic equations and quadratic inequalities. She is supposed to discuss with her teacher, and the external examiner is only allowed to pose a few short questions. Ruth did not understand all the details of the proof of the solution to the quadratic equation, which she admits wanting to continue to her own example of a quadratic inequality. But the system will not allow her to continue before she has reproduced the grand story. Even the external examiner forgets all about rules and takes active part in the efforts to make Ruth follow the ritual before telling her own story.

The examiner asks me how I can say what is the case when d is zero. I say that I cannot explain it, I don’t know it, but he keeps on and keeps on an keeps on returning to it, so while the examiner keeps on returning to it and starts to give me explanations that is supposed to help me, my teacher does the same and in the end,

> eh, I am close to giving up, for when they begin to talk both of them, the examiner starts to say something and the teacher starts to say something, it all just starts to run together. You hardly know what they want you to do, so it went really wrong. Repeatedly I say I cant explain it. I would like to continue with the second part of my question and then return to it later, but apparently they think this is something fundamental

Prevented to tell her own story Ruth decides to file a complaint.

> I complain about the behaviour of the external examiner, I complain about not being allowed to present my own example of a quadratic inequality because there was no time for that since both my teacher and the examiner and in the end mostly the examiner kept on returning to the same thing.

In the future Ruth will avoid mathematics and look for "something quite opposite".

> Future plans? I really lost my courage after the second year. It has completely gone, no more math in the future even if I use to like it. My father had an education as an engineer specialising in ice-cream machines, where enormous amount of math should be applied. I was really fascinated by this. I have always been fond of mathematics, but, eh, I can say it ten thousand times, my courage disappeared after the teacher we had the second year and his teaching methods. My future plans have jumped from one ditch to the other, from the world of mathematics and numbers to something quite opposite, culture or religion or something like that.

Conclusion

Considering mathematics socially constructed might provide a solution to the enrolment problem. Post-modern reflexive students having to construct their own biographical narratives will shun the traditional grand story of abstract mathematics and ask for explanations and authenticity instead of authority. This wish is met by contextual mathematics explaining concepts as social constructions to human needs.

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A FEW THOUGHTS & MANY QUERIES ABOUT MATHEMATICAL CULTURE AND MATHEMATICS EDUCATION

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In this paper an attempt is made to discuss the impact of culture on teaching and learning mathematics at the school level. All over the world research findings reveal examples of students rejecting mathematics, fearing it, disliking it, given a chance preferring it as last resort, even if they learn resorting to rote and instrumental methods. To what extent is culture the cause of it? How can enculturation help students in enjoying mathematics and in transferring the problem solving skills to new situations? Many more queries and thoughts about the influence and impact of culture on mathematics education at school level are put before you for discussion.

In the later part of the 20th Century, there have been intensive efforts to improve mathematics education in schools. Educationists, subject experts, teacher-educators, teachers, parents, psychologists, policy makers, text book and resource material producers and many more are showing their concern about the state of mathematics education at the school level. Many attempts were made in this regard. After more than a quarter century of attempted reforms, there appears to have been little improvement. Where are we still going wrong? Why is mathematics still a problem subject for so many? The problem is widespread. If we cannot find at least a partial answer to this question, there is no reason to expect that future efforts will be any more successful than past. I don’t think that there is a single answer to the problem, nor that any one person knows all the answers. The mathematical culture that is adapted may be one of the causes and mathematical enculturation is one of the solutions.

The Present Mathematical Culture in Secondary Schools

The importance of teaching mathematics as an integrated subject is recognised every where. But still we are sticking on to the idea of ‘teaching mathematics’. It is better to move from ‘teaching mathematics’ towards ‘mathematics education' (Bishop, 1988), though educating the pupils mathematically is more difficult, challenging and complex than teaching them some mathematics.

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In most of the countries including India the mathematical culture adapted in schools has the following characteristics
* Curriculum of procedures, methods, skills, rules and algorithms which insist on 'doing' mathematics rather than 'thinking' mathematics.

* Quantum of content of mathematics subject matter, completion of stipulated syllabus in the given rigid time, examinations by giving less or no importance to the abilities, interests and cognitive level of the learners.

* Text books written by authors mostly who have less or no chance of knowing about the pupils, teachers and mathematics classroom.

* The assumptions made such as 'mathematical knowledge flows from higher level to lower level'. Here the teacher is considered as at 'higher level' and student as at 'lower level' ignoring the interpersonal relationship. Mathematics is considered as optional, pupils can drop themselves whenever they find it difficult, meaningless, or of less useful to them.

* Normally 'what to teach', 'how to teach', 'how much to teach', 'how long to teach' etc., are dictated to the teachers.

* Teachers generalise the learner ability i.e., teacher plans the lesson, teaches the lesson aiming at the average generalised ability of the learner considering it as every student's ability, resulting that it is not suitable to anybody because the average represents no body in the group.

* Mathematics teaching is dominated by dehumanisation, depersonalisation and decontextualisation. School pupils many times confused why are they learning about algebra, trigonometry theoretical proofs of theorems. They mean that if Apollonius theorem or Pythagoras theorem is true every where, so what? Why should they learn about universal truths? If the teaching is not contextual, the aims are not realised by the pupils, mathematics learning is meaningless to them.

* The focus is on 'how many sums the pupil did', 'how many topics are learnt', 'how much portion the teacher completed' and not on what is going on in the minds of the pupils. If the pupil gets the correct answer there ends learning. It is rarely attempted to know if the pupil does it correctly, how could he arrive to the correct answer, and if a child fails, where has he gone wrong, what misconceptions led him to arrive a wrong conclusion. There is no scope given to get multiple answers to the same task. Emphasis is more on the product rather than on the process.

The Values of Mathematical culture

The values of mathematical culture are providing different dimensions and directions to mathematics teaching at school level. Greenfield and Bruner(1966), offer the idea that "some environments push cognitive growth better, earlier and longer than others. What does not seem to happen is that different cultures produce completely divergent and unrelated modes of thought".

* Since the time of the Egyptian and Hellenistic civilisations Rationalism. logic and reason have become basis for mathematics education. Rationalism, with its focus on deductive reasoning challenged the trial-error pragmatism, and inductive reasoning. The main aim of mathematics teaching was to develop logical, rational, abstract and theoretical thinking. The mathematical ideas are developed by proofs, extensions, examples, counter-examples, generalisations and abstractions.

* In the western culture it could be observed that Objectism as the driving force in the development of mathematics. The focus is on the origin of ideas which form with the interaction of the environment and it is material objects which provide the intuitive and imaginative bases for these ideas (Newman 1959). Pythagoreans developed the abstract idea of numbers starting from concrete objects like stones and pebbles and proceeded to particles and points. Therefore, to the Pythagoreans numbers were objects, literally.

* While Rationalism insists on logic and Objectivism on materialism, the third dimension of mathematical culture focuses on the aesthetic pleasure and satisfaction that a learner experiences when a pattern is suddenly revealed by organisation or structuring a messy collection of number facts, or a set of random shapes.

* Another aspect is the feelings of growth, of development, of progress, and of change which led to the alternativism - the recognition and valuing of alternatives. It means finding multiple solutions to the same task or problem. Multiple solutions or ways out of a problem definitely leads to progress. Horton (1967) wrote: "in traditional cultures there is no developed awareness of alternatives to the established body of theoretical tenets; whereas in scientifically oriented cultures, such an awareness is highly developed". This may be the reason that the 'Investigations' and 'Investigatory approach' are picking up the attention even in mathematics teaching.

* The sociological component is concerned about the examination of mathematical truths, propositions and ideas, where from the mathematical ideas come from and who generate them.

* The technological value imposed on mathematics due to the introduction of calculators, computers and ICT

Mathematical Enulturation and Mathematics Education

The idea of enculturation entered interestingly in the field of mathematics education. Enculturation, is a creative, interactive process among those who live in it with those who born in it. Though the ideas, norms and values may be similar to the previous generation, it would be inevitably different due to the re-creation role of the next generation. The mathematics education of the child is affected at three levels, *informal, formal* and *technical*.

As cultural transmission is certain, the mathematics teaching is affected informally. Many times this interaction is informal and so mathematical culture can play an important role in informal enculturation. Initially before the children go to formal schooling many mathematical concepts like many and few (number concept), size (big and small), shape (regular, irregular, plane, curved), area, volume, capacity (more, less and measurement), distance (far, near, length, breadth, height, depth, perimeter), time (tense and measurement), weight (heavy, light and measurement), are formed and attained through informal inter action with the elder members of the society. Adults transmit the mathematical culture by approving and disapproving the actions based on mathematical logic. They also develop symbolic ideas through stories, music, discussion and examples from day to day life. These reactions with social experience develop among the pupils the abilities of rational thinking, perceiving the logic, finding the relation between cause and effect and reason for every action. By insisting on regularity, punctuality, following the rules with reason, rational thinking, pupils develop mathematical attitude. The social interactions also vary according to the different personality and interests of the children. No two children are identical and therefore the ways they interact will differ. As a result, no two people will develop an identical conception of their shared mathematical culture. So the adults who share the mathematical culture play an important role in informal mathematical enculturation.

At the formal level the mathematical culture is to be filtered and the best of the mathematical cultural heritage is tried to be transmitted. Formal mathematical enculturation is possible through 'formal education'. Therefore it should help to explain and understand various aspects of informal mathematical culture and should make it more structured by refining it. The formal enculturation should take place at an appropriate level intentionally and explicitly. Though this responsibility is entrusted to the schools, the situation is far beyond satisfaction. Mathematics is a universal language of symbols and concepts. So the main goal of formal mathematical enculturation is to induct the children in to the symbolisation, conceptualisation and values of mathematical culture. The child and culture should be given equal importance. One cannot be over emphasised at the cost of the other. But what is observed in the developing countries like India, the mathematical culture is given more importance ignoring the individual and the suitability of the culture to the individual. The curriculum and subject content planned for a different society with different social, cultural and technological environment is adapted and at the same time the process is not adapted. We cannot confine to the process oriented because of the culture's frame of knowledge, nor we can just concentrate to that knowledge, since education is more than just imparting knowledge. So the teacher education Institutions should take the role of liaison-fare and guide the future teachers. There should not be any conflict between informal and formal mathematical enculturation, they should supplement each other.

Due to modernization and technological developments school mathematics is affected at the technical level. The advent of Computers and calculators and other technological developments broadened the mathematical applications in society. As such the mathematics education should be modified according to the industrial, technological and societal requirements. It should no longer remain as a product of pure mathematics. Here also the under developed countries and developing countries, which cannot afford heavy investment and without laying a sound suitable environment, the syllabuses and curricula of more-technological societies are considered as models and facing utter failure. It is because these changes are in-appropriate to the social and cultural environment of the child. Here again there is a conflict between mathematical enculturation at informal level and at technical level. We cannot neglect the individual personalities. A cultural perspective on Mathematics education must surely recognise the existence of individual differences and we can no more consider 'children' as 'child'. Cultural learning is a creative and re-creative act on the part of every person. Cultural learning is thus no simple one-way process from teacher to learner. Therefore it is necessary to think about the type of mathematical enculturation we would like to bring. Then the questions rise how should it be done? and at what level? Should it be done at an informal level? or at the formal level? or at the technical level? The teacher who is the key person and the Mathematical Enculturator in the formal enculturation process should establish a proper rapport among these three levels of enculturation by eliminating the opposing forces. Teacher education Institutions have to take the major responsibility in inculcating these abilities in mathematics teachers.

The next question is what should be the approach in the curriculum? Should we follow the Behaviourist Approach aiming at improving learning by a 'task analysis', or the 'New-Math approach insisting on a 'systematic description of mathematics', or the structural approach based on the theories of Bruner and Diene, or the Formative approach that focuses on cognitive abilities and affective and motivational attitudes of the pupils, or the Integrated-Teaching Approach which is based on problem-solving process.

The Behaviourist approach insists on sequential learning and the main objective is mastery of specific mathematical content. In this approach only cultural transmission is possible but not enculturation. The New-Math approach is just like 'an old wine in a new bottle'. It reorganises and describes the mathematics content with common uniform and precise language. The structuralist Approach also gives importance to the mathematics subject content. The Kilpatrick's Formative approach goes beyond the subject matter and aims at the development of cognitive abilities and motivational attitudes which describe in terms of personality traits. The Integrated approach insists on the flexibility of the curriculum and problem solving processes. The combination of Formative and the Integrated teaching approach focusing on the process may be the best solution for mathematical enculturation since it provides flexible curricular units and open-ended processes for the learner according the psychology of the individual. (the suggestions offered are not final, subjected for discussion).

What cultural components should be considered for enculturation? Is it the symbolic component that is based on 'rationalism' and 'Objectivism'? or the societal component that insists on the uses of mathematical explanations or the cultural component that insists on alternativism and openness? What activities are to be planned accordingly? In case if we would like to have the integration of all these, how to balance them in the curriculum? A combination of all these three components namely symbolic, societal, and cultural may supplement each other and can bring mathematical enculturation. The symbolic component helps in developing the intellectual (cognitive) abilities and Objectism in explicit exploration, the societal component takes care of the applicative value and the cultural component will look into the technical and alternatives of the existing phenomenon.

How to bring the Enculturation process in action? Should it be interpersonal and interactional? Should it be formal, institutionalised, intentional and accountable? Should it be concerned with mathematical concepts, meanings, processes and values? or should it be suitable to the social context? or should it be for all?

Above all, according to Bishop, J (1988), the 'enculturation' was focused on values with an insist of moving away from a 'transmission' image of mathematics education. According to Bishop enculturation can not be done by one person to another, culture is not a 'thing' which is transmitted from one person to another, nor is the learner merely a passive recipient of culture from the Enculturator.

Enculturation is an interpersonal process and therefore it is an interactive process between the teacher and the taught. There should be a strong relationship between teacher and Mathematics Teacher Educators.

Since mathematical enculturation is an intentional, shaping process, the teacher's task is to create a particular kind of social environment for the learner and it is the learner's task to construct ideas and modify them in interaction with that environment. The psychologists, Educationists, the subject experts and curriculum framers should provide the supporting system.

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POTENTIAL SOURCES OF INEQUITY IN TEACHERS' INFORMAL JUDGEMENTS ABOUT PUPILS' MATHEMATICS

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Abstract *Data about the informal assessment practices of thirty UK mathematics teachers were collected through unstructured interviews. A theoretical description of such practices revealed that they were complex and integrated with other aspects of teaching and pupil-teacher interaction. Analysis within descriptive categories showed many contradictions within individuals' practices, or between teachers. Some of these could, theoretically, lead to different assessment decisions being made by different teachers about similar situations. In this paper the author describes many of these potential differences, and concludes that in the UK situation, where differential grouping is the norm and teacher assessment is included in high-stakes assessment, more attention needs to be paid to evaluating such decisions.*

Introduction

Teacher assessment of mathematics is a component of statutory assessment in the UK at ages 7, 11, 14 and 16. Results of statutory assessment can be used to determine access to particular mathematics classes, schools, examination tracks and progress to further education or employment. Hence teacher assessment is high-stakes assessment. In addition, decisions about access to the curriculum through differentiated teaching groups are made throughout a child's school career, by grouping within classes in primary schools and by tracking in secondary schools. Teachers' informal assessments contribute to these decisions.

During the last ten years all teachers have been trained in the use of assessment techniques against National Curriculum statements, and record-keeping, pupil-monitoring and reporting have become important parts of their work.

Underlying these formal and paperwork aspects are the human interactions and judgements made in the ordinary day-to-day practice of the classroom. Interactions are subject to interpretation; mathematics is heavily dependent on interpretation since it is communicated through a variety of forms, not all transparent. The teacher's response depends on her interpretation of the children's

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attempts to communicate their mathematical understanding. It was to learn about these processes that the research study was set up.

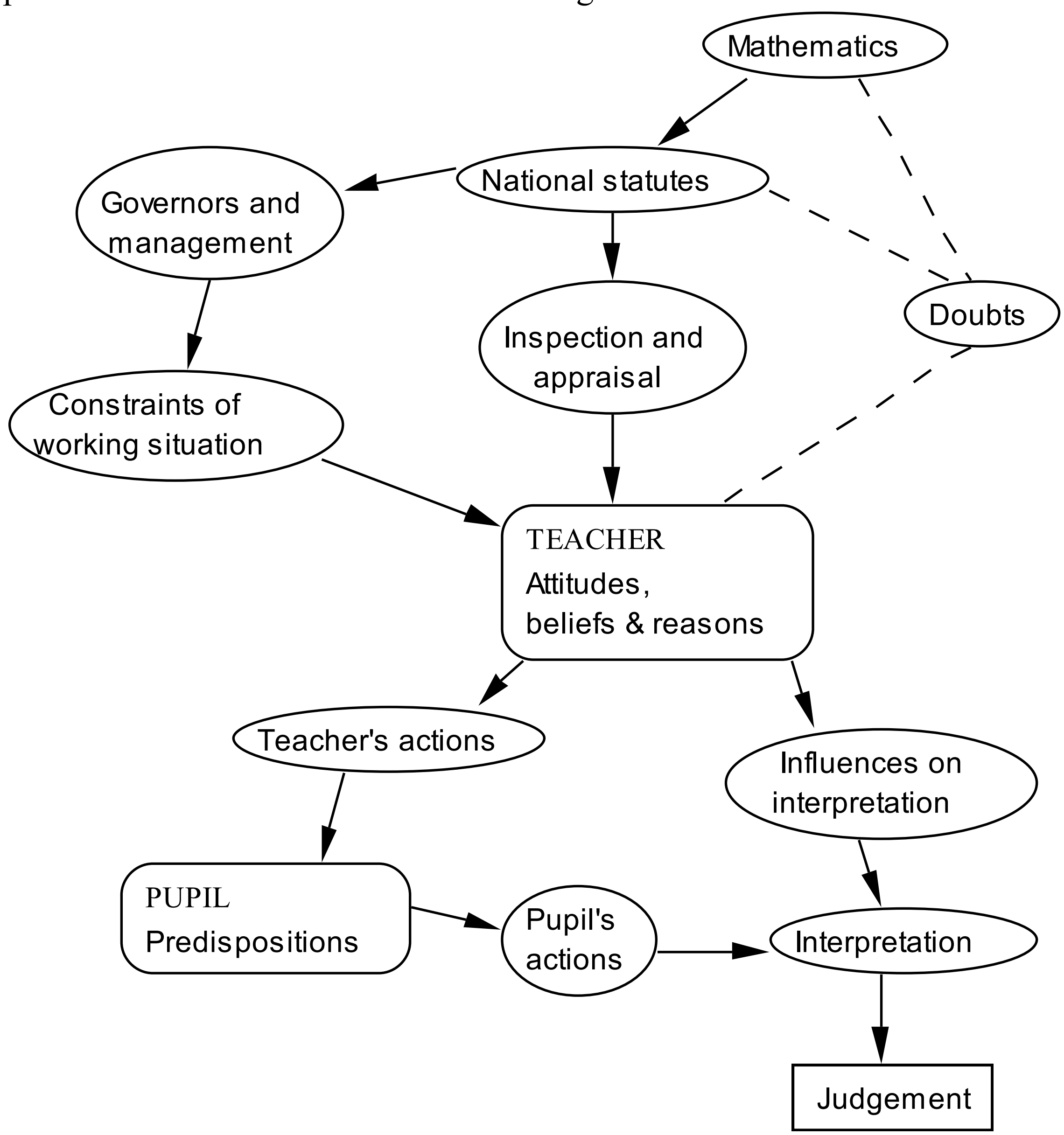
Methods and analysis

A report of the methods and analysis follows, this being necessarily brief in order to devote most of the paper to the findings.

Thirty teachers of 10, 11 and 12 year-olds were selected from a range of schools and interviewed about their practices. Interviews were unstructured and based around core questions about assessment methods and descriptions of the mathematical understanding of pupils chosen by the researcher after classroom observation. Pupils were chosen for their apparently unexceptional behaviour and response in class. Interview transcripts were analysed for content to create a detailed story of the range of practices and issues accumulated by all the teachers; the aim was to provide descriptions of what might be going on in teaching mathematics in general, not portraits of individual teachers. Interviewing and analysis were undertaken concurrently, a liberal approach to coding was taken, transcripts and data were re-read frequently and several theoretical frames were tried in order to produce a credible view of the field. These techniques to develop grounded theory were broadly comparable to those suggested by Glaser and Strauss [1967] and are described elsewhere [Watson,1998].

A description of practices was developed, parts of which have been published elsewhere [Watson,1995 & 1996]. So far, the work had been largely descriptive, but during analysis it was found that a more critical stance could be taken towards teachers' methods. The critique stemmed from a range of literature suggesting that teachers' expectations might be based on social and behavioural aspects of children's classroom performance, rather than mathematical achievements [McIntyre et al 1966; Lorenz, 1982; Ruthven, 1987]. Also important were examinations of fairness in assessment [Gipps & Murphy, 1994]. Apart from a few cursory suggestions that teachers should be wary of bias, and that assessments should be moderated somehow in schools, this literature appears to have been largely ignored by the bodies generating training materials to support teacher assessment [SEAC,1991]. Emphasis in the official advice was on recording and reporting rather than examining judgements. Further research showed that even aware and careful teachers who were being observed could conceivably make biased judgements [Watson,1997]. The power of first impressions to influence future interpretations was also a factor in making judgements [Nisbett & Ross,1980].

It was decided to analyse the interviews and produce a full list of possible sources of contradiction and inequity. The original descriptive categories were grouped into a network which reflected the relative positions of teacher and pupil in a hierarchy of power which was seen to operate in schools in the field of assessment. Teachers-as-assessors were seen to be answerable to governors, headteacher, inspectors and appraisal systems as well as to a view of mathematics represented by the National Curriculum (NC). They also had their own doubts, beliefs and attitudes. These factors combined to influence their actions and decisions as assessors. Other raw material for assessment decisions included, of course, the pupils' predispositions and actions; teachers have powerful positions as assessors of pupils. Further elaboration can be found in Watson [1998]. These power vectors are summarised in this diagram:



With this power structure in mind the raw data was re-read and re-sorted. It was found that differences and contradictions in practice were revealed at every level, and similar issues showed up in several places. Return to the raw data after structuring according to power proved to be very fruitful in terms of revealing sources of inequity. As a result of this stage of the research a small scale study of in-house moderation meetings was conducted, the outcomes of which are referred to below.

The rest of this paper will report the potential sources of inequity so found.

Problems made explicit by teachers

Problems and issues of informal mathematics assessment mentioned explicitly by teachers included a broad concern with not having enough time to do assessment properly, according to what they believed to be required or important:

- time to talk and listen with pupils enough;
- time to assess fully each individual pupil;
- time to find out what pupil is thinking when they appear to have made an error.

Also prevalent were comments about the inaccuracy of statutory tests and the difficulty of having to make summative decisions:

- tests do not give a true picture of achievement;
- NC criteria are not always clear;
- there is not enough time to prepare pupils for tests and teach for understanding;
- what a pupil can do today they may not be able to do tomorrow;
- there is a gap between being able to "do" and being able to "write" maths.

A further area of disturbance was a recognition that pupils appear differently to different teachers:

- previous teachers "under-" or "over-" assess (sic);
- different pupils respond to different teachers differently.

Apart from the last area, to which I shall return below, the other areas of doubt are all about existing systems, rather than the teacher's own role within the system. Any of these problems could lead to different decisions being made by different teachers, particularly in contrasting teaching situations, and hence to potential inequity.

Implicit problems in teachers' descriptions of practice

This section reports on problems which were not mentioned explicitly by teachers but were apparent in the analysis.

Although oral interactions were far and away considered the most important way to assess individual understanding, and the skills of reading and writing were seen as barriers to understanding and to communication of understanding, written outputs were valued more highly as evidence of achievement, and the value increased for higher stakes assessment. In particular, extended written demonstrations of understanding were valued throughout primary and secondary schools, although some of the subject-specific features of mathematics are brevity, essence, symbolism and compactness of argument.

Most teachers expressed their distrust of tests as providers of anything other than a flawed snapshot. There was a feeling that understanding was contextually and temporally specific, and that tests would therefore not show the whole picture. Nevertheless, there was widespread use of tests of various kinds for various uses in school; reasons included parents' needs, pupils liking getting ticks, they provide "firm" evidence even if it is not complete, they allow monitoring of what pupils are learning etc. No one suggested that the level of understanding they sought through their *own* tests would *not* be time and context specific, although this was frequently applied to other kinds of assessment including SATs, and hence teacher-devised tests were assumed to reveal an ultimate level of understanding.

Different views of learning led to radically different teaching approaches. Some teachers gave practical work first, followed by skills and abstract work, while others taught the other way round. Since most teachers agreed that *use* was the ultimate evidence of understanding, these differences will lead to different assessment decisions. However, the adherence to tests suggests that teachers will take *performance* as evidence if opportunities for use cannot be assured. Very few examples of use of mathematics were given as illustrations; very few were observed to be deliberately offered in classrooms.

Teachers' descriptions of an individual pupil's mathematics were largely about behaviour and attitudes to learning. Little use was made of descriptions of mathematical thinking in Ma1, although some teachers clearly valued some aspects of mathematical thinking. Although all said they valued "understanding" above "right answers" the extent to which they took this in their teaching and assessment varied hugely. There was also variation in what kinds of approaches to work were valued; for instance, some teachers might insist on step-by-step formal workings of a conventional type being given where others might accept intuitive, imaginative and short-cut arguments. An exploration of the characteristics teachers said were important for pupils to be good at mathematics revealed a little beyond good work habits. Some teachers were specific about kinds of thinking thought necessary to be successful at mathematics, but only a few mentioned anything that was special to mathematics. Many teachers regarded as essential certain traits which are useful for organisational purposes (speed, accuracy) but which have not been found to be essential in advanced mathematics [Krutetskii,1976].

In nearly all the schools visited pupils were grouped and offered different curricula for at least part of the time according to teachers' judgements and test results. The degree of flexibility in grouping varied a little; some teachers regarded different aspects of mathematics as bringing out different strengths, others that ability in mathematics was largely homogeneous across number, shape, reasoning and data-handling. There were differences in what was regarded as innate and what was regarded as changeable; for instance, "ability" was usually talked about as if it is innate, yet one or two teachers did not believe this and chose instead to talk about gaps in knowledge, lack of suitable mental images, a need for more confidence and so on as if ability could change given appropriate teaching. In general, however, grouping and setting were fixed for most pupils, and I took these to be high-stakes decisions because access to different curricula affects pupils' futures.

Different decisions could be made by different teachers based on their own judgements, their own views of what is valuable in mathematics and their own views of what is changeable in their pupils.

Teachers' views of fairness in their judgements varied, at one extreme some teachers believed that the only fair assessment would be the same test for all pupils of the same age; at the other, some teachers believed that the only fair assessment was their own judgement made on the strength of their knowledge and observations. Holders of each view felt the other would give skewed pictures of pupils. When asked, all teachers were convinced their judgements were fair, and many described how any statement of assessment was only a snapshot, and that pupils' knowledge and achievement were not permanently measurable qualities. Others talked of how they used as much evidence as they could to ensure fairness, but it was noticeable that these checks were always from other aspects of their own judgements rather than against evidence from elsewhere.

Differences in the teachers' own levels of mathematical knowledge did not seem to affect their assessment practices, or views of mathematics. Similar variations in practice were found among primary and secondary teachers. None of the above remarks apply more to one phase than another apart from a very slight increase in direct reference to mathematical thinking encountered among secondary teachers, and a decrease in the use of the word "confidence". More important was the similarity of problems about implementing the NC described by all teachers [Askew et al,1993]. More worrying than this agreement were the differences, such as those above, which could directly affect pupils' futures and which occurred in all phases.

These findings show that SEAC's view [1991], that teacher assessment should be a "combination of professional judgement and common sense in the use of available time" [p.19], is over-optimistic about the operation of both "professional judgement" and "common sense".

Further potential sources of inequity

The latter stages of the research revealed more causes for concern, and potential sources of inequity. It was shown that even committed, aware teachers were capable of making hasty judgements based on partial information and then viewing the pupil's subsequent behaviour in the light of their first impression, sometimes being reluctant to change even when there is a lack of evidence to support the view. The normal standards of inter-personal judgement which people use in their daily interactions appeared to be used also in the classroom. The presence of a researcher inevitably made the teachers more aware about the conclusions they drew, but even then their views of most of the focus pupils could be challenged, if not contradicted, by more detailed evidence. I commented above that teachers appeal to the breadth of their evidence to support their judgements, and of course this may be adequate if a teacher is prepared to change her mind. Tests were sometimes seen as a safeguard, yet often a pupil has already been offered a differentiated curriculum before the test so that an unfair decision may already have been made. Also it was found that a teacher could dismiss or excuse a test result which does not accord with her own judgement.

School moderation meetings could provide a safeguard of professional discussion about assessment decisions, but teachers' views and prior decisions about pupils were used as extra evidence in such meetings rather than as the focus of examination. A kind of circular self-justification was used to explain assessments so that " he's one of my brighter ones" (an assessment decision) was used to justify "so his work is an A grade" (another assessment decision). It is fair to add that most of the time moderation meetings were about agreeing the meaning of, and evidence for, certain criteria, yet arguments such as the one above were noticeable because they were the only times that teachers' informal judgements were mentioned. Only a few such meetings were visited, so I cannot generalise from the results, but the data supports the raising of the issue of whether teachers' informal judgements, which carry so much weight for a pupil, are *ever* professionally examined, challenged and systematically justified in terms of mathematical achievement.

Can assessment of mathematics be fair?

This research shows that teachers, in good faith, can fail to act justly for individuals because of the nature of their informal judgements and, ironically, their emphasis on individual treatment of pupils. Perhaps attention to pupils as a group, rather than as individuals, might result in fairer decisions. Or perhaps "fairness" should be in terms of the community as a whole rather than individuals.

So if teachers' socially acceptable attention to individuals can result in injustice, how can this be ameliorated?

In the research it was found that teachers rarely showed awareness of the potential for flaws in their judgements. One step forward could be to become aware of these processes, so that judgements are doubted and teachers become self-critics, doubting themselves as well as the systems and requirements with which they work. Instead of this being a private activity, it could be part of professional life. Another step could be for teams of teachers to demand more justification of each other's decisions, and less accepting of commonplace phrases which may mask judgements. A further step would be to review all decisions frequently, avoiding circular self-justification which uses previous decisions to justify current ones, but possibly testing other possibilities to ensure that pupils have not been needlessly limited by what is offered to them.

This last suggestion indicates that accepted hierarchies of mathematical knowledge might be questioned as part of professional life. There are many examples in history, and in the data, and in Krutetskii's work [op cit.] to show that uniform progress in all areas is not necessarily a pre-requisite for mathematical success. Nevertheless school mathematics is usually organised as if it were.

Of course flawed judgements do not matter if they do not affect pupils' futures. Another area for change would be to avoid the incorporation of unexamined or interpersonal judgements, or judgements dependent on local circumstances, into high-stakes assessments. By this I mean avoidance of semi-permanent grouping or setting decisions which result in different curricula and hence different opportunities for progress, as well as summative assessments at the end of courses or in career selection. Increased setting in schools [Ofsted,1994], often partly dependent on teacher assessment decisions, is a manifestation of the frequency of high-stake decision-making in mathematics.

It is recognised, by teachers in the case of external measures and through this research in the case of teachers' own measures, that all measures of mathematical achievement are flawed and hence potentially unjust if used to make decisions about futures.

Conclusion

Since teachers' judgements influence pupils' mathematical progress on many levels, in particular the comparatively early selection and differentiation of treatment peculiar to mathematics teaching, it is crucial that these are regarded as high-stakes decisions.

It is therefore recommended that:

(i) teachers incorporate systematic self-criticism into their casual judgements, informed by awareness of how they form such judgements;

(ii) schools incorporate systematic examination of teachers' judgements, including exploration of other possibilities, into their moderation procedures;

(iii) irreversible decisions and actions based on teachers' judgements be avoided;

(iv) accepted hierarchies of mathematics teaching and learning be questioned as a normal part of professional practice;

(v) that the culture of individualism, judgement, selection and elitism in school mathematics be replaced with a culture of professional self-doubt and an open-minded approach to the potential achievement of all.

It is my belief that such developments would enhance the professional life of mathematics teachers and improve mathematics teaching and learning.

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How to Do Things with Assessments: Illocutionary SpeechActs and Communities of Practice

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Abstract

In this paper, it is suggested that many of the difficulties arisingfrom the use of open-ended and investigative tasks in 'high-stakes' assessments of mathematical achievement arise from anover-emphasis on interpreting these assessments in terms of an individual's past, present or futurecapabilities (perlocutionary speech acts). As an alternative, it isproposed that high-stakes assessments of open-ended and investigative workinmathematics be regarded as illocutionary speech acts, which inaugurateindividuals into communities of practice.

Introduction

In the 1955 William James lectures J L Austin, discussed two differentkinds of 'speech acts'—illocutionary andperlocutionary (Austin, 1962). Perlocutionary speech acts are speech acts*about* what has, is or will be. In contrast, illocutionary speech acts are *performative* (Butler,1997)—in other words, by their mere utterance they bring into beingwhat John Searle calls *social facts* (Searle, 1995). For example, the verdict of a jury in a trial is an illocutionary speechact—by its utterance, it does what it says, since the defendantbecomes innocent or guilty simply by virtue of the announcement of theverdict—the jury's announcement creates a social fact (in this case, the guilt or innocenceof the defendant). Once a jury has declared someone guilty, they *are*guilty, whether or not they really committed the act ofwhich they are accused, until that verdict is set aside by another(illocutionary) speech act. What the judge says about the convict'scrime, however, is perlocutionary, since it is a speech act *about* thecrime.

Another example of an illocutionary speechact is the wedding ceremony, where the speech act of one person (the personconducting the ceremony saying "I now pronounce you husband andwife")brings into being the social fact of the marriage.

In my view a great deal of the confusion thatcurrently surrounds educational assessment arises from the confusion of these two kinds of speechacts. Put simply, most educational assessments are treated as if they wereperlocutionary speech acts, whereas in my view they are more properly regarded as illocutionary speech acts.

The validity of educational assessments

In the predominant view of educational assessment it is assumed that theindividual to be assessed has a well-defined amount of knowledge,expertise or ability, and the purpose of the assessment task is to elicit evidence regarding the level of this knowledge, expertiseor ability (Wiley & Haertel, 1996). This evidence must then be interpretedso that inferences about the underlying knowledge, expertise or ability canbe made. The crucial relationship is therefore between the task outcome (typically the observedbehaviour) and the inferences that are made on the basis of the taskoutcome. Validity is therefore not a property of tests, nor even of testoutcomes, but a property of the inferences made on the basis of these outcomes. As Cronbach noted over forty yearsago, “One does not validate a test, but only a principle formaking inferences” (Cronbach & Meehl, 1955 p297).

Within this view, the use of assessment results is perlocutionary, becausethe inferences made from assessment outcomes are statements *about* the student. Inferences within the domain assessed (Wiliam, 1996a) can beclassified broadly as relating to achievement or aptitude (Snow, 1980).Inferences about achievement are simply statements about what has been achieved by the student, while inferencesabout aptitudes make claims about the student’s skills or abilities.Other possible inferences relate to what the student *will* be able to do, and are often described as issues of predictive orconcurrent validity (Anastasi, 1982 p145).

More recently, it has become more generally accepted that it is alsoimportant to consider the *consequences* of the use of assessments aswell as the validity of inferences based on assessment outcomes. Some authors have argued that a concern with consequences, while important,go beyond the concerns of validity—George Madaus for example usesthe term *impact* (Madaus, 1988). Others, notably Samuel Messick in his seminal 100,000 wordchapter in the third edition of *Educational Measurement*, have argued that consideration of the consequences of the use ofassessment results is central to validity argument. In his view, “Test validation is a process of inquiry into the adequacy andappropriateness of interpretations and actions based on test scores” (Messick, 1989 p31).

Messick argues that this complex view of validity argument can be regardedas the result of crossing the basis of the assessment (evidential versusconsequential) with the function of the assessment (interpretation versususe), as shown in figure 1.

llx9424_result interpretation		result use
evidential basis	construct validity A	construct validity and relevance/utility B
consequential basis	value implications C	socialconsequences D

Figure 1: Messick’sframework for the validation of assessments

The upper row of Messick’s table relates totraditional conceptions of validity, while the lower row relates to theconsequences of assessment use. One of the consequences of theinterpretations made of assessment outcomes is that those aspects of the domain that areassessed come to be seen as more important than those not assessed,resulting in implications for the values associated with the domain. Forexample, if open-ended and investigativework in mathematics is not formally assessed, this is often interpreted as an implicitstatement that such aspects of mathematics are less important than thosethat are assessed. One of the social consequences of the use of suchlimited assessments is that teachers then place less emphasis on (or ignore completely) those aspects of the domainthat are not assessed.

The incorporation of open-ended and investigative work into 'high-stakes' assessments such as school-leaving and universityentrance examinations can be justified in each of the facets of validityargument identified by Messick.

A Many authors have argued that an assessment of mathematics that ignoresopen-ended and investigative work does not adequately represent the domainof mathematics. This is an argument about the evidential basis of resultinterpretation (such an assessment would be said to under-represent the construct of 'Mathematics').

B It might also be argued that leaving out such work reduces theability of assessments to predict a student's likely success inadvanced studies in the subject, which would be an argument about theevidential basis of result use.

C It could certainly be argued that leaving out open-ended and investigativework in mathematics would send the message that such aspects of mathematicsare not important, thus distorting the values associated with the domain(consequential basis of resultinterpretation).

D Finally, it could be argued that unless such aspects of mathematicswere incorporated into the assessment, then teachers would not teach, orwould place less emphasis on, these aspects (consequential basis of resultuse).

The arguments for the incorporation of open-ended and investigative work inhigh-stakes assessments in mathematics seem, therefore, to be compelling.However, the attempts to introduce such assessments have been dogged byproblems of reliability. These problems arise in three principle ways (Wiliam, 1992):

disclosure:	can we be sure thatthe assessment task or tasks elicited all the relevant evidence? Put crudely, can we be sure that "if they know it they show it"?
fidelity:	can we be sure that all the assessment evidence elicited bythe task is actually 'captured' in some sense, either bybeing recorded in a permanent form, or by being observed by the individualmaking the assessment?
interpretation:	can we be sure that the captured evidence isinterpreted appropriately?

By their very nature, open-ended and investigativetasks take longer to complete than traditional assessments, so that eachstudent attempts fewer tasks and sampling variability has a substantialimpact on disclosure and fidelity. The number of tasks needed to attain reasonable levels ofreliability varies markedly with the domain being assessed (Linn & Baker,1996), but as many as six different tasks may be needed to attain thelevels of generalizability required for high-stakes assessments (Shavelson, Baxter, & Pine,1992).

The other major threat to reliability arises from difficulties ininterpretation. There is considerable evidence that differentraters will often grade a piece of open-ended work differently, although,as Robert Linn has shown, this is in general a smaller source ofunreliability than task variability.

Much effort has been expended in trying to reduce this variability amongstraters by the use of more and more detailed task specifications and scoringrubrics. I have argued elsewhere (Wiliam, 1994a) that these strategies arecounterproductive. Specifyingthe task in detail removes from the student the need to define what,exactly, is to be attempted, thus rendering the task more like an exercise,or, at best, a problem (Reitman, 1965). The original impetus for open-endedwork—that the student should have a role in what counts as a resolution of thetask—is negated.

Similarly, developing more precise scoring rubrics does reduce thevariability between raters, but only at the expense of restricting what isto count as an acceptable resolution of the task. If the students are givendetails of the scoring rubric, then their open-ended task is reduced to a straightforward exercise, and if theyare not, they have to work out what it is the teacher wants. In other wordsthey are playing a game of ‘guess what’s in teacher’shead’, again negating the original purpose of the open-ended task. Empiricaldemonstration of these assertions can be found by visiting almost anyEnglish school where lessons relating to the statutory ‘coursework’ tasks are taking place (Hewitt, 1992; Wiliam, 1993).

These difficulties are inevitable as long as the assessments are requiredto perform a perlocutionary function, making warrantable statements aboutthe student’s previous performance, current state, or futurecapabilities. Attempts to ‘reverse engineer’ assessment results in order to make claims aboutwhat the individual can do have always failed, not least because of theeffects of compensation between aspects of the assessments.

However, many of the difficulties raised above diminish considerably if theassessments are regarded as serving an *illocutionary* function. To seewhat this would entail, it is instructive to consider what might beregarded as one ofthe most prestigious of all educational assessments—the PhD.

Assessments as illocutionary speech acts

In most countries, the PhD is awarded for a 'contribution tooriginal knowledge', and is awarded as a result of an examinationof a thesis, usually involving an oral examination. Although the award istechnicallymade by an institution, the decision to award a PhD is generally made onthe recommendation of examiners. In some countries, this can be thejudgement of a single examiner, while in others it will be the majority recommendation of a panel of as many as six. Theimportant point for our purposes is that in effect the degree is awarded as the result of a speech act of a single person (ie the examiner where there is just one, or the chair of the panel where there is more than one). The perlocutionary content of this speech act isnegligible, because, if we are told that someone has a PhD, there are veryfew inferences that are warranted. In other words, when we ask "What is it that we know about what thisperson has/can/will do now that we know they have a PhD?" theanswer is "Almost nothing" simply because PhD theses are so varied. Instead, the award of a PhD isbetter thought of not as an assessment of aptitude or achievement, or evenas a predictor of future capabilities, but rather as an illocutionaryspeech act that *inaugurates an individual's entry into a community of practice*.

The notion of a community of practice is an extension of the notion of aspeech community from sociolinguistics, and has been used by authors suchas Jean Lave to describe a community that, to a greater or lesser extent,'does things the same way' (Lave & Wenger, 1991). New members begin as peripheralparticipants in the community of practice, and over a period of time, byabsorbing the values and norms of the community, move towards fullparticipation.

Attempts to make sense of the assessment of open-ended tasks such as PhDs,and, more prosaically, mathematics portfolios, in terms of the traditionalnotions of norm-referenced and criterion-referenced assessments have been unsuccessful (Wiliam, 1994a).There is no well-defined norm group, and even if there were, there would beno way of ensuring that the norm group represented the range of allpossibilities for a PhD. There are also no criteria, apart from the occasional set of 'guidelines' which are never framed precisely enough to ensure that they areinterpreted similarly by different raters. Consistency in the assessment ofPhDs, to the extent that it exists at all (and this, of course, isdebatable),is not in any sense objective. There is no explicit reference to a normgroup, nor is the judgement based on reference to a set of criteria. Itmight be argued that many universities require that a PhD represents"a contribution to original knowledge, either by the discovery of new factsor by the exercise of independent critical power", but thisis far too imprecise to be regarded as a criterion, and in any case, thecriterion is never interpreted literally. For example, the number ofcharacters and words in this paper is not known to anyone at present, so asimple count of these would be'new facts', but it is certain that this would not be awardeda PhD in any university. PhD assessments are therefore neither norm- norcriterion-referenced. Instead, any consistency in the judgement of PhDexaminers exists by virtue of a *shared construct within the community of practice*. For this reason, Ihave termed these *construct-referenced* *assessments*. The judgements areneither objective nor subjective—rather they areintersubjective—and the evidence is that they can be made dependable, even with relatively newmembers of the community (Wiliam, 1994b).

John Searle (*op. cit.*) illustrates this by an interview between abaseball umpire and journalist who was seeking to establish whether theumpire's judgements during his career had been objective orsubjective:

Interviewer: Did you call them theway you saw them, or did you call them the way they were?

Umpire: The way I called them *was* the way they were.

The umpire's calls bring into being socialfacts because the umpire is *authorised* (in the sense of having boththe power, and that use of power being regarded as legitimate) to do so.The extent to which these judgements are seen as warranted ultimatelyresides in the degree of trust placed by those who use the results of the assessments (for whatever purpose) in the community of practice making the decision about membership (Wiliam, 1996b).

The arguments sketched out above apply equally well to mathematicseducation. The assessment of students' open-ended and investigativework in mathematics can be assessed in the same way that anapprentice's 'work sample' is assessed.

An apprentice carpenter, nearing the end of her apprenticeship, will beasked to demonstrate her capabilities in making (say) a chair or a chest,and a student nearing the end of a particular phase of theirmathematical education could be asked to assemble a portfolio of theirwork. Decisions about how much time is allowed, how much support is given,and to what extent a mathematical portfolio is required to be theindividual's unaided work will vary from community to community (for an interesting discussionof the extent to which T.S. Eliot's poem The Wasteland can beattributed to him, rather than as a joint effort with Ezra Pound, seeWineberg, 1997). In some communities it may be felt important to establish an individual's ability toact alone. In others, it will be far more appropriate to establish theindividual's ability to work with others in arriving at asolution.

While the portfolio will provide some information about the student's past achievements and future capabilities, this will be limited by thevariability in the circumstances under which the portfolio was prepared. However theportfolio will be capable of indicating the extent to which the individualcan be regarded as a member of a community of practice.

These aims do not conflict at all with the aims of certifying students forfurther stages of education or employment and they are often much moreconsistent with the demands of industry than the individualistic approachesso favoured ineducational systems in Western societies. Indeed, if we take seriously the arguments emerging from work on socially-sharedand socially-distributed cognition (for example Resnick, Levine, & Teasley,1991; Salomon, 1993), we would be less interested in what an individualcould achieve on their own, but more interested in what they could achieve as part of a community. If wefurther accept that it does not make sense to talk of knowledge being‘inside the individual’s head’, but constituted in the social interactions between individuals, as isincreasingly being accepted (Clark 1997, Hendriks-Jansen, 1997), we wouldno longer speak of ‘intelligent individuals’ but ‘individuals intelligent incommunities of practice’.

Summary

In this paper, I have argued thatregarding the assessment of open-ended and investigative work inmathematics as illocutionary, rather than perlocutionary, speech acts substantially alleviates many of the problemscommonly encountered in the assessment of such work. The score or markgiven to a piece of work indicates the extent to which the individual (orthe group) has acquired thevalues and norms of the community of practice, and therefore the extent towhich they are full or peripheral participants in that community. Suchjudgements are neither norm- nor criterion-referenced, but rather *construct*-referenced, relying for their dependability on the existence of a sharedconstruct of what it means to be a full participant.

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Looking at mathematics as technology: implications for numeracy

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Abstract: *A numeracy education which enables people across different socio-cultural groups to develop and participate in more numerate discourses is needed. In articulating this need, and thinking about how such discourses may evolve, the paper will focus on the the pervasiveness of mathematical models in our socio-political spheres. In analysing the nature of the presence of mathematics in people's lives, the paper views mathematics as a form of technology, and suggests that numeracy ought to be seen as part of a broader critical technological literacy.*

Introduction

It has been said that "structural engineering" is the art of modelling materials we do not wholly understand into shapes we cannot precisely analyse so as to withstand forces we cannot properly assess in such a way that the public at large has no reason to suspect the extent of our ignorance.

[The "technical approach" assumes that] experts have a predominant role in decision-making and citizens are see as 'consumers' who are incapable of exerting ethical or practical concerns about the environment ... [while the political approach] adopts a critical view of industrial market society with its growth imperatives and focuses on alternative economic and social strategies which may involve less exploitative values towards the environment.

When we talk about risks, I give them numbers. They give me sociology. There is NO discourse.

The first quote appears in a set of guidelines for construction engineers published by the professional body of Australian engineers (Institution of Engineers, Australia, 1994). The second appears in an article which contrasts the technical approach with a political approach in social impact assessments of technological projects (Craig, 1990 cited in Norrie, 1990, p 31). The third was a statement made by an engineering colleague in a conversation about different discourses about public risk.

What these quotes suggest is that approaches to thinking about some of the significant problems which impact on the whole of society are classified as *either* technical *or* non-technical, and that these approaches are incompatible. Assumptions are also made about the superiority of one over the other, depending of course, on whether the person making that judgement belongs to the "technical"

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or the “social/ non-technical” community. There is also a sense of futility in trying to engage people in a shared discourse across the technical/ non-technical divide.

An area where the lack of a numerate discourse is noticeable is in public disputes about health risks associated with technological developments such as cellular phones and high voltage transmission lines. These disputes may be eventually “resolved” at the legal or policy levels, but often are not resolved at the socio-cultural and personal levels. The community is accused of arguing along “emotive” lines; the technologists along reductionist and complex mathematical lines. Another example is the lack of meaningful dialogue between managers who concoct quantitative (and often complex) workload and performance measures, and those whose workplaces are prescribed by these models of “work”. Is it not the role of numeracy educators to be concerned about the absence of discourse between “communities of mathematical practitioners” from different backgrounds, and to address the gap in our numeracy education which may be contributing to this.

This paper emerges out of a reflection of my work as a teacher of mathematics in engineering courses, and of numeracy and numeracy education in adult basic education (ABE) teacher training programs at a university in Australia. The unusual combination of student groups in my work has presented me with the challenge of looking at numeracy more broadly than something which resides in ABE alone, and finding ways of theorising numeracy in ways which make real links with critical education.

My purpose in this paper is to examine ways in which mathematics “works” in society, and to consider ways in which discourses can be created around these workings towards inclusive social actions. I will argue that looking at mathematics as a technology is useful in this regard, and that therefore numeracy might be viewed as an aspect of technological literacy, which in turn needs to be conceptualised within the framework of critical education.

How does mathematics work in society?

Skovsmose (1994) talks about mathematics having a “formatting” power. He expands on the central role played by mathematical models in “giving not only descriptions of phenomena, but also by giving models for changed behaviour” through technologies, including energy, social and information technologies (p 55). Davis (1991, p 2) talks about the descriptive, predictive and prescriptive functions of mathematical models, and how their “ability .. to provide frameworks of reality and of action, and .. to change what is, is very great”. So when we talk about the social power (and sometimes violence) or what Skovsomse calls the “formatting power” of mathematics, we are talking about mathematical *models*, such as economic models, models of risks, models of work and school performance.

Skovsmose shows how mathematical models underpin technologies, and how those technologies in turn exercise social and political power in society. I would suggest that mathematics itself can be viewed as a technology, although it is clearly not associated with an artefact such as computers are with information technology, solar panels are with power generation technology, and ticket machines are with transport technology. It is, as Postman calls, an "invisible technology" (1993). And in its power to create distances between people with different mathematical experiences, it is as Porter calls, a "technology of distance" (1995). When we examine the ways in which public safety and risks, welfare support and access, workplace performance, definitions of "full-time" employment, intelligence levels are prescribed by "numbers", both of the technological metaphors of mathematics seem appropriate.

Mathematics as an invisible technology

Mathematical models, as experienced by the community at large is an invisible technology. It is invisible not only in the actual numbers which the models produce as "solutions", but invisible in its derivation. Figure 1 shows a model of a mathematical modelling process. Risks, pay rises, new tax rates, cut-off scores for university entrance are all prescriptions which mathematical models generate. These models are constructed by people who want an answer to *a problem they see*, based on *their assumptions*, using methods that *they deem appropriate*. What the general public sees is typically the answer only. Rarely do they even see the original question that drove the modelling process. Yet, they become co-opted as consumers of these model generated solutions.

The public's relationships with mathematical models is akin to the public's relationships with many other technological developments. Many of us find ourselves in the position of having to "upgrade" our perfectly functional computers to more powerful, faster and bigger machines to accommodate the software which will replace the perfectly functional software that we have been using. We may not have seen or felt any problems with our software or the computer, but a "solution" to someone else's problem is imposed upon us, and we have little choice but to upgrade our(?) technology.

There are some mathematical models which produce solutions which the public will reject. An example of an area in which this occurs is in decisions about personal and environmental risks. In many of these cases, the community is also ignorant of the mathematics which has gone into measuring the risk factor of, say, a technological development. The question may be assumed to be whether or not use of cellular phones causes brain cancer, or whether or not living under high voltage transmission lines causes leukaemia. But typically, the specific questions which drive the decision making processes for the different interest groups are quite different. What is rejected by the community may not really be what they thought they were rejecting.

The community is typically looking at risk versus no risk; safety versus danger. They are searching for "answers" which would give them some definitive assurance or a disaster signal. The technical experts are looking at *minimising* risks within a complex "systems framework", not focused on the particular individual incidences, but on statistical information and subjective probabilities based on *their assumptions and knowledge.* But within that systems framework, there is never the possibility of achieving a zero risk solution (unless the project is brought to a halt altogether). The critical issue here is not whether one approach is more valid than the other; rather it is the invisibility of the magnitude of differences in motivation behind the questions being asked by the two groups.

To resolve the conflict between the two groups, it is not sufficient, nor is it appropriate to focus only on the "solution" that the expert's model produce. Ideally, there would be a process of developing a shared understanding of the question, engaging in a shared process of modelling the problem - mathematically or otherwise, which includes an agreed set of assumptions, using a methodology which takes into account information which all parties grow to accept as relevant to the problem. An answer, or a set of answers thus derived would not only be better understood by all concerned, but also have shared ownership. So long as the process of modelling employed by the technical experts are invisible, especially in the early stages where the question itself is being specified, and assumptions are made, this shared understanding and ownership (and with it responsibility) of the decisions based on the models would not be possible.

Mathematics as a technology of distance

How does the technology of mathematics create distances within social groups? It is often said that the perception of objectivity of mathematics also contributes to the lack of contestation against mathematically prescribed decisions. Porter puts forth the thesis that objectivity has "to do with the exclusion of judgement, the struggle against subjectivity. ... this, more than anything else accounts for the authority of scientific pronouncements in contemporary political affairs. ... In science, as in political and administrative affairs, objectivity names a set of strategies for dealing with distance and distrust" (p ix). He presents a thesis that the high level of discipline in the discourse of (formal) mathematics helps to make it something that is well suited for communication that "goes beyond the boundaries of locality and community" (p ix). But where does it go?

The authority of mathematics, based on the perceived objectivity of what "truths" it can convey, is at one level, a critical part of explaining how mathematics creates a distance (and in some cases gulfs) between the "haves" and "have nots" of mathematical knowledge. But it doesn't fully explain how mathematical models work as a technology in the wider community. More maths will not necessarily enable people to challenge expert claims on risk and safety, economic policies, or workload formulae. In order to understand this better, it helps to look at some of the theories of technology.

Kranzberg (1997) illustrates a constructivist view of technology with a set of six "laws" which he (unashamedly) calls "Kranzberg's Laws". Two of these laws are helpful in explaining how mathematics can be seen as a technology of distance. They are his -

Third Law: Technology comes in packages big and small. (p 10)
and
Fourth Law: Although technology might be a prime element in many public issues, nontechnical factors take precedence in technology-policy decisions. (p 11)

The Third Law seeks to explain how a technological innovation is embedded within complex systems, which in turn interact with other systems. He presents as an example, the Ford assembly line. One could view the assembly line as composed of different technological elements, such as conveyor lines, which are integrated within a comprehensive system (p11). The assembly line itself can also be seen as an integral part of the "technology" of manufacturing systems, which exists in particular ways within a larger socio-economic system. This then leads to his Fourth Law which explains how technological capability is only one of many factors which determine the ways in which technological infrastructures and policies are realised in society. These include the nature of the economic systems, dominant social values, and environmental contexts which determine the location of the technological system within the society. In terms of how mathematical models are embedded within a tight and socially non-inclusive system, the following quote about a dominant ethos in engineering is illustrative -

Systems engineering is a choreographed "dance" between the client and the project team. The client is the person/organisation who "owns" the project. It is the client that has the need for the results of the project, who will pay, and who will put the results of the project into operation. The project team is the technical group that will actually develop the system. The client knows what they want (or at least would like to think that they do), while the project team knows how to build it (or at least claims competence in building such systems). Systems engineering is the process of matching the client's understanding of what is needed with the technical competency necessary to build a complex system. (Drane & Rizos, 1998, pp 17-18)

What this definition of systems engineering shows is that it is not the lack of mathematical knowledge which makes technological decisions impenetrable for the general public. It is the way in which problems/projects are conceived within a client/patron relationship which has little visible accountability to the society at large. Gilchrist (1995) reveals another example where an Australian Government bureau's mathematical model of greenhouse gas emissions led to misguided policy recommendations, because it was not immediately revealed that the model was funded by major Australian coal producers whose immediate interests clearly would not have been served by tighter greenhouse policies. While a reasonably sophisticated level of mathematical knowledge would most likely be needed to decode the workings of the model, both technical and non-technical details of the model were unavailable for scrutiny. The public in these cases are therefore distanced firstly and more significantly by the boundaries of the "system" agreed upon by the technical/economic interest groups, than by the possible lack of mathematical knowledge. In these cases, “more mathematics” would not help to penetrate the boundaries set by the patron/client systems.

Penetrating the impenetrable - the role of numeracy education?

[Numeracy] is a social consciousness reflected in one’s social practices which bridges the gap between the world of academic maths and the real world, in all its diversity. The consciousness enables one to challenge the boundaries and the role of mathematics in social contexts. For this reason, numeracy is not linked to any level of knowledge in the hierarchy of academic mathematics; there is a need for numeracy associated with all levels of mathematical knowledge. Further, being an ingredient in the expansion of social justice, if it isn’t political, it’s not numeracy and if it’s not in context, it’s not numeracy.(Yasukawa & Johnston, 1994, p 198)

A “noble” definition of numeracy. But how can numeracy education tackle an invisible technology which excludes and distances the community from decisions and policies which affect all of us? What does it mean for numeracy education to expand social justice? What pedagogical frameworks are available to help us find direction in *critical* numeracy education?

For some years now, there has been effective “translation” of constructivist learning theories into effective mathematics pedagogy by applying what has been observed in socio-cultural studies of mathematics and mathematics education (Lave, 1988; Nunes, et al, 1993; D’Ambrosio, 1995; Walkerdine, 1990). Teaching in contexts relevant to the learner, celebrating different ways of doing maths, and negotiating the curriculum are some of the ways in which as teachers, we have tried to make mathematics more meaningful to our learners. But is there a danger that teaching only in the learner’s contexts may leave the maths “trapped” in the learner’s immediate and personal contexts, and thereby also the learner? Is there a bigger danger in learners defining their contexts *through the maths* they perceive themselves capable (and incapable) of doing?

Many teachers have adopted the approach of teaching “in context”. For teachers who work in the ABE sector, this may mean using contexts such as shopping, utility bills, and social statistics in their teaching (Helme, 1995). For teaching engineering students, it can mean, as I have done, engaging students in project work which makes explicit the role of mathematics in a great variety of engineering processes and systems such as energy demand forecasting, traffic control, product reliability analysis, and so on (Yasukawa, 1995).

But where in our education system do we educate students who will become engineers, economists, business managers who respect the ethics and practical concerns of the wider community? Where do we give students in the non-technical areas the strategies to critique the technical approach effectively, rather than simply criticise its “narrowness” and “reductionist approach”? How can we ensure that “critical numeracy” education for the wider community does not end with “a bit more maths in context", resistance to learning statistics because "statistics lie", and perhaps a letter to the editor about media's selected use of data?

Numeracy, technological literacy and critical education

The ramifications of decisions based on models can be too broad and far-reaching, both in terms of decisions based on public policy and private investment, for society not to take an active role. (Leet and Wallace, 1994, p 243)

If the subject labelled Technology is to be largely focused on practical aspects of designing and making, then it cannot possibly bear the sole weight of responsibility for enabling students to make sense of technology. To achieve the latter aim, other subject areas must take technology seriously. However, an arrangement by which responsibility for practical capability rested with Technology, and for critical awareness with subjects ... where values had been driven into exile from out of Technology, would be undesirable. This would tend to confirm Technology as a ghetto for ingenious, specialist tinkerers, and the Humanities as the natural home for anti-technologists. (Barnett, 1994, pp 62-63)

[Critical education] must assume an active role in identifying inequalities in society, in identifying causes for the emergent sociological and ecological crises and in explaining and outlining ways of dealing with such problems .... [as Giroux states] schools must be defended as an important public service that educates students to be critical citizens who can think, challenge, take risks, and believe that their actions will make a difference in the larger society. (Skovsmose, pp 40-41)

Leet and Wallace advocate that society has a role in developing an ethics of modelling. They recommend that "the discipline of Applied Ethics be employed to provide the knowledge upon which we can begin to prescribe ethical behaviour for model builders. ... to sensitize practitioners to where ethical dilemmas arise in practice, ... and to encourage reflective decision making in order to arrive at a consensus that satisfies a 'greater moral good'" (pp 243-244). But how can this be realised in a way that is a meaningful and practical part of the education for model builders and users, rather than an ethical "prescription"?

I have learned from my experience of teaching a subject called Mathematical Modelling to senior engineering students that there is a great dilemma in saying, on the one hand, "be aware that your view of reality is not the only view ... don't presuppose that a valid solution to the problem you wish to investigate is a mathematical one" and on the other, to say that they must complete a project which meets the subject objectives, one of which is "application of mathematics in a 'real' world problem". I can teach them "the right line" about ethics in modelling, but within a program which primarily emphasises technical expertise, it has been difficult to develop strategies which students can use to turn a social critique of their technical approach into a change in their own practice of engineering in the community.

There is also a problem, as Barnett points out in having subjects which isolate the critique from the practice of technology. Society which is divided into a group of tinkerers and another group of critics can never properly challenge the formatting power of mathematical or any other technology. What is needed is a technological literacy education which is strongly grounded in the project of critical education, that is educating those people who as Giroux's quote by Skovsmose states, can "challenge, take risks, and believe that their actions can make a difference in society".

In order to challenge, people need to understand what they are challenging. As I have attempted to describe earlier, the formatting power of mathematics is not determined by mathematics alone. It is determined by the nature of the "contracts" by and the "systems" within which models are built, and by the specific questions which drive the modelling projects. Before people can meaningfully challenge prescriptions delivered by models, they need to understand what Lemke and others call the "community of practice" (CoP) of these model builders, that is the "ecology..., meanings and things" with which they deal (1997, p42). Equally, the model builders would need to develop an understanding of the different communities of practice who are affected by what they do, and what their "things" prescribe to them.

In considering a pedagogy for such understandings, we can draw from the theories we already have about "meaningful" mathematics learning. We have varieties of constructivist theories, all of which are underpinned by the notion that learners negotiate meanings, and meanings cannot be imposed by transmission. That is why many of us try to teach in ways which engage students in hands-on, interactive learning activities. Is it too optimistic to suggest that meanings can be negotiated across people who identify with different communities of practice?

Situated cognition theory has used ways of researching mathematical and literacy practices, ways of understanding people's practices in relation to their socio-cultural contexts. But Lemke also suggests that we must "look at networks of interdependencies among practices, activities and CoPs to understand the dynamics of ecosocial systems" (p49). This, he says, is because each individual changes through participation in a CoP, and is a member of more than one CoP at any one time, and through life. Could we not make the research methodology available to our students, so they may understand not only the CoPs with which they themselves identify, but also the CoP of other groups in society? Is it not possible that through that, people of different CoPs can evolve a new CoP in which they all belong, and which can engage in a common discourse about technologies?

What is a hopeful ingredient for this vision of technological literacy, is the recognition that every individual identifies with different CoPs at different times.

For example, an engineering student may identify her/himself as an engineer when s/he is tackling a problem about traffic control technology in say, a Computer Systems Analysis class. But the same student also experiences aspects of the transport system in her/his everyday life, and most likely without applying formal analytical methods to negotiate them.

A pre-requisite for a technological literacy program which can educate active citizens is an educational framework which makes explicit the holistic influence that education has on individual learners, that is not only the technical/ discipline specific expertise which the students are expected to develop. It needs to make visible the different communities of practice which will interact with students in their professional and private lives, and promote critical reflection on how they can interact effectively in these networks of CoPs for the greater social good. It needs to engage students in a discourse "within themselves" as members of different CoPs and challenge them to reconcile their technical approaches and their needs and issues in their personal lives. We need to build on this reflection and engage students in discourses with other student groups and wider community groups on real social issues so that they may evolve a new CoP which represents, respects, and negotiates, rather than build walls against different interest groups. We can engage students from non-technical areas in "tinkering" with design projects of their choice, so make visible that even in their own lives, they use analytic methods to deal with uncertainties and risks. This may be less visible, but similar to the formal methods used by "professional" risk practitioners. If we as professional members of society cannot bridge gaps between different CoPs, we have to ensure that at least our students become agents for developing new practices and discourses.

It is one thing to acknowledge and respect people's different realities. However, we all belong to the one planet, and while the "degree" of social injustice and environmental degradation on the planet may be debatable, social injustice and environmental degradation are real. We as educators must recognise this, and be actively seeking ways to educate people who are going to help us move towards a more socially equitable and environmentally sustainable future. Numeracy for a sustainable society has to be much more than maths in context, and criti*cism* of other people's maths. It must ultimately be about building numerate discourses across different communities of practice. We as numeracy educators need to extend beyond our traditional communities of numeracy educators, and be active participants in this project.

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Classroom Interactions and Linguistic Capital: A Bourdieuian Analysis of the Construction of Social Difference In Mathematics Education

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Goodenough (1976) asked the question "What do people need to know in order to operate in a manner that is acceptable to others in a society?" This question provides the stimulus for this paper where the question is rephrased to: "What do students need to know in order to operate in a manner which is acceptable in the mathematics classroom?" Such a question is not without political implications and so needs to be extended to include questions about the consequences of participation in mathematics classroom. To answer this question, I appropriate Gore's (1990) notion of "pedagogy as text" and develop the argument that mathematics pedagogy is a text which students must be able to read in order to be constructed as effective learners of mathematics. These texts, however, are not apolitical and as Bourdieu (in (Wacquant, 1989) has argued persuasively, language is a form of capital which can be exchanged for other forms of capital - social, economic or cultural. Combining these two frameworks, I argue that students enter the mathematics classroom from a range of socio-cultural backgrounds whereby students whose socio-cultural background is congruous with that of the culture represented in and through the practices embedded within the mathematics classroom - including linguistic practices - are more likely to be constructed as successful students.

Language as a Form of Capital

Bourdieu (in Bourdieu & Wacquant, 1992) argues that access to legitimate language, in this case mathematics, is not equal and that linguistic competence is monopolised by some. In considering the case of mathematics, this suggests that access to the discourses and discursive practices of mathematics is differentially accessible. For those students who enter the mathematics classroom with a competence in the discursive practices, access to mathematics is made more easily. Simultaneously, such students are more likely to be constructed as successful students based on the teacher's judgement of their ability. Within this context, language background is a form of capital which can be converted to academic reward.

Linguistic competence – or incompetence – reveals itself through daily interactions. Within the mathematics classrooms, legitimate participation is acquired and achieved through a competence in the classroom dialogic interactions. Students must be able to display a discursive competence which incorporates a linguistic competence, an interactional competence along with a discursive competence if they are to be seen as competent learners of mathematics. Classroom interactions are imbued with cultural components which facilitate or inhibit access to the mathematical content. To gain access to this knowledge, students must be able to render visible the cultural and political aspects of the interactions.
Bourdieu (in Bourdieu & Wacquant, 1992) argues that

> Linguistic competence is not a simple technical ability, but a statutory ability.

> …what goes in verbal communication, even the content of the message itself, remains unintelligible as long as one doe not take into account the totality of the structure of the power positions that is present, yet invisible, in the exchange. (p. 146)

From their early years, students are located within family structures and practices which will facilitate the development and embodiment of particular cultural features, least of which is language. For these students, the embodiment of their cultural background into what Bourdieu refers to as the habitus, predisposes them to think and act in particular ways. This embodiment of culture includes a linguistic component. Students whose linguistic habitus is congruent with that of the discursive practices represented in mathematics classrooms are more likely to have greater access to the knowledge represented in and through such practices.

From this perspective, language must be understood as the linguistic component of a universe of practices which are composted within a class habitus. Hence language should be seen to be considered as another cultural product - in much the same ways as patterns of consumption, housing, marriage and so forth. When considered in this way, Bourdieu proposed that language is the expression of the class habitus which is realised through the linguistic habitus.

> Of all the cultural obstacles, those which arise from the language spoke within the family setting are unquestionably the most serious and insidious. For, especially during the first years of school, comprehension and manipulation of language are eth first points of teacher judgement. But the influence of a child's original language setting never cease to operate. Richness and style of expression are continually taken into account, whether implicitly or explicitly and to different degrees. (Bourdieu, Passeron, & de saint Martin, 1994a) p. 40

To this end, the linguistic habitus of the student will have substantial impact on his/her capacity to make sense of the discursive practices of the mathematics classroom and hence their subsequent capacity to gain access to legitimate mathematical knowledge along with the power and status associated with that knowledge. The processes through which the schooling procedures are able to value one language and devalue others must be systematically understood. Through this process, we will be better understand how mathematical pedagogy both inculcates mathematical knowledge and imposes domination. In order to understand how the linguistic practices of the mathematics classroom position and hinder the effective participation of some students, the notion of pedagogy as text is proposed.

Mathematics as a Text

Gore (1990) has argued that pedagogy can be seen as a text which can be read and interpreted by the reader. Texts can be read in a multiple of ways, so that the student entering the mathematics classroom will be required to read, interpret and make sense of what transpires in the mathematics classroom - not only of the mathematical content, but also the pedagogical approaches within which the content is relayed. To be able to read these texts, the students must have some linguistic competence in the reading of social texts and discursive practices.

The interactions that occur within the classroom have been subjected to ethnomethodological approaches and have been found to have highly ritualised components with clearly identifiable discursive practices (Lemke, 1990; Mehan, 1982a). They argue that these components are not explicitly taught but are embedded within the culture of the classroom. The highly ritualised practices of classroom interactions can be seen in the types of interactions which occur across the various phases of the lesson. For example, the most common form of interaction consists of a practice in which the teacher initiates a question, the students respond and the teacher evaluates that response which Lemke (1990) refers to as "triadic dialogue". This interactional practice can be observed in the following:

T: What does area mean?
S: The outside of the square
T: Not quite, someone else? Tom?
S: When cover the whole surface, that's area.
T: That's good

Lemke (1990) argues that this practice allows teachers to keep control of the content and flow of this phase of the lesson. While Lemke focussed on the science classroom, the style and purpose of this interaction can be just as readily applied to the mathematics classroom and is aptly summed up as follows:

> Triadic dialogue is an activity structure whose greatest virtue is that it gives the teachers almost total control of the classroom dialogue and social interactions. It leads to brief answers from students and lack of student initiative in using scientific language. It is a form that is overused in most classrooms because of a mistaken belief that it encourages maximum student participation. The level of participation it achieves is illusory, high in quantity, low in quality (Lemke, 1990, p. 168)

This practice is not made explicit to students, rather it must be learnt through implicit means. To participate in the classroom interactions effectively, students must have knowledge – either intuitive or explicit – of these unspoken rules of interaction.

Furthering the work of Lemke, Mehan (1982b) has identified three key phases of a lesson - the introduction, the work phase and the concluding/revision phase. In each of these phases there is a shift in the power relations between the students and teacher which permits different forms of interactions to occur (Mehan, 1982b; Schultz, Florio, & Erickson, 1982). For the purposes of this paper, it is my intention to discuss the introductory phase only.

Mehan (1982) argues that during the intodutory phase of the lesson, the teacher maintains tight control over the students, initially to ensure that the students are ready for the content of the lesson. Once control has been established and attention gained, the lesson can then proceed. Triadic dialogue is commonly observed in this phase in order to keep control of the academic content of the lesson and the control of the students. Dialogue between students and between teacher and students is not generally part of this phase. If the teacher initiates a question but the student is not able to respond, it is not appropriate for students to express their lack of understanding since this will interrupt the flow of the phase. If there is a misunderstanding or lack of understanding, it is more appropriate for this to be voiced in the work phase of the lesson.

What is lacking from this corpus of knowledge of classroom interaction is the failure to recognise that these interactions recognise a particular linguistic form which will be more accessible to some students than others. In this sense, the interactions within the classroom can be considered as another cultural product which is more familiar to some students and not others. The linguistic habitus of the students will or hinder a students capacity to render visible the mathematical content embedded in the pedagogic action. The

> …such cues [IRE] are not necessarily "understood" by all participants, but they are certainly part of the "functional conflict" between dominant and dominated languages in (and out of) educational settings. (Collins, 1993, p.131)

In the previous sections, I have drawn on the work from a number of traditions – theoretical and methodological – and have proposed that the social background of the student will the construction of a particular linguistic habitus. The field of mathematics, having its own regulatory discourses and discursive practices, will recognise and value some linguistic practices and not others. These practices are socially biased.

Method

An ethnography of two classrooms was undertaken in which mathematics lessons were videotaped. The two classrooms were located in socially divergent sites - one an independent school which serves a middle- to upper-class clientele (Angahook). The other classroom was in a state school serving a predominantly working-class clientele (Connewarre). The classrooms were in the second last year of primary school and most students were approximately 10-11 years old. The video-taped lessons were transcribed and analysed. Extracts from one of the lessons from each classrooms will be used as examples for this paper.

Angahook

This school serves a middle- to upper-class client group. The mathematics teaching, learning, assessment and curriculum are relatively conservative with a strong emphasis on rote learning, preparation for examinations and teacher-directed pedagogy. The class sizes are small with only 12 students in classroom observed. In the lesson presented here, the students were undertaking an activity from the Mathematics Curriculum Teaching Package. Prior to the extract shown, the teacher (Helen) has used a number of short mental arithmetic tasks. The following is the introduction to the lesson.

T: You are asked to judge the diving for the Olympics, you will need to know the degree of difficulty because what if someone did just a plain dive and did it perfectly and got full marks for it and what if someone else did a triple somersault, back flip, side swinger double pike and knocker banger and only got half marks for ti because they entered the water and made a bit of a splash. Is that fair?

C: No

T So we have to talk about degrees of difficulty. What do you think that means? What does that actually mean? Robert?

Robert You have to add a bit more to the score because of the degrees of difficulty.

T Good boy. Yes, good. Daniel?

Daniel Well the performance of their dive, how they dive and well like they might have a very good dive and make a very big splash and may even get off

T Right, good. OK you are on the right track. What do you want to say about degree of difficulty Cate?

Cate How hard it is?

T How hard it is. Tom what would you like to say about degree of difficulty? That's not a word we use much in our everyday language..... degree of difficulty.

Tom The percentage of how hard it is

T Good. Because you're focussing on the word degree though aren't you. So a really hard dive. Now you can see on this sheet they're talking about DD which is short for degree of difficulty and a really hard dive. What would be a really hard dive? What would be the highest number for a degree of difficulty be? Have a look at your sheet. Try and work out the degree of difficulty. Vicky?

Vicky8

From this extract it can be seen that the teacher follows the triadic dialogue identified by Lemke. The teacher retains control of the content and interactions through the use of the three phases of interactions. Using this approach she is able to control the flow of the lesson as can be seen in the last interaction where Tom has mentioned "percentages" which she then takes as a cue for linking percentage and degree in a way which suits her purposes.

Examining the flow of the interactions indicates that there is a complicit agreement between the teacher and students to participate in the interactions. There are no transgressions or challenges to the teacher's authority. This allows for substantive content to be covered.

The teacher is able to maintain control over both the form and content of the lesson and the students through a mutual compliance with the implicit rules by both the students and the teacher. She has used Triadic Dialogue to structure the interactions and students infrequently transgress the rules. This allows her to retain the focus of the lesson and in so doing, the students are exposed to a significant amount of mathematical knowledge that is embedded in that dialogue. The teacher's capacity to deliver the lesson in this way allows for her to use a very rich mathematical language as she discusses the mathematical content. In other words, the students are exposed to mathematical language and concepts in a style which takes for granted their linguistic background. The work of Brice-Heath (1982) has shown that middle-class students are more likely to be familiar with these forms of school interactions due to their similarity with the linguistic patterns of the home environment. This familiarity has facilitated a linguistic habitus which is similar to that of the formal mathematics classroom and hence permits access to the codes and signifiers of school mathematics.

Connewarre

Connewarre is a large government school which is located within a large housing commission estate. The clientele of the school is predominantly working class with many of the parents receiving government support. The classrooms are smaller than Angahook with approx 25-30 students in each class. The teacher introduces the mathematics lessons with problem solving activities which the students undertake as small groups. They are able to be physically involved in the activities and it is not uncommon for the students to draw on the carpet with chalk to represent the task or physically construct the problem. The mathematics has a strong emphasis on real life situations. The following extract is the introduction to a lesson in which the teacher has drawn a net on the board which the students will have to draw onto card and then construct. Students are then required to develop a number of nets for nominated prisms.

T So if I put those together we start talking more about a shape I am talking about. It's sort of a rectangle on the sides, all the way round but you don;t call it is a rectangle, because a rectangle is jus the flat surface. What do you call the whole thin if that was one whole solid shape. What do you call that?
C A cube
T He said a cube. Don't call out please.
C A rectangular rectangle
T You're on the right track
C A 3D rectangle
T Three dimensions, technically I suppose you're right.
C A rectangular
It's a rectangular something. Does anyone know what it is called?
C A parallelogram
T Put your hand up please.
C [unclear]
T No
More calling out
T I guess you could have a rectangular parallelogram, but no. A rectangle is a special parallelogram.,
C A rectangular oblong
T The word we are looking for is prism
C Yeah that's what I said
T Say the word please
Cs Prism
T Not like you go to jail "prison", that's prison. Excuse me, could you return those please.
[calling out]
T So one thing that we think about with rectangular prisms and that this shape on here is, excuse me...Now you can leave them down please. You need a little bit of practice at lunch because you can;t stop fiddling. This shape here is drawn out on the graph, this grid here [net for a rectangular prism]. We 're going to try and do the same thing. Draw the shape and then cut it out. If you look at the shape, it's made up of rectangles and squares.

In this extract, it indicates that the flow of the lesson and content is hampered by the challenges to the teacher's authority. The triadic dialogue does not serve the same purpose as noted by Lemke (1990) and found at Angahook. The field of mathematics education has particular unspoken rules of interaction which have not been appropriated by the students at Connewarre, or may be resisted by the students.

There are many transgressions of the implicit rules of classroom interactions. The flow of the lesson is fragmented as students challenge the teacher's control for the floor and content of the lesson.

The linguistic habitus of the student implies a propensity to speak in particular ways which, as can be observed in the case of the interactions in this extract, works to exclude students from the mathematical content. The students are not as competent in the linguistic exchanges of the mathematical interactions as their middle-class peers thereby marginalising them in the process of learning. The teaching of mathematics in this way tacitly presupposes that the students will have the discursive knowledge and dispositions of particular social groups, namely the middle-class. The students are not as complicit in the classroom practices and in so doing are being excluded from active and full participation in the mathematics of the interactions. In this way, students have been exposed to the symbolic violence of formal education.

Conclusion

Using data from mathematics classrooms, Voigt (1985, p. 81) has argued, "The hidden regularities, the interaction patterns and routines allow the participants to behave in an orderly fashion without having to keep up visible order" so the idea is far from new. However, what I have sought to uncover using an interactionist approach is the ways in which some students are able to gain access to mathematical content and processes more readily than others. I have proposed that one subtle and coercive way is through the linguistic habitus of the students and the practices of classroom interactions whereby some students enter the formal mathematics classrooms with a habitus that is akin to that which is valorised within that context. These students will be able to participate more effectively and efficiently than their peers for whom the patterns of interaction are foreign to their habitus, thereby making the habitus a form of capital which can be exchanged for academic success within this context.

The predominantly implicit codes of curriculum and classroom interactions take as a given that students will have a familiarity with the legitimate linguistic practices of the mathematics classroom, but neither curriculum nor pedagogy render that language visible. gaining access to mathematical knowledge is facilitated, or hindered, but a match or mismatch of codes. Rather than perceive this a function of language deficiency, but as systemic through which the dominant classes are able to maintain control:

> pedagogies that tacitly select the privileged and exclude the underprepared are not regrettable lapses; they are systemic aspects of schooling systems serving class-divided societies. (Collins, 1993, p.121)

The linguistic habitus of the middle-class students predisposes them so that their possession of knowledge of what constitutes "appropriate" classroom linguistic exchanges is similar to that which the system values thus allowing them to participate in effective classroom practice. Alternatively, the linguistic habitus facilitates the appropriation what the system offers. The dispositions, as per the linguistic habitus, each of the classes have facilitate or hinder their acquisition of mathematics. The linguistic habitus is differentially valued within the mathematics classroom so that for some students the linguistic code with which they are familiar and use within the classroom becomes a form of capital which can be exchanged for other cultural goods - in this case, grades and the subsequent academic success conveyed to the individual. "The more distant the social group from scholastic language, the higher the rate of scholastic mortality (Bourdieu, Passeron, & de saint Martin, 1994b, p.41).

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