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Critical Mathematical Competence for Active Citizenship within the Modern World

Sikunder Ali
University College Buskerud and Vestfold Norway

In this paper, I aim to contribute, through a state-of-the-art literature review, to exploration of critical features that mathematics as a tool offers for reading and writing the world. Our world is crowded by scientific and technological cultural practices, among which mathematics constitutes an essential and integral part. In this review, I look at interactions between developments within mathematics as a discipline (especially mathematics as applied in various fields such as statistics, economics, finance, engineering) and mathematics as a subject taught in schools, specifically the secondary and higher secondary levels. The product of this critical review is to formulate a set of proposals meant to bring a critical eye to bear on mathematical structures taking away citizens' critical capacity to read and write the world. These structures render citizens docile, in-active, and un-able to make their input into the authoritative characteristics that these mathematical structures achieve through the processes of abstraction and objectifications within the context of the constitution of the modern world through actions of techno-science and technology.

Context of the Research Project

Modern societies are built around complex mechanisms to regulate matters such as trade among different countries and regions, through negotiation and dialogues in different dimensions such as political, economic, cultural, and other related domains. Within a framework of nation states, peoples compete, hopefully to co-operate and to create opportunities for mutual co-existence in an increasingly globalized world. Capitalism in one form or other is holding sway in governing societies around the world. Under the strong influence of the capitalist economies, the ways of living of people are getting affected as well. For example, people develop calculated rationalities to organize their daily living. This process could start from organizing a monthly budget, to organizing the education of their children—in general, planning how to use resources such as time, money, and human effort to achieve different objectives of their lives.

Governing patterns have also changed greatly in modern societies. Societies have developed different kinds of strategies to govern the population within a frame of the state (Baber, 2007; Foucault, 2003; Hacking, 1999, 2001, 2002; Neocleous, 2003; Prewit, 1987; Rose, 1999; Rose & Miller, 1992). For instance, Foucault (2003) detected a shift in unit of government from family to governing population.

Furthermore, statistics presupposes the "… emergence of the calculating rationality of the bourgeoisie class" (Neocleous, 2003, pp. 52-53). Specialized techniques of statistics have been utilized to study the social phenomena. In this way, the complexity of social phenomena has been reduced to some manageable units or variables, and constant attempts have been made to posit some causal connections among the phenomena. Therefore, complex phenomena have been put into the machine of generating quantified generalized knowledge wherein some numbers appear on the social planes with their significance embedded in the socio-political space within society. The practice of standardization can be one form of generating a condition of interaction among diverse interests.

> …standardization is not just a matter of the imposition of a system of bureaucratic regulation. Rather, it is a condition for interaction in diversified societies with an expanded division of labour, requiring common means of "trading" between different sectors—that is to say, requiring something that will provide a certain "translatability". Stable standards thus enable the coordination of commercial activities across wide time-space zones, producing a means by which widely dispersed activities can be made commensurable one with another (Rose, 1999, 206-207).

It has also been argued that quantification created room for democratic public spaces where people were not judged according to status but judged through numbers (Alonso & Starr, 1987). This process of quantification has generated standardization which has led to objective exchange of goods and services without dependence on personalities. The significant point is that the officials who use the statistical calculations are also constrained by the calculative apparatus they use.

> Quantification is significant because it standardizes both its object and its subject—the act of exchange is no longer dependent on the personalities or statuses of those involved. … Hence, while quantification is certainly bound up with the emergence of a specialist elite who calculate in terms of numbers, this is not simply a matter of the rise of technocracy. The officials who use these statistical and calculative methods are themselves constrained by the calculative apparatus they use. And this means that quantification produces a certain type of objectivity (Rose, 1999, 206-207).

Baber (2007) has demonstrated that numbers and functioning of liberal democracy are linked together; that is, the extensive uses of numbers have made possible for liberal democracy to function the way it functions in the current political landscape. Furthermore, he has shown how numbers are being used in making subtle government of citizens within the liberal democratic tradition possible, and how this governmentalization process conceals its own governing strategies. This double nature of relationship between governmentalization and the numbers raises the importance of developing a critical approach, mathemacy, needed for reflective judgement of citizens within democratic societies in today's increasingly globalized world. Under these conditions, literacy and numeracy become two important aspects that prepare citizens to become effective in handling varied demands of their lives in complex modern societies.

Moreover, in order to face a fast changing-world and become actively engaged, citizens require both critical literacy and critical competence in mathematics, and both can play important roles. For example, Morgan (1997) describes critical literacy as follows:

> Critical literacy encourages students to challenge taken-for-granted meanings and "truth" about a way of thinking, reading and writing the world. It works against the notion that meaning is transparent, neutral, and unproblematic. Critical literacy also questions the neutrality of power relations within the discourses.

> In pedagogic terms, students should be encouraged to develop enquiring minds that question the cultural and ideological assumptions underwriting any text. They also learn to investigate the politics of representation in the discourse, interrogate the unequal power relations embedded in texts, and become astute readers of the ways texts position speakers and readers within discourse (Morgan, 1997, p. 259).

Similarly, critical mathematical competence can be looked at in terms of empowerment of citizens as individuals and citizens as part of society. This empowerment can be achieved by developing mathematical power in terms of overcoming the barriers to higher education and employment and creating critical competence among citizens via mathematics (Ernest, 1996, p. 1). In this way, critical literacy and critical mathematical literacy can go hand in hand to develop the power of citizens to challenge the taken-for-grantedness of the text, or any claim made through it. Through the tools of critical literacy and critical mathematical literacy we can uncover the power relations that are responsible for the creation of the text. In this way, focus will be brought to the cultural and ideological assumptions behind the text production. The same holds for the ways in which messages are created by different media technologies.

Moreover, recent economic crises (or financial crises) in Europe and the USA have raised the importance of bringing a critical eye to bear on the mathematical models that banks or insurance companies are/were selling in order to make profits. On the other hand, citizens simply become/became passive actors in the crises as, due to their lack of knowledge or competence, they were unable to make informed decisions on important matters such as mortgages for homes. By implication, they become dependent on expertise/advice of financial counsellors who guided/misguided them (intentionally or unintentionally), and ultimately this lack of critical competence among citizens led them to suffer greater losses (financially, socially, and psychologically). This brings us to the need to create an active and agentive citizenship that would afford citizens access to their entitlements, in significant part by gaining competence in mathematics while becoming critical towards the mathematical structures and models which are formatting the modern world we inhabit. Here citizens exercise their rights via entitlement to "mathemacy" for reading and writing their lifeworld critically, and ultimately to shape their lifeworld with informed decision-making.

There follow three examples that illustrate a need for citizens to understand socio-mathematical situations within their lives that are formatted through mathematical structures.

Example 1: Payment of Taxes as an Obligation on Part of Citizens

Over time, different societies and cultures have evolved varied sets of mechanisms in order to regulate themselves. Forms and strategies of governance/regulation of societies are evolving/changing constantly. One of the important aspects of governance has been economic sustenance of a society. Here imposing/collecting taxes has been instrumental for economic viability of any society. The state as an instrument for the economic sustenance of society has created different formulae and structures to provide a constant flow of resources in order ensure good economic health. The state uses taxation as an important instrument to take out a portion of the income (personal income or income of a company) to generate financial resources for running the affairs of a society. The fixing of a particular level of taxation by any state has been an expression of political compromise that a particular polity of the particular society has agreed upon. Historically, the regime of taxation (include evolution of particular percentages within particular levels of taxation as a whole) have evolved. Often the determination of a particular level of taxation has also been an act of struggle. Many precious human lives have been compromised because of this struggle to achieve an agreement on a particular degree of taxes within a society. One can learn about these fierce economic and political struggles in societies such as UK, France, Germany and USA. Correct payment of taxes has been conceived of as an important obligation on the part of citizens in meriting citizenship within a particular polity.

A related realization is that citizens should be able to get a good education whereby they have possibilities to develop set of interpretations about taxation principles and the ways these principles are formulated and agreed upon. The ways different groups or actors make their claims inform getting discounts on their taxation structures and their systematic or unsystematic struggles and strategies to achieve their goals. That is, it is an right of citizens to achieve good understanding of the processes involved in the formulation of principles and structures behind the taxation structures in a society. In this way, citizens would participate actively, not only in understanding the principles or logic behind the taxation structures, but also in acquiring sufficient knowledge to enable them to have an active input in making processes of formulating taxation more democratic in a society. Here various questions arise: Are practices of mathematics education at different levels of schooling in a society making citizens aware of their entitlements of knowing principles/structures through which a taxation system in a particular society operates? Can we find examples that can show us such educational practices in action? How can one characterize these examples from the point of view of their effectiveness? What gaps can we find in the effectiveness of these exemplary practices? What should we do so that we can improve entitlements of citizens to become able to evaluate critically these exemplary practices?

Example 2: Buying a House while Understanding the Market, and Motives and Models of Decision-making of the Significant Factors

Housing is one of the basic human needs, for which one wants to find a good solution. That requires one to take into consideration factors such as earning power of a person or family, the locality where the person lives or wants to live, housing market (buying and selling values of houses), the seasons for which houses are sold or bought, interest rates on housing loans, accessibility or factors determining the availability of a housing loan, general banking policies and government policies concerning housing in a particular society, size of the family, and so on. In other words, finding a reasonable house to rent or own is a complex matter to handle. If one is not knowledgeable about the factors that determine the conditions of the housing market, then there are multiple possibilities that citizens can make decisions that can waste their valuable income resources. The negative effects of the housing market have been clearly demonstrated by recent economic crises in countries such as Spain. The property boom dburst, and it destroyed not only the citizens who were buying the houses, but also many companies who were engaged in selling the housing products based on certain trends and models. These mathematical trends and greedy profit motives created conditions for speculations which inflated the prices of housing fictitiously, and subsequently buyers of the houses were trapped by the ramifications of these speculations. In other words, speculation on the housing market brought resulted in economic catastrophes for the companies as well as for the citizens who were buying the houses, all on the basis of spurious mathematical models. Many families were destroyed because of not having the capacity to read the housing trends critically. This outcome clearly raises the importance of citizens having an entitlement to an education that supports them in exercising their critical capacities to make informed decisions such as buying or not buying a house and how the housing market is organized.

Example 3: Taking Health Insurance for Life and the Politics of Body Mass Index

Societies are increasingly operating in insecure environments. The nature of the problems facing us requires us to cope with uncertainty. Here, tools with deterministic calculations might not help us much to deal with situations of uncertainty and risks. For example, securing health in a modern society is receiving increasing attention. There are different insurance companies that offer a variety of products to provide health coverage for a person. Often these companies operate on models devised through statistical methods or knowledge derived through empirical methods. The companies use several indices to inform their decision-making processes in formulating packages suitable for securing health coverage of people within different age groups. One of the common indices the health insurance companies use is Body Mass Index (BMI). This is defined as ratio between mass in kilograms divided by the square of the height in square meters. This index is typically used to quantify the obesity of the population of a country. Usually this norm of obesity in an individual case is used as a reference point to compare the body mass index of the particular person in order to estimate chances of longevity or early or later death of the person. For example, in the USA, there are companies that require a person who seeks their insurance product to monitor their BMI on a regular basis. If this measure is not within an acceptable limit, then they should make decision on changing their lifestyle or eating habits so that they bring down their BMI to an agreed level. If not, then they have to either pay an expensive premium or leave the company health coverage. In other words, socio-mathematical models bring implications for the securing of health coverage. Hence the question arises: how can one support citizens so that they can develop their entitlements to read and interpret their world in a critical way? How they find opportunities to give an active input to decision-making processes of companies and the state concerning security of their health or minimizing risks to their healthy life?

Critical Mathematics Competence and Democracy

Democratic society requires that its state, government, and legal framework should provide conditions for equitable distribution of resources within and among states, regions, groups, and individuals, for the increase/regeneration of economic resources. In this regard, mathematics is conceived of as important in creating conditions of democratic functioning while potentiating citizens/regions to settle issues such as resource allocation through logic based on rationality. Mathematics is thought of as important in providing objectivity, whereby everything is measured in one way or other. The intensive pressure of scientific and technological advances has led to foregrounding the act of measurement so that one can quantify current resources (human, time, financial, and material) and their utility in future through planning processes. Through mathematics, one can formulate conceptual objects namely categories (through codings/identifying variables) and relationships (in the form of models).

This process of formulating mathematical objects or mathematical structures in forms of models is not a neutral process. It brings a host of ethical issues in modern society. The intentions of modellers, and the social and technological contexts, shape the very purposes for which mathematical objects and structures are applied or used. This is very much a political process, therefore it requires all citizens to not only get an access to this process of objectification of mathematical objects and structures but to acquire knowledge/skills in order to have the possibility of an active understanding of, and control over, these mathematical processes. Often mathematical structures act as action tools but these structures hide their own principles on which they operate. Technical experts and machines usually hold control on the formation and implementation of these processes. Experts and machines do the calculations for us, but normally do not tell us about the underlying principles on which these mathematical models are based. These underlying principles are normally "black-boxed". It requires active engagement of citizens not only to understand these black-boxed principles but also to see them in the social contexts that create conditions for existence and possible uses/misuses of these underlying mathematical principles by various social actors who invoke these mathematical structures in order to promote their own proposals/projects. Through bringing a critical eye to bear on these black-boxed principles, citizens would add to the quality of democracy of the societies they live in. This could generate possibilities of dialogues between citizens, states, and other important actors in society for working together for the greater promotion of democracy in all walks of life.

One finds a gap in Mathematics Education, wherein there has not been many serious attempts made to prepare citizens to become critical of the world colonized by numbers and models with increasing intensity, controlled by experts encrypting their claims into language that is inaccessible to people. In this situation, it becomes imperative that there are opportunities available for citizens whereby they can develop their competency in giving input to authorities making claims embodied in mathematical structures. This also raises the importance of combining literacy and numeracy together to form a mathemacy (Baber, 2007) or to bring attention to critical mathematics education Skovsmose, 1993, 1994, 2005). Against this background, we want to investigate the following research questions:

- In what ways do mathematical structures, through the process of objectification, constitute the lifeworld of citizens in societies under conditions of modernity and globalization?

- How can one develop educational processes that can facilitate citizens to become critical towards the formatting processes of mathematical structures as part of their entitlement to critical mathematical competence (namely mathemacy)?
- What are the implications for preparation of future mathematics teachers in making them critical on these formative mathematical structures?

This research can open up more opportunities to develop new lines of enquiry for curriculum development for better understanding the relationship between the self and the other (human and non-human) for creating sustainable conditions for living and the quality of living in the world. This critical consciousness can also develop organizations that would be more responsive to the needs of the users of the services of these organizations. In this way, the quality of service provisions can also be enhanced to a greater extent, and the connection between learning and organization can be brought to bear on the quality of services for the people and for the overall enhancement of the quality of provisions for society. This frame of learning can be geared to meet the needs of learners under the regime of globalization, wherein bringing a critical eye to technologically formatted structures is essential.

Significance of this Project

This project will bring genealogical insights: how mathematical knowledge has been constructed historically and socially and is used as a part of power technology. This understanding of the relation between mathematical knowledge as historically and socially constructed and power technology would further open up new investigative fields in mathematics education, especially keeping in view the relationship between governmentality and mathematics in formatting lifeworlds of citizens in modern societies such as Norway.

This project will help citizens to link mathematical knowledge with society and its power relationships. This, in turn, will help citizens/learners to become conscious of their lives being constructed through numbers and mathematical models.

Methodological Considerations

Here I want to pursue the following theoretical and empirical investigations (with considerations of sociological, historical, comparative perspectives):

- Conceptual history of accountability and audit society—investigations on objects such as evidence, objectivity, validity, trustworthiness, justification, research (Foucault, 1995)
- Balancing qualitative features with quantitative patterns (i.e. politics of large numbers)
- Sociology of money and taxation
- Learning and teaching of Mathematics with focus on development of active citizenship through development of critical competence in mathematics (through development of pre-service and in-service mathematics teachers)
- Mathematics and decision making in the context of ICT

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Researchers and Researched as Other within the Socio-p/Political Turn

Annica Andersson and Kate le Roux
Stockholm University, University of Cape Town

This paper discusses the use of theoretical tools from the socio-p/Political turn for viewing the positionings of researchers and the researched in mathematics education. We focus on mathematics education contexts where differences in these positionings have the potential to stigmatise the Other. The research writing that provides the substance for the discussion is drawn from two research projects focusing on inclusion and exclusion in mathematics, one in Sweden and the other in South Africa. We argue that the socio-p/Political theoretical turn provides a common lens for viewing and working productively and ethically with the troubles of researcher/researched and researcher/researcher relations in and across contexts.

Introduction

Talking about his background in an interview, Luthando, a black African university student, located his home in a South African "township". He said his "coloured" high school was "disadvantaged", lacking computers and maths teachers but having large, noisy classes. He speaks isiZulu at home and learnt mathematics in English. Using community resources and "cutting out" classroom noise, Luthando "never failed".

(le Roux, researcher, teacher, female, English-speaking, middle class, white, South African)

Talking about his ongoing problems with mathematics, Ara, 17, referred to his background; growing up with eight siblings in a Kurdish immigrant family in Sweden, speaking Kurdish at home but learning mathematics in Swedish. Ara said that after failing year nine mathematics he attended a compulsory summer school "som min farbror undervisade" [that my uncle taught]. With his improved grade he qualified for upper secondary school. However, in addition he now also needed to work late nights at his brother's pizza restaurant.

(Andersson, researcher, teacher, female, Swedish-speaking, middle class, white, Swede)

Relations between participants in the mathematics education community are the subject of ongoing debate, with researcher/practitioner, researcher/researched, researcher/research community, and researcher/researcher relations variously in view (e.g. Adler & Lerman, 2003; D'Ambrosio et al., 2013; Foote & Bartell, 2011; Skovsmose, 2006). In addition, international conferences and ease of communication in many countries provide enabling conditions for collaborative relations across countries and continents. Indeed, a researcher's international collaborations in English, the lingua franca in the research community (Meaney, 2013), convey a level of status. However, internationalisation in mathematics education research brings with it conflicting discourses concerning equity, plurality, complexity and values (Atweh & Clarkson, 2002). Some researchers express reservations about what they have to offer participants in other contexts (e.g. Wagner, 2012). For some, collaboration requires publishing in English as a second or third language.

The reference style in the introductory quotes in this paper is deliberate, because this discussion paper is about us (Kate and Annica) as two researchers, about two mathematics students, Luthando and Ara, who are participants in research projects in South Africar and Sweden respectively, about mathematics education research in these contexts, and the relations between these. The commonalities between the two projects suggest possibilities for collaboration. The empirical focus of both studies is how students narrate themselves and are narrated by mathematics education discourses as included or excluded. We both describe the student interviews in our projects as deeply troubling, troubles related to the positionings of the researcher and researched as Other in mathematics education in our particular contexts. *Troubling* here signals that these research challenges are not just technical but personal and contextual. The projects also share a theoretical location within the socio-p/Political turn (a particular naming we discuss in this paper). Where the projects differ is the contexts of the mathematics education research, differences that position, using Janks' (2010) words, Kate in the p/Political south, and Annica in the p/Political north.

The question we address is this discussion paper is, how do we work productively and ethically with this web of similarities and differences? To answer this question we pursue the argument that researchers can, in a powerful way, offer the mathematics education community appropriate theory and analysis that gives voice to others (Gutiérrez, 2013; Valero, 2014). We argue that tools from the socio-p/Political turn provide a common lens for viewing and working with the troubles of the Other in researcher/researched and researcher/researcher relations in different contexts.

We begin by explaining what we mean when we identify our theoretical work as located in the socio-p/Political turn. Next, we present troubling extracts from our research writing. We produced this writing, which is the substance for the discussion, over time as we explored our understandings of tools from the turn. This interaction between the empirical and theoretical allows us to consider knowledge production in contexts where differences in positionings have the potential to stigmatise the Other.

The Socio-p/Political Turn

Our choice of the word *turn*, rather than *perspective*, signals our awareness of the danger of fixing what or whom is included in a community and that we do not offer an exhaustive review. The tools in view are those we find productive in our particular socio-p/Political contexts, and hence our personal choice of the term *socio-p/Political* in this paper. Indeed our choice of references is itself a political act.[2]

Tools from the Socio-p/Political Turn

Mathematical practices are re-created in social and cultural conditions and are thus *political.* These practices are networked with other practices. *Power* is not an intrinsic and permanent characteristic of participants or practices, but is situational, relational and in constant transformation (Valero, 2004; 2007). Power works between practices in the network and we use *Political* to signify these macro-level processes.[3] Power also works at the micro-level in actions between participants, and hence our choice of *political*. The relation between a socio-Political practice and participants' micro-level socio-political actions is dialectical. On the one hand, the former practice gives meaning to micro-level actions, offering certain subject positions for participants. On the other hand the participants' actions give meaning to the practice – participants have agency (Stinson, 2008) and position themselves in ways that are reflexive, relational and contextual (Wagner & Herbel-Eisenmann, 2009). Thus, all participants are implicated in the construction and circulation of power and mathematical practices are the sites of both reproduction and resistance (Gutiérrez, 2013). Our use of the term *positionings* recognises that positions are given meaning in multiple ways at the macro- and micro-level.

These tools have implications for what we as researchers choose to bring into view and how we do this. They also provide a view of researcher positionings in the network of practices. We discuss these methodological and ethical entailments next.

Methodological and Ethical Entailments

Since macro-level socio-Political practices are not just background to but give meaning to participants' actions, it follows that both micro-level actions and macro-level practices, as well as the relations between these levels, are in view (de Freitas & Zolkower, 2009; Valero, 2007). This includes making careful discursive choices when giving voice to students at the micro-level (Gutiérrez, 2013; Stinson, 2008).

In addition, research in mathematics education itself is part of the network of socio-Political practices. This practice constitutes the objects of research (Valero, 2004). Since the researcher herself is positioned in the network (Valero, 2007), she is both constrained by the practice and positions herself in the practice. Thus the researcher's choices - the questions, the tools, what and how to write and in what language, the knowledge produced - are not neutral (d'Ambrosio et al., 2013). Thus the researcher is called to account for her own role in the research, rather than simply saying who she is for transparency (Chronaki, 2004; Valero, 2004).

Before discussing our use of these theoretical tools, we develop our stories about Luthando and Ara, the two students represented in the introduction. This writing represents a particular moment in the ongoing interaction between our understandings of the tools and our empirical work with the interviews.[4]

Our Research Writing

Luthando's Talk about University

(le Roux, researcher, teacher, female, English-speaking, middle class, white, South African)

> Luthando described how, as a school student, he told himself he was "definitely" going to the "best" university in South Africa. His application to study engineering was not successful. Instead, this university classified him as having the potential to succeed in science but educationally disadvantaged, and placed him in a support programme. Luthando described his initial "struggles" with mathematics, but within a few months his position was "good" and he was getting "all A's". Finding money to travel home for the vacation was difficult, but on campus he could use his financial aid to buy food and books. At that stage he said additional support was an "advantage". However, after completing two years in the programme he encountered "definitions" and "proofs" in advanced mathematics and felt excluded from the classroom conversations:
>
>> … the lecturer is bouncing ideas around and you don't know what the hell they are talking about and you get students, really smart students, students who really, really love maths and […] you get them interacting a lot with the lecturer and you are totally lost.
>
> Luthando described other university students as "really, really disadvantaged", but his and his family's concerns about his

difficulties securing a study bursary recurred in the interviews. In his fourth year at university he lost his "love" of mathematics, stopped attending classes, and failed one of his final courses. However, the next year he was told he could transfer to an engineering programme if he passed mathematics. With this career focus he secured a bursary and he spoke about providing his family and himself with a "good life" when qualified as an engineer. He felt motivated to pass mathematics, but said the support programme had been a "disadvantage" for his progress in mathematics.

Ara's Talk About Mathematics in Upper Secondary School

(Andersson, researcher, teacher, female, Swedish-speaking, middle class, white, Swede)

At the end of Ara's first semester in upper secondary school, Ara expressed concerns for not passing the compulsory mathematics course. After receiving a written warning [IG-varning], Ara gave an impression of resignation and worry:

> I did not fix the tests so very well […] but Elin will help me with extra assignments, […] she said I had to just do it. I need to spend more time on it (sighs) and just hope for the best.
>
> [Jag fixa inte provet så jättebra […] jag kommer att få hjälp av Elin med inlämningsuppgifter […] hon sa jag måste ta itu med det. Måste lägga mer tid på det (suckar) och hoppas på det bästa.]

When asked about the reasons for his bad results, he referred to his home situation. He had not informed his parents which he was expected to do, as he was afraid of their response. His parents were poor when they came to Sweden and they wanted a different life for their children. However, what Ara raised as his main problem was his older brother who expected Ara to start early and work late every weekend at his restaurant. This affected Ara who became constantly tired and it reduced his study time.

When asked about his future, Ara told that he desperately wanted to move from his brother and the impact his brother has on his life and his responsibilities to earn money for the family. He gave a stressed impression in the way he tightened his shoulders and tramped with his foot when he reflected:

> Must have my pass grade, it will not be a good life for me because mathematics is important. I hope it goes well. (sighs)
>
> [Måste ju ha betyg, det kommer inte att gå bra för mig i livet, matte är viktigt. Jag hoppas det går bra. (suckar)]

Ara said that it was difficult to concentrate in the classroom. Classmates disturbed him and he was always tired. Hence the tests become problematic and he failed:

> It is as if my brain mixes up different things, it don't recognise the stuff I have studied when it comes on the math test. Then you're history.
>
> [Hjärnan blandar ihop olika saker, den känner inte igen det jag pluggat när det kommer på matteskrivningen. Då är det kört.]

Lastly, with an angry voice he stated that he had the "Worst things at home to think of, and my teacher just talks about mathematics, she does not understand anything, she only thinks about mathematics". He concludes that this situation is not unique; this is how it is in many immigration families. "We all know it but we don't talk about it with you."

Locating our Writing in the Socio-p/Political Turn

The interviews with Luthando and Ara and other students in our projects are a remarkably rich source of student voice on their experiences of being mathematics students in the two socio-Political contexts of South Africa and Sweden. However, as noted, these interviews are also a source of deep methodological and ethical trouble for us as researchers, trouble related to the positionings of the students and ourselves in these contexts. In this section we use the tools of the socio-p/Political turn to discuss our troubles with the empirical data and how we respond in our writing. The tools bring into view the similarities in our work as mathematics education researchers, with the differences, our Otherness, lying in the contextual differences of the two countries.

Troubled Writing about the Other

As researchers we recognise the material, discursive and psychological load on students' lived experiences of particular positionings in our contexts. For example, in South Africa positionings like "black African", "township" and "English second-language" in turn position students like Luthando as "disadvantaged" re-accessing mathematical practices and the related symbolic and material rewards. For Ara in Sweden, positionings like "immigrant" [blatte] and "second-language student" position him in turn as "marginalized" relative to mathematical practices. These are loads that we, Kate and Annica, have not experienced and which position us as Other relative to students like Luthando and Ara.

There is a need for us as mathematics education researchers to give meaning to the complex ways that Luthando and Ara work with these positionings and their associated load in time and space. However, responding to this need means not using these positionings analytically in ways that become reified and have the potential to contribute to further stigmatisation and harm. This means, for example, taking care to represent the financial struggles of the two students' families and their wish to overcome these through education in ways that are meaningful and respectful in South Africa and Sweden respectively. Responding to this need also means foregrounding particular positions for Luthando and Ara and backgrounding others not visible in the interviews. For Kate, it means taking care not to represent Luthando's "success" in university mathematics as a "good news story", a representation that backgrounds the relative lack of "success" of many students with backgrounds like Luthando, and potentially leaves the structure unchanged.

Bringing into view Luthando and Ara's complex work also requires writing about how they position themselves differently within one interview and also across longitudinal interviews. For Kate this means representing, in a linear way for publication, how Luthando over five years variously represents as "dis/advantaged" his home and school background and the support programme. For Annica, this means writing about, on the one hand, Ara's talk about wanting to be good in maths, wanting his parents to be proud of him and to be a good Muslim, but on the other, his stories about stealing video-films and cheating on maths tests. It means writing about how Ara variously positions her, the researcher, as an authority in mathematics, as a trusted confident, and as the Other, that is, as a Swede with whom immigrants do not share certain things: "we don't talk about it with *you*".

The socio-p/Political turn acknowledges students as "whole" individuals (Valero, 2004) and emphasises the importance of power relations at the macro- and micro-levels, positionings and discourses – in other words how we fluently relate to each other in particular contexts (Gutiérrez, 2013; Wagner & Herbel-Eisenmann, 2009). In the rest of this section we use these tools to account for our *discursive* and *p/Political* choices of what in the interviews to bring into view and how to do this.

Constraints on Discursive and Political Choices

Firstly, our words *discursive* and *Political* signify the theoretical view that the choices of Kate, Annica, Luthando and Ara are constrained in different ways by the wider contexts. As noted, positionings related to race, socio-economic class and language in our backgrounds have been identified as having effects on educational opportunity and performance in mathematics both in South Africa and Sweden. Thus, our choice to write about these positionings and how the students position themselves in mathematics education is not idiosyncratic, but identifies the researchers and the researched as Other in ways that matter in mathematics education in these contexts.

These positionings in our backgrounds also constrain our discursive and Political choices in the present and future. Luthando is identified by the university as having the potential to succeed, but on the basis of his race, school and language background is placed in a support programme with other students positioned in this way. Being a support programme student in turn defines his opportunities to be a university mathematics student. Luthando himself looks back at this positioning as a "disadvantage". Ara's positionings do not qualify him for special education in Sweden, but he indicates that he might fail just because of these positionings. Thus, both students regard their current positionings as closing opportunities to be "successful" mathematics students in the future. Indeed, the importance of this success for the students' futures figures large in the interviews; both Luthando and Ara suggest that their "success" in mathematics has implications for how they will be positioned in education, work, and family practices in the future.

The tools of the socio-p/Political turn bring into view how power works at the macro-level by offering particular positionings for students in particular contexts. Thus we can ask whether these positionings act in reproductive ways. The turn also brings into view how we as researchers, who are positioned differently to Luthando and Ara in the contexts of our research, may be complicit in that reproduction when we write. However, the socio-p/Political turn also brings into view participants' agency and how power works reflexively and relationally at the micro-level. We turn to this next.

Acting through Discursive and Political Choices

Our use of the words *discursive* and *political* signifies the theoretical view that Kate and Luthando's choices and Annica and Ara's choices also act in the context of opportunity and performance in mathematics education in South Africa and Sweden respectively. The quotes Kate chooses for her writing about Luthando's interviews indicate that positionings related to race, socio-economic class, language and disadvantage are part of his language for talking about being a mathematics student with a particular home and school background in a support programme at university. Annica's choice of quotes indicate in a similar vein that positionings about being an immigrant, Kurdish speaking and marginalised are part of Ara's talk about his experiences in mathematics education. At the micro-level, these students' choices are discursive and political as they act agentically within the set of power relations to position themselves in various practices. These actions may involve reproducing, redefining or rejecting the positionings they identify in their respective contexts. In our writing we aim to bring into view the complexity of the actions of students like Luthando and Ara as they work to become mathematics students.

The socio-p/Political also brings into view how the discursive and political choices of both researcher and researched act in the interviews themselves. In particular we recognise how Luthando and Ara use language reflexively to position themselves politically at a particular moment relative to the past and future and relative to the researcher as Other. Ara positioned Annica as somebody he trusted and hence reproduced the positioning of a student who possessed important knowledge about mathematics education that should be shared with others. However, he also positioned himself as Ara, about to fail in mathematics. Ara positioned Annica as an authority and Annica recognised that he initially told stories he might believe she wanted to hear. However, later in the conversations he shared stories about stealing video-films, cheating on maths tests etc. And then, at the end of the second interview he positioned Annica as one of the Others, the "*you*"; one of the Swedes as opposed to the Immigrants when explaining "we don't talk about it with *you*".

This description of Annica's interviews with Ara illustrates well the reflexive, relational and contextual nature of the power relations in the interview process. Yet the nature of these power relations shifts when the researcher comes to write about the interviews. Recognising her own discursive and political power as a researcher in this context, Annica makes further careful choices aimed at building a rich and caring description of the Other. Firstly, in her wider study Annica asked participants to select their own pseudonyms. However Ara chose not to. Thus, Annica turned to a Kurdish language teacher at Ara's school who suggested and explained different Kurdish male names. Annica chose "Ara", meaning Wind, to represent the elusiveness of her research relationship with Ara at that moment in time. Secondly, following Meaney (2013), Annica represents Ara's talk both in Swedish and in English. This move positions her and Ara in the context; Ara speaks Kurdish at home, Annica interviewed Ara in Swedish and transcribed in Swedish, Annica translated the Swedish into English for publication. Since English is not Annica's home language, this final translation itself positions her in another set of power relations within the research community.

Finally, our self-identification in the introduction to this paper serves as a constant reminder of the inequities in the discursive and political choices of researcher and researched in our contexts and our responsibility to write about the interviews with care. It also foregrounds the importance of our work as researchers in giving voice to the Other. We cannot draw on our "lived experiences" Gutiérrez (2013) as marginalized or disadvantaged in our own contexts, but our "bearing witness" and "orienting" experiences (Foote & Bartell, 2011) in our own contexts makes this work our only choice. Our reference style also resists the dominant way of writing in the research community.

Conclusions

In this paper we (Kate and Annica) have discussed research troubles that stem from meaningful differences in researcher/researcher and researcher/researched relations within and across contexts in mathematics education. Drawing on interviews from research projects in Sweden and South Africa we have surfaced the trouble of accounting for what we bring into view and how we do this in our writing about the Other in each of these contexts. We have also surfaced the troubles of collaboration between researchers who share common research interests, but whose contexts position them as Other relative to one another.

We have used the interaction between the empirical and theoretical in our writing to argue that tools from within the socio-p/Political turn enable us as researchers to view and to work productively and ethically with this web of similarities and differences. That is, we can view relations of Other within and across contexts in terms of power relations and positioning in the dialectic between macro-level socio-Political practices and micro-level socio-political actions. These tools enable us to account for our discursive and p/Political choices of what and how we write about Luthando and Ara respectively. These tools also provide a framework for us to work productively and ethically with one another as researchers, as these tools open up the wider socio-Political contexts that define our Otherness and inform how we work with one another. They also foreground how our discursive and political choices position one another in our collaboration. Using interviews from research projects in our own contexts to explore our understandings of the socio-p/Political turn leads us to suggest that working within this turn is not just about using particular tools, but also about assuming an "attitude" that seeks for consistency between those tools and our actions as researchers in researcher/researched and researcher/researcher relations.

Notes

1. This project is financially supported by the Andrew W. Mellon Foundation.

2. We reference researchers specific to mathematics education, but acknowledge that their tools draw on wider social theory. We refer the reader to the referenced work for more detailed exposition of the tools and their antecedents.

3. The discursive move to use upper and lower case characters to signify power relations at different levels of the social is inspired by, but not conceptually consistent with, other such moves (c.f. Andersson, 2011; Gee, 2005; Janks, 2010).

4. The texts in this paper are necessarily short, but are elaborated more fully in a further paper.

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Questions from Ethnomathematics Trajectories

Annica Andersson and David Wagner
Stockholm University, University of New Brunswick

The purpose of this discussion paper is to explore and critically reflect on trajectories of ethnomathematics in research practice. This begins with our own narratives followed by an overview of ethnomathematics within MES conversations as empirical data. From these, we raise questions focusing on the place of ethnomathematics research and practice and on what this says about our academic culture/community.

Introduction

Our conversation (David and Annica) about ethnomathematics began at the international conference MES7, where we attended the same focus group discussing Swapna Mukhopadhyay's (2013) plenary talk, which was based on her ethnomathematical work with ship builders. We noticed that the two of us shared similar experiences of mathematics, culture and (not conducting) ethnomathematical research. We continued our conversation and decided that the place of ethnomathematics in the field warrants more discussion at MES. The questions we asked ourselves initially were "Why are we not doing ethnomathematical research, given our interest in it?" and "In what other ways have we tried to address cultural issues in our 'adapted' or 'accepted' research and teachings?" These questions developed over time. In this paper we draw on our own research trajectories in relation to ethnomathematics and connect these with the trajectory of ethnomathematics research in the field, focusing on papers accepted in the prior MES conference proceedings. We use this as a platform for making observations and raising questions about the research discourses that dominate the research field today.

Narratives of Experience With(out) Ethnomathematics

We begin with the narratives we wrote for each other — accounts of our trajectories in relation to ethnomathematics, including the questions that arose from our experiences. We include accounts of context in our narratives in recognition that our experiences and ideas are culturally situated. The third narrative gives an overview of development of ethnomathematics as expressed within MES conference

David's Trajectory

When people outside the field ask me about my work in mathematics education, I usually position my work in relation to the observations that propelled me into focused reflection on my work as a mathematics teacher. For example, this excerpt comes from my autobiographical statement on my website (http://davewagner.ca).

> Prior to doing my PhD […], I taught grades 7-12 mathematics in Canada for six years and in Swaziland for two and a half years […]. It was the experience of teaching mathematics in Canada, then Swaziland, then Canada that *alerted me to the highly cultural nature of mathematics teaching, which I had thought was culture-free and values-free.* This experience prompted me to leave teaching to investigate the cultural nature of mathematics.

My commitment to investigating the cultural nature of mathematics was strong enough to convince me to take the financially difficult step to become a graduate student, which involved giving up my secure, enjoyable teaching position.

At the outset of my graduate studies, I was most drawn to the work of Ubiritan D'Ambrosio and Ole Skovsmose. D'Ambrosio's articulation of the cultural nature of mathematics bolstered my confidence that this warranted attention, and he gave me words to talk and think more about this ethnomathematics. Skovsmose, a critic of ethnomathematics (though I didn't know it at the time), did similar work to D'Ambrosio in my eyes. He challenged the orthodoxies of mathematics education (and here I refer to widespread societal views of mathematics teaching and learning but his challenges also applied to the research community).

I come from a rich tradition of orthodoxy challengers. In addition to my mathematics education identity, I self-identify as Mennonite. This is a Christian denomination that emerged after the reformation in Europe. Reformers challenged the authority of the Roman Catholic Church and set up contesting churches, still structured hierarchically but aligned with different political powers. The Radical Reformers, including Mennonites, said that the Reformation did not go far enough because the reformers depended on and sustained political/military authorities. Early Mennonites rejected the shackles of even the new authorities. This ethic permeated my upbringing; my parents regularly questioned traditions vocally in private and respectfully in public. I have also come to realize that academic mathematics carries a rich tradition of orthodoxy challenges. I have written about this in a few places, including my essay "If mathematics is a language, how do you swear in it?" (Wagner, 2009). Here I characterize mathematics as a tradition that questions conventional ways of seeing, invents new spaces with alternative rules, and often finds that these invented spaces have descriptive power in the "real world" (e.g., Wagner, 2009).

I notice that my orthodoxy challenger spirit has continued to underpin my research and writing. But I ask what sidetracked me from my strong interest in ethnomathematics after I observed its tenets myself, and read about it voraciously in my early graduate studies. I blame the charisma of David Pimm for this. "Blame" is probably not the right word. His insightful observations on language practices, supported by his generally attentive, generous, and respectful character, got me seeing that language is the medium of cultural development. While ethnomathematics notices differences among cultures and their forms of mathematics in particular, where and how would these distinctions arise? These forms of mathematics would have arisen in community interaction, which is by nature mediated through language.

Nevertheless, immediately after completing my dissertation research, which was focused on language practices in the classroom, I embarked on some ethnomathematical research. At the same time I continued with my interest in language and communication. I found publishing the work on language much smoother than the work on ethnomathematics. So I reflect here on the reasons for my difficulties in developing the ethnomathematics stream of my work.

The first challenge I encountered was the need to develop deep relationships with the people with whom I would work. Looking back, I feel naïve for not realizing the (extent of the) necessity of this. Ethnographic work, including ethnomathematics, takes a much larger time commitment than other forms of research. This discrepancy had a significant impact on choices of where to devote my limited temporal resources. Early career positioning, with the expectation to show results, exacerbated this pressure. I was fortunate to have a PhD student (Lisa Lunney Borden) who had a long-standing connection with Mikmaw (an Aboriginal group on the Atlantic coast of North America) communities and who embraced ethnomathematics. Her relationships facilitated my/our ethnomathematical work.

Second, doing the ethnomathematical work, I myself had questions about the validity and appropriateness of it. I found that these questions aligned with some of the critiques in mathematics education literature from people not doing ethnomathematical work. Lisa and I tried to address the challenges in our approach to the work, and also wrote about them from within the work. However, we often felt short-changed because our choices in relation to those tensions cut us off from the data that would have made publishing much easier. In our conversations with elders and teachers, together we chose to encourage community children to interact directly with elders regarding their mathematics instead of us doing this ethnomathematical work and positioning ourselves as mediums of this community knowledge (Wagner & Lunney Borden, 2012). Publication was challenged because we were not the ones doing the ethnomathematics. Instead we wrote about the development of the conversation about negotiating this space.

Third, in Canada and elsewhere there is a view among many First Nations scholars (and argued against by others, including Battiste, 2013) that it is inappropriate for research relating to First Nations communities to be published without a First Nations author. Lisa and I found this frustrating because our submissions to journals that were most appropriate for our work were discounted at the outset by this ethic. We understood the viewpoint and found it somewhat valid, because it is dangerous for outsiders to research and write about a community without the participation of local people. However, in our work the community leaders wanted us to publish and did not want to co-author (except for one book chapter that is still in process), especially not as a token author! Again, publication was challenged.

Fourth, I have heard scholars, whom I otherwise respected, discount ethnomathematics, saying it has been roundly critiqued. I am disturbed by the weight of these critiques done by people who do not conduct ethnomathematical research.

Ethnomathematics addresses from an equity standpoint the heart of the question that plagues equity researchers in mathematics education: where is the mathematics? Indeed, ethnomathematics turns that question around and asks it of the mainstream. Thus it saddens me to see marginalized this idea that has the potential to strike at the fundamentals we may not feel ready to challenge.

Annica's Trajectory

Yesterday I slowly walked around admiring the beautiful art pieces and handicrafts made by Canadian First Nations artists at the UBC Museum of Anthropology in Vancouver. While I was moving through the museum I reflected on my personal deep interest in culture and anthropology and what impact this interest has on the work I do and have done as a researcher in mathematics education, a teacher educator and a former upper secondary school teacher. I will in this narrative strive to make a reflection of how these different contexts affect my trajectory as becoming researcher, the decisions I took along the way, and why these decisions were taken at particular points of time. Ethnomathematics is part of that story.

During all my years teaching mathematics in Swedish secondary schools there were issues that disturbed me. I met a large number of 15-year old students whom, on the very first day I met them, told stories about not feeling well in mathematics classrooms, disliking the subject, or even hating it. These students had an interest in social sciences, humanities, and language but usually disliked mathematics and did not recognise the possibilities that mathematics knowledge may offer them. I asked myself why so many students had these not-so-good experiences of prior mathematics education. Being a teacher I continued to reflect on pedagogy — why is mathematics not naturally connected to other school subjects or present societal topics?

Especially in the social sciences where there are such rich possibilities to connect mathematics with culture, humanities, and the arts.

At that point in time I strived to develop my teaching whenever possible, and place the mathematics in a cultural or societal context to show the cultural and societal connections. Once I even managed to raise some money to bring two classes to an art exhibition focusing on Australian Indigenous people's maps as art. The students explored different aspects of maps, scales, time, and time flow and how these can be expressed mathematically from diverse cultural perspectives. Social science and language teacher colleagues labelled me "the crazy math-teacher" with a smile. I took that as a compliment, but it also prompted me to reflect: in their subjects teachers are expected to contextualise the subject outside the classrooms, but when done in mathematics, people reacted as though this is strange. I loved the students' comments, such as, "It is actually fun to have a teacher who *knows* something other than maths." Such comments indicated their view both on the subject and on us as mathematics teachers. For me, to be able to reach these students, I explicitly showed and discussed in class that mathematics is a culturally developed subject (as is the teaching), its societal importance, and the power that is connected to the subject. The insight grew in me that mathematics and mathematics education are not values-free.

These insights challenged me to go back to University and hence I became a post grad student. In a master program thesis I further explored relationships between culture, society, and mathematics teaching. With energy and happiness I read the work of Ubiritan D'Ambrosio, who gave me arguments to support ideas I had been developing already. Alan Bishop's book on mathematical enculturation, and Ole Skovsmose's critical writings became food for thought. These readings were eye-openers but also gave me a language to express my concerns of mathematics and for the teaching of mathematics as a culturally developed and situated subject.

My first experiences of mathematics education research hence connected with my cultural interests, anthropology and ethnography. The studies of ethnomathematics, ethnomathematical theories, and the critique of ethnomathematics in combination with a developed way of teaching made me fortunate too. I received two large scholarships within two years. The first made it possible to attend the International Conference on Ethnomathematics (ICEM) in Auckland in 2006. I was challenged by the deep and honest interest in culture and its connections with teaching of mathematics and the important associated political questions discussed at this conference. I believe that ICEM is the conference that gave me inspiration for continuing into further academic work and research. Second, I got the opportunity to be a research assistant with Kay Owens and spent time with her in remote villages in Papua New Guinea. Her research interest was geometry, specifically area calculations. Hence, we interviewed and filmed house building, gardens/farming, etc. to gather knowledge to be used in Papua New Guinean teacher education to connect the school mathematics with the children's experiences from home. There is a large body of ethnomathematical research with these purposes from all over the world, but also a growing body of critique, especially political critique that is important to recognise. Summing up, for me personally this was the time when I discovered the impact of understanding mathematics teaching as the culturally developed subject it is and the politics of mathematics education.

So, I started to formulate research questions and write research-funding applications. However, I was advised by caring professors in the field to leave my ethnomathematical ideas out if I wanted funding, even though I showed awareness of the political issues and complicated concerns when conducting ethnomathematical research. It seemed that ethnomathematical research was not accepted. When Paola Valero asked me if I wanted to join her research group in Aalborg I decided to leave ethnomathematics for the time being. I was grateful for this opportunity in my life to address the societal and political concerns for mathematics teaching. I became a PhD fellow in Denmark and wrote a thesis focusing students' identities and agency in mathematics education contexts inspired by concerns raised in critical mathematics education (Andersson, 2011b; Andersson & Valero, 2014).

In 2011/12 I had the opportunity to teach in a diploma program for in-service mathematics teachers in Greenland. Again, I worked in a different culture — sharing and discussing research in mathematics education with Inuit teachers. The issues they brought forward where in some cases very different from the ones recognised in "Western" research literature. I would have liked to conduct research with these teachers further; they had a number of ideas on topics they wanted to research concerning cultural and language challenges. However, in order to do that, administrative support was needed from both Greenland and Denmark, which was not possible for me to get at that time, as a Swedish national, an outsider.

I continue to reflect on the challenges I experienced. First, ideas, results, and discussions originating in ethnomathematical and cultural research are also applicable in "Westernised" mathematics classrooms. I developed these ideas together with Elin, a mathematics teacher who talked about herself as being a "Curling teacher" (Andersson, 2011a). My experiences allow me to argue that there is value in raising discussions in schools and universities about understanding mathematics and mathematics teacher education as culturally developed and situated.

Second, following from point one, I believe that research addressing ethnomathematics from various perspectives should be discussed in teacher education courses *in parallel with* other research that is already prominent and mainstream. Becoming teachers should have had rich and colourful possibilities to reflect on the cultural development of *both* mathematics and mathematics education. This is not the case in the Swedish university contexts I know today.

Third, I reflect more generally on experienced resistance for ethnomathematical research. There seems to be a resistance for granting funding for ethnomathematical research. There is also a resistance to address specific ethnomathematical research in mathematics teacher education courses as discussed above. Ethnomathematical research might be more accepted when rephrased as language research, diversity research, critical/political research, social justice, equity research, etc. The ethnomathematical umbrella covers diverse topics from cultural and anthropological perspectives. The important critique has been heavy and well accepted in our community however the result seems to be a resistance towards ethnomathematics, a resistance I believe neither serves further theorisation, epistemological discussions, nor the critique of ethnomathematics. The resistance does not serve becoming mathematics teachers either. All these are political dilemmas — tensions and possibilities that compel further open and lively discussions.

Ethnomathematics Trajectory at MES

The first MES conference was convened in 1998 (using the acronym MEAS to emphasise the word "and" in Mathematics Education and Society). This was followed by six conferences located in Europe, Australia and Africa. There have been three plenaries that explicitly focused on ethnomathematics; D'Ambrosio's (1998) plenary at MEAS; Powell's (2002) reflections on "Ethnomathematics and the challenges of racism in mathematics education" in MES3 and Mukhopadhyay's (2013) talk "The mathematical practices of those without power" at MES7. However, a number of the plenaries addressed ethnomathematical research in an implicit way; cultural aspects of mathematics education and learning are overtly addressed in almost all plenaries. At MEAS there was a symposium on "Ethnomathematics and Critical Mathematics" led by Powell, Knijnik, Gilmer and Frankenstein. This symposium discussed tensions within ethnomathematical research, specifically tensions regarding the "exotical" and the "critical" strands, to use the authors' words. In a very powerful discursive way, these tensions seem to be underpinning the accepted papers, plenaries and critical discussions throughout the MES conferences.

Regarding the papers accepted for proceedings in the conferences, we first identified the papers that had the word "ethnomathematics" and its various grammatical forms as a way of tracing the development of it within the conference. We found no clear pattern or trend, as the successive conferences had 18%, 5%, 11%, 3%, 18%, 16%, and 10% of their papers including the word or its stem. Nevertheless, these numbers should be read cautiously. A high number of the papers in which we found the word "ethnomathematics" were not focusing on ethnomathematical research in particular; the word was only used once or twice in these papers. In an even higher number of papers the word or a form of it was present *only* in the reference list.

We became increasingly interested in the papers that did not use the word "ethnomathematics" though they might have. In other words, papers that report on work that could be connected to ethnomathematical work but in which the authors did not mention ethnomathematics intrigued us. Thus we considered other words we might search on to identify patterns in such papers as various forms of "culture" and "anthropology". By contrast to the word "ethnomathematics", the word "culture" and/or "cultural" are present in almost all papers. It has been used in a variety of contexts, usually in a nominal way, with no theorization of culture or the place of culture in mathematics. "Anthropology" was hardly found at all.

We took the analysis one step further in those papers where "ethnomathematics" was found more than one time in the text body. Some different possibilities emerged for categorisations. Not surprisingly, the largest number of these papers used ethnomathematical research to justify and/or position their own research, and/or, in a few sentences, show awareness that ethnomathematics exists. Fewer in number specifically grounded the research in ethnomathematical theories. There were also papers that talk *about* ethnomathematical research; they may argue that ethnomathematical research is important, should be done with care and awareness, or raise concerns — for example against possible exoticism or, as in the case of South Africa, concerns about the ghettoization. What we can conclude is that almost all papers raise concerns about cultural aspects and/or particular cultural groups, however, the number of papers addressing, critiquing, or discussing ethnomathematical research explicitly is low at the MES conferences.

Reflecting on the Trajectories

Our distinction between work that is *overtly* versus *possibly* ethnomathematical required consideration of a working definition of the term. We talked about the definitions given by the progenitors of ethnomathematics and the various ambiguities in these definitions. For example, D'Ambrosio (1985) referred to a "defined cultural group" and we noted the challenges of trying to draw bounds around a culture to clarify whether or not a practice is particular to the culture. Though Bishop (1991) does not refer to ethnomathematics explicitly, his list of practices worth investigating to find mathematics is used extensively by ethnomathematicians: counting, measuring, locating, explaining, designing, and playing. We chose to use these categories to identify papers in the category of "possibly ethnomathematical."

In order to be even possibly ethnomathematical we felt that research needed to address both culturally-specific practices and values. To do this we challenged the boundaries among Bishop's categories. For example, we, and our students, have wondered what makes counting distinct from measuring. One way to make a distinction is between counting discrete objects and counting an artificial comparison (i.e., units of measure). Thus categorization is at the heart of counting because we have to decide which objects are "like" each other in order to group them. Categorization can be done in various ways, and it is often (perhaps always) political — Who counts? Who doesn't? — and thus an expression of some cultural milieux. Measuring is also political because it involves direct comparison among things or indirect comparison to some normative unit. This too is political and an expression of some cultural milieux. Locating requires points of reference; either a normative point such as an origin, or relative positioning (see Wagner & Herbel-Eisenmann, 2009 for this distinction used metaphorically). Positioning then is political because it involves choices of what to use as reference points. These fields of activity cut across values that are expressed in language, which includes language registers (and other forms of representation) in design, which indexes the values of desire (what do we want?), and in play, which indexes the values of aesthetics (what do we find beautiful?). This all relates to local resources that comprise the context of a culture because sensory experiences with landscapes and materials impact choices and constructs.

We use this elaborated definition to reflect on the larger narrative of our research presented in this paper. To analyse ethnomathematics in the field, we *counted* papers that mention ethnomathematics. This was easy enough to do with search engines and thus seemed rather clear … until we started looking at the papers. We felt that other papers did ethnomathematical-like work without mentioning the word, and we knew that a discussion of these papers is part of the story, just as our individual non-ethnomathematical research (as described in our personal stories) shared common values with ethnomathematics. In order to count papers, we had to categorize in some way and thus distinguish among nominal use of words, significant use, critique, etc. Whatever counting we did could not be objective — it referenced our scholarly values of what is desirable or appropriate. Our counting is situated in an academic culture. It is an ethnomathematical practice.

We do not want to reserve ethnomathematics for the analysis of exotic cultures. Critical mathematics education is also ethnomathematical in several aspects. For example, Gutstein (2010) reports on his action research in a classroom in which "reading and writing the world — with mathematics — were very much the agenda" (p. 272). He carefully described cultural aspects of the classroom, thus he led a particular form of ethnomathematics. This may be different from analysing ethnomathematics in a culture in which the researcher is not actively taking part. As another example of interpreting culture broadly for potential ethnomathematical work, we would place any work on language register as ethnomathematical. For example, Zolkower & de Freitas (2010) "guided [teachers in the] deconstruction of whole-group interaction texts selected as paradigmatic instantiations of this genre" (p. 509). Their attention to a genre, which is a part of the mathematics classroom register, would seem to place the work in a culture, but their reporting does not connect the language practice to the culture. We wonder whether this omission disqualifies the work as potentially ethnomathematical, or whether it would be more accurate to describe it as a poor example of ethnomathematics (and we note that some of our own work would have to be in that same category).

Our thoughts about the heart of each practice also prompted questions about locating. In particular, we asked where is ethnomathematical work done and where is it critiqued? More accurately speaking, we were interested in who (or what *category* of researcher) was saying what in relation to ethnomathematics. Generally speaking, explicitly ethnomathematical work has been done in colonized settings. By contrast, also speaking generally, the critique of ethnomathematics has been done by people of colonizer cultures (with the exception of people from South Africa, who have raised concerns about the ghettoization of people groups, arising out of their experiences of Apartheid). To save space and face, we will not point to particular papers to justify these two generalizations, leaving the possibility of affirmation or exception in discussion at the conference. Nevertheless, we need not save our own face and thus we point to ourselves as examples of White people of European ancestry interested in and having done ethnomathematics in colonized settings.

Questions for Discussion

Our narratives of our two separate trajectories in relation to ethnomathematics, along with our field's trajectory within MES and our joint analysis of these trajectories lead us to summarize questions that we believe warrant discussion at MES:

1. Looking behind/underneath the critiques, what motivates educators to resist explicit reference to ethnomathematics in the academy and in schools?
2. For mathematics educators and students who are drawn in by ethnomathematics (such as ourselves), how does it satisfy deep needs/values?
3. In what ways do current critiques of ethnomathematics also apply to other mathematics education research (especially socio-cultural research)?

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Shaping a Scientific Self: A Circulating Truth within Social Discourse

Melissa Andrade-Molina, Paola Valero
Aalborg University

In this paper we illustrate how a truth circulates within social discourse. We examine a particular truth reproduced within science, that is: through the understanding of Euclid's axioms and postulates a person will gain the access to all human knowledge. We deploy a discourse analysis that helps us to understand how a truth is reproduced and circulated among diverse fields of human knowledge. Also we show why we accept and reproduce a particular discourse. Finally, we state Euclidean geometry as a truth that circulates in scientific discourse. We unfold the importance of having students follow the path of what schools perceive a real scientist is, not to become a scientist, but rather to become a logical thinker, a problem-solver, and a productive citizen who uses reason.

Introduction

We want to tell a story about a circulating truth that has been shaping a scientific self since before science was called science. Even though there are many truths within scientific knowledge, this particular truth seems to resist every attack, seems to win every fight. Within social discourse, it is believed that mathematics is a powerful knowledge that will enlighten people.

> All adults, not just those with technical or scientific careers, now require adequate mathematics proficiency for personal fulfilment, employment and full participation in society. [… Students should] be able to apply them to solve problems that they encounter in their daily lives (Organization for Economic Co-operation and Development, 2014, p. 32).

Then, in order to be the productive citizens that society requires, it becomes important that students develop mathematical thinking.

Students should be able to

> reason mathematically and use mathematical concepts, procedures, facts and tools to describe, explain and predict phenomena. It assists individuals in recognising the role that mathematics plays in the world and to make the well-founded judgements and decisions needed by constructive, engaged and reflective citizens (Organization for Economic Co-operation and Development, 2014, p. 28).

Therefore, mathematics becomes the tool to solve problems from everyday life. In fact, one of the areas that PISA measures is *Mathematising*, this is the ability to move between the, so called, 'real world' and the mathematical world. Thus, schools address the development of this particular ability by connecting everyday life to mathematics. But this link does not always work, because the 'real world' of school is not the physical world. The models that we have to link both are outdated (Burgin, 1987).

School has been developing a 'school space', where everything has a coordinate in a two or three-dimensional Cartesian system. This 'school space' is rooted in Euclidean geometry. In other words, it is shaped by Euclid's axioms and postulates. At school, straight lines are always straight, they do not curve at the horizon, parallel lines are in fact parallel and the sum of the interior angles of a triangle is 180 degrees. However, 'school space' is a space that has been modelled by mathematics which is different from the world the students live in, precisely because geometry provides the materials for models of the physical world, models that are abstract analogies and not the world itself (Ray, 1991).

In this paper, we examine a particular truth reproduced within scientific thinking. The belief is that through the understanding of Euclid's axioms and postulates a person will gain the access to knowledge, not just the access to geometrical or to mathematical knowledge, but to all human knowledge. We are going to deploy a discourse analysis that will help us to understand how a truth is reproduced and circulated among diverse fields of human knowledge. Also it will show why we reproduce a particular discourse through our own language.

How are Discourses Reproduced?

Discourses are not impositions; we are not forced to believe in them. But sometimes, something sounds very reasonable to us, so self-evident, so logical, so common sense that we agree with it and we start to reproduce it through our own language. Who will go against the idea that we need mathematics in our daily life? But we perceive these 'truths' as common sense just because we are inserted in a particular time and place, spatio-temporal conditions, with a particular rationality. We are *subjected* to those self-evident truths.

Foucault claims that "taken-as-truth" statements circulate within social discourses, discourses that are produced because we reproduce them through language (Foucault, 1982). At the same time, these discourses are not isolated; they are produced by the interaction of different spheres of social life and are shaped by statements and their related truths (Foucault, 1972). In other words, discourses do not materialize from thin air, nor are they commandments by a superior force, such as a God or government.

Therefore, truths become a discursive formation, and there exists diverse rules for what is considered to be true and false (Jørgensen & Phillips, 2002). In other words, there are regimes of knowledge determining what is accepted as meaningful and true and what is not. As Deleuze stressed, only statements may be repeated, but these statements "are not visible not hidden" (Deleuze, 1988, p. 10).

For instance, a circulating truth could be that 'school provides tools to achieve success in life'. Some people might agree with this truth and reproduce it, and some people might be against it. In fact, educational sciences have been providing these tools, with the promise of a better future, for the fabrication of a 'cosmopolitan child' (Popkewitz, 2008). If we analyse school mathematics, it was believed that by a 'mathematics for all' it was possible to create this brighter future (Valero, 2013), and that belief has not changed though time. But, why do we say that it is a 'circulating truth'? Recall the PISA quotation above: *All adults require adequate mathematics proficiency for personal fulfilment, employment and full participation in society.* This implies that to be 'productive' and 'successful' one must know mathematics and science.

There are many naturalised truths circulating in the discourse of diverse scientific fields, and such truths constitute unproblematized understandings of its practices. One of these truths is to believe, for example, that Euclid's axioms and postulates became a universal key to access human knowledge (Sbacchi, 2001). In the same fashion, that Euclidean geometry began to appear as a dominant perspective within scientific knowledge (Majsova, 2014). Or, that Euclid's *Elements* are so necessary to every science that we must believe in them as its basis, principle and fundamental elements (Guarini, 1968). So, a particular truth within scientific knowledge has been reproduced.

Building from this truth, what makes Euclid's *Elements* so important? Are the Elements important because it was the only 'recognized' form of geometry until the 19th century? Harrison (1919) stresses that for a great period of time Geometry and the *Elements* of Euclid were considered as synonymous. But, is it the only reason?

> In the 1630s, Descartes's Discourse set out philosophical reasons for seeing Euclid's Geometry as an intellectual model for theories in other areas of inquiry. Fifty years later, Newton showed that this model was not just formally rigorous, but empirically powerful: i.e., it resolved problems that had plagued European thinkers ever since the publication of Nicolaus Copernicus's de Revolutionibus (1543) (Toulmin, 1998, p. 330).

Apparently Euclid's *Elements* were more than just books summarising the geometrical knowledge of his time, Descartes and Newton recognized them as an intellectual, rigorous and powerful *model*. So, the question left is: how has Euclidean geometry been operating in the development of scientific knowledge?

How Can Truths on Scientific Discourse be Analyzed?

Now, we have a truth: "Euclid's *Elements* are a key to access human knowledge". But, how are we going to deploy a discourse analysis to understand how has Euclidean geometry been operating through scientific discourse?

As we stated above, discourses are not isolated, they are produced by the interaction of different spheres of social life. This means that diverse spheres will evolve around certain truths, a certain statement will be shaped. But, at the same time, the same statement might be repeated in other spheres. Bang (2014) adds a new insight to this 'equation', he presents a new framework employing an image of 'quasi-self-similar fractals' to trace entanglements between multiple semi-autonomous fields.

> A new image of thought employing quasi-self-similar fractal—an image better suited to clarifying the issues and understanding the transversals and influences among multiple fields. This new image of thought is an attempt to represent the strange universality or 'universal mechanisms of fields' one encounters (Bang, 2014, p. 54).

To understand how a truth is circulating among social discourses, which means, to understand how 'Euclid's *Elements* are a key to access human knowledge' is navigating within different spheres of scientific discourse, we have to think outside the box of causality (e.g. Daston & Galison, 2007; Popkewitz, 2008). We have to trace the entanglements across scientific fields --- a 'quasi-self-similar fractals' image. This truth moves between the different spheres and also between spatio-temporal conditions. For example, Valero (2013) examines a "taken-as-truth" statement on mathematics education research, this statement is: a "mathematics education for all" is needed. She uses a discourse analysis strategy, which implies a rhizomatic analytical move (Deleuze & Guattari, 1987).

> My analytical strategy involves visiting a number of interconnected spaces that without any linear or strict logical connection […] map different aspects of the statement under examination. […] I also move in the connection of ideas in time and space. As mathematics education research is thought as an international field of inquiry, and probably because for many of its practitioners mathematics is still conceived as a universal activity, […] In keeping my eye on the ideas that circulate across nations I try to make evident how a field of inquiry generates truths that seems to be transferable from place to place and from time to time, contributing in this way to the reification of mathematical ability as a human ability and right that equates with reason, and

> with that installs one unified logic of being (Valero, 2013, p. 6)

Therefore, to analyse how the *Euclidean truth* is operating, we are going to move outside the field of mathematics, then, we are going to be able to see the entanglements across fields, the rhizome.

Is there a Euclidean Truth?

It is possible that one might think that we are forcing the Euclidean truth to appear. As if we were searching for a suspect, then everyone would be guilty. As we previously mention, discourses are produced within different spheres of social interactions. So, how will you react if we state that Euclidean geometry was not only the root for the development of mathematical knowledge? Well, the answer could be simple: physics. But no! Euclid's *Elements* have been entangled across diverse fields, such as architecture, literature, religion, philosophy, political science, and so much more. A 'quasi-self-similar fractal', a rhizomatic web where everything is connected by, the one and only, Euclidean thinking.

By deploying an analysis on the discourse of scientific research fields, about their 'roots', the existence of some beliefs about Euclidean geometry emerged. A truth that states Euclid's *Elements* as a method, the *Euclidean model*, that it was considered the "standard pattern for any "hard" science" (Toulmin, 1998, p. 336).

> Euclid's geometry, for instance, is notable for its rigor in demonstration […] is distinguished for its orderly "progression from the simple to the compound, from lines to angles, from angles to surfaces, and so forth," a method that particularly "contributes to the enlargement of mind and makes us think with precision" […He] develops the propositions of geometry in response to a natural need to know or to a spontaneous order of inquiry […] Any of these three systems increases the student's capacity for reasoning, for understanding ideas, which properly understood are "notions determined by relations" […] It is "nothing more than the faculty of arranging, *facultas ordinatrix*" […] The desire for order leads to the ideas of truth, goodness and beauty (Frank, 2007, p. 251)

From this, it is clear that Euclidean geometry is understood as a rigorous model of demonstration, as a model for 'organizing' knowledge as a progression and, finally, as a response to a natural need to know. These three aspects of Euclidean geometry will increase the capacity of students for reasoning and so forth. It is possible to think that this sort of statement derives from school mathematics discourse, or that it was a result from mathematics education research. But no, it was stated within the field of theology, in 1765, where it was argued how a man, through reason, becomes a man (Griffiths & Griffiths, 1765).

> Euclid's reasoning described a method which […] provided the foundation for all true reasoning, an abstract scientific method, through which the world becomes intelligible by a means of reasoning which is entirely independent of sense perception (Vinnicombe, 2005, pp. 670-671). Geometry is central to the great philosopher's thought in two quite distinct ways: as methodological guide and example, and as the most basic of all branches of knowledge, from which "synthesis" might deduce, step by step, the immutable laws of social justice (Grant, 1990, p. 151).

In the 17th century, it was believed that a new political science could be established that relied upon the principles of Euclidean geometry, an abstract scientific method, which developed the model for organizing human behaviour. Hobbes, who was proposing this connection, believed that 'geometry was central', a knowledge cannot subsist without a proper method and the key to achieve that method was held by Euclid's reasoning (Grant, 1990).

And those statements about the Euclidean truth have been entangled in other fields,

> In Architettura Civilequite often the elements of geometry become the elements of architecture *tout court*. For Guarini, for example, a wall is a 'surface' and a dome a 'semisphere.' […] the problem, for him, was not 'how to build' but 'how to draw.' Therefore, not only Euclidean geometry has become a part of architectural theory but it has also carried with it its implied linearis essential (Sbacchi, 2001, pp. 30-31).

It is clear that this quotation above is from architecture. These notions of Euclid's *Elements* were formally introduced in the 15th century by the *Trattato di Architettura Civile e Militare*. Euclidean geometry began to appear as "a good alternative to more complicated numerological calculations […And was probably] the preeminent one among the masses and the workers" (Sbacchi, 2001, p. 27).

It is possible to think that these quotations are old and that they recognize Euclidean geometry as the *model* for science simply because non-Euclidean geometries were not "formalized" until the 19th century. But, in the 21th century, literary education is being rooted in Euclidean geometry, not in the axioms and postulates, but in the model of order, of organizing knowledge, from the self-evident to the most complicated abstractions. Where the self-evident technical literary terminology are assumptions as the role of an "unreliable narrator" in a book or a play, there is no need to define it (Rabinowitz & Bancroft, 2014).

> We are proposing Euclid as a model because we believe that literary education should begin with the fewest possible number of initial assumptions, and that more complicated interpretations, in later years, should come from increased development and subtler manipulation of those assumptions, rather than from introducing entirely new concepts (Rabinowitz & Bancroft, 2014, p. 4)

I Completely Agree with It! Do You?

So, how has this *Euclidean truth* been accepted in the development of scientific knowledge? To reproduce a truth is not to repeat it incessantly, rather to reproduce implies acceptance and agreement. No scientist was forced to think *Euclideanly*, they accepted the *Euclidean model* because it seemed reasonable for them. In other words, they are *subjected* to the self-evident truth of Euclidean geometry's consistency, simplicity, rigour, "progression order" and so on.

This is the second time that the word *subjected* appears. According to Foucault (1982), there is not a domination of the self; no one is forced to do or believe anything by imposition. That is how he understood power. So, this power implies 'the other' as a person who acts on his/her own. Hence, this power depends on the freedom of the subject. Here is when the term *subjected* comes to play; to be subjected could mean "to be shaped in a particular way" or "to be shaped to become a particular self". And every spatio-temporal condition has a 'rationality', a way of thinking and behaving, as codes that are transmitted to people. The game of "being part of" requires the acceptance of that spatio-temporal discourses, not a simple repetition; one has to believe in it.

So, let's return to Euclidean geometry. How has this *Euclidean truth* been accepted in the development of scientific knowledge?

> Since seventeenth- and eighteenth-century natural philosophers took their Platonist ambitions from Galileo and Descartes [...]. From the start, formal systems modelled on Euclid had a charm that carried people's imagination over into fresh fields: if the world of nature exemplified in Newton's dynamics had a timeless order, this could presumably be extended to the world of humanity as well (Toulmin, 1998, p. 353).

> The mathematical method of deduction from axioms had a decisive effect on the social sciences of the Enlightenment. [...]. Find the axioms of human nature, deduce from them in the approved Newtonian manner, and a complete science of man became a possibility (McClelland, 2005, p. 290).

> One of the issues which played a major rôle in most of the discussions of the Theory of Relativity was the simplicity of Euclidean geometry. Nobody ever doubted that Euclidean geometry as such was simpler than any non-Euclidean geometry with given constant curvature [...] Euclidean geometry is the only metric geometry with a definite curvature in which similarity transformations are possible (Popper, 2005, pp. 129-130).

Here the acceptance is not in order to accept the brilliance of Euclid, or to agree to only use Euclidean geometry. Neither is a matter of stating that a science will become science depending on how much Euclidean geometry was used in the development of their field of knowledge. It is an acceptance of Euclidean Geometry as an axiomatic, scientific *model* (Hartshorne, 2000), an acceptance that through a

Euclidean way of thinking people will become a scientific self.

We are aware that not everybody blindly accepted Euclidean geometry. For example Einstein demonstrated that this geometry was only thought to be applied in a void, not in the *real world,* where space is inseparable from matter (Woods & Grant, 2007); "where mass tells space-time how to curve, and space-time tell mass how to move" (Wheeler, in Sweeney, 2014, p. 826). Einstein was referring to Euclidean geometry as a mathematical model of space; however, he was interested in a geometry that provided him tools to understand the physical space. For instance, "it was much later, with Einstein's general relativity, that it was shown that the geometry of the universe is not Euclidean but curved" (Hirsch, 1996, p. 62). We want to be clear that the discussion is deeper than that. We are not against Euclidean geometry. We are drawing awareness to Euclidean geometry as a truth that circulates in scientific discourse and performs, as an effect of power, a scientific self.

So, am I a Scientist Already?

> What Euclid did that established him as one of the greatest names in mathematics history was to write the Elements. [...] Euclid' great genius was not so much in creating a new mathematics as in presenting the old mathematics in a thoroughly clear, organized, and logical fashion" (Brodkey, 1996, p. 386)

Indeed Euclid was a great geometer, probably the most recognized of all time. The *Elements* deploy a schematic *order* from the basic definitions to the most abstract *formalizations* (axiomatics). But this sacredness was not eternal. Not only mathematicians, but also researchers of others fields of knowledge tried to show that Euclidean axioms are in opposition with our optical perception of space. These studies concluded that visual space is far from being Euclidean (Suppes, 1977). But it is possible to find that almost all Western school geometry is based on Euclid's work (Burgin, 1987; Ray, 1991). So, how can we explain this Euclidean resistance? As stated in the previous section, Euclidean Geometry is not just formulas and axioms that need to be applied to solve problems. This geometry is being operated in a completely different fashion.

Does this mean that I will become a scientific self if I accept Euclidean geometry as the 'basis' of all knowledge? It is not as simple as that. Subjectivity does not imply only the repetition of a truth; the acceptance of this discourse will operate in an interesting way. For {Daston, 2007 #33@@author-year;Foucault, 1982 #92}Foucault (1982), human beings become subjects through the objectifying effects of scientific knowledge. At the same time, the practice of knowing generates effects in the form of knowing and in the subjects who know (Daston & Galison, 2007). Therefore, subjects must train themselves to become part of a practice; in other words, they have to conduct their own conduct. Such *subjectification* pursues to fabricate a scientific thinking.

How is this Euclidean truth prompting to a scientific self? The method deployed by Euclid is shaping a deductive and axiomatic way of thinking, a method that "contributes to the enlargement of mind and makes us think with precision [and increases the] capacity for reasoning and for understanding ideas " (Frank, 2007, p. 251).

In the End…

Let's return to school, school geometry is rooted in Euclidean geometry, but this was not intentionally. It was not because someone wanted it there. This geometry is an important part of school due to the circulating truth within 'scientific discourse'. Currently, to become a scientist means to become a productive citizen. Therefore, if we want to have a brighter future we have to be *subjected*, in Foucaultian terms, by schooling.

> Educational sciences have provided the tools for fabricating the cosmopolitan child through being a cornerstone of the planning of social life for the promise of a better and brighter future. […] I connect the statement of the need of a mathematics education for all for creating a brighter future with the way in which educational sciences in the 20th century have produced the elements for the reasoning making possible such statements (Valero, 2013).

Euclidean geometry is much more than a particular way of seeing space or a formalization of the metrics of the earth; it is much more than just learning a set of mathematical concepts and rules. Euclid's elements are deploying a deductive system, rooted in proofs and demonstrations. Euclidean geometry becomes the template or the path to become a scientific self. So, in order to become this scientific self, students must follow the path of what schools perceive a real scientist is, not to become a scientist, but to become a logical thinker, a problem solver, who uses reason! The desired cosmopolitan child (Popkewitz, 2008), as described by the Chilean Ministry of Education when stating:

> School mathematics curriculum aims to provide students with the basic knowledge of the field of mathematic, and, at the same time, helps students to develop logical thinking, deductive skills, accuracy, abilities to formulate and solve problems and abilities to model situations [...] The learning of mathematics enriches the understanding of the reality, facilitates the selection of strategies to solve problems and contributes to an autonomous and individual way of thinking (Ministry of Education of Chile, 2010, p. 3, our translation).

This discussion is not about how Euclid's axioms and postulates are the easiest for children, cognitively speaking. The discussion is that Euclid's Elements are a consistent, deductive and progressive system that shapes the way of thinking of a scientist. School geometry also operates by constructing its subjects; it shapes in students a way of visualizing the world and a way of thinking about space and reality. If the method deployed by the *Elements* was the basis of almost all scientific knowledge, then it does not seem such a bad idea to teach Euclidean geometry at schools, right?

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Morality and News Media Representations of Mathematics Education

Richard Barwell & Yasmine Abtahi
University of Ottawa, Canada

Mathematics education is often in the news and is often presented in controversial terms. The nature of news reporting influences public opinion, public policy and the context of mathematics teaching. In this study, we looked at how mathematics education is framed in Canadian newspaper reports after the publication of recent PISA results. We analysed 26 articles published in two clusters, in two national newspapers. We particularly focused on the moral dimension implicit in this corpus of news reporting. Our analysis revealed a dominant framing derived from the binary of traditional teaching vs. discovery learning, which is portrayed to be the prevailing cause of students' decline in PISA ranking. The dominant framing also includes a sense of moral decline related to both educational and economic trends.

Introduction

Mathematics education is often in the news. A cursory search reveals coverage of, among other things, the results of international comparisons, national tests, curriculum changes or periodic reports produced by various bodies, generally showing a decline in standards, skills or quality of students or teachers. For many mathematics educators, such reporting is of professional interest, perhaps a source of frustration at times, but ultimately not that important. In this paper, we start with the assumption that news reporting of mathematics education actually matters quite a lot; such reporting, along with associated commentary and debate, shapes the way in which mathematics education is understood by the general public. The general public includes, of course, parents, taxpayers, employers, politicians, student teachers and current teachers. All of these people have an interest in mathematics education.

In what follows, we report our analysis of part of a corpus of Canadian news media reports relating to mathematics education.

Our goal is to understand how mathematics education is portrayed in news media. We report specifically on a group of articles that appeared in December 2013 and January 2014 triggered by the release of the international PISA mathematics findings (OECD, 2013, 2014). Our analysis is based on a critical discourse perspective and draws specifically on the media theory concept of framing. For this paper, we focus, in particular, on the moral dimension implicit in the news reporting we have analysed.

Literature Review

When education is the focus of the popular press, the media has a strong influence on public opinion; it influences "the dynamics of opinion expression and formation" (Scheufele & Tewksbury, 2007, p. 10). More specifically, the way in which particular news is framed in the media significantly influences the assessment and judgment of the audience: "framing guides the audience on how to think about an issue" (Kee et al., 2012, p. 17). As a result, the relationship between the education system and the public is, in part, mediated by images portrayed in the media, which may (or may not) be indicative of day-to-day classroom activities.

Research on the framing of education in news media suggests that educational stories are mostly presented in a negative light. Camara and Shaw (2012), for example, criticise educational media coverage, arguing:

> Press coverage of educational issues tends to be biased and provides greater attention to the negative rather than positive side of stories, presents an incomplete or too simplistic view of educational issues, exhibits a major lack of understanding of statistics and educational research. (p. 34)

This kind of negativity has been noted specifically in relation to mathematics education. In particular, publication of PISA results generates extensive and controversial headlines in different participating countries. Pons (2012) referred to this media coverage as "PISA shock"; that is, the popular media discourse that focuses primarily on how each country is positioned in the PISA rankings, producing and disseminating "shocking" headlines, as well as the way in which this coverage leads to particular ways in which people analyse, translate, and reinvest the results of PISA. For example, the publication of the first round of the PISA results in December 2001 had a "Tsunami-like impact" (Gruber, 2006, p. 195) in Germany, affecting educational policy-making discourse (Waldow, 2009) and fundamentally changing the educational discourse in Germany (Ertl, 2006).

We have noted similarly strong headlines in our corpus, including "Canada's fall in math-education ranking sets off alarm bells" or "Math wars: The division over how to improve test scores". Mathematics educators may find such headlines irritating or frustrating, since they can appear to give an unbalanced and misleading view of mathematics teaching, learning or curriculum. In our research, however, we have become interested in the implicit moral aspect that such headlines hint at. What we look at in this writing, then, is the not the PISA results themselves, but the "PISA-shock" – that is the reaction to PISA – in two Canadian national newspapers.

Theoretical Framework: Discourse and Frames

Our research is broadly framed by a critical discourse perspective (e.g. Edwards & Potter, 1992). From this perspective, language is not seen as a neutral medium for describing a pre-existing reality. Descriptions always involve choices about what to highlight, what to leave out, and what specific words to use. The language of news reporting constructs particular versions of the world. These versions of the world are seen as reflecting particular interests and as designed for particular audiences (Edwards & Potter, 1992). News texts are not created in a void, however. They draw on prevailing discourses and ideas about any given topic.

Critical discourse frameworks have been used in mathematics education since Walkerdine's (1987) study of the discursive construction of rationality and associated discourses of the autonomous child found in much constructivist writing about mathematics learning. Her analysis of this prevailing idea of rationality relates to a view of society based on individual choice and normativity. Abstract reasoning is seen as the 'normal' endpoint of individual intellectual development, and as the means with which individuals can then function in a capitalist society.

Walkerdine's (1987) analysis has been extended in subsequent work, including Appelbaum's (1995) examination of constructions of mathematics education in popular culture, including news media, in the USA in the 1980s. He argued, among other things, that mathematics teaching was popularly constructed as a kind of heroic, individual endeavour. This construction makes it more difficult to think about mathematics education in social or political terms. The discourses about mathematics teaching identified in Appelbaum's (1995) research draw on discourses circulating in society, but also contribute to the reproduction of these discourses. More recently, Lange and Meaney (2014) noticed that a sample of media texts collected in Australia reflected a discourse of commodification of children in which children were seen as objects of investment (of time, of money, etc.) and to which "value needed to be added" (p. 392), again reflecting the link between mathematics education, rationality and a capitalist society.

Framing

The term 'framing' refers to "modes of presentation that journalists and other communicators use to present information in a way that resonates with existing underlying schemas among their audience" (Scheufele & Tewksbury, 2007, p. 12). At the same time, framing is based on the assumption that the way an issue is characterised in news reports can have an influence on how it is understood by audiences. Moreover, through framing, news is characterised in terms of pairs of concepts that readers come to see as connected (Price & Tewksbury, 1997). News often frames this connection by establishing predominant labels (e.g. the description of politics in terms of left and right) to show how forces and groups in society are shaping public discourse (Scheufele & Tewksbury, 2007). For example, in the framing of PISA in news reports, the forces and groups that are shaping the public discourse are actors such as parents, governments and ministries, teachers, and curricula and the established predominant labelling are 'back to basics' and 'discovery learning'.

Entman (1993) proposes four elements of a given news framing. The frame includes a *problem definition*: it defines particular aspects of an event as problematic situations. It establishes a *causal relationship*: the frame identifies forces creating the problem. There is a *moral* *judgment*: that is, an effort to "personalise the news, dramatise or emotionalise it, in order to capture and retain audience interest" (Semetko & Valkenburg, 2000, p. 96). The moral judgment is not simply a judgment of what is equitable, it is a judgment that is "laden with emotions" (Gamson, 1992, p. 7). Finally, the frame includes *endorsing remedies*: it offers and justifies treatments for the problem and predicts their likely effects.

Entman (1993) explains that framing involves selection and salience. It is through framing that particular aspects of an issue are highlighted and others are ignored (Matthes, 2007). Hence framing guides the audience by the omission of potential problem definitions, explanations, moral evaluations, and recommendations as much as they do by their inclusion. In summary, the dominant framing consists of the problem, plus the causal, moral evaluative and treatment interpretations that are most likely to be noticed, processed, and accepted by the most people. So the dominant framing is a framing of a situation that is most heavily supported by the text and is, ideally, congruent with the most common audience interpretation of the situation.

Based on this theoretical perspective, we established the following research questions: How are the PISA results framed in the corpus of news articles? How is mathematics education constructed through these frames? What moral judgments are implied by these frames?

Methods

We examined three national print publications: the Globe and Mail, the National Post and Macleans (a weekly publication), to represent a range of national news coverage. We collected all news articles on mathematics education in these publications within a six-month period (September 2013–March 2014). Altogether we found 64 articles: 45 in the Globe and Mail, 16 in the National Post and 3 in Macleans. In this paper, we report our analysis of a subset of 26 articles, all published within a few days of each other, that covered the then recently released PISA results. This subset consists of two clusters of articles: a) 15 articles in a 4-day timeframe: 2–6 December 2013, and b) 11 articles in a 5-day timeframe: 7–12 January 2014. The first cluster appears to have been triggered by the publication of the PISA 2013 results. The second cluster appears to have been triggered by the announcement of a $4 million fund for teacher professional development for mathematics teaching by the Ontario Ministry of Education in response, in part, to the PISA findings. We focus on these two clusters to investigate framing with respect to the PISA results.

To analyse the subset of articles, we first examined each article in terms of the four aspects of framing. We read all the pieces thoroughly, and re-read them several times. In each reading we looked for a coherent sets of phrases and statements that seemed to represent or talk about any of the four aspects of framing (i.e., general problematic situation, causes, moral judgements, and treatments). To ascertain how certain aspects of the PISA results are highlighted and others are ignored, we looked for words, phrases and metaphors used to describe a particular image: for example, phrases such as 'math wars', 'battleground', 'national emergency', 'watering down discovery learning', 'avalanche of concern' and 'downward progression'. We then looked for commonalities in how these different aspects of framing appeared across the corpus, as well as instances that ran counter to any general patterns. In all the readings we kept asking ourselves what images or ways of thinking about PISA results are presented and/or denied by the framing, and if we could identify a dominant framing. While there is some variation across the selected articles, we were able to identify a broad overall frame to which most of the articles were oriented (occasionally by adopting an explicitly alternative stance). In the next two sections we first summarise the nature of the dominant frame we identified in the set of articles and then examine the moral dimension of the frame in more depth.

The Dominant Frame: Traditional Teaching vs. Discovery Learning

The dominant frame that emerged from our analysis used the PISA mathematics results to portray the decline in Canada's ranking as a problematic situation. The following statements were used in many articles to manifest this prevailing situation:

An increasing percentage of Canadian students are failing the math test in nearly all provinces. (Globe and Mail, 7 January 2014) [1]

PISA results were not a disaster for Canada. But they were a giant, flashing amber light. Canada's student performance, formerly well above the OECD average, is now considerably less so [...] our students are doing decidedly worse in math than they did a decade ago. (Globe and Mail, 9 January 2014)[2]

With respect to this problematic situation, different articles identified different causes, including the way mathematics is taught, the mathematical competence of school-teachers, and lack of societal expectations. Not surprisingly, perhaps, we found that the dominant framing particularly suggested a strong connection between the decline in Canada's PISA ranking and the ways in which mathematics is taught. This causal relationship is established through a contrast between traditional, back-to-basics teaching and discovery learning. These ideas were represented by two groups of terms, including: "conventional math", "fundamentals that parents were taught" and "basic math algorithms" versus "learning by investigation, problem solving, and open ended questions", "conceptually based" learning, and "discovery learning". For example:

For one, straight long division isn't on the curriculum anymore; at least not as it once was. The old ways of learning — rote strategies and "math facts" — have been replaced by so-called "discovery math" and "inquiry-based" teaching methods that focus on word problems, strategies and estimations. (National Post, December 5, 2013)[3]

Gone are the days, in much of the country, of long division, mad-minute multiplication, addition with a carry and subtraction with a borrow. Today, children in provinces that have introduced the Western and Northern Canadian Protocol (WNCP) curriculum – a vast swath of the country – learn instead by investigating ideas through problem-solving, pattern discovery and open-ended exploration. (Globe and Mail, 10 January 2013)[4]

Quoting Professor Donna Kotsopoulos, the article referred to in this last quotation stated: "If you look at what's been happening, predominantly over the last decade, there's been an unprecedented emphasis on discovery learning". Citing Robert Craigen, a University of Manitoba mathematics professor "who advocates basic math skills and algorithms", the article noted:

> Canada's downward progression in the international rankings, slipping from 6th to 13th– coincides with the adoption of discovery learning. (Globe and Mail, 10 January 2013)[4]

The dominant framing suggests various treatments to the problematic situation. These treatments were closely allied to the identified causes and included: extending teacher education program and including more emphasis on mathematics; changing the curriculum; and following Quebec's education system. This last point is related to Quebec's relatively superior performance in PISA in Canada. In many articles, Quebec is referred to as the province that "adds" while the rest of "Canada subtracts on its math scores" (Globe and Mail, 3 December 2013)[5]. The reporting of Quebec's performance fits within and contributes to the same binary frame of 'traditional' vs. 'discovery learning'. For example:

> Quebec, with its intensive training and teachers who apparently refuse to shirk algorithms despite reforms, enjoys the best scores in Canada and is now at the centre of math-education research. (Globe and Mail, 10 January, 2014)[4]
>
> Moreover, the dominant framing suggests a strong connection between the need to change curricula and a Manitoban parent-led campaign and petitions reported in other provinces. That is, grassroots movement of parents and educators are portrayed as "pressing provincial governments across the country to make immediate changes to the way math is being taught" (Globe and Mail, 8 January 2014)[6].

Overall, the dominant framing we have described based on the full set of articles draws on the binary of 'traditional' teaching vs. 'discovery learning'. It relates a clear problematic situation (Canada's lower PISA ranking) to a cause (not enough traditional teaching) and an associated treatment (more traditional teaching) through, among other things, curriculum changes to include more emphasis on arithmetic facts and procedures. These features of the framing form the backdrop to the aspect we found most interesting: the moral dimension.

Moral Judgments

We noticed two aspects in the moral dimension of the dominant frame: the choice of words to describe mathematics education and the use of anecdotes, often featuring parents, to illustrate the news reports.

The moral aspect of word choice can be seen in many of the quotations in the previous section. For example, the choice of the word 'failing' in the first quotation ("increasing percentage of Canadian students are failing the math test"[1]) carries an implicit moral judgment: failing is bad (the claim that students are failing is, of course, questionable). When combined with the causes included in the dominant frame, the implication is not just that failing is bad, but that discovery methods are also bad. Across the set of articles, a diverse set of word choices combine to paint a picture of moral decline. Canada's performance is a "giant flashing amber light", a "downward progression" and "slipping", with students who are "doing decidedly worse" (it is the word 'decidedly' that hints at moral failure). This moral judgement is explicitly linked to the binary division of traditional teaching from discovery methods. For example, the contrast "Gone are the days" (of traditional methods) with "today [...] vast swathes of the country" (are using discovery methods)[4] suggests the replacement of something reasonable with a rather more dubious approach: "vast swathes" implies a marauding invader, rather than the spreading of something positive. This link between morals and causes, and indeed treatments, also appears in the depiction of Quebec, where teachers "refuse to shirk" traditional methods, "despite reforms" and so "enjoy" success[4]. Finally, several articles captured this sense of moral decline and linked it to broader economic concerns:

> This is something that should send shivers of fear down all of our spines. If Canadian students cannot master basic math skills early on, there is no question that we will fall behind in

economic competitiveness. And it will happen quickly. (Globe and Mail, 6 December 2013)[7]

The genie is now out of the bottle," said Paul Cappon, a former head of the Canadian Council on Learning who is now with the University of Ottawa. "Not only is Canada mediocre at best, we now know that our future in learning, and therefore our prosperity, is more clouded than ever." (Globe and Mail, 4 December 2013)[8]

> This is on the scale of a national emergency," said John Manley, CEO and president of the Canadian Council of Chief Executives, which has sounded the alarm on the shortfalls in our education system. (Globe and Mail, 3 December 2013)[5]

Hence, failing and falling scores are related to "shivers of fear", "falling behind" economically, mediocrity, loss of future prosperity and a "national emergency", with the clear, even incontestable point that negative economic performance is morally bad. It should not be forgotten that PISA is a project of the OECD, an organisation that works for economic development, not education.

Moral judgment was also conveyed through anecdotes featuring parents. Here are two examples:

> Ms. Murray said she felt compelled to speak up after tutoring students in math who haven't mastered basic skills. "I see the same problems everywhere I go. Serious gaps with addition, subtraction, multiplication, division. More complex arithmetic is almost completely missing," she said. "Any parent I have ever spoken to understands the problem, but feels helpless – that no one will listen to them." (Globe and Mail, 7 January, 2014)[1]

> Tara Houle, a mother of two in North Saanich, says she has seen firsthand the confusion so-called discovery-based teaching techniques can lead to. "[My daughter] was being taught using Sudoku math puzzles [in Grade 3]. They had computer games in the classroom to learn the times tables," she said. "They were these methodologies to, I guess, conceptualize and make children think in different ways to come up with the answers. We don't have an issue with [that]. However, there wasn't a

> lot of emphasis on, say, learning the multiplication table." Her daughter continued to struggle with these concepts until she was enrolled in an after-school math program. "The transformation was incredible," Ms. Houle said. "After understanding their simple and effective methodology to solve math problems, it made her embrace math again. We went back to the basics." (Globe and Mail, 9 January, 2014)[2]

In these two accounts, the moral judgments relate to parents' feelings of "helplessness", observing their children's "confusion" and "struggle" followed in one case by an "incredible" transformation when traditional methods were provided. These accounts are constructed in terms of lack (for example, of "basic skills" and of "learning the multiplication table") arising from "so-called" discovery methods. In these accounts, moral decline is linked to the challenges faced by parents (which most readers would be) who are constructed as wanting to address this decline through their own actions. This portrayal suggests a kind of 'meta-moral' problem in which the badness of decline is compounded by a loss of agency on the part of parents to tackle the decline themselves.

Discussion and Conclusions

Our study of the two clusters of Canadian newspaper reports of the result of PISA revealed a dominant organising framing in which the different elements – problematic situation, cause, moral judgments, and treatment – are clustered and held together through a narrative of decline. This frame is derived from the binary of traditional teaching vs. discovery learning. The different dimensions support each other to sustain an overall portrayal of mathematics education riven by the dispute between proponents of these two approaches to teaching. It is important to underline that we do not necessarily accept the distinctions and categories used in the news articles. Indeed, it is important to contest the idea that all mathematics teaching falls neatly into one of these two categories, rather than being, for example, a mixture of these and other ideas. Mounting such a challenge is, however, likely to be difficult. The binary underpinning the dominant frame is largely taken for granted, coming close to being a 'regime of truth' (Foucault,

1979) in that even alternative positions must conform to these general categories.

Similarly, the news reports carry a fairly strong moral tone related to the general sense of decline. Again, this moral tone is related to deeply embedded and largely unquestioned assumptions. After all, who is in favour of educational or economic decline? It is important, in fact, to note this connection with economic prosperity as both a key purpose of education (in this portrayal) and a key source of moral judgement, since increasing prosperity is assumed to be a good thing in capitalist societies.

How, then, as mathematics educators, can we respond, if at all? It is difficult to know how to respond to frames that are based on assumptions that come close to the status of regimes of truth. Any response or attempt at rebuttal will tend to be framed in the same terms. Indeed, one or two articles illustrate this, challenging the binary distinction, and so reinforcing it. The reflexive relationship between news framings and public understanding of issues means that it is difficult to change these framings. It is part of the nature of frames to use binaries and to simplify complex situations to make them more digestible. Attempting to explain the complexity of mathematics learning and teaching is difficult to do within the constraints of a news frame. Appelbaum (2014) has engaged with this challenge, arguing for something like guerrilla-style "tactics" (another part of the battle metaphor) in which we try to disrupt the frames and so make people think. For our part, we do not have any simple suggestions; instead we welcome discussion on this issue.

Notes

1. Math wrath: Parents and teachers demanding a return to basic skills. By Alphonso, C. & Maki, A. Globe and Mail, 7 January 2014.

2. Canadian education: The math just doesn't add up. Editorial. The Globe and Mail, 9 January 2014.

3. Math isn't hard. Teaching it is. By Urback, R. National Post, 5 December 2013.

4. Math wars: The division over how to improve test scores. By Carlson, K. B. The Globe and Mail, 10 January 2014.

5. Quebec adds, Canada subtracts on its math scores. Editorial. The Globe and Mail, 3 December 2013.

6. Ontario unveils $4-million math upgrade plan. By Morrow, A., Alphonso, C. & Maki, A. The Globe and Mail, 8 January 2014.

7. The double danger of low math scores. By Hirsch, T. The Globe and Mail, 6 December 2013.

8. No time for educational complacency, Canada. By J. Simpson. National Post, 4 December 2014.

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A Derridean Critical Contribution to Social Theories in Mathematics Education Research

Marcelo Salles Batarce
Universidade Estadual do Mato Grosso do Sul

In "Whither social theory?", Pais and Valero (2014, p. 246) considered that social theories in mathematics education research leave "mathematics untouched and outside the possibility of being deconstructed". In this paper, I discuss this claim from a philosophical point of view. My theoretical reference is the work of Derrida.

A New Question for Social Theory in Mathematics Education?

In "Whither social theory?" (Pais & Valero, 2014), one can read two fundamental critiques to social theory in mathematics education: (a) that not taking "research itself as a social structure that provides ways of thinking and doing the teaching and learning of mathematics in schools" (p. 244), researchers disavow "a critical reading of their own role and of the research they produce in the problems they identify" (p. 244); and (b):

> ... there is a strong limitation in the use of sociological theory when researchers still behave as uncritical ambassadors of mathematics. To leave mathematics untouched and outside of the possibility of being criticized and deconstructed is creating a limit to our understanding. (p. 246)

Both critiques indeed make together the kernel of my doctoral thesis (Batarce, 2011). They appear in Chapter 2 under the title "The limits of mathematics educators' critiques to mathematicians". Also, part of my effort in the same chapter is to show how the two critiques are linked. The issues have also been partially framed in Mattos and Batarce (2010).

I have presented the second critique from a Derridean perspective and therefore in a very philosophical way. After the paper of Pais and Valero's, my considerations might be of interest to a critical philosophy of social theory in mathematics education. My claim is that mathematics education research's adhesion to "mathematics" is much deeper rooted than one seems to realize and it is, after all, an adhesion to a metaphysical tradition of knowledge.

The Meaning of Mathematics: An Old Question in Mathematics Education Research

If one were to review every paper, chapter, or book in the mathematics education literature that includes any reference to discussions about the meaning of "mathematics", one would probably gain a good understanding of the production of mathematics education research as a whole. If one wanted to look at this question through the historical developmental lens of mathematics education research then the reviewer would have a wealth of historical material to follow. Names and classical texts such as Freudenthal (1978), Davis and Hersh (1980) and Ernest (1993) became pertinent and influential, in part, through their approach to this theme. Could not the same be said in respect of developments such as ethnomathematics, for example, where challenging the concept of "mathematics" appears to be at the heart of the program?

In recent years, the theme of the meaning of mathematics still influences a wide range of trends in mathematics education research. A good example of this is "the professional formation of teachers" as it was named in the Symposium on the occasion of the 100th Anniversary of ICMI in 2008. Drawing on Shulman's notion of pedagogical content knowledge, Deborah Ball became a leader amongst those who ultimately attempted to delineate a "pure" content knowledge for mathematics teaching that differed from that afforded by mathematicians' mathematics.

Coincidentally, the theme of PME (2006) was "Mathematics at the Centre".

> The theme of this year's PME conference is Mathematics at the Centre. This theme is chosen with the intention of going back to the roots… at the heart of every effort to make mathematics teaching comprehensive, useful, interesting and thrilling must be mathematics itself and this is not to be neglected… We believe … that many of us share this concern for mathematics (Novotná, 2006, p. 1_liv)

But this theme appears to have caused discomfort inside the Conference itself. Romulo Lins (2006) led a plenary panel with the theme "A Centre and a Mathematics" and his words certainly did not completely mirror those of the welcoming Conference address. He asked whether mathematics was at the core of our root system or whether it had been relocated to the centre of mathematics education, hence becoming our last line of defence.

> …having being absent from PMEs for a while, I couldn't help but consider the possibility that, given the theme of this 2006 conference, "Mathematics in the centre", this might be our last trench: mathematics. After 17 years—and I am not counting what might have happened before I join PME community—the inner centre seemed to have been moved to "mathematics" (p. 1/67)

The Word "Mathematics"

But after long meditations, one may consider that, mathematics, first of all, is nothing more than a word. Or, it cannot be anything if there is not the word. Here and there, mathematics education researchers have scratched this conclusion.

In the same PME paper quoted above, Lins visibly mistrusts the word "mathematics".

> … the very word "mathematics" is something that, in our western or westernised cultures, floats above all of us, better, it fills, in a sense, some cultural "air" we are immersed in, something whose presence does not depend on the mention of any specific content or area. (p. 1/68)

This strange power of confusing the senses appropriated by the word "mathematics" would become more apparent when the constituent parts are read within the term "mathematics education". It is almost as if "mathematics" failed to refer to the body that is "mathematics" at all!

> When Susie's paper mentions a keen interest in what she calls "subject cultures", I think she is precisely acknowledging that there is a sense in which "mathematics" in "mathematics education" does not need to mean a reference to specific topics and the teaching and learning of those topics, although it may, of course, be meant in this way. (p. 1/68)

But none presented the question as straightforwardly and clearly as Anna Sfard (1998) did. She appears also to have grasped a question, which seems to be fundamental as a basis for mathematics education's critiques of mathematicians: "If a mathematics educator studies mathematics is it the same object for him or her as it is for a mathematician who studies mathematics?" (p. 491).

Not by chance, that same issue, the reduction of "mathematics" to a word will upset Sfard's analysis of an irreducible difference between mathematicians and mathematics educators. She was aware that the issue lies in the question:

> To what meaning of the word "mathematics" do people subscribe when they identify themselves as researchers in the field of mathematics education? (Sfard, 1998, p. 495)

If, on the one hand, she seems to cast doubt on the possibility of concealing this difference with respect to the concept of "mathematics":

> Moreover, it seems that trying to fill in the gap in an attempt to make the two mathematics into one would be pointless. (p. 505)

… she appears, at the same time, to trust in an objectivity of "it", beyond differences, as a word:

> It seems that the first step necessary … is to clarify what the word "mathematics" means to them [mathematics educators] in relation to what it means to mathematicians. (p. 492)

… and everything is based on the premise that the word "mathematics" is not a unit. Could it be possible to write an irreducible difference of meanings but yet maintain identical features of their referents? Perhaps, but only with a trick of writing: the use of hyphens.

> That the Typical-Mathematician's-mathematics and the mathematics-education-mathematics come to be worlds apart seems undeniable. (p. 505)

Wittman attempted another example, where instead of hyphens, he played with capital letters:

> I suggest the use of capital letters to describe MATHEMATICS as mathematical work in the broadest sense; this includes mathematics developed and used in science (Wittmann, 1998, p. 90)

One may suggest that the existence of a plurality of meaning for "mathematics" supports, in general, the arguments of mathematics educators against mathematicians. That is, the possibility of many facets of mathematics, as promulgated by Sfard in her chapter "The many faces of mathematics: Do mathematicians and researchers in mathematics education speak about the same thing?" (1998); the difference between academic mathematics and street mathematics, as developed by Terezinha Nunes; the form of mathematics in different cultures and D'Ambrosio's program on ethnomathematics; Alan Bishop's on anthropology; the postmodern approaches and the concept of mathematics constructed through discourses; the different epistemology of mathematics in Paul Ernest; or even the most traditional psychology of mathematics education and Begle's concept of mathematical behaviour. It follows that the essence of the meaning of mathematics is held in mathematics education itself. Conversely, the very difference between the word "mathematics" and its meanings, in its plurality, is bound to a tradition of metaphysics, which appears to limit the effectiveness of the critiques of mathematics education practitioners.

> But to these metaphysico-theological roots many other hidden sediments cling. The semiological or, more specifically, linguistic "science" cannot therefore hold on to the difference between

signifier and signified—the very idea of the sign—without the difference between sensible and intelligible, certainly, but also not without retaining, more profoundly and more implicitly, and by the same token the reference to a signified able to "take place" in its intelligibility, before its "fall," before any expulsion into the exteriority of the sensible here below. As the face of pure intelligibility, it refers to an absolute logos to which it is immediately united. This absolute logos was an infinite creative subjectivity in medieval theology: the intelligible face of the sign remains turned toward the word and the face of God. (Derrida, 1976, p. 13)

In fact, one might suggest that a criticism of mathematicians' idealism of mathematics will only be accomplished through the elimination of those sediments referred by Derrida. In this paper, I do not intend to exam this suggestion in depth. Rather, I intend only to provide the bases whereby a critique might be developed. In order to do so, I shall now return to Derrida and his discussions about word, writing, signifier and signified

Mathematicians and Mathematics Educators as Unities of Consciousness

The first question to be raised following the previous sections is: "What is a 'word'?" But this question does not have a straightforward answer and rather than being a simple enquiry, the question itself may entail a response. It is in this way that Derrida refers to Saussure's project:

> The form of the question to which he [Saussure] responded thus entailed the response. It was a matter of knowing what sort of word is the object of linguistics and what the relationships are between the atomic unities that are the written and the spoken word. Now the word (vox) is already a unity of sense and sound, of concept and voice, or, to speak a more rigorously Saussurian language, of the signified and the signifier. This last terminology was moreover first proposed in the domain of spoken language

> alone, of linguistic in the narrow sense and not in the domain of semiology ("I propose to retain the word sign [signe] to designate the whole and to replace concept and sound-image respectively by signified [signifié] and signifier [signifiant]" p. 99 [p.67]). The word is thus already a constituted unity, an effect of "the somewhat mysterious fact ... that 'thought-sound' implies divisions" (p. 156) [p. 112]. Even if the word is in its turn articulated, even if it implies other divisions, as long as one poses the question of the relationships between speech and writing in the light of the indivisible units of the "thought-sound," there will always be the ready response. Writing will be "phonetic," it will be the outside, the exterior representation of language and this "thought-sound." It must necessarily operate from already constituted units of signification, in the formation of which it has played no part. (Derrida, 1976, italics in the original, p. 31)

As recognised by Derrida, Saussure's project entails a notion of a word or sign which "primordially" refers to the spoken-word. That is, the signifier has a phonetic nature. The following passage from Saussure, as quoted by Derrida, illustrates this: "The linguistic object is not defined by the combination of the written word and the spoken word: *the spoken form alone constitutes the object*." (Saussure in Derrida (1976), p. 31, Derrida's italics). This "privilege" of voice over writing, which is at the very heart of Derrida's critique, is not only a result of Saussure's project but it also refers more overarchingly to the concept of sign in the western tradition, a tradition that has always placed the concept of phonè above that of writing:

> Language and writing are two distinct systems of signs; the second exists for the sole purpose of representing the first. (Saussure in Derrida, Derrida's italics p. 30)

It is this very fact that writing is considered to be subordinate to language that Derrida will explore when confronting Grammatology and Linguistics. He will challenge the notion of presence as an aspect imperative to the constitution of a science of language, with the concept of writing. While the presence of the speaker seems to be essential to speech, writing disseminates without the presence of its author. The notion of presence which Derrida confronts is the idea of consciousness and authorship as a full diktat of meaning. The absence, as an essential feature of writing, challenges this full diktat of meaning. Let us, at this point, revisit the question as it appeared framed by Anna Sfard.

> It seems that the first step necessary … is to clarify what the word "mathematics" means to them [mathematics educators] in relation to what it means to mathematicians. (p. 492)

For Sfard, the two subjects, mathematicians and mathematics educators, appear to be the guarantors to the meanings of the word "mathematics" in each case. The differences of meanings are posterior to the subjects already represented. Their presence represents therefore the consciousness (of a community or the like) of a completed meaning (we might say "understanding") in each case for the word "mathematics".

The scenario in which Sfard's arguments appears to emerge would be a particular time when mathematicians and mathematics educators have not yet confronted their divergent meanings for the word "mathematics". Each cohort, as independent subjects with their own history, consciousness, and knowledge has independently constructed their own meaning for this word. Sfard's arguments appear exactly at the time when these opposing meanings could be fruitfully and vigorously explored and debated to eliminate the almost inevitable misconceptions arising from such autonomous positions behind held.

But, what about if mathematics educators' concept of mathematics gains its actual meaning already and only in relation to mathematicians' meaning of mathematics and vice versa? That is, if each meaning for mathematics was constituted exactly and only at the point of their differences? And the subjects mathematicians and mathematics educators (the consciousness, history and so on of each) are constituted not beforehand the meaning that each one assigns to "mathematics" but on the contrary, the subjects are constituted in their relation to the meanings of mathematics. From this point of view there cannot be a third and impartial position which, looking from above, could see the differences and establish fairly (above the two positions) the correct relation and alliance between the two. Instead, in principle, the differences and the constitution of the subjects (mathematicians and mathematics educators) was the outcome of their differences.

What is Written Here, "Mathematics"?

In Habermas's (1987) critique of Derrida we read:

> [For Derrida] writing makes what is said independent from the mind of the author, from the breath of the audience, as well as from the presence of the objects under discussion. The medium of writing lends the text a stony autonomy in relation to all living context. It extinguishes the concrete connections with individual subjects and determines situations, and yet the text still retains its readability. Writing guarantees that a text can always repeatedly be read in arbitrarily changing contexts. What fascinates Derrida is this thought of absolute readability. (p. 166)

Setting aside the fact that Habermas appears to overlook the possibility that Derrida's project attempts to affect the meaning of "be said", he is correct in affirming that Derrida insists on the necessity for the concept of writing not to be attached to an individual subject or any determined situation. This very motive appears constantly in Derrida and is the very theme developed in "Signature Event Context":

> ...a written sign carries with it a force that breaks with its context, that is, with the collectivity of presences organizing the moment of its inscription. This breaking force [force de rupture] is not an accidental predicate but the very structure of the written text. (Derrida, 1988, p. 9)

Notably, Zizek, however, interprets Derrida completely to the contrary of what Habermas says. While Habermas critiques Derrida for his ideas of writing—for it "extinguishes the concrete connections with individual subjects [what one says or intends to say, to use Zizek's words] and determined situations, and yet the text still retains its readability [effectively written]". Zizek, on the other hand, suggests that, in Derrida's work, it would be possible to measure the coincidence or not of what one intends to say and what is effectively said. That is for him the very distinction between Derrida and Lacan; in Derrida, he states that the text is tied:

> ...every text, however metaphysical, always produces gaps which

> announce breaches in the metaphysical circle: the points at which textual process subverts what its "author" intended to say (Zizek, 1989, p. 154).

Indeed, if one considers that Derrida is constantly reminding us that "Writing is read; it is not the site, 'in the last instance,' of a hermeneutic deciphering, the decoding of a meaning or truth" (Derrida, 1988, p. 21) one can see that Habermas's point is much closer to Derrida than Zizek's. Consequently, one can posit that the conclusion which Zizek arrives at regarding the distinction between Lacan and Derrida is incorrect:

> In Seminar XI he [Lacan] begins one of his sentences: 'But this is precisely what I want to say and what I am saying – because what I want to say is what I am saying…' In a post-structuralist reading, such phrases prove that Lacan still wants to retain the position of Master: 'saying what I want to say' lays claim to a coincidence between what we intend to say and what we are effectively saying—is not this coincidence which defines the illusion of the Master? Is Lacan not proceeding as if his own text is exempt from the gap between what is said and what he intended to say? Is he not claiming that he can dominate the signifying effect of his text? In Lacanian perspective it is, on the contrary, precisely such 'impossible' utterance—utterance following the logic of the paradox 'I am lying'—which keep the fundamental gap of the signifying process open and in this way prevent us from assuming a metalanguage position. (Zizek, 1989, p. 156)

In fact, contrary to Zizek's analysis, Derrida's line of thought does not challenge "the coincidence between what we intend to say and what we are effectively saying" but as Habermas realised, Derrida takes it much further:

> Inasmuch as Derrida replaces grammar as the science of language with grammatology as the science of writing, he intends to make the basic insight of structuralism even more pointed. (Habermas, 1987, p. 166)

Derrida (1976) brought attention to this point himself in "Of

Grammatology":

> It has sometimes been contested that speech clothed thought. Husserl, Saussure, Lavelle have all questioned it. But has it ever been doubted that writing was the clothing of speech? For Saussure it is even a garment of perversion and debauchery, a dress of corruption and disguise, a festival mask that must be exorcised, that is to say warded off, by the good word: 'Writing veils the appearance of language; it is not a guise for language but a disguise' (p. 51) [p.30] (Derrida, 1976, p. 35)

It is in the very sense of an "impossible" utterance, as suggested by Zizek when referring to Lacan, that one can recall Derrida's "absolute readability" as claimed by Habermas. In Derrida, the "impossible" utterance would be translated into "what one is reading is not actually what is written", or in paraphrasing Zizek: "the text is lying". That interpretation is impossible for Derrida. If Lacan claims that "what I want to say is what I am saying", Derrida would retort: "what one reads is what is written".

One should recall that, independently of Derrida's concept of writing, the common perception of writing actually allows for a notion of absolute readability. It is the common notion of writing that says that, although different meanings may be granted for a text, the printed text itself never deviates. In other words, if one person reads a text and gives "it" to another person to read, both can be free to disagree about its meaning but they will be in no doubt that it is absolutely the same text. This line of reasoning follows Saussure's distinction between signified and signifier, as I have mentioned in the previous sections.

It was in this sense that the two mathematics; the mathematicians' mathematics and the mathematics educators' mathematics, although they may differ in terms of meaning, are always identical in phonetic terms. That is, this same force, when it frees itself from the concept of the word "mathematics," ultimately depends upon the idea of the "shape" of a word or the phonetic component of that word—Saussure's concept of "signifier". It is in this very locus that, I suggest, there remains a pure, a priori, and firm core of mathematics for mathematics educators.

Following Habermas's tenet in the notion of absolute readability one could ask: "Does not Derrida's concept of writing imply that mathematics is always mathematics?" I shall answer this by saying both "Yes" and "No". Here Zizek's position regarding Lacan is very welcome and, as I have suggested, in Derrida it is more pointed than Lacan's "I am saying what I want to say'. If what I am reading (interpreting and understanding) in a book is exactly the content of the book (absolute readability for Derrida) it means that if I read it again tomorrow and read "there" another "meaning", as this "new" meaning it is always what is indeed written in the book (absolute readability in Derrida's sense) and it implies that the same book in the most extreme, radical, and literal way possible is always really another book. It is as if the book with its printed mark and identifiers was always literally "re-written". What Zizek has postulated about Lacan rings even more true with Derrida. That is, the fact that for Derrida writing is read, or what Habermas calls Derrida's "absolute readability", maintains "the fundamental gap of the signifying process open [and] in this way [it] prevent[s] us from assuming a metalanguage position". Derrida's formula here is that "fundamentally nothing escapes the movement of the signifier".

Using Derridean concepts, since the difference between the "two" mathematics is readable, it is also therefore written. It is not the interpretation of writing but it is the writing itself. In other words, as long as one continues reading it, it is indeed written there. However this difference does not imply the impossibility of equality. One may argue that both "mathematics" are identical but it depends a priori on the existence of two separate connotations rather than just one. The very point is that the difference is always situated prior to any notion of equality.

The simple implication which one can observe here is that the unity of mathematics' meaning never existed. Whenever one writes "it" again, one writes another thing. Whenever one reads "it" again, one reads another thing. At this point let me bring again Lins's statement that " 'mathematics' in 'mathematics education' does not need to mean a reference to specific topics" (Lins, 2006, p. 1/68). If one can read a non-reference it is because it is written.

It is at this point that any commensurability a priori between the mathematics of mathematicians and mathematics educators' mathematics must be totally eradicated. The mathematics of mathematics educators' is not more or less cultural or social a priori than the mathematics of mathematicians. Whilst the "logic of the word" highlights certain commensurability; the sameness as the principal of repetition; the sameness in the different occurrences of the same word, Derrida introduces a new logic for writing; the logic of dissemination. For Derrida, writing is never the representation of a truthful event located in time and space or the code of true meaning, and this capability of not being anchored to a meaning, an event or whatever, is the essence of dissemination. In fact, it is against the logic of a "word" and all that it presupposes (form, shape, unit, noun, appellations etc.) that Derrida writes:

> Nonphonetic writing breaks the noun apart ... The noun and the word, those unities of the breath and concept, are effaced within pure writing (Derrida, 1976, p. 26)

The meaning of "mathematics" and the word "mathematics" (even the quotation marks are unhelpful here) are not crucially different for Derrida, since meaning is always situated in the position of a signifier. The meaning "mathematics" has no value before or after its "written form"—either mathematics or mathematics (education). However, if, on the one hand, Derrida's concept of writing may accomplish the very object of mathematics education's critiques of mathematicians, on the other hand it also challenges the purity of a concept of mathematics education and therefore the rationale of a project enclosed within the supposed boundaries of a pure domain of research. It is this paradox that, perhaps, persuades the reader of the appropriateness of the scope and consequences of this paper.

No community, no science, no research domain, no political position, whether in their ontological or epistemological boundaries buried in cultural spaces, dominates "mathematics" if this is to be understood as writing using the Derridean concept of writing. For the same reason, Derrida's concept of writing does not attempt to correct those ontological unities and discover another essential meaning to mathematics education. By contrast, it attempts to disable all of them before or after they are written. The meaning of "mathematics" is neither affixed after nor before the text "mathematics (education)". That is the simple statement which Derrida encourages us to accept.

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Building a Case for Understanding Relational Dimensions in Mathematics Classrooms

Dan Battey & Luis Leyva
Rutgers, The State University of New Jersey, USA

Research on mathematics instruction often focuses on issues of problem solving, explanations, and discussions. However, relational aspects of classrooms may be just as important, particularly in understanding the success or failure of underserved students. The paper briefly looks across four studies that examine dimensions of relational interactions in mathematics. This research details a case study, builds a framework for understanding relational dimensions of mathematics classrooms, and uses regression to study links with student achievement. The work builds the case that relational dimensions of classrooms are critical in understanding student learning and engagement with mathematics.

In examining instruction in urban schools, mathematics education research often focuses on dimensions such as problem solving, mathematical discussion, student explanations, and cognitive depth. However, this focus on content instruction may overlook relational dimensions that impact learning. We think there is much to gain in attending to this as a mechanism that impacts student learning in mathematics. As a field, we may be underestimating the impact of instruction on student learning when not considering mechanisms such as relational dimensions in the mathematics classroom (Lubienski, 2002). While quality content instruction is necessary, it may be insufficient in generating the kinds of student understandings that the field aims for, particularly for underserved students. We begin by reviewing literature on relational dimensions of mathematics classrooms and then briefly discuss four related studies building a case that relationships are a critical element for the field to examine in more detail.

Teacher-Student Relationships in Mathematics

Scholars have approached the study of teacher-student relationships in mathematics in a number of ways. For example, Hackenberg (2010) builds off of Nel Nodding's (1984) work on an ethic of care to understand how teachers can form caring relationships with students, both with respect to their mathematical ideas and their emotions. In working with four students, she shows how teachers can build caring relations through mathematical support. Bartell (2011) adds to this work by conceptualizing "caring with awareness", which explicitly addresses cultural and racial aspects of relationships. In theorizing caring with awareness, she considers that to care for many Latino and African American students, who are often marginalized in the US educational system, teachers must take on student perspectives. In doing so, teachers take on explicit stances that challenge stereotypes about who is mathematically competent. Across this work, caring for student contributions, emotions, and cultural backgrounds are central.

Cultural backgrounds are also critical in attending to behavior. African American and Latino students endure more conflictual relationships and behavioral discipline than their white peers (Jerome, Hamre, & Pianta, 2009). In general, research shows that teachers overreact when addressing behaviors of African American and Latinos, placing a strong emphasis on physical control (Monroe, 2005). Monroe (2005) determined that teachers with a limited understanding of students' cultures tended to issue severe behavioral sanctions. Even when controlling for economic status, Gregory and Weinstein (2008) found that teachers interacted more negatively with African American and Latino students by issuing sanctions. How teachers address student behaviors is critical then in teacher-student relationships.

Boaler and Empson highlight the importance of attending to student contributions in mathematics. Boaler (2006) documented teachers that highlight the intellectual value of student contributions through explicit statements, questioning, or asking students to share their mathematical thinking. Through these teacher actions, students were framed as competent mathematically, disrupting the low status and fixed notions of ability that are common in mathematics. Empson's (2003) research also speaks to the importance in framing students' contributions as having value. She found that for the lowest achieving mathematics learners, positive interactions with the teachers served to enhance their mathematical identity and performance. Both of these scholars highlight how acknowledging the mathematical ideas of learners framed students as mathematically competent.

Gorgorió and de Abreu use social representations to understand teacher-student interactions (Gorgorió & de Abreu, 2009). In this work, they highlight the way in teachers dismiss different ways of thinking mathematically or misinterpret various cultural representations within mathematics. Similar to Bartell, this work on social representations highlights cultural aspects of mathematics classrooms that impact student engagement. In contrast, Civil (2007) focuses on classrooms where teachers value the cultural knowledge of parents and students. In blurring the boundary between the school and home, teachers valued the everyday practices that students brought to mathematics. These scholars emphasize cultural perspectives as central to understanding students' representations and knowledge.

Finally, Setati and Adler draw attention to the overlap between culture and language (Setati, Adler, Reed, & Bapoo, 2002). In their research, code-switching serves as both a tool to move between informal and formal talk as well as across mathematical discourses. Relatedly, Moschkovich (2002) highlights the importance of language in bridging relationships with students and how language can construct mathematical competence. She discusses the importance of giving students access to mathematical discourse, defined more broadly than vocabulary. These researchers highlight the complexity in supporting students' language practices, their cultural nature, and implications for mathematics learning.

This work raises the complexities involved in teacher-student relationships within mathematics including dimensions such as acknowledging student contributions, providing emotional support, highlighting student competence, and attending to linguistic and cultural resources. However, this work does not look across these relational dimensions. The studies discussed below extend this research by examining relational interactions across dimensions.

Study 1: A Case Study of Relational Interactions

This case study of a 4th grade classroom examined mathematics instructional quality and teacher-student relational interactions. We define relational interactions as a communicative action or episode between teachers and students, occurring through verbal and non-verbal behavior that conveys meaning (Battey, 2013). In considering "good teaching" in mathematics, scholars usually refer to teacher content knowledge and instruction that promotes understanding (Wilson, Cooney, & Stinson, 2005). However, these two elements are often not as prevalent in urban contexts, a space where high percentages of African American and Latino students are educated (Lubienski, 2002). Using video, field notes, and an interview, the case study found an urban teacher that engaged students in substantive mathematics, but a number of relational interactions seemed to disrupt access to mathematics. Across two classroom lessons, the study found four dimensions of relational interactions that mediated access to mathematics: addressing behavior, framing mathematics ability, acknowledging student contributions, and attending to culture and language (Battey, 2013). The two research questions were: 1) How does an elementary teacher exhibit mathematics knowledge and instructional practices in her classroom? 2) How do relational interactions shape access to mathematics for African American and Latino students?

The teacher, Ms. Spencer, displayed quality content instruction across the lessons. She moved students to more sophisticated numbers and encouraged generalizations using a variety of pedagogical strategies. Sometimes her relational interactions were aligned with these mathematical goals. Her positive interactions encouraged student strategy use, affirmed students' mathematics ability, and connected the mathematics to familiar contexts. On the other hand, a number of interactions contrasted with her mathematical goals. In these negative episodes, Ms. Spencer isolated students, questioned students' ability, ignored student thinking, used sarcasm, withheld instruction, and focused on language issues to the detriment of mathematics learning. What is particularly interesting in this case is that relational interactions were mostly orthogonal to the quality of content instruction.

This case study speaks to the fact that our conceptions of "good" mathematics teaching are often too narrow, particularly for underserved students. What seems missing in Ms. Spencer's interactions and comments is a deeper understanding of why these episodes take place. Her relational interactions enable or restrict access to quality mathematics, regardless of the form of instruction provided. When she praises students, affirms student ability, and encourages them to go deeper into the mathematics, we see a teacher who is using relationships to support the practices we so often seek in urban mathematics classrooms. When Ms. Spencer uses sarcasm, withholds instruction, treats students as invisible, and misses student contributions, we see a teacher who may be holding deficit views of students. But research in mathematics education does not have a framework to bring more understanding across relational dimensions.

Study 2: A Framework for Relational Interactions

The second study extends the first by developing a framework for relational interactions across seven 4th and 5th grade classrooms in one urban school. To further this line of research, the study aimed to look at the association between relationship interactions and instruction across classrooms. The two research questions were: 1) What are the types, frequency and intensity of relational interactions in elementary mathematics classrooms? and 2) How do relational interactions relate to the quality of mathematics instruction?

A fifth dimension of relational interactions was identified in this study, setting the emotional tone (Battey & Neal, resubmitted). This dimension builds on Hackenberg's (2010) work around attending to students' emotions. The interactions that constituted this additional dimension spoke to broad messages that teachers passed on to students, sometimes centering on the need to struggle through mathematics or affirming multiple ways to practice mathematics. Additionally, interactions such as framing mathematics ability and attending to culture and language, while infrequent, sent intense messages to students. These infrequent dimensions are ways in which teachers reproduce or challenge broad discourses about who can or cannot engage mathematics.

Addressing behavior and acknowledging student contributions were more frequently occurring dimensions though the former was more negative and the latter positive (see Table 1). The frequency

Table 1
Relational Interactions: Dimension, Frequency, and Intensity

DIMENSION	TEACHER							ALL
	Ms. B	Mr. J	Mr. D	Mr. L	Mr. G	Ms. S	Mr. T	
Behavior								
Positive	2(1.0)[a]	0	0	2(1.0)	0	0	1(1.)	5(1.0)
Negative	1(1.0)	0	3(2.7)	23(1.8)	0	6(1.7)	18(1.9)	51(1.9)
Ability								
Positive	0	0	0	2(1.0)	6(2.5)	2(2.5)	2(1.5)	12(2.1)
Negative	2(3.0)	0	2 (2.0)	0	0	2(2.5)	1 (2.0)	7(2.4)
Contributions								
Positive	5(1.6)	4(1.3)	6(2.0)	3(1.7)	8(2.1)	18(1.5)	11(1.2)	55(1.6)
Negative	11(2.5)	3(1.3)	3(2.3)	2(2.0)	0	19(2.2)	1(1.0)	39(2.2)
Culture								
Positive	0	0	0	1(2.0)	4(2.3)	2(1.0)	0	7(1.9)
Negative	0	0	0	1(1.0)	0	1(3.0)	0	2(2.0)
Tone								
Positive	0	3(1.0)	0	0	3(1.7)	0	0	6(1.3)
Negative	0	0	0	0	0	4(1.0)	0	4(1.0)
Total								
Positive	7(1.4)	7(1.1)	6(2.0)	8(1.4)	21(2.2)	22(1.5)	14(1.2)	85(1.7)
Negative	14(2.6)	3(1.3)	8(2.4)	26(1.8)	0	32(2.0)	20(1.9)	103(2.0)
Interactions/ minute	0.43	0.31	0.44	0.71	0.75	0.55	0.74	0.56

[a] *Average intensity*

and intensity of the interactions differed significantly across the classrooms, as did the quality of instruction. Rates of relational interactions varied between .30-.44 per minute in classrooms with more traditional instruction to around .70 per minute in classrooms with more reform-oriented instruction. This means almost twice as many interactions occurred in more reform-oriented classrooms, which are largely based on discussions. This increased rate provided more opportunity to acknowledge student contributions – a dimension that was more positive than negative. However, the increased rate of interactions sometimes meant that rates of negative interactions in these classrooms were higher as well. The implication is that while mathematics educators might try to increase reform-oriented content instruction in classrooms, this potentially could increase negative relational interactions as well.

The increase in interactions raises a critical issue of changing behavioral expectations as instructional norms shift in classrooms. When teachers take on more reform-oriented practices, as they were supported in doing during this study, teachers may struggle with communicating behavioral expectations for students. An often-overlooked issue in supporting teachers as they change instruction is how this change makes it necessary to communicate different behavioral expectations and therefore requires teachers to develop different strategies for managing students. While this may not seem like an issue for mathematics educators, if behavioral issues result in school discipline (see Gregory & Weinstein, 2008), and in turn removes a student from instruction, it heavily impacts mathematical access.

Interestingly, despite differences in instruction and rates of relational interactions, no relationship existed between the quality of relational interactions and instruction for these seven teachers. The finding that relational interactions do not necessarily parallel instructional quality raises the need for future research.

Study 3: Successful Relational Interactions

The conceptualization of relational interactions as within instruction allows for a better conceptualization of what constitutes high-quality mathematics instruction, particularly for urban African American and Latino students. To that end, this study identified teachers who succeeded with students based on high student performance, quality content instruction, and building strong relationships (Battey, Neal, Leyva, & Adams-Wiggins, under review). Using video data of seven 2nd and 3rd grade teachers, the study explored one research question: How do urban elementary teachers, who successfully support their students, engage in relational interactions within their mathematics classrooms? The study offers a key contribution to the literature by detailing strong teacher-student relationships in urban elementary mathematics classrooms within one district.

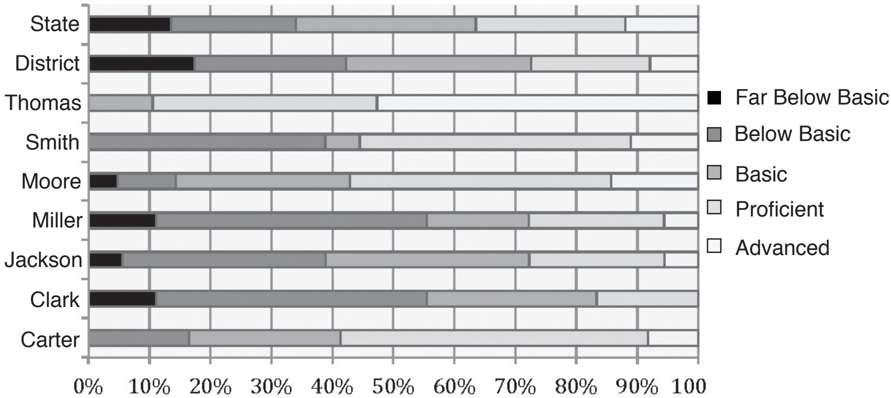

Figure 1: Achievement Levels of Classrooms on the CST

Quantitative data from state assessments and classroom video were used to select successful teachers. After selecting two teachers (see Thomas and Moore in Figure 1) based on achievement data, instructional quality, and positive relational interactions, the analysis detailed the relational interactions between these teachers and their students. Almost 90% of Thomas' students achieved at the proficient or advanced level on the state achievement test, with no students scoring at the two lowest levels. Her students performed statistically better than students in all of the other classrooms except for Moore's. Moore's had over 20% more students achieve proficiency than the state and they performed statistically better than all but Carter's students. All of the teachers scored high on instructional quality, but only Thomas, Moore and Jackson had overall positive relational interactions with students. Therefore we detailed Thomas and Moore's relational interactions.

A number of patterns were evident in unpacking Ms. Moore and Ms. Thomas' relational interactions in the mathematics classroom. Both teachers made their expectations clear for students' mathematical engagement with peers. Additionally, Ms. Thomas consistently recognized positive models of behavior and neither teacher escalated episodes when noting students' off-task behavior. Both teachers pushed beyond answers, requiring justification and consistently pressing for more complete explanations. Ms. Moore was particularly adept at extracting the important mathematics even when students had incorrect answers. Ms. Moore and Ms. Thomas framed students as competent in contrast to stereotypes of the mathematical abilities of African American and Latino students. Ms. Moore also actively incorporated student-generated mathematics problems, giving students more ownership of the content. Ms. Thomas, on the other hand, openly showed vulnerability with her students during instruction and praised students for their thinking and fluency with the mathematics.

Consistent with study 2, some of the teachers in this study had an extensive negative focus on behavior. However, Moore and Thomas were balanced in addressing student behavior. In fact, both teachers noted positive behavior more than misbehavior. Additionally, they set clear expectations for how students should engage with each other in the mathematics classroom. This is noteworthy given that this study was performed in the last quarter of the school year and teachers were still restating their expectations to students.

Table 2
Frequencies and Average Intensity of Relational Interactions

TEACHER	DIMENSION									
	Behavior		Ability		Contribution		Culture		Tone	
	Pos.	Neg.	Pos.	Neg.	Pos.	Neg.	Pos.	Neg.	Pos.	Neg
Moore	16(1.31)[a]	13(1.31)	2(1.5)	0	24(1.88)	3(1.33)	2(1.0)	0	9(1.56)	0
Thomas	7(.143)	4(1.25)	7(1.)	0	35(1.4)	4(1.25)	0	0	3(1.67)	0

[a] *Average Intensity*

Another connection between this study and the extant literature are the caring mathematical relationships that Bartell (2011) conceptualized. The lessons did not provide clear examples of how either teacher drew on culture or language in instruction besides Ms. Moore drawing on student-generated mathematics problems. It is possible that if more lessons had been videotaped, more examples of this dimension may become apparent. However, both teachers shared power in the classroom and exhibited elements of care in terms of assigning competence to students that are typically negatively stereotyped mathematically. The design of this study does not allow the authors to say whether or not these were intentional in challenging racial narratives, but they certainly run counter to the deficit narratives that are so pervasive about mathematics achievement among students of color. The study offers a key contribution to the literature by detailing strong teacher-student relationships in urban elementary mathematics classrooms.

Study 4: Relational Interactions and Achievement

Study four examined how teacher-student relational interactions affect students' mathematics achievement. Analyzing the seven 2nd and 3rd grade teachers and their 137 students in study 3, this paper explored two research questions (Battey & Leyva, 2013): 1) How do relational interactions explain variance in student achievement in elementary mathematics classrooms? and 2) How do relational interactions differ based on sex and ethnicity?

We entered the relational interactions dimensions as independent variables with the state achievement test as the dependent variable into a linear regression. Across all of the students, the only significant relationship was Acknowledging Student Contributions ($F = 21.57$, $p < .01$). It explained 13.4% of the variance in students' mathematics test scores. Since only the third graders took the test the prior year, a linear regression analyzed third grade scores with the added predictor of prior achievement. Prior achievement accounted for 61.4% of the variance in students' subsequent scores. The only other variable that accounted for a significant part of the variance was setting the emotional tone, accounting for 12.6% of the variance in scores. The effect size for the model was high ($F = 63.63$, $p < .01$).

Teachers engaged in acknowledging student contributions at a higher rate for females than males across both grades ($p < .01$). This raises concerns about whether the mathematical contributions of Latino and African American boys were missed in classrooms. It is not surprising that attending to culture and language and framing mathematics ability did not produce any significant results. This was probably due to two issues: (a) the infrequency of these relational interactions and (b) tendencies to engage in these dimensions with the whole class rather than with individual, resulting in a lack of variance.

These findings point to the impact of teachers' roles on student achievement by way of attending to students' mathematical ideas. The results also speak to a needed reconceptualization of mathematics instruction as both an academic and social mechanism affecting equitable opportunities.

Conclusion

The approach of detailing specific relational interactions builds on prior work that looks at dimensions such as language, emotion, and culture within mathematics classrooms. From a case study showing how relational interactions can be orthogonal to content instruction to links with change in achievement, the need to examine relational dimensions of mathematics classrooms grows. We have been intentional in making a case, building a conceptual framework, using the framework to elaborate practice, and linking interactions with student achievement. In looking across relational dimensions, a focus on interactions holds potential in adding to our understanding of access to mathematics for different student populations. For one, looking at relational interactions for various student groups could create comparative contexts to see if teachers' attitudes about groups change the frequency and intensity of interactions. For instance, do African American and Latino students experience more negative relational interactions than their white peers? This might explain a piece of the puzzle in terms of classroom mechanisms that impact student learning. While this area of research is still developing, we think it provides a way to codify observable classroom behaviors to tease out the relationships that students are building with teachers and the mathematics.

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"Archaeology" of Measurement Knowledge: Implications for School Mathematics Learning

Arindam Bose and K. Subramaniam
Homi Bhabha Centre for Science Education, TIFR, Mumbai, INDIA

This paper explores measurement knowledge that middle-graders from low-income families gain from out-of-school contexts and the implications of such knowledge for classroom learning. Work and other out-of-school contexts entail rich and diverse "funds of knowledge" about measurement. Such knowledge includes conceptual elements which may be fragmented or hidden, but if unpacked (archaeology) can support classroom learning. The out-of-school measurement-related experiences have been analyzed to show the underlying conceptual constructions and their diversity in terms of measures, systems of units, and measurement tools. The paper discusses possible connections between classroom learning and specific aspects of out-of-school measurement knowledge using a characterization that marks such connection.

In the diverse contexts that comprise everyday living, the notion of measurement occurs frequently. It is used in diverse ways in workplaces, in economic exchange, and in homes. The topic of measurement is also a compulsory part of the school curriculum. Much of the literature on out-of-school measurement knowledge has explored the contours of measurement knowledge—diverse measurement tools, modes and units, ways such knowledge is acquired, and its difference from school mathematics. While such studies communicate a promise of reshaping school maths education based on what was known about out-of-school knowledge, there is still a lack of clarity about the implications of such studies for school learning. This paper unpacks the diversity of measurement knowledge embedded in work and other out-of-school contexts and also possessed by school children from an economically active urban low-income neighbourhood, dotted with a micro-enterprise economy in a South Asian developing world context. Many students in our study either participate in, or are aware of, the work-contexts where measurement plays a role. Our purpose is to inquire into the implications of such knowledge for school learning.

Archaeology of Embedded Mathematics

We have argued that many measurement tools used in everyday contexts, such as measuring scales and templates, have embedded in them mathematical ideas and elements which remain hidden even from those who frequently use such objects. Surprisingly, the mathematics textbooks or curricula do not require students to explicitly uncover the hidden or embedded mathematics from such objects. Such uncovering or unpacking of the underlying conceptual constructions is what we refer to as "archaeology" of embedded mathematics (Subramaniam, 2012). To begin with, unpacking of the embedded mathematics in measuring scales can be the starting point of an "archaeological" exploration, that can lead to learning about length measurement and its uses such as the notions of construction of new or sub-units, chunking, equi-partitioning, iterative covering, etc. Such archaeology can have an important role in supporting the mathematical learning of students who gather, as evidenced from our study, fragmented and obscure mathematical knowledge from their work-contexts. We discuss these connexions below.

Funds of Knowledge

Children in low-income conglomerations are often bound in social relationships and work practices from an early age and the broad features of their learning develop in their homes as well as in their surroundings. Households and surroundings contain resources of knowledge and cultural insights that anthropologists have termed as *funds of knowledge* (González, Moll, & Amanti, 2005). The "funds of knowledge" perspective brings to mathematics education research insights that are related to, but different from, the perspectives embedded in studies of "culture and mathematics". In contrast to restrictive and sometimes reified notions of "culture", the concept of "funds of knowledge" emphasizes the hybridity of cultures and the notion of "practice" as "what people do and what they say about what they do" (*ibid*, p. 40). *Funds of knowledge* are acknowledged to be broad and diverse, and embedded in networks of relationship. When they are not readily available within households, then they are drawn from community networks. This concept emphasises social inter-dependence. From this perspective, children are active participants, not passive by-standers.

We have used "funds of knowledge" as a guiding notion in analysing the work contexts that students are exposed to, and in illuminating the nature and extent of everyday mathematical knowledge available within the community of the classroom. We look at "funds of knowledge" as a resource pool that emerges from people's life experiences and is available to the members of the group, which could be households, communities, or neighbourhoods. In a situation where people frequently change jobs and look for better wages and possibilities, members of the household need to possess a wide range of complex knowledge and skills to cope and adapt with the changing circumstances and work contexts. Such a knowledge base becomes necessary to avoid reliance and dependence on experts or specialists, particularly in jobs that require maintenance of machines and equipment.

Children's participation in work, either within the household or in the neighbourhood, allows a closer integration with the social networks that generate funds of knowledge, and makes this knowledge present and available in the classroom. Educational philosophers, such as Gandhi, thought of productive work as central to education, and developed a vision of education centred around work. At an education conference in India in 1937, he argued that "the proposition of imparting the whole of education through the medium of trades (crafts) was not considered [in earlier days]. A trade (craft) was taught only from the standpoint of a trade (craft). We aim at *developing the intellect also with the aid of a trade or a handicraft* ... we may ... educate the children entirely through them" (National Council of Educational Research and Training (NCERT), 2007, p. 4, italics in original). In the present Indian context, this perspective has had an influence on the new National Curriculum Framework (NCF) (2005) which urges educators to draw on work experiences as a resource for learning. It points out that "productive work can become an effective pedagogic medium by connecting with life experiences of children; by allowing children from marginalised sections of society, having knowledge and skills related to work, to gain a definite edge; and by facilitating a growing appreciation of cumulative human experience, knowledge and theories by building rationally upon the contextual experiences" (NCERT, 2005, p. 6).

Measurement in the Out-of-School Context

Previous research on measurement within work-contexts or in other everyday settings was carried out alongside or within the research on out-of-school mathematics, with a particular focus on the alternative ways of thinking in different everyday contexts. Such research provided evidence of how mathematical ideas were developed and framed within work-contexts. These studies have highlighted the use of different measurement modes and units (e.g., Lave's study (1985) with Liberian tailors); mental estimation techniques markedly different from those learnt in school; extensive use of conventional mathematical concepts like congruence, symmetry, proportional reasoning, optimisation, and use of spatial visualisation (Millroy's study (1992) with South African carpenters in their everyday woodworking activities); multiplicative thinking in everyday work-contexts using proportions and inversion techniques, and use of scale-drawings which drew on measurement knowledge and proportional reasoning (study of Nunes, Schliemann, and Carraher (1993) with construction foremen); use of spatial visualisation, estimation skills and indigenous tools in Mukhopadhyay's work (2013) on "vernacular boat-making" in India. Saraswathi's study (1989) on agricultural labourers' measurement practices reported use of variety of measurement modes and units to describe the linear dimensions of routine objects used in everyday contexts. The units were standard (old British, metric) and non-standard (body parts, indigenous units). Linear dimensions often served as an object's identity and description. Estimation skills depended more on experience and mental measurement.

Most of the above studies have focused on participants' measurement knowledge in their singular work-contexts. We have not come across studies that looked at the varied contexts in the everyday settings that students from low socio-economic backgrounds are exposed to and the affordances of these settings for school learning about measurement. The implications of the above studies have led to a cumulative understanding of the skills, procedures, and strategies based on mathematical principles that are acquired in out-of-school work contexts. In this paper, we take a broader view of not only what our participants know or can do, but also what they have observed and are familiar with, even if the mathematical knowledge associated with these aspects is partial and fragmented. Our perspective is to explore what aspects can serve as starting points or building blocks for mathematical exploration in the classroom and unpacking the underlying mathematical concepts embedded in measurement practices. We are also interested in how mathematical learning can strengthen the understanding of measurement practices in real world contexts.

Measurement Learning as a School Curriculum Topic

Research on the teaching and learning of measurement as a school curriculum topic has been influenced greatly by the work of Piaget. Measurement refers to the quantification of an attribute of interest for purposes of comparison and for using in a calculation. Piaget stressed the key notions of conservation, transitivity, equi-partitioning, displacement, and iterative covering as underlying length measurement. Subsequent research has added the notion of accumulation of distance and additivity and the role of the origin on scales (Sarama & Clements, 2009). These ideas have also been extended to the learning of area and volume measurement. A look at textbooks prescribed by the central and state governments (followed by the vast majority of students in India) reveals that the dominant emphasis is on acquiring measurement skills and on knowledge of the international system of units for measurement (e.g., Maharashtra state mathematics textbooks for Grades 5, 6, 7 (Maharashtra Textbook Bureau, 2006). Conceptual issues are dealt with briefly under the rubrics of "use of non-standard units" and "need for standard units", before the treatment moves over wholly to the development of skills. These include familiarity with common measurement instruments, use of standard measurement procedures, interconverting between smaller and larger international units, and computing with units. Classroom teaching in the schools that formed part of the study revealed that there is even greater emphasis on paper and pencil computation skills with very little treatment of either conceptual matters or even of practical measurement.

We note that the curriculum and research agenda also need to include concepts that connect with and illuminate the diversity of measurement-related practices encountered in work and everyday contexts. It needs to focus on the idea that quantification is at the heart of measurement and quantification is achieved in different ways for different attributes and for different purposes. It needs to develop an appreciation of the difference between scientific measurement and measurement in the everyday world. This paper, therefore, argues for the inclusion of conceptual aspects that have so far not been included either in the curriculum or in the research on measurement learning. We argue that the diversity of measurement experiences in work contexts and everyday settings justifies inclusion of these aspects in the curriculum and that the knowledge that children bring into the classroom from out-of-school contexts supports learning of these ideas.

The Study

The large ethnographic study was conducted in a low-income neighbourhood in central Mumbai that has as vibrant economy household based micro-enterprises and small scale manufacturing units, which provide employment to the dense population living in the locality Even within a single class, we find students engaged in a variety of income-generating work both within house-holds and in the neighbourhood. Some common micro-enterprises that students participate in are embroidery, *zari* (needle work & sequin stitching), stitching and garment-making, making plastic bags, leather goods (bags, wallets, purses, shoes), dyeing, button-stitching, making of *rakhi* (decorative wrist bands) and stone-fixing work on ornaments. Recycling work is also a major occupation in this locality. Being an old and established settlement, it receives immigrants from different parts of India, mostly unskilled workers who find jobs in the workshops and some of them become apprentices in the small factories.

The study done over two and a half years time, which forms the setting for this paper, was conducted in several phases. Beginning with the ethnographic exploration and classroom observation of Grades 5 and 6 of two municipal corporation-run schools, the researcher did informal discussions with the students to understand the nature and extent of their everyday mathematical knowledge. It helped in knowing about the opportunities available to gather such knowledge and the extent of their involvement in economic activities. In the next phase, data was collected through semi-structured interviews of a representative sample of 31 students (one-third of the two Grade 6 classes) to understand their family-background, socio-economic status, parental occupations, productive work done at home/elsewhere, and student's involvement in them. The interview included questions aimed at understanding students' basic arithmetical knowledge. In the third phase, a sub-sample of 10 students and an additional 7 students from the same grade who volunteered, were interviewed to obtain a detailed understanding of their work-context knowledge. Interviews were transcribed and transcripts were coded at first and second levels to review what they indicated about the nature of work students are involved in, and what they know about aspects of the work. Students have been designated with the letter "E" or "U" (for English and Urdu medium school respectively) followed by a numerical subscript. The data used for this paper is drawn from the interviews for measurement aspects and from other phases of the study including informal visits to the house-holds, manufacturing units and discussions held with adults in these locations.

Characterising Out-of-School Measurement Experience

Features and nature of students' involvement in work practices shape what the contexts demand of the students and the richness of the knowledge that they acquire. Diversity of out-of-school settings gives rise to diverse experiences of measurement. A characterisation of such diverse knowledge is presented below from the point of view of portraying the inherent richness of concepts implicated in such experiences.

Comparison and Estimation in Measurement

Measurement in everyday contexts including work and domestic settings is different from measurement in the scientific world. Precision and accuracy are not as important as convenience. In many situations approximate measurements suffice. However, many of the processes and concepts that underlie measurement in the everyday world are centrally relevant to a conceptual understanding of measurement. Everyday measurement contexts present diverse and extensive use of comparison and estimation, and varied processes of quantification. Templates for length measurement are often used in tailoring and leatherwork. Tailoring work begins by cutting "*futta*" - stiff fabric or a canvas cut as per the dimension specifications of the garment to be stitched and made into a template called "*farma*". *Farma* of shirt-collars, pockets, of wallets and purses are commonly used. Comparison is done following a *farma* and its design and specifications for making new products. Wallet making often involves cutting square shaped leather pieces of dimension 4" × 4" referred to as "*desi*" often cut from a rexin piece of size 33" × 39". A "*desi*" is a template and also used as a measuring unit. Although "*desi*" is an area measure, it is used as a discrete length unit and often leather pieces are measured in terms of number of *desi*. For example, "*nau desi se ek foot banta hai*" [nine desis make a foot]" implying 9 desis cover and are equal to a square foot. What may seem improper or ambiguous use of measurement units is commonly used and understood in the community, possibly from the context.

In everyday contexts, estimation is a common measurement mode used with continuous as well as discrete attributes. Children like adult workers learn different kinds of estimation skills based on their work requirement. Work-contexts like *zari* (decorative sequin stitching on garments) entail frequent use of estimation in choosing the quantity of sequins to be stitched in a marked area or a specified design laid out on a garment-part. Similarly, in leather and tailoring work, estimation skill is used while deciding the amount of adhesive to be used or while choosing the needle of a certain grade (called number) and amount and types of threads for stitching. "*Chindhi*" (garment recycling) work uses both estimation and visual comparison skills while sorting. Cloth pieces of similar size are sorted and collected together and the weight of the collection is estimated. Other work like textile printing requires estimating the lengths of cloth pieces on which block printing is done and choosing a suitable "stopper" (i.e., printing block) whose dimensions are known to the workers. During the interview with U23 (engaged in textile printing work), he gave detailed explanation about the estimation of the quantity of colour required in printing designs on cloth-pieces of different dimensions. For example, he said in simple designs, one kg colour is sufficient to print the design on 2000 small cloth-pieces. The researcher observed that some students like U23 had a strong estimation sense and were skilled in estimating the dimensions of different objects lying around. Estimates of quality are also a part of some work contexts, although these are rarely quantified. An exception is the practice of "grading" in plastic recycling work in which visual estimation and tactile senses are used to designate numbers to plastic wastes based on their quality.

Quantification and Construction of Measurement Units

All measurement depends on the use of measurement units. In school learning, children largely encounter standard units that are pre-given in the form of measuring instruments (tapes, weights, etc.). The choice of a unit and the construction of a convenient unit are the first steps towards quantification of an attribute, and are important aspects of the concept of measurement. In the classroom, these steps are rarely emphasised. In many classrooms, they may at best be explained verbally. However, there are several out-of-school contexts where children encounter construction of a unit and other abstract notions embedded in measurement processes.

Use of Body Parts in Measurement

The use of the body for purposes of length measurement using hand-spans, finger bands or finger widths is commonly practised. In tailoring, "finger band" (phalanx) and "finger width" are commonly used to estimate length and length intervals. E6 who regularly visits his father's button-stitching workshop and also manages its running at times, mentioned the use of finger bands to quantify and measure the distance between every two buttons – about four-seven fingers width distance is maintained between them. E6 knew that one “inch” is roughly equal to one “finger band” length.

Equi-partitioning of Units

Construction of sub-units from bigger units by equi-partitioning is a common feature in work-contexts, for example, convenient weight “templates” for small weights (50, 100 or 250g). Construction of convenient units or templates derived from standard units is a conceptually rich activity, since it may involve partitioning, combining or otherwise manipulating a given standard measure. It is a step beyond using ready made measuring instruments that are pre-encoded with standard units, in the direction of understanding measurement conceptually rather than learning it merely as a skill.

Iteration and Discrete Quantification

Most students were familiar with artefacts like measuring tapes and their iterative use in quantifying a length measure. Some were also familiar with folding of rope to make smaller lengths using equi-partitioning. Students also knew about templates (*farma*) and their use in the iterative covering of an area, for example for carving out smaller pieces of rexin from a bigger piece and to quantify it. Similarly, discrete quantification is also common in work-contexts, viz., the garment-sizes marked with a letter or a number. Although most adults and many children are familiar with these sizes, whether and how these numbers are obtained through measurement is not clear to most people. Students in our study interpreted these numbers as unrelated to any units like inch or centimetre, and as merely indicating increasing sizes. Only some tailors were aware that this indicates the person’s chest measurement (not chest measurement of the garment, which is larger) in inches. Here we have an instance of a measure familiar from experience, but whose origin in quantification is obscure.

Diversity of Objects, Measurement Instruments, and Units

Students are familiar with and handle diverse objects in a range of contexts with a variety of measurement units and tools. Length as a salient attribute of an object is measured in unary as well as in multiple dimensions. As a unary dimension, length may refer to length or distance, for example, length of a strap sewn on a bag, distance between two buttons, or the depth of a pouch or a bag. Sometimes area measures are indicated by specifying two length dimensions, as for example, when a rectangular textile printing frame is indicated by specifying the length of its sides (16"×12") or when different sizes of rectangular plastic packets are given by their dimensions "*satrah paanch*" (seventeen by five), "*pandrah dus*" (fifteen by ten). Here the underlying connection between length of sides and area of a rectangle is implicit.

> Volume is commonly measured using both standard and informal units. Volume measures are often interchangeably used with weight measures. The word "kilo" commonly means "kilogram" and is a unit of weight. However "kilo" is often used as a synonym for "litre", a unit of volume. For example, E_8 and U_{23} referred to kilos of milk and colour used in everyday shopping and textile printing work respectively, although they actually meant "litres". Another common practice is to measure some quantities by volume instead of weight; for instance, shops sell *mutthi* or fistful of tea powder and grocery items. *Mutthi* is also a unit used in measuring sequins for *zari* work apart from other weight measures.

In micro and small manufacturing units and in everyday contexts, students encounter a diversity of measuring instruments in the work-contexts. Weight measurement, for example, is done with the help of spring balances, two-pan balances of various designs, beam balances, electronic single pan balances, and platform weighing machines for large weights. Besides the use of tapes marked in both inches and centimeters for length measurement, shops and workplaces use steel rulers, which may contain other kinds of markings.

Shops selling cloth use steel meter scales with usually with markings for every 5 or 10 cms. Steel rulers often contain binary divisions of the inch up to 1/32 of an inch. Volume measures used to measure grain, oil or milk come in a variety of shapes and sizes. Often such volume measuring instruments are not properly calibrated or marked. There are diverse measurement units in practice that are both standard and non-standard, scientific and indigenous units (international, old Indian and British units) and known to the students. In most measurement practices (weight, length or volume), the underlying mathematical constructions remain implicit and disconnected between practices.

Opaque Quantification, Fragmented Knowledge

Students' familiarity with diverse measurement modes and instruments do not necessarily translate into sound knowledge, rather the knowledge remains fragmented and their understanding unclear. Although most students were familiar with the measuring tapes, they were unclear about the meaning and construction of the markings on the tape. In some instances, even if the measurement is fully quantified, the quantification remains opaque, and the measurement remains critically dependent on the integrity of the artefact. For example, as we noticed, some plastic scales that students were using perhaps were not marked with proper calibration. Understanding the construction behind a measuring scale, its meanings, and inter-connections between the different markings was not required. What has become important now in the school curriculum is to learn to use the scale and be able to measure a length. However, in this study we came across students (viz., U23, E6) who did not know the connection between inch and cm but still had a fair estimation of how much distance both signify. Archaeology of concepts can connect such skills for better learning.

In everyday contexts, ways of quantification are diverse. It is important to make sense of the quantified attributes. We argue that by drawing on students' familiarity with the range of objects and attributes that are quantified, students can explore questions such as what is common and what is different in how we quantify different attributes? How is an abstract attribute like monetary (exchange) value quantified? How do we quantify different aspects of labour such as time, effort and expertise? Such questions are important to build a holistic understanding of the measurement concept that students get to handle in different domains of their lives and in different manner.

Implications for Classroom Learning

We argue that archaeological exploration resists the processes of demathematization as well and stresses on the comprehension of the hidden underlying concepts. Such explorations therefore have strong potential to become effective pedagogic modes. Generalised forms of knowledge are neither about abstraction without the concrete content, nor is it about mere induction from a number of instances. Rather, generalisation is all about arriving at or holding an idea or a construct that can illuminate and be applicable in diverse instances. Valuing generalizability as an outcome of school learning in fact places greater importance to the diversity of out-of-school experiences, for such diversity actually creates contexts for school learning. From this standpoint, we understand that mathematical aspects are present in the work-contexts as hybridized and opaque embeddings and it would not be correct to look at such practices as reflecting mathematical thinking and understanding. At the same time, we argue that it would be fallicious to look for elements of school learning in a particular work-context or to expect school mathematics to illuminate such similar practices. We claim that formal mathematical learning can illuminate the diversity of practices as a whole and strengthen the understanding, not the practice.

A second aspect of out-of-school knowledge that makes for potentially powerful connections with school learning is the fact that artefacts and practices from everyday settings represent a sedimented and embodied form of mathematics. The measuring tape embodies the processes of unit construction, unit iteration and counting and partitioning of units into sub-units. These processes are however hidden from view and are opaque. The redundant inclusion of a second system of units in the form of inches and feet on the measuring tape incorporates a part of historical reality, and highlights the arbitrariness of the choice of the basic unit of length. The purpose of such embodiment is precisely to make the mathematical thought and processes behind the construction of the measuring scale unnecessary, and to reduce the practice of measurement to the simple act of reading off the scale. As long as we treat the learning of measurement as merely the learning of a skill, unpacking the mathematical ideas that are embodied in artefacts will remain unnecessary. However, if we view the learning of measurement as conceptual understanding, then such material artefacts present an opportunity for unpacking the mathematical constructions sedimented in them (Subramaniam, 2012). Such "archaeology" may have an important place in providing opportunities to learn powerful mathematics that illuminates the diverse aspects of everyday experience. An approach to the teaching and learning of measurement that aims to connect out-of-school knowledge with school learning will hence need to draw on the implicit as well as the explicit conceptual constructions that underlie measurement experiences in the real world.

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Education for Whom? Word Problems as Carriers of Cultural Values

Anita Bright, Ph.D.
Portland State University

Is the mathematics presented in textbooks, trade books and standardized tests neutral? Drawing from critical theory and feminist epistemologies, the purpose of this research is to examine mathematics curricular material through two questions: "What is valued?" and "Knowledge for whom?" Findings indicate that mathematics texts contain multiple examples of problems that reify hegemony, the exploitation of people and a marked disregard for the environment. This paper includes ways mathematics educators can reconceptualize mathematics texts as inextricably linked to cultural reproduction and furthermore, to use these insights to build ways that mathematics educators can disrupt the current narratives and replace them with more equitable, inclusive, sustainable, and critical perspectives.

Introduction

Is the mathematics presented in textbooks, trade books and standardized tests neutral? For many K-12 educators, the answer is obvious: Of course; it's just numbers, and mathematics texts (textbooks, curricular materials and standardized assessments) are totally objective. However, what this knee-jerk response may fail to consider is the rich complexity and contextualization that mathematics texts carry (Bright & Wong, 2009; Gutstein, 2006; Boaler, 2009; Moses & Cobb, 2001). Although it's entirely possible that the contexts presented in mathematics texts are purposefully selected to convey a particular frame, I posit that the field of mathematics educational materials is simply part of a more insidious, unproblematized facet of institutionalized hegemonic educational practices. Speaking to this possibility (if not probability), Greer and Mukhopadhyay (2012) state, "mathematics and mathematics education are implicated in various forms of interpersonal dominance and in ideological struggles" (p. 229).

With the exception of vanguard educators like Greer & Mukhopadhyay (2012), Gutstein (2006), Boaler (2009), and Ball, Gofney & Bass (2005), few researchers have focused on mathematics as a carrier or transmitter of hegemony. Framed around the questions, "What is valued?" and "Knowledge for whom?" the purpose of this research is to highlight the ways mathematics educators can conceptualize mathematics texts as inextricably linked to cultural reproduction (Bourdieu, 1986), and use these insights to build ways to disrupt the current narratives of inequity and the privileging of particularly narrow perspectives in mathematics education and replace them with more equitable, inclusive and critical perspectives (Freire, 1982).

The mathematics educators described in this research critically analyzed (Kubota, 2004) mathematics items –word problems—selected from their classroom mathematics materials. They practiced uncovering the ways mathematics education is decidedly not neutral, but is instead politically and socially situated within a particular agenda. Using these new perspectives to examine this corpus, the educators were surprised to unearth hundreds of hegemonic examples. They plan to use their new insights to actively disrupt the hegemonic narratives and, with their students, co-create counternarratives intended to empower the learners.

Theoretical Framework

Informed by critical theory, this work "recognizes power—that seeks in its analyses to plumb the archaeology of taken-for-granted perspectives to understand how unjust and oppressive social conditions came to be reified as historical "givens"" (Cannella & Lincoln, 2012, p. 105). This term, "givens," serves well in the context of this research, as the use of this term in mathematics traditionally means "known." By employing critical theory, the intent of this work is to scratch away at these givens—particularly the most insidious examples in the canon of mathematics education— and cast light into what may have been not only the intentions of the original authors in invoking these givens, but also in recognizing that the most insidious forms of hegemony are those that are so far below the surface they may be considered *unintentional* by the authors. Further, this work seeks to reframe these assumptions in ways that may be more emancipatory for all K-12 mathematics students in the U.S.—not just those already enjoying various forms of privilege.

Building upon these ideas, this work also draws from feminist epistemology, in that the situated-ness of the knowledge of mathematics signals a masculinity that is often unnamed and unchallenged. Invoking Anzaldua's (2002) concept of the *nepantlera*, which she describes as those who "facilitate passage between worlds" and who engage in thinking that seeks to "question old ideas and beliefs, acquire new perspectives, change worldviews, and shift from one world to another" (p. 1), this work frames the work of participants as active and agentic, questioning and challenging.

Methods

In an effort to provide an agentic opportunity for educators to begin to conceptualize (and then re-formulate) mathematics texts through a critical lens, this research is decidedly qualitative, and draws from the lived experiences and perceptions of the participants. This research is centered in the collaborative work of 58 graduate students (teachers and future teachers) who agreed to participate in this exploration. The participants were enrolled in one of 3 sections of a graduate mathematics methods course for educators, which focused heavily on a critical implementation of pedagogical content knowledge in mathematics.

To this end, participants were invited to first participate in an interactive critical discourse analysis experience (described below), and then identify three "problematic" examples of word problems from their own mathematics materials and generate a written analysis of each. Participants were then invited to participate in focus-group discussions to both share their insights and deepen their own understandings by considering the findings and perspectives of other participants. Finally, participants will take their word problems to their students, and generate contexts and situations that maintain the mathematics content while focusing on more socially just situations. Grounded theory (Glaser & Strauss, 1967) was used to generate codes and themes.

To begin, the participants took part in a collaborative analysis of a mathematics text, a picture book titled, "The Dot and the Line:

A Romance in Lower Mathematics" (Juster, 1963). The text, featuring 3 non-human characters, is described as, "a supremely witty love story with a twist that reveals profound truths about relationships—both human and mathematical—sure to tickle lovers of all ages" (Amazon.com, 2012).

Using Burbles' (1986) work, "Tootle: A Parable of School and Destiny" as a model for deconstructing this superficially innocent-seeming children's book, the participants were asked to consider the following in relation to The Dot and the Line:

> Where the text implicitly assumes certain social circumstances that can be raised to question; where it colors certain conditions with an evaluative shade, or makes outright judgments about them; and where it distorts, misrepresents, or offers a partial, incomplete version of social events, it can be subject to criticism (Burbles, 1986, p. 240).

Working from this perspective, participants readily identified examples from the text of sexism, heterosexism and heteronormativity, racism, violence against women, linguicism, and white privilege. Building from this experience, participants were then asked to look at their own curricular materials, either in their teaching or student-teaching settings, and select 3 examples (of word problems) to scrutinize using some of the same critical stances. Drawing from what Burbles terms "ideology analysis" or "ideology critique," students were asked to engage in, "an attempt to hold a portrayal accountable to social reality" (p. 240). They were asked to consider the following questions as they crafted their critiques:

- What is valued in this problem? Who has power?
- What is not mentioned/ missing/ assumed in the problem?
- What prior knowledge (aside from mathematics) is assumed for this problem?
- Does this problem contain or promote "aspirational" cultural values?

After completion of the activity, participants engaged in focus group discussions to both provide commentary on the process and also to discuss insights gained during their analyses of their chosen problems. Information from these focus groups, along with the written analyses of textbook items, were included in this research. Finally, participants shared these examples with their students and collaboratively generated more appropriate, socially just scenarios.

Findings

One of the most common themes that emerged in the examples participants identified in the mathematics examples was that of consumerism and acquisitiveness. Dozens of problems were identified that focused on purchasing items, with the stated goal often being to acquire the maximum quantity for minimum cost. The problems were usually rooted in the perspective of the consumer, serving to normalize and routinize the act of shopping, reinforcing the ideals of capitalism and framing the readers / students as buyers. Here's an example from *Saxon Math Course 2* (Hake, 2007):

> Sasha had $500.00. She purchased four shirts that cost a total of $134.00. If each shirt cost the same amount, what is the cost of one shirt?

Another example is found in an extended activity called "Hawaiian Dream Vacation," found in the blackline masters for *Bridges in Mathematics, 2* (Snider and Burk, 1999), which includes the following squares on a game board for second grade students:

> You call home: $8. You check into your condo: $155. You buy a camera: $35. Charter a plane back to Oahu: $78. You rent a beach umbrella for the day: $35.

Some of the other items commonly featured in "buying" problems included laptops, televisions, jackets, cars, bicycles, and occasionally mildly baffling, no-picture-included things like a problem about a "snowskate." "A boy asked me what it was, and I had to go Google it," explained one of the participants.

Related to this, participants also identified dozens of examples promoting middle- and upper-middle-class values as highlighted in consumative acts. These examples (typically with a stated focus on calculating area and/ or perimeter) centered on re-carpeting, re-tiling, or re-painting rooms, walls, or other surfaces. A typical 5th grade problem, found in on the website for Everyday Mathematics, (The Center for Elementary Mathematics and Science Education, 2015), reads:

> Regina wants to cover one wall of her room with wallpaper. The wall is 9 feet high and 15 feet wide. There is a doorway in the wall that is 3 feet wide and 7 feet tall. How many square feet of wallpaper will she need to buy?
>
> Another example is found in *Glencoe Pre-Algebra* (Malloy, 2003):
>
> Ashley is going to retile a part of a wall in her shower… The area of the square section to be retiled is 36 square feet. If each square tiles covers an area of .25 square feet, how many …tiles will she need?

What participants found troubling about this was the ways in which "re-anything" implies a disdain for not only an environmentalist orientation, but also the idea that it is framed as normal to keep up with current fashion in home decor. Participants also took issue with what they interpreted as classist ideals, in that those who elect to re-work parts of their homes are typically homeowners and not renters, and have the disposable income to support decorative projects. One participant explained her thinking on this, stating, "These problems tell me that it's "normal" to be a homeowner, and…I am expected to be constantly striving to "improve" my space in ways that cost money."

The other most commonly identified themes that participants found included middle and upper middle class examples of leisure activities. Common examples include problems like this one from *Big Ideas Math* (Boswell & Larson, 2010):

> It costs $175 to rent a jet ski for 2 hours. It costs $300 to rent a jet ski for 4 hours. Write an equation that represents the cost y (in dollars) of renting a jet ski for x hours.

Here's another example from *Primary Mathematics Textbook: Standards Edition, 2b* (Cavendish, 2009):

> David's swimming lesson started at 9:10 a.m. and ended at 9:50 a.m. how long was the swimming lesson?

Perhaps most worrisome are the examples that emphasized getting "cheap labor" and calculating ways to pay "the help" as little as possible. Here's an example from a school-endorsed website, mathhelpforum.com (2009):

> An orange grower in California hires migrant workers to pick oranges during the season. He has 12 employees, and each can pick 400 oranges per hour. He has discovered that if he adds more workers, the production per worker decreases due to lack of supervision. When x new workers (above the 12) are hired, each worker picks $400 - 2x^2$ oranges per hour.

The layered status-orientations in this problem may be seen as insulting and painful, while also reinforcing damaging stereotypes, framing migrant workers as being in need of supervision to work effectively.

Upon scrutiny, it's relatively easy to identify problems that, without naming it, seem to hint at race or racialized ways of knowing and being. One common example is illustrated in problems that focus on meals, like this one from *Algebra 1* (Larson, 2010):

> You want to plan a nutritious breakfast. It should supply at least 500 calories or more. Be sure your choices would provide a reasonable breakfast.

Table 1

BREAKFAST FOOD	CALORIES
Plain bagel	195
Cereal, 1 cup	102
Apple Juice, 1 glass	123
Tomato Juice, 1 glass	41
Egg	75
Milk, 1 cup	150

First, the way the problem is worded indicates that breakfast consists of options, and that the reader has a choice in what to select for the meal. While this is true for some students, there are also many students who receive free or reduced price meals at school, and have no choice in what they are served. Although this breakfast is typical of what might be eaten in some U.S. households, is it common for people to have 3 different drinks at the same meal? It was noted that few of the options seem to be whole foods (except perhaps the egg or maybe the cereal), with the emphasis being instead on processed foods.

Additionally, what's emphasized in this problem is not the nutritional content, but rather, the calories associated with each food. The instructions, using the words "nutritious" and "reasonable" seem to assume some collective, baseline agreements of what these terms might actually mean in practice.

Finally, the inclusion of milk (assumed to be cow's milk) on this list of options for a "nutritious" breakfast fails to recognize the fact that the majority of people on the planet (~60%) are lactose intolerant, and it is primarily white folks (people of European descent) who are able to digest cow's milk (Itan, Jones, Ingram, Swallow, & Thomas, 2010).

Although most participants described experiencing a series of epiphanies around issues of social justice education and the subtle ways hegemonic thinking can creep into mathematics problems, as the result of their participation in this project, a handful of participants (4) instead had a different reaction that ranged from indifference to strenuous defense of the entire canon of mathematics problems discussed. One wrote, "I am inherently skeptical of reading values, ``cultural aspirations", and power dynamics into everything. In particular, I think most math textbook problems are made with little or no thought, and with the attempt to make it "relevant to students"." Drawing heavily from the work of Lockhart's (2009) generalized critique of "word problems" in mathematics, this participant went on to state, "I think to have borderline paranoia about how we as teachers are somehow perpetuating an oppressive system by assigning word problems that may involve a male carpenter instead of a female one is fairly ridiculous." So although the majority of participants in the research gained new insights into how mathematics educational materials may perpetuate worldviews and norms that may be damaging, insulting or otherwise excluding to some students, a few participants found that engaging in this research reinforced further solidified their complicity with or perhaps indifference to hegemonic thinking.

The next steps in this project involved the participants taking their selected mathematics problems back to their K-12 students for the purpose of re-working, re-framing, or re-conceptualizing their chosen problems into examples that will more accurately suit the beliefs and ideals that will best serve the students themselves. Gutstein (2007) advocates for this form of co-construction of new meanings with students, stating, "While we cannot always directly or immediately affect macro political and economic structures, although that is an essential part of creating a more just society, we do have agency ourselves" (p. 438).

Discussion

As the literature on the ways current mathematics discourses may serve hegemonic ideals is only newly emerging, this work is significant in that it identifies an engaging and accessible means for educators to deepen their critical perspectives and undertake agentic activities that work against hegemonic patterns of discourse in schools. By locating social justice work in the critical analysis and purposeful re-shaping of mathematics contexts, this work broadens the field of opportunity for creating a more democratic and critical liberatory pedagogy (Freire, 1982; Frankenstein, 2009). The initial findings from this research suggest that given a supportive and collaborative forum, educators may be equipped to challenge the oft-replayed examples used in mathematics education and craft new and more socially just substitutes.

In exploring how engaging in this activity changed the thinking and professional practices of participants, several themes emerged. Initially, many participants expressed a sense of disappointment or shame at never noticing the preponderance of "troubling" math problems before. Once beyond this initial wave of guilt, some participants expressed outrage aimed in two directions: first, outrage directed at their own teachers for never identifying or challenging the hegemonic examples in textbooks and problems, and second, outrage directed at the authors, editors and publishers of the materials. However, most participants recognized that understanding of hegemony and the insidiousness of cultural reproduction is not part of the common conversation in mathematics education— if anything, it's avoided. Pennycook (2006) explains, "Any model of relation between language and society will only be as good as one's understanding of society" (p. 117). For authors, editors and publishers who have never been asked to consider their work through the lenses offered in this paper, the problems identified as classist, sexist, heterosexist, racist, xenophobic or consumerism-oriented seem only natural. So where do we go from here, if anywhere?

First, and perhaps most obviously, I believe that as educators, we should strongly consider broadening our lenses to consider how different kinds of frames (mathematics contexts) may be interpreted and experienced by our students. What seems normal or neutral to me may be foreign, uncomfortable or even offensive to my students. But of course, this raises the concern with meeting the needs of all learners—how might I possibly account for and incorporate the range of conflicting and possibly confusing perspectives shared by my students? At root, I posit that the solution to this is to know one's students, and to create a classroom climate wherein challenging the status quo is accepted, normalized and encouraged. Educators can create classroom climates wherein it's normal for students to make note of what sits uneasily, to call out what may be seen as classist or sexist or racist, to identify and respond to what feels oppressive or colonizing in some way— even if these thoughts and ideas aren't at the point they can be fully articulated and outlined. Perhaps we can craft classroom communities where it's all right and normal to say "I feel uneasy about this, and although I can't exactly say why, there's something about it that feels wrong or off." Setting this space, where the students are authentically agentic, may provide educators with insights into how they might re-shape the mathematics contexts we ask students to engage with. In other words, this iterative process may better equip teachers to select more appropriate problems in the first place.

I posit that from this centering of the student's lived experiences, this centering of student voices, careful listening may provide educators with rich educational opportunities to expand their understandings of the kinds of things students notice, the kinds of things students bristle at, and the kinds of things students identify as problematic. Thus, this situation may set up a scenario in which teaching is symbiotic, and in the purest Freirian (1973) sense, the students inform the teacher and the teacher responds in kind, making better selections for the students the following year. Also, by asking students to intentionally re-shape their own curricular materials, their level of engagement with the actual content (as the need for fidelity to actual mathematics objectives will remain) may in fact deepen student understandings— for example, when recrafting a problem about calculating the perimeter of an irregularly shaped room, it will be important for the student to present another context that focuses on the same mathematical objective. This may serve to benefit the students even more deeply that by simply completing the assigned problems.

Additionally, with this kind of grass-roots focus on the contexts presented in mathematics, it's entirely possible that a class of students (or even individual students) may wish to reach out to textbook authors and curricular material publishers with specific feedback on the ways their examples and wordings may be unwelcome or unsettling for students. This may, in turn, help to reshape the overall quality of examples textbooks choose to include.

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Maintaining *Standards*: A Foucauldian Historical Analysis of the NCTM Standards Movement

Erika C. Bullock
University of Memphis

The National Council of Teachers of Mathematics' Standards documents represent the most recent and most enduring reform movement in mathematics education. These documents have formed a discourse that has guided mathematics education through the 1990s and beyond. This study uses Foucault's archaeology and genealogy and his concept of the author function to explore Standards-*based mathematics education as a discursive formation and the complex power relations that made it possible for the formation to become The discourse of school mathematics. Data for the exploration includes the Standards documents, earlier histories of the NCTM Standards movement, literature surrounding the documents, and oral history interviews with several of the writers of the NCTM documents.*

Introduction

Since the mid-20th century, there have been several reform movements within mathematics education in the United States; each movement has been subject to its own unique socio-cultural and –political forces. The National Council of Teachers of Mathematics (NCTM) has positioned itself as the voice of mathematics educators in the United States, assuming an increasingly active role in mathematics education reform conversations. This activity reached a seminal point during the most recent and most enduring mathematics education reform movement—the *Standards* movement. The NCTM *Standards* documents—*Curriculum and Evaluation Standards for School Mathematics* (*CESSM*; 1989), *Professional Standards for Teaching Mathematics* (*PSTM*; 1991), *Assessment Standards for School Mathematics* (*ASSM*; 1995), and *Principles and Standards for School Mathematics*

(*PSSM*; 2000)—provided the basis for the *Standards* movement.

Collectively, the *Standards* documents have formed a *discourse*—*Standards*-based mathematics education—that has guided U.S. mathematics education through the 1990s and beyond. This paper represents a portion of a larger investigation (Bullock, 2013) that presents a history of the *Standards* movement in mathematics education based on a premise that the *Standards* were not simply documents, but monuments of a discourse (Foucault, 1972). In the spirit of the sociopolitical turn-moment in mathematics education research (Gutiérrez, 2013; Stinson & Bullock, 2012), this historical study combines Foucault's archaeology and genealogy as the methodological foundation to explore *Standards*-based mathematics education as a discourse. This study problematizes *Standards*-based mathematics education by positioning it as a discourse created and maintained through power relations. By representing a singular approach to mathematics education, this discourse has made certain statements possible and impossible, subsequently limiting ideas that fall outside of NCTM's purview.

Theoretical and Methodological Concepts

Discourse.

In education, the term "discourse" most often refers to talk and semiotics. However, Foucault's (1972) use of discourse is "more" than the signification of semiotics and "it is that 'more' that we must reveal and describe" (p. 49). The "more" that Foucault alludes to includes what is spoken, written, thought, and enacted, as well as what is silent, unthought, or unactionable (Walshaw, 2007). The term "has come to be used to embody both the formal system of signs *and* the social practices which govern their use" (Codd, 1988, p. 242). In this study, discourse is "a *system of possibility* which makes a field of knowledge *possible* [and impossible]" (Usher & Edwards, 1994, p. 90). This system includes the codes, mores, traditions, taboos, and habits of language that we accept in our daily lives and that govern the possibilities [and impossibilities] for thought, speech, and action under particular socio-political and –historical conditions. Functioning within discourse becomes second nature making it "difficult to think and act outside it" (St Pierre, 2000, p. 485). Although we exist in and are surrounded by various discourses, we often remain unaware of their effects. For this reason, discourse often operates without critique.

Power/Knowledge.

For Foucault (1990), power is not an object or structure. Instead, it is a strategy used to exercise control in both positive and negative ways. He does not deny that power can be used as a means of oppression, but he does not limit power as a sovereign and constraining force. It can also work in productive ways to create and maintain knowledge and discourse (Fox, 1998). Power is not *a* thing that can be "possessed, seized or shared" (Foucault, 1990, p. 94), but rather *some*thing—"the multiplicity of force relations immanent in the sphere in which they operate and which constitute their own organization" (p. 92)—that circulates within relationships, hence the phrase *power relations*, and makes knowledge possible.

Foucault (1972) links knowledge and discourse by highlighting that "there is no knowledge without a particular discursive practice" (p. 183). Knowledge, therefore, is not without bounds; as the number of possible statements within a discourse is finite, so is the knowledge produced therein. While the power relations that maintain discourse also dictate knowledge production, Foucault also proposes the reverse: the knowledge produced also presupposes some conditions that make it possible (St Pierre, 2000). He popularized the term *power/knowledge* to illustrate this symbiotic relationship between power and knowledge.

Archaeology and Genealogy.

Foucault's methodological concepts of archaeology (1972) and genealogy (1995) provided a framework through which a decentered analysis of *Standards*-based mathematics education as a discourse becomes possible. Both of these approaches gather around statements as the building blocks of discourse. In archaeology, the guiding question is "How is it that one statement appeared rather than another?" (Foucault, 1972, p. 27), making apparent Foucault's intent to locate (or attempt to locate) the "conditions of possibility" (Walshaw, 2007, p. 10) for the appearance of a statement. Archaeology allows the historian to investigate how statements become true within discourse and what discursive "rules of formation" (Foucault, 1972, p. 227) make that truth possible. The archaeologist studies the archive of discourse and "how 'things said' come into being, how they are interpreted, transformed and articulated" (Cotton, 2004, p. 220). In the spirit of the disciplinary archaeologist, the Foucauldian archaeologist begins by treating the texts that she or he approaches as monuments ripe for excavation.

Genealogy adds to archaeology "a new concern with the analysis of power" (Kendall & Wickham, 1999, p. 29). Foucault (1980) defines genealogy as "a kind of attempt to emancipate historical knowledges from subjection" (p. 85). He destabilizes the historical narrative and views it as "a field of entangled and confused…documents that have been scratched over and recopied many times" (Foucault, 1984, p. 76). Genealogy rejects history as such a metanarrative and favours historical accounts that are replete with incongruity, thus disrupting the romantic nature of historical narrative that is characterized by a clear beginning, middle, and end. Genealogy's goal is "to locate a precontext, to plot a particular historical 'surface of emergence,' to sketch a complex of events and circumstances" (Hook, 2005, p. 14) as opposed to the linear path of causation that connects the present to some "singular or determinant" (p. 14) origin that is the product of mainstream histories.

There is no consensus among scholars about the relationship between archaeology and genealogy, but I see them as complementary methodologies (Walls, 2009). The combination of archaeology and genealogy in this study is aimed at disrupting discourses that exist and function by excluding and subjugating knowledges that do not align with what is acceptable within them. Combining archaeology and genealogy allowed me to address not only the question of how *Standards*-based mathematics education became a discourse (through archaeology), but also of how that discourse became the dominant discourse (through genealogy).

Data Collecton and Analysis

Data for this exploration included the *Standards* documents and literature published in response to the *Standards* documents in newspapers, magazines, and academic journals. Additional data came from 25, one-hour, semi-structured oral history interviews with several writers of the *Standards* documents. Narrators consented to forego anonymity after reviewing, editing, and approving the transcripts. I used holistic/descriptive coding and writing as a method of inquiry (Richardson, 1994) as analytical tools for the study.

Standards-based Mathematics Education as Discourse

Nearly all of the narrators in this study stated that the purpose of the NCTM documents was to provide direction for the field by establishing guidelines for what high quality mathematics curricula, teaching, and assessment should look like. Gary Martin described the *PSSM* as a rallying point: "It's like being the standard bearer in the old middle ages army or the guy with the flag in the Civil War and we're all gonna rally around the flag" (G. Martin, interview). The *Standards* could be no more than guidelines for curriculum because "education is the purview of the states" (D. Briars, interview). With no power to set state curricula, NCTM utilized its influence to create an environment in which the *Standards* became the basis for many states' mathematics curricula. In other words, they created a discourse: *Standards*-based mathematics education.

The data reflects that the process of creating the *Standards* began as a means of allowing mathematics educators to claim authority in mathematics education. I assert that neither the sole purpose for creating this discourse was the elevation or preservation of NCTM as an organization or any affiliated person or group of people, nor that the discourse of *Standards*-based mathematics education has formed in the way that the NCTM leadership may have intended. Nevertheless, my suggestion that the NCTM leadership did intend to create a discourse is not controversial. Romberg (1998) wrote:

> The vision of what mathematics students should have an opportunity to learn, how mathematics should be taught in classrooms, and how students and programs should be assessed and evaluated has been described in three documents prepared

by NCTM: [*CESSM*] (1989), [*PSTM*] (1991), and [*ASSM*] (1995). (p. 8)

Although he may contest the language, it appears that Romberg is describing a discourse of school mathematics based upon the *Standards* in which what "should be" included on all fronts is clearly defined.

NCTM has been the leading voice of mathematics teachers and mathematics educators since its founding in 1920. NCTM's research journal, *Journal for Research in Mathematics Education* has been the flagship research journal in U.S. mathematics education and a leading journal internationally (Johnson, Romberg, & Scandura, 1994). The practitioner journals provide current information and instructional ideas for teachers that are not available elsewhere. Through these publications and its national, regional, and state meetings, NCTM has created a platform from which it has been able to direct the conversation within mathematics education.

NCTM used its organizational structure and assets, along with political positioning and media, to craft and promote its *Standards* as the guiding documents not only for the organization but also for school mathematics writ large. In several ways, the NCTM managed what I was able to know about the process by (a) not maintaining archives of primary data that are available to the public or to dues-paying members;[1] (b) publishing the only extended histories of mathematics education with the exception of the *Bold Ventures* study (McLeod, Stake, Schappelle, Mellissinos, & Gierl, 1996); and (c) publishing the majority of literature available related to the *Standards*. These observations may seem like an exercise in finger pointing, but these mechanisms of discourse management help to make my case rather than work against it. NCTM has done what any organization would do with its intellectual property: it has protected its investment in the *Standards* process by constructing a discourse around the process whereby the stories told about the *Standards* must be told, in large part, from NCTM's vantage point or with its endorsement.

Apple's (1992) description of the *Standards* as a *slogan system* based on three criteria provides additional support for positioning *Standards*-based mathematics education as discourse. First, the *Standards* were vague enough to create an umbrella large enough to cover those who may disagree with the message. Second, the *Standards* were specific enough to give the audience something tangible in the moment. Finally, the *Standards* were charming, providing a call to action that inspired the mathematics education community to sustained action. This analysis is compatible with my discussion of *Standards*-based mathematics education as a discourse. The discursive representation addresses the *limits* that maintained the discourse while the slogan system described the *strategy* for maintaining the discourse.

Maintaining the Discourse

In its brief existence, NCTM has been through many changes. It has become "a recognized leader and a driving force in mathematics education" (Gates, 2003, p. 750) by engaging in a larger political project marked in 1966 by a measure that allowed it to take a more active stance on "controversial professionally related topics" (Gates, 2003, p. 749). As NCTM has changed, so has its political volume. The organization has been able to leverage its political connections in favor of *Standards*-based mathematics education. In the *Standards* movement, it experienced a shift where involvement in NCTM leadership took a notably political turn. Skip Fennell, former NCTM president, described:

> When we were on the Board you never spent time talking about mathematics, we were seemingly always talking about policy. That's a policy job.... I say to everybody that the job of the NCTM president is in Washington, DC. It's all about policy surrounding your subject. (F. Fennell, interview)

The NCTM leadership had a message that it wanted to deliver through the *Standards* documents and made personnel and editorial decisions accordingly. These measures were a form of *discourse management*. Discourse management is a mechanism of preservation or a strategy for making decisions that form and re-form a discourse to keep it viable and prominent. It is a step beyond keeping up with the pulse of the discipline; it also entails changing the pulse when necessary to redirect it to the desired discourse. NCTM engaged in discourse management through sponsorship, oversight, and dissemination of knowledge.

Managing Discourse through Sponsorship.

NCTM assumed sole authority over the contents of the *Standards* based upon its role as financier of the *Standards* documents. Aside from a small ($25,000) grant from the AT&T Foundation (McLeod, 2003) to begin the work of the *CESSM*, NCTM financed this initial standards effort. McLeod (2003) asserts:

> The lack of outside funding allowed NCTM an independence that other curriculum areas did not always have. Although other curriculum areas received up to $3 million in federal grants to develop standards, most NCTM leaders were pleased that they did not have to follow federal agency guidelines for such a project. (p. 772)

Lee Stiff echoed this point: "Unlike everyone else who created standards at this time who had federal government money to help them do that, the council paid for the creation of the standards document out of its own budget, which was millions of dollars" (L. Stiff, interview). Sales of the *Standards* documents helped to replenish NCTM's coffers and to support the later *Standards* efforts.

Managing Discourse through Oversight

The chief means of discourse management in the *Standards* movement was the selection of writers. Lee Stiff, NCTM president from 2000 to 2002, commented:

> In the guise of a democracy what that means [is] the people who were in charge didn't make themselves in charge…. The [NCTM] Board [of Directors] picked those people and the Board would know who those people are and what their perspectives are. So that when I pick you … I know who you are. I know what you've written in the past. I know what your perspective is … I believe in a very real sense the Board orchestrated this…. So in that sense the Board is creating the document in its own vision. It's just not writing the words. In reality … the dynamics will have outcomes that you may not have fully expected but in

> broad terms it's exactly what the Board and the president foresaw because they picked the people. (L. Stiff, interview)

Stiff's comments suggest a sort of secondary authorship assumed by the NCTM leadership as commissioners of the documents and conveners of the writing groups.

Olson and Berk (2001) posit that "[the *Standards* represent] the collective best thinking of the mathematics education community" (p. 306). Burrill (1997) argued that the *Standards* provided a framework to "ensure that discipline experts have a voice in helping states and districts make interpretations" (p. 335). Juxtaposing these assertions with Stiff's position calls into question if the *Standards* truly represent the community's best thinking or the best thinking that aligned with the organization's strategic plan. Stiff's comments demonstrate that the organization selected writers who were great minds that would likely legitimize the work and limit the threshold of variation from the message that NCTM wanted to bring to market.

In each of the *Standards* documents, NCTM exercised its ability to construct the document and the conditions of its public presentation. Mary Lindquist (2003) wrote of increased oversight from NCTM with each document:

> The different NCTM Boards during the five-year period of planning and developing [*PSSM*]…were much more involved in the process than previous Boards had been. They were no longer content…to react to a draft just as other NCTM members did and then to wait for the final version. (p. 831)

As NCTM changed as an organization over the years, so did its position with respect to the documents and their writers and its need to manage opportunities to deviate from its mission.

Managing Discourse through Knowledge Dissemination

NCTM's role as publisher of the *Journal for Research in Mathematics Education,* one of the leading mathematics education research journals, makes it the gatekeeper for knowledge dissemination in mathematics education. Through this venue, the organization "influences the direction of mathematics education research" (Langrall, Martin, Ellerton, Hertel, & Fain, 2013, p. 338). Throughout the *Standards* movement, the NCTM practitioner journals functioned as supplementary instructional materials (Seymour & Davidson, 2003). In addition, NCTM has maintained a viable publishing arm that produced books that support the *Standards* agenda. The NCTM imprint is an indicator that it maintains controlling interest. Therefore, NCTM directs, in large part, knowledge production in mathematics education.

It became an unwritten rule for those who wrote for NCTM publications to demonstrate a link between their ideas and those values espoused in the *Standards*. Gerald Rising named this issue as a negative consequence of the *Standards:*

> The NCTM journals have been extremely strongly affected by [the *Standards*] ... If you write an article for [an NCTM] journal the first question they ask it "Does this fit the Standards?"... Once again it's saying, "Look, if you're doing anything that's different from the Standards, forget it." (G. Rising, interview)

It is evident that Rising perceived the *Standards* to be the arbiter of what is (im)possible for publication in NCTM's outlets. A glance through more recent NCTM journals reveals that Rising's sentiments still ring true (although the referent is increasingly shifting to the Common Core State Standards). In fact, the *Standards* became more than an obligatory reference; they have defined a movement of *Standards*-based mathematics education in which anything that has been thought, spoken, or acted upon must line up with the *Standards'* perspective in order to be considered true or valid (Parks, 2009).

Current Conversations

Efforts to problematize existing structures or ideas can be perceived as cynical or pessimistic and there may be some truth in that perception. However, it is not my intent to disparage NCTM in any way. To claim that *Standards*-based mathematics education has been "good" or "bad" is reductive; school mathematics is too complex for such simple claims. Rather, the issue here is that positioning *any* single discourse as "right" for mathematics education is *dangerous.* Foucault (1983) explains:

> My point is not that everything is bad, but that everything is dangerous, which is not exactly the same as bad. If everything is dangerous, then we always have something to do. So my position leads not to apathy but to a hyper- and pessimistic activism. (pp. 231–232)

My pessimism, therefore, is more a healthy skepticism that keeps me from being lulled into complacency, believing that *Standards*-based mathematics education has solved any problems that it was designed to address or that it has not caused its own share of problems.

I charge the mathematics education community to maintain this sense of pessimistic activism. Although benefits are important and new ideas should breed excitement, we must exercise greater care in counting costs. However, we must watch for the moment when those ideas show potential for physical, psychological, or intellectual harm to *any* child. At that moment, we must be prepared to act. Maintaining this level of preparation means stretching the boundaries of mathematics education research so that we will be prepared with new possibilities to address existing problems (Bullock, 2012). It also means maintaining flexible curricular, instructional, or assessment structures that allow us to must maintain the humility of spirit required to abandon our individual and organizational agendas for the children's benefit.

Notes

1. I contacted NCTM as a part of this study to request access to official records from *Standards* era. I was told that the organization does not maintain such records.

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Changing the Game while Playing the Game: Teaching Social Justice Mathematics

Jessica Hopson Burbach, 2-5 students from Portland YouthBuilders

Portland YouthBuilders, Portland State University

In this paper, I will share my goals as a social justice mathematics teacher, which include to deconstruct mathematics, to connect mathematics to students' lives, to honor multiple approaches to thinking mathematically, and to encourage student leadership of the learning in the classroom. I will describe and reflect on a social justice unit I designed and taught, which explores whether police traffic stop data is proportionate to racial demographic data. By critiquing my social justice unit, I will offer ideas for improving students' understanding of proportionality and for empowering students to see themselves as agents of change. Finally, I will argue that a social justice mathematics curriculum gives students from traditionally marginalized communities greater access to mathematical literacy, while developing their social consciousness and agency.

Introduction

> Changing the game of mathematics means partly rewriting the narrative about who contributes to mathematics and who does well in it. It also means offering new ways of envisioning school mathematics by considering home communities and other ways of viewing the world. Changing the game of mathematics also means students are able to use mathematics to acknowledge hegemony in society and can address social and political issues of importance in their communities. (Gutiérrez, 2009, p. 11)

The mathematics achievement gap between White, middle-class students and students in marginalized communities (including students of color, working class students, and English language learners) has been well documented by U.S. education researchers. As a result, the focus on the achievement gap has emphasized the failure of marginalized students in mathematics and normalized the achievement of White students as the standard for all students (Martin, 2012). Standardized tests measure achievement in school mathematics and not the mathematical thinking and learning done in marginalized communities. In fact, Tate (1994) argues that a barrier to African-American students' learning and success in mathematics is that the curriculum does not reflect their ways of thinking mathematically and their own cultural and community experiences. A mathematics curriculum must address the diverse racial, cultural, socio-economic, gender, and sexual orientation identities of students. A social justice mathematics curriculum allows students to explore and analyze important social, political, and economic inequities in their communities (Ukpokodu, 2011). Students from marginalized communities are empowered because their identities and experiences are reflected in a social justice curriculum and because they are given the opportunity to counter the prevailing narrative about their lack of achievement in mathematics. Thus, one way to change the game of mathematics education, as Gutiérrez (2009) suggests, is for mathematics educators to embrace the teaching of mathematics through a social justice curriculum.

The purpose of this paper is to describe my goals and learning process in designing and implementing a social justice mathematics unit. By using mathematics as a lens for analyzing an injustice in their community, I hoped that students would engage deeply with the mathematics content while being encouraged to see themselves as successful in mathematics and as leaders in their school and communities. My specific goals for the social justice unit are described in detail later on in this paper. It is important to note that creating a social justice curriculum is not easy and can feel risky. In this paper, I will honestly reflect on these difficulties and critique my social justice unit based on the goals I set for myself. In fact, I will offer changes to the unit so that it better meets my goals. At the time of the conference, I will have taught the unit, with the changes described in this paper, at least twice more and will be able to provide further feedback on it. Additionally, I plan to invite several students to present with me. Above all, my commitment to the success of all students and, in particular, my belief that students in marginalized communities can learn mathematics as well as any other students are what drive me to advocate for the teaching of mathematics using social justice.

Context

I work at an alternative school in Portland, Oregon called Portland YouthBuilders (PYB). It is a vocational school, where students gain valuable skills in construction or in technology and media while achieving high school completion. All students that enter PYB are low-income, between the ages of 17 and 24, and on average have been out of school for two years. Students enter PYB with the burden of many labels that barely scratch the surface of who they really are, including "high school drop out," and "at-risk youth." I have learned the importance of getting to know them beyond these labels and of continually working to build strong relationships with them. As their teacher, I focus on the learning not the labels (Aguirre, Mayfield-Ingram, & Martin, 2013). PYB students have shown an incredible resilience in their outside lives and astonishing courage to decide to return to school despite the negative educational experiences they have had in the past. I work with students to rewrite the single story of their underachievement and failure in mathematics and school by emphasizing their success and resilience (Martin, 2012).

Goals for Social Justice Mathematics Teaching

In order to connect with my students, to engage them in thinking deeply about mathematics, and to empower them to be leaders in their communities, I have created several goals that I strive to achieve when I teach. It is through a social justice mathematics unit that I am best able to meet these goals. The following sections explain my goals and how a social justice framework helps me to achieve them.

To Deconstruct Mathematics

Traditionally mathematics is seen as culturally-neutral and objective (Tate, 1994). On the contrary, mathematics knowledge is culturally-situated and it is not a universal truth. For example, mathematics problems involving roller coasters, skiing, down payments, and/or pumpkin pie may prevent students from showing their mathematical knowledge or disengage them because the context is not culturally relevant nor part of their lived experience. Ethnomathematics studies how different cultures have different ways of thinking, learning, and doing mathematics (D'Ambrosio, 1997). Moreover, students from different cultural backgrounds may have different ways to approach, think, reason, and process mathematically. It is important that teachers understand this and dispel the misconception that there is only one way to do mathematics (Ukpokodu, 2011). I make sure to explicitly deconstruct the universality of mathematics with students and we discuss how it is culturally-influenced. Using a social justice curriculum allows students to view mathematics in a new way: as a useful tool for exploring and analyzing injustices in their communities and in the world.

To Connect Mathematics to Students' Lives

By connecting mathematics to students' lives, students see the relevancy of what they are learning, they are more engaged, and they are more likely to identify with being a thinker and doer of mathematics. Aguirre et al. (2013) identifies drawing on multiple resources of knowledge, including students' cultures and communities, as one of the five equity-based practices in the mathematics classroom. These practices serve to improve learning and encourage students to develop positive mathematical identities. When students explore a social justice issue that affects their community, they become an important source of information as do other members of their community. Thus, when mathematics is presented in a familiar context, students from marginalized communities are empowered to counter the prevailing deficit narrative because they contribute knowledge in the mathematics classroom.

To Explore Multiple Ways of Learning Mathematics

Martin (2012) argues that in order to understand how Black children reason and think mathematically in school, researchers must explore "the variety of ways these children engage in mathematical thinking and knowledge construction in relation to the demands of their everyday lives" (p. 57). Students may successfully use mathematical thinking strategies in their everyday lives, but not in the classroom because the strategies they use outside of school are unlike those that are shown in class by the teacher. A culturally responsive teacher rejects the notion that there is only one, right way to solve a mathematics problem and is open to students' divergent thinking and problem-solving styles (Ukpokodu, 2011). Hence, I often ask students: Did anyone solve this in a different way? In my classroom, students are encouraged to share their own strategies and we challenge ourselves as a class to represent as many ways to solve a problem as possible. When students' lives and communities are already brought into the classroom in a social justice curriculum, students are more likely to believe that the mathematical strategies they use outside of classroom will also be accepted.

To Allow Students to Take the Lead

An important aspect of teaching is empowering students to "see themselves as legitimate and powerful doers of mathematics" (Aguirre et al., 2013, p. 14). I model openness to multiple problem-solving strategies by asking students to present their reasoning and thinking to each other. Students have agency and choice to use the strategy that works best for them. In my classroom, students are often in the front of the room, teaching us about their solution strategy or mathematical thinking process. In fact, my students have taught me many of the unique and successful strategies for thinking about mathematics that I currently share with students. When defining critical pedagogy, Paolo Freire (1970) speaks to the joint responsibility of both students and teacher to be both learners and teachers. "The teacher is no longer merely the one-who-teaches, but one who is himself taught in dialogue with the students, who in turn while being taught also teach" (p. 80). This shared responsibility for teaching and learning is especially important in a social justice oriented curriculum since multiple perspectives are required for a deep analysis of the injustice. A social justice mathematics unit builds collective mathematical agency (Aguirre et al., 2013) because both students and teacher work together to solve mathematical problems and explore inequities. In addition, students may be inspired to be agents of change by taking action against these injustices in their communities.

To Reflect on My Own Assumptions

Teaching is about inspiring students and about student learning, but it is just as much about a teacher's personal and professional empowerment and growth. "Professors who embrace the challenge of self-actualization will be better able to create pedagogical practices that engage students, providing them with ways of knowing that enhance their capacity to live fully and deeply" (hooks, 1994, p. 22). The more I strive to fulfil my own potential, the better prepared I am to open-mindedly address the wide array of student needs and interests. I engage in deep personal reflection on my practice as a mathematics teacher and seek to uncover my own weaknesses, assumptions, and biases. In particular, as a White teacher, I challenge myself to deconstruct the White, middle class values that I have normalized in order to continually de-center whiteness. I reflect on how students walk in the world differently and similarly to myself because of their racial, cultural, socio-economic, sexual orientation, and gender identities. For teachers to support the creation of positive mathematical identities in their students, teachers need a deep understanding of the multiple identities and social realities of their students (Aguirre et al., 2013). By exploring a social justice issue in students' communities, I am able to gain better insight into my students' lives because their experiences become important sources of information to the project. As a result, students' identities are validated and recognized in the classroom.

Social Justice Mathematics Unit on Traffic Stop Data

I chose to deepen students' knowledge of fractions, percentages, and proportionality through a social justice unit that analyzed traffic stop data. The Portland Police Bureau had recently released a report to respond to concerns from community stakeholders about racial disparities in the stop and search data and potential racial profiling (Stewart & Covelli, 2014). Additionally, I had heard from multiple students, and in particular students of color, about times that they were pulled over without reason by police officers while walking or driving. Hence, traffic stop data was not only directly relevant to many of my students' lives, but also an important issue to students as citizens of the city of Portland. This unit was an opportunity for students to work together to see how mathematics could be used as a tool for exploring issues that affect them. Ultimately, my hope was that students would be empowered to get involved in advocating for changes to the Portland Police Bureau's stop and search policies. Paolo Freire (1970) asserts, "Students, as they are increasingly posed with problems relating to themselves in the world and with the world, will feel increasingly challenged and obliged to respond to that challenge" (p. 81). Although my intention was to encourage student leadership and involvement, I will describe how my first attempt at this social justice unit did not let students take the lead nor feel a sense of agency. I will offer changes to my social justice unit that allow students more choice in how to explore the data and to affect change in their communities around police stops and searches. No doubt I will have further reflections and critiques at the conference of what I can continue to improve.

At the start of the unit, I asked students to write about a time when they were unfairly accused of something. Additionally, I asked students to respond to the following questions: Were the police or a security officer involved? How did this make you think or feel? Did you do anything to prove it was unfair? Why or why not? Then, I had students share these stories with each other. Starting with students' experiences helped them to see the connection between the unit and their everyday lives. It also sent the message to students that their experiences were valuable data. Next, in small groups, students looked at the racial breakdown of traffic stop data in Table 1 from the Portland Police Bureau's report (Stewart & Covelli, 2014). I left out the percentages and had students calculate them. I also asked students to use estimation to create fractions with the number of people of each race pulled over as the numerator and the total number of stops as the denominator. Then, students had to write a sentence using either the percentage or the fraction to represent approximately how many people of each racial demographic in the table got pulled over. For example, about one out of every 20 people pulled over was identified as Asian. We talked about how it was important to use the phrase "was identified as" because Portland police officers entered the race in the computer system, so it might not represent how the person pulled over actually identifies. Letting students react to the race categories used by the Portland Police Bureau was important in validating students' racial identities. For example, my student who proudly identifies as Pacific Islander wondered how he would have been identified by police officers and students who identified with multiple races felt that these racial demographic categories were highly limited.

As the next step, I gave students a challenge: To write down as many facts, comparisons, surprises, and thoughts that they had as they looked at the data. Afterwards, each group shared with the rest of the class. It was important that we had a multitude of perspectives and thoughts about the data. While each group presented, I asked students to think about the question: Are the Portland Police pulling people over fairly? When I led students in a conversation about fairness, I directed students toward the need to compare the traffic stop data to the racial demographics of the city of Portland. However, students had other ideas about how to measure fairness, for example to look at the racial demographics of who drives in Portland or to look at how many traffic stops lead to tickets, arrests, or seizures of drugs and/or guns. Some of this data is even included in the Portland Police Bureau report and would be easy for students to explore. Nevertheless, I was so tied to my plan for the next day to explore the racial breakdown of the Portland population, I did not allow students to think mathematically about fairness from multiple approaches. Later I will address how changes to this social justice unit will allow students to explore alternate perspectives of fairness in the traffic stop data.

Students engaged with U.S. Census Bureau data from 2010 on the racial demographics in Portland in much the same way as the traffic stop data. Students calculated percentages based on the total population in Portland and the number of people identifying as each race. It is important to emphasize for students that the race and ethnicity categories are defined differently on the Census than in the Portland Police report. For example, the U.S. Census Bureau (2014) defines Hispanic/Latino as an ethnicity and not a race and in addition people can identify as two or more races. I asked students to write a short paragraph about their reaction to the demographic data using specific percentages and fractions to support their claims. Several students were surprised that the percentage of people identifying as White was over 75% in Portland (U.S. Census Bureau, 2014). This observation led to a conversation about how different neighborhoods in Portland would reflect different racial demographics. Although students were curious to explore the racial demographics of their own and other neighborhoods in Portland, they were not given the chance to satisfy that curiosity in this social justice unit.

The next day we began a discussion about proportionality and how to decide whether the traffic stop data was proportional to the racial demographic data of Portland (the measure of fairness we used). Upon reflection, I believe I led students to the definition of proportionality and handed them the process of comparing the percentages without a deep conceptual understanding of proportionality nor why it could serve as an indicator of fairness. To wrap up the exploration of traffic stop data, I had students define racial profiling and I asked students to write a paragraph about whether they thought the traffic stop data showed that racial profiling was a problem in the Portland Police Bureau. Students were allowed to use both the data and their personal experiences. Since my students' voices often go unheard, I wanted to give students the opportunity to share from their own experiences and include that in their explanations as valuable data. I also asked students to come up with one or two aspects of this data that they would like to study further. Several students mentioned they wanted to look at where the majority of the traffic stops were occurring because they suspected that traffic stops were concentrated in specific neighborhoods in Portland. Other students wanted to explore how socio-economic status plays a role by looking at the condition of the cars being pulled over. They hypothesized that more traffic stops involved cars that were old and looked beat up. However, due to time constraints, after students shared their conclusions about racial profiling and ideas for further study in a large group discussion, I brought the social justice unit to a close.

Although I felt proud to have tried a social justice lesson with students, I was disappointed that the unit did not create a student-led learning environment where students could explore the traffic stop data driven by their own curiosity. Students did not see themselves as agents of change because they were not asked to think about ways to engage with this issue in their communities, nor were they provided with examples of ways that members of their community are involved in this issue. One of my African-American students struggled to come up with ideas for what he would have liked to explore further. When I checked in with him individually, he sighed heavily and said, "This is hard. It's about me. What can I do?" I had underestimated the emotional impact that looking at this data might have on my students of color, particularly the African-American men. As a White woman, I could look at this traffic stop data, which shows that a disproportionate number of African-Americans get pulled over, and think about racial profiling from a very intellectual place. In contrast, for my African-American students, these statistics represented very real and painful everyday experiences. More importantly, my student had expressed hopelessness in changing these statistics. Clearly, I had not achieved my goal of letting students take the lead and did not empower students to affect change in their communities.

Another critique was that the unit defined proportionality for students, instead of allowing them to develop their own understanding of proportionality. According to Eric Gutstein (2003), who also developed a project with his students to analyze racially disaggregated data on traffic stops, the mathematical concept of proportionality is key in understanding racial profiling. In order to build students' conceptual understanding of proportionality, Gutstein (2013) gave groups of students a small bag with colored cubes to represent the racial breakdown in the city. Students took a cube from the bag without looking, recorded its color, and replaced the cube 100 times. After every 10 picks, students recorded the total cumulative picks for each race/ethnicity and calculated the fraction and percentage of each. Next, students created their own simulation based on data about the number of Latino motorists and compared their simulation to the actual percentage of Latino drivers pulled over.

Next time I give my social justice unit on traffic stop data, I will have students explore the racial breakdown of Portland using the colored cubes. Then, I will give students the racially disaggregated data on motorists in Portland and ask them to create their own simulation using cubes (Stewart & Covelli, 2014, Table 3). Although I will give students clear directions, I will keep it open-ended so that groups of students can choose to focus on data from one or more races/ethnicities depending on their interests and identities. Students will use the percentages from their simulations to compare to the percentages of each race/ethnicity pulled over at traffic stops in Portland. There are several advantages to these changes. First, students are developing a stronger understanding not only of proportionality, but also experimental and theoretical probability. Second, groups of students are allowed to create their own simulation, instead of being told how to show proportionality, which gives students more ownership of their learning. Third, by drawing cubes out of the bag, students are engaged in a different approach for thinking about percentages and fractions.

Finally, I will not be satisfied that I have achieved my goals with this social justice unit unless it helps students develop a "belief in themselves as conscious actors in the world" (Gutstein, 2003, p. 40). For students to develop a sense of agency, they must see that they can play a part in interrupting injustices in their communities. Students can research what is already being done to counteract the inequities they studied and get involved or start their own projects. In the case of my social justice unit, I will have small groups of students explore one other social justice issue represented in the Portland Police Bureau Report on traffic stop data that interests them. Students will use mathematics to create a visual model or simulation to expose the inequity in the data. They will research what is currently being done to respond to either the injustice they explored or the issue of racial disparities of who gets pulled over in Portland. In doing so, students will generate a list of possible solutions and ways to get involved. As a whole group, we will decide our next step in supporting the end of racial profiling. For example, students could decide to bring in a police officer to answer students' questions or to write a letter to Portland's Independent Police Review, a division responsible for the civilian oversight of the Portland Police Bureau. Lastly, I will ask students to reflect on how mathematics helped them in the social justice unit. In doing so, I will explicitly ask students to deconstruct mathematics and to show how it can play a powerful role in revealing injustice and in helping to stop it.

Conclusion

> To provide schooling for everyone's children that reflects liberal, middle-class values and aspirations is to ensure the maintenance of the status quo, to ensure that power, the culture of power, remains in the hands of those who already have it. (Delpit, 1995, p. 28)

When students do not see themselves or their own life experiences reflected in the curriculum, it devalues their home cultures and disengages students from the learning. Students of color and poor students are made to feel that they must change themselves to meet

White, middle class standards. Bright and motivated students from marginalized communities are denied access to what Delpit (1995) calls the culture of power, or the set of tools, language, behaviors, and knowledge of the dominant culture, which are required for upward mobility and full participation in the U.S. democracy. Robert Moses, the creator of the Algebra Project, a national math literacy program serving a predominately African-American community, would agree. He believes mathematics literacy is a civil right and inexorably linked to "the on-going struggle for citizenship and equality for minority people" (Moses & Cobb, 2001, p.14). In effect, marginalized communities are being disenfranchised from the political process when they don't achieve mathematical literacy.

However, in order to achieve equity in mathematics education, teachers need to prepare students not only to play the game of mathematics education, but also to change the game of mathematics education (Gutiérrez, 2009). A social justice mathematics curriculum changes mathematics education by deconstructing traditionally held views of mathematics, connecting mathematics to the lives of marginalized students, presenting multiple approaches to thinking and learning mathematics, and by allowing students to develop leadership and agency within the curriculum and in their own lives. Gutstein (2003) defines mathematical power as developing an understanding of the complexities of the systems of power and privilege while seeing mathematics as a tool for uncovering and resolving these injustices in the world. In other words, a social justice mathematics curriculum gives students in marginalized communities greater access to mathematical literacy (as part of the culture of power) and mathematical power and agency.

I have set as a goal to reflect on the assumptions I bring to my teaching practice, in particular how my identity as a White, middle-class, heterosexual woman impacts the teaching and learning of my students. While constant reflection is important, so is the transformation from teacher to teacher-activist. Mathematics is not a politically neutral activity (Gutiérrez, 2009). By teaching a social justice curriculum, I am taking a political stance in shifting the way mathematics is taught in my classroom, acknowledging hegemony, and addressing social and political issues of inequity in my classroom. As Delpit (1995) suggests, I strive to be "unafraid to ask questions about discrimination, and voicelessness of people of color, and to listen, no hear, what they say" (p. 47). Developing students as agents of change means that teachers must model activism in their daily lives. By becoming involved in organizations seeking to end social and political injustices, teachers can more easily connect and involve students in organizations making change in their communities. Thus, students are empowered themselves to be leaders and activists in their schools and communities. As a social justice mathematics teacher, I will continue to challenge myself to create stronger social justice lessons, but more importantly I am committed to becoming a stronger advocate for my students and activist in my school and community.

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Teaching Mathematics for Social Justice: Linking Life History and Social Justice Pedagogy

Susan M. Carlson-Lishman, Indigo Esmonde
Independent Researcher, University of Toronto

In this case study, we consider the relationship between life history and classroom pedagogy for one secondary mathematics teacher who participated in professional development to support equitable mathematics teaching. Within this teacher's life history, we identified three primary themes: being the other, being mainstream, and orienting experiences, which we then linked to the teacher's discourse about equity, social justice, and mathematics pedagogy. We found that these experiences informed the teacher's pedagogy and served as a strong motivation for him to address issues of equity and social justice within his classroom.

The increasing diversity of the North American population and a growing achievement gap in mathematics underscores the need for high-quality, equitable mathematics instruction. In many school districts across the U.S. and Canada, teachers are overwhelmingly middle-class and White, while students are much more varied in socioeconomic status and race/ethnicity (Howard & Aleman, 2008). As a result, it is critical to study how teachers (especially those from privileged groups) can learn how to effectively and equitably teach students with a diversity of backgrounds and experiences.

In mathematics, as in other subject areas, teachers' experiences and perspectives (who teachers are) affect how they teach (Wager, 2010). As Wager points out, there are many ways to conceptualize what (equitable) mathematics teaching should look like, and teachers' prior experiences with injustice inform their starting points as they try to take up equitable practices. Foote and Bartell (2011) presented similar results in their study of life histories of mathematics education researchers. Their descriptions of researchers' positionality could equally well describe teachers: "life experiences impact the positionality we bring to our work—they inform the questions we ask, the data we choose to gather, and the interpretations we draw" (p. 63).

In this paper, we report on one case from a study of a year-long professional development in which teachers used action research to investigate how to teach mathematics more equitably. We sought to understand how mathematics teachers' personal histories and experiences influenced their discourse on equitable mathematics pedagogy.

Theoretical Framework

We use a sociocultural perspective for this study, in particular drawing on repertoires of practice. Such a perspective considers individual development and disposition within—not separate from—cultural and historical context. (Gutiérrez & Rogoff, 2003).

Repertoires of Practice

The notion of repertoires of practice (Gutiérrez & Rogoff, 2003) is rooted in cultural-historical activity theory. An individual's repertoire of practice can be considered their toolkit—the skills, knowledge, perspectives, and expertise that they develop over time through their participation in a variety of formal and informal settings and contexts (Gutiérrez & Larson, 2007). In relation to teaching, repertoires of practice might be developed through early experiences with schooling as students, and as teachers in formal and informal educational contexts. Repertoires of practice is an appropriate conceptual tool for this study because it shifts the focus from the individual in the moment to the individual in social and historical context. Further, it considers the ways in which the individual and the multiple contexts in which they operate are mutually constitutive. That is, repertoires of practice allow us to gain insight into the historical experiences and circumstances that are "relevant to an individual's likelihood of acting in certain ways" (Gutiérrez & Rogoff, 2003, p. 22).

Professional development (PD) has often been conceptualized as a one-way transfer of skills from the PD context to the classroom. However, a sociocultural perspective on PD suggests that an individual's participation in classroom and PD contexts are co-constitutive and mutually inform their repertoire of practice (Kazemi & Hubbard, 2008). Further, in keeping with the work of Wager (2010) and Foote & Bartell (2011), teachers' life histories shape the way they engage in the classroom, and the PD, but the way they make sense of their life histories can also be influenced by classroom and PD contexts. All three of these contexts mutually inform one another, as we will show in our data analysis.

Narrative Inquiry

Narrative inquiry is a research method and a way of interpreting data. In this study, we use narrative inquiry in both ways. As a research method, narrative inquiry gives voice to the participants (Riley & Hawe, 2005). They describe their life history and experiences from their own perspective. When interpreting data,

> narrative analysis begins from the standpoint of the [participant] … From here we analyse how people, events, norms and values, organizations, and past histories and future possibilities, are made sense of and incorporated into the [participant's] interpretations and subsequent actions (Riley & Hawe, 2005, p. 229).

By examining *what* events the individual describes, *how* they describe these events, and even what they do *not* talk about, we gain insight into the ways in which the individual makes sense of their experiences and what they value.

Methods

Data for this analysis were taken from a year-long research project on a professional development (PD) study group. The study group focused on equitable mathematics teaching and action research. Eight teachers and five university researchers participated in the group, which met 9 times from October 2010 to May 2011.

Participants

The teachers were all volunteers. All were White and all worked in schools in Toronto, Ontario and the surrounding area. For this paper, we focus on one teacher, Rico (a pseudonym), a 26-year-old White man.

Rico had attended Catholic elementary and secondary schools and then studied mathematics and physics at a university in his home city. He worked for one year as the Diversity Director for the student union at his university and then completed a one-year initial teacher education program. At the time of the study, Rico was in his first year of teaching. He had been hired on a long-term occasional contract to teach mathematics and physics at a large urban secondary school.

PD Context

The teachers examined their own classroom contexts, identified challenges, then each developed their action research project to address one or more of those challenges.

Rico focused his action research project on his Grade 12 "Foundations for College Mathematics" class. (In Ontario, "college" generally refers to 2-year colleges that focus primarily on vocational and technical skills, so this course was considered to be at a lower level than Grade 12 university-preparatory courses.) In this class, he described challenges with student attendance and motivation. Rico chose to explore the impact of a social-justice context on student motivation and mathematics learning in the data management unit of this course. To begin, he had students participate in an activity called the "power flower" to introduce them to the ideas of power and privilege. Students were then given data, collected by the school board, on racial achievement trends in their school, which they organized and analysed.

Data Analysis

For this analysis, we drew from Rico's interviews at the beginning, middle and end of the year, his written reflections, and the video record of his participation in meeting 1, specifically in an activity in which he described his life story to the other PD participants. We examined the data and identified themes in relation to Rico's life history, drawing from and extending the themes found in Wager (2010) and Foote & Bartell (2011). We went through the data multiple times to refine the themes and look for connections among Rico's life experiences as he described them, his stated conceptions of equity and social justice, and his pedagogy. In keeping with narrative inquiry methods, we present our results through Rico's own words, as much as possible.

Results: Rico's Life History

We will begin with a brief description of Rico's life history, elucidating three themes that build on Foote and Bartell's analysis. The three themes that we identified were: being the other (Foote & Bartell, 2011), being mainstream, and orienting experiences (Foote & Bartell, 2011). We will then describe Rico's social justice and mathematics pedagogy, and show how Rico made links between his life history and his pedagogy.

Being the Other

Experiences of being the other involve being stigmatized or marginalized on the basis of some difference from societal norms (Foote & Bartell, 2011). Rico identified as a gay White Canadian man of Italian heritage. He described feeling alone during his K-12 school years:

> I think that high school is such a huge part of a lot of people's lives. It wasn't a big part of my life because I had to hide the whole time … there was regret in me not to get involved in high school, I never did and I never got involved in university because I was kind of scared about everything and how I'd be perceived. (interview 1)

> I think that if I would have known that there was someone I could have talked to about it or just seen that, hey there are gay

> people out there and they're talking about it with a smile on their face, I would have loved that because my teachers in high school made fun of it. There was open homophobia from the teachers. (interview 1)

Rico struggled with his identity in high school and university and he talked about how he tried to bury himself in mathematics as "a way to escape the reality." (interview 1)

> I learned being in Catholic schools that there are some things that we just don't talk about. I didn't like that; it didn't settle right, but I was struggling with my own identity. I didn't know what to do about it so I agreed with them: let's just not talk about it and things will get better. (meeting 1)

During his fourth year of university, Rico decided that he had to be true to himself and he came out to his parents: "my parents call it the 'bombshell,' not anymore, but they call it the bombshell that I dropped on them. But it really did shake up my entire life." (meeting 1) After this, he set aside plans to do a master's degree and he took a year off to "get to know" himself and decide what he wanted to do. (meeting 1)

Being Mainstream

Being mainstream involves membership within a dominant group, and enjoying the associated power and privilege. Rico grew up in a "predominantly Italian" neighbourhood in a large Ontario city, and attended schools within his neighbourhood. Rico described the population of his elementary school as quite homogeneous and he stated that high school was the first time that he "experienced people of other cultures outside of the television screen." (meeting 1)

Rico described having supportive parents and he characterized his background as "higher middle class." (interview 1) He recognized that he was "on the privileged side" (interview 1) in terms of social economic privilege and White privilege.

Orienting Experiences

Orienting experiences involve becoming aware, through third party accounts, of the ways in which people are marginalized or oppressed. These experiences may be formal (academic experiences) or informal (conversations or reading) (Foote & Bartell, 2011). During the year he spent getting to know himself, Rico took a job as the Diversity Director for the student union at his university. He described this year:

> …that was my first taste of social justice, diversity, and everything that comes along with it. It was an amazing year and I learned unbelievable amounts and I met so many great people. And then I came to [pre-service teacher education] right after; I knew I had to do it. I had to go back into the classroom. I learned as the Diversity Director and at [pre-service teacher education] that we need to be talking about these things—that they're not going to just go away; we need to create some dialogue. (meeting 1)

During his pre-service teacher education, Rico learned more about equity and diversity and he was exposed to Cochran-Smith's (2004) six principles of teaching for social justice. Rico stated that in his teaching, he thought often about the six principles. Rico also talked about the realizations he made during pre-service teacher education about economic (dis)advantage and opportunity, the importance of building on what students bring to school with them, and about race as a potential means by which students are marginalized. He stated, "one of the things that really stuck with me in terms of equity was the idea of responding to the 'isms'" (interview 1) such as racism and sexism.

Results: Linking Life History and Pedagogy

During the year of PD, Rico's conception of equity and social justice seemed to remain relatively stable, although he appeared to formalize his understanding and make firmer connections to Cochran-Smith's (2004) principles. Rico had a good sense of what he wished to accomplish, but expressed that, as a new teacher, he needed support to learn how to accomplish it. In interviews and at PD meetings, Rico described his approach to social justice and equity pedagogy in detail. Here, we elaborate on three themes that arose in his discussions of pedagogy. We characterize each theme briefly, and then draw connections from the pedagogical theme to Rico's life experiences being the other, being mainstream, and orienting experiences (including the PD).

Being a Role Model

Many times, Rico described the importance of being a role model for his students. In particular, he wanted to be a role model of an openly gay man, and he wanted students to be able to talk to him about issues that are often taboo in high schools (e.g., cigarette smoking).

He directly connected his desire to be a role model with his experiences being the other. He remembered feeling as if he had to hide while he was in high school, and stated that he did not want his students to feel the same. He said,

> I made a commitment to myself to bring social justice into the classroom. I wanted to be that role model that I never had in high school for all those unsung kids. (meeting 1)

He also connected his desire to be a role model with his orienting experiences as Diversity Director and in pre-service teacher education. Through these experiences, Rico had come to realize the importance of creating awareness and talking openly about difficult issues. He elaborated,

> just talking about the issues, I feel like that's one step to being a role model for the students, even though I'm not necessarily connecting one-on-one with them. They're seeing someone standing up and saying that there are things that need to change and there are people that support you. I think that's a huge step in itself for some students. (interview 1)

Rico did not make any connections to his experiences being mainstream in relation to this pedagogical theme.

Social Justice Issues

Rico felt strongly that it was part of his responsibility as a teacher to address social justice issues and confront the "isms". He stated, "even though the curriculum may not address it, I think there is always room to discuss these things." (interview 3) Rico described three ways in which he addressed social justice issues in his teaching. The first way was as a special lesson. He said,

> I've brought it [social justice] into the classroom sometimes as a special lesson. We talked about anti-homophobia and watched some videos about it, we had class discussions, we've brainstormed, and we've just talked about how the words affect people. (interview 1)

A second way that Rico addressed social justice issues was by interrupting class when an unanticipated issue came up.

> I will often stop classes if someone raises a really interesting point or observation about the way people are acting or what they say to get students to think critically about information. If they saw something in the news or we read something in the textbook that I feel requires a little bit more thinking or there is something deeper inside, I want to spend more time. I think it's a very valuable lesson for them. (interview 3)

Finally, Rico also sought to address social justice issues by modifying the existing curriculum and resources. In the data management unit, Rico had students explore data, collected at their school, on student achievement by race. In the textbook, he explicitly worked to de-gender problems, using the pronoun "they" in place of "he" or "she," so as not to reinforce or perpetuate gender stereotypes. He also asked his students to examine their textbooks for gender stereotypes.

Rico connected his social justice pedagogy to experiences of being the other, and of being mainstream. Because of his experience being the other, he was passionate and knowledgeable about addressing issues of gender and sexuality. Because of his mainstream experiences, he felt less comfortable about addressing economic or racial issues. He explained:

> Do you start with what's important to you or what's more important to the greater good? Because I think I'm going to focus a lot on gender equity and sexuality, those are something I'm passionate about, so that's something I feel confident to bring into the classroom. But when it talks about social economic privilege or White privilege, I am on the privileged side of those so I have to do a lot of deconstruction on my own side before I can teach it. (interview 1)

In addition to his discomfort addressing economic and racial issues, Rico described other challenges. One big challenge he described was figuring out how to integrate social justice issues into the mathematics curriculum. (interview 1) He talked about the challenge of "getting good math (that was at an appropriate level of difficulty) to work with social justice" (interview 1). Rico was also concerned about "presenting social justice issues and then doing it wrong, or not considering the other side" (interview 1). Rico cited these challenges as a primary motivation for seeking out new orienting experiences such as this PD group.

After describing the lessons he had taught about gender and sexuality, Rico mentioned that there was a teacher at his school who was transgender who conducted "trans workshops" with students. His relationship with this colleague represents an orienting experience that helped Rico develop his repertoire of practice about social justice pedagogy.

Equitable Mathematics Pedagogy

Rico conceived of equitable teaching as meeting students' needs and giving each student the best possible education. In keeping with this focus, Rico sought to get to know his students. He did this in several different ways: he asked students about their future plans and he used surveys, free-writes, and exit interviews. Early in the year, Rico discovered that many of the students in his grade 12 Foundations for College Mathematics course had not chosen to take the class, but had been placed in it due to timetabling issues. He stated that student "motivation is the biggest challenge" (interview 1) and that "attendance is always a big problem" (interview 1). Rico found that the students "didn't express math interests" and have "very weak math skills" (interview 1). Many of his students said they planned to go to college, but not for anything math-related.

To meet his students' needs, Rico had multiple strategies. He used project-based learning that he hoped would emulate the college experience. He tried to address multiple abilities and to make mathematics relevant and useful for students by using mathematics as a tool to better understand social issues. He built in what he called 'checkpoints' to scaffold longer, project-based tasks. Rico tried to address students' apparent lack of motivation by designing "worthwhile tasks" and by building on what students bring to school with them (here he referred to Principle 2 of the six principles for social justice teaching, Cochran-Smith, 2004). He tried to "work with" students' inconsistent attendance.

These students did not act as Rico had in high school, when he was a high-achieving, university-bound high school student who enjoyed mathematics. Perhaps this is why he did not refer to his personal experiences as a young person (either being the other or being mainstream) when he described his equitable teaching strategies.

Instead, Rico referred many times to orienting experiences, especially Cochran-Smith's six principles for teaching for social justice, which he had encountered in pre-service teaching and in the PD. The action research format of the PD itself was also designed to be an orienting experience, in which Rico could take action in the classroom and study the results, learning more about his students and his teaching methods in the process.

In reflecting on the results of his action research project, Rico stated,

> The action that I did in class informed me about my students and their preferences. It also offered me a lens into what tasks they found interesting and what tasks are better left undone … I learned that getting to know how my students are thinking and what works and doesn't work for them was exactly the answer I was looking for. (written reflection)

Thus, according to Rico, the PD served as an orienting experience that shifted his thinking about who his students were, and what teaching practices might best serve them.

Discussion and Implications

Experiences being the other, being mainstream, and orienting experiences influence the development of an individual teacher's repertoire of practice by shaping their perspectives and by providing models of how to teach or how not to teach. According to Gutiérrez & Rogoff (2003),

> An important feature of focusing on repertoires is encouraging people to develop dexterity in determining which approach from their repertoire is appropriate under which circumstances. (p. 22)

We could extend their argument to say that another important feature is helping people identify when they need to seek new orienting experiences in order to expand their repertoire.

Kazemi & Hubbard (2008) theorized that teachers' participation in classroom and PD contexts co-evolved, such that teachers' participation and experiences in the classroom influenced their involvement in PD and vice versa. Wager (2010) and Foote & Bartell (2011) found that individuals' life histories, particularly the ways in which individuals make sense of their life histories, influence their practice. We propose a model (see Figure 1) that synthesizes both of these findings. This model shows how individuals' life histories, PD, and classroom experiences mutually influence one another and shape the development of individuals' repertoires of practice.

Whether they are aware of it or not, teachers' life histories influence their classroom practice. In Rico's case, his experience being the other allowed him to know directly what it is like to be marginalized. His orienting experiences (in his role of Diversity Director and his

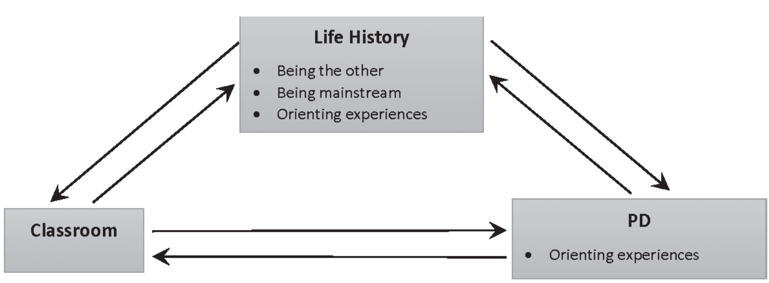

Figure 1: Interdependence of life history, PD, and classsroom experiences.

pre-service teacher education) introduced him to other ways in which people are marginalized. Together, these experiences informed his pedagogical decisions. When Rico privileged social justice goals over math goals in this particular class, he cited his experience being the other as a strong motivation for this pedagogical choice. Perhaps this choice was reinforced partially because of his own experience (being mainstream) as a high-achieving mathematics student (who did not need much support with mathematics). Another reason he cited for this choice was that, having gotten to know his students and their identified goals and interests, he felt this approach was appropriate for these particular students (orienting experience through teaching and PD).

Life history also influences teachers' engagement with PD. First, life experiences may motivate teachers to seek out PD. For example, from his experiences a student, Rico did not have a good model of how to teach equitably. His experience in pre-service teacher education gave him the beginning of a model and he sought PD to help him develop further. Second, life history shapes teachers' participation in PD. For this PD, Rico sought to learn effective ways to bring social justice issues into the mathematics classroom and to learn more about other ways that people are marginalized.

The influence of life history on teachers' classroom practice and their engagement in PD is not unidirectional. Experiences in the classroom or in PD may offer a teacher a new perspective or insight into their own life experiences, providing a means for them to reinterpret the meaning of their experience or reevaluate the importance of a particular experience.

According to Kazemi & Hubbard (2008), teachers' "experimentation in classrooms changes the nature of their conversations in PD and their changing participation in PD leads to new enactments of practice" (p. 432). It is particularly helpful when the PD structure is designed to align with teachers' classrooms. This allows teachers to make sense of the PD in terms of their own particular teaching context and to transform theory learned in PD into classroom practice. Rico's participation in this PD allowed him to realize the importance of getting to know his students better: their strengths and weaknesses, interests, and motivations. Rico also made connections between the iterative nature of action research and the process of changing his classroom practice,

> The action helped me get to know the students in this particular class. Then with the lessons learned, it will help inform my teaching in the next classes. And the cycle goes on. (written reflection)

This study shows that it is important and useful to have teachers articulate and reflect on their own life experiences (direct and indirect) with equity and diversity. Given the relative homogeneity of the teaching population and the relative heterogeneity of the student population, it is also useful to design professional development that provides teachers with orienting experiences. This will afford teachers opportunities to identify and reflect on any mismatches between their own life experiences and those of their students and allow teachers to better meet the particular needs of their students. Further research will allow for deeper understanding of the interdependent influences of life history, PD, and the classroom experiences.

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Ethics and Solidarity in Mathematics Education: Acts of Creative Insubordination

Beatriz S. D'Ambrosio,
Celi Espasandin Lopes
Miami University, Oxford, OH, USA;
Universidade Cruzeiro do Sul, SP, Brazil

In this report we describe the professional life-histories of one teacher built from auto-biographical narratives. Solange shared an experience about a project developed with her students. Using her narratives we identified how the project about toys and games resulting from her acts of creative insubordination promoted the ethics education of her students. Through a pedagogy of inquiry, problem posing, and problem solving, young children contemplated issues of social equity and grappled with understanding the rights of children and the struggles of society to assure the well-being of all children. The students proposed and implemented activities within their control, demonstrating solidarity toward children less privileged than themselves, and showing evidence of ethical and moral development.

Introduction

In this article we discuss the pedagogical practices of a teacher of seven year-old children in a Brazilian school. Using a project centered on the Declaration of Children's Rights, she enacted her creative insubordination with respect to curriculum, transforming it in ways that engaged students in reflecting on what happens in society and in the world in which they live.

According to D'Ambrosio and Lopes (2014) the first discussions of creative insubordination appear in 1981 when Morris et al describe an ethnographic study conducted in the Chicago schools involving 16 school principals. In that report, the authors defined acts of creative insubordination as occasions when the principals made decisions that were contrary to the mandates of the school districts. In general, the need to be disobedient occurred with the intent of diluting the dehumanizing effects of certain rules, protecting the professional decisions of the teachers that were based on the best interest of their students. The disobedience occurred to preserve ethical and moral principles, or to assure pedagogical practices grounded in principles of social justice.

Instances of breaking rules for the benefit of those one serves are called acts of "responsible subversion" in the nursing literature (Hutchinson, 1990). In the education literature, primarily in studies of school administrators, the preferred terminology is "creative insubordination" (McPherson & Crowson, 1993; Haynes & Licata, 1995). More recently, in the specific area of mathematics education, Gutiérrez (2013) describes teachers' creative insubordination as necessary to set in motion the political action of mathematics teachers. For Gutiérrez, the political actions of a mathematics teacher would include acting in opposition to the standard curriculum, to evaluation practices, to rules and guidelines, when any of these seem to be unfavorable to the learning of one's students.

Teachers are the key to creating a classroom environment with rich opportunities for learning. It is their responsibility to propose and organize tasks and to coordinate developmental learning activities for their students.

> The teacher acts as a person and her actions constitute her professional self. Teachers' actions are both product and process and correspond to their unique individualities. Behind their actions are their body, their intelligence, their feelings, their aspirations, and their ways of understanding the world. All of this is projected in each teacher's undertakings, thus constituting the biography of the agent (Sacristán, 1999, p 31, *authors' translation*).

In this sense, each teacher is unique, and defines his/her practices based on personality traits, feelings, beliefs, and expectations. When teachers are moved to enhance student learning and invest in improving the conditions under which such learning occurs, they create and put into motion standards and procedures that are aligned with their professional identity. At times these attitudes are responsibly subversive, and result in acts of creative insubordination.

As we considered teachers as protagonists in the construction of their practices and identities, we chose to conduct a study through the narrative of Solange, understanding, as does Clandinin (2013), that narratives reveal how humans experience the world and assist in understanding how they characterize their human experiences. Since education is the construction and reconstruction of personal and social histories, narrative research allows for greater clarity about the phenomenon being investigated.

Solange's narrative reminds us that "the essence of the ethics of diversity is respect for, solidarity with, and cooperation with the other (the different). This leads to quality of life and dignity for all" (D'Ambrosio & D'Ambrosio, 2013, p. 21).

As Freire (2003) suggests the social and political solidarity is essential to our construction of a society that is less reprehensible and distressing, in which individuals can be themselves. It will be through a democratic educational process that individuals will come to understand themselves as social and historical beings, and dare to imagine and work toward a better society with dignity for all.

Methodology

In this study, we analyze both the oral and written narratives of Solange, understanding that the narrative takes on the dimension of the phenomenon under investigation and the method of investigation: the data are the (auto) biographical reports, shared with the researchers through interactive dialogue and in written form. Through (auto) biographical narratives the teacher shared specific dimensions of her individual life, as requested by the researchers. The narratives were then analyzed, according to specific criteria and procedures, to give meaning to the story (Bolívar Botia, Domingo Segovia, & Fernandéz Cruz, 2001). The criteria identified for analysis in this study were the acts of creative insubordination used by the teacher including: breaking from the prescribed curriculum; placing students at the center of the educational process; attending to students' understanding in light of the complexity of the topic; posing the challenge to students of experiencing the problem; venturing to place students in a situation where they could more readily "live" the unfamiliar reality and intervene in it; encouraging students to draw their own conclusions as they made sense of reality and shared their ideas with others.

The interviews that generated the narratives were conversations, rather than formal interviews. In two conversations for a total of four hours Solange was asked to tell the researchers about her professional life history and was prompted for clarification or for greater details about the events as they were described in an effort to engage both interviewee and researchers in co-constructing the narratives (Clandinin, 2013). As a follow-up Solange provided the researchers with a written journal including several entries that specified the details of her lesson and included several pieces of student work that she had used to assess their learning.

The interviews were transcribed and all the material analyzed with several cycles of readings and hermeneutic interpretations. These cycles resulted in conjectures about Solange's actions of creative insubordination. According to Bertaux (2010) in the analysis of (auto) biographical narratives it is not important to extract all of the details that it contains, but instead it is important to consider only those elements that are relevant to the specific research study. Based on Bertaux's considerations, for this study, we extracted the indicators of creative insubordination that allowed us to better understand the components of the teacher's actions as she reinvented her practice in order to better meet the needs of her students.

Data Analysis

Introducing Solange

Solange studied to be an elementary school teacher and had the opportunity to take a course from Paulo Freire. To this day she teaches young children early literacy and numeracy and feels very successful in her profession. Throughout her career, she sought opportunities for professional development, by participating in courses, attending events, and listening to lectures. She participated in classes, study groups and research groups regarding mathematics teaching and learning.

Solange's Narrative

Solange teaches 7 year-old children in a private school in the interior of São Paulo, Brazil. In this school, one of the pedagogical approaches

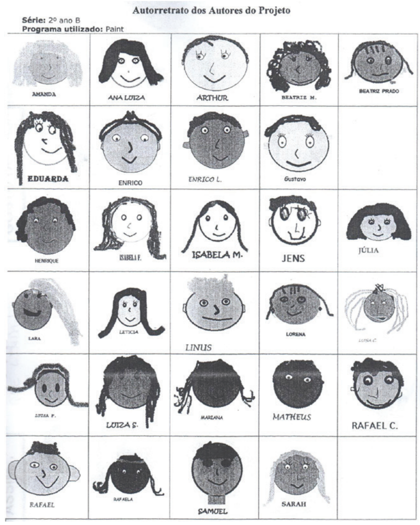


Figure 1: Children's self-portraits. Source: Solange's written narrative, 2014.

used is called "Class Projects", where teachers become co-investigators with their students. The focus of the project to be developed in the classrooms is chosen by the team of teachers and administrators, and is based on an issue or problem raised in the Universal Declaration of Human Rights, or in the Statute of Children and Adolescents[1]. The topics chosen relate to life within the community, people's lives, their needs, their interests and are appropriate for the age group of the students. Each year the children experience a unique Class Project appropriate for their age.

In this way a "landscape of investigations" (Skovsmose, 2014, p. 45) is created. This landscape permits students to explore the world in which they live learning that the interactions and the relationships with knowledge are essential components of life in society. In her narratives, Solange describes how the students asked questions that were of interest to them and that would guide their inquiry during the Class Project titled: Children's Play. After discussing the coherence, pertinence, and adequacy of their questions the class settled on the following list to guide their inquiry:

1. How can we build toys and how are the toys that we buy actually made?

2. Which spaces in school are adequate for playing? Why are they adequate?
3. Are rules important in play? Why? How can we respect them?
4. How can we care for ourselves and others during play?
5. Why are there children who can't play? How could we help them? (Solange's written narrative, 2014)

In this article we draw from the teacher's narrative to discuss question number 5. The children, along with their teacher, developed an action plan to investigate the question. In particular they sought to: learn about the Declaration of Children's Rights; understand each other's life histories; observe similarities and differences in their histories and human condition; learn about the stories of children who work and cannot play; draw conclusions. (Solange's written narrative, 2014).

The students dove into their inquiry project by researching the 7th Principle of the Declaration of Children's Rights.

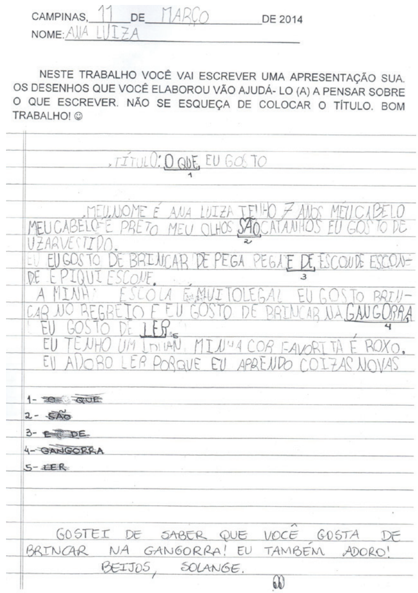
> The child is entitled to receive education, which shall be free and compulsory, at least in the elementary stages. He shall be given an education which will promote his general culture and enable him, on a basis of equal opportunity, to develop his abilities, his individual judgment, and his sense of moral and social responsibility, and to become a useful member of society. The best interests of the child shall be the guiding principle of those responsible for his education and guidance; that responsibility lies in the first place with his parents. The child shall have full opportunity for play and recreation, which should be directed to the same purposes as education; society and the public authorities shall endeavor to promote the enjoyment of this right. (United Nations, 1959)

Having read this principle and worked to understand the vocabulary in it, the children discussed it in light of their lived experiences, learning of the life histories of each child and searching for similarities and differences in their stories. In this activity the teacher worked with the children's literacy development as suggested by the Freirean perspective in which reading isn't walking among the letters, but rather interpreting the world and being able to use words to interfere in the world through actions. In this way literacy development in language and in mathematics occurs in the act of coming to know.

Translation:

Title: What I like

My name is Ana Luiza. I am 7 years old. My hair is black, my eyes are brown. I like to wear dresses. I like to play chasing and hide-and–seek. My school is very cool I like to play at recess and I like to play on the teeter-toter. I like to read. I have an…. My favorite color is purple. I love to read because I learn a lot of new things.


CAMPINAS, 11 DE MARÇO DE 2014
NOME: ANA LUIZA

NESTE TRABALHO VOCÊ VAI ESCREVER UMA APRESENTAÇÃO SUA. OS DESENHOS QUE VOCÊ ELABOROU VÃO AJUDÁ- LO (A) A PENSAR SOBRE O QUE ESCREVER. NÃO SE ESQUEÇA DE COLOCAR O TÍTULO. BOM TRABALHO! ☺

TÍTULO: O QUE EU GOSTO

MEU NOME É ANA LUIZA TENHO 7 ANOS MEU CABELO MEU CABELO É PRETO MEU OLHOS SÃO CATANHOS EU GOSTO DE UZAR VESTIDO.
EU EU GOSTO DE BRINCAR DE PEGA PEGA E DE ESCONDE ESCONDE E PIQUI ESCONE.
A MINHA ESCOLA É MUITO LEGAL EU GOSTO BRINCAR NO REGREIO E EU GOSTO DE BRINCAR NA GANGORRA
EU GOSTO DE LER.
EU TENHO UM INDIAN. MINHA COR FAVORITA É ROXO.
EU ADORO LER PORQUE EU APRENDO COIZAS NOVAS

1- ~~O QUE~~
2- ~~SÃO~~
3- ~~E DE~~
4- ~~GANGORRA~~
5- ~~LER~~

GOSTEI DE SABER QUE VOCÊ GOSTA DE BRINCAR NA GANGORRA! EU TAMBÉM ADORO! BEIJOS, SOLANGE.

Figure 2: Ana Luiza’s text. Source: Solange’s written narrative, 2014.

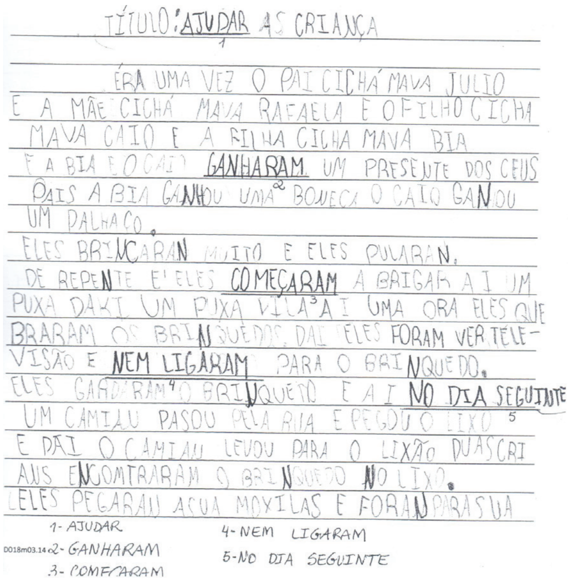
We interpret Solange’s actions of working with the children to understand the legislative language of this official document as an act of creative insubordination. Solange found a way to convert the complex language of the document into language that was accessible and comprehensible to these very young children, who were just beginning to learn to read and write. She helped them to make sense of text typically belonging to the world of adults.

On a day following the discussion of the Declaration of Children’s Rights, the children created a collage of their self-portraits as seen in Figure 1.

The development of the self-portrait occurred in an effort to produce a text with the central theme: “Who am I?”, an example of which is shown in Figure 2.

Each child read his or her text to the group so that they could get to know each other a bit better. They each chose one of their favorite toys from when they were younger, to bring to the classroom and include in their stories. The goal was to identify the similarities and differences in their life histories and social condition.

The children learned that the first social group to which they belonged was the family and as they grew they became part of several



TÍTULO: AJUDAR AS CRIANÇA

ÉRA UMA VEZ O PAI CICHÁ MAVA JULIO
E A MÃE CICHÁ MAVA RAFAELA E O FILHO CICHA
MAVA CAIO E A FILHA CICHA MAVA BIA
E A BIA E O CAIO GANHARAM UM PRESENTE DOS CEUS
PAIS A BIA GANHOU UMA BONECA O CAIO GANOU
UM PALHACO.
ELES BRINCARAN MUITO E ELES PULARAN.
DE REPENTE E ELES COMEÇARAM A BRIGAR A I UM
PUXA DAKI UM PUXA VILA A I UMA ORA ELES QUE
BRARAM OS BRINQUEDOS. DAI ELES FORAM VER TELE-
VISÃO E NEM LIGARAM PARA O BRINQUEDO.
ELES GARDARAM O BRINQUETO E A I NO DIA SEGUINTE
UM CAMIAU PASOU PELA RUA E PEGOU O LIXO
E DAI O CAMIAU LEVOU PARA O LIXÃO DUAS CRI
ANS ENCOMTRARAM O BRINQUEDO NO LIXO.
ELES PEGARAN A SUA MOXILAS E FORAN PARA SUA

1- AJUDAR
2- GANHARAM
3- COMEÇARAM
4- NEM LIGARAM
5- NO DIA SEGUINTE

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Figure 3: Ana Luiza's written text. Source: Solange's written narrative, 2014.

other groups, such as classroom, school, club, swimming class, ballet, gymnastics, play group, etc. This stage of the project enabled the children to identify the different groups shaping their identities.

During the next stage of the project, which involved learning the stories of some children who work and can't play, the teacher launched the conversation with a piece of children's literature titled "Brinquedos" [Toys] (Neves, 2006). The text-less picture book depicts the story of two children who receive gifts: a girl is given a doll and a boy is given a clown. After playing quite a lot with their toys, the toys get old and are thrown in the garbage. The old toys end up in the city's garbage dump yard. There, two children going through the garbage to find anything salvageable, find the toys, fix them, and begin to play with them. The very expressive illustrations convey the characters' expressions of delight upon finding the old toys and their expression of attention as they carefully fix the toys. Having "read" the story as a whole group, each child wrote paragraphs to retell it based on the illustrations, as seen in figure 3.

The experience with the book helped the students to become aware of the reality of the lives of other children with whom they do not normally interact and to realize that they share the same desires to play and to have toys, and that they valued the toys they were able to have. Solange contemplates:

> In this context, the children have the opportunity to realize that there exist social inequities in our society and that the children of the dump also have the desire to play and that they do play. This experience resulted in a perception that in our society not all children live in the same conditions. The children realized that there are life conditions that are very different. (Solange's narrative, 2014)

The teacher recognized an opportunity to delve more deeply into the topic, suggesting that they read another piece of children's literature titled: "Serafina and the child who works" (Azevedo, Huzak & Porto, 2005). Her goal was to help the students better understand the human condition of many children in the world, who have to work. Solange wanted to sensitize them to their condition of privilege, and to an understanding that these inequities must to be resolved by society, that it is a problem for all of us to consider and strive to solve.

However, Solange did not want the students to simply read about these realities, so she planned a field trip to a public school for children ages 3 to 5, which caters to a very poor community. Solange once again turned to creative insubordination as she took the children beyond the boundaries of the school, in order for them to experience an unfamiliar reality. With this subversive act she challenged the children to contemplate the reality of others, less privileged than themselves. The age of the children and their socio-economic status as members of an economically privileged social class had hindered their opportunities to get to know children of different economic status living in poverty stricken communities.

She states that her goal was to "reduce the sentiments of indifference and discrimination towards the children living in such social conditions, those from such a different socio-cultural reality" (Solange's oral narrative, 2014).

In preparation for the trip, each student chose a book and practiced reading it, in order to read the book to a child during the school visit. In addition, the students planned to gather data in order to unveil the types of play activities that are part of the lives of the young children. For this, the students developed interview questions to ask of the children during the school visit. The students also built, with the help of an artist who crafts toys, four toy cars made of wood. The interaction with the toy-maker provided students with a perspective on how human creations can result in objects that bring joy and a sense of well-being to others.

The experience of visiting a school that was very different from theirs was very significant for the students. Solange shares that her students were able to perceive some similarities with the children they met, like the fact that they all enjoy playing and creating games. They were surprised that some of the children were unfamiliar with some of their favorite games. During the visit, they had lunch at the school, eating the free lunch that is fed to children in the public schools. Coming to know the reality of others with less privilege than themselves permitted Solange's students to reflect on their own reality, to appreciate the foods that are available to them and that they take for granted, and to reflect on the meaning of wastage and the value of what they have.

Solange's daring act of taking her students to experience an alternative reality, and to co-develop with them a plan to intervene in that reality is evidence of her responsible subversion She seized an opportunity for developing in the students the reflective tools needed for responsible citizenship. The students had the opportunity to learn that legislation alone is not enough to ensure the well-being of children. Instead, it is possible and important to intervene in reality with actions of solidarity. She enticed her students to imagine and create acts of insubordination toward the social status quo.

Solange shared that upon returning to school, the students composed a collaborative text in which they discussed what they had learned. They stated that they enjoyed meeting new children and teaching them new games and reading them stories unfamiliar to them. They realized that their visit brought joy to the children they visited and the students themselves expressed joy and pleasure in the work they had accomplished.

The development of the collective text provoked the students to communicate their feelings, their perceptions, and their conclusions. The teacher acted with responsible subversion as she encouraged the students to share their ideas and listen to the ideas of peers, co-constructing meaning based on the readings of the world that they were able to do at this young age.

DO QUE ELES GOSTAM DE BRINCAR NA ESCOLA?

NOME DA BRINCADEIRA	QUANTIDADE DE ALUNOS
Pega pega	4
Boneca (o)	4
Carrinho	4
Quebra cabeça	3
OUTRAS BRINCADEIRAS: Esconde esconde Parque Patati patatá Cabelereiro Ferramentas Escorregador Bola	

DO QUE ELES GOSTAM DE BRINCAR EM CASA?

NOME DA BRINCADEIRA	QUANTIDADE DE ALUNOS
Carrinho	2
Correr	2
Boneca (o)	5
Bicicleta	2
OUTRAS BRINCADEIRAS Ursinho de pelúcia Amarelinha Pular corda Polly Max Stell Barbie Homem aranha Desenhar Nada Pingue Pongue Esconde esconde	

COM QUEM ELES COSTUMAM BRINCAR?

QUAIS PESSOAS?	QUANTIDADE DE ALUNOS
Papai	1
Mamãe	1
Sozinho	1
Amigos	13
Primo	1

QUAIS ATIVIDADES VOCÊ FAZ FORA DA ESCOLA?

NOME DA ATIVIDADE	QUANTIDADE DE ALUNOS
Desenho	2
Pintura	1
Futebol	1
Capoeira	1
Karatê	1
Não faz atividade	14

Figure 4: Interview data tabulated by the children. Source: Solange's written narrative, 2014.

To enhance the student's reading of the unfamiliar reality, the data collected were analyzed. The students tabulated the responses to the interview questions as shown in figure 4. The organization of the information in tables provided another means of determining similarities and differences in comparing the realities of the lives of the children with their own. For example, from the interview data the students found that while they play in groups when at school, they typically play individually at home. Individual play occurs with commercial toys and electronic games. On the other hand, the children of less privilege tend to play in groups both at school and at home. Having no access to electronic games and little access to commercial toys, they are more likely to play together, to play outside, and to play cultural games such as chase, and hide-and-seek.

Another significant fact that the students discovered was that, differently from themselves, the children they visited did not engage in out of school activities, like ballet lessons, swimming lessons, music lessons, foreign language lessons, clubs, etc. This realization further added to their understanding of socio-cultural differences, social inequities and injustices. They came to appreciate the privileges of their lives and the conditions often unavailable to others. This is the first step towards building solidarity.

Solange claims that the visit to the public school was "a way to make real the right of children to play and enjoy themselves, as is stated in the 7th principle of the Declaration of Children's Rights" (Solange's oral narrative, 2014). The narrative shared by Solange illustrates the possibilities of developing the ethical dimension of mathematics education in problematizing with students issues of social justice present in the culture of childhood.

Conclusions

In several instances throughout the experience Solange had the students pose and solve problems (Lopes, 2011). Solange's views that the learning of mathematics and statistics is enriched when children engage in solving problems that they themselves identify within the real world, shaped how she provided opportunities for students' learning, as they collected information that became data for analysis and understanding. At the heart of Solange's instruction was the doing of mathematics and statistics as students communicated, reasoned, investigated, collected information, and used the data to analyze and understand, or "read," the reality of the world around them.

Throughout the experience, as the students unearthed information and attempted to make sense of it, they found themselves immersed in a world of differences and contradictions. Through Solange's acts of insubordination towards the prescribed curriculum, the students contemplated economic and cultural realities that were unknown to them. The pedagogical moves taken by the teacher resulted in the collective construction of knowledge with emphasis in humanizing children as they acquired moral consciousness regarding their childhood in contrast to the childhood of others.

This type of experience creates a foundation for children's understanding of societies in which humans are living in inhumane conditions. It will encourage children to engage in the deep and robust critical reflection necessary for solving the social problems, the economic problems, and the problems of preservation of the world's natural resources in order to envision a better world with dignity for all (D'Ambrosio, 2014). The teacher's practice promotes the development of a generation of human beings who may be able to overcome the limitations of the current adult generation that has been unable to deal with the problems of society and the world. The experiences of this group of children encouraged their development as ethical human beings, with compassion, solidarity, and understanding social justice as resulting from actions that are much more complex than simple acts of charity.

Notes

1. The Statute of Children and Adolescents is an official document (Law 8,069 of July 13, 1990) in Brazil that legally assures the human rights of all children.

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Challenging Ableism in High School Mathematics

Rossi D'Souza
Homi Bhabha Centre for Science Education, Tata Institute for Fundamental Research, Mumbai

Although disability is case of oppression wherein disabled people are marginalized, denied access and full participation in society, it is generally perceived simply as something wrong with a person. Ableism, a central part of disability, manifests in mathematics education when mathematics is presented, taught, and even defined in terms of the visual (assuming that all can see). Education research focussing on disability rarely challenges ableism in the curriculum and focuses mostly on how disabled children learn or how they can be taught. This contributes neither towards liberation nor empowerment. Using critical ethnography in my fieldwork, I argue for an expanded understanding of mathematics for social justice as well as for justice towards mathematics as a discipline.

Introduction

Disability means different things to different people. Although it needs to be addressed as a case of social oppression wherein disabled people are marginalized, denied basic human rights, access to knowledge and full participation in society, disability is popularly understood simply as something wrong with an individual.

This "*individual model* of disability" identifies the causes of disability as being limited to the functional limitations or psychological losses, which are assumed to arise from impairment (Oliver, 1990). Such understandings also underpin the *charity model* where disability is perceived as a sad event that happens to unfortunate people who are rendered abnormal and deserve sympathy. The *medical model* adds on to the charity model by assuming that disability is a medical condition suffered by "damaged" or "disfigured" or "abnormal" individuals who need to be "normalized" or "fixed". The attitudes shaped by understandings of disability are reflected in the actions of people like doctors and teachers who work with disabled people.

Dominant approaches towards working with disabled people rarely recognize disability as a form of oppression. Even when abled people (who advocate visual methods of educating) discover themselves to be oppressors, they do not really stand in solidarity with disabled people but, as Freire says, "Rationalize (their) guilt through paternalistic treatment of the oppressed, all the while holding them fast in a position of dependence..." Rarely do educationists attempt to develop pedagogies in solidarity with disabled people.

Charlton (2000) argues that due to having internalized their oppression, disabled people have developed a false consciousness. For example, during my field visits I'd have people blaming blindness for their inability to excel in mathematics. A false consciousness prevents disabled people from recognizing that, as Charlton (2000, p. 27) says, "their self-perceived pitiful lives are simply a perverse mirroring of a pitiful world order." Elisabeth Fiorenza (2001) highlights the acceptance for self-alienation by stating, "The self-alienated consciousness is accepted because it is seen as natural and common sense. It convinces people that situations of oppression and dehumanization are normal" (p.111)

Models and Official Definitions of Disability

The dominant model of disability is essentially inclined towards the individual model that focuses only on, and puts the onus of disability related problems on, the individual. The social model, on the other hand locates the source of the problem of disability in society, how it is organized and the attitudes of society towards the physically/mentally impaired.

Inclusion

With regard to education, the philosophy of Inclusion acknowledges that all children are different and that all children can learn. Idol (1997) elaborates that in an inclusive school "a student with special education needs attends the general school program, enrolled in age-appropriate classes 100% of the school day. This is as opposed to *mainstreaming* where a *student with special education needs* is educated partially in a special education but to the maximum extent possible is educated in the general education program (Idol, 1997). Thus, with regard to visual impairment and the curriculum, an inclusive school should acknowledge that the emphasis on the visual in the curriculum would disadvantage some students (who need not be visually impaired). An inclusive school would also not identify individuals for whom the schools would need to be inclusive. In fact, identifying a student as "included" is a form of exclusion.

The social model of disability can also inform discourses around issues related to formal education. For example, as partially/non-sighted students have the option to select the 7th standard level mathematics examination paper for their 10th standard exams, a lot of them are encouraged (or not discouraged) to do so Here, due to society's inability or insensitivity or impatience to teach non-sighted students, society's action is rationalized by blaming disabled students. Consequently, "not having passed the 10th standard Mathematics exam" is used to justify their exploitation by, say, paying them a smaller salary for as much work or not being offered a job or the opportunity for higher studies that demands 10th standard level of mathematics.

In most cases, non-sighted people are employed but not assigned any work. However, companies are provided benefits for having disabled people on board.

Theoretical Framework

I advocate a Freirean perspective on education wherein authentic education is a means towards humanization. Such education is carried out in a dialogic manner in which there is no dichotomy between teacher and students, but both work together towards learning the causes of (an oppressive) reality and thus transform that reality. In my work, this problem is the Disability oppression in which the visual hegemony present in mathematics education plays a significant role.

I argue that Ableism assumes an (idealist) essence of the "Abled Body". However, such an Abled Body does not exist in the *material* world. Every body is in essence a continuous struggle of being simultaneously abled and disabled. While social structures are designed presupposing this "Abled Body" no one really fits this category albeit some benefit over others due to it. Furthermore, each person holds simultaneously a dual oppressor-oppressed consciousness in a constant dialectic conflict.

I claim that mathematization is a part of being human. However, the norm is to teach and define mathematics in a visually dominated manner that works towards alienating many people, especially those who are not sighted. However, this visual hegemony also dehumanizes the (predominantly sighted) benefactors of the education system. For example, sighted students are compelled to compete against their disabled friends in a dog-eat-dog world of a market controlled education system. And the winners of this rat race often celebrate this feat. This is not human. Thus, the oppressor and the oppressed are both manifestations of dehumanization. Dehumanization occurs at even more levels, for example when attention is aimed at understanding reality (so as to adapt to it) rather than the *causes* of this reality since, *being human* implies the conscious transforming of reality rather than merely adapting to it. In schools, presupposing the oppressive reality, while adapting children (especially disabled children) to it, is often the norm.

Without a good understanding of the nature of education, mathematics, etc., one may presuppose that these are essentially, activities in which the able-bodied excel, thereby assuming the need to opt for a lesser ambition. This assumption is also a contributing factor towards (disability) oppression. Critically recognizing the causes of disability mediates in the emergence of a consciousness of a human being in the process of achieving freedom. This is as opposed to "being" either disabled (and in need of special schooling) or abled (and in the need of normal schooling).

Liberation would also include the effort to reclaim the right to own and legitimate abstractions developed through the labour of mathematization, rather than have it dismissed while blindly purchasing that mathematics which is sold by the dominant elites. Recognizing their rights would mean a change in consciousness – from that of those that cannot excel in mathematics to those whose rights to mathematize have been stolen.

The revolution process of transforming an unjust reality can only be carried out by oppressed since it is only they who truly understand the brunt of the injustice meted out upon them and "can better understand the necessity of liberation" (Freire, 1970). The role of a teacher-learner can only be to engage in critical dialogue with the learners.

Research Methodology

I used a participatory research methodology within the domain of critical qualitative research as elicited in Cohen, Manion, & Morrison (2011). Critical ethnography is one such research methodology which addresses issues of oppression, empowerment, and liberation. As opposed to ethnography, which deals with *what is*, critical ethnography deals with what *could be*. Here the researcher has an explicit agenda to *achieve liberation* of people. In critical ethnography, the move is from describing a situation, to understanding it, to questioning it, and to changing it (Cohen, Manion & Morrison 2011, p. 243). In critical ethnography, the role of the researcher is to "use the resources, skills, and privileges available to her to make accessible—to penetrate the borders and break through the confines in defence of—the voices and experiences of subjects whose stories are otherwise restrained and out of reach." (Madison, 2004)

Trueba (1999) argues that "all education is intrinsically political" and critical ethnography must "advocate for the oppressed by: (a) documenting the nature of oppression; (b) documenting the process of empowerment - a journey away from oppression; (c) accelerating the conscientization of the oppressed and the oppressors - without this reflective awareness of the rights and obligations of humans, there is no way to conceptualize empowerment, equity, and a struggle for liberation; (d) sensitizing the research community to the implications of research for the quality of life—clearly linking intellectual work to real-life conditions; and (e) reaching a higher level of understanding of the historical, political, sociological, and economic factors supporting the abuse of power and oppression, of neglect and disregard for human rights, and of the mechanisms for learning and internalizing rights and obligations."

As I believe and hypothesize that the visual hegemony in mathematics education contributes to the oppression of disabled children, the documentation and conscientization included (albeit not be limited to) the assertion that the nature of mathematics is not inherently visual.

Fieldwork

Background

My fieldwork involved meetings with students of a school officially named Vivek Education Foundation School for the Blind. It is a registered NGO that was set up in 2005 and made official in August 2010. The school is essentially a study centre, which facilitates the learning of partially/non-sighted students most of whom attend regular schools (with blackboards and teachers with no knowledge of Braille). Some of the students do not attend any regular school but are preparing for their open schooling exams. Although a total of 45 students are registered with the school, around fifteen attend regularly. My fieldwork was carried out working with these fifteen students. Every student has a different history with regard to his/her eyesight. While five students are congenitally blind, one lost her eyesight due to glaucoma, another is losing it due to the same. Others have either retinal or optic nerve related problems.

Field Visits

I began visiting the school for the blind since the last weekend of June 2013 just for recreation (then, not intending to pursue my fieldwork here). Being a musician, I was offered a permanent slot 2 hours per week where I would teach music among other things. We would have discussions related to mathematics, science, and social issues. These activities helped me to gain acceptance into their group. In the following months, in addition to the Saturday visits, on weekdays, I would volunteer to read out their textbooks so as to help them study (not as part of my fieldwork). I would also (on their request) prepare notes for them to memorize. During these times, the tutoring would lead to discussions, and incidents of disability oppression would come to light. This led me to read about disability oppression and reflect on what should be done. I finally began my official fieldwork, which was planned as a series of sessions in mathematics. After discussing which topic should be learned, we narrowed in on divisibility. We also had brief discussions on the nature of mathematics. I audio-recorded our sessions (including those that involved just reading textbooks to them) and along with two colleagues, noted my experiences.

Recognition of Oppression

Our conversations contained indications that the students recognized their oppression and were conscious of being dehumanized and were yearning for freedom and justice. This included statements (by a student), "If mathematics is something done in the head, why is there such a heavy emphasis on using a paper and pencil?" This 9th std. student also expressed his discomfort with questions based on diagrams given in the book and unnecessarily long equations in Algebra. There were also instances wherein a student suggested we protest against an incident wherein a group of disabled students were denied entry into a boat. This was despite the fact that the incident was presented just to justify booking a bus for an outing.

On one occasion, one of students (a 9th standard visually impaired girl) spoke of how she felt included in her old school and experienced exclusion when she had to be shifted into a new private school (although both were "normal" schools). She said that:

> … in the new school during exams, my writer and I were made to sit outside since my reader would be a source of disturbance to the other students. The bench there was very uncomfortable. The teacher was concerned about the other students getting disturbed with no concern towards what I was going through.

She elaborated on the role played by her friends on whom she depended for taking notes. A similar observation was made during a conversation another student (who had passed her 12th std. exams) who used to have partial vision before losing all her sight. She said:

> I could not see (the blackboard) even (while) sitting on the first bench. So then what my friends would write that only I would copy. Taking books, seeing from their note books like that.

On another occasion, she spoke of how her hobby was to read books and that she read one book a day. After losing her sight, she stated that she lost her passion for reading. She said, "I can't read books.

Braille books are there but print books... I used to read one book in one day, so that passion is gone." Having passed her 12th, she was seeking admission to colleges that would accept visually impaired students. Although she preferred St. Xavier's College which is well equipped with a resource (and research) centre for the visually challenged, called XRCVC, she expressed her compulsion to opt for a less preferred college since she would need help reaching college. This restricted her access to even XRCVC.

Alienation

There was also a case of a partially sighted student being labelled as a slow learner who expressed his inability and hence, refusal, to learn mathematics.

Pity and Dehumanization

There was no observed instance of abled teachers recognizing the process of dehumanization that came with reference to imposing an ableist curriculum on disabled children or otherwise.

Sometimes during our sessions, there would be visits by elite guests. The students would introduce themselves and talk about their impairments. The school owner would then speak of the children's achievements. He would then speak of the "charity" work done by volunteers (including me) for the "unfortunate" children. On certain occasions there were cultural events held at either National Association for the Blind (NAB) or sometimes at The Blind School where the students were expected to perform. The students would prepare a dance routine and sometimes a drama. We would also sing together while I played the guitar. These events were generally sponsored by either individuals or some organization. A sponsor would then speak about the efforts done by his organization to benefit the blind students. In one of the speeches the speaker mentioned, in front of the students that they had nothing before being part of NAB. On another occasion, a student who began to speak of disability was interrupted by a volunteer who repeatedly said, "You are not disabled!" During these events I would speak to the visitors. One of them told me that on the following day a math teacher would be visiting to teach them Mathematics to help them do well in the banking examination. On another occasion, one of the teachers of the school told me that banking is the best option for the children. While emphasizing on the effort to help the children to get employed in bank, she said, "The bank is the best place they can get a job since their options are less." She continued, "In my office also there are a lot of openings and many people who work there don't do anything since there is less work to be done." This teacher was herself visually impaired. However, the aspirations of the children did not include banking or allied disciplines. One of the congenitally blind student wishes to pursue higher mathematics. Another student has an interest in languages but decided to pursue management since that would "ensure her getting a job". One wishes to be a lawyer and another a (natural) scientist.

I visited a "special school" in Bangalore that worked with "students with learning disabilities". The school was equipped with a loom used to knit fabric. This was to make students employable. A school authority spoke of how her ex-student was content with a (current) monthly salary of 2000 rupees stating that, "His parents don't expect much from him, so he's happy".

On Their Learning of Mathematics

On the first day of the teaching, there were ten students present. Their ages ranged from 9 to 20 years. Age did not turn out to be an issue regarding what would be an "appropriate level" of mathematics. We began by asking the students which topic they found difficult. Most of them said "steps." (It took me weeks to understand that this "difficulty" underlay a case of ableism, after being pointed out by a student) After an hour of discussions and deliberations with the students, we decided to carry out daily sessions beginning with the topic of multiplication, which later reshaped into the topic of divisibility. On asking whether they had any specific difficulties, one of the students asked the rationale behind putting the number zero below the units place while multiplying two digit numbers. To elaborate her query she gave the example of 42 into 42. On deciding to explore the problem, all the children there expressed their desire to work it out. Five students used the *Taylor mathematical slate* (an instrument used to perform mathematical operations on a grid using hexagonal pegs whose orientations indicate different mathematical objects) to work out the problem, four used their notebooks while one just gave out the answer and explained how he did it. This student was 15 years old and in the 9th standard his explanation was "42 2's are 84, hence 42 20's would be 840 and double of that is 1680. Now since we are left with 2, we add 84 giving the answer, 1764."

As time progressed each one of the students arrived at the same answer independently and all explained their procedures through narratives. All the students were fluent in performing multiplication of three digit numbers with either the slate or using their notebooks or without any tangible objects (i.e. purely through thought). They did, however, rely on their peers for explanation of the questions posed. On asking the rationale behind certain procedures, however, most did not understand the question. So to be specific, I asked whether successive addition would yield the same result i.e., if the multiplication table were extended till 42, would the result be the same? A few answered in murmurs. One student did not understand what it meant to have a multiplication table going beyond 10. Her understanding of the tables was that it ended at 10 with no relation among the numbers in the tables. It turned out that a few other students also thought this. This issue was sorted out during the rest of the session. We focused on the right hand side of the multiplication tables and got the children to speak it out. Through discussions, each child understood the rationale behind the construction of the multiplication tables. There were times when only a few children could comprehend what I was saying. But these instances were resolved by letting the children discuss amongst themselves. It was quite common for a child to explain to another on the latter's palm. This was done not just for multiplication tables, but generally for communicating. As we moved on to multiplying bigger numbers, the children seemed as capable of multiplying them with as much ease as was with smaller numbers. However, unsure if they could see any patterns in multiplication of any two numbers I decided to focus only on the multiples of numbers. Many were fascinated by the observation that multiples of ten ended with a zero. Much more so was on the divisibility condition for nine. Discussing the reason behind this was interesting albeit not fascinating for them. They did not consider "proofs" as a part of mathematics. Nonetheless, I continued to discuss divisibility of other numbers. We did reach a consensus on the ways of determining whether a number is divisible by 2, 3, 4, 5, 6, 9, 10, 11, and later on, other composite numbers. (Divisibility test for 9 was proved using the abacus, which some children were fluent with). In the course of time, all the older children were competent in justifying why a particular number is divisible by another. For numbers less than 1000, they would perform the division algorithm in their minds and give out the answer as a justification for divisibility. To test our understanding, I presented an example: "Is the number made of ten 3s followed by 15 (i.e. 333333333315) divisible by 15?". Since many didn't understand the question, I chose smaller numbers, 315 and 3315. When they still didn't understand, I asked those who did understand to explain to those who did not. This worked well. But still the students would work out the solution by actually dividing the number by 15 and giving out the answer (although in their mind using long division; and it was rather quick). Impressive as it was, I did not know whether they would use their understanding of divisibility rules for divisibility by 3 and 5. Hence I chose a bigger number: Ten 3s followed by 15. After presenting this problem and the usual two minutes for the children to explain to their peers, the correct answers along with explanations did emerge. On ending that day's session, one of the students (a 15 year old girl) gave me some homework (which I did not do). She asked me to solve thousand 3s and 15 i.e. 333...315 ÷ 15; that is, a 1002 digit long number where the first thousand digits are made of 3s followed by one and five which had to be divided by 15. We left for the day. It was Friday. We met again on Monday and I forgot about my homework. I was reminded about it. I tried solving it audibly saying, "I will first divide the number by 3 and the result by 5". I asked the children whether that would yield the correct answer. In a minute, they answered in the affirmative. I continued, "So 333333 thousand times followed by 15 divided by 3 would yield 111111 thousand times followed by 05. This number divided by 5 would be, let's see, 11 divided by 5 is 2 and leaves a remainder 1 which moves on to the next 1. Thus continuing, I will have a thousand 2s followed by 1". She immediately said I was right. Being surprised that she was so confident, I asked her to explain and she said that "315 divided by 15 equals 21, 3315 divided by 15 equals 221 Therefore, the answer is a thousand 2s followed by 1". Although I was impressed, I couldn't help feeling a little uneasy for what I considered inductive reasoning in mathematics. I articulated my discomfort. Her reply indicated she, in fact, did not use inductive but rather deductive reasoning. Her reasoning was precisely that dividing the first 33 would yield a quotient 2 and leave remainder 3 that would pass on to the neighbouring 3 and so on thereby having as many 2s in the answer as 3s in the question. This would result in the last 3 tagging onto 1 which when divided by 15 would yield a quotient 2 and remainder 1 which would tag onto 5 thereby leaving 15 which gives the last digit as 1 leaving remainder 0.

After discussing a few more numbers we ended the day with homework to think of numbers that would divide 3600. The next day we explored what the children had to say. Each child presented 2 numbers whose product was 3600 and explained to the rest how they arrived at a particular number. The responses were of the form, "Since it ends with 0, its divisible by 10. Hence it is also divisible by 360" another said "100 and 36" yet another answered "9 and 400", etc. I answered "75" and justified saying that "Since the number is divisible by 100, it must be divisible by 25" Since it is also divisible by 3 and considering that 3 is not present in 25, 3600 is also divisible by 75. We had already reached a consensus that if a number is divisible by another then it is also divisible by a factor of that number. There was no confusion on "75" being a factor. I divided 3600 by 75 and got 48. So my answer was "75 and 48". I used this mode of reasoning to introduce the idea of relative primes. The students got a clear idea about: Divisibility rules for 2, 3, 4, 5, 6, 8, 9, 10, 11, and numbers made up of a product of those numbers (e.g., 15, 35, etc.), idea of primes, relative primes, LCM, GCD, the rationale behind writing the number 0 for multiplication, intuition of mathematical induction, etc.

On my last session, we discussed the nature of mathematics. All reached a consensus that it is about calculation. Proof was not considered a part of mathematics. They also said that even people who have never been to school know mathematics citing examples of young vendors. On future visits after finishing the fieldwork, a discussion with a student enlightened me on various aspects of visual dominance in school Mathematics. However, the concepts which were taught in a visual manner when presented from fundamental principles were easily accessible to him. During a session on Euclidean geometry, the student was able to represent a cube and even a 4D hyper cube on my palm. Here, the 2D shapes were discussed using *wikisticks*. Beginning with a point and then going on to a 1D segment and then to a 2D square (by first drawing the segment and replicating it while joining the corresponding sides), we extended the same to represent a cube as a 2D square followed by a replica of the same followed by joining the corresponding vertices keeping in mind that each side represented unit length while each angle a right angle. Here, it was not necessary to mention that the eyes make square look like parallelogram when looked at from the side (as is assumed to be the understanding behind drawing a cube in "normal" schools). On asking about a 4D extension, he was able to represent it on my palm.

Summary and Reflections

Although I had initially read significant material on disability oppression, the same literature made more sense as it became contextualized. Theoretical concepts like dialectics, oppression, praxis, comradeship, dehumanization, exploitation, cultural imperialism, hegemony did not seem so abstract anymore. I got a better understanding of what it meant to say that disability is form of oppression. I realized that mathematics is not inherently visual (although the way the eyes distort objects can be mathematized). However, mathematics being taught through an ableist narrative is an oppressive practice while having students learn in a manner accessible to them can be empowering. Through asserting that disability is a form of oppression and working in solidarity with disabled people, I have learned to believe that the visual hegemony in mathematics can be challenged. Although I began with questions like, "How do I teach non-sighted students mathematics?" I realized that my approach legitimized ableism. My current questions now include questions, "How do we work towards a praxis for solving the problem of ableism in mathematics education?" I have proposed as my PhD thesis to use critical ethnography to develop a counterhegemonic pedagogy in mathematics with my visually challenged students.

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Recognising Gender in Mathematics Identity Performances— Playing the Fool?

Lisa Darragh
Universidad de Chile

This paper uses the metaphor of a theatrical performance to understand, analyse and present students' mathematical identities in the context of secondary school in New Zealand. Three vignettes are presented: a scene description, a short play, and a monologue. Each vignette is analysed in terms of gender; looking at the positioning of girls within their mathematics learning experiences. In the second part of the paper this gender recognition is problematized. A notion of the researcher positioned as audience is employed to question the unintended effects of recognising gender in identity performances.

Introduction

Identity is a concept that is currently enjoying something of a surge in popularity within mathematics education research (Chronaki, 2013; Cobb, Gresalfi, & Hodge, 2009; Lerman, 2012). MES7 in Cape Town, South Africa included a large number of papers and presentations on this topic. Identity has been referred to as the "missing link" in the "complex dialectic between learning and its sociocultural context" (Sfard & Prusak, 2005, p. 15) and it is seen as related to issues of power (Gutiérrez, 2013), access (English et al., 2008) and opportunity to learn mathematics (Esmonde, 2009).

However it is a concept that suffers from conflicted, and at times incoherent, definitions within the literature of our discipline (Cobb et al., 2009; Lerman, 2012). The way we define identity has implications for the way we recognise identity of our research participants, our students, and groups within the wider population. This recognition works power on these individuals and groups by labelling and categorising them in particular ways. As such it is important that we take care with both our definition and recognition of identities. With this paper I question my own recognition of gender identity from the position of researcher.

Identity as Performance

In an attempt to resolve the conflicts described above, I draw from Goffman's (1959) theatre metaphor and Butler (1988) in defining identity as *performative*. Our identities are constituted through repeated acts. To clarify I do not see identity as *reflected* in performance; rather that identity *is* the performance. This aligns more closely with Butler's meaning rather than that of Goffman. The identity exists only in the moment it is performed. However these performances are embodied over time, and the collection of performances work together to constitute an enduring sense of self, a performance *repertoire*. Furthermore positioning theory, which draws from and extends Goffman's work (van Langenhove & Harre, 1999), can be used to help understand the interactions between people or the identity performances given.

Our mathematics identity performances might include putting a hand up to offer an answer during a lesson, persevering to solve a problem, arguing or justifying a solution given. We may perform by working silently and individually or by giving up on a problem after a single attempt. Such performances are enabled or constrained by many factors, including the classroom, the teacher, peers, parents, possible future performances and not least by past performances. And yet we will be recognised in these performances as being particular kinds of people; the good learner, or the unconfident student for example.

We can extend this metaphor of performance to consider the stage, the direction, the audience, the scripts and the notion of co-performance. The stage, such as the classroom, is the immediate context for the performance. The teacher may be seen as a director of performances, exerting influence in direct ways. The audience forms part of the context as those who witness an identity performance respond to it and influence future performances. The researcher is also an audience member – their presence may affect the performance and they are certainly responding to and recognising the performances as being indicative of being a certain kind of person. We perform from scripts, that is, expected ways of being, that are drawn from wider society.

These scripts may be stereotypes and they may provide only limited acts for the performer. And yet we may perform impromptu and in doing so generate new scripts for performance available for ourselves and others to follow. Finally co-performances occur when an individual enacts more than one identity at the same time and with the same act. Sometimes co-performances work well together – performing confident, performing boy, performing good mathematics student – and sometimes they do not.

Context and Methods

The data to follow has been taken from an 18 month study looking at students' mathematical identity performances as they transitioned from primary to secondary school in a New Zealand city (see Darragh, 2014). The study was undertaken from 2011 to 2013.

The transition to secondary school occurs at around the age of 13 years for most New Zealand students and brings with it a number of changes to students' experiences of mathematics learning. It was an assumption of the study that with this change there would be change in the ways in which students' performed mathematics identities.

Twenty-two students were followed and 'visited' four times over this transition. The visits occurred: at the end of Year 8 (the final year of primary school), at the beginning of Year 9, midway through Year 9, and at the beginning of Year 10. At each visit the student participants were observed during mathematics lessons and interviewed afterwards for approximately 15-20 minutes. The students' Year 8 and Year 9 teachers were also interviewed.

The following data is taken from the observations of two of the Year 10 mathematics lessons and an interview with one of the Year 9 teachers. It is part of the data from three different girls in the study: Emily, Belinda and Lauren.

Three Vignettes

In line with the theatrical metaphor for identity I present my data in a series of vignettes: a scene description, a short play, and a monologue.

A Scene

> The curtain rises to reveal a secondary school classroom. Desks are spaced in rows and columns, all facing the front of the room towards a whiteboard. The teacher sits at his desk to the front and right-hand side of the room. There are 8 girls and 20 boys seated at the desks. The girls take the positions in desks along the furthest edge of the room, far from the teacher's desk, at the margins.

There is much research about the marginalisation of girls in mathematics education (see for example: Mendick, 2005; Solomon, 2007; Solomon, Lawson, & Croft, 2011; Walkerdine, 1998), but in this lesson I saw a physical manifestation of this. When I interviewed Emily after the lesson she was unable to give me any insight into the seating choices of her classmates and herself. She explained that the form classes were re-streamed for mathematics and that the girls knew each other more than they knew the boys, indicating that this was why they had chosen to sit together. Why they sat far from the teacher's desk remained unknown to me.

During my observation of this lesson I noted that no girl put up a hand to answer or ask a question at all, despite there being many opportunities for discussion, to which the boys contributed. The girls sat at the margins and they also performed marginalised mathematics identities.

A Play

SCENE: *A classroom filled with students of around 14 years of age. Students are seated at desks in groups of four. One group, containing BELINDA, two other girls and one boy, is at centre stage under the spotlight…*

TEACHER: Before we start, are you familiar with trig ratios? If not, you need to pay attention. (She writes up a trigonometry question on the board.)

BELINDA: (Raises hand) It's eight.

TEACHER: Very good. Do we need another example? Right, now make sure your desks are joined so you can discuss. The winning group will get the pizza next week. (Teacher hands out a whiteboard marker, small whiteboard and question sheet to each group.)

(BELINDA takes the marker and the board. BOY grabs the question sheet and starts to work it out on his own.)

GIRL 1: (Snatches the question sheet back and starts to read the question out to the others.) A ladder is rested against a wall…

(BELINDA draws a triangle on the whiteboard and labels the sides and angle with numbers. She takes the BOY's calculator. He takes it back. She writes in a four.)

GIRL 2: Don't do the four like that, make sure it is all joined up.

(BELINDA hands the board to the teacher.)

TEACHER: Well done, here is the next question.

(BOY takes the question sheet and starts to write the question and answers on the board. He uses his calculator occasionally.)

GIRL 1: Read it out!

(BOY continues to work, ignoring GIRL 1.)

GIRL 2: So did you see that movie last night?

GIRL 1: Nah, I've seen it before.

GIRL 2: What about that thing they do with the knife?

BELINDA: I can do that too.

(BOY hands in the board to the teacher and gets the next question sheet.)

BELINDA: Read it out!

(BOY continues to work on his own. GIRL 1 grabs the question sheet. BOY grabs it back, writes an answer and takes it to the teacher.)

TEACHER: This group is doing really well. (She gives the BOY the third question sheet).

(BOY takes his calculator and starts inputting numbers.)

GIRL 2: Can you really do it?

BELINDA: Let me show you. (She picks up a pen and spreads her fingers out on the desk. She stabs the spaces between each finger and gets faster and faster.)

GIRL 1: Woah!

(BOY hands the third answer to the teacher. The teacher shakes her head.)

BELINDA: Let me see, you got it wrong. (She takes the question sheet and starts to read it.)

(The teacher comes over to the group and starts to explain the problem. BOY takes the question sheet back from BELINDA. The teacher leaves the group.)

GIRL 1: I hate my ears. I have to wear my hair like this 'cause my ears are so big.

BELINDA: I've got really small ears – they look stupid though, see!

GIRL 2: You have a small nose at least – look at mine.

GIRL 1: My tongue is really long – I can touch my nose.

(The three girls all stretch out their tongues and try to touch their noses. Meanwhile BOY writes down an answer and hands it to the teacher.)

TEACHER: Great, now the last question requires you to make a nice poster showing all your working and the answer.

BOY: My desk is too messy for this. (Turns to BELINDA) Can you move so I can do this on your desk?

(BELINDA does not move. BOY tidies away his belongings into his bag. He starts to write on the poster.)

TEACHER: The poster that is the nicest with good working will get extra points.

BOY: (Hands the poster over.) I've done the maths now you guys finish it.

BELINDA: Ok, we'll make it pretty.

BOY: Yes you three need to make it pretty. Here, use my felt pens. You have ten minutes.

(The three girls enthusiastically bend over the poster and start colouring. BOY stands over them, watching and micro-managing.)

TEACHER: Well done, this is the winning group!

It is useful to use positioning theory here to understand this play; particularly the way in which "each of the participants always positions the other while simultaneously positioning him or herself" (van Langenhove & Harré, 1999, p. 22). Before the group work began, Belinda positioned herself as an able mathematics student; she answered a number of questions, publicly demonstrating that she knew how to work out trigonometry problems. Once the group activity began it appeared that BOY was also positioning himself as the one who knew the answers. Belinda and BOY engaged in a power-play for control of the marker, question sheet and recording sheet. Controlling resources endows advantage in controlling the discourse for that student (Barnes, 1998). One difference in their actions was that while Belinda tried to include the others, BOY wanted to complete the task alone. Half way through the activity Belinda was drawn into a discussion with the other two students, GIRL 1 and 2 who appeared to be her friends. BOY continued the mathematics work, returning it to the others when it was time to present and make it "pretty".

For BOY and Belinda, positioning themselves as the mathematics expert required positioning the other as not. This positioning appeared not to impact on the other two girls. However they were actively positioning Belinda into the role of 'friend' and 'girl' by drawing her into a discussion about the problems with their facial features. In doing so they positioned Belinda within a figured world of friendship that was also gendered (see for comparison Esmonde & Langer-Osuna, 2013).

When I tried to engage Belinda in a discussion about this activity during the following interview, she revealed her perceptions of the influence of the others in the group. When I asked about what might have changed with different group members she said they "probably would have worked together … or not done it at all (laughs)" (Belinda, Year 10 interview). This demonstrates an effect of peers on performances (see also Ingram, 2008) and raises the question of how much agency Belinda (and the others) had in this group learning situation.

A Monologue

> Algebra [is the most important topic]. Definitely algebra, you know, um, especially girls they find it quite hard. And measurement too. Algebra, well ... girls, um, well I don't want to make it like offensive, but yeah, comparing with the boys, they are not as logical as are boys. Or, ah, I mean, boys are just - they are born with that kind of um, ah, like a skill, when they, it comes with them, like area or measurement, they just know. I mean, just get the concept really easily. But girls, it takes time, and I guess for them it is hard to, um, you know, handle it. I just feel that [algebra is] a boys verses girls sort of thing. […] I think I know why girls they are not really good at algebra, because it can be not really practical, if it is practical they can understand better, but girls they are not really, like if it is like ... abstract, it is hard to understand it, for them - I think. I might be wrong. (Lauren's

Year 9 teacher interview)

Clearly this teacher recognised gender in the performances of his students. His comments were made say-able by his drawing from societal scripts which often describe mathematics as masculine (Mendick, 2006) and ability as natural (Solomon, 2007). However his interview responses are somewhat surprising given the situation of a female interviewer. None of the other teachers interviewed for this study made similar suppositions regarding gender.

Ironically this teacher taught at a single-sex girls' school. In fact, his entire (three year) teaching career had been at this school and any experiential based comparison between girls and boys learning mathematics was based on his more limited practicum and tutoring experiences. My own interpretation of the situation was that this teacher had experienced difficulties in the teaching of the more abstract topics like algebra and seemed to have attributed his lack of success to gender.

Discussion

In the theatrical production that is this paper, the teacher described above can be seen as playing the role of the jester. He displays unconscionable attitudes to girls' competence in mathematics, arguably acting the 'fool'. And yet the jester in a theatrical production may in fact serve to send a message to the audience. The message I received concerns the recognition of gender and the possible mis-recognition of identity performances.

The teacher recognised gender in his students' difficulties with algebra. And I recognised gender in each of the vignettes reported above. The girls were marginalised, they were positioned into superficial and non-mathematical roles in the group learning situation and they were recognised as incompetent by their teacher.

However I would like to problematize this gender recognition. Do we as researchers, like the fool, recognise gender when it may just as well be other identity performances that we are mis-recognising instead? My recognition of gender during data collection and analysis derived in part from my own experiences of being a girl in mathematics classes and from my engagement with the research literature on girls as marginalised in mathematics education (Burton, 2003; Mendick, 2006; Solomon, 2007; Walkerdine, 1998). Although such research is often problematised itself, I arrived at the theatre ready to recognise gender.

Consider, for example, the play above. In my labelling of the other characters in the play (and in my field notes) as BOY and GIRL, I indicate my own immediate interpretations of this as a gender issue. Before I had a chance to formally analyse my data I had already viewed it through a gender lens. I saw Belinda and BOY as competing to be the mathematical authority, and the boy as ultimately winning in this. Had I labelled the other characters as STUDENT 1, 2 and 3 then I would be highlighting the co-performances of friend and student that Belinda was forced to try and merge together.

However, due in part to my labels, when I heard the girls talking about their appearance, worrying about their ears being too big or too small I recognised this as a gender performance drawn from the figured world (Holland, Skinner, Lachicotte, & Cain, 1998) of gender relations or perhaps of beauty and sexual stereotypes. When BOY passed the mathematics work to his three female group-mates and told them to make it “pretty” I recognised this as positioning them within gender discourses where the males do the thinking and the females do the superficial work.

Was I right to recognise gender here?

> [R]arely if ever do researchers raise the question of the relevance of the specific categorisations and identities chosen to represent the participants to the research. Consequently, particular identities are taken for granted and assumed to reflect the identities actually realised, felt, or believed by the participants. (Stentoft & Valero, 2009, p. 61)

Belinda did not engage in an explanation of gender during the follow-up interview. Furthermore the character of BOY was more than just a boy for Belinda. He held a historically constituted social position in the class and she would have been responding to him within a much more complex web of relationships than that which I observed.

There were certainly multiple performances required of Belinda. How easy was it for her to co-perform able mathematics learner, girl, and friend along with any other identities invisible to me? And how might recognising her performance in terms of gender work to render those other identity performances less visible?

The social world can be conceived of as performed and created through the repetition of performative acts. Law and Urry (2004) use this terminology. The act of research in particular creates the reality it seeks to investigate. These authors apply their arguments to methodology specifically. "Our argument is that [research methods] are performative. By this we mean they have effects; they make differences; they enact realities; and they can help to bring into being what they also discover" (Law & Urry, 2004, pp. 392 - 393, italics in original). By recognising gender issues in identity performances do I help to produce them?

"Power not only *acts on* a subject but, in a transitive sense, *enacts* the subject into being. As a condition, power precedes the subject" (Butler, 1997, p. 13, italics in original). Recognition works power through categorising and labelling students as particular types of mathematics learners and it also works power through recognition of students through other categories. But perhaps the most insidious form of this power is the way it works through the students' own self-recognition. "Power acts on the subject in at least two ways: first, as what makes the subject possible, the condition of its possibility and its formative occasion; and second, as what is taken up and reiterated in the subject's 'own' acting" (Butler, 1997, p. 14).

This means that if we recognise people in a certain way it may lead to them self-recognising in that same way. If we see girls as a 'problem' in mathematics learning they may come to see themselves similarly (Rodd & Bartholomew, 2006; Walkerdine, 1989). It is important to note that none of my research participants reflected on their mathematics learning in such a way as to suggest being girl meant being marginalised – not once in the 18 months of the study. This leads me to question: at what point is it useful and at what point is it further discriminatory to recognise a performance as being part of one's gender identity?

Re-analysing Belinda's identity performance to be that of 'friend' rather than 'girl' enables me to consider whether she was able, in this situation, to co-perform friend and good mathematics student. It appeared that this was a difficult co-performance and although she began with a good student performance she ended up drawn into performing good friend instead. Recognising Belinda's identity in this way enables a way forward, a consideration of how the stage of the classroom, the group organisation, and the teacher direction could be re-thought to enable her to co-perform good mathematics student alongside other identities important to her.

I do not wish to naively suggest that we abandon our recognition of gender in understanding marginalised mathematics identity performances. Indeed it is difficult to re-analyse Emily's situation (the first vignette) in another way. Even Belinda's performance of friend is in part constituted by and through her gender. Rather I wish to suggest that we consider carefully what gender recognition may achieve. In some cases recognising gender may blind us to other identity performances that impact on performing good at mathematics. And we must also be aware that recognising gender may lead to girls self-recognising in a similar way and writing themselves out of future mathematics performances.

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How Do Non-indigenous and Indigenous (Mathematics) Teachers, Jointly, Contribute to the Revitalization[1] of the Native Language?[2]

M. do Carmo Santos Domite,
Valéria de Carvalho
Universidade de São Paulo-USP,
Universidade Paulista-Unip

This study may be understood as a set of ideas and proposals about possible directions for indigenous teacher education when the purpose is revitalization of indigenous language in general and the use and valuation of indigenous language in mathematics education in particular. Taking as a point of departure the vision and needs of indigenous Terena teachers relative to native language fluency, we worked with them in one Terena village, producing materials for learning language and mathematics. Our work focused on indigenous language classes, bilingual schooling, immersion in communal cultural practices, and the involvement of native-language-speaking elders in elementary education.

Introduction

In the last ten years, a number of research studies (e.g., Reyhner, 1990; McIvor, 2009) have been published about the cultural movement of indigenous peoples toward the revival of their native tongues as well as the means, methods, and working styles of this revitalization. Researchers argue that since the mid-twentieth century, indigenous peoples have begun to reclaim their languages and work toward their revival and use, debate some aspects of the endeavor, and discuss possible ways to revive and give a sense of continuity to indigenous languages generally.

As part of our effort to better understand the potential to revitalize the Terena language in two different Terena villages, in particular the intricacies and complexities of the mind-frames of indigenous speakers, we worked with Terena to better understand their own comprehension of the revival and continuation of indigenous languages as well as this particular objective. We addressed learning materials in language learning, including research, language classes, other subject classes (especially mathematics), bilingual schooling, immersion practices, and early- childhood focus. We found that many questions and concerns require further investigation.

This research was grounded in the knowledge that our group acquired in the Intercultural Undergraduate Indigenous Teacher Course (FEUSP, 2005–2008), which included several extended visits to groups of different indigenous ethnicities. From that field experience we began to develop a draft of what could be called an amalgamation between the principle of an indigenous peoples' bilingual education program and an initiative to investigate related issues at a deeper, more academic level.

Our research can be justified by the Terenas' own perception, which was shared by the non-indigenous language teachers of the Intercultural Undergraduate Indigenous Course, that Terena language in São Paulo villages is seriously endangered because it is no longer being transmitted to new generations. Surely the objective of maintaining Terena indigenous language (or any other indigenous language) is especially important for maintaining a culture, as D'Ambrosio has eloquently point out:

> We are interested that a tree flourishes, then we would be taking good care of its branches, but it will never be strong if its roots are not good, deep... It is useless to try straightening the things of a society if you don't give to the elements that will work in that cultural root, which will grant them strength. If you don't give it, they may be easily manipulated… You can only enter this reflection if you have deep roots and think: "I know myself and know that I am as much a human being as that other one. I know that my culture has as many accomplishments as the other's culture." (cited in Carvalho 2007, p. 264)

Background

During 2002-2003 and 2005-2008, in a partnership between the College of Education of the University of São Paulo-USP and the Secretary of Education of São Paulo, Professor Maria do Carmo Domite coordinated both the Indigenous Teacher Education Course for the Elementary Level and the Intercultural Undergraduate Indigenous Teacher Course for Elementary and Middle Schools at the College of Education of the University of São Paulo. The basic goal of these courses was to generate a proposal for learning/teaching for the elementary schools of the indigenous villages in the State of São Paulo, but to do so in such a way that those teachers could take over the village schools. Eighty indigenous teachers from five different ethnic groups (Guarani, Tupi-Guarani, Kaingang, Terena, and Krenak) living in different villages took part in this project.

We, as non-indigenous teachers, were responsible for co-planning lessons in mathematics aimed at the revival of the Krenak, Kaigang. and Terena languages—under the advice of linguists and anthropologists—as part of the bilingual perspective of the aforementioned Indigenous Education Program.

The status of fluency differed for each people indigenous languages. The Guarani, in general, and the Guarani teachers in particular are fluent. The Tupi are not as fluent, but are aware of this situation and strive for enough creativity to maintain and revive their language. Among the Terena, Krenak, and Kaingang, few adults are native speakers and there are almost none in the youngest generation. As they studied the preservation of their indigenous languages, those teachers began to systematically approach the revival and use of these languages.

In terms of native-language communication and mathematics, the course was based on understanding of indigenous ways of interpreting and enacting linguistic/mathematical relationships. Synthesizing the theorizations of literacy (Freire, 1983, 1994; Freire & Macedo, 1987), matheracy (D'Ambrosio, 1999; Skovsmose, 2001) and ethnomathematics (Barton & Alangui, 2004; D'Ambrosio, 2001; Domite, 2006) we aimed at approaching indigenous children's mathematical mind-frame by contextualizing these concepts within their native educational environments.

Specifically, we defined *literacy* as the indigenous people's capability of processing information (which included written and spoken language as well as signs and gestures), whereas we defined *matheracy* as the capability of dealing with codes and numbers, inferring, proposing hypotheses, drawing conclusions from data, and orienting in space. Ethnomathematics focuses on how quantitative and spatial relationships are identified by culturally differentiated groups, according to their own rationalities and in their own terminologies. Indeed, the theoretical basis we propose for the learning and teaching process in this indigenous education program comprises the understanding that:

> … different mathematical relationships and practices can be generated, organized, and transmitted informally to solve immediate needs, as occurs with language. This way mathematics is incorporated into the core of the learning-by-doing processes of the community and thus mathematics is part of what we call culture. From this standpoint, ethnomathematics is concerned not only with the cultural roots of mathematical knowledge, but also quantitative and spatial relationships generated within the cultural community, which often compose mathematics as well. (D'Ambrosio & Domite, 2007, p. 201).

Research Questions

With a focus on these views of indigenous education, this study aimed to provide evidence for the following research question: In terms of our personal and professional experiences as mathematics educators with indigenous Terena teachers, which directives and possible interventions might be used to construct a contextualized proposal for indigenous language revitalization?

Method

The aim of this qualitative study was to investigate the use of certain activities in revitalizing native-language use among indigenous Terena teachers and, in turn, among their pupils. Meetings and discussions about the contents presented to the indigenous teachers had the result of allowing us to understand the emotional and intellectual preoccupations of the participants, generate questions, and provide context for the analysis.

We gathered information about how these indigenous teachers made progress in using their native languages through a sort of action research process, the aim of an action researcher being to bring about development by analyzing existing practices, identifying elements for change. In our work, we combined two types of actions: indigenous teachers reflected on their practices and non-indigenous teacher educators investigated their support processes. First, both the indigenous Terena teachers and the non-indigenous teacher educators agreed to work on this revitalization process. Second, the act of finding evidence helped us to understand our process as researchers, not only in terms of what we proposed to do to solve these problems, but also in terms of the factors that affected what we were doing.

In order to understand how the professional development of Terena indigenous teachers can be best supported by educators from outside that culture, we felt that we must acknowledge certain conceptual problems pertaining to psycho-epistemological issues. For example, we as non-indigenous teacher educators seemed to know that we were expected to listen to and understand the problems of the indigenous teachers in light of the education of their own people (Domite, 2010, p.308).

Toward the Indigenous Village: Terena Language in Action

The experiences we describe here took place, in the Terena village of Ekeruá during three different working meetings. After our arrival at the village, we spent some time with the chieftain—who demonstrated a broad view of linguistic revival—asking him for permission to gather as many indigenous teachers as possible. Next, in collaboration with our indigenous colleagues and the chieftain, we gathered in a classroom and concocted a work plan for the three days of our stay. Having arrived with a set of suggestions, we shared these thoughts and considered them, as a group, in terms of the perspective of the local teachers.

Our informal discussion with the eight Terena teachers and their chieftain included telling them that one of the aims of this revitalization movement for their language was to expand their education as indigenous teachers so that they could work in a bilingual context – Terena and Portuguese. Then, jointly, we tried to reach an understanding of what it means to work in a bilingual context (e.g., alphabetizing the children in native and Portuguese language, stimulating bilingual fluency, and so forth). At this point, some contradictions appeared. For example, some of the indigenous teachers wanted to emphasize indigenous-language conversation, whereas others preferred to teach reading and still others thought that the progression of listen-speak-read-write was most important.

The atmosphere on this first day was relaxed and informal, possibly because of our conscious creation of a social and affective approach. During the second encounter the teachers displayed a more involved and participatory attitude—perhaps because of the nature of the activities we proposed—arriving with a different degree of involvement, such as deciding to take a deeper and keener look at the actual requirements for the success of the linguistic revival project. The third day was invested with the importance of sharing experiences of the implementation of the proposed tasks as well as initial, general findings.

The activities we proposed in order to construct evidence that could be used for analysis were based on our concern with the development of positive attitudes on the part of the teachers as well as their relationships as learners and self-elaboration in dialogical processes which language and mathematics subjects assume a larger conceptual understanding.

The analytical perspective of the empirical activities we proposed and discussed with the indigenous teachers is theoretically supported in works concerning Freire's viewpoint—we found data and evidence in the *other*'s answers and did not arrive with a fixed set of ideas about the *other*.

Investigative Activities: Arguments and Empirical Results

The indicators that made it possible to understand the variables related to our goals were obtained with two instruments: the investigative activities themselves, and dialogue with the Terena teachers. The latter was focused upon their feelings about and predisposition to the revitalization of their own language, either by means of scholarship or by social, cultural and linguistic empowerment. The following activities were carried out.

Walking in Several Directions According to Instructions

In order to engage indigenous teachers in the process of reflecting upon their teaching and learning of geometry we encouraged them to talk about representations they perceived via the observation and representation of movements and constructions in terms of their living spaces. The proposed activities were intended to lead students to coordinate visual information taken from the world around them, including their perceptions of the body in action (displacements, constructions) and speech about their representations.

A very important finding—in terms of language and mathematics communication— during the applications of these activities was that when a Terena teacher was asked to tell a second one, in Terena Language, to turn left or right, he did not know what to say. And, during a further discussion a female Terena teacher, who was more fluent in Terena language than most of her colleagues, explained that in Terena there is no single words that mean left/right. Instead, native speakers of Terena use body language to indicate physical markers that they use to spatially orient themselves.

Similarities and Differences

In this hypothetical exercise, a student would throw a ball to another student while saying loudly, in Terena language, one similarity between them (e.g., "white T-shirt"). Then the student who had caught the ball would throw it to another one while loudly observing one similarity between them (e.g., "brown hair"). And so on.

Mental Calculations

The teachers were invited to choose, with total freedom, strategies for mental calculations in ways that would accomplish at least two goals: students' acquisition of a more autonomous relationship with their mathematics thought and student understanding of arithmetical calculations not just as an exercise in mathematical thought but also in terms of social, political, or economic situations in daily life (Domite, 1996; Skovsmose, 2001). By giving them some arithmetical calculations, we noticed that the teachers were loudly reciting different/proper calculations, using some words in Terena language but most in Portuguese.

Vocabulary Practice

Students would prompted by the teacher to research names of different things, places, ideas and so forth, in order to bring out their natural curiosity and develop vocabulary and linguistic skills in their native tongue.

Immersion Practices

Based on the indigenous teachers' report that they were already used to meeting with students on Saturdays to play guitar and sing as well as to rehearse theatrical productions, we proposed the concept of a "Language Saturday" during which all members of the village would be motivated to practice in their native language.

Bilingual Schooling

From our knowledge of other researches, we concluded that one valid suggestion to enhance the usage of native ways of communicating among indigenous students is to split language use between the two periods in which they attend school (Bello, 2000; Reyhner, 1990).

Focus on the Children

From our observation that many adults and/or elders were Terena-language-fluent speakers, we led the indigenous teachers group to consider that the presence of village elders in the children's classrooms would be very important for cultural crossover.

Indigenous Teachers as Researchers

The native teachers were invited to raise awareness by producing research that would elucidate why their language, in their community and in their vicinity, suffers degradation and is even in danger of extinction, whose results should be made available to the all other Terena villages from São Paulo.

In summary, at the end of the three days, we decided -together with these indigenous teachers—to produce didactic materials for learning indigenous language and mathematics (mental calculation, measurements, spatial orientation, etc.) that would focus on five actions: a) involvement of native-language-speaking elders in elementary education; b) dedicated indigenous-language classes; c) bilingual schooling; d) immersion activities in communal cultural practices; and e) research by indigenous people of the reasons that so few people are speaking the native language.

Findings

The issues of language revitalization that arose in this indigenous teacher education context led us to focus our attention on three categories of analysis: a) requirements that would enable Terena indigenous teachers to be able to conveniently enact tasks related to the usage of their native language, and successful ways of doing so; b) the strenuous manner in which Terena people negotiate their relationship with the bilingual perspective of the Intercultural and Bilingual Indigenous Education Program; and, c) differences of receptivity and expectancy in terms of mathematics activities intended to improve Terena language fluency.

It is our finding, based on the different pedagogical actions and manifestations on the part of the indigenous teachers, that when given the appropriate guidance and instruments, the parties involved experience an awakening in terms of language fluency. It seems to us that the indigenous teachers, their students, the students' parents, as well as village or community elders and, in this case, the chieftain, were thinking about the possibility of classes being taught primarily in their tongue.

We also conclude that the reversal of the imperilment of an indigenous language should not only prove to be highly feasible by teaching indigenous students, but also should prove quite effective in associating the idiosyncrasies of this manner of linguistic revival, by means of all the specific techniques that could be involved, with the particularities of the construction of knowledge of basic mathematics. More expressive forms of receptivity were noted in the activities that involved mental calculation, in that the self-capacity—autonomy—of calculating by one's own means seems to be a possibility of significative learning.

One aspect that provided a way to diminish the level of degradation of the Terena language was the attempt to recover the interest of the youth by increasing their natural curiosity about the ways of the elders by resuming the regular evening practice of sunset gatherings at the House of Prayer. This powerful practice has been, unfortunately, sidelined due to the competition provided by, for example, soap operas on TV.

In terms of what would be required for teachers to perform this reversal, the elders would have to be fully available to answer students' inquiries about the old ways, tales and general ancient knowledge. Such interactions would be mostly put into practice in the original native tongue, as a means to generate curiosity among the youngsters. We found that these initiatives generate some discrepancies. A certain perceived difference of expectations seems to have been located in the teachers' own perceptions of improving fluency, such as the extent of the lack of fluency and/or the value given of revitalizing the mother tongue.

Conclusions

At the core of our research rested the conviction that it should be not merely possible, but is in fact quite necessary, for the native groups we advised throughout our study to develop their traditional linguistic proficiencies to a functional stage. This conviction is in accordance with the official directives provided by the educational indigenous movement in Brazil. Gradually, we noticed—perhaps due to our predispositions to learning the old ways—hat this initiative not only strengthens the intergenerational bonds but also instills a sense of ancestrality in the older generations and a sense of belonging in the younger ones.

We came to realize, with a great sense of self-esteem and joy shared by our indigenous colleagues, that the more we all persevered in the proposals, discussions, exercises, and practices aimed at raising awareness of linguistic revitalization, the more names of the past emerged.

It is important to emphasize our recognition, as teacher educators foreign to this indigenous culture, the constant need to question the role and the meaning of the dominant language (Portuguese) and its consequences—which include recognizing that the Brazilian Federal government sometimes aims for assimilation and, at other times, recognizes its obligation to support Indian language preservation. Indeed, the indigenous policies in Brazil have always been a national project inside a larger one that has not taken plurality into consideration because it has contained the unspoken goal of unifying (i.e., erasing) cultural differences, which does not attribute value to linguistic diversity and indigenous knowledge.

Finally, from the perspective of (mathematics) indigenous teacher education, we were unsure about how we could apply mathematics activities in order to transform the indigenous language revitalization process. In other words, the value of orienting teachers to motivate fluency in Terena language among their students, whether the activities involved concerned mathematics, arts, or other fields of knowledge, remained unclear—as did the actual roles to be played by these teachers. It seemed to us that the focus of teacher education in terms of the revitalization of the Terena language must be upon problematizing the processes by which the cultural knowledge of the teacher educators as well as of the indigenous teachers themselves become the heart of instruction.

The factor that gained the most clarity was the transformation of the way of being, as a learner, a member of the Terena people during this teacher education process. We perceived this growing clarity as the indigenous teachers posed questions and as we, as teacher educators, questioned them. The questions of the Terena indigenous teachers about their fluency in their own language became significantly more solid. In fact, they began to problematize their own ways of thinking about these linguistic cultural dynamics during the course of our meetings.

Notes

1. To understand the terms *revitalization* and *revival,* as well as to be able to choose the appropriate set of instruments to implement such initiatives, one must carefully assess the current state of these languages. A language that has no native speakers is called dead/extinct. A language that has no native speakers in the youngest generation is called moribund and the one that have few native speakers is called endangered/imperiled.

2. A very similar paper has been already accepted by Revista Internacional de Educación para la Justicia Social (RIEJS) (Universidad Autónoma de Madrid), which gave us permission to send it to MES8.

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The Role of Rational Dependence in the Mathematics Classroom

Ander Erickson
University of Michigan

This conceptual piece defines rational dependence, or the reasoned dependence on the knowledge of others, and provides a rationale for its inclusion in the mathematics classroom. I proceed to argue for the use of problems that require to students to seek out and evaluate information outside of the classroom as a way of providing an opportunity for the exercise of rational dependence. This is followed by an illustrative example drawn from a qualitative study in which the author observed the implementation of such problems. The example illustrates the perils of encouraging a critical stance towards quantitative claims without encouraging students to seek out the expertise of relevant epistemic communities as part of their analysis.

Teachers of mathematics are increasingly confronted with students who have access to varied sources of information outside of the classroom. The premise of a word problem can be challenged by a quick glance at Wikipedia, calculations that were formerly impossible to carry out on a graphing calculator can now be performed with ease using an answer engine like Wolfram Alpha, and students can track down discussions of test problems on various Q&A web-sites. Teachers have the option of either ignoring this reality, treating it as an imminent threat to be guarded against, or encouraging students to engage with *information-based problems* (Walraven, Brand-Gruwel, & Boshuizen, 2008) or those problems that require "students to identify information needs, locate corresponding information sources, extract and organize relevant information from each source, and synthesize information from a variety of sources" (Walraven et al., 2008, p.623). In this paper, I argue that the use of information-based problems is not just preferable, but that it is also necessary if mathematics instruction is to help prepare students for the quantitative arguments that they may expect to encounter in their everyday lives (Paulos, 1988; Steen, 2001). Following this argument, I will describe an example drawn from my dissertation research that exemplifies one of the problems that can occur when students critique a quantitative argument without attending to the nature of the epistemic community that the argument stems from.

Students have always been able to draw on information outside of the classroom if they so choose, but a convergence of demands bound up with three different conceptions of literacy are serving to push this issue to the forefront. (1) A move to attend to how reading and writing is conducted in the disciplines and to use this as a way of thinking about how to teach those disciplines means that mathematics teachers must think more carefully about how those who work in STEM fields locate and evaluate mathematical resources (Schleppegrell, 2007; Moje, 2007; Shanahan & Shanahan, 2012). (2) While mathematics instruction has long been charged with an instrumental role with respect to training in the STEM fields, there have been more recent moves to develop curriculum that works toward a mathematical proficiency that can serve any individual in their everyday lives. This is sometimes framed as quantitative literacy (Steen, 2001), mathematical literacy, quantitative reasoning or granted a more limited scope as a type of statistical literacy (Cullinane & Treisman, 2010; Watson, 2013) and has seen some realization in courses offered by many colleges for non-STEM majors who need to fulfill a mathematics requirement as part of their liberal arts education. (3) Researchers in the information sciences have long recognized the importance of instruction in order to facilitate college and high school students' ability to research topics at the library (Rader, 2002).

Information-Based Problems in the Math Classroom

What does all of this mean for mathematics instruction? While the importance of information-based problems for disciplinary literacy is easy to justify as long as one accepts that information-seeking is an important part of practice in the disciplines, it requires a little more unpacking to explain why this type of instruction might have a place in mathematics instruction specifically. As a start, we can imagine an applied mathematics problem where students are given an editorial in which the author argues that federal guidelines on fuel efficiency will end up costing the country more money than it will save (Diefenderfer, 2009). Students are asked to read thid editorial and then provided with several guiding questions that encourage the students to analyze the numerical argument contained in the article. While this activity is a legitimate applied mathematics problem, the real-world context (see Figure 1) suggests other directions that such a problem could be taken.

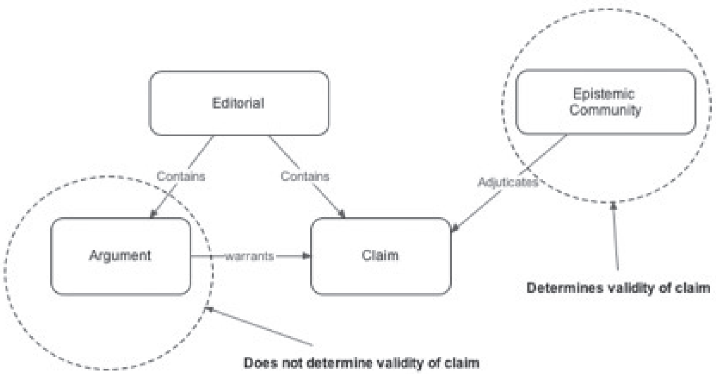

Figure 1: Relationship between Editorial, Claim, and Epistemic Community

For example, if a reader were to actually want to determine whether the editorial's claim was true or not, they would want to locate the relevant *epistemic community* (Haas, 1992), or that community that possesses the expertise to tentatively rule on the truth of the claim. The problem as originally stated does not afford the student an opportunity to seek out and evaluate those sources of information that might either corroborate or challenge the argument found in the editorial. An information-based problem, on the other hand, with its requirement that the student seek out and evaluate sources outside the classroom, would give the student the opportunity to challenge and/or corroborate the premises of the argument presented in the editorial. It may turn out that numbers were misrepresented by the author or that straw-men arguments were used to mischaracterize the opposition. It could even be the case, as it often is in the inevitable compression characteristic of newspaper articles, that an invalid argument was presented in support of a position that was nonetheless held to be true by the relevant epistemic community for other legitimate reasons. None of these aspects of the situation could be discovered without recourse to information sources outside of the article itself.

Why Rational Dependence?

Why should information-based problems be introduced to a quantitative literacy course? Looking at the situation from the other way around, one can ask a different question: How much can we learn about highly contextualized problems that are mathematical in full or in part without relying on pertinent experts and expert communities? The epistemologist John Hardwig (1985) argues that we are very limited in this respect with his notion of *epistemic dependence:*

> The layman's appeal to the intellectual authority of the expert, his epistemic dependence on the expert, and his intellectual inferiority to the expert (in matters on which the expert is expert) are all expressed by the formula [...]: [The layman] has good reasons to believe [the expert] has good reasons to believe that p. But the layman's epistemic inferiority and dependence can be even more radical—in many such cases, extensive training and special competence may be necessary before [the layman] could conduct the necessary inquiry. And, lacking this training and competence, [the layman] may not even be able to understand [the expert]'s reasons, or, even if he does understand them, he may not be able to appreciate why they are good reasons (Hardwig, 1985, p. 338)

I would only supplement this description with three points. First, while the layperson/expert dichotomy may sound divisive, it should be remembered that we are all in the same boat. That is to say, an expert is only an expert in a highly circumscribed domain and the moment she steps beyond that limited sphere her status reverts to that of a layperson. Thus, we do not live in a world that consists of two classes: experts and laypeople. Rather, we are all laypeople who are occasionally granted the opportunity (or responsibility) to contribute our thinking on those few matters that touch upon the areas of our expertise. Second, while Hardwig (1985) refers to individual experts, he also acknowledges that anybody who might be called an expert also belongs to and depends on a community that enables, shares, and affirms the expert's status as an expert. Any scholar must trust in the findings of other scholars whether their object of study is particle physics or Milton. Thus, it may make more sense to speak of an epistemic community (Haas, 1992) with respect to a given topic rather than isolated experts.

Finally and crucially, as Harvey Siegel (1988) noted, an individual's awareness of her epistemic dependence does not give her leave to relinquish her rationality, it only highlights that she must use her reason to decide between sources of information rather than relying on her direct knowledge of the field. In his paper responding to Hardwig (1985), Siegel spelled out many of the considerations that would occupy a hypothetical layperson, Smith, prior to accepting the opinion of a hypothetical expert, Jones, on a topic of import:

> She must, first, determine whether this is a question about which she ought to defer to expert opinion: Does she have the ability to become expert and reason about the matter directly? If so, is it rational for her to expend the requisite time and effort? Supposing that one of these questions is answered in the negative (rationally so answered, of course), Smith must ask herself: Is this question one concerning which there is likely to be reliable expert opinion? Is it likely to admit of evidence which an expert might come to recognize? Is the relevant field of inquiry sufficiently developed and sophisticated that there is reason to regard its practitioners as experts? And what about Jones: What sort of expert is she? Would other experts agree with her? Is she rightly regarded as an expert? Does she have a conflict of interest in this case? Is there any other special circumstances which renders her expert judgment in this case problematic? (Siegel, 1988, p. 4)

By way of contrast with the educational goal of intellectual independence (Norris, 1997), I hereafter refer to skills displayed by Siegel's hypothetical student as *rational dependence*, or the ability of an individual to rationally seek out and draw from relevant epistemic communities. The situation outlined is a confrontation between a layperson and an expert but we can nuance that by recalling what was stated above about claims being warranted by epistemic communities (Adler & Haas, 1992; Collins & Evans, 2008). It is also worth noting that Kuhn's (1991) investigation of personal epistemologies suggested that it would be misleading to talk about trust in an absolute way. Kuhn found that college students tend to move from a position of absolute trust in experts to a position of radical relativity where no one's perspective is privileged and finally to a point of view that acknowledges that there exist good reasons to at least provisionally privilege the opinions of some individuals over others when it comes to subjects that fall within their area of expertise. These considerations do nothing to mitigate the essential problem: people necessarily come by much of their knowledge of the world around them through trust in a variety of epistemic communities rather than through direct investigation. In my dissertation research, I investigated the potential for introducing this type of activity into the classroom through the introduction of information-based problems. While my dissertation presents a more thoroughgoing cross-case analysis of what I found, I will present here a single illustrative anecdote in order to better illustrate some of the perils of encouraging a critical stance in one's students without fostering a concomitant sense of rational dependence.

The Vaccine–Autism debate

Ivan[1] is a mathematics teacher who was teaching a topics in mathematics course for non-STEM majors at Delta University, a regional research university. The majority of his students were juniors and seniors fulfilling their mathematics requirement prior to graduation. I collaborated with Ivan to develop a couple of information-based problems that he introduced to his class, the first of which required his students to choose a controversial topic and to analyzing the sampling strategy used by some relevant polls and research articles -- the students had just begun a unit on statistics and experimental design. Ivan's primary goal for this information-based problems was to, in his own words, instill a "critical sense" (Ivan, 4/17/14, Line 150) in his students. He described this as "something that should immediately click that says 'something fishy going on here'" (Ivan, 4/17/14, Line 149) when a student encounters a mathematical claim that does not make sense. I offer up this specific episode without making any claims about typicality, as there was only one time that anything like this occurred during the two information-based problems. Rather, it serves to illustrate a potential outcome of an assignment in which students are encouraged to examine published research with a critical eye.

Ivan began this activity by demonstrating what he wanted students to do by displaying an article on the board and discussing where it met and fell short of a proper experimental design. As it happened, the first presenting group adhered exactly to this format on a superficial level. The students described the article that they would go on to critique:

> Taylor: We ran across this new article and it's this one up here, the project by the CDC, government funded, and the claim of it is that there is no causal relationship between vaccines and autism rates in children. With this being said—
>
> Michael: Only 3 of the 8 managed care organisations were chosen for the study. A thousand, roughly, children participated. 256 had autism and 752 did not. And which I thought the eligibility for being selected was that you had to be born January 1st, 1994 to December 31st and you had to be previously enrolled in one of the NCOs from birth until your seventh birthday. Currently enrolled at the same time of sample selection, and you had to live within 60 miles of the study. You had to be between 6 and 13 years old at the time the study was selected and had to live with your biological mother since birth and, lastly, you had to speak good English. And you were excluded if you had any of the links to autism. (First Session, First Presentation, 3/3/14, Lines 22–33)

After spending a little more time describing the different sections of the article, Taylor criticized the choice of sample size by saying,

> It has 1008 participants, and I don't know about you guys but I feel like that's pretty low for a sample size. You're trying to

> sample a huge autism outbreak and you want to find out what causation or correlation is, you're not just going to sample a thousand from a 60 mile radius. You're going to try to get a nice randomized sample. It never says in any of the articles how it was randomized. It never—it just says that they were picked from the 1008 and it says they were picked from a 60 mile radius, so you wonder how they got these people. It never explained in the actual research article. (Taylor, First Session, First Presentation, 3/3/14, Lines 45–51).

According to Taylor, the problem with the sample size is that it "feels [...] pretty low" given the claim that was being made. This concern could be a result of the common misconception that a larger population requires a larger sample size or that the sample size should be thought of as a fixed proportion of the population (Huck, 2009). The student's subsequent concern about how "it never says in any of the articles how it was randomized" could be an honest concern although it is not clear what would count as sufficient evidence of proper randomization.

Immediately following her critique of the sampling method that Taylor used this evidence to level an accusation at those who funded the study:

> And it never said about the 256 people that did contract autism from these vaccines, they never talked about, even though they did, why it wasn't a cause. Also that kind of led to a sample bias because it's such a small sample size and, I think, well what we thought, was that maybe the government could be trying to cover up something because they didn't want us to think that they're vaccines which are regulated through the government, they wanted to make everything seem okay, but this study just seemed a little bit fishy. With all the, 'you can't be in the study if you're linked to autism', you can't be in the study if you don't live within 60 miles, and only taking a thousand and some people, I think that - Definitely a larger sample size would be more accurate and not have bias and it was very problematic to put so many restrictions on who could be in the study or not. (Taylor, First Session, First Presentation, 3/3/14, Lines 51–61).

While "sample bias" was one of the concepts that had been covered in the lectures that led up to these presentations, these students were not using it in the proper sense. While they said "that kind of led to sample bias", the referent of "that" is not apparent. It may be that the speaker is suggesting that the various restrictions related to recruitment resulted in a sample that was not actually random and that it was instead designed to be biased in order to exclude people who had been diagnosed with autism and whose inclusion would reveal the "true" link between autism and vaccination.

During my interview with Ivan after the class session, he expressed great frustration at how that particular presentation had played out. While the students had satisfied the terms of the assignment with respect to the tasks that they were asked to engage in (i.e., describe and critique the sampling methodology of a study), they had misapplied the statistical concepts that they were being taught. They had critiqued the sample size without actually having grounds to do so and complained about the restrictions placed on the sample without giving a reason for why it would be problematic. Ivan stated that he was frustrated because the students did not draw on what had been covered in the previous lectures when they criticized the paper. Further, while he did not say so explicitly in class, there is evidence that he disagreed with the conclusion that the students were arguing. During the questioning period, he contested a claim that was made about the chemical thimerosal being used as an additive in vaccines by pointing out that thimerosal had not been added to vaccines in the last ten years. This is notable because it is the only time that he called into question a point of fact that was not mathematical in nature. He told me after the class that he was aware that the students had "an agenda" (Ivan, 3/3/14) but he did not know how to intervene.

While, as noted above, there was a mathematical critique to be made of the students' presentation, it is notable that these students followed the directions of the assignment and that they engaged with the assignment to a degree that other teachers with whom I worked, and even Ivan in other circumstances, would have found creditable. The problem was not the students' engagement or enthusiasm, rather Ivan appeared to be upset because the students were failing to acknowledge the recognized consensus of the relevant epistemic community (Plotkin, Gerber, & Offit, 2009), i.e. those involved in epidemiological research. I submit both that Ivan's frustration is an understandable byproduct of an attempt to create a more contextualized mathematical problem without encouraging students to look beyond the articles at hand in order to assess the state of the debate writ large and that there exist other activity structures that could give students the motive along with the opportunity to engage in just that type of investigation. For example, I describe elsewhere (Erickson, forthcoming) a classroom in which students were given a very similar problem to the one described above, but in this case they were required to discuss the credibility of the sources that they found with their peers and to deliver a verdict on the most credible of those sources. Some of these students also chose the vaccine-autism debate but the subsequent conversations touched on the controversy surrounding Andrew Wakefield's retracted paper for *The Lancet* (Wakefield et al., 1998) as well as the understandable fears of parents confronting with the media's interpretation of the controversy. I do not make any claims here about the relative efficacy of the two different activity structures for encouraging rational dependence in classrooms other than those that I observed first-hand. Rather, I present these examples in order to (1) demonstrate that the reasoned dependence on the expertise of others is an important component of a productive critical attitude and to (2) show that teachers with similar goals and supports may nonetheless provide very different opportunities to carry out the exercise of rational dependence based on how information-based problems are operationalized in the classroom.

Conclusion

As long as mathematics instruction is directed not just towards the preparation of future STEM-professionals but also intended to empower students to engage with the mathematics that they will encounter in their everyday lives, I argue that students must both be encouraged and supported in the development of rational dependence. There is nothing to be served by allowing students to think that they can, or should, critically evaluate issues on their own if only they had the necessary background knowledge. This is both an unreasonably high bar that can serve to exclude rather than include people in public debate, and it misrepresents the collectively evolving nature of scientific knowledge itself. And there is everything to be gained by helping students realize that they depend on others and providing them with tools to help them navigate the cacophony of voices that they will encounter in the real world. Finally, I want to stress that an acknowledgment of our individual limitations ought to be seen as empowering. When it is allowed that we can (and should) cede certain judgements to expert communities, we are broadening our world in an essential way. This attitude empowers individuals to make more decisions of consequence and brings them into contact with the greatest of human accomplishments in all their plurality.

Notes

1. All proper names of people or places are pseudonyms.

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Respect for Ethnomathematics: Contributions from Brazil

Mariana Leal Ferreira
San Francisco State University

"Mapping time, space and the body: Indigenous knowledge and mathematical thinking in Brazil" (Ferreira, 2015) brings people, land, and numbers together in the fight for justice. In this extraordinary voyage through ancestral territories in Brazil, the Xavante, Suyá, Kayabi, and other nations use mapping as a tool to protect their human rights. Mathematics activities inside the classroom and in everyday life help explain how Indigenous Peoples understand the cosmos and protect its living beings. Ethnomathematics makes another contribution to a growing literature on the mathematical and scientific thinking of Indigenous Peoples around the globe. It makes mathematics alive and culturally relevant for students of all national backgrounds worldwide.

Indigenous Mathematics in Brazil

Squatting on the white sandbanks of the Xingu River, Chief Carandine Juruna carefully sorts out the bamboo arrows he has just exchanged for pottery with the upriver Kayabi. As he sets aside the different fish, game, and bird-hunting weapons, according to specific arrowheads, the 60-year-old man makes sure each household that contributed with animal-shaped clay pots receives its share of goods. Large families are privileged in the quality and number of arrows they get, and so are good hunters, hard-working pottery makers, elders, and the Juruna to whom the Kayabi were previously indebted.

The Juruna nod approvingly as their leader distributes the arrows, commenting on the quality of the bamboo, feathers, wax, and tree-bark ties employed by the Kayabi. The transmission of wealth is only one element of a much broader and enduring contract between the Juruna and the Kayabi. It embodies and records the entire credit structure of the community, including symbolic, interpersonal, economic, and emotional associations that reach far beyond the sole exchange of property—a "system of total services" (Mauss, 1990, p. 5). The fairness of the leader's distribution of goods is not put into question, nor is there a concern for immediate material profit, in this vast system of services rendered and reciprocated.

Meanwhile, an employee of Funai, the Fundação Nacional do Índio (National Indian Foundation) in Brazil, standing nearby, nervously operates his calculator, attempting to stipulate a price for each arrow he intends to buy from the Juruna and resell in Brasília, the country's capital. Antonio's reasoning is based on the monetary profit he customarily earns from the sale of indigenous "art craft." Waving the number of Brazilian bills that represents a "fair" price for the 20 arrows he wants, Antonio is outraged when Tarinu Juruna, Carandine's son, remarks that "only 7 arrows are for sale," and that he himself will calculate their monetary value. The reasoning behind the "exorbitant" price Tarinu arrives at is far beyond Antonio's comprehension. Unwilling to accept or understand a system that attributes different values to goods, Antonio angrily tosses the piece of paper listing Tarinu's calculations and shouts in indignation:

> I came all the way from Brasília to help you guys and you want to cheat me? Where on earth does 7 times 5 equal 125? I've pacified more than 500 Indians in my life, I've caught malaria more than 100 times in 20 years and you guys want to charge me $125 for 7 arrows! I could get arrows just like those anywhere in Brasília for as cheap as $2.50 a piece! You lazy Indians know nothing about money, about buying and selling. It's true what people say, that Indians are too stupid to learn math. [January, 1982]

A few miles downstream, along the Xingu River, shaman Intoni Suyá is busily drawing the 12 underwater creatures whose environmental knowledge provide the Suyá people guidance to fight court cases against insatiable mega-landowners. Horse fish, stork fish, needle fish, a couple of *human fish*, as well as the anaconda and sting rays, and other beings, feature prominently in Intoni's narrative, too. The shaman resorts to the animals' ancestral erudition in identifying the sacred locations of Suyá territory. In this respect, Suyá practices of land occupancy clash abruptly with non-Indigenous ideas of land ownership and jurisprudence. The Suyá are most interested in creating, negotiating, and redistributing resources within broad territories, in the generous spirit of a true gift economy. Their cosmology and everyday practices show local and extended families supporting one another in an economy of gift exchange in which the goal is to circulate, rather than accumulate resources. The discourse of land ownership according to Brazilian jurisprudence relies heavily, instead, on the basic tenet of capitalism: the expropriation and accumulation of wealth. Multiple Brazilian map books produced since the early 1980s convey very clearly Amerindian perspectives of land occupancy, which differ greatly from the complicated and convoluted process of "demarcation" of Indigenous territories by the Brazilian federal government.

Swimming downstream, the 12 hybrid creatures (Fig. 1) dodge floating debris dumped into the Xingu River by cattle ranchers, timber and mining companies. Patches of bubbly brown foam sail alongside big and small logs, on their way to sawmills farther north into the Amazon. Determined to protect the underworld of animal existence, the sacred fish-like animals work in unison, conferring with each other on how to best school the Suyá people about fighting for their rights and protect the land from its greedy invaders. Shaman Intoni Suyá explains in detail the incredible knowledge conveyed to humans by the sacred underwater beings:

Figure 1. The 12 underwater creatures. Intoni Suyá, 1999.

1. The *diacuí* or flute fish. It has a lot of bones and only moves and swims when other fish do. Suyá men should learn from the *diacuí* that they should never act in isolation, but act collectively in trying to regain disputed territories.
2. This little blue creature is the boss of the *diacuí* flute fish. This means that eventually a political or religious leader can tell a Suyá to act in isolation.
3. The black fish, *peixe preto*, is just like a canoe, but it is a fish. When it wants to surface from the depths of the water, it does so as a canoe. The Suyá people treasure their canoes and should learn that no matter how appealing cars or airplanes appear to be, canoes are the valued means of transportation because they are genuinely Suyá.
4. The needle fish, *peixe agulha*, floats on the water's surface. It has the capacity of being in the water while surveying from

inside what's going on outside above the surface, without raising any suspicions. Suyá men should act accordingly in the court case, trying not to raise unnecessary suspicions about, for example, the eventual use of violence if the results of the court case are not successful.

5. The sieve fish, *peixe peneira*, is full of holes but it is still a fish. This fish can swim swiftly against the current, because the water will pass through him and push him even further upstream. Suyá people must have this ability of fighting against strong currents.
6. The anaconda, *sucuri*, which in the beginning of times broke the waters loose from the grips of the humming bird, who kept the precious liquid hidden in secretive underground hideouts. Suyá people need the strength of the *sucuri* to fight against the white men, *caraíbas*.
7. The *peixe cavalo*, horse fish, is just like a horse, but a bit different. It is yellow and black and can live underwater. Unknown to the Suyá before contact with the *caraíbas*, the horse is strong-smelling and therefore a dangerous animal that is able to survive in hostile environments, underwater. Surviving hostilities is a key strategy today for the Suyá.
8. The *socó*, stork fish, which has the same ability as the *peixe cavalo*, but enhances its flying ability with a swimming one. Same traits the Suyá should cultivate during the court case: show the expert [myself] the Suyá's ability to envision their territory from above, as well as from an underwater perspective.
9. The gourd fish, *peixe cabaça*, which looks like a gourd but is a fish. He lives in the bottom of the river, but is still full of mud. Despite its dirty appearance, the gourd fish is very clever because it is deceptive. It pretends to know nothing, but it is very knowledgeable.
10. This is a human being, *ser humano*, but a water creature as well. It can tell all the fish to move around at any given time. Humans are more powerful than animals, so when they feel weaker than animals, they need to regain their strength by resorting to the power of the human fish, *peixe humano*.
11. It is a sting ray, *arraia*, that lives under the ground, but also inhabits the rivers. The *arraia* can be deceptive and sting those

who are unaware of its ability to become invisible. The Suyá should only resort to violence against *caraíbas* when really needed, since stinging can cause the *caraíbas* to spread diseases, throw bombs, and engage in other acts of violence that can decimate the Suyá population.

12. This one is also a human being, *ser humano*, but also lives in the water. All humans come from the waters, and therefore the importance of the disputed rivers and lagoons to the Suyá people. All our headwaters are in the hands of the *caraíbas*, big farmers, the gold miners that make the waters dirty with poison that kill the fish, and make our children sick.

Two thousand miles down South, outskirting the São Paulo metropolitan area, Guarani children of the Itaóca Village place their bodies on the frontline of this rich industrial state. They demand that the physical demarcation of their ancestral territories respect the 2007 United Nations Declaration on the Rights of Indigenous Peoples (UN DRIP). Guarani mapmaking ascertains the Itaóca villagers' right to the occupancy of their land. In their intricate and detailed maps, the youth show how knowledgeable they are about using mathematics to ascertain their human rights to land, quality education, food security, and health services.

Guarani children are indeed 21st century Indigenous Brazilians, associates of the largest Indigenous nation in Brazil, the 40,000-member Tupi-speaking Guarani. Their mapping activities demonstrate a systematic response to the dehumanizing conditions under which their parents live and work on the reservation, at dumpsites, in hospitals, and as cheap labor force for missionaries, farmers, tourists, and government officials. The Land-without-Evil, which they aspire for, depicts an apocalyptic vision of time and the body familiar to anthropologists and other social scientists. The youth reject the passivity of their elders, who seem to have been bludgeoned into accepting the continual assaults and violations of their dignity as human beings that have become part of everyday Guarani life in the Itaóca Village, just next door to the buzzing activity of São Paulo City.

Within the sprawling limits of São Paulo megalopolis, lies the Instituto Cajamar—a learning community center founded by Paulo Freire and other revolutionary educators in the 1970s. This is where young Guarani leaders, in the company of other Indigenous and non-Indigenous educators, engage in map-making activities to secure ancestral land rights. The young Indigenous leaders commemorate the first two decades of Indigenous Peoples, from 1995 to 2014, with a Book of Maps, bringing together people, land, and numbers in the fight for social justice. During the course of the workshop, our conclusion, reached by more than 60 teachers, was that "Mathematics is important for the autonomy of Indigenous Peoples." Self-determination, which includes the right to free, prior, and informed consent (FPIC), detailed in the UN DRIP is, indeed, of particular importance to Indigenous Peoples worldwide.

In Brazil, more than 3,000 Indigenous schools are now operating and dozens of universities are sprouting throughout the country. They are successfully strengthening Indigenous systems of knowledge by using map-making as a pedagogical tool. Most of these schools use their own intercultural and multilingual curricula to teach mathematics, history, geography, biological sciences, and language arts. These curricula not only meet state and federal standards, but also bring mathematics education to a whole new level. The curricula have increasingly been designed by, or with strong input from, the communities and the local teachers themselves. States and municipalities, often supported by the federal government, have passed important legislation supporting community-based calendars and cultural pedagogies.

Map-books and atlases have figured prominently in the Indigenous Education Movement, which gained force in the 1980s with the fall of the military dictatorship in 1985 and the Brazilian Constitution of 1988. Maps of several kinds were crucial for the protection of ancestral lands. The right to ancestral lands was enshrined in the new Constitution, a victory of Indigenous Peoples' own making. The Movimento Organizado Indígena put up a massive fight alongside nongovernmental organizations to protect ancestral territories. Different kinds of maps mentioned in *Mapping Time, Space, and the Body* have been at the forefront of this liberating rights-based movement. Many of these maps have been produced in Indigenous schools and in teacher-training programs throughout Brazil and South America. This work has spiked a much higher interest in school-taught mathematics as well as Indigenous mathematical knowledge. Mathematics was by far the favorite subject at Indigenous schools where I taught in Brazil, mostly because the mathematics we studied in school had real-life implications.

Real-life Mathematics Earns Respect for Indigenous Knowledge

The arrow problem featured in the opening of this paper, where Funai employee Antonio tried to buy arrows from Tarinu Juruna for far less than their value, is best understood in the context of real life. Of course 1 + 1 in the abstract equals 2. But in the context of Juruna, Suyá, and Kayabi peoples of the Xingu Indigenous Park, central Brazil, who practice a gift economy, 1 + 1 was *not* an accurate mathematical model for the real-life problem at hand. Nor was 7 × 5 the correct model because more was involved than paying $5 cruzeiros for each of 7 arrows. The men were requiring Antonio to pay his debt of $72 + $18 cruzeiros (the currency at the time). And they wanted a fair price of $5 for each of 7 arrows for a sum of $125, as Tarinu explained to his classmates in school. So there were two different models clashing. Antonio's model was 7 × $2 = $14 and no payment of his debt. Tarinu's model, in turn, added up to $125. Since Antonio refused to pay that amount, the final answer to the arrow problem was $0. Here, the clash was between two real-life economic models: capitalism *versus* gift-exchange.

Other real-life examples of great practical importance documented in *Mapping Time, Space, and the Body* were the map-making models of Indigenous territories that won official recognition as demarcated, protected federal lands. Shamanic map-making activities in the Xingu Indigenous Park relied on the lived experience and shamanic knowledge of spiritual leader Intoni Suyá. In their successful court case, the Suyá people won the possession of a section of their traditional landbase, the *Terra Indígena Wawi,* adjacent to the Xingu Park in central Brazil. The people later disputed farmers' challenges to that decision, winning again in court in 1998. Intoni evoked the power of humans and animals over the environment, its goods and resources, applying Suyá mathematical knowledge to this real-life problem and its hands-on solution. The information provided allows us to construct models, for mathematical, environmental, and other sciences, by using different kinds of maps to represent the impact of deforestation on and around Indigenous lands in Brazil and throughout the Americas.

In addition to mapping territories, other real-life situations have generated intense interest in mathematics among Indigenous Peoples in Brazil. Worldwide, real-world applications have also been credited for boosting interest in mathematics, as reported by the *New York Times* in its series "Number Crunch" (*New York Times*, 2013). Like other communities across the planet, Indigenous Peoples in Brazil are able to identify important mathematical models in practical situations and map their relationships using such tools as diagrams, tables, graphs, and flowcharts. They have also made extensive use of photographs, drawings, and paintings all along, as shown or mentioned throughout my aforementioned book, *Mapping Time, Space, and the Body*. Brazilian Indigenous Peoples can analyze those relationships mathematically to draw conclusions, focusing on their own mathematical knowledge, often referring to Indigenous rights as human rights. Indigenous Peoples routinely interpret their mathematical skills in the context of the situation they live in and help create. In addition, they reflect on whether the results make sense, possibly improving mathematical models that can help improve mathematical achievement in mathematics education. What these models have in common is that real-world applications, such as designing maps; building houses, schools, fences, and canoes; negotiating sales and purchases; cooking; and reckoning with family relationships, reveal that mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.

The latest news and reports show that despite Brazil's economic and social inequities, the country is improving its international education standards. Brazil is considered a strong performer and successful reformer in education. By setting individual quality goals and then leaving schools, Indigenous or not, free to choose how best to achieve them, Brazil's Education Development Plan has "effectively transformed the country into a giant laboratory of best education practices," according to a Pearson Foundation report. "Improving the education of its citizens is vital for Brazil's future economic development. With children under 15 years of age accounting for one-fourth of its population of 200 million, the challenges are huge. But progress is being made" (www.pearsonfoundation.org/oecd/brazil.html). *Mapping Time, Space, and the Body* shows that Indigenous Peoples' mapping activities in Brazil are contributing to the country's overall successful performance in mathematics literacy today.

Notwithstanding Brazil's wide-ranging educational reform, the country still has much to improve. The United Nations Children's

Fund (UNICEF) office in Brazil points out that inequities still persist in the country, especially in relation to regional, ethnic and racial, and socioeconomic differences. So that the country's success rates in education can effectively benefit its minorities and the poor, UNICEF believes that public policies that reduce poverty and inequality in all its dimensions need to be implemented. Nevertheless, Brazil's experience with education in the last decade shows how a country facing major challenges in teacher preparation, infrastructure and student commitment, and with a highly decentralized education system, can use national and international benchmarks to identify problems and propose solutions. Indigenous Peoples have played a big part in these advances. In fact, in July 2014 the 66th Annual Meeting of the *Sociedade Brasileira para o Progresso da Ciência* (Brazilian Society for the Progress of Science, SBPC) inaugurated the *SBPC Indígena* to discuss the Indigenous universe in its scientific program (see http://www.sbpcnet.org.br/site).

Improvements in Brazilian Indigenous education supply a useful lesson for mathematics education in the U.S. the *New York Times* (*NYT*) Editorial "Who Says Math Has To Be Boring?" (www.nytimes.com/2013/12/08/opinion/sunday/who-says-math-has-to-be-boring.html) asks: Why are girls and minorities missing from science and mathematics classes? The answer is disturbing, but not surprising: "A big reason America is falling behind other countries in science and mathematics is that we have effectively written off a huge chunk of our population as uninterested in those fields or incapable of succeeding in them" (*NYT*, 2013, p. A28).

Just like Indigenous Peoples in Brazil, women and most ethnic minorities in the United States have internalized the false belief that they are incapable of mastering Science, Technology, Mathematics and Engineering (STEM) fields as well as men. More than half of the U.S. population will be made up of minorities in 2043. It seems imperative that one of the most dynamic sectors of the U.S. economy no longer remain a male and largely white and Asian domain.

There is plenty of evidence that Indigenous Peoples in the U.S. also developed mathematical ideas based on their real-life experiences. Many of these examples can be useful in instructional materials to meet Common Core State Standards for Mathematics. This most recent renaissance in Indigenous scholarship, associated with information documented by mathematicians and social scientists, has spurred a variety of pedagogical materials. These include lesson plans and activities involving problem-solving and critical thinking, frequently in the context of real-life situations. Such didactic publications refer to either the acclaimed Curriculum and Evaluation Standards for School Mathematics (National Council of Teachers of Mathematics, 1989), or else the controversial Common Core.

Multicultural Mathematics: Critical Thinking and Real-life Situations

The truly multicultural history of mathematics also provides useful material for today's classrooms. For example, the children's book, Senefer, *A Young Genius in Old Egypt* (Lumpkin, 1992), tells the story of the African people of Egypt who developed the first ciphers or symbols for numbers. The events in the book, inspired by the life of Ah'mose, the scribe, and Senmut, an advisor to the female Pharaoh Hatshepsut, took place 3,500 years ago. Senefer grew up learning how to model with mathematics, raising obelisks and designing temples in Egypt. Can Senefer's wisdom help our mathematics classes today? Can we use multicultural examples to encourage students of color and broaden the cultural outlook of all students? A number of scholars, starting in the 1960s, dedicated enormous effort to make that happen. They were inspired by Ubiratan D'Ambrosio and Claudia Zaslavsky, pioneers in ethnomathematics. Take a look at the International Study Group on Ethnomathematics (ISGM; see http://isgem.rpi.edu/) for many excellent resources. Based on real-life situations, the works of ethnomathematicians offer a multitude of multicultural classroom-ready resources for mathematics teachers and students (e.g. Lumpkin, 1997a, 1997b; Zaslavsky, 1987a, 1987b).

Improving the representation of African Americans, women and minorities, Indigenous Peoples included, in the U.S. would also enrich American scientific research and development, because these populations would add a different perspective to workplaces currently dominated by white men. As the *NYT* article "How Teacher Biases Can Sway Girls From Math and Science" recently argued, entrenched stereotypes about who does well in science and math also work against minorities in classrooms (Miller, 2015). Many teachers give up easily on girls and minorities because they are not expected to do as well as white students.

These entrenched stereotypes begin to give way as teachers and students realize the value of Indigenous mathematics. Some examples of integrating Indigenous knowledge of mathematics into the standard mathematics curriculum were presented here. Others are in my book *Mapping Time, Space, and the Body*. But the question remains: How to integrate Indigenous mathematics and other non-European sources of mathematics into STEM programs? That is the challenge still facing all of us today.

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Politics of Ethnomathematics: An Epistemological, Political, and Educational Perspective

Karen François[1], Carlos Monteiro[2], Liliane Carvalho[2], Eric Vandendriessche[3]
Free University of Brussels VUB, Belgium[1]; Federal University of Pernambuco, Brazil[2]; Paris Diderot University, Sorbonne Paris Cité, France[3]

This article discusses the concept of ethnomathematics from three perspectives: the epistemological, the political, and the educational. We use a theoretical toolkit that borrows the concepts of "discourses" and of "disciplining" from Foucault but also the concepts of "language games" and of "family resemblance" from Wittgenstein in order to analyze the pedagogical program of field education in Brazil. A philosophical analysis enables us to investigate the failure of the government's educational policy as well as resistance and social movements among "field populations". Basing ourselves on an anthropological perspective, trying to understand how cognitive acts underlie local activities and how these activities are embedded into a cultural environment, we suggest that this approach might have a positive impact.

Introduction

Throughout the last decades, research carried out in the field of ethnomathematics have aroused lively debates of an epistemological nature, discussing whether or not mathematics are culturally determined, and thus questioning the "possibility of the simultaneous existence of culturally different mathematics" (Barton, 1999). The debate has often amounted to a discussion between essentialist and non-essentialist mathematicians about the universal nature of mathematics (Pais, 2011). Without entering into this debate, we would like to address here some epistemological, political, and educational issues raised by the (potential) use of "traditional" knowledge in local mathematics curricula. In this paper we use the word 'mathematics' in the plural to emphasize that there are more than one 'mathematics' as shown by the anthropological evidence of diverse mathematical practices on which we will elaborate. Since the 1970s, many educational approaches aiming to teach mathematics to students living in small-scale societies by taking into account the cultural context have been carried out and described by educators. In most cases, these educators have suggested teaching some mathematical concepts in classroom through specific cultural activities involving numbers, logic or spatial configurations that could be used as pedagogical supports (Pinxten & François, 2007, 2011). These teaching practices led governments to implement specific mathematics curricula among different populations, each with their own specific needs and cultural background. In this article we will focus on the epistemological, political, and educational issues raised by such ethnomathematical teaching practices. First, we will focus on some epistemological issues that need to be addressed—in our view—to better understand the nature of (mathematical) knowledge thus taught to students. In the second and third parts of this paper, we will expand on the political, and educational issues, by investigating the case of government policy on field education in Brazil.

Epistemological Reflection

In small-scale societies, mathematics does not generally appear as an autonomous category of indigenous knowledge. However, as it has been confirmed by numerous ethnographies of various societies of oral tradition, a form of rationality occurs within various practices, such as the making of calendars or ornaments, navigation, games, kinship systems, string figure-making, sand-drawings, traditional housing, etc. These activities have generally been used as teaching resources in math classroom through two main different methodological approaches. First, some ethnomathematicians have proposed to use the figures or artifacts produced through such "traditional activities"—but without taking into account the processes involved in the making of these artifacts—as a base to teach usual mathematical concepts. Gerdes (1999) proposes such an approach, using sand-drawings, baskets, etc., as a mean to introduce geometrical concepts (e.g. symmetry) and arithmetic properties (e.g. sum of an arithmetic sequence), but also as a geometrical support to demonstrate some fundamental theorems such as Pythagoras' Theorem. This pedagogical practice's major disadvantage lies in its disconnection from the actors' viewpoint(s) and from the cognitive acts involved in the creation of such artifacts. The connection between this pedagogical practice and the cultural context is thus highly questionable. The second methodological approach consists in including some specific cultural activities "as such" in the (local) mathematics curriculum (Murphy, 1998; Nkopodi & Mosimege, 2009). The epistemological issue is then to determine whether these activities relate to mathematical practices, and how. Philosophy in general, and the philosophy of mathematics in particular, does not yet offer efficient conceptual tools to tackle this issue, as it often takes for granted all that is mathematical. However, regarding the issue of determining whether or not an activity is related to mathematics, it can be noticed that there is a significant difference between activities that involve numbering and/or measuring, and those dealing with geometrical forms. The counting of yams with a basket as it is practiced in Melanesia—in the Trobriand Islands (Papua New Guinea), for instance—is quite readily recognized by academia as mathematical, as it involves the use of a particular counting method. By contrast, other practices that require "geometrical" abilities (such as basketry or ornamental frieze making) are usually not—or not so readily—regarded and analyzed as mathematical by scholars. This question needs to be further investigated. Although a few ethnomathematical studies have been carried out in that perspective in the last decades (Ascher, 2002; Chemillier, Jacquet, Randrianary, & Zabalin, 2007; Vandendriessche, 2015), there is—to our knowledge—a lack of ethnomathematical in-depth study, aiming to analyze the form of rationality that underlies these "(ethno) geometrical" practices. Progress on that issue would allow us, on the one hand, to better understand the kind of mathematics students could learn while practicing activities such as string figure-making or basket-weaving in math classroom, and, on the other hand, to contribute—in some way—to an epistemological reflection on the nature of mathematics.

Political Power

Methodological approaches that aim to include specific cultural activities in school contexts can be questioned from different perspectives. We will shortly comment on the perspectives of cultural diversity and political hegemony, and finally on the anthropological approach. A first perspective is in line with the above comments made on the methodological approach which consists in incorporating specific culture-related activities into the mathematics curricula without taking into account the connection to the cultural context or to the actor's viewpoint. Within the framework of field education—on which we will elaborate in the next section—government policy on the implementation of standardized pedagogical programs is criticized because of the disconnection from the specific needs and interests of the targeted groups. Moreover a top-down implementation of the pedagogical programs failed to take into account the teachers' viewpoint(s) (i.e. the actors' viewpoint) in setting up and implementing those pedagogical programs. A second perspective on the implementation of pedagogical programs in specific school contexts reflects the philosophical and political notion of hegemony based on Foucault's (1966) concepts of discourses as 'disciplining'. Mathematical curricula and mathematical pedagogical programs can be analyzed as discourses that facilitate the teaching process, but at the same time they regulate and delineate the practices of knowledge. These mathematical discourses can both enable and limit the teaching process, but also the teachers' and pupils' practices and thoughts. Therefore, the mathematical programs that are developed and implemented in specific school contexts turn out to be tools of power (François, Coessens, & Van Bendegem, 2014). Knijnik (2012) analyses ethnomathematical practices from a philosophical perspective that articulates Foucault's theoretical notion of school mathematics as disciplinary. She claims to go beyond the naïve understanding of mathematical diversity as an acknowledgement that there are different ethnomathematics. We have to introduce the notion of power to understand and to analyze the politics of knowledge and how it operates in schooling processes, in the implementation of pedagogical programs and, more particularly, in mathematics curricula. The politics of power has to be understood as the imposition of meaning produced by a *double violence* (Bourdieu, 2003 as referred to in Knijnik, 2012). The first violence consists in imposing one culture on another. The second one is the power of naturalizing the first violence by accepting the dominant culture as the only possible way of being or, at least, as far more preferable to the former one. In the case of mathematical knowledge, we notice the first move of imposing the Western body of academic abstract mathematics upon all mathematics curricula. The second move is the violence caused by establishing this Western body of academic abstract mathematics as the highest achievement of human culture and as the necessary condition of full citizenship.

The third perspective is the anthropological one, which is central to the discussion of the existence of many ethnomathematics and of their respective values. In line with D'Ambrosio's use of the notion 'ethno' as the 'very different and diversified cultural environments, i.e., in the diverse ethnos' (D'Ambrosio, 1990: 369) we argued that Western mathematics are themselves also considered as having been developed—and as still being developed nowadays—within a particular contextual reality. The research interests of ethnomathematics pertain to the development, analysis and teaching of mathematical knowledge as dynamic processes embedded in their socio-cultural context. The Western body of academic abstract mathematics itself cannot be understood or taught separately from its cultural environment and power mechanisms (François & Van Kerkhove, 2010). Knijnik (2012) refers to the later work of Wittgenstein (1975) to explain the existence of many different ethnomathematics. Wittgenstein abandons the essentialist concept of language and therefore denies the existence of a universal language. Languages—or 'language games' as Wittgenstein calls them—immerse in a form of life, in a cultural or social formation and are embedded in a totality of communal activities. This idea gave rise to the notion of understanding rationality as an invention or as a construct that emerges in specific local contexts. Thanks to the notion of 'family resemblance', one can understand the existence of different kinds of mathematical knowledge and call it ethnomathematics that coexist. Wittgenstein (1975) used the concept 'family resemblance' (*Familienähnlichkeit*) to refute the idea that words have a single and fixed meaning by standing for objects in reality. He also claimed that words acquire meaning from the thoughts of those who are using them. Instead, to Wittgenstein, words are connected by a series of overlapping similarities—as it is the case with family members, among whom no single feature is common to all of them. In the next section we will discuss both the relevance of epistemological reflection and the notion of political power as related to mathematics teaching. Therefore we will look into the case of field education in Brazil. We will argue that the government's inadequate response to the field populations' demand for an educational approach that respects their cultural identities could be partly explained by (i) denying epistemological constraints and (ii) political hegemony. We will illustrate how new pedagogical programs aim to take into account the cultural context in field education, but at the same time fail to implement it by directing field teaching towards a homogeneous approach. Besides we will illustrate how the governmental education system used pedagogical programs as a "device" (Foucault, 1966) to impose urban curricula to field populations.

Field Education

In Brazil the concept of *field education* is related to the notion of *field population*, which is used to identify specific groups who are involved in certain economical activities that have their own cultural specificities—e.g. family farmers; salaried farmers; vegetal extractive workers; forest people; artisanal fishers; riverbank people; coast inhabitants; settlers and encamped of agrarian reform; slaves of African descent who live in the remaining *quilombo* communities throughout Brazil; and various others who work in rural areas to produce their material conditions of existence (IBGE, 2010). Brazilian social movements and organizations, as well as the governmental education legislation, use the term 'field education' when referring to the teaching and learning processes which are developed to meet the needs of these populations. There is a strong ideological connotation in the term *field*, which was constructed by Brazilian social movements and NGOs during the last decades of the 20th century. The theoretical and ideological debate about the use of the concept *rural education*, as opposed to *field education*, emphasizes the fact that field education cannot be limited to geographical and demographical references. According to Operational Guidelines for Basic Education in Field Schools (Ministério da Educação, 2002), the term 'field' emphasizes the notion of *field of possibilities*. According to the Brazilian statistical report on demography (IBGE, 2010), the total population of Brazil was 190,755,799 inhabitants in 2010 and 29,830,007 people (15.6% of the total population of Brazil) lived in rural areas. The figures for schooling show that 41.8% of the people aged over 15 in this rural population are functionally illiterate, while in urban areas this rate is much lower, at 17.2% (IBGE, 2009). Historically, the organization of schooling brought to field populations in Brazil did not take into account their particularities (Monteiro, Leitão, & Asseker, 2009). Educational policies were not concerned with schooling that considered field reality. In this sense, the curricular content, teaching methodologies and pedagogical proposals of the urban schools were implemented, unchanged, in the rural ones. Therefore, one of the major claims among field populations was an educational approach that would respect their identities. This approach was called 'field education' to underline that it had emerged from the field. The aim was to build a curriculum perspective that would give value to the specific knowledge and needs of field populations (Arroyo, Caldart, & Molina, 2004). In order to meet the claims of social movements, the federal government proposed pedagogical programs even before field education became an official public policy. In 1997, the Ministry of Education proposed the Active School Program (ASP) (Programa Escola Ativa) with the objective to improve the quality of early elementary teaching in the field schools throughout more of the densely populated rural areas of North, Northeast and Midwest regions of Brazil. The ASP prescribed a pedagogical approach designed for multi-grade classes that made a combination of elements such as: teamwork, self-learning, teaching through specific textbooks, community participation, monitoring of students, and continuing education for teachers. Over the years, the ASP was criticized in terms of stereotyping, hegemony and imposition of pedagogical and didactical programs. Monteiro and Alves (2011) conducted an analysis of the ASP math textbooks, and identified that most of their contents were related to numbers and operations. The texts books offered only few discussion and tasks about contents that could be associated with geometry and statistics. Such perspective reinforced the stereotype that mathematics is purely a matter of calculation. Additionally, the textbooks contained examples of a stereotyped field context that was limited to agriculture and animal husbandry. They were illustrated by photographs and drawings of people—supposed to be field folks—that went barefoot, wearing cheap checkered shirts and straw hats, and using carts. When Knijnik and Wanderer (2013) analyzed the ASP, they based their study on official documents and ASP pedagogical publications, especially those associated with the teaching and learning of mathematics. They also collected data from a questionnaire that was answered individually by 150 teachers, all of whom were responsible for teacher education courses developed in different regional ASP centers in the State of Rio Grande do Sul (southern Brazil). Knijnik and Wanderer (2013) emphasized that the ASP pedagogical guidelines were presented in detail: these guidelines covered not only the contents that were to be taught but also the didactic procedures that are to be followed. This aspect is analyzed as an underlying process of subjection that regulates the teachers' conduct, in order to ensure that their students' learning should be developed within a specific rationality. Although the ASP documents underlined that cultural, political, economic, and social characteristics needed to be considered, the teachers' pedagogical procedures were conducted so as to not include those specificities. Knijnik and Wanderer (2013) argued that an analysis of the teachers' responses suggested they tended to value less the knowledge and experiences of people who lived in field contexts. For example, the teachers' responses showed that they saw field people as people "without culture" or "with little culture" (p. 219, our own translation). Monteiro, Carvalho & François (2014) drew similar conclusions from an analysis of the teachers' opinions about field realities. Teachers tended to describe field students as restricted and inferior compared to students who live in urban areas. The outcomes of their study also suggested that the pedagogical organization of field schools does not yet consider the particularities of field education. Generally speaking, the teaching of mathematics has a peripheral status within their pedagogical planning and less time is spent in class on this subject than on others.

A "New" Governmental Program

Such criticism of the ASP resulted in occasional reformulations. One of the recent changes in the program was the substitution of textbooks. In 2012 the National Program of Textbook organization (Programa Nacional do Livro Didático – PNLD) approved two new collections of textbooks, that were considered as more suitable for field schools and intended to replace previous ASP textbooks. In 2013 the ASP was itself replaced by the Program School of Land (PSL) (Programa Escola da Terra), in accordance with a ministerial ordinance (Ministério da Educação, 2012). According to the Minister of Education, the PSL aims (i) to promote school access for field students and *quilombolas,* (ii) to maintain them at school, and (iii) to improve their learning in their communities by supporting teacher education. Teacher education should meet operating needs that are specific to rural schools and to those located in *quilombolas* communities by making new resources available: PNLD Field textbooks and a *Pedagogical Kit*. There are no available PSL pedagogical publications that explicitly give guidelines on the processes of teaching in field and *quilombola* schools. Therefore, the only one is the ministerial ordinance signed by the federal government. An analysis of that official document allows us to identify similarities to the ASP. Akin to the ASP, this new PSL program aims to consider the specificities of field school, but its *Pedagogical Kit* directs field teaching towards a homogeneous teaching approach. One year after the signature of the official document, the PSL begun to implement a pilot project in four major regions of Brazil with the support of seven federal universities. It seems that the university staff involved in this implementation has already begun to question the PSL. The universities decided not to accept the Pedagogical Kit, and they now intend to produce specific material for each State in which the PSL will be developed. The Field Education became a public policy when a ministerial ordinance was published (Ministério da Educação, 2012). As a consequence of this policy, all state school networks had to follow the official prescriptions. However, many Secretaries of Education of federated States and municipalities are not familiar with the Field Education principles. Thus, the main criticism about governance and subjection, as described by Knijnik and Wanderer (2013), is still very relevant in many field education contexts, because the people in charge of this public policy are not conscious enough of its actual motivation, which is to give value to field people. Between the end of the ASP (2012) and the implementation of the new Government Program for Field Education (by 2014), the Research Group in Mathematics Education in Field Education Contexts (GPEMCE) of the Federal University of Pernambuco (Souza, 2014) conducted a study. The central research question of this study was how the teaching of mathematics was developed in field schools at a period when the federal government had no specific pedagogical guidelines (i.e. 2013). The data were collected in the municipality of Igarassu and its educational network of 45 schools, among which 23 were classified as field schools. 104 teachers from those field schools completed a questionnaire with open and closed questions related to (i) professional background, (ii) conceptualization of field education, and (iii) resources used in the teaching of mathematics. The first analysis of the data was a descriptive statistical analysis (using SPSS) of frequencies. The preliminary results show that an overwhelming majority of teachers (90%) responded positively when asked whether they knew what a field school is, and whether they had already heard about field education. However, it was identified that very few teachers (4%) made an explanation that considered the economic and cultural aspects related to the concept of field school. The vast majority of participants (68,4%) based their answer on a demographic notion of the field that was associated with a rural context. Almost all of the teachers did not live in field contexts and, similarly to the findings of Knijnik and Wanderer (2013), and of Monteiro, Carvalho & François (2014), teachers seemed to have a narrow perspective about their students and the field life. From our observations, we could infer that teachers and pedagogical coordinators do not conceive field education as an important approach that could benefit to field people and to their communities. Generally, the educators involved are not conscious of the political and cultural issues associated with field education. Therefore, the actual pedagogical approach seems to contradict both the official documents and the critical perspective proposed by the social movements.

Discussion

On the grounds of a philosophical analysis of the politics of ethnomathematics, we will give meaning to the gap between governmental policy and the organization of schooling and mathematics education in specific social and cultural contexts. Therefore we use the concept of hegemony and Foucault's concepts of discourses as 'disciplining' to investigate the governmental mathematical pedagogical programs. They can be understood as discourses that try to facilitate the teaching process, but at the same time they regulate and delineate the practices of knowledge. In the case of field education in Brazil, we observe the resistance of social movements that criticize the educational policies of implementing in field schools the urban schools' curricular content, teaching methodologies and pedagogical proposals. Field populations demand an educational approach that respects their cultural identities. This educational approach has to emerge from the field, from local practices and from local knowledge. In tune with Wittgenstein's conception of language games (1975), field populations demand an educational approach in their own language games, starting with their own 'ethnomathematics'. At this point we see that an anthropological approach can play a central role in the field of ethnomathematics. Ethnomathematicians seek to articulate the conceptual approaches of local mathematical practices (mathematical modeling, historical, philosophical, and/or pedagogical perspectives) with an ethnographical approach that would enable them to collect new data about activities that involve mathematical ideas. At the same time, they try to reach a better understanding of the cognitive acts that underlie these activities and how these activities are embedded in the social organization and symbolic systems of the societies within which they are practiced. We suggest that this approach should have a positive impact in the sphere of education. It might incite policy makers and teachers to introduce the mathematical aspects of activities practiced in a particular society, in an attempt to better take the cultural context into account. Further research should be undertaken to analyze how pedagogical programs and educational materials would be accepted and implemented by local educational actors, as well as by populations concerned, if their elaboration was based on the second stage of an in-depth epistemological and anthropological/ethnographic research on local (ethno) mathematical knowledge.

Notes

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Challenging Stock Stories of Mathematics Education: Meritocracy and Color-blindness Within Teachers' Beliefs

Mark Franzak
New Mexico State University

Although mathematics is often described as race-neutral and color-blind, data on the achievement gap and lived experiences of marginalized populations indicates otherwise. This article problematizes widely held views of meritocracy and color-blindness in mathematics education, exploring the origins of these beliefs, and the potential to disrupt them. The author utilizes his own experience in evolving his classroom practice as a function of his beliefs of mathematics, and teaching and learning mathematics, to provide a specific context in which to situate the literature base. Critical race theory is incorporated as a lens to understand the power structures associated with these stock stories.

Introduction

Research on mathematics teachers' beliefs has a history more than four decades long, with a myriad of different foci. Thompson's (1992) summary of this body of research from 1916 to 1992 delineates the history of research into mathematics teachers' beliefs in parallel with the aspects of research in social psychology. The study of people's beliefs held strong interest among social psychologists in the early 20th century, which then faded in the 1930s, and then was revived with cognitive psychology in the 1970s (Thompson, 1992). Subsequently, "the 1980s witnessed a resurgence of interest in the beliefs and belief systems among scholars from disciplines as diverse as psychology, political science, anthropology, and education" (Thompson, 1992, p. 129). Thompson (1984) noted the connection between mathematics teachers' beliefs and their practices, although she indicated that the relationship was complex, involving many factors.

As the body of research in mathematics education focusing on teachers' beliefs grew in these decades, so did the varying conceptualizations of belief, and commensurately definitions of the term (e.g. McLeod, 1992; Thompson, 1984). Pajares (1992) attempted to gather the variety of conceptualizations in the literature base to reduce confusion and clutter regarding varying definitions of beliefs, and has been cited in more than 5,000 scholarly works[1]. However, mathematics education in the 1990s occurred in an era that has become known as the Math Wars (Schoenfeld, 2004), in which academics fought over epistemology and pedagogy of mathematics. Teaching methodologies and mathematics curricula were central research topics in the last decade of the 20th century. The 2002 publication, *Beliefs: A Hidden Variable in Mathematics Education?* further opened the academic conversation to an aspect of research that has received less attention than more prominent mathematics topics at the end of the 1990s. Subsequently, the publication in 2009, *Beliefs and Attitudes in Mathematics Education: New Research Results* added to the body of knowledge, but did not address at least one critical aspect. Lacking from this work, and virtually all other mathematics education research on teachers' beliefs, is the study of race.

Mathematics is widely perceived as neutral and accessible to all (Battey & Franke, 2013; Nasir, Atukpawu, O'Connor, Davis, Wischnia, & Tsang, 2009; Tate, 1994), yet this perception is in direct contrast with the lived experiences of minority[2] students (Berry, 2005; Martin, 2006). The widely researched achievement gap (Leonard, 2009) is one indicator of inequitably structured education, consistently showing a drop in performance between white/Asian students and Black, Latina/o, American Indian, and Alaska Native students. However, merely noting the differences in performance on standardized tests can result in perpetuating inequity[3] through deficiting[4] minority students (Gutierrez, 2008; Gutierrez & Dixon-Roman, 2011), without significantly adding to the understanding of institutionalized barriers. The identification of the gap in performance between white and minority populations, without situating this aspect within the larger consideration of race in education, allows for the gap to be explained through deficits in minority groups (e.g. Herrnstein & Murray, 1994).

Deficit thinking in the United States has a substantial historical lineage, with social impacts that span the centuries from colonial America to the present day.

> In sum, the roots of deficit thinking are inextricably tied to racist discourses that evolved from the early 1600s to the late 1800s. Out of these discourses came beliefs that racial minorities were physically, cognitively or culturally inferior to whites… In the area of American education, most racial minorities were denied the privilege of being schooled until the early 1900s, and the rationales used to justify this practice ranged from ideological beliefs based upon racist assumptions to economic interests supported by whites (Menchaca, 1997, p. 37).

Specifically applied to education, deficit thinking is manifested in the development of intelligence testing through racist ideologies (Valencia, 1997b). Intelligence testing was utilized to restrict access to the highest quality of education on the basis of the purported credibility of measurable intelligence: "The alliance of intelligence testing, deficit thinking and curriculum differentiation helped to legitimize the meritocratic ideology of 'equality of opportunity' (one gets in curriculum what one deserves, based on innate ability)" (Valencia, 1997b, p. 82). The devastating impact of deficit thinking is relevant, especially when race is undertheorized in mathematics education research, and is instead reified as a categorical unit with causal implications.

Hence, research into aspects of how mathematics education impacts the lives of minority students is called for (Gutierrez, 2008; Martin, 2009), with particular attention to how race is conceptualized (Martin, 2013).

Personal Experiences as a Teacher Impact Belief Structures

I had been teaching secondary mathematics for fourteen years before my beliefs and teaching practices underwent significant transformation. In those initial years, I believed that mathematics was a discipline removed from questions concerning bias and subjectivity. Mathematics was a matter of thought, not a matter of interpretation, and if there were problems with accessibility to mathematics, I did not perceive them to be rooted in content or instruction. My beliefs in mathematics as a unique and pure discipline meshed with my beliefs about education in general, and specifically about learning mathematics. I professed the values into which I was acculturated (Harro, 2000), and believed strongly in the majoritarian narrative[5] of rugged individualism and work ethic (Cammarota, 2006; Lawrence III, Matsuda, Delgado, & Williams Crenshaw, 1993), which "privileges Whites, men, the middle and/or upper class, and heterosexuals by naming these social positions as natural or normative points of reference" (Solorzano & Yosso, 2009, p. 135).

To provide insight into this story, I utilize critical race theory[6] to name my students, and myself and to provide context to understand the underlying structures of power that permeate my story (Ladson-Billings, 2009). I am white, male, and middle class, and I exist in a society that bestows privileges on people with these characteristics (Bell, 2010; Delgado & Stefancic, 2001). One of these privileges is that of majoritarian stories, which establish and reify norms based upon white privilege (Solozano & Yosso, 2009). "Stock stories are the tales told by the dominant group, passed on through historical and literary documents, and celebrated through public rituals, law, the arts, education and media" (Bell, 2010, p. 23). Two stock stories figure prominently in this discussion: meritocracy (Bell, 2010; Delgado & Stefancic, 2001) and colorblindness (Bell, 2010; Delgado & Stefancic, 2001; Kendall, 2006). I believed strongly in the majoritarian narrative that espoused "you can accomplish anything if you work hard enough at it." Two values emanate from this ideology: work ethic and individualism – one must work hard, and one must work alone to succeed, as anything else would be cheating. Although I have been greatly concerned about racism my entire life, I willingly embraced the narrative of colorblindness when it came to instruction and my classroom. I believed mathematics to be a race-neutral subject, and the learning of mathematics was not just imminently connected with the work one was willing to put in, but was inseparable from that.

I teach in a school district on the border with Mexico in which 95% of the students are Latina/o and 100% of the students receive free lunches, yet my adherence to colorblind and class-blind ideologies allowed me to alleviate myself of responsibility for their widespread failure in my classroom. As I reviewed the capabilities of my students, I freely engaged in deficit thinking (Valencia, 1997a), noting to myself and my colleagues that "if they would only do their homework/take notes during class/ask questions/pay attention/try harder," they would learn the material and their grades would be higher. I fully situated their acquisition of mathematics as *their responsibility.* Those students who excelled, I applauded; and those who failed, I blamed for a spectrum of faults (Valencia, 1997a). I decried their lack of effort, and their unwillingness to accept personal responsibility. I perceived their devaluing of individualism through their continual desire to work together and lamented that with group work I could not ensure who had actually earned the grade, and who was benefiting from others' knowledge without commensurate effort. By and large, the ways that my students wanted to learn in the classroom conflicted with my beliefs about learning mathematics. My colleagues in the mathematics department did not challenge my teaching methods, results, or construction of a race-neutral meritocratic identity as a teacher. Instead, as my colleagues historically held identical views, these were the norm for mathematics instruction at my school. "The story of meritocracy affirms an image of fairness and justifies positions of dominance as rightly earned, while simultaneously holding those who are not successful accountable for their own failure" (Bell, 2010, p. 34). Many of my students failed, and I applied the stock story of work ethic to shoulder them with the burden.

My beliefs in mathematics as a field accessible to all who were willing to put forth the effort were challenged by several personal experiences. After a decade and a half of teaching, I returned to school to earn my Master's degree and later began my doctorate. I continued to teach during graduate school, and courses in my Master's program focused on student-centered learning and inquiry-based instruction. As I began to explore how to implement these pedagogies, I saw students who had previously been rendered silent now more active in my classes. I continued to modify my practice, learning how my students wanted to engage in their own learning. Their renewed involvement in mathematics provided counterstories[7], contesting my previously unchallenged beliefs about mathematics – beliefs that until this point melded nicely with the majoritarian narrative.

Through these events, my beliefs about mathematics and teaching and learning mathematics changed forever. The foundations I had relied on for nearly a decade and a half had been cracked, and over the following years would be razed. I began to question my assumptions about accessibility to mathematical knowledge, with the burgeoning understanding that my belief in the unbiased nature of mathematics was inherently flawed. I came to understand that access to mathematics was impeded by my practice, and that my belief system created unequal opportunities to learn, marginalizing students through deficiting. It was not a matter of effort – it was access!

I experienced cognitive and emotional dissonance as I considered how I had erected barriers that impeded the access of many of my students, and how social structures of race and power allowed me to do so with impunity. Kendall (2006) and Bell (2010) both discuss the methods that dominant groups implement to remain ambivalent to or unaware of their complicity in perpetuating inequities. "We anesthetize ourselves and dissociate, feeling as though we detach from both ourselves and the situation that we find unbearable" (Kendall, 2006, p. 34). Bell (2010) describes this as "the selective editing of reality that allows white people to disengage from the racial advantages we enjoy" (p. 19). Even now, I find it difficult to accept that I had not only perpetuated inequity in my practice, but that I did it through willful disregard. However, the foundation of my belief structure had been so damaged as to prevent its repair. Once I became aware of my complicity, I could "never not see it again" (Kendall, 2006, p. 13). My identity as a race-neutral teacher previously allowed me to perpetuate practices that ensured the continuance of poor performance of my students, supporting the stock stories that I held to tightly. Through this cognitive and emotional dissonance, I had two choices: (1) anesthetize myself and rejoin with my colleagues; or (2) recreate myself as a teacher engaged in anti-oppression and anti-racism within my own classroom.

As I explored the aspects of my teaching that privileged some of my students and disadvantaged others, I constructed a new set of beliefs of mathematics, and teaching and learning mathematics. My development of concern for social justice[8] and equity led to my evolution of beliefs of mathematics education encompassing anti-racist and anti-oppressive pedagogies. Over time, I began to comprehend how gender, race, ethnicity, fluency in English, socioeconomic status, and other factors were essential components in students' (in)accessibility to mathematics in my classroom and in the broader spectrum of public education in the United States. My students' lived experiences (silenced by oppressive practices, and later empowered by student-centered pedagogy) were the impetus of the change of my beliefs about mathematics and teaching and learning mathematics, and thus my practice, too.

Racialized Classrooms in Contrast to Race-neutral Perceptions of Mathematics

As counterstory to the race-neutral characterization of mathematics education that permeates the research base, Martin (2006) asked, "In what ways can mathematics learning, participation, and the struggle for mathematics literacy be conceptualized as *racialized forms of experience* – that is, as experiences where race and the meanings constructed around race become highly salient?" (p. 198). This overt stance is a necessary response to the resistance to change the status quo. The meritocratic view of mathematics education perpetuates a minimalistic conceptualization of race, rooted in deficit theory in which minority students are painted as deficient, often based on non-academic factors such as the ability to sit still during instruction (Berry, 2005). This deficit notion is pervasive, and research has not shied away from asserting a genetic basis (Herrnstein & Murray, 1994). Positioning Black, Latina/o, American Indian, and other minority students as inherently less intelligent and lacking the work ethic to succeed shifts the focus away from the schools and onto the students, their families, and their communities.

Countering this distraction is the focus on racialized classrooms—situating the research into mathematics classrooms in which the structure of the class is informed not only by mathematics content, but also by the social constructions of race:

> While the bulk of research on mathematics teaching and learning views it as largely a culture-neutral enterprise, mathematics classrooms (as students experience them) are not separate from the broader social worlds of schools, communities, and societies. In the United States, as elsewhere in the world, these social worlds and communities, and the social arrangements by which particular students arrive at them, are deeply informed by issues of race (Nasir et al., 2009, p. 231).

Race is a factor of schooling manifested in a number of ways, including teachers' expectations of students. Low expectations of the performance and capabilities of Black students is prevalent in American schools (Landsman, 2004), and is a factor in access to advanced academic classes (Corra, Carter, & Carter, 2011; Thompson & Lewis, 2005). Pizarro found that "teachers use racial profiles to determine who will and will not benefit from opportunities to excel in school" (cited in Nieto & Bode, 2008, p. 80). These findings underscore the racial nature of education, in which access to knowledge is a function of color. Martin (2006) highlights this within mathematics education, noting, "African Americans do not readily identify with mathematics as 'ours'" (p. 213). Mathematics education exists in racialized contexts, with differentiated accessibilities.

The race-neutral perception of mathematics privileges the status quo, and marginalizes not only minority students, but also research into race. As a result of the positioning of mathematics as neutral, race has been excluded from research and from the national conversation on mathematics education (Martin, 2009). Similarly, Gutierrez (2008) notes that concerns about achievement gap research perpetuating deficit notions is largely a matter of faculty of color and those with a vested interest in equity. Berry (2005) summarizes these concepts, stating, "too often race, racism, and social justice are relegated as issues not appropriate for mathematics education, when actually these issues are central to the learning and teaching of mathematics for all students" (p. 18). The academic discourse on race in mathematics education is silenced as a result, extinguishing the potential of considerations of race (and especially the social construction of race) as a factor. The stock story of meritocracy, in which blaming the victim is central, is not only reified, but also amplified.

Notes

1. Determined through Google Scholar.

2. Minorities are defined as groups who have lower social and political status in society. Minority establishment can include, but is not limited to bases of race, class, and native language. Marginalization of minorities occurs not through these differences but through dominant societies' values of these differences (Ogbu cited in Nieto & Bode, 2008).

3. Inequity is framed within the discussion in critical race theory of equality, and problematizes the notion of equal opportunity in lieu of equal results (Delgado & Stefancic, 2001). Equity in education is conceptualized as creating environments in which there is "*the real possibility of an equality of outcomes*" (Nieto & Bode, 2008, p. 11).

4. Deficiting students is the act of explaining failure or lower performance through negative perceptions and stereotypes, often associated with race, class, lack of fluency in English, or culture. Specifically, students are positioned as having internal deficiencies such as "limited intellectual abilities, linguistic shortcomings, lack of motivation to learn and immoral behavior" (Valencia, 1997a, p. 2).

5. Majoritarian narrative is one of many equivalent terms that refer to stories that are told by dominant society – demographically white, male, and middle/upper class – reflecting values and perpetuating privilege for the dominant group. Other terms include monovocals, master narratives, standard stories, and majoritarian stories (Solorzano & Yosso, 2009, p. 135), and stock stories (Bell, 2010).

6. Critical race theory is a theoretical lens, which evolved out of legal theory and has been used in education and other fields to explore race and power within social structures (Delgado & Stefancic, 2001).

7. Counterstories challenge "the validity of accepted premises or myths, especially ones held by the majority" (Delgado & Stefancic, 2001, p. 144).

8. Social justice "means affording each person the real—not simply stated or codified —opportunity to achieve their potential by giving them access to goods, services, and social and cultural capital of a society" (Nieto & Bode, 2008, p. 11).

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Social Class and the Visual in Mathematics

Peter Gates
University of Nottingham

This is a discussion and theoretical paper arguing for a social class analysis of the use of the visual in mathematics teaching. Whilst each topic has a considerable literature, they are rarely linked in the mathematics education literature. However I argue for an inextricable connection between non-textual modes of thinking and communication and the SES gap in mathematical achievement.

Introduction

Each academic year, the prospect of yet again teaching fractions to a class of low achieving challenging adolescents strikes abject frustration in mathematics teachers throughout the world. Yet the reality is many young people fail to understand even basic mathematics. How we get to this position where, after 9-10 years of compulsory schooling, we are still trying to convince some children that 1/4 = 2/8 is nothing short of an international scandal. Worryingly, this is after decades of curriculum reviews, policy changes and millions spent on research. One feature is that achievement is not equitably spread throughout society; children from less affluent homes do disproportionately worse than those bought up in relative affluence. This after all is the *raison d'etre* of MES. Much research has attempted to articulate this relationship, whilst much more has ignored it, through denial, or the misguided belief that by supporting the affluent all will benefit through the *"trickle down"* principle. There is one feature of school mathematics that is perhaps more common than most: a dependence on *language* and textual communication to the near exclusion of other modes of communication—most notably the *visual*. But whilst *"language is a marvelous tool for communication, but it is greatly overrated as a tool for thought"* (Reed, 2010, p. 1).

The importance of how children mentally represent mathematics is widely acknowledged. Mental representations are ways of constructing mental models, and the process of teaching is to support that construction. To argue, think and reason *visually* is something not yet at the forefront of teachers' conceptions of mathematics learning. Hence the use of diagrams and visuals may currently play a very superficial, insignificant or minor role. Some go further suggesting teachers discourage visual and diagrammatic forms as less valid than the symbolic (Morgan, 2004). Others take an alternative stance arguing that visual thinking is epistemologically central to mathematics (Giaquinto, 2007); others offer a neurological justification:

> Visualisation extends working memory by using the massively parallel architecture of the visual system to make an external representation function as an effective part of working memory. (Crapo, Waisel, Wallace, & Willemain, 2000, p. 220)

It is well known that we receive, hold and process information in various ways - for example through an auditory stimulus, or a visual stimulus and that these channels combine in a non-trivial way. However, there still appears to be a lack of attention given to the construction and manipulation of mental models especially those drawing on visual-spatial processes. Dawe puts as an imperative for teachers to *"consciously link visual images, verbal propositions and memories of activities, involving the manipulation of physical objects"* (Dawe, 1993). Whilst visualisation and mental imagery are cognitive processes evident from birth, there would appear to be different levels of individual facility yet little evidence of explicit instruction at school. These skills are open to enhancement through classroom activity—e.g. with evidence of gains in higher education after explicit training in visualisation (Lord, 1985, 1990). The need to be proficient in visualisation is important in many fields such as engineering, medicine, construction—in fact it may be difficult to find a field of employment where it is not. The lack of direct instruction in visual facility in school mathematics is therefore worrying.

Visual skills useful in learning and doing mathematics might include: constructing a mental image or mental model of a mathematical artifact or process, developing and using a mental representation, constructing representative diagrams, describing (representing) images and models, mental rotation. Here there are further claims of gender and class differences in spatial skills (Linn & Peterson, 1985) and age effects (Bishop, 1978). Linn and Peterson argue there is evidence of males using a holistic approach, with females taking an analytical approach. Bishop argues the there is a developmental process moving from topological, through 2d to increased sophistication in 3d. The widespread use of computer gaming by young people may also be enhancing their visual and spatial ability. However as with all developmental processes, environment and social factors play a significant part.

Those pupils who do well at mathematics tend to be able to use a range of strategies that help them develop the capacity to engage at a number of levels within a range of topic areas. Mathematical thinking engages us in a wide range of cognitive and social practices and so defies a clear well bounded definition - one learns to use strategies, to see relationships and structures in a wide variety of forms. Furthermore in participating in these practices, we engage in logico-deductive thinking, use language to express ideas, talk to communicate and convince but in addition we also use a spatial, visual mode of thinking which helps us see, experience and master various mathematical objects and relationships.

The Importance of Economics

However all this ignores the fact that children do not start school —or life—on an equal footing. It is well known in an extensive literature that there is a significant difference in the levels of achievement of children from different social backgrounds (or social classes), and these early differences expand during the course of compulsory schooling (Alexander et al., 1988; Geary, 1994, 2006). For most pupils, visualization is not instinctive but *"one learns to 'see'"* (Whiteley, 2000, p. 4) though we might observe in many mathematics classroom pupils learning to *repeat* or learning to *say*. Children from disadvantaged backgrounds have forms of knowledge that do not allow them to fit so well into the expectations of schools as do those from more affluent or middle-class homes. Whilst this seems to be true generally, there seems to be specific differences in learning of mathematics (Case, Griffin, & Kelly, 1999) where the most significant and consistent predictor of academic achievement in school seems to be the parental income, which has an effect stronger even than parental educational background. Where ethnicity and gender are factors, they are usually confounded with SES (Jordan, Huttenlocher, & Levine, 1992, p. 652); in the first two years of formal education, school makes little difference to this (Stipek & Ryan, 1997, p. 721). Yet, one major impediment to the amelioration of mathematical teaching and learning around the world is that much work in mathematics education is so politically focused as to ignore the social class basis of mathematics learning. Whilst this is lamentable, it is not surprising; indeed it would be surprising if the field of mathematics education were quarantined from the left-right/radical-conservative dispositions that exist everywhere else.

Crucial to understanding the influence of class on learning is specifying the types of mathematical knowledge on which the discrepancy is present (Siegler & Ramani, 2009). On nonverbal numerical tasks, preschoolers' performance does not vary significantly with economic background (Ginsburg & Russell, 1981; Jordan et al., 1992; Jordan, Levine, & Huttenlocher, 1994). However, on tasks with verbally stated or written numerals, the knowledge of preschoolers and kindergartners from low-income families lags far behind that of peers from more affluent families. The differences are seen on a wide range of tasks: recognizing written numerals, reciting the counting string, counting sets of objects, counting up or down from a given number other than 1, adding and subtracting, and comparing numerical magnitudes (See Ginsburg & Russell, 1981; Griffin, Case, & Siegler, 1994; Jordan et al., 1992; Jordan, Kaplan, Olah, & Locuniak, 2006; Siegler & Ramani, 2009; Starkey, Klein, & Wakeley, 2004; Stipek & Ryan, 1997).

Significant is the argument that the problem lies deep within the way in which schools divorce children from the informal intuitive forms of understanding they had experienced before formalized education. Ginsburg and Russell (1981) investigated the associations of social class and race with early mathematical thinking arguing that early mathematical thought develops in a robust fashion regardless of social class and race and that school failure, specifically in mathematics cannot be explained by initial cognitive deficits (p. 56) a finding in conflict with many early years teachers' beliefs. However, Ginsburg and Russell (1981) argue that it was cognitive *competence* not a cognitive *deficiency* that might be in existence. Specifically, low-income children seemed to have a less developed set of what Case and Griffin call "*central conceptual structures*" (1990a) that went on to underpin future cognitive development specifically of mathematical and numerical processes (Griffin et al., 1994, p. 36), that without these detailed structures early on, children would go on to develop a "rote" approach to learning which would limit the scope of their level of achievement (p. 47). However, through a taught "Rightstart" programme focusing on conceptual bridging, multiple representation and affective engagement, Griffin et al. (1994) were able to demonstrate elimination of differences. The importance of looking at the competencies of children very early on is the more significant neurological influence of the developing brain, since

> children's early mathematical capacities show a considerable degree of differentiation by social class during the years when the neurological circuitry on which they depend is showing its most rapid development (Case et al., 1999, p. 148)

Case et al. go on to argue that whilst SES differences are not observable at birth they do begin to appear around 3 years old (Ginsburg & Russell, 1981), but by kindergarten this had become a year and a half difference in capabilities (Case et al., 1999, p. 131). These early differences in mathematical knowledge have lasting effect as preschoolers' performance on tests of mathematics is predictive of mathematical achievement at age 8, 10 and 14 and even in later in upper secondary school (Duncan et al., 2007; Stevenson & Newman, 1986). This stability of individual differences in mathematical knowledge reflects to some extent the usual positive relationship between early and later knowledge, but the stability of individual differences in mathematics is unusually great. This might be because mathematics is something of a secret garden, avoided by low SES parents (Siegler & Ramani, 2009):

> Observations of homes and preschools, as well as the self-reports of teachers and parents, suggest that the home and preschool environments provide children with relatively little experience where their attention is focused on mathematics, far less than literacy-oriented experience (Siegler & Ramani, 2009, p. 558)

However, there is some evidence, that social class effects upon the development of mathematical skills is more marked for verbal than non-verbal forms (Jordan et al., 1992). Children from middle-income families do better when the mode of representation is verbal. Yet where the mode of representation is visual or nonverbal, the social class gap is much reduced possibly because verbal and written forms of communication are less prioritized in working class families. Alternatively for working class families *"knowledge that has been constructed directly from their own actions on objects as well as their observations of the world"* applies equally to development of visual and nonverbal modes (Jordan et al., 1992, p. 651). Siegler and Ramani (2009) take this need for privileging of the non-verbal further but argue that whilst pre-school children from more affluent backgrounds perform better on some numerical tasks than disadvantaged children, this differential performance can be partially alleviated by regular playing of linear board games - consistent with the hypothesis that playing board games contributes to differences in numerical knowledge among children from different backgrounds, children from middle-income families reported playing far more board games (though fewer video games) than their low-income peers, indicating part of the gap between low-income and middle-income children's mathematical knowledge when they enter school is due to differing play experiences (Ramani & Siegler, 2008; Siegler & Ramani, 2009, p. 557). Given that these same disadvantaged children report playing board games at home with much less frequency than the affluent children, Siegler conjectured that this might be partially influential in not providing the cognitive experience that would move them forward (see also Dehaene, 2011):

> …board games provide a physical realization of the mental number line, hypothesized to be the central conceptual structure for understanding numerical operations in general and numerical magnitudes in particular" (Siegler & Ramani, 2009, p. 546)

Allocation of blame to working-class parents is common amongst politicians and some researchers, yet interviews with parents in low-income families indicate that many believe the primary responsibility for teaching mathematics lies with the professionals in schools (Holloway, Rambaud, Fuller, & Eggers-Pkirola, 1995; Tudge & Doucet, 2004) a perhaps not surprising position given the self-importance with which the teaching profession surrounds itself. Indeed Tudge and Doucet (2004) studied children's exposure to explicitly mathematical activities in their own homes, other people's homes, and child care centres, supporting this assertion.

> A majority of children from working class backgrounds were observed engaging in mathematical play or mathematical lessons in 0 of 180 observations. If it is indeed correct that working-class parents look to preschool settings to provide children with mathematics experiences . . . our data suggest that they are mistaken—we found no evidence that children are more likely to be engaged in mathematical activities . . . in formal childcare centers than at home (Tudge & Doucet, 2004, p. 36).

The role of schools and preschools then might usefully be expanded by building stronger and more explicit links between school and home activities that support mathematical thinking and visualisation.

Conclusions

For many, the claim that economically disadvantaged children do less well at school will be hardly controversial, or new. Yet the next stage of that argument often escapes some. This is the *"so what"* question. A damaging stance is to take a deficit perspective, that *"these children"* need remediation, that they miss out of stimulation in the home, that both children and parents *"lack aspiration"*, and even worse, they need a more practical curriculum for a practical future, focusing on *"the basics"* reinforced though repetition. In a study of 262 US preschool children, Stipek and Ryan (1997) argue that economically disadvantaged preschool children very quickly developed a more negative view of their own competencies and negative attitudes to school, both which lead to a decline in motivation leading to potential future depression of achievement (p. 722).

> Disadvantaged children are every bit as eager to learn as their more economically advantaged peers. They do however have much further to go in terms of their intellectual skills and, as schools are presently organized, they do not catch up. (Stipek & Ryan, 1997, p. 722)

In a society—and school system—that extols only the virtues of the rich, famous and successful, this is perhaps quite iniquitous but not surprising. Their suggested alternative is to develop instructional methods that will decrease the gap in cognitive competencies specifically targeting the self-esteem and interest of disadvantaged children. This is not an easy policy to enforce, especially since narrowing the gap acts against the social and economic interests of those who benefit from being at the head of the gap. Many studies have indicated ways in which parents might support children in seeing and thinking more mathematically, yet the practices being advanced might be more readily seen in middle class families: taking advantage of opportunities to practice spatial thinking (Joh, Jaswal, & Keen, 2011; Newcombe, 2010; Pruden, Levine, & Huttenlocher, 2011); playing construction games that challenge children to recreate a design from a sample or design (Ferrara, Golinkoff, Hirsh-Pasek, Lam, & Newcombe, 2011), encouraging children to gesture when they think about spatial problems (Cook & Goldin-Meadow, 2006; Goldin-Meadow, Nusbaum, Kelly, & Wagner, 2001), playing with tangrams and jigsaw puzzles (Levine, Ratliff, Huttenlocher, & Cannon, 2012), creating and explaining maps (Kastens & Liben, 2007), practicing mental rotation skills including through computer games (Terlecki, Newcombe, & Little, 2008; Wright, Thompson, Ganis, Newcombe, & Kosslyn, 2008). Studies of early cognition do suggest potentially useful strategies which might benefit learners of mathematics from more disadvantaged backgrounds. Whilst much work on the links between disadvantage and achievement look to generic social structures, a look toward studies of cognitive development point toward more specific aspects of how that process is operationalized. As a consequence, there is sufficient evidence to consider a greater examination of the mode of communication and representation as playing a significant role.

Diagrams can be important in grasping mathematical concepts and solving mathematical problems but diagrams are only one of the ways we come to “see” things. If learners are going to keep learning new ideas and new structures in mathematics they have to be able to imagine objects that they can’t actually see, but also need to be able to visualize all sorts of processes and objects and interpret visual information. However, interpreting diagrams and helping pupils visualize is rarely explicitly taught in school mathematics lessons and the result is that pupils see and experience school mathematics as written, largely formulaic, with symbols that need to be textually manipulated.

There is another element to this. It is also well known that young people from disadvantaged backgrounds find it harder to succeed at school mathematics than those young people who have experienced relative economic privilege. Schools don't make this any easier for them by placing all such pupils together in the same mathematics groups and restricting their curriculum and linguistic opportunities, but also restricting the development of alternative forms of representation. Research has consistently shown that young people specifically from low SES backgrounds do less well (Noble, Norman, & Farah, 2005), and on spatial tasks the SES difference is confounded with gender (Levine et al., 2005). This may be due to their experiences as young children, the toys they have (or don't have!) the use they make of maps etc. Hence there is a need to explore the use of visualization in teaching and learning mathematics and how teachers and pupils can be supported to develop imagery and mental manipulation as a natural part of mathematics—which after all gets increasingly abstract the further you go.

This forces us to ask, what use is made of visualisation in teaching mathematics and do groups at different levels get the same experience, particularly those in lower-attaining groups populated by pupils with low SES backgrounds? Low teacher expectations can influence the methods that teachers use and can limit the access of these groups to higher order skills. Children from low SES backgrounds may well have an impoverished mathematical experience before school but their progress may be restricted further if teaching methods do not allow them access to the appropriate opportunities for development. If visualisation is potentially beneficial to mathematical development then how and when is this taught in schools, but more importantly, how might it reduce the SES gap in achievement?

The use of visual methods in early education does require some understanding and interpretation of simple visual representations, such as gesture, and these have a social context. In using these representations teachers make assumptions that may put certain social or cultural groups at a disadvantage. Pupils may hear the same words and see the same gestures but the resulting message may not be the same. If maths students from Papua New Guinea failed to interpret certain features of mathematical diagrams in a 'conventional' way because of their cultural experience (Lean & Clements, 1981) then socio-cultural differences may well affect students' in all our schools. Their perceptions of visual representations may well be very different to those expected by their teacher making it difficult for them to then grasp the associated mathematical concepts. The difference between individual mental images and those held by the teacher lies in the variation in interpretation of the shared external representation, which, itself depends on the image generated by memory and their existing underlying conceptual understanding. A visually rich prior experience and a socio-cultural background that embraces the norms of the school culture may be influential in the early development of visualisation for individuals in schools.

In conclusion, there is a need to examine two key issues in mathematics which are too often kept apart by looking at the spatial and visual capabilities of pupils from low SES or less affluent backgrounds. We probably can't do much about improving their social and economic backgrounds; we might however be able to do something about enhancing some of the key skills which they have not previously been required to focus on—visual acuity, visual reasoning, and mental representations.

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Gender, Culture and Ethnomathematics

Thomas E. Gilsdorf
Instituto Tecnológico Autónomo de México (ITAM)

In this paper we contemplate three considerations with regard to gender, culture, and ethnomathematics: The long-term trend in Western culture of excluding women and women's activities from mathematics, an analysis of some activities typically associated with women with the goal of showing that mathematical processes are frequently involved in said activities, and an examination of some specific examples of cultural tendencies in which certain skills of women are highly regarded. The cultures we will consider are from the regions of the Andes of South America, India, and Mesoamerica.

Overview

Let us begin with a quote. "On the one hand you have the typically feminine, gentle and woolly world of needlework and, on the other, the exciting but incredibly unwoolly world of hyperbolic geometry and negative curvature" (Higgins, 2010). This is a quote by Horace Bent, the administrator of the Diagram Prize, about the book *Crocheting Adventures with Hyperbolic Planes* by Daina Taimina (2009), which was 'awarded' the "World's Oddest Book Title" in 2009. This quote reveals something cultural. There is an assumption being made that an activity like crocheting, which is typically associated with women, has no place in mathematics. In this paper we will see that the exclusion of women from mathematics in Western culture has existed for at least several hundred years. Second, we will examine several examples of work associated with women's activities in which there is mathematical thinking involved. The examples we will consider come from traditional weavers in Mesoamerica: P'urhépecha, Otomi, Mazahua, Aztec, an Andean culture: The Inca, and from the Tamil Nadu region of India: Tamil culture. Third, we will see how the skills and work by women, including the fact that it involves mathematics, has been considered important in cultures from these three regions. As a concluding example from the Otomi culture of what is now central Mexico, we will compare how the work and skills of women were treated before the arrival of Europeans- and hence the arrival of Western culture, and how such work and skills were then considered differently afterward.

The Long and Winding Road to Acceptance of Women in Western Mathematics

There are plenty of examples of how women have been formally and informally excluded from Western mathematics, until recent times. In the first place, women were prohibited from enrolling in Western universities for any reason until the early Twentieth Century. Second, there are well-documented cases of women who were specifically excluded from formal mathematical activity based solely on gender. Details of several such cases can be found in Burton (2007). I will briefly describe one here.

We can consider the life of (Marie) Sophie Germain (1776-1831). During her lifetime, Sophie was formally prohibited from attending university classes. She wrote several mathematical papers, but wrote under the pen name Monsieur le Blanc in order to hide her gender. In 1831 the University of Gottingen granted her an honorary degree, however, Germain died of breast cancer before she could receive the diploma.

Returning to our initial example of *Crocheting Adventures with Hyperbolic Planes*, by Daina Taimina, there is a sense of disappointment that, even in current times, there are still people who believe that women and mathematics do not belong together.

Mathematics and Activities Associated with Women

There are activities typically associated with women in which mathematical thinking is involved. I would like to look at traditional art as one example of such activity. By “traditional”, I mean that the artwork is mainly done by hand, and represents skills and practices that have been handed down for generations. The first example is textile art, specifically weaving and embroidery. Of course, creating textiles has been part of cultures in many parts of the world, so several of my comments here are likely valid for many cultures. My focus is on contexts and cultures in which the weaving and embroidery is done mainly by women. If we look closely at the process of traditional textile art, we will notice that there is complicated mental thinking involved, and there is mathematics, too.

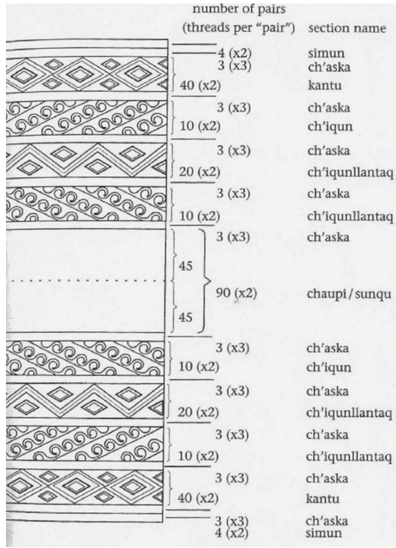
Typically a traditional (textile) artist begins to learn the craft in childhood and continues learning lifelong. Regarding the Mazahua culture of central Mexico, Romaní (2005, p. 60) states that "Desde pequeña la mujer Mazahua assume que bordar es parte de su vida", which translates to: "From the time she is little, a Mazahua woman assumes that embroidery is part of her life." (See also Ascher (1991, Ch. 6) and Gilsdorf (2012, Ch. 5) for further details. In short, becoming a traditional textile artist is not something a person learns in, say, a few days. Careful observation of traditional textile artists making their products reveals that they do not look at diagrams, nor do they use rulers to measure distances while manipulating the threads. The textile project is done entirely from memory. Making textile art in the traditional way takes a long time to learn and requires recalling the details of the entire project by memory.

In fact, we can discover mathematics in the processes. To begin with, let us consider weaving. Many traditional weavers base their projects on extremely large counts of threads. Urton (1997) carefully studied the process of thread counts within the context of women weavers from the Inca culture in Bolivia. Figure 1 shows an example of the thread counts done in the process of weaving in this context.

It is clear from Figure 1 that hundreds of threads are counted as part of the weaving process. Moreover, the patterns of counting are not just raw number counts. Indeed, in order that a stripe of a certain color and width at the top match a corresponding stripe at the bottom, the number of threads must match exactly. More discussion of thread counts can be found in Romaní (2005, p. 62) with regard to the Mazahua culture, and in (1937/1993), with regard to Otomi culture.

Our next view of traditional textile art is embroidery. A main feature of embroidery is that it often includes patterns of symmetry. Thus, the person who embroiders such a pattern has used symmetry,

Figure 1: Example of thread counts of a female Inca weaver from Bolivia, Urton (1997).



but we would like to know if there is a way we could verify that the embroiderer understands the mathematical aspect of symmetry. In several interviews I have done with traditional textile artists from the P'urhépecha culture (Gilsdorf, 2011)), when I ask how the artist created a symmetric pattern, the kind of reply I get is something like "I just made it so it would look right." We can ask: If a person does not describe the mathematical definition of a certain concept, can it be that the person understands that concept? Let us examine how a symmetric pattern is formed. To create vertical symmetry, for instance, one must first create half of the pattern on one side, say, the left side, then create the other half, in the *opposite orientation*, so that it matches the left side. See Figure 2.

Hence, if an artist creates such a pattern in this way, the artist understands what it means to be vertically symmetric, even without a formal mathematical explanation of the definition of vertical symmetry. In Ascher (1991), you can read that symmetry concepts include non-trivial mathematical thinking. Here is a separate piece of evidence of how traditional textile artists understand symmetry concepts as they make textile projects. In his description textile of artists of the Otomí culture, Soustelle (1937/1993) states, "… es necesario … hacerlo en dos etapas, una mitad sobre el borde derecho de la banda, la otra sobre el izquierdo" (p. 94). This translates to, "… it is necessary … to make it in two parts, one half on right part of the band, the other on

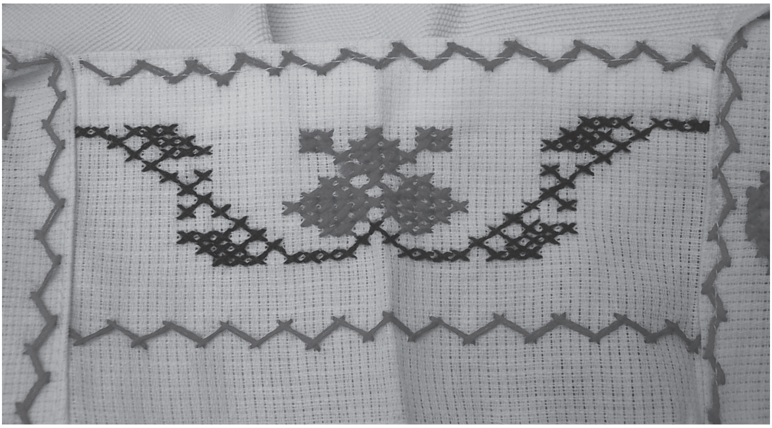


Figure 2: A vertically symmetric embroidery pattern, P'urhépecha culture. Photo ©2011, Thomas E. Gilsdorf

the left." Here we see that the artist creates each half, presumably adjusting the orientation in order to create a symmetric pattern.

The second type of traditional art to consider is *kolam*. Kolam is the art of making designs out of rice powder (or comparable substances), primarily from the Tamil culture of South India and northern Sri Lanka; kolam and similar art forms are done in other parts of Asia (see Ascher, 2002, Ch. 6). The art of kolam is done almost exclusively by women. An example of a kolam design is shown in Figure 3.

As with textile art, learning to make kolam designs begins in childhood. As Ascher (2002) explains, "Girls are taught the designs and techniques by their mothers. It is an important part of girl's training" (p. 163). Kolam figures often include symmetry patterns as you can see. They also frequently include the use of concepts from calculus and computer science. For example, a *continuous curve* in two dimensions is a graph in which there are no holes or breaks and for which the beginning and ending points are the same. Such curves appear in many kolam designs (see Ascher, 2002). Moreover, kolam designs have been studied by computer scientists because of the intricate patterns

Figure 3: A kolam example from outside a house in Tamil Nadu, India. Photo ©2009, Ontology2.

of vertices that can appear in the final product. Finally, kolam patterns have connections to the mathematical concept of fractals – see Ascher (2002, p. 176). An excellent discussion of kolam art, the mathematics involved in it, and its connections to gender roles can be found in Ascher (2002). There is no doubt that the women who create kolam designs have an understanding of nontrivial mathematics concepts involved in the art process, even if they would not express them in textbook mathematical terms.

We can summarize this section by putting the skills of the traditional artists we have considered here in terms of mathematical skills. Let us examine those skills within the framework of "Bishop's Six", that is, six categories of mathematical activity such as, *counting, locating (identifying), measuring, designing, playing (experimenting)*, and *explaining* (Bishop, 1988). It is possible to recognize each of the six categories for the activities done by the artists: In all cases, the artists *measure*, and typically without using any particular measuring tools. Also, the textile artists *count* threads, while the kolam artists keep track of counts of vertices, numbers of curves, and so on. Recall that traditional artists usually make their products from memory. That is, they *identify* the patterns and designs, including geometric and symmetric patterns, and in the case of the kolam artists, patterns that constitute continuous curves and/or vertex and sides patterns that also appear in other areas of mathematics. All artists *design* in the process of creating their products, that is, they determine ahead of time the overall design of the project. The category of *playing* includes the idea of trying different combinations or patterns within the framework of the activity. The general environment of making art as a creative activity indicates that artists often vary the designs and/or the details (e.g., colors) of the designs, hence engage in the activity of play. Finally, traditional artists learn their skill from previous generations and pass those skills to future generations. In other words, the artists participate in *explaining* their craft. In the case of textile artists, for example, a typical situation is that of a group of women who weave or embroider together. With them are daughters, nieces, granddaughters and other girls or young women of the community, learning to be textile artists.

Examples of Cultural Contexts in which Women's Skills are Highly Regarded

As we saw at the beginning of this paper, in Western culture there has been a tradition of excluding the mathematical skills of women. We can look at some examples in which the cultural tendency is to appreciate certain skills of women, hence appreciating the mathematical skills of the women who possess said skills.

The first example is from the Andean culture of the Inca from South America. In traditional Inca culture, a woman who has reached a high level of expertise as a weaver is referred to as a *mama* Urton (1997). This word *mama* is not the Spanish word for *mother*, rather it is a term from Quechua, the language of the Incas that refers to an expert female weaver. In this context, a woman who is an expert at weaving, and as we have seen, uses mathematical skills, is highly regarded in traditional Inca culture. The weaving skills, and indirectly, the mathematical skills, of those women are highly regarded.

Let us now consider the Tamil culture and the art form of kolam. According to Nagarajan (2007, p. 85), regarding the significance of the kolam within the context of female gender roles, "These women's ritual designs also mark time: dawn and dusk; the month of the winter solstice or Markali; and the abundant rice harvest festival called Pongal". Ascher (2002, p. 161) describes the connection between making kolam designs and its social importance as follows: "More than simply a folk art, the kolam tradition is closely tied to, and expressive of, the values, rituals, and philosophy of the people of Tamil Nadu". We can see from these references that making kolam, including its mathematical aspects, is intimately tied to the roles of women in Tamil culture, and the importance of the meanings of kolam, such as marking time and harvests, indicates that this work of women is appreciated and considered important in Tamil society.

Next, we go to Mesoamerica to look at the Aztecs. In Aztec culture the importance of weaving, as a female activity, is also very strong. As Brumfiel (1996, p. 456) explains, "Spinning tools served as symbols of womanhood; they were given to a female infant at her birth and placed with a woman at her death". Moreover, Brumfiel (1991, p. 226) indicates that "Thus, cloth was a primary means of organizing the flow of goods and services that sustained the Aztec state, and cloth production was women's work". Again, we can conclude that in traditional Aztec culture, the skills of women as weavers was regarded with respect and valued highly.

Thus, in each of the cultures we have considered, skills of women are considered to be important cultural knowledge. For further details about connections between gender roles of women and traditional art in the cultures we have looked at a number of resources: Aztec and Maya (textiles): Brumfiel (1991, 1996), Hendon (2006), Joyce (2000), and McCafferty & McCafferty (1991); Otomi and Mazahua (textiles): Gilsdorf (2012), Gilsdorf (2015), and Romaní (2005); Tamil (kolam): Ascher (2002), and Nagarajan (2007); Wari and Inca: Costin (1995, 1998), and Urton (1997).

Before and After: An Example of Change in Perspectives

As one last, but concrete example, let us compare how skills of women had been considered in the Otomi culture before the arrival of Europeans, and how that changed afterward. We can first consider the Aztec document *Codex Mendoza* (Ross, 1978), regarding tributes paid by cultures that were under Aztec control. Figure 4 shows a folio from the codex.

Figure 4: A folio from the *Mendoza Codex*. From Ross, 1978.

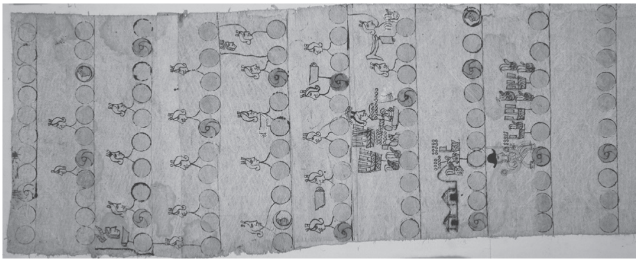


Figure 5: A page from the *Tributos de Mizquiahuala.* From the Biblioteca Nacional de Antropología e Historia, Mexico City.

The icons along the left are symbols of communities that paid tributes to the Aztecs. The icon that is fourth from the bottom is that of the community of Mixquiahuala (which has other spellings, including Mizquiahuala), and the third from the bottom is the symbol for Ixmiquilpan, both locations being part of Otomi territory. Meanwhile, observe the kinds of products that were delivered: At the top are six square shaped icons with what appear to be feathers above them. The icons represent woven cloth products while the feather -like parts on top of those icons represent the number 400. See Gilsdorf (2012, Chapter 9) for more details. The other person- shapes and shield icons represent war suits and war shields, respectively. The flag symbols that are attached to two of the war suits represent the number twenty. There are 400 jars of honey shown in the bottom right corner and two boxes of goods to the left of that. An estimate of the total number of goods that were delivered would be 2400 woven cloths, plus 448 other items; that is, 2848 items. The number of woven cloths represents about 84% of the total. Thus, most of the goods delivered as tributes were textiles and textile related products, and the vast majority of those cloths were made by women. As we saw in the previous section, the Aztecs considered weaving to be an important activity and directly associated it with gender roles of women. We can conclude that there was many cloth products made by Otomi women, those products were delivered to the Aztecs, and the Aztecs considered that work to be valuable.

Now let us jump to after the arrival of the Spanish. Figure 5 shows a fragment of tributes that were delivered to the local Spanish ruler, some time between the 1600s and about 1740. The location is Mixquiahuala, so we can assume that the tributes were made by Otomies.

The figure is sideways, so the fragment is read from left to right. There are several icons of women shown and the one icon of a man represents the local Spaniard. The symbols of women represent household maid and cooking services that were provided. Notice that not one single woven cloth is shown. (There are several other fragments from the collection of tributes, some showing food and other items, but none showing woven cloths (Hermann Lejazaru, 2001)). We can conclude that the role of Otomi women in this location changed dramatically. No longer were their skills at weaving sought, rather, the local Otomi women were valued for their service as housekeepers. This is an example of how the local culture changed.

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Dealing with Diversity in the Mathematics Classroom

Guðný Helga Gunnarsdóttir and Guðbjörg Pálsdóttir
University of Iceland

Mathematics teachers in Iceland are teaching in an inclusive school and in this study the focus is on analysing how they deal with the diversity in their classes. Data from a study on teaching and learning in Icelandic schools, where 51 mathematics lessons were observed, are used. The findings reveal that the teachers are using an individualistic approach. The students are using most of their time working with assigned problems in the teaching materials and little time is used on public interaction. Most common is that the students are working with the same topic. The teacher and an assistant are mostly assisting the students by their desks. The focus is on helping all students even though the assistant is there because of students with diagnosed difficulties.

Introduction

In this paper we report on a study of how teachers deal with the diversity of students in an inclusive school. Data from observations of 51 mathematics lessons in a big study on teaching and learning in Icelandic schools was used to identify how the teaching was organized in order to meet the diverse needs of the students.

In Iceland the development towards an inclusive education started in 1974 with the approval of a law for a nine-year comprehensive school. In the law education for all was mandated regardless of the children's learning abilities and social background. It was a requirement that classes should be heterogeneous. The discourse of this period focused on integrating students with special needs into the mainstream schools. Through the years the discourse and ideas about inclusive education has been changing and it can be seen as a process focusing more and more on the right of every child to get quality education in their neighborhood school. One turning point was the

Salamanca declaration which was confirmed in 1994, translated into Icelandic by the Ministry of Education and sent to all schools in Iceland (Menntamálaráðuneytið, 1995).

Background

In a new Icelandic school legislation for all school levels set in 2008 it is clear that inclusive education is what is aimed for. In the compulsory school act it is stated that all students have a right to go to their neighborhood school regardless of their physical or mental abilities and that their needs should be met in an inclusive school (Lög um grunnskóla nr. 91/2008). In the national curriculum guidelines it is stated clearly that inclusive education is a coherent process with the aim of providing good education for all students. Diversity, different needs, abilities and characteristics of students should be respected and emphasis should be put on eliminating all kinds of differences and segregation in schools. Inclusive education is defined in a regulation about students with special needs in compulsory school. There it says that an inclusive school is a school in the students neighborhood where their learning and social needs are met in a regular school with human values, democracy and social justice as the guiding light.

Even though the aim is clear the practice is sometimes different. In 2013 there were three special schools operating in the country and only 0.37% of students were attending these schools. But 27% of all students in compulsory schools (grades 1-10) were getting some kind of special education or support and 36% of them were getting this support in a special education classroom, 16% in the classroom and approximately 48% both in class and in a special education classroom (Statistics Iceland, 2014). So there is still some segregation within the schools. The biggest threat against the inclusive school comes possibly from within the schools (Sigurðardóttir, Guðjónsdóttir & Karlsdóttir, 2014). Teachers find it difficult to deal with the diversity within the class and a recent study shows that only 49% of teachers found it important that all students attended their neighborhood school and 83% felt they were not prepared to meet students with diverse needs (Björnsdóttir & Jónsdóttir, 2010). According to international comparisons like PISA equality is considered to be on a high level in the Icelandic school system. However in the PISA 2012 results there are for the first time some indications that socioeconomic status could be affecting students' outcomes (Halldórsson, Ólafsson & Björnsson, 2014).

The city of Reykjavík has for the last ten to fifteen years been promoting a policy called individualized and cooperative learning with the aim of better meeting the needs of all students. Their definition of individualized learning is:

> The organisation of learning that builds on the abilities of individuals but not on a group of students or whole classes in a compulsory school. Students are not learning the same thing at the same time but can be working on different content or tasks alone or in groups. Students are responsible for their own learning and their learning is built on an individualized learning plan (Menntasvið Reykjavíkurborgar, 2007).

This definition is much narrower than the definition of inclusive school in law and regulations and it implies that if teachers are to meet the different learning needs of students an individual learning plan has to be made for each student. There are many indications that teachers find it difficult to fulfil this goal (Gunnþórsdóttir, 2010; Gunnþórsdóttir & Jóhannesson, 2013). They feel they have not been prepared properly in their teacher education for dealing with the diversity of students in regular classes and they also feel they need more support within the schools.

In the study *Teaching and Learning in Icelandic Schools* (Óskarsdóttir et al., 2014) one of the main goals was to establish the schools´ progress in the development towards individualized learning. In the study, a measurement tool on individualized and cooperative learning developed by educational practitioners in Reykjavík School District, was used as a framework. The framework consists of six strands; internal structures, learning environment, attitudes towards students learning, teaching strategies and practices, students activities and responsibilities and parental involvement (*Measurement tool on individualized and cooperative learning*, 2005). According to the study only small steps have been taken towards individualized learning, where teaching practices are based on individual learning plans for students, different assignments, learning strategies and assessment. The biggest progress is probably in assessment but in most lessons students in the same class or learning group are working with the same tasks at the same time (Sigurgeirsson, Björnsdóttir, Óskarsdóttir & Jónsdóttir, 2014).

Guðjónsdóttir and Karlsdóttir (2009) studied the implementation of inclusive education in Icelandic schools. In a study in four of the largest school districts they found that three districts had stated a formal school policy on their web site and they all emphasized that all students should have equal access to education but only one used the term *skóli án aðgreiningar* (e. *school without segregation*) which is the Icelandic term used for inclusive schools or education. In this school district the policy is more clearly stated and it is made clear that all children have the right to attend their neighborhood school if the parents wish so. When looking more closely on the websites of 68 individual schools in these school districts only 52 display an inclusive policy, and in stating how to meet students' diverse needs emphasis is put on special education in or out of class, special classes and learning centers for students with difficulties.

Karlsdóttir and Guðjónsdóttir (2010) also studied the implementation of inclusive education in five schools in Iceland. They interviewed teachers, special education teachers and school leaders in all schools and observed teaching in grades three, six and nine, in total nineteen observations. According to their findings more students with special needs getting support within the classroom either by special educators or an unqualified assistant. Those who leave their home classroom for special support have some specific needs that cannot be met within the classroom. Within the class children with special needs most often get individual support from a special education teacher or an assistant. The focus is on students with some kind of difficulties but rarely on the more able students. There are positive attitudes towards inclusion in the schools and close cooperation between teachers teaching the same year group and also between the classroom teachers and the special education teachers. Teachers find it challenging to meet the diversity in the class and they would like to get more support for instance in the form of courses on different teaching approaches and assignments. Teachers make individual learning plans for students with special needs in cooperation with special education teachers and other specialists. There seems to be little room for cooperation between the teacher and teaching assistants even though they are supposed to work under the guidance of the teacher (Karlsdóttir and Guðjónsdóttir, 2010).

In a study conducted in 2012 on mathematics teaching in grades eight through ten in eight schools in Iceland 102 lessons were observed (Þórðardóttir and Hermannsson, 2012). The most common form of support was to get an extra teacher into the class. It was also common to assign an extra teacher to a year group with for instance two classes and then dividing the whole group into three, sometimes according to ability with the weakest group the smallest one. Some of the schools had special learning centers where students would go for support. Here there were usually less than ten students together getting individual support in the center. In the study it was noted that in 56% of the lessons the students were working individually with their textbooks and the teacher(s) were walking between the desks assisting them.

Inclusive education is still in some countries in the world mainly about providing effective education to all children (Ainscow, 2005; Ainscow & Miles, 2008). In others the main issue is how ensure that children with disabilities have their needs met within a general education system. Internationally inclusive education is seen more broadly as a reform that welcomes and supports diversity among learners. The aim is to eliminate social exclusion based on for instance diversity in social class, gender, race, ethnicity, religion or ability. Education is seen as basic human right and a foundation for a more just society (Ainscow & Miles, 2008)

According to Meijer (2010) all European countries have ratified the Salamanca declaration from 1994 and most have signed the UN Convention on Rights of People with disabilities from 2006 where it is stated that all states shall ensure an inclusive education system at all levels. However dealing with difficulties and diversity is, according to Meijer, still one of the biggest problems within European schools. But as he points out the success and failure of inclusive education depends on the practices that teachers in ordinary classrooms use in order to deal with diverse groups of learners. There are various approaches that have proven to be effective among them co-operative teaching and learning, collaborative problem solving, heterogeneous grouping and more general teaching based on high expectations and assessment for learning.

In most European countries individual education programs, focusing on how to adapt a mainstream curriculum for certain categories of students, are used for students with special needs (Meijer, 2010).

According to Ainscow and Miles (2008) this preoccupation with individualized teaching draws the attention away from developing ways of teaching that suits all learners within a class and creating conditions within schools that will encourage such developments. The introduction of teaching assistants, who work alongside teachers, often assisting students having special needs, has created some problems because teachers sometimes feel they can no longer cope if the assistant is not present or available. Ainscow and Miles (2008) say that we need to move away from the emphasis on individualized planning to more personalized and active engagement with the whole class. They also claim that there is no need for separate special needs pedagogy.

Adequate funding and availability of support within mainstream schools is one of the important factors influencing inclusive education and by many considered a barrier (Meijer, 2010). Ainscow (2005) claims that the development of inclusive schooling has to be seen in relation to factors that can help or hinder the progress. According to him the definition of inclusion and the evidence used to measure educational performance are the two most important factors. A debate of important elements like: inclusion being an ongoing process, about identification and removal of barriers, the presence and achievement of all students especially those who may be at risk of marginalization, exclusion or underachievement, is crucial for a wider understanding of the principle of inclusion within a community. When it comes to gathering evidence we need to make sure that we "measure what we value" and that it is about the presence, participation and achievement of all students. We have to create disruptions in order to see and explore new possibilities (Ainscow, 2005). According to Ainscow and Miles (2008) there are many possibilities in increased cooperation within and between schools, all kinds of networking across contexts and collection of relevant data. There is no need to invent new strategies and techniques.

In Iceland the discourse about inclusive education has been influenced by the definition of individualized and cooperative learning put forward by the local authorities in Reykjavík and adopted by many others. Both within the schools and in society there are individuals who are questioning the policy, how it has been interpreted and put into practice, and whether the schools are really able to meet the needs of all students. These voices have grown louder due to less funding to schools as a result of the recent financial crises in Iceland

(Sigurðardóttir, Guðjónsdóttir & Karlsdóttir, 2014). When we got access to observation protocols from 51 mathematics lessons from the study *Teaching and Learning in Icelandic Schools* we therefore found it interesting to look into how Icelandic teachers were dealing with the diversity in mathematics classrooms. We had these questions in mind:

- How is the teaching organized?
    - Who is present in the class and what are their roles?
    - What is the teacher focusing on during public interaction sessions?
    - Are the students working on the same topics or are they working through curriculum materials at their own speed?
- Are there any special arrangements made for students with special needs or more able students?
    - Are there examples of ability grouping?

Data and Data Analysis

In this study we use data from a study on *Teaching and Learning in Icelandic Schools* (age levels 6-15) conducted in 2009–2010 (Óskarsdóttir et al., 2014). The study was done in cooperation with many stakeholders in education. The study focused on many aspects of teaching and learning like the learning environment, student learning, teaching strategies and internal structures. A special focus was put on the development towards individualized and cooperative learning advocated by school authorities both on local and national level according to a framework developed by local school authorities in Reykjavík. Data was gathered by using multiple methods including observations, interviews, focus groups, questionnaires and action research in 20 schools in four municipalities. At this time 175 schools were operating in the country. In total 518 lessons in all school subjects were observed (Óskarsdóttir et al., 2014).

In this study observations from 51 lessons mathematics lessons in grades one through ten were analysed. None of the researchers had specialized knowledge about mathematics teaching and learning. The observers made detailed notes in an observation protocol during the lessons. The focus of the observations was on the progress of the lesson and the activity of those present during the lesson. This has its limitations but nevertheless we feel the observation protocols give us an idea of what is happening in the classroom, how the teaching is organized and how teachers deal with the diversity of the students. We, the authors of this paper, have been actively engaged in teaching mathematics teachers and making mathematics curriculum materials for a long time and are therefore well known to most mathematics teachers in Iceland. We felt that by using this data we could gain some information about mathematics teaching in Iceland without collecting the data ourselves and thereby probably influencing the results.

In our analysis we started by reading carefully all the observation protocols. We then formed some categories on basis of the data and the ideas of categories used by Savola (2010) and Johansson (2006). We have used the same data in another study on instructional practices in mathematics classrooms in Iceland (Gunnarsdóttir & Pálsdóttir, submitted). We made a diagram of each lesson using the categories: (a) non-mathematical work, (b) teacher's public interaction with the whole class including presentation of new material and checking and assignment of homework, (c) individual seat work, (d) assessment, (e) group work, and (f) playing of games. We also described with few words what was happening in each part of the lesson for instance whether the students used textbooks or not. In our analysis we also noted who was present in the classrooms, their roles and interactions with the students. For this study we used the lesson diagrams but we also went back to the observation protocols to get better ideas about how the teachers and other assistants were working with the students trying to meet their different needs. We analysed the data with the research questions in mind.

Findings

There were 19 observations from grades 1–4, 13 from grades 5–7 and 19 from grades 8–10. Two thirds of the lessons were 40 minutes long and the rest 60–80 minutes. In about half of the lessons at all grade levels there was some public interaction between the teacher and the students. There was usually a short presentation in the beginning of the lesson cantered on guiding students through the textbook, explaining or demonstrating procedures or reviewing homework. The public sessions were usually only 5–15 minutes followed by individual work of the students. In the lowest grades there were several examples of the teacher working through the pages with the children by using an overhead. During the students individual work the teacher was circling in the class assisting the children.

In one third of the lessons the students were working individually during the whole lesson. Here they were usually working at their own speed through some textbooks or worksheets. In most cases they were working within the same textbook. In grades 8–10 there were examples of the students working individually according to a plan set for a certain period of time or a chapter and then taking tests when they had finished the module.

In about half of the lessons in the lowest grades (1–4) an unqualified assistant or a social educator was present in the class with the teacher. In most cases they were walking between desks and assisting the children alongside the teacher. In one case the social educator was working with one child with some disabilities and in one case the teacher was working with a small group who needed special support and the assistant was assisting the others. There was only one mention of a special education teacher working in class with the teachers and no example of children being referred to a special teacher not working in class. In the lowest grades there were two examples of ability grouping. In one of the classes the teacher was working with the middle group and the children were working on different multiplication tasks at four work stations and in the other two teachers and one assistant were working with 29 children in two groups.

In the middle grades an assistant was present and assisting in general in two lessons. There were two examples of small groups of four-to-six students leaving the class for special assistance by another teacher and one of a group getting special assistance by a social educator in class. There was one mention of a single child not following the others and working on an individual plan. In two lessons it was noted that students who were quick to finish their tasks in the lesson either got to leave the class to work on a special assignment or got to work with computers. In two classes the teacher was working with children who had been grouped by ability.

At the lower secondary level there were four examples of another teacher coming into class working with a small group or individuals needing special assistance and working with different materials than the rest of the class. There were also examples of more able students working on their own with teaching materials from upper secondary level. In two lessons the teacher had divided the class into two groups where one group was working on their own and the teacher was working publicly with the other group reviewing or introducing new material. In one school two classes were observed where the students were working on collaborative problem solving in mixed age groups from grades 8–10.

It seemed to be most common that in the main textbooks the students were working within the same chapter or even the same pages or problems in class. In the lower grades the students sometimes had extra workbooks were they could work at their own speed when they have finished the problems for the day. But here there were also three examples where the students were working at their own speed through the textbook and some were just on the first pages while others were on page 50 or 80. Both in the middle grades and in grades 8–10 there were examples of the curriculum being divided into modules, monthly plans or plans for each chapter where the students work through the problems listed at their own speed and taking tests at the end of the unit. At lower secondary level there were examples of the entire curriculum being divided into modules the students work though on their own.

Discussion

From the data it seems evident that the teachers put a lot of emphasis on assisting individual students. This is also the main role of the teaching assistants and reflects the belief that you do not have to know much about mathematics and its teaching and learning to be able to assist students in grades 1–7. In grades 8–10 in almost all cases the extra resource is a qualified teacher (presumably with some knowledge of mathematics) and not an assistant or a special education teacher. There it seems to be the belief that some knowledge is required.

The idea about individualized learning seems to indicate that the students have to work on their own through teaching materials and learn by themselves. In the organization of the teaching the curriculum materials play a central role; the teaching relies on them and it is assumed that completing the tasks in the textbooks constitutes the learning.

From the data it cannot be concluded that there is an emphasis on assisting the less able students by a special education teacher in class as stated in the study done by Karlsdóttir and Guðjónsdóttir (2010). The assistance provided in class is rarely focused on the more able students. But the presence of teaching assistants indicates a need for extra resources in the class based on diagnosed problems of some sort.

By focusing on individual instruction teachers can be using their knowledge of students and differentiating their teaching and meeting their diverse needs. But this cannot be concluded based on this data. There are only a few examples of ability grouping and it has to be noted that ability groupings in Iceland are usually flexible and are often only used in subjects like mathematics and Icelandic. It is also not evident whether the students are all working towards the same goals or not. In most classes they seem to be working on the same topic at the same time and the main difference between individuals seem to be that you can work faster or solve more tasks.

Instructional approaches are not very varied and the teachers mainly try to make some variations by organizing work stations where students use hands on materials, play games or solve problems but often these activities are only drill and practice put in another format. There seems to be an emphasis on creating a warm and positive atmosphere where all students have their space and are a part of a community or as Ainscow (2005) puts it, can be included in a community. The teaching and learning is individual for each student and it is presumably based on the belief that everyone is special. In most of the classrooms the students´ work is organized by the teacher in way that all are working on the same topic. This gives an opportunity for opening the social community and make it into a learning community. It is also likely that in many classrooms small learning communities will be created. This situation gives opportunities shift emphasis from individualized planning to more personalized and active engagement with the whole class.

From the data we can conclude that there are often two persons assisting the students in class, either a teaching assistant or another teacher. The teacher focuses on assisting the students in completing the tasks in the textbook. In most cases the students are working on the same topic. Students with some difficulties seem to be getting support mostly within the class. From the data it is difficult to say if this also applies for more able students or students with other needs, but the main focus is on assisting individuals. This strong focus on individualized learning raises some questions for us, the authors of this paper. We are authors of almost all curriculum materials used in Icelandic schools at the time of data gathering for this study. The materials are written with more collective an inquiry based teaching approaches in mind where discussions between students and teachers play a central role. There is tension between how the materials are used in practice and the ideas about mathematics teaching and learning they are based on. We doubt that our curriculum materials (if any) are suited for this kind of use in classrooms.

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The Mathematical Formatting of Obesity in Public Health Discourse

Jennifer Hall, Richard Barwell
University of Calgary, University of Ottawa

Rising rates of obesity are of widespread public concern and are targeted by public health policy around the world. In this paper, we examine the origins of the most common definition of obesity, known as the Body Mass Index (BMI). We draw on Skovsmose's concept of formatting, combined with a historical examination of the origins of the BMI, to show how obesity is a form of realized abstraction. We discuss how mathematics therefore formats obesity, indicate some of the consequences of the particular way this occurs through the BMI, and suggest some possibilities for mathematics teaching arising from this work.

Introduction

Obesity rates have been widely reported to be increasing in both children and adults, leading to higher rates of mortality and diseases like heart disease and diabetes. Societal effects include a less healthy population and increased pressure on health care systems. In response, governments around the world have developed public health policies, including school-based and public education programs[1]. Public and educational discourses tend to focus on the negative effects of obesity, contrasted with the positive effects of exercise and a healthy diet. In schooling, obesity, diet, and fitness are typically addressed in health and physical education curricula. Obesity discourse is, however, partially based on mathematical constructs. In this paper, we focus on a standard measure of obesity: the Body Mass Index (BMI). We examine the history and recent use of this measure as an example of the mathematical formatting of society (Skovsmose, 1994). We conclude with reflections on the role of mathematics education in empowering students and citizens to participate critically in the production and consumption of public health information.

The Obesity Epidemic

The discourse of an "obesity epidemic" has infiltrated the collective cultural consciousness. For example, a recent U.S. poll found that childhood obesity was the most common childhood health concern reported by the adult participants, with 55% of participants selecting this option, compared to 52% for bullying (which, arguably, may be weight-related in many cases) and 49% for drug abuse (Allen, 2014). In Canada, the overall prevalence of obesity has increased from 9% to 21% over the past 25 years (Elgar & Stewart, 2008). More specifically, the 2013 edition of the Canadian Community Health Survey (CCHS), which uses self-reported height and weight data, found that 18.8% of adults were obese (Statistics Canada, 2014). When considering men and women separately, marked gender differences were seen: 62% of men and 45% of women were considered to be either overweight or obese. Although the proportion of overweight or obese women has remained stable since the 2010 edition of the CCHS, the combined rate of overweight/obesity in men had a statistically significant increase from the 2012 edition of the CCHS (Statistics Canada 2014). While increases in the percentage of the U.S. population in the "obese" category, as measured by the BMI, were seen from 1976 to 2000, this percentage has stabilized in recent years: Flegal et al. (2012), using nationally-representative data from over 5,000 U.S. adults, found that there was no statistically significant difference in the proportion of men or women who were "obese" between 2003 and 2008.

Similar trends have been seen with regard to obesity in children and adolescents: substantial increases in the proportion of obese young people in the 1980s and 1990s, followed by a plateau in the 2000s (Ogden, Carroll, Kit, & Flegal, 2012). Using a nationally representative sample of over 4,000 U.S. children and adolescents, Ogden and colleagues found no statistically significant difference in the proportion of obese young people between 2007-2008 and 2009-2010. However, when considering this dataset from 1999-2000 to 2009-2010, statistically significant increases in the proportion of obese children and adolescents (ages 2 to 19) were seen for boys, but not for girls. In contrast, research involving adolescents in Ontario, Canada (McCrindle et al., 2010) found a statistically significant increase each year from 2002 to 2008 in the proportion of adolescents, both girls and boys, who were obese. The most recent edition of the CCHS found that levels of overweight/obesity in Canadian youths aged 12 to 17 are stabilizing: 20.7% of participants were found to be overweight or obese in the 2013 survey, which is similar to the percentage reported in the 2008 survey, although an increase over the percentage (18.7%) reported in the 2007 survey (Statistics Canada, 2014). The self-reported levels of physical activity in the 2013 survey were not statistically significantly different between the "normal" weight participants and the overweight/obese participants.

A Critical Mathematics Education Perspective

Understandings of obesity are heavily influenced by medical and epidemiological research, fields that are both based, in part, on mathematical foundations. Mathematics is not simply a neutral tool in these endeavours. For Skovsmose (1994), mathematics can be understood as formatting society—in this case, public health discourse. This idea does not simply mean that mathematics is used as a tool to understand obesity. Rather, it means that mathematics contributes to the creation of obesity as a concept and as a problem. Skovsmose uses the term "realised abstraction" to describe the way in which "thinking models" become in some sense "real". This idea is similar to discourse theories that emphasize how language is not simply a means of describing the world or of transmitting thoughts between people; language influences what it is possible to say and to see, providing categories, structures, and ways of organizing experience. Moreover, both discourse in general and mathematics in particular are not simply tools; their use reflects the interests of those who use them (e.g., Edwards & Potter, 1992). They contribute to and reflect prevailing ways of understanding the world to the extent that they become largely invisible—reflecting what Foucault (1979/2008) calls regimes of truth.

Realised abstractions arise, of course, through historical processes. As Skovsmose (1994) suggests:

> Every society and every culture has developed a realm of realised abstractions. But from what sources? They must be brought into existence by some creative act. We may, for instance, be able to

> trace some realised abstracts back to ideological structures or to metaphysical systems. However, as realised abstractions they have obtained the status of laws and principles for the formation of certain social entities. They have to be taken into consideration as part of reality. They are not any longer just models for our thinking. [...] the concepts come to life (p. 52).

This observation suggests a method with which to examine the role of mathematics in the production of realized abstractions in society (not dissimilar to Foucault's method of genealogy, although in our case not nearly so extensive in its execution). In the case of obesity, a concept that is now widely used to organize public, education, and health policies, we can trace the origins of its definition, paying attention to the way mathematics has shaped its development and implantation in public discourse. We can also, through this process, uncover the interests at stake and the social effects of the particular way the concept of obesity has been realized.

What is BMI?

BMI is calculated using the formula BMI = weight/(height)^2.[2] According to the Centres for Disease Control and Prevention (2014a), BMI values fall into four categories: Underweight (BMI < 18.5), Normal weight (18.5 ≤ BMI ≤ 24.9), Overweight (25.0 ≤ BMI ≤ 29.9), and Obese (BMI > 30.0). For example, an individual who is 1.63 m tall would need to weigh less than 49 kg to be considered underweight, 49 to 66 kg to be considered normal weight, 66 to 79 kg to be considered overweight, and more than 79 kg to be considered obese. For children (ages 2 to 19), the BMI formula is used in a different way than it is used for adults, due to children's rapidly changing growth patterns, and gendered differences in body fat percentage (which, oddly, the BMI does not consider for adults). A child's BMI is calculated using the standard formula, and then the BMI and age are plotted on a graph (one for boys and one for girls) that shows percentiles (Centres for Disease Control and Prevention, 2014b; Dieticians of Canada, 2013). Children are considered underweight if their BMI/age data are less than the 5th percentile, normal weight if their BMI/age data range from the 5th to less than the 85th percentile, overweight if their

BMI/age data range from the 85th to less than the 95th percentile, and obese if their BMI/age data are greater than or equal to the 95th percentile (Centres for Disease Control and Prevention, 2014b).

It is apparent that the definition of obesity, based on the BMI, involves a number of assumptions, including the selection of weight and height as variables, the ratio weight:height^2, and the cut off points for the different weight categories. On what, then, are these assumptions based?

History of the BMI Formula

The Body Mass Index (BMI) formula was formerly known as the Quetelet Index, in deference to Adolphe Quetelet (1796-1874), the Belgian statistician who derived the formula. Quetelet was a polymath, with a strong interest and talent in both the arts and sciences, and he made significant contributions to a variety of fields, including statistics, meteorology, visual arts, mathematics, and astrology (Eknoyan, 2008). In 1835, he published *Physique Sociale, ou Essai sur le Development des Facultés de L'Homme* (published in 1842 in English as *A Treatise on Man and the Development of his Faculties*), in which he sought to describe l'homme moyen (the average man), in terms of a variety of social and physical characteristics (Eknoyan, 2008; Faerstein & Winkelstein, 2012). Quetelet used population census data of men from the Netherlands as the basis for his statistical work, creating formulas to fit the existing data (Brody, 2014). He struggled to fit the relationship between weight and height data using a Gaussian (bell) curve, a pattern followed by many of the other variables he examined. Quetelet (1842) noted the following:

> If man increased equally in all his dimensions, his weight at different ages would be as the cube of his height. Now, this is not what we really observe. The increase of weight is slower, except during the first year after birth; then the proportion which we have just pointed out is pretty regularly observed. But after this period, and until near the age of puberty, weight increases nearly as the square of the height. The development of the weight again becomes very rapid at the time of puberty, and almost stops after the twenty-fifth year. In general, we do not err much when we

> assume that, during development, the squares of the weight at different ages are as the fifth powers of the height; which naturally leads to this conclusion, in supporting the specific gravity constant, that the transverse growth of man is less than the vertical (p. 66, emphasis in original).

Due to his recognition of this relationship, Quetelet's Index was derived as the formula now known as BMI. Despite the index's recent, widespread use as an indicator of obesity, "In developing his index, Quetelet had no interest in obesity. His concern was defining the characteristics of 'normal man' and fitting the distribution around the norm." (Eknoyan, 2008, p. 49). Notably, Quetelet's "normal man" was based on data from men in the Netherlands in the 1800s, a rather homogeneous Anglo-Saxon population. As a result, the widespread use of this index to describe obesity in both men and women in a worldwide population is problematic.

Quetelet's Index was not well-known by the general public for more than a century after its publication. Indeed, obesity was not a major societal concern until the 20th century, when the relationship between obesity and ill health became a focus, particularly by the insurance industry (Eknoyan, 2008). In 1943, Louis Dublin, a statistician and vice-president of the Metropolitan Life Insurance Company, developed tables of "ideal weights" for men and women based on height and weight data from insurance clients (Jarrett, 1986). These tables used data from individuals aged 25 to 29, as they tended to have the lowest mortality rates (Wildman & Medeiros, 2000). Presuming that this low mortality rate was linked to body weight is flawed statistical reasoning. Arguably, lower rates of disease and age-related ill health (most of which are unrelated to being overweight or obese) are a more likely explanation for young adults' low mortality rates. As with the dataset upon which Quetelet's Index was based, the Metropolitan Life tables were based on a rather homogeneous dataset comprised mostly of individuals of Caucasian descent (Pekar, 2011). Data were included from people who were wearing clothing and shoes during the measurements, and 20% of the data were self-reported (Jarrett, 1986); both of these factors decrease confidence in any claims derived from the dataset.

The Metropolitan Life tables were separated by frame size, with the acceptable weight for each height being divided into thirds, relative to small, medium, and large frame sizes. Problematically, these divisions were not based on any measurements of bone structure; rather, Dublin noticed that "healthy" weights (i.e., those associated with low mortality) encompassed a range of up to forty pounds, and then, in order to account for this range, he divided it arbitrarily into thirds, based on his explanatory notion of bone structure (Gaesser, 2002; Pekar, 2011). Weights falling into the lowest 20-25% and highest 30% for a height were considered undesirable with regard to insurance purposes (Eknoyan, 2008). Similar to the use of the BMI as a measurement of obesity, the Metropolitan Life tables were used out of context: These tables were initially intended for use as an actuarial tool. However, Dublin did promote the link between excess weight and early mortality (Oliver, 2006). The widespread adoption of the Metropolitan Life tables has been cited as the impetus for a dieting frenzy that began in the 1940s and continues to this day, as people—mostly women—sought to match the "ideal" weights promoted by the tables (Crossen, 2003). The tables were revised in subsequent decades using more recent actuarial data from Metropolitan Life clients, and actual measurements of body frame (elbow breadth) were incorporated, but many of the aforementioned problems remained, including the lack of consideration of age or ethnic background (Crossen, 2003; Gaesser, 2002; Himes & Bouchard, 1985).

Dublin's tables remained in common use well into the 1970s, when a study by Keys, Fidanza, Karvonen, Kimura, and Taylor (1972) shifted the focus of the obesity measurement conversation, by confirming the validity of Quetelet's Index. In so doing, these researchers challenged many of the assumptions underpinning the Metropolitan Life tables. For example, they argued that the sample used as the basis of the tables was not random and thus should not have been generalized, since "certainly persons examined in connection with application for life insurance are far from being a random sample of the population" (p. 330). Other ratios, such as weight/height, $(\text{weight})^{1/3}$/height (ponderal index), and weight/height^3 (Rohrer index), were considered in this study, but Quetelet's Index best fit existing weight and height data. Keys and colleagues suggested referring to this index as the body mass index, the first known use of this term. To examine this index, these researchers used data from nearly 7,500 men in Japan, South Africa, the United States, and several European countries. As with Quetelet's original dataset on which his index was based, Keys and colleagues' dataset also only included men, a major oversight since the BMI is applied to both women and men. The men in this study were subjected to skinfold measurements (to show subcutaneous fat) and density measurements (to show body composition). When comparing these precise measurements to the aforementioned ratios, Keys and colleagues found that "the body mass index seems preferable over other indices of relative weight" (p. 341), and noted that it was simplistic in its application.

This simplicity in application arguably underpinned the promotion of the BMI as a tool for measuring (or at least estimating) a person's "fatness". The World Health Organization has been using the BMI to calculate worldwide obesity statistics since the early 1980s (Marchand, 2010), while the U.S. National Institutes of Health (NIH) began using the BMI in 1985 (Singer, 2009). At this point, the NIH defined overweight individuals as those who were in the 85th percentile of BMI by gender: 27.8 for men and 27.3 for women (Singer, 2009). However, in 1998, the NIH changed their BMI "cut-off" points to 25 for overweight and 30 for obese individuals, with men and women now grouped together despite their differences in body fat (Singer, 2009); as Cohen and McDermott (1998) reported, 25 million Americans who were not considered overweight previously were suddenly placed in that category, simply due to the NIH's new "cut-off" points for the BMI measurement. Cohen and McDermott pointed out a few of the key problems with the BMI: lack of consideration of body composition, gender, and frame size. These issues, and others, will be considered in the next section.

Critically Interrogating Obesity and the BMI

It is apparent from our brief historical account of the BMI that a number of assumptions, generalizations, and simplifications were included in the development of the formula and its application. We summarize the most significant issues and highlight some of their implications.

1. The BMI does not take into account body composition (Cohen & McDermott, 1998). Fat, bone, and muscle all have different densities (fat = 0.9 gm/mL, muscle = 1.06 gm/mL,

and bone = 1.85 gm/mL), but these differences are not measured by the BMI (Devlin, 2009). Consequently, individuals may have the same BMI, but very different body compositions and thus, different health risks. A bodybuilder, athlete, or other highly muscular person (who has a low percentage of body fat) would have the same BMI as a same-height individual with a far higher percentage of body fat, as long as they weigh the same amount. Arguably, the former person is the less "overweight" with regard to obesity-related health concerns, but both individuals would have the same BMI and thus would be considered equally "overweight".

2. Quetelet's formulation of the BMI, as well as subsequent verification (Keys et al., 1972), was based on measurements of White European men. As such, the BMI has been generalized from one category of person to apply to both genders and all racial backgrounds. In similar fashion, the Metropolitan Life tables were based on White men and women in their twenties (who applied for life insurance), but were applied to the population as a whole. Again, these data were used to make generalizations about a more diverse population, including people of all ages.
3. The formula for BMI (weight/height^2) overestimates "fatness" in tall people and underestimates "fatness" in short people (including children) (MacKay, 2010; Marchand, 2010). That is, for a given weight, BMI is inversely proportional to height^2. Since the BMI formula is used for people of all heights, these issues with scaling imply that for a given body shape, a taller person is less healthy (i.e., more obese) than a shorter person (MacKay, 2010).

Quetelet's goal was not to define obesity, but to describe a population. His formula, now known as the BMI, was descriptive—and descriptive of a very specific population. Mathematics, however, derives its power in part from generalization. In the case of what is considered "normal" weight, these generalizations take particular kinds of people as "normal". When indices like the BMI or the Metropolitan Life tables become widely used (facilitated by information technology, mass communication, and mass health care systems), these generalizations become realized abstractions. That is, what was originally developed as a description has shifted to become *prescriptive*. These norms are now used to define what people *should* weigh and, indirectly, what they should look like.

Our summary of the problems with defining obesity comes from the health literature, so it is clear that health researchers are well aware of the limitations of the BMI. The literature shows that other indices have also been considered, so there is no sense that BMI is *the* definitive measure. Nevertheless, when the mathematical force of the BMI as the "preferred" index, partly due to its simplicity, is implanted into health care systems and public health and education policy, these kinds of subtleties tend to be smoothed off, leaving only a definitive (i.e., It defines people) formula, with various cut-offs based on statistical analyses for what constitutes obese, overweight, normal, or underweight. The BMI, moreover, is not simply a technical tool; it has real consequences for real people. For example, the implementation of "BMI report cards" for students in elementary and secondary schools has been reported in the U.S., the U.K., and Malaysia (Flaherty, 2013).

Our point, then, is not that obesity does not exist or that obesity is not associated with health risks. Our point is that the certainty of science, through the use of mathematics, turns a fuzzy and complex phenomenon into a normative, prescriptive abstraction, which in turn leads to concrete interventions, in the form of advice, medication, and penalties. This normativity, in turn, is likely to feed into wider discourses relating to such topics as body image, femininity, masculinity, and identity.

Implications for Mathematics Education

The learning and teaching of mathematics in school can address the role of mathematics in society (Skovsmose, 1994). In the case of obesity, for example, students could study the history we have recounted in this paper, collect data of their own, test different indices, and discuss the validity of their findings. This kind of work relates to curriculum goals for statistics, algebra, and other mathematical topics. Such explorations could also be developed as cross-curricular topics, so that, for example, health education and mathematics education are combined. In this way, the role of mathematics in defining health can be discussed.

We do not see mathematics education, however, as something confined to schools and curricula. In an increasingly information-rich society, there is scope for a stronger form of public mathematics education. Citizens must engage with information about health in general and about themselves in particular, in order to make decisions about their lives, their families, and their communities. Public mathematics education would not presume that citizens are incapable of using information effectively; rather, it would prompt them to look "beyond the data" rather than accepting the prescribed role of passive consumers of health information.

Notes

1. For example, in the U.K., Public Health England emphasizes the negative effects of obesity on children and promotes various policies including school health plans, healthy diets, and physical activity for children (See www.noo.org.uk/LA/tackling/education). In Ontario, Canada, the government has set a goal of reducing childhood obesity by 20% over five years (See www.health.gov.on.ca/en/public/programs/obesity/.)

2. An alternate formula, using the Imperial measurement system (pounds and inches), is: BMI = weight/(height)^2*703, with 703 being a conversion factor. Consequently, the units for the BMI are either kg/m^2 or lb/in^2, although these units are rarely cited with a BMI value.

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Frame Analysis in Mathematics Education

Victoria Hand
University of Colorado Boulder

As the field of mathematics education attempts to marry interpersonal and informational aspects of mathematics teaching and learning, conceptual tools from social disciplines (e.g., sociology and sociolinguistics) are being (re)visited. This paper offers Goffman's theory of framing as a source of analytic tools for studying prompts for and orchestration of social arrangements in classroom mathematical activity. As interactional roadmaps, frames guide the construction of local social interaction and positions in relation to broader activity. Here, frames are considered in relation to storylines and figured worlds, and the benefits and limitations of frame analysis are explored.

Introduction

A teacher asks for a volunteer to lead the class through the next homework problem, which involves finding the area of an oblique triangle drawn on a Cartesian graph. A student volunteers and walks the teacher through a solution strategy, which the teacher writes on the overhead. The solution involves creating a rectangle around the triangle, and sectioning the rectangle into pieces to find the area of the triangle inscribed in it. The following interaction ensues:

ESME: Am I doing this wrong?
TEACHER: I don't know. We are gonna talk about it in a second. So this is what we have. Is that an okay representation? And Esme is saying that what I drew… it's going to give us draws a triangle. If I am right, Esme, it's gonna give me just this right here. (Teacher colors part of the triangle red.)
ESME: Yeah. Well no. I'm gonna get the area right there. (points to a different part of the rectangle)
TEACHER: Okay
ESME: And then I'm gonna divide it in two.

TEACHER: Okay. And that's to get this this red part here? (Esme-*mmm hmm*). Okay, so what do we think, folks? Is that right? (students shouting things) So she takes the area of this whole triangle? Let's do it! So what is the area of the rectangle Esme, not the whole rectangle, but the blue rectangle (writes A_{Δ} = on the overhead).

ESME: 2 times 5, 10

TEACHER: 2 times 5. Do we all agree with that part? (a student says loudly "*Yes*") 2 times 5. And then she says she's going to divide it in half. She says which gives us 5, right? Is 5 that red part? Is that what you think is 5?

ESME: Yes

TEACHER: Does everyone agree with that? (Esme looks to classmates)

STUDENT: (loudly) That's how you do it?

TEACHER: She says that 5 is the red part. Remember this is the triangle I screwed up when I showed you the first time, right?

STUDENT: I got 4 ½.

TEACHER: You got 4 ½ for that red part? You can go ahead and sit down Esme. We will discuss it.

One way to interpret this exchange is as a case of a student presenting a solution to a mathematics problem. A closer analysis reveals that the student, Esme, is exercising agency over mathematical concepts, and that the teacher is positioning her with the authority to do so (Gresalfi, Martin, Hand, & Greeno, 2008; Pickering, 1995). Yet, there are other aspects of this brief exchange that supported students like Esme, a Latina who began the class believing that she was poor at mathematics, to view themselves as mathematically capable (Hand, 2003, 2012).

The teacher's talk in this episode also seems to have a coaching feel to it. How do aspects of these discourse practices relate to the coaching feel? What is the relation between the "feel" of a mathematics classroom and mathematics learning? The aim of this paper is to illustrate how aspects of social interaction can cue the organization of particular "frames" (Goffman, 1974) or "scenes" (Burke, 1941), that have a recognizable feel to them. These frames, in turn, invite social arrangements, which over time influence the ways that individuals view themselves, others, and their joint activities.

Cuing Social Arrangements

In the last twenty years, the field of mathematics education has turned to social theories of learning and development to explicate the how pedagogic practices and systems influence and are influenced by social processes and structures (Lerman, 2000; Morgan, 2014; Valero & Zevenbergen, 2004). In particular, researchers have been concerned with classroom discourse practices and texts that are characteristic of particular kinds of communities and member identities (Gee, 2000; Wenger, 1998). Not satisfied with accounts that treat local classroom communities as isolated units, researchers concerned with issues of equity and power are also attempting to bring socio-political structures and technologies that function broader levels of human activity into view (Gutiérrez, 2007; Skovsmose, 1994). According to Morgan (2012),

> It is necessary to take such [higher-level social] structures into account in order to be aware of the ways in which phenomena apparent within a particular social practice may arise from or have impact upon the lives of the participants beyond that practice and to be able to consider the possibilities of more equitable practices. (p. 182)

Morgan echoes what a growing number of researchers in mathematics education internationally regard as a crucial direction for research in mathematics education.

To date, researchers have drawn on a number of theorists and philosophers of social theory to prod the field in this direction. As described in a recent special issue of *Educational Studies in Mathematics* (2014), prominent among the social theorists informing critical studies of mathematics education are Basil Bernstein, and French philosophers Michael Foucault and Pierre Bourdieu (Morgan, 2014). Loosely categorized as critical theories provide important perspectives on the reproduction of dominant power structures through pedagogic practices and systems. Methods used to analyse these relations, however, have been less readily available for researchers focused on classroom mathematical activity. As a result, analytic techniques such as discourse analysis, critical discourse analysis (CDA) and structural functional linguistics (SFL) have been employed to operationalize critical theories with respect to classroom mathematical activity

(Herbel-Eisenmann, Choppin, Wagner, & Pimm, 2012). These methodologies enable researchers to study the construction of classroom structures and member positioning in moments of classroom discourse. I offer yet another methodological approached based on the theories of framing by Erving Goffman (Goffman, 1967, 1974, 1981).

Frame analysis has been employed by sociologists (Benford & Snow, 2000), cognitive psychologists (Bateson, 1972; Tversky & Kahneman, 1981), sociolinguists (Tannen, 1993), political scientists (Coburn, 2006; Oakes & Rogers, 2006), and learning scientists (Engle, 2006; Hammer, Elby, Scherr, & Redish, 2005) to understand how individuals come to orchestrate activity together. Frames signal to participants in the activity (often tacitly) what it is that they take themselves to be doing (Goffman, 1974). They position certain actions and responses as reasonable, while rendering others nonsensical or even detrimental to desired outcomes (Tversky & Kahneman, 1981).

In this paper, frames are conceptualized as co-constructed in interaction (Greeno, 1989). They do not proceed social activity, but instead are bid for, ignored, contested or orchestrated in moments of interaction by interlocutors (Tannen, 1993). At the same time, frames point to social practices and processes at a broader levels, which lends familiarity to them among groups of individuals. Cuing a frame, then, also prompts the organization or dismantling of socio-political hierarchies and structures in local social arrangements.

This paper explores the advantages and limitations of employing frame analysis to study the relation between classroom mathematical activity and broader socio-political processes.

Frames as Interactional Roadmaps

Frames are interactional roadmaps in the sense that they cue participants to the goals, routines, and expectations under which they are interacting in a particular context (Bateson, 1972; Goffman, 1974). For Engle (2006),

> …a context has been framed when someone uses meta-communicative signals that help establish what the participants are doing together in it, when and where they are doing it, and how each person is participating in it… (p. 456)

By signalling a particular framing of on-going social interaction, an individual makes a bid for the activity to be interpreted and to proceed in a particular way. This bid may or may not be taken up by other parties in the interaction. Similarly, the construction of the frame does not dictate the interaction that follows within it. Similar to a dramatic "scene" (Burke, 1941), a frame invites participants to embody roles and actions that render the scene recognizable to others, while allowing for flexibility and improvisation.

Frames are recognizable to groups of people in part through their connection to broader social imaginaries (Taylor, 2004). A social imaginary represents the systems of meanings, symbols, and ideals that are implicit, but ubiquitous in the collective life of individuals (Taylor, 2004; Thompson, 1995). In a sense, they are "backdrop" that renders engagment in constellations of social practices in a particular place and time coherent and relevant.

Frames offer connections between broader scales of collective social activity, or *figured worlds*, and the trajectories of participation that particular (groups of) individuals assemble in their local activity with respect to these worlds, or *storylines*.

Frames, Storylines, and Figured Worlds

Holland, Lachiotte, Skinner & Cain's (1998) notion of figured worlds is a popular analytic framework for producing accounts of broad spheres of collective activity that individuals point to and call up in joint social interaction. In part imagined and in part real, figured worlds enable researchers to account for relatively stable and broad scales of social activity, circumscribed by mutual experiences, interests and lifestyles, (re)made in the everyday activities of individuals. Analysing the construction of worlds in moments of social interaction, however, is challenging. Frames offer an analytic "middle ground," whereby researchers can study and theorize particular worlds through the scenes that comprise them.

One way to conceptualize the type of scenes/frames that make sense in a particular figured world is through the concept of storylines (Harre & van Langenhove, 1999). According to Harré & van Langenhove (1999), storylines articulate particular subject-positions for participants that are telling them through their interactions.

Wagner & Herbel-Eisenmann (2009) view multiple storylines as characteristic of figured worlds, as they relate to relationships among individuals participating in and configuring them. Storylines, then, guide the negotiation of different scenes in the world, and the direction these imagined scenes take serve to reshape storylines at various scales of activity.

The opening clip illustrates the proposed relations between these concepts. The mathematics class featured in the episode was the subject of a larger study that examined the nature of opportunities to learn mathematics for US high school students from non-dominant ethnic, racial, and socioeconomic backgrounds (Hand, 2003, 2012). Analysis of transcripts of social interaction captured through video over the course of a school year revealed that features of the teacher's discourse were characteristic of what Hoyle (1993) has describes as a *sports commentary frame*. Drawing on Goffman, Hoyle identified discursive techniques that were constitutive of this frame, which included: a) extensive use of action verbs, b) third-person narrative, c) distinctions between, "you", "us", and "them", d) vivid and detailed descriptions of action, e) if-then projections, and f) physical shifts to direct attention to plays (or in this case, mathematical productions). These techniques can be seen, for example, when the teacher narrates Esme's solution strategy, describes her mathematical actions in detail, and re-voices the justifications Esme produces for them. While the sports commentary frame necessarily points to a figured world of professional sports, it's unclear how this world relates to classroom mathematics learning.

It is helpful to focus on particular storylines in the figured world of professional sports, through which the sports commentary frame may be getting organized. How are the students getting positioned and positioning each other in this episode? What are they entitled and expected to do with respect to mathematical activity? One potential storyline that figures the world of sports is that players are engaged in making strategic decisions about their plays, based upon their read on a situation. This storyline positions players as skilful and rational actors. Highlighting aspects of the players' decisions and the intended and unintended consequences can lead to improved performance on the part of individuals and the team. Within this storyline, players are viewed as members of a team that depend on each other to be successful, and who are entitled to participate in the broader sports community. (Of course, numerous other storylines are perpetuated in the figured world of sports, some of which are less affirming of players.)

How does this storyline function with respect to the teacher's pedagogical goals? One of her explicit goals was to invite students to articulate and evaluate each other's mathematical reasoning as a community. This activity resonates with the storyline above, about students being capable, entitled and obligated to recount the rationale behind their solutions strategies. By cuing the sports commentary frame, then, the teacher was *reinforcing* a less common storyline in the figured worlds of mathematics for groups of students who are typically marginalized, with one from the figured world of professional sports. The sports commentary frame reinforced positionings of students with respect mathematics that were less stable than those typically organized in mathematics classrooms. Put another way, the frame served to disrupt traditional status hierarchies around race, ethnicity, language, and class that are perpetuated in and through mathematics teaching and learning.

The next section explores the advantages and drawbacks of frame analysis for mathematics education research.

Analyzing Framing

One strength of frame analysis is the discernment of subtle meta-discursive signals that trigger interpretations of activity (which may or may not be intended by the interlocutors). For example, frames "at play" in mathematics classrooms may illuminate why participants are cued to move in and out of different social arrangements (Hammer et al., 2005). For example, the teacher may not have intended to organize a coaching frame, but doing so had important implications for students' uptake of opportunities to reason mathematically.

A second strength of frame analysis is the ability to conjoin analyses of "relational" aspects of mathematics classrooms with content learning. To separate discourse within the sport commentary frame from mathematical discourse would be a difficult and unproductive task, since they are constitutive of each other. Frame analysis maintains the glue between social positioning and what individuals come to know and do (Greeno, 2009).

A third strength of frame analysis is observing how frames perpetuate or disrupt broader social hierarchies in local activity. Social arrangements that are organized through a frame often involve storylines for participants based on social categories such as gender, race, class, etc. (Hand, Penuel, & Gutiérrez, 2012). Shifting frames may offer new storylines for individuals who are typically marginalized through the activity.

A significant limitation of frame analysis is identifying when and what is a frame, and who gets to decide. Frames are necessarily partial, cultural objects. Fragments of frames are continually being assembled and disassembled from differing points of views in interaction (Voigt, 1994). They are also connected to particular social and cultural communities, and thus, marked to one group of people and unmarked to others.

A second limitation is that frames will necessarily overlap with and interpenetrate each other. It's complicated to analyse to which frames individuals are responding in a given moment, and to which figured worlds particular frames are pointing. Thus, as the field attempts to make this social turn, the concepts described above will require careful consideration and deliberation. However, seeking answers to these questions may enable researchers to discern features of mathematics classrooms that give them a distinct feel and offer particular possibilities to their learners.

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Uncertainty in Texts about Climate Change: A Critical Mathematics Education Perspective

Kjellrun Hiis Hauge, Richard Barwell
Bergen University College, University of Ottawa

Skovsmose's concept of the formatting power of mathematics refers to the power of mathematics to influence our reality. The way uncertainty is formatted in mathematized problems influences our perception of the limits of such quantifications. In this paper, we apply concepts from post-normal science to analyse how uncertainty is described and constructed in texts on climate change. We have chosen texts that all refer to a figure developed by the Intergovernmental Panel on Climate Change (IPCC) showing projections of temperature changes, including the source document, a newspaper report and a discussion involving master's students. Uncertainty is addressed quite differently in each of these texts. We discuss how insights into such differences can be valuable for developing critical citizenship.

Introduction

Climate change is one of the most urgent global issues of the 21st century. Despite the huge amount of scientific research on climate change, it is full of uncertainty. There is uncertainty in predictions of how climate change will progress, in predictions of the impacts it will have, and in foreseeing the possible effects of any measures we may take to reduce or mitigate the effects. Mathematics is used to describe climate change, to predict climate change and to communicate climate change (Barwell, 2013). Uncertainty has been identified as a significant mathematical problem in climate change research (Nychka, Restrepo & Tybaldi, 2009). Our attention in this paper is on both the prediction and communication of climate change, with a particular focus on the treatment of uncertainty. We examine how a small sample of texts communicates the projections of the future increase in global temperatures produced by the Inter-governmental

Panel on Climate Change (IPCC, 2013a; 2013b). We trace how information presented in one graph, along with the accompanying text, is taken up in a set of related texts. Uncertainty is a key feature in each of the texts, but is treated in subtly different ways.

Deciding how to tackle climate change requires the engagement of all citizens. To participate in the necessary discussion and debate, however, requires some level of understanding of the mathematics of climate change, as well as, crucially, the role of mathematics in shaping our understanding of climate change and in contributing to the causes of climate change. Mathematics education has an important part to play in preparing citizens for this participation. Based on our analysis, we consider how mathematics teaching might address uncertainty in the future.

Literature Review

Uncertainty is related to the mathematical topics of probability, statistics and mathematical and statistical literacy. Research in mathematics education on these topics is extensive and diverse. For this paper, we highlight a few key points. Research on the critical interpretation of graphs has been interpreted as an important aspect of mathematical and statistical literacy, including in relation to understanding publicly available information:

Several researchers emphasise the importance of a critical approach to interpreting data and graphs. In particular, Monteiro and Ainley (2004) proposed the notion of critical sense, to highlight the importance of relating information in a graph to the context in which it is used. Critical sense can be facilitated by students' existing expertise in the subject of the graph (Langrall et al., 2011).

There are few specific mathematics education studies that consider uncertainty. Pratt et al. (2011), however, reported on their development of a software environment in which students could explore and learn about aspects of risk and uncertainty, although their emphasis was more on risk than uncertainty. Nevertheless, they highlight some important points. They refer to research, for example, that shows that lay people tend to over-estimate risk and suggest that this may be because they draw on contextual information that scientific analyses would not consider:

> Estimations of impact are subjective and may be based on sources of information that are unknown to the expert and these, in a rational way, lead to different reasoning about risk. Real problems are extremely complex in their context-dependence, and generally dependent in reflexive ways on the subjective perceptions of different participant groups. It is often not possible to develop comprehensive quantitative analyses. Nevertheless, the scientific/mathematical viewpoint has tended to dominate over the intuitive and informal viewpoint (p. 328).

Pratt et al. found that participants' reasoning about risk in realistic situations with which they had some familiarity not only drew on mathematical inferences (e.g. of probability); they also drew on personal experience of similar situations and affective responses to the situations.

These findings echo research on the public understanding of climate change. Again, there is research that suggests that lay people have a poor understanding of climate change, but this work often examines strictly scientific understanding and can be characterised as deficit-oriented. Bulkeley (2000), however, in a study involving parents and children, found that:

> …public understandings of global environmental risks involve local knowledges, personal values, and scientific information. In these understandings both the social relations surrounding the issue and the physical risks of climate change are important. Despite the scientific uncertainty and claims and counter-claims that have surrounded the issue of climate change, survey respondents and focus group participants continued to place faith in science and education as the most reliable sources of climate change information (p. 329).

This picture is further complicated by the involvement of multiple actors even in the communication of climate change. Weingart et al. (2000) examined scientific, political and media discourses of climate change in Germany, drawing on texts collected from 1975 until 1995. They observed significant differences in these different domains.

In particular, they noted "scientists politicized the issue, politicians reduced the scientific complexities and uncertainties to CO_2 emissions reduction targets, and the media ignored the uncertainties and transformed them into a sequence of events leading to catastrophe and requiring immediate action" (p. 280).

Theoretical Framework

Environmental issues where "facts are uncertain, values in dispute, stakes high and decisions urgent" are referred to as post-normal situations (Funtowicz & Ravetz, 1993, p. 86). Climate change matches this description. Funtowicz and Ravetz (1993) point out that post-normal situations are about socio-political and ethical issues rather than problems which scientists can solve alone (p. 99). Post-normal science includes uncertainty and values as new dimensions of science. They argue that uncertainty and values are interlinked and should be brought to the centre of debates.

In traditional science, uncertainty has been seen as something to control, either by reducing it or by quantifying it through statistical measures. Funtowicz and Ravetz (1993) argue that in a policy situation, the nature of uncertainty is crucial. They divide uncertainty into three forms: inexactness, unreliability and ignorance. Inexactness denotes uncertainty that can be controlled by quantification (e.g. error bars, probabilities). Unreliability refers to uncertainty where interactions of concern are known, but where its quantification is associated with uncertainty. Ignorance is the most severe form of uncertainty, since the unknown cannot be controlled in terms of quantities. These three forms of uncertainty do not denote the 'size' of uncertainty, but rather the degree of the ability to control uncertainty with quantification. In practice, uncertainty can be seen as a combination of these three forms.

Funtowicz and Ravetz also highlight the significance of values in post-normal science. Their significance is also apparent in some of the work reviewed in the previous section, including Bulkeley's (2000) and Weingart et al.'s (2000), which showed that people draw on different forms or sources of information (e.g. personal experience, emotional response, media, etc.). In post-normal situations, then, uncertainty may not be controllable through quantities (e.g. in the case of climate change, there are things that are not known about the climate system). Moreover, the way in which uncertainty is handled may relate to particular value perspectives (e.g. scientists prefer mathematical treatments). Funtowicz and Ravetz, (1993) therefore propose the idea of extended peer review. That is, they argue that the scientific process needs to involve a wider range of participants in order to ensure the inclusion of a range of value systems, sources of knowledge, treatments of uncertainty etc. Extended peer review is a form of quality control performed by non-expert citizens, to review the definition of the problem and the quality of expert advice. This approach presumes that multiple views, interests and knowledge bases are valued. This idea implies a role for education to prepare citizens to participate in this process.

Our thinking about how mathematics education can take up this role is informed by critical mathematics education. Skovsmose (1994) introduced the concept of the formatting power of mathematics, to account for the way mathematics shapes reality. He further argued that the ability to recognize this formatting power and reflect on it is an essential democratic competence in order to balance the experts' influence on politics and society. Barwell (2013) proposed the use of a critical mathematics education perspective to theorise the role of mathematics in conceptualising climate change. In particular, climate change can be understood as what Skovsmose calls a 'realised abstraction': that is, the political response is based on a mathematical version of climate change, developed through the collection and analysis of statistical data, and through the production and use of complex mathematical models. In producing these models, of course, choices are made, based in part on the values of science which privilege some kinds of information (e.g. measurements) over others (human experience). A critical mathematics education involves students engaging with the way mathematics is used to shape our world, as well as engaging with the often hidden human choices that go into this mathematical work. This kind of work prepares them to be critical citizens who can engage in the extended peer community posited by post-normal science.

Research Design

The research we report in this paper was inspired by the study of Weingart et al. (2000) of climate change in different discourse domains. Specifically, we wanted to examine how uncertainty is constructed in a range of discourse domains. For this present study, our research questions were: How is uncertainty in climate change constructed in different discourse domains? What differences are there across different domains?

We began with a particular figure from the recent IPCC (2013a) report, which Hauge had already used in a master's course in mathematics education at Bergen University College (see Figure 1). The figure shows projections for temperatures until 2300 based on four different emissions scenarios (RCPs). We sought texts with explicit reference to this figure. We therefore included: the original presentation of the figure in the IPCC report on the physical basis for climate change, the technical summary of the same report, the IPCC's Summary for Policymakers, a news report that refers to the figure and includes a simplified version of the graph, and a transcript of the master's students' discussion.

In our analysis, we differentiated between explicit uncertainty statements and implicit uncertainty statements. The former include assessed uncertainty, either quantitative or qualitative, and statements where uncertainty is otherwise addressed. Assessments include spread

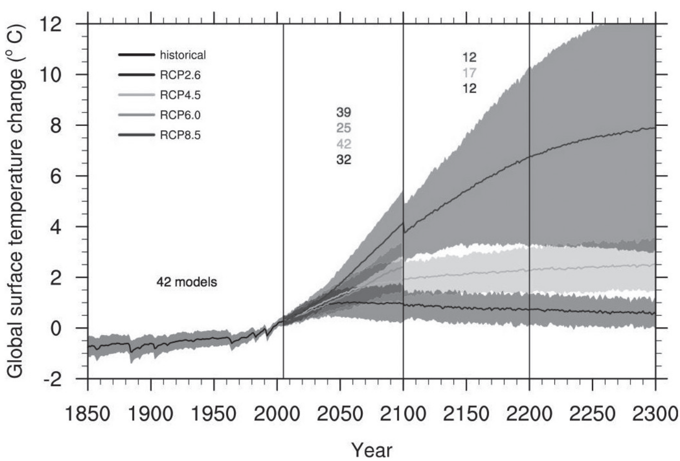

Figure 1: Our selected graph (IPCC, 2013a, p. 1054; original is in colour)

and probabilities, but also qualitative categories for describing uncertainty. Explicit uncertainty statements also include comments on uncertainty or statements on unknown features. Implicit uncertainty statements cover statements where the uncertainty is not explicitly addressed, but the reader can infer that there is associated uncertainty from the context. These uncertainties were then assessed according to the sort of uncertainty the text communicates them to be, based on the forms of uncertainty proposed in post-normal science. Since, however, it can be difficult to distinguish unreliability and ignorance, we merged them into a single form: epistemic uncertainty.

Results

In this section, we present the main patterns observed for each text.

IPCC Working Group Report on the Physical Basis for the Climate Assessment

The IPCC working group explicitly communicates uncertainty in three ways: by quantitative assessments, qualitative assessments and statements about uncertainty. The graph contains quantitatively assessed means and 90% uncertainty ranges related to the predicted scenarios for temperature changes. These intervals are also qualitatively assessed in the text. The following paragraph contains a number of uncertainty statements:

> For these long-term projections, the 5 to 95% ranges of the CMIP5[1] model ensemble are considered the likely range, an assessment based on the fact that the 5 to 95% range of CMIP5 models' TCR[2] coincides with the assessed likely range of the TCR […]. Based on this assessment, global mean temperatures averaged in the period 2081–2100 are projected to likely exceed 1.5°C above 1850-1900 for RCP4.5, RCP6.0 and RCP8.5 (high confidence) (IPCC, 2013a, p. 1055).

The "5 to 95% ranges" refers to quantitative uncertainty measures, while "likely" represents a qualitative uncertainty measure, although it can be linked to quantities. The IPCC applies a likelihood scale for assessing the certainty in results, ranging from "virtually certain", "very likely", down to "exceptionally unlikely" (IPCC, 2013a, p. 36). Each element corresponds to a probability range; for example "likely" represents a probability between 50 and 100%. These probabilities are either based on statistical analysis or expert judgement (p. 36). The 5 to 95% ranges in Figure 1, and described in the cited paragraph, can thus be associated with a probability of 50–100%. This likelihood seems to be based on judgment since it makes little sense to statistically assess the likelihood of an interval. "Confidence" is a qualitative measure of the validity of a finding, based on the degree of evidence combined with the degree of agreement between experts (IPCC, 2013a, p. 36). For example "high confidence" can either refer to high agreement and medium evidence or to medium agreement and robust evidence (p. 36). The two metrics of confidence and likelihood provide a way to communicate that there is uncertainty in the temperature projections, and imply that uncertainty cannot be fully controlled through quantification. Epistemic uncertainty is thereby implicitly communicated, and differentiated through the degrees of confidence and likelihood.

The IPCC offers several explicit statements related to uncertainty in addition to the two certainty metrics. For example: "The likely ranges for 2046–2065 do not take into account the possible influence of factors that lead to near-term (2016–2035) projections of global mean surface temperature […] that are somewhat cooler than the 5 to 95% model ranges […] because the influence of these factors on longer term projections cannot be quantified" (IPCC, 2013a, p. 1057). This statement also expresses the factors as the origin of uncertainty.

Uncertainty is also suggested implicitly. From Figure 1, a reader can see that the mean is constructed from up to 42 models, which means that the models do not necessarily produce the same results or the same uncertainty measures. Indeed, the report states: "some CMIP5 models have a higher sensitivity to GHGs[3] and a larger response to other anthropogenic forcings (dominated by the effects of aerosols) than the real world (medium confidence)" (p. 1055). This suggests that because mathematical models are constructed in different ways (different data, different variables, different computation techniques, different statistical approaches, different drivers), they behave differently and produce different results. Other examples of implicitly addressed uncertainty include statements that justify findings by comparing them to previous studies. This implies that "facts" are constructed and settled when there is consistency in findings across research communities. All the implicit references to uncertainty suggest the presence of epistemic uncertainty and that part of the uncertainty cannot be controlled through quantities.

Technical Summary of the IPCC Report on the Physical Basis

The IPCC report described above has a summary section where Figure 1 is also included and accompanied with a brief text. The technical summary presents the main findings in a similar manner to the first excerpt from the report, presented in the previous subsection. The qualitative descriptors of confidence and likelihood are frequent, but implicit uncertainty statements as exemplified above are not included. Neither are sources of uncertainty an issue in the summary, apart from the reference to the number of models in the figure caption. The figure caption in the summary (which is not the same as the caption in the main report) makes an explicit comment about the graph that is not in the main report:

> Discontinuities at 2100 are due to different numbers of models performing the extension runs beyond the 21st century and have no physical meaning (IPCC, 2013a, p. 89).

The discontinuity is thus presented as an artefact (without a physical meaning) resulting from a reduced number of models and model output. Drawing attention to this artefact makes the implicit uncertainty clearer.

The figure is also referred to in the Summary for Policymakers (IPCC, 2013b), when considering the effect of making future energy use within transport more efficient:

> Projected energy efficiency and vehicle performance improvements range from 30 – 50% in 2030 relative to 2010 depending

> on transport mode and vehicle type (medium evidence, medium agreement). Such mitigation measures are challenging, have uncertain outcomes, and could reduce transport GHG emissions by 20 – 50 % in 2050 compared to baseline (limited evidence, low agreement) [...] [Figure] 12.5 (IPCC, 2013b, p. 23; Figure 12.5 is shown as Figure 1 in this paper).

The figure is treated as a baseline without supporting uncertainty statements. Rather, the paragraph emphasises uncertainties associated with mitigation measures. This represents another layer of uncertainty, as the uncertainty associated with the figure relates to the effect of emissions on temperature change, while the citation addresses uncertainty related to the effect of mitigation measures on emissions. How uncertainty from one layer affects the other is not addressed in this text.

Newspaper Report

The news text was published in the UK's Guardian newspaper[4] shortly after the IPCC's report was released. It provides a summary of the key points in the report. The language of the report includes little that explicitly indicates uncertainty. Indeed, much of the language implies certainty: for example, "without 'substantial and sustained' reductions in greenhouse gas emissions we will breach the symbolic threshold of 2C of warming". In this kind of expression, 'will' is an expression of certainty. Much of the report follows the same tone, although in one place, 'could' is used rather than 'will'. The report also includes a graph derived from Figure 1. It shows only the range RCP6 until 2100, labelling RCP6 'business as usual'. By stopping at 2100 and only showing one scenario, the graph removes much of the implicit uncertainty in Figure 1, including the 'break' at 2100, and the presence of multiple scenarios. Finally, a section of the report addresses the idea that warming has slowed in recent years, stating that the IPCC report "rebuffed the argument made by climate sceptics that a 'pause' for the last 10-15 years [...] was evidence of flaws in their computer models." In this case, the possibility of uncertainty is 'rebuffed' with a defence of the models. Overall, it is noticeable that there is much less indication of uncertainty in the report and the expression of uncertainty is largely not mathematical, although the graph does include temperature ranges and the report includes the statement "global temperatures are likely to rise by 0.3C to 4.8C by the end of the century depending on how much governments control carbon emissions." This statement echoes in simplified form similar statements in the IPCC report.

Students Discussing Temperature Change

In the master's course, the students did not read the report or the summary where the figure is presented. Kjellrun Hiis Hauge briefly explained the meaning of RCP and that the figure represented future global temperature projections related to four RCP scenarios. Previous to the following excerpt, the students were trying to make sense of the figure and the meaning of the coloured shading.

TOR INGE: Where is the limit for where the measurements fall within?

MARIA: It depends on which scenario is taken into account. If you - if they have calculated with an RCP value of 8.5, and that resulted in - those who achieved the lowest values at the … lowest one, right? And then they calculated with the same model with a value of 2.6. And that could have resulted in one of the lowest values there, and - so - while some with a high value could have ended down there. But - what … you can see here is that there's a lot of variation here. That here it seems that they disagree much more.

TOR INGE: More uncertain?

MARIA: Yes, while at the blue, they quite agree all the way, in a way. They are more certain.

Maria is referring both to RCP values and to models, so she seems to have understood that each colour represents one RCP scenario, and that the consequence of each scenario is predicted through a combination of models. Her use of "disagree" indicates that she pictures that at least some of the models do not produce the same predictions. It is not clear whether she pictures stochastic or deterministic models, but in any case she characterises the uncertainty derived from the deviating results as disagreements. She thereby expresses an awareness of an uncertainty that is not controlled.

In the next excerpt, Hauge draws the students' attention to the sudden break in the graph.

KJELLRUN: But if we look at year 2100, what is happening there?
[pause 16 sec.]
ELISABETH: [inaudible] is a break?
KJELLRUN: Yes, why is that?
ELISABETH: At least the red one.
KJELLRUN: Yes, at least the red one, that's very distinct.
ELISABETH: There are fewer models, you know.
[…]
TOR INGE: I'm thinking that the most critical until 2010 do not continue further in the models.
KJELLRUN: Yeah, well that's true.
TOR INGE: So that the curve isn't as steep when it continues.

Elisabeth explains the break by stating that "there are fewer models". By that she indicates that the number of models influence the features of the graph. Tor Inge stresses that the break is a drop by saying "the most critical until 2010 do not continue". He thereby indicates that the projections could have been more severe. Both indirectly express that uncertainty is not controlled and Tor Inge also suggests that uncertainty can influence our understanding of the severity of the situation.

Discussion and Conclusions

The four main texts (including the students' discussion) display some differences in how they treat uncertainty. In the IPCC report, there is much technical uncertainty in highly mathematized form, with some indications of epistemic uncertainty. In the technical summary and the summary for policy-makers, some of the detailed mathematical treatments of uncertainty are reduced. In the news report, there is much less evidence of technical uncertainty in mathematical form, or indeed of epistemic uncertainty. Finally, in the class discussion, the students show some awareness of epistemic uncertainty in their interpretations of the graph. These differences are consistent with the findings of Weingart et al. (2000).

The shifts that occur from the original IPCC report to the summary texts and the news reports suggest that the mathematical treatment of uncertainty in climate science becomes much less visible in texts meant for more general audiences. We suggest that uncertainty is itself formatted by mathematics; it is constructed and interpreted by scientists in largely mathematical terms, which then become much less visible and much less human in subsequent reporting. The effect of this formatting is to construct uncertainty as controllable, while de-emphasising epistemic uncertainty. Thus the news report only really indicates uncertainty implicitly and gives no hint of epistemic uncertainty. Interestingly, in their discussion of the original graph, the students seem able to identify possibilities for epistemic uncertainty.

Acknowledgments

We would like to thank IPCC for granting permission to reprint the figure.

Notes

1. CMIP5: Coordinated Modelling Intercomparison Project phase 5, a joint activity of scientists at a certain stage.

2. TCR: Transient Climate Response: the temperature rise at the time of CO_2 doubling.

3. GHG: Greenhouse gases.

4. "IPCC: 30 years to climate calamity if we carry on blowing the carbon budget." The Guardian, 27 September 2013.

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Beginning Early: Mathematical Exclusion

Ola Helenius[1], Maria L. Johansson[2],
Troels Lange[3], Tamsin Meaney[3], Anna Wernberg[4]
National Center For Mathematics Education, NCM[1],
Luleå Technical University[2],
Bergen University College[3], Malmö University[4]

In this paper, the distinction between Bernstein's horizontal and vertical discourse is used to show how two children are restricted in their possibilities to learn mathematics. The social relationships set up within contexts, both of the problems being solved, and between participants, contributed to the horizontal or vertical discourse being employed. In a circular motion, these discourses then reinforced the social relationships that could come into play. It is argued that mathematical exclusion can occur when social relationships, not only within problem contexts but also within interactions, miscue the kind of discourse which is foregrounded. Children can become confused over the sort of discourse that contributes to mathematics learning.

Mathematical Exclusion

It has been known for some time that certain groups of students become alienated from mathematics and thus are excluded from the opportunities that having mathematical qualifications might bring (Stinson, 2004). Often the cause for this exclusion is debated in regard to whether it is the lack of qualifications that exclude people from accessing further education and well-paid jobs or whether it is institutionalised racism or other discriminatory practices that ensures that some groups of students do not have opportunities to gain such qualifications or even if they do gain them, still find themselves excluded (Knijnik, 2002). Approaches to mathematics education may well differ according to which cause is accepted as the root of the problem. However, whatever the cause, groups of students will suffer from social exclusion.

Klasen (2001) suggested that social exclusion was "socially generated barriers that reduce the ability of the excluded individuals to interact with society" (p. 416). Mathematics education, because of its role as a gatekeeper (Stinson, 2004), can be considered a socially generated barrier that reduces the ability of individuals to interact with society. Stinson (2004) discussed the issue of tracking of minority students as one example of how exclusion occurs while Lerman and Zevenbergen (2004) discussed how classroom practices act to exclude students from working-class backgrounds because of their unfamiliarity with those practices. Social exclusion, in which the way that mathematics is presented to children leads to them being excluded from it, we call mathematical exclusion.

Presently, there is much attention on the perceived lack of mathematics of young children from disadvantaged backgrounds who are beginning school. Reports such as those by Arnold, Fisher, Doctoroff, and Dobbs (2002) have found that the mathematics that children know on entering school will have an impact on their school learning. This has led to calls for more mathematics to be introduced to children in preschools (see Lange, Meaney, Riesbeck, & Wernberg, 2012) to overcome the likelihood of a disadvantaged school experience. These approaches have been critiqued for their underlying deficit assumptions about children (Meaney, 2014). Nevertheless, how mathematical exclusion operates in micro events of learning situations is not well identified, particularly in regard to young children.

In this paper, we compare and contrast two interactions which included the same two children to consider what characteristics of the situations contributed to them being mathematically excluded. To do this, we use Bernstein's theory of vertical and horizontal discourse.

Theoretical Framework

Over several decades, Bernstein developed a systematic sociology of education which included the development of many different ideas. In his later years, he concentrated on the distinction between horizontal and vertical discourses (Knipping, Straehler-Pohl, & Reid, 2012). Bernstein (1999) defined these discourses by the forms of thought that they develop within individuals in regard to the kinds of knowledge that were valued within a wider society - "the structuring of the social relationships generates the forms of discourse but the discourse in turn is structuring a form of consciousness, its contextual mode of orientation and realisation, and motivates forms of social solidarity" (p. 160).

Horizontal discourse is equated with common sense forms of knowing, essential for solving specific issues that are highly relevant to the solver, but not easily transferred to other situations (Bennett & Maton, 2010). On the other hand, vertical discourse, often equated with educational knowledge (Knipping et al., 2012), can be generalised to a range of situations. Thus, "the meaning of educational knowledge is given by its relations with other meanings rather than its social context" (Bennett & Maton, 2010, p. 327). Although Bennett and Maton (2010) suggest that social context or relationships are unimportant in regard to vertical discourse, Bernstein suggests that social relationships have a strong influence on the forms of discourse that operate. Therefore, it does not seem valuable to suggest that the social context of the vertical discourse should be ignored as unimportant in the development of meanings.

In this paper, we explore the tensions that arise when the social relationships, either through being maintained or disregarded, affect young children's opportunities to engage in mathematics learning. Our argument is that one way that mathematical exclusion can occur is when social relationships, not only within problem contexts but also within interactions, miscue the kind of discourse which is foregrounded.

Data Collection and Analysis

In the first half of 2013, video recordings were made on four different occasions in one preschool class in Sweden, when most children were about six years old. The episodes used in this paper come from a free play interaction and a more formal teacher-led interaction. The first episode was analysed in depth in Helenius et al. (2014), whilst some of the second episode was used in Helenius, Johansson, Lange, Meaney and Wernberg (2015, forthcoming). Here, we focus on two children, known as Teo and Klara in Helenius et al. (2014) and as Teo and Lova in Helenius et al. (2015, forthcoming). The two children appear in both episodes and so we analyse their interactions from the perspective of vertical and horizontal discourses. Although these two children differ according to gender, we do not present them as examples of how particular groups of children become mathematically excluded but rather use their interactions as examples of how the process of mathematical exclusion operates.

The interactions in which they participate seemed at first glance to be inclusive but the earlier analyses showed that this was not the case. In order to consider how mathematical exclusion operates, we examine the relationships between the participants in both episodes and the relationships set up within the problem contexts that the children are interacting around.

Episode 1—Buying a Popsicle

The problem at the heart of the free play episode is the cost of a popsicle, represented by a piece of LEGO. After some joint LEGO construction, Teo initiates this interaction by situating Tom as the seller and himself as the buyer (*Får man köpa nåt här?*). Tom first refused Teo's request to buy something but then demanded all of his money (play kroner notes). Figure 1 shows Tom indicating that Teo would need to hand over all of his money (*Den kostar alla dom*).

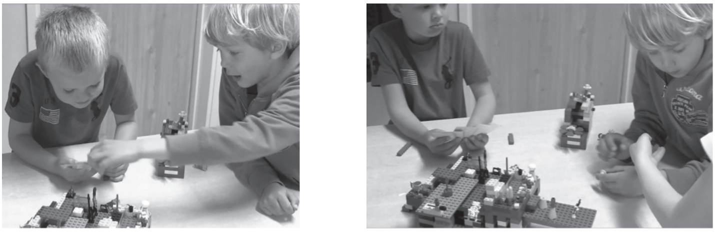

Figure 1: Free Play Exchange. Left to Right: Teo, Tom and Klara

Klara had been involved in the original LEGO construction, but left before Teo asked about buying the popsicle. She returned soon after Teo protested the need to hand over all of his money. At this point she tries to interrupt the discussion about the popsicle by indicating that her brown piece of LEGO was a piece of chocolate. Neither her first or second attempt to gain the boys' interest in the chocolate was successful (see Figure 1).

Later, Klara attempted to trade a car for some other constructions that she helped to build with the others (*Byter ni den här bilen mot alla de här och mitt bygge?*). There was some discussion around this suggestion, but Tom rejected the possibility of a transaction which was made by both Klara and another child, Patrik. Tom's rejection was based again on them not having "enough" money (*Men var är pengarna då?*).

At the end of the episode, Teo makes another request to buy the popsicle but Tom again vetoes it, first outright and then by suggesting that Teo will die from eating the popsicle (*Varför måste du ha piggelinen, då dör du ju*). When Teo repeated that he wanted to buy the popsicle, Tom then said it would cost 40 000 kroner (*Det där är fyrtiotusen, I så fall får du betala, vänta de här också.*). After a further exchange about it costing too much, Teo stated "You are so mean, why does it have to cost that much?" (*Ni är så elaka varför måste det kosta så mycket?*). At this point, Tom capitulates and agrees to Teo buying it for only one of his play money notes.

Discussion

In our original analysis (Helenius et al., 2014), we considered that Tom and Teo had been involved in mathematical play as it included participation, creativity, and rule negotiation. On the other hand, it appeared that although Klara (and Patrik) tried to participate in similar ways, their suggestions were rejected, suggesting that they did not have the same opportunities to be involved in mathematical learning. By considering this episode using understandings about the differences between vertical and horizontal discourse that were in circulation, it is possible to clarify how Klara's mathematical exclusion occurred whereas Teo's did not.

Although the problem appeared to be an everyday one about buying something, for Tom and Teo the discussion focused on general understandings about ideas to do with enough and too much and how their meaning changed depending upon who used the terms. As both boys were only six years old, it is unlikely that either of them knew precisely how much 40 000 kroner was. Although Tom initially indicated that this is what he wanted Teo to pay, both boys recognised it as being more than what Teo could possibly have in play bank notes. Such considerations were generalizable to other situations and so can be seen as belonging to the vertical discourse.

It is not until Teo calls Tom "mean" at the end of the episode that social relationships gain prominence in the discussion. They had been evident earlier, but implicitly, when Klara and Patrik tried to enter the discussion and Tom, and to a lesser extent Teo, did not respond. Tom was able to make such decisions because he was accepted as controlling the situation by the others. This may have been because, in the imaginary play situation, he had the role of seller, who had the power to determine the price of the things to be sold or exchanged. The context of the problem, set in a shopping scenario, seemed to set up a particular social relationship with someone in power. This kind of relationship contributed to the vertical discourse being used as it enabled one person to determine what knowledge was relevant, in a similar manner to teacher-class relationship. As is the case in this example, such a relationship within a play situation can contribute to generalizable concepts, rather than the specific problem, being discussed. Although Teo could overcome the power structure by appealing to his real-world friendship with Tom, in doing so he contributed to the ending of the play situation. In this case, the role of relationships within the problem context contributed to the vertical discourse being foregrounded with the everyday relationship between participants, contributing to the horizontal discourse becoming backgrounded.

Klara, in trying to navigate the vertical discourse within the play situation, choose not to bring in her real world friendship with Tom, but instead tried to participate in the same way as Teo. When her efforts to participate were rebuffed by Tom, she was unable to affect what knowledge was discussed and in what ways and so can be said to have been mathematically excluded. On the other hand, Teo was not mathematically excluded, even though he accepted Tom's right to make the decisions in the same way that Klara had. Whereas Tom did not accept any of Klara's attempts at contributing to the discussion, Tom accepted that Teo had a right to contribute and also to have a different opinion of what was enough and what was too much. Thus, although Teo's contributions were different to those of Tom's, he had a more active role in the vertical discourse than Klara. The negotiation of the meanings would have contributed to Teo's understandings of enough and too much and also to how to engage in negotiations within the vertical discourse, thus supporting his possibilities for learning mathematics.

Episode 2—Splitting 10 into 3 Groups

The second episode was a fairly typical mathematics lesson initiated by the teacher with a group of eight children. It began with a warm up activity to do with pairs of numbers that added to ten. Then a problem was posed about ten children in a small preschool class who were to be sent to three activities—woodwork, baking, and painting. The children were given time to determine individually how many children should be in each group, with the teacher stating that there are no wrong or right answers. She also said that it might be possible to distribute the class evenly or it could be that one group had more children or another group had no children (*Tre grupper och så de tio barn. Nu vill jag att ni tänker ut hur de delade sig. Hur delar man tio barn i tre grupper? Det finns inget rätt, inget fel. Ni bestämmer själva. Om det kanske bara är ett barn i en grupp, och det kanske är inget barn i någon grupp och det kanske delar något så att det blir lika i varje*). The children were told that they could record the distributions on paper in any way they liked. As the children worked, the teacher moved around the class asking them about their distribution. At the end of the session, the teacher had the children fold their papers and sit down in a horseshoe so that they could explain the way they had distributed the class.

In the warm-up activity, Klara was the first to stand up and try to find who had the pair number for her 2. Nevertheless, it was her partner who told the teacher about their pair. On the other hand, Teo was one of the last to identify the child who had the other half of his pair, but it was he who took hold of both cards. Both children answered in chorus when the teacher asked for their answer (seven and three, *sju och tre*). The teacher wrote the solutions on the board using standard number symbols.

As the teacher described the problem of sharing ten children in the three groups, Klara could be seen using her fingers to work out

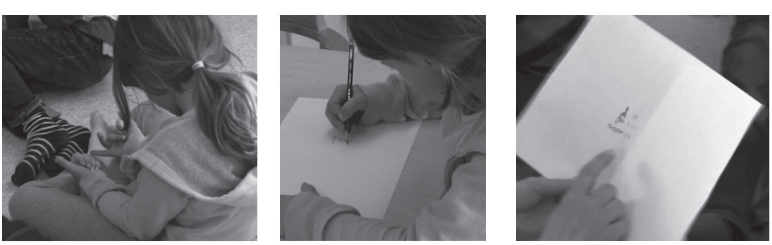

Figure 2: Klara's Problem Solving - 2 Painting, 5 Woodwork and 3 Baking

possible solutions (see Figure 2). Before she collected her paper, she shared her solution with another child who also used his fingers to find a solution. When working on the problem, Klara used first her fingers and then tally marks (see Figure 2). After she put one round of tally marks next to each group symbol, she counted them before putting down the next round.

Teo spread his hands and first counted the fingers on one hand before separating his fingers on both hands into groups so that he seemed to have found a 4, 3, 3 solution (see Figure 3). Still, his initial drawing showed that he had only 9 tally marks and he seemed stuck. Nevertheless, by the time the teacher talked with him, he had given the baking group, represented by a rolling pin, a fourth member.

Figure 3: Teo's Solution Process

Although the teacher usually presented each child's solution. Klara was asked to present a proposal (see Figure 3).

Teacher:	Klara, what proposal do you have? Oh, okay what is there? Can you tell me?	Klara, vad har du för förslag? Oj okej vad står där? Kan du berätta för mig?
Klara:	Three, five and one. Three, five and two.	Tre, fem och ett. Tre, fem och två.
Teacher:	Let me see, three, five and two, okay. It is, let's see here.	Jag får se, tre fem och två okej. Det är, ska vi se här.
Klara:	Three in one group, five in another.	Tre i en grupp, fem i en.
Teacher:	But where are the three, is it in the baking group?	Men var är de tre, är det i bak gruppen?
Klara:	Yes.	Ja.
Teacher:	And then it's five in the woodworking group and two in the painting group.	Och så är det fem i snickargruppen och två i målargruppen.
Klara:	Yes.	Ja.
Teacher:	Why did you decide this?	Varför bestämde du dig för det?

Klara:	Because the woodworking group, it's much more who like to do woodwork, less who like to paint and in the middle those who like baking.	För att snickargruppen, det är mycket mer som tycker om att snickra, mindre som tycker om att måla och mittemellan som tycker om att baka.
Teacher:	So they could choose for themselves in that class, okay.	Så dom fick välja själva i den klassen, okej.

In presenting what she had done, Klara focused on the numbers. However, the teacher shifted her attention, by asking which group had three children in it. Klara happily responded by discussing the popularity of activities and providing details about why she had split the ten children as she had. After Klara presented her proposal, Teo was asked to present a solution. His initial response was to deny that he had one.

Teacher:	Teo, do you have a solution?	Teo, har du någon lösning?
Teo:	I just do not know what the solution is.	Jag bara inte vet vad lösningen är.
Teacher:	Can you open it so I can see how you did it?	Kan du öppna den så jag ser hur gjorde du?
Teo	I shared it so.	Jag delade den så.
Teacher:	Three, three and four, okay. Why was it four? You told me. Why are there four for baking? It was a little funny.	Tre tre och fyra, okej. Varför blev det fyra? Det berättade du för mig. Varför är det fyra på baket. Det var lite roligt.
Teo	Nah, I changed.	Nä, jag ändrade.
Teacher:	You changed it. I thought it was a funny thing, you said about there being four in baking.	Du ändrade det. För jag tyckte det var en rolig sak du sa att de var fyra på baket.
Children:	What did he say?	Vad sa han?
Teo:	I changed.	Jag ändrade.
Teacher:	Then it would become many more buns.	Att det skulle bli mycket mycket bullar.
Teo:	I changed.	Jag ändrade.
Teacher:	But you decided three, three and four?	Men du bestämde tre tre och fyra?
Teo:	Yeah, I just wanted that.	Ja, jag bara ville det.
Teacher:	You just did that, okay. Thanks for that, okay.	Du bara gjorde det, okej. Tack för det, okej.

In this exchange, the focus is on finding a solution, whereas in Klara's case, it was about presenting her proposal. Teo's suggestion that he did not have a solution seemed to mystify the teacher who had previously talked with him. It may be that the word solution (*lösning*) suggested to Teo that she was looking for one specific solution. Having heard several different suggestions, some of which the teacher had already stated were brilliantly solved (*strålande löst*), Teo seemed uncertain that his solution which was different could also be correct. Regardless of his reasoning, Teo was not interested in the teacher's questions about which group had which number of participants. Instead, he kept repeating that he had changed, which seemed to mean that he had changed the group with four members because he remained adamant about the 3, 3, 4 distribution.

Discussion

Although it could be expected that the social relationship between the teacher and the children would be similar, the type of problem that children were engaged in solving and the roles that they took up in presenting their solutions altered their social relationships with the teacher. The differences in social relationships affected and were affected by the discourses that were utilised in the interactions.

The problem itself allows for discussions about the different combinations of three numbers which add up to ten. As the teacher had begun the lesson with an activity based on two numbers that added to ten, it could have been expected that this would have been the teacher's focus. However, the setting of the problem within a preschool class and the distribution into three activities allowed the teacher and the children to become involved in a discussion about children's preferences for particular activities. In asking about children's preferences, the teacher moved from being an expert determining the correctness of the mathematics under discussion to an interested enquirer into what activities the children saw as the most interesting. By changing her role in the social relationship, she also changed those of the children. These changes led to the discussion being within the horizontal discourse where the focus was not on generalizable ideas but on the specific situation. In the original analysis of Klara's role (Helenius et al., 2015 forthcoming), we could see that Klara's opportunities to discuss mathematics were restricted by the discussion of preferences. However, the traditional use of a vertical and horizontal discourse analysis to focus on what was being discussed and in what ways did not provide information about how the social relationships contributed to this restriction.

Although Klara was happy to discuss her distribution of children into the three groups, Teo resisted this shift. He remained committed to his 4, 3, 3 distribution but refused to enter into a discussion about his preferences for the groups. It is unclear whether he was confused over the kind of discourse which he felt should have been in operation. If the teacher had asked him about his reasoning to do with the numbers adding to ten, perhaps he would have been more willing to engage as he had done in the warm-up activity around the pairs of numbers adding to ten. His resistance in responding to the teacher's questions about the distribution into groups meant that he adopted a different role in the exchange and this affected the social relationship with the teacher. Having taken on the role of resister, he was not drawn into the horizontal discourse of the teacher.

In this example, the teacher's role moved from being the expert in the warm-up activity to a gentler enquirer about the children's preferences in the presentation of solutions or proposals. Her shifting role affected the social relationships with the children and contributed to a move from the vertical discourse to a horizontal discourse. Although Klara moved with her, Teo did not. Nevertheless, as neither child had possibilities to engage in vertical discourse around how different combinations of numbers could be formed to make ten, both could be said to be mathematically excluded. It could be said that Teo in taking on the role of resister had the best opportunities of changing the discourse to a vertical one by refusing to consider that the teacher should have a different social relationship with him. In not accepting the teacher as a gentle enquirer who had the right to find out about his group distribution preferences, it might have been possible for him to force the teacher back into her role as expert on mathematics learning. Although in this case, this did not happen, in the earlier analysis (Helenius et al., 2015 forthcoming) when another child gave a combination of 5, 5 and 5 earlier in the interaction, the teacher had moved back into her role as expert. However, that child did not have the requisite mathematical knowledge to discuss the mathematics with her.

Conclusion

Watching her work on the solution of splitting ten into three groups, it appears that Klara had skills and interest in mathematics which could have been developed. Yet, she was excluded from mathematics in both interactions because she tried to act in accordance with the social relationships that those in control had put in place. As a result in the free play episode, she could be excluded from the vertical discourse by Tom, who being the seller and thus in charge, was not required to accept her suggestions. In the exchange, when the teacher moved to a horizontal discourse, Klara moved with her, and was then confined there.

On the other hand, Tom accepted Teo's right to be involved in the popsicle discussion, enabling Teo to take part in the vertical discourse and to learn about abstract notions such as enough and too much. In the episode with the teacher, Teo resisted being brought into the horizontal discourse around his preferences for the group distribution. As a result, he may have become more aware that mathematics was about being in the vertical discourse. This is in contrast to Klara who, if she does not gain experiences of being in the vertical discourse, is likely to associate mathematics learning only with the horizontal discourse.

Mathematical exclusion has been noted in many different situations, even if it has not been labelled as mathematical exclusion. However, insights about the impact of social relationships on the use of vertical and horizontal discourses provide a richer understanding of how mathematical exclusion comes into existence.

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Conceptualizing Societal Aspects of Mathematics in Signal Analysis

Reinhard Hochmuth, Stephan Schreiber
Leibniz Universität Hannover, Leuphana Universität Lüneburg

This paper focuses on different possibilities for conceptualizing didactically relevant aspects of advanced mathematical subject matter in such a way that fits with subject scientific categories considering mathematically learning as a societal mediated process according to Holzkamp (1993). This concern requires studying subject matter, teaching and "university" not as conditions that cause reactions but as meanings in the sense of generalized, societal reified action possibilities. We consider the following three theoretical lenses: an epistemological-philosophical analysis, Anthropological theory of didactics and elements of Bernstein's theory. After rather short characterizations of these approaches we illustrate their application to the introduction of the delta distribution in a signal analysis text book.

Introduction

This paper contributes to an ongoing major research project that describes and analyzes form and content of advanced mathematics and its teaching and learning from a "subject scientific" point of view. This approach grounds in the so-called "Critical Psychology", worked out in Holzkamp (1985), also see Tolman (1991). Recently this theory becomes internationally more known within the mathematics education community due to Roth & Radford (2011), who value "German Critical Psychology" as a further development of the culture-historical approaches by Leontjev (1978) and Vygotsky (1978).

The main features of "Critical Psychology" and its subject scientific point of view are well elaborated psychological categories for describing and analysing subject related experiences, in particular thoughts, actions and learning, in such a way that major societal aspects are inherently incorporated. Within this framework there is so far not much (if any) research done that relates to mathematical learning in the context of higher education. This paper focuses on different opportunities for conceptualizing (specific aspects of) advanced mathematical subject matter in such a way that it is "viable" to descriptions and analyses based on subject scientific categories, i.e. that allows to consider mathematical learning in a specific way as a societal mediated process or activity. The main connecting link between the subject scientific approach and subject matter is a specific understanding and use of the concept "meaning". Consequently, a criterion for the adequacy of an approach for the analysis of subject matter is to blend with crucial aspects of the subject scientific concept of meaning. In view of this criterion we exemplarily discuss the application of analysing tools from anthropological theory of didactics (ATD), Bernstein's discourse analysis and materialistic oriented epistemological studies by means of the introduction of the delta distribution in signal analysis.

The structure of the paper is organized as follows: At first we sketch concepts from the subject scientific theory that are relevant for an embedding of our further observations; in particular we describe shortly their specific concept of meaning. Then we present a few details about an introduction of the delta distribution in signal analysis and demonstrate in the following, how an epistemological-philosophical approach, ATD and elements of Bernstein's theory could contribute to working out basic facets of corresponding meanings. A summary and remarks on further research steps conclude the paper.

Subject Scientific Approach ("Critical Psychology")

Critical Psychology claims to present a scientific discussable and criticizable elaboration of basic psychological concepts (categories). The starting point is a historical-empirical investigation of general historical-specific characteristics of relations between societal and individual reproduction. Within the context of this paper there are two important points to notice: First, the actual historical-specific form of subjectivity is characterized by the so called "possibility relation" with respect to the societal reality, which gives and includes in particular the basic experience of intentionality and makes consciousness to a prerequisite for the societal reproduction. Second and connected to the first, the specific modality of subjective action experiences comprises a certain discourse form ("I" speak about my "own" actions in terms of subjective reasonable actions and of premises in the light of "my" living interests.) that characterizes to some extent the specific subject scientific standpoint.

According to this modus, world conditions are given in terms of meanings in the sense of generalized societal action possibilities. Meanings of reality aspects, which are relevant for "me", become premises. Consequently, subject scientific considerations are essentially given by premises-reasons-relations.

In "Critical Psychology" meanings and their mediation role do not only represent social-interactive but also societal aspects grounded in relations between production and reproduction. Via meanings, human activities, like teaching and learning, can be thought as societal mediated. An analysis of subject activities regarding its societal mediation requires an adequate conceptualization of the objective situation of the subject. As representations of subjectively relevant objective conditions they have to be describable and analyzable as generalized action possibilities, hence meanings.

Since meanings appear (via objective-subjective premises) to some extent as the medium within which subjective action reasoning is grounded, their study is a prerequisite for describing and analyzing related cognitive, motivational and emotional processes as aspects of subjective activities like learning under concrete societal conditions.

Meanings in the indicated sense are relevant for acting and thinking, but do not determine them. Furthermore, they are not only of linguistic-symbolic nature, but objective-societal objects, to which symbols relate. Of course, in particular in mathematics and science, symbols are objects by their own and acting with them underlies rules that are epistemologically and institutionally determined and are also determined as elements of a scientific or pedagogical discourse.

In the following, we exemplarily consider the introduction of the delta distribution in signal analysis and apply three different approaches shedding light on different aspects of meanings in the indicated sense of generalized societal action alternatives.

Higher Mathematics and Signal Analysis: The Introduction of the Delta Distribution

Students in engineering courses learn mathematics in at least three contexts. First, they have to pass courses in higher mathematics. Here the students learn mathematical concepts from analysis, linear algebra and sometimes elementary numerical analysis. These topics are mostly presented in a more or less theoretical mathematical setting. Second, students must apply mathematics in their basic engineering courses. Since most of the mathematical concepts required in these courses have not been presented in the higher mathematical courses until that moment (at least in Germany), often additional seminars are offered that accompany the engineering courses. While exact mathematical definitions and/or justifications for the mathematical concepts used in the basic theory-oriented engineering courses are often presented later in the higher mathematics courses, this is generally not the case for more advanced mathematical concepts applied in the third context, e.g. courses like signal analysis. For example, a concept like delta-distribution is typically not covered in the mathematical courses attended by an electrical engineering student. It is not clear how, if at all, students are able to integrate these variations of mathematics. To study this problem, it would be helpful to have research-methods that represent, relate and reflect these variations of meaning.

Next we sketch exemplarily the introduction of the delta-distribution in one textbook, Frey and Bossert (2008): Continuous linear and time-invariant systems are generally described by differential equations. In view of discontinuous signals that are not differentiable in single points, the authors introduce so called "generalized functions" or "distributions" (p. 108). This leads to the problem introducing some kind of derivative to a non-differentiable function, which is exemplarily considered by means of the Heaviside function

$$\varepsilon(t) = \begin{cases} 1, & t > 0 \\ 0, & t < 0 \end{cases},$$

that is even not continuous at t=0. Supported by graphical illustrations it turns out that the Heaviside function can be represented as the pointwise limit of the parameterized sequence of differentiable functions

$$f_a(t) = \frac{1}{\pi}\left(\arctan\left(\frac{t}{a}\right) + \frac{\pi}{2}\right),\quad a > 0,$$

for a→0. These functions are formally differentiated and under the not further justified assumption that function limits and differentiations are commutable it is obtained

$$\frac{d}{dt}\varepsilon(t) = \frac{d}{dt}\lim_{a\to 0} f_a(t) = \lim_{a\to 0}\frac{d}{dt} f_a(t) = \lim_{a\to 0}\frac{1}{\pi}\frac{a}{a^2+t^2}.$$

Here the convergence of the starting function sequence can be understood pointwise which fits to the convergence concept in higher mathematical courses for engineers. Instead, the limit of the differentiated sequence turns out to be a new object that mathematically has to be interpreted as functional on a space of test functions. Furthermore, the related limit has to be understood as distributional, which is distinctively different to the limit concepts introduced in higher mathematical courses for engineers. Considering the graph of the differentiated function for a→0 the notion

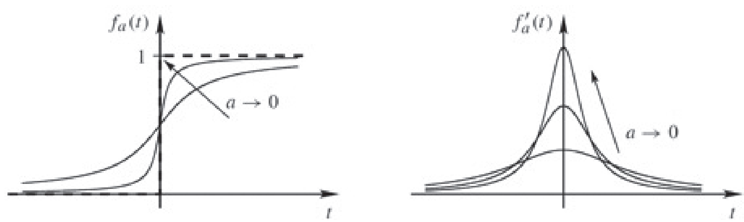

Figure 1: Representing the Heaviside Function and the Dirac Impulse as Limit (Frey & Bossert, 2008, s. 109)

"impulse" is suggested and for the "limit impulse" the symbol $\delta(t)$ with name "Dirac impulse" is introduced as the pointwise limit function

$$\delta(t) = \lim_{a\to 0}\frac{d}{dt} f_a(t) = \begin{cases} 0, & t \neq 0 \\ \infty, & t = 0 \end{cases}.$$

Additionally, the effect of the Dirac impulse is symbolically represented by the integral $\int_{-\infty}^{\infty}\delta(t)\cdot\phi(t)dt = \phi(0)$, for $\phi(t)\square C_0^\infty(\mathbb{R}^n)$, which relates to the concept image (Tall & Vinner, 1981) of an integral as infinite sum of infinitesimal small pieces weighted by the "function" $\delta(t)$.

From a strong mathematical point of view severe mathematical deficits in the foregoing argumentation are obvious. Taking into account the, in the 20th century, well-developed theory of distributions another and mathematical correct presentation could be given. E.g., integrals with respect to distributions might be introduced that have (nearly) nothing to do with the Riemann integral taught in higher mathematical courses that represents the reference concept here. In the context of this paper we refer to those deficits in another way: We argue that there are very specific reasons for those inconsistencies. E.g. in the following section, we refer to epistemological-philosophical ideas claiming that they root (in a certain sense) in necessities that are related to the application of mathematics to reality, i.e., the use of mathematics in empirical sciences as a mean for developing knowledge facilitating measurements and calculations.

Epistemological-Philosophical Analysis

Epistemological-philosophical analyses contribute to a deeper understanding of applied sciences like, for example, electrical engineering, mathematics and their relations. Here we compile a few basic general theses from a materialistic oriented theory about relations between mathematics and physics, a field which is in particular considered within the philosophy and history of science. In the following sections we will demonstrate how these ideas blend with the other theoretical approaches we refer to.

In Wahsner and Borzeszkowski (1992) it was claimed that in physics (and we believe: similarly in electrical engineering) "objects" appear as "moving" (or "behaving") and that "moving" is considered in certain dualistic forms like geometry and dynamics or particle and field, which express the categorical unit of "object" and "behavior". The dualistic forms root in the necessity that measuring, and physics implicates to measure, do not only require to differentiate moments but to separate them. In contrast to physics, mathematical "objects" appear merely as "position" within functionally structured systems that presuppose their existence. In other words, mathematical "objects" are subsumed under the "as relation thought property" (p. 133). This handling of "objects" is the specific characteristic form, which allows mathematics to be without inherent contradictions, but, at the same time, disable mathematics to make assertions about real "movements". Consequently, mathematics needs physics (or another empirical science) to be applied to "moving". On the other hand, physics needs mathematics for measurements, calculations and formulating dynamic interactions by laws.

In philosophical and concrete historical studies Wahsner and Borzeszkowski (1992, 2012) figure out those conceptual and experimental-objective preparations within physics that facilitate to use mathematics as mean for recognizing real "moving" and to link mathematics with "measuring". The following both aspects are in particular important:

1. Since only "finite distances" are measurable, (conceptual) contradictory identifications of infinite or infinitesimal quantities, which arise in modern mathematical structures, with finite quantities are enforced. In context specific derivations of model equations, e.g. the heat equation, or relations like the Gaussian theorem, infinitesimal quantities are typically replaced by or used as sufficiently small but finite quantities, which leads to (in a strong logical sense) inconsistent argumentations. The particular context dependent adequate inconsistent use of mathematical concepts is historically one of the most original achievements of physics.
2. Only effects of properties of objects are measurable and not dynamic interactional relations. This leads to the question, which behavior can be transformed to a property. Related answers could be found studying the complicated historical genesis of physical measured quantity.

Whereas physics expresses basic possibilities of nature, theories in engineering sciences represent concretizations in view of subject related aims and embedded within culture-historical as well as socio-economical processes, which implicates that generally the sketched ideas remain relevant.

Within our signal analysis context, a) and b) might be related to the following observations: The use of the Dirac impulse is not only accompanied by the visual representation as an infinitesimal narrow, in $^{\mathrm{R}}$ the support is in fact$\{^{0}\}$, and unbounded and infinite "high" function, but also as a function with sufficiently narrow support and sufficiently high values near^{0}. In principle, the latter functions, resp. "impulses", can technically be realized and for those functions the given integral representation is reasonable. Within the electrotechnical context this argumentation is "sufficient", since the "limit" Dirac impulse does not represent a realizable impulse. Analogous arguments apply to the commutation of function limits and differentiation.

In other words, signal analysis considers "mathematical objects" also as electrotechnical quantities, which allows to operate with those objects in argumentations and calculations in a, as such, mathematically inconsistent but electrotechnical adequate way. Of course, the mentioned inconsistencies do not have to become conscious, although they are inherently enclosed in teaching, learning and problem solving activities.

In contrast to the societal aspects sketched later in ATD and by elements of Bernstein's discourse analysis, the moments highlighted within the epistemological-philosophical point might be understood as reified results of historical specific societal processes and as objective and (in a certain sense) logic prerequisites for those in some sense "actual" societal aspects uncovered by ATD and Bernstein. It should be clear that these differentiations are of analytical nature: Neither the epistemological-philosophical highlighted aspects would "exist" if they were not realized and constituted in actual institutional practices nor could the institutionally established practices be understood without the involved historically reified mathematical and electrotechnical concepts together with their basic inner relations.

From our point of view epistemological-philosophical considerations contribute to a deeper understanding of differences or even contradictions between different mathematical practices. The indicated somehow contradictory (or "dialectic") relations between different realized possibilities are constituted within discourses related to different institutional contexts: The mathematical use is addressed in higher mathematics, and the context specific use is addressed in engineering courses such as signal analysis. Here, ATD comes in, since it allows describing and analyzing certain crucial aspects of practices in different institutional contexts.

ATD Analysis

ATD (Chevallard, 1992, 1999; Winslow, Barquero, Vleeschouwer & Hardy 2014) aims at a precise description of knowledge and its epistemic constitution. Behind this approach is the conviction that a cognitive-oriented approach tends to misinterpret contextual or "institutional" aspects as personal dispositions. This aim fits perfectly to the subject scientific approach.

A basic concept of ATD are praxeologies, which are represented in so called "4T-models (T,τ,θ,Θ)" consisting of a practical and a theoretical block. The practical block (know how, "doing math") includes the type of task (T) and the relevant solving techniques (τ). The theoretical block (knowledge block, discourse necessary for interpreting and justifying the practical block, "spoken surround") covers the technology (θ) explaining and justifying the used technique and the theory justifying the underlying technology (Θ). Finally ATD ends up with local and regional mathematical organizations that allow contrasting and integrating practical and "epistemological" aspects of mathematical objects in view of different "institutional" contexts. ATD is in particular helpful in analyzing mathematical knowledge and its different institutional realizations within different learning contexts. This expectation is supported, e.g., by related but differently focusing ATD analyses in (Castela and Romo Vázquez, 2011) considering teaching signal analysis topics in mathematics and two control theory courses.

In terms of ATD the related task T in our example under consideration is to introduce some kind of derivative to a non-differentiable function, which is exemplarily considered by means of the Heaviside function. Applying the sequence of differentiable functions etc. could be seen as τ (technique), "justifying" the limit of the infinitely narrow and infinitely high functions or impulses resp. via aspects of the concept image of the "limit"- and the "integration"-discourses represent θ (technology, discourse). This kind of plausibility arguments is rather typical and important in the context of mathematical argumentations within the electrotechnical discourse. As theory Θ there is a mostly self-contained electrotechnical distribution theory that differs from the mathematical one, for example in its discursive structure. Facets of this electrotechnical theory are related to graphical visualizations that are connected with the effect idea of integration and in particular symbolically based "analogies" to former learned mathematics.

The crucial point is that the signal analysis technique τ does not fit with higher mathematical discourses (technologies). With the epistemological-philosophical considerations as background the electrotechnical technique and discourse has to be qualified as inconsistent but fits in its plausibility and efficiency perfect with the electrotechnical phenomena under consideration.

Applying Elements of Bernstein's Theory

According to our observations about the epistemological-philosophical background and the ATD analysis of an introduction of the delta-distribution, the students have somehow to accept a certain type of inconsistencies and to "learn" that they should neglect specific aspects from those discourses, for example they have to ignore concept definitions (The Riemannian integration of the Dirac impulse is not possible etc.) and at the same time, they have to realize aspects from them, for example specific parts of the "concept image" of integration and limits. In other, (Bernstein's, 1996) words: The students have to understand the context specific principles of knowledge classification (recognition rules) and to apply correctly related "realization rules" in view of solving tasks. For arguments for the relevance of Bernstein's theory in the context of mathematics in engineering courses we refer to Jablonka, Ashjari and Bergsten (2012) and for its general relevance for describing and analyzing teaching we refer to Gellert and Sertl (2012).

Our further empirical investigations show that sometimes tasks send "wrong" signals, that is, they look like mathematical tasks from higher mathematical courses for engineers. Novices in the field then try to apply arguments and techniques from the mathematical discourse but often fail in solving the tasks because of the arising complexity. It requires time and experiences until the students recognize that the electrotechnical discourse establishes some different but subject dependent more efficient techniques (realization rules) that leads to more satisfactorily results. Recognition and realization rules are obviously in strong relation to the selection of premises as well as to contents and structure of reasoning processes. With this observation a link back to our subject scientific approach is established.

Summary

We presented a few observations obtained by considering a specific mathematical topic in signal analysis by three different lenses. We explained their principal usefulness and in particular their compatibility with a subject scientific approach allowing the description and analyzing of learning experiences in view of historical specific societal mediations. Our main observation is that the three applied lenses figure out substantial aspects of the mathematical-electrotechnical subject matter in such a way, that they can be injected as facets of meanings within the subject scientific approach. In particular, they can be seen as generalized societal action possibilities, which, among others, were potentially reflected in subject related reasoning schemes as premises. This kind of meaning exploration has to been seen as a non-trivial and essential first step within subject scientific research. In terms applied in Roth and Radford (2011) and taken in particular form Leontjev and others, these considerations are part of an analysis of "activity" in the sense of "Tätigkeit". Within the dialectic of "action" and "activity" it concerns the societal side. Within a subject-scientific embedding cognitive and affective-emotional aspects have to be understood in an integrated way, such that their non-separable dialectic interrelation will be respected. In other words, premises or actions could not be understood as the result of a cognitive recognition and realization of meanings alone. This was also strongly emphasized by Roth and Radford and in detail illustrated in their empirical investigations. We expect that this is also true in our context where the mathematical object is more complex and the institutional embedding is rather different. We also expect that all three lenses will contribute to further analyses although in different ways. ATD analyses will inform microanalyses of task solution processes and contribute to answers on questions about concrete institutional techniques and their justification. Elements of Bernstein's theory could inform the analysis of specific selection processes between discourse possibilities and the difficulties students will have in recognizing whether a task has to be understood as a mathematical or an electrotechnical one. The philosophical-epistemological considerations could inform a deeper analysis of the relations between these both discourses. Hence these approaches allow unfolding different aspects of the meaning of mathematics in signal analysis. They are relevant for describing and analyzing related "activities" and appear to some extent as the medium, within subjective action reasoning grounds. They represent action alternatives but do not determine them, since there is an "active" unconscious-conscious step of selecting, neglecting or highlighting meanings in view of the subjectively noticed "life interests".

In a next step one has to reconstruct typical premises-reasons-connections as part of the general causes-reasons-connections, which will enclose concrete empirical research questions regarding tasks and solution processes and where among others video and interview data will be used. It is clear that the presented approaches have also potencies for the reconstruction of the "subjective" side of premises: ATD with respect to previous knowledge, which influence for, e.g., how deep meanings can be recognized; Bernstein's theory with respect to predominant accessible discourses and their recognition and realization rules; epistemological-philosophical analyses with respect to the general "world view", e.g. the mediation by societal work considering the cultural-historical nature of knowledge.

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History of Mathematics Textbooks and the Construction of Mathematical Subjectivity

Sara Hottinger
Keene State College

In this paper, I consider how our ideas about mathematics shape our individual and cultural relationship to the field. Specifically, I argue that histories of mathematics can play a role in the construction of a mathematical subjectivity that is both gendered and racialized. I examine a best-selling history of math textbook which uses a biographical approach to tell the story of the mathematician-hero. I show how this approach to the history of mathematics results in the construction of a mathematical subjectivity that limits who can see themselves within it. My paper ends with a consideration of a textbook that constructs a much more expansive mathematical subjectivity by teaching readers how to engage in the process of creating their own accounts of the historical development of mathematics.

One of the key ways we come to understand mathematics is via the histories that we tell of its development as a field of knowledge. There are many who argue that the history of mathematics should be included in standard mathematics curricula (Calinger, 1996; Katz, 2000). At the very least, many mathematics textbooks have small sidebars that give brief histories of famous problems and biographies of the mathematicians associated with them. In this paper, I argue that the way we tell the history of mathematics often results in the construction of a mathematical subjectivity that is normatively masculine and white. David Stinson (2013) calls this normative mathematical subjectivity the "white male math myth," and demonstrates the negative impact it can have on African American mathematics students. In order to show how this normative mathematical subjectivity is constructed in history of mathematics textbooks, I examine the work of David Burton (2010), whose *The History of Mathematics: An Introduction* is the best-selling history of mathematics textbook in the

US (Smoryński, 2008). I use Michel Foucault's theory of the author function to analyze Burton's account of one of the most well-studied periods in the history of Western mathematics, Newton's discovery of the calculus.

Foucault's (1998) central argument in his essay, "What is an Author?" is that the author is a subject position produced by discourse and that the figure of the author works to regulate the proliferation of meaning by limiting who is allowed to speak and what is allowed to be said. He begins by arguing that, "The coming into being of the notion of "author" constitutes the privileged moment of individualization in the history of ideas, knowledge, literature, philosophy, and the sciences" (p. 205). Enlightenment notions of the self—essential, self-contained, autonomous, rational—were exemplified by the idea of the author. The image of the author is represented by what Foucault calls the fundamental category of "the-man-and-his-work"—the author sits in a room of his own, creating his next literary masterpiece (p. 205). Foucault argues that the author is a discursive construction that allows us to maintain the illusion that we are autonomous selves in control of our own destinies. Histories of mathematics like Burton's perpetuate this illusion, constructing a figure of the mathematical author that epitomizes the author function. Consider the following description of Newton's work from Burton's (2010) textbook:

> While Newton was forced to live in seclusion at home [due to the plague], he began to lay the foundations for his future accomplishments in those fields with which his name is associated—pure mathematics, optics, and astronomy. During these two "golden years" at Woolsthorpe, Newton made three discoveries, each of which by itself would have made him an outstanding figure in the history of modern science (p. 391).

Just in this short passage, key elements of the Enlightenment construction of the author appear. The idea of the "the-man-and-his-work" is central to the story of Newton's discoveries and Burton establishes the image of the mathematical author, sitting in a room of his own with the phrase "forced to live in seclusion." A component of this trope is the idea that history does not act on the subject, rather the subject acts upon history; he makes history. This is apparent in the reversal we see in the above passage. Newton might have been forced by circumstances to live in seclusion, but he utilized that time to do work that "made him an outstanding figure in the history of modern science."

With this phrase, a Western ideal is constructed—a rational, productive individual, whose work not only garners him a place in history, but effectively makes history. This is a common trope in biographical history writing. Jean-Michel Raynaud (1981), in his essay, "What's What in Biography," notes:

> All biographies that you can read deal with the same story of which the hero is not one particular individual, but the Individual as such manifested as being the powerful agent acting on everything, on groups, on events, on history. Biography is therefore the story which reveals the Individual, the essential myth of our European society (p. 93).

By embedding biographical information into the history of mathematics, a mathematical author gets constructed in the same heroic way that the subjects of most biographies are constructed—as capital-I Individuals, whose life and work serve to change the course of history. This is similar to the construction of the author that Foucault (1998) critiques. As both the source and origin of a text, the author stands outside the text; he is beyond it. This corresponds with the biographical construction of the individual—the powerful agent acting upon history and, as such, standing outside of history. Foucault identifies how problematic this construction is, arguing that the concept of the author functions to regulate who can see themselves within it. He uses literary texts, specifically focusing on narratives, to analyze how the author function operates via discourse, but he acknowledges in numerous places throughout his essay that a variety of texts work to construct an author.

By analyzing the ways in which a mathematical author is constructed in history of mathematics textbooks, we can begin to understand how a normative mathematical subjectivity gets created. According to Foucault (1998), "the manner in which [discourses] are articulated according to social relationships can be more readily understood…in the activity of the author function" (p. 220). There is a connection between the social relationships that determine our understanding of who can occupy the position of author and the discursive construction of the author figure. Foucault goes on to argue that the subjectivity that is constructed is a privileged subjectivity, clearly stating that "one could also, beginning with analyses of this type, re-examine the privileges of the subject" (p. 220). He suggests a set of questions that get at the relationship between the author function and the construction of subjectivity:

> How, under what conditions, and in what forms can something like a subject appear in the order of discourse? What place can it occupy in each type of discourse, what functions can it assume, and by obeying what rules? … What are the modes of existence of this discourse? Where has it been used, how can it circulate, and who can appropriate it for himself? *What are the places in it where there is room for possible subjects? Who can assume these various subject functions?* [emphasis added] (Foucault, 1998, pp. 221-222).

Analyzing the ways in with the figure of the author is discursively constructed gives us insight into how subjectivity itself is discursively constructed. If we can identify the ways the mathematical author function limits who can "appropriate it" for him or herself, then we can gain insight into who "can assume these various subject functions."

So when thinking about the figure of the author, we need to move beyond the simple attribution of a text or a mathematical theorem to a specific individual. Rather, according to Foucault (1998), the idea of the author results from various cultural constructions, in which we choose certain attributes of an individual as "authorial" attributes, and dismiss others. What attributes signify the mathematical author? Suzanne Damarin (2000) argues that there are two conflicting discourses that work to construct the figure of the mathematically able in Western society: a discourse of power and a discourse of deviance. The power associated with mathematical ability is both cultural and economic. In the highly technological society we live in, mathematical achievement can translate into economic success in the form of job skills that lead to success on the job market. But even more so, mathematical achievement brings with it cultural capital in corporate, political and academic circles. Heather Mendick (2006), in her study of the discursive construction of mathematics found that mathematics is variously framed as, "a route to economic and personal power within advanced capitalism," "a source of personal power," and "the ultimate form of rational thought and so a proof of intelligence" (p. 18). Yet there is associated with mathematical achievement a degree of deviance as well. Because mathematics is understood to be the ultimate form of rational thought, those who engage with mathematical knowledge are often thought of as removed from normal human occupations and leisure. So much so, that the trope of the mentally ill, yet brilliant, mathematician, is quite common, whether we are discussing the "beautiful mind" of John Nash or the dangerous insanity of Unabomber Ted Kaczynski (Damarin, 2000). Damarin describes the major factors that mark the mathematically able: "brilliant but remote from reality, different from 'the rest of us', and bearing bodily marks, to wit, 'it's in the genes'" (p. 77).

This description of the mathematically able corresponds with our cultural understanding of mathematics itself. Mathematical knowledge production is conceived to be an individual cognitive activity, where the mathematician, working in isolation, discovers a mathematical theorem using logical, rule-based reasoning to develop and modify the work of those who came before him. According to Foucault (1998), "these aspects of an individual which we designate as making him an author are only a projection, in more or less psychologizing terms, of the operations that we force texts to undergo, the connections that we make, the traits that we establish as pertinent, the continuities that we recognize or the exclusions that we practice" (p. 213). We can see this projection happening in histories of mathematics that embed biographical information within the story of the historical development of mathematical knowledge. Our cultural understanding of the mathematician is intimately connected to our understanding of mathematics itself. According to Paul Ernest (1992), we have in our culture a popular image of mathematics as "difficult, cold, abstract, ultra-rational, and largely masculine" (n.p.). I argue that in Burton's (2010) history of mathematics textbook, a mathematical subjectivity is constructed, via the figure of the mathematical author, that reflects this popular image of mathematics.

We can see elements of this image throughout Burton's textbook—the ideas of deviance and power; the construction of mathematical work as difficult, cold and abstract; the celebration of rationality above all else; and the characterization of rationality as masculine. Burton certainly refers to the various ways in which Newton is constructed as deviant and I will show this in just a moment. But far more prominent than any description of Newton's deviance are descriptions of his influence, the power that he wielded and his status in seventeenth-century England. Burton's characterization of Newton follows that of most histories of mathematics. It is simply understood that Newton is a hero—a key figure in the history of the West; his mathematical work shaped Western history for all time. Consider the hyperbole in Burton's (2010) introductory paragraph to the section on Newton:

> It remained for a still greater mind, Isaac Newton, to give the scholarly world the synthesis for which it yearned. Newton's *Philosophiae Naturalis Principia Mathematica* (1687) was the climax of the soaring intellectual thought that marked the seventeenth century, the Century of Genius. Probably the most momentous scientific treatise ever printed, it aimed, in Newton's words, "to subject the phenomena of Nature to the laws of mathematics" (p. 386).

The cultural capital that Newton gains from his mathematical achievements becomes clear in Burton's account. After the publication of his *Principia*, Newton receives a royal appointment as warden, then master, of the British mint. He becomes the president of the Royal Society in 1703 and is re-elected to the role, without opposition, until his death twenty-four years later. In 1705, Queen Anne knighted Newton, a farmer's son and the first scientist to be so honored. Yet alongside this cultural capital, there is ample evidence that Newton suffered from mental illness, which many attribute to the strain of his mathematical genius. According to Burton (2010),

> The severe mental exertion of composing the *Principia* took its toll. Newton began to suffer from insomnia and lack of appetite, and by 1692 his mental health had deteriorated to the point where he was afflicted with some sort of nervous illness (p. 406).

The difficulty of genius-level mathematical thinking is established. The coldness of such work is clear in Burton's constant references to Newton's obsession with his work, his desire to work alone, his propensity to see colleagues as enemies, and his lack of a family. The abstract nature of Newton's work is well-established. Burton frequently refers to the fact that few of Newton's contemporaries could understand his work. Newton's lectures during his time at Cambridge University as the Lucasian Chair of Mathematics were so rigorous and "severely mathematical" that they were largely unattended (Burton, 2010, p. 392).

Finally, the characterization of mathematics as masculine becomes clear in Burton's (2010) description of the lack of women in Newton's own life—his mother abandons him as a young boy, he never marries—and in Burton's attempt to discuss the only two female mathematicians associated with Newton: Maria Agnesi and Émilie du Châtelet (pp. 430-432). Burton does not incorporate biographical information about these women into the sections on the history of the calculus. Rather, he creates a separate section for them at the end of the chapter. He briefly describes their peripheral role in the discovery of the calculus: Agnesi wrote one of the first textbooks on the calculus and du Châtelet translated Newton's *Principia* into French. While their work is certainly considered important vis-à-vis the work of Newton, nowhere does Burton describe them with the aplomb he reserves for the heroic Newton. In fact he ends his section on du Châtelet as follows: "It may be said of Émilie du Châtelet that she was more an interpreter of the accomplishments of others than a creator of original science" (Burton, 2010, p. 432).

Burton's biographical writing about Newton solidifies the construction of mathematics and mathematical subjectivity as difficult, cold, abstract, and ultra-rational, but also as largely masculine. This construction of mathematical subjectivity serves a purpose, according to Paul Ernest (1992). If mathematics is understood in this way:

> … then it offers access most easily to those who feel a sense of ownership of mathematics, of the associated values of western culture and of the educational system in general. These will tend to be males, to be middle class, and to be white. Thus the argument runs that the popular image of mathematics described above sustains the privileges of the groups mentioned by favouring their entry, or rather by holding back their complement sets, into higher education and professional occupations, especially where the sciences and technology are involved (n.p.).

History of mathematics textbooks contribute to this phenomenon, constructing a normative mathematical subjectivity that limits who can identify themselves within it, thus limiting who can access mathematics itself. In the last part of this paper, I examine a history of mathematics textbook that challenges this construction of mathematical subjectivity.

Luke Hodgkin, in his (2005) *A History of Mathematics From Mesopotamia to Modernity*, argues that most textbooks present the history of mathematics as a series of facts and that within those textbooks "the live field of doubt and debate which is research in the history of mathematics finds itself translated into a dead landscape of certainties" (p. 4). Hodgkin very clearly states in his preface that his intention in the writing of this textbook is to engage students, not just in the history of mathematics, but in the making of historical narratives. In this way, Hodgkin approaches the history of mathematics very differently than does Burton. Burton (2010) presents the history of mathematics as an factual narrative of intellectual development in Western culture. He chooses to include biographical elements in this narrative in order to engage the interest of the reader, arguing that "there is no sphere in which individuals count for more than the intellectual life" (p. x). According to Burton, because mathematicians "stand out as living figures and representatives of their day, it is necessary to pause from time to time to consider the social and cultural framework in which they lived" (Burton, 2010, p. x). The construction of the normative mathematician-hero is a result of the choices Burton makes in his textbook. In contrast, Hodgkin introduces his textbook with a discussion of the contingent and constructed nature of scholarship in the history of mathematics, differentiating for his readers between the absolutist histories that are normally presented in textbooks and professional research in the field. Hodgkin

> … hope[s]to introduce students to the history, or histories of mathematics as constructions which we make to explain the texts that we have, and to relate them to our own ideas. Such constructions are often controversial, and always provisional; but that is the nature of history (Hodgkin, 2005, Preface, para. 2).

To this end, Hodgkin begins each chapter by outlining the field of historical literature, both primary and secondary sources. He then asks readers to evaluate and interpret this literature on their own—to engage in the act of history-writing.

This powerful approach to the teaching of the history of mathematics acknowledges what Michel-Rolph Trouillot (1995) argues is a key element in the study and creation of historical knowledge: figuring out how history works by studying how it is produced. Trouillot articulates two sides of historicity—what actually happened and the narrative of what happened—and argues that a focus on the process of producing history is the only way to "uncover the ways in which the two sides of historicity intertwine in a particular context" (Trouillot, 1995, p. 25). This very much corresponds with Hodgkin's approach in his history of mathematics textbooks. He explicitly states in his introduction that "the emphasis falls sometimes on history itself, and sometimes on *historiography*: the study of what historians are doing" (Hodgkin, 2005, p. 4). By inviting readers into the process of producing history, Hodgkin both exposes the pluralistic and contradictory nature of historical knowledge and invites readers to generate their own interpretations. By encouraging his readers to create history—to work with primary and secondary texts, to create their own stories about the mathematicians and mathematical knowledge they are studying—he is empowering readers to actively engage with the knowledge, both historical and mathematical. In so doing he positions his readers to ask some of the questions with which Foucault (1998) ends his essay on the author function: What are the places in [this discourse] where there is room for possible subjects? Who can assume these various subject functions?

Hodgkin's (2005) approach in his history of mathematics textbook results in a more pluralist construction of mathematical subjectivity, allowing more people to see themselves within it. Because of how closely intertwined mathematical knowledge is with histories of its development, producing historical narratives about mathematics necessarily involves working with mathematical knowledge. By inviting readers into the process of constructing historical narratives, Hodgkin makes room for possible subjects in ways that Burton's textbook does not; he opens up the possibility of assuming mathematical subjectivity. Trouillot (1995) argues very clearly that the process of producing history influences the construction of subjectivity; writing about what happened empowers the writer as a subject. I consider two ways that Hodgkin expands possibilities with regard to the construction of mathematical subjectivity. First, I look at the way he challenges the trope of the mathematician/hero by portraying figures like Isaac Newton with all of their human foibles intact. Though he might have invented the calculus, Newton was not a very likable character, nor was he the ideal hero that Burton's text makes him out to be. I then look at the way Hodgkin invites readers into the assessment and interpretation of historical evidence and mathematical argument. In this way, Hodgkin takes historical and mathematical knowledge that is characterized by Burton as monumental in scope, and, instead, asks that his readers reinterpret those historical narratives and work with the mathematical knowledge in order to critically engage with it.

Perhaps the most striking part of Hodgkin's (2005) chapter on the calculus is that he characterizes what Burton (2010) calls, "the climax of the soaring intellectual thought that marked the seventeenth century, the Century of Genius" (p. 386) and "the most momentous scientific treatise ever printed" (p. 386) as follows:

> Any mediocre person can break the laws of logic, and many do. What Newton and Leibniz did was to formalize the breakage as a workable system of calculation which both of them quickly came to see was immensely powerful, even if they were not entirely clear about what they meant (Hodgkin, 2005, p. 162).

Hodgkin is referring here to the use of infinitesimals, those infinitely small quantities that are continuously vanishing as you work a calculus problem. By characterizing the invention of the calculus in this way, Hodgkin humanizes it and makes the work that Newton and Leibniz did relatable. In a similar vein, Hodgkin calls Newton and Leibniz's work an invention, asking "so what was it that Newton, and later Leibniz, invented?" (p. 169). Calculus becomes not a Platonic truth discovered by extraordinary genius, but the work of humans who were not quite sure what it was that they were toiling away at, only that it seemed to provide a new tool for solving some age-old problems.

Hodgkin not only characterizes this moment in the history of mathematics in a very different way than do most historians of mathematics, he also uses the exercises in his chapter to ask the reader to engage with the mathematical knowledge as it was presented within the original texts, not just as passive problem solvers, but as active assessors of that knowledge. For example, he asks readers to consider a passage from one of Newton's papers and then assess the strengths and weakness of the mathematical argument within that passage (Hodgkin, 2005, pp. 171-2). In a similar fashion, Hodgkin asks readers to engage in the production of historical knowledge, as well. He provides an excerpt from one of Leibniz's papers on the calculus and asks readers to give "a historical take" on the paper by posing the following questions: 1) what was Leibniz trying to communicate?; and, 2) how might this communication have been received by a reader? (p. 173). Hodgkin then discusses how a reader might answer these questions, teaching the reader how to work through Leibniz's original text. Not only is he enabling his reader to engage with the original texts, he is also providing them with the historiographical tools to assess that text and the impact it might have had at the time of publication. This allows readers to put the invention of the calculus into context, not as the most monumental discovery of human history, but as an innovative, but highly confusing and contentious, invention.

This approach to the history of mathematics makes room for readers to understand themselves as part of the process of producing knowledge about mathematics and the history of mathematics; they enter into subjectivity as they read through and engage with Hodgkin's text. Hodgkin's textbook powerfully challenges normative constructions of mathematical subjectivity by challenging the trope of the mathematician-hero constructed in most history of mathematics textbooks and by expanding who can understand themselves as engaging with mathematical knowledge. It is important, as we move forward, that we continue to challenge those history of mathematics texts that perpetuate a very limited cultural construction of mathematical subjectivity and that we encourage the publication of more histories of mathematics that invite readers to engage directly with that history and with mathematical knowledge itself. In doing so, we will construct a much more expansive mathematical subjectivity that may allow those from marginalized groups to understand themselves as mathematical subjects.

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